Universidad de Valladolid

Facultad de Ciencias

TRABAJO FIN DE GRADO

Grado en Física

Estudio de la estructura de la parte bosónica de la supergravedad en D=11

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To my family.

Contents

Agradecimientos

Este trabajo de fin de Grado es el resultado del camino recorrido a lo largo de los últimos cuatro años, en los que muchas personas y la institución de la Universidad de Valladolid han ayudado a que sea posible.

En primer lugar, me gustaría dar las gracias a mi tutor, José Manuel Izquierdo, por la ayuda prestada durante estos meses y por su dirección, sin la que no habría sido capaz de realizar este trabajo.También por darme la oportunidad de colaborar y aprender con él durante estos dos últimos años.

Quiero agradecer a mi familia todo el apoyo recibido estos años.

Agradecer a cada uno de los profesores de la Universidad de Valladolid que he encontrado en mi camino, y a mis compañeros y amigos por el cariño mostrado.

Gracias a todos.

Acknowledgements

This degree Project is the result of the long way walked during the last four years, in which many people and the institution of the Universidad de Valladolid have helped fot this to be possible.

Firstly, I would like to thank to my tutor, José Manuel Izquiedo, the assistance provided for the last months and for his direction, without which I could have not been able to perform this work. As well for give me the opportunity to collaborate and learn with him for the last two years.

I would like to thank to my family the support I have reeived this years.

Thanks to every teacher of the Universidad de Valladolid that I found in my way,and to my partners and friends for the affection shown.

Thanks to everyone.

Resumen

Este trabajo de fin de grado se basa en el estudio de la supergravedad en $D = 11$, que juega un papel importante en la Física Teórica moderna porque:

1. Es el límite en bajas energías para la teoría M [1].
2. $D = 11 = 10 + 1$ es la dimensión más alta compatible con la supersimetría [2], [3].
3. La supergravedad en $D = 4$, $N = 8$ que se obtiene por reducción dimensional de la supergravedad en $D = 11$ es posiblemente una teoría finita en todos los órdenes de la teoría de perturbaciones [4].

Las teorías de Chern-Simons (CS) también son relevantes. Se trata de teorías que consisten en acciones construidas exclusivamente en términos de los campos de gauge de algún (super)grupo, con la condición de que sean invariantes gauge. Sólo existen en dimensión impar. Por ejemplo, se sabe que la (super)gravedad en $D = 3$ es también una teoría de CS [5] , aunque eso ya no es cierto cuando $D > 3$, D impar. Sin embargo, se ha conjeturado que:

1. (Cremmer-Julia-Scherk) El grupo de simetrías de la supergravedad en $D = 11$ es $Osp(1|32)$ [6].
2. (Horava) La teoría M es en realidad una teoría CS basada en $Osp(1|32) \otimes Osp(1|32)$. Esto implicaría, en particular, que la supergravedad en $D = 11$ es un límite de bajas energías de dicha teoría de CS [7].

El estudio busca analizar la posible relación de la supergravedad en $D = 11$ con una teoría de CS basada en el grupo $Osp(1|32)$. Para ello, consideraremos únicamente el caso bosónico (*i.e.* tomaremos el campo del gravitino ψ igual a cero) y construiremos una teoría de CS modificada con un campo dado por una 3-forma $\mathscr{A}$.

Abstract

This degree project is based on $D = 11$, which plays an important role in modern Theoretical Physics because:

1. Is the limit of low energies for the M theory [1].

2. $D = 11 = 10 + 1$ is the highest possible dimension compatible with supersymmetry [2], [3].

3. The supergravity in $D = 4$, $N = 8$ which is obtained by dimensional reduction of supergravity in $D = 11$ is possibly a finite theory at every order of perturbation theory [4].

Chern-Simons (CS) theories are also relevant. These theories consist on a set of actions exclusively constructed in terms of the gauge fields of some (super)group with the condition of gauge invariance. They only exist with odd dimensionality. For example, it is known that (super)gravity in $D = 3$ is also a CS theory [5], although this is not true when $D > 3$, D is odd. However, it has been conjectured that:

1. (Cremmer-Julia-Scherk) The symmetries group of supergravity in $D = 11$ is $Osp(1|32)$ [6].

2. (Horava) The M theory is a CS theory based on $Osp(1|32) \otimes Osp(1|32)$. That would imply, particularly, that supergravity in $D = 11$ is a low energy limit of that CS theory [7].

This study analyzes the possible connection of supergravity in $D = 11$ with a CS theory based on the group $Osp(1|32)$. To this end, we will only consider the bosonic case ($i.e.$ we will take the gravitino field ψ equal to zero) and we will build a CS theory modified with a field given by a 3-form $\mathscr{A}$.

Chapter 1

Introduction

"Imagination will often carry us to worlds that never were, but without it we go nowhere."

Carl Sagan

1.1 Motivation

We move in our daily environment without understanding almost anything about the world. We spend a little time thinking about the mechanisms which make life possible , and in the forces which make them possible, as gravity, that binds us to the Earth. In the atoms which compose our bodies and whose stability is fundamental for our lives. Few of us dedicate time to make us questions about why nature is as it is, and where the cosmos came, or wether it has been always there, definitely, about the existence of fundamental limits on which humans can learn about.

It is very common in our society that the receptor of this kind of questions would reply just shrugging or with some religious reference. However, much of the philosophy and science has been guided by such issues. At present, our knowledge about the fundamental laws of physics is not only an incomplete whole, but it is not even an self-consistent whole.

Usually we start doing physics studying isolated phenomena, explained through ad hoc disjointed theories. Over time we observe that actually these theories can correspond to different aspects of a more general unification theory. It is for example the case of the four fundamental interactions. Quantum electrodynamics and electroweak interaction were the first unified theories (after electricity and magnetism of course). After that, quantum chromodynamics was unified to them within the standard model frame. Nevertheless, gravitational force resists the attempts to be unified with the other interactions.

Probably, one of the most promising approaches with this objective has been that of string theory, quantum gravity, loop quantum gravity and non-commutative geometry. From among these four, string theory is particularly appropriate to achieve the objective of unification. Within its frame, the four fundamental interactions correspond simply to each different vibration modes of only one solid fundamental object called string.

There exists a problem when we try to formulate the theory in only one way. Actually, we can formulate it in five different ways: type I, type IIA, type IIB, heterotic SO (32) and heterotic $E_8 \times E_8$; which are related through a net of dualities. The type IIA theories have as a limit of low energies supergravity IIA in $D = 10$, the type IIB theories have as a limit of low energies supergravity IIB in $D = 10$ and the heterotic SO (32) and heterotic $E_8 \times E_8$ theories have as a limit of low energies $N = 1$ supergravity / Yang-Mills with SO(32) and $E_8 \times E_8$ group respectively. . In 1995, E. Witten introduced a possible explanation [8] (see also [1])which says that supergravity IIA in $D = 10$ can be obtained from a formulation supergravity in $D = 11$ – called M-theory by Witten - whose low energy limit is CJS supergravity, related with strings theory by dimensional reduction.

No one knows the defining principles of M-theory, only the compactification and low energy limits. A conjecture was put forward by [9], called matrix theory, in which space-time was substituted by a space of matrices. Later, in 1997, Horava [7] suggested that M-theory could after all be a Chern-Simons field theory.

The present work explores the second conjecture. We will analyze the relation between this supergravity in $D = 11$ and a CS theory based on the $Osp(1|32)$ group considering only the bosonic case.

1.2 Work schedule

The process we will follow after constructing a CS theory modified with a field given by a 3-form $\mathscr{A}$ is to introduce one scale factor with dimensions Λ which allows to take the low energy limit and to choose as a Lagrangian the term in Λ^9 of the development of the original Lagrangian that results from the scale change (Horava argues that this Lagrangian corresponds to the low energy limit). After that we compare the equations of this Lagrangian with that of the bosonic case of supergravity in $D = 11$.

What we can observe is that we reproduce the equations for some selection of the free parameters, and then, the role of the auxiliary field which has to be present in the formulation of the first order of supergravity in $D = 11$, is played by the gauge fields of certain $Osp(1|32)$ generators. that would provide evidence supporting the existence of a relation between supergravity in $D = 11$ with a CS theory. But we don't know the precise form of this relation because, when introducing the ψ field, the CS equations are not the same that that of the supergravity in $D = 11$. If we reach this connexion, it would imply that $Osp(1|32)$ plays almost a role in M-theory, whose basic equations are not known at the moment.

Chapter 2

Initial considerations

In this section we will introduce the fundamental aspects of the theory that we are going to develop.

2.1 ¿What is supergravity?

Supersymmetry[2] (see also [3]) is, by definition, a symmetry between fermions and bosons. Supersymmetric theories allow us to relate fermion properties (matter) with boson properties (force carriers) in both plane or curved spaces (supergravity). A supersymmetric field model cosists on a set of quantum fields and of a Lagrangian for them which exhibit such a symmetry. The Lagrangian determines, through the action principle, the equations of the motion and hence the dynamical behaviour of the particles. A supersymmetric model which is covariant under general coordinate transformations or, equivalently, a model which possess local ("gauged") supersymmetry is called a supergravity model.

Supersymmetric theories describe model worlds of particles, created from the vacuum by the fields, and the interactions between these particles. Supersymmetry manifests itself in the particle spectrum and in stringent relationships between different interaction processes even if these involve particles of different spin and of different statistics. Both supersymmetry and supergravity aim at a unified description of fermions and bosons, and hence of matter and interaction. Supergravity is particularly ambitious in its attempt at *unification of the gravitational with the other interactions.* All supersymmetric models succeed to some degree in these aims, but they fail in actually describing the world as we experience it and thus are models, not theories. We are still striving to find some contact between one of the models and physical reality so that that model could become an underlying theory for nature at its most fundamental level.

As we have introduced, supergravity is the supersymmetric theory of gravity, or a theory of local supersymmetry. It involves the graviton described by Einstein's gravity (general relativity), and extra matter, in particular, a fermionic partner of the graviton, called the gravitino. By itself, Einstein's gravity is non-renormalizable, so its quantization is one of the most important problems of modern theoretical physics. Supersymmetry is known to alleviate some of the ultraviolet divergences of quantum field theory, via cancellations between bosonic and fermionic loops, hence the UV divergences become milder in supergravity. In fact, by going to an even larger theory, string theory, the nonrenormalizability issue of quantum gravity is resolved, at least order by order in perturbation theory. At energies low compared to the string energy scale (but still very large compared to accelerator energies), string theory becomes supergravity, so supergravity is important also as an effective theory for string theory.

We start with some of the basic concepts as the Maurer-Cartan equations for the Lie algebra sp(32), which will give us information about the curvatures of our model.

2.2 Maurer-Cartan equations for the Lie algebra sp(32)

Connections are usually defined on principal bundles. We shall consider only the trivial bundle case, so, for us, connections will be one-forms*. A connection form associates to each basis of a vector bundle a matrix of differential forms. The connection form transforms in a manner that involves the exterior derivative of the transition functions, and the main tensorial invariant of a connection form is its curvature form. We can introduce the condition for the vanishing of the curvature for the one-form f as follows:

$$df^{\alpha}{}_{\beta} = -f^{\alpha}{}_{\gamma} \wedge f^{\gamma}{}_{\beta} \quad , \qquad df = -f^2 \quad , \tag{2.1}$$

where

$$f_{\alpha\beta} = f_{\beta\alpha} \quad , \qquad f_{\alpha\beta} = C_{\alpha\gamma} f^{\gamma}{}_{\beta} \quad , \qquad C_{\alpha\gamma} = -C_{\gamma\alpha} \quad , \tag{2.2}$$

where $C_{\alpha\gamma}$ is the simplectic metric.

f can be expanded in the basis of symmetric antisymmetrized products of γ matrices:

$$f = f_a \gamma^a + \frac{1}{4} f_{ab} \gamma^{ab} + \frac{1}{5!} f_{a_1 \ldots a_5} \quad , \tag{2.3}$$

where

$$\{\gamma^a, \gamma^b\} \equiv \gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab} \quad , \tag{2.4}$$

η^{ab} is the Minkowski metric in D = 11,

*or three forms in the case of supergravity, defined on space-time

$$\gamma^{ab} = \frac{1}{2!}(\gamma^a\gamma^b - \gamma^b\gamma^a) \quad ,$$

$$\gamma^{a_1\dots a_5} = \frac{1}{5!}\sum_{\sigma\in S_5} \epsilon(\sigma)\gamma^{a_{\sigma(1)}}\gamma^{a_{\sigma(2)}}\dots\gamma^{a_{\sigma(5)}} \quad . \tag{2.5}$$

i.e. The weight are antisymmetrized products of γ matrices $\gamma^{a_1\dots a_4}$, $\gamma^{a_1\dots a_3}$ and $\mathbb{I}$ are excluded because they are not symmetric; $(\gamma^{a_1\dots a_3})_{\alpha\beta} = -(\gamma^{a_1\dots a_3})_{\beta\alpha}$, etc.

2.3 Gauge algebra

We now say that the Maurer-Cartan equations are no longer satisfied. Then, the curvatures are the quantities that express the failure of the one-form f to satisfy the Maurer-Cartan equations.

$$\Omega \equiv df + f^2 \quad . \tag{3.6}$$

The differential of Ω is given by:

$$d\Omega = \Omega f - f\Omega \quad . \tag{3.7}$$

We can also expand Ω in the basis of symmetric antisymmetrized products of γ matrices:

$$\Omega = \Omega_a\gamma^a + \frac{1}{4}\Omega_{ab}\gamma^{ab} + \frac{1}{5!}\Omega_{a_1\dots a_5}\gamma^{a_1\dots a_5} \quad . \tag{3.8}$$

2.4 Gauge transformations

These transformations are given by:

$$\delta f = fb - bf + db \quad ,$$

where $b = b^\alpha{}_\beta$ is a zero-form gauge parameter:

$$b = b_a\gamma^a - \frac{1}{4}b_{ab}\gamma^{ab} + \frac{1}{5!}b_{a_1\dots a_5}\gamma^{a_1\dots a_5} \quad . \tag{4.9}$$

The variation of the curvature Ω is:

$$\delta\Omega = \Omega b - b\Omega \quad . \tag{4.10}$$

Chapter 3

Chern-Simons form and action

A CS theory is special because its action is proportional to the integral of the three-form of CS. In our case we will define a "CS-like" form in order to make possible this procedure.

We can write the 12-form H as a function of the curvatures Ω as follows:

$$H = Tr(\Omega^6) = Tr(\Omega \times \Omega \times \Omega \times \Omega \times \Omega \times \Omega) \quad , \tag{0.1}$$

which is a closed and gauge invariant form. That is:

- $dH = 6Tr(d\Omega\,\Omega^5) = 6Tr((\Omega f - f\Omega)\Omega^5) = 0 \quad ,$
- $\delta H = 6Tr(\delta\Omega\,\Omega^5) = 6Tr((\Omega b - b\Omega)\Omega^5) = 0 \quad .$

Since $dH = 0$, $H = dB$ for some 11-form B . And this form is quasi-invariant, *i.e.*,

$$\delta B = d\Lambda \quad . \tag{0.2}$$

Now, one can define a Chern-Simons action I_{CS} as follows:

$$I_{CS} = \int_{\mathscr{M}''} B \quad , \tag{0.3}$$

where $\mathscr{M}''$ is the $D = 11$ space-time. I_{CS} is gauge invariant up to topological effects that we will not consider.

3.1 Modified “CS-like" form

Since the bosonic part of $D = 11$ contains a three-form field, we will put it by hand.

$$\mathscr{F} = d\mathscr{A} \ , \quad \delta\mathscr{A} = 0 \ , \tag{1.4}$$

where $\mathscr{F}$ is a four-form and $\mathscr{A}$ is the three-form field we introduce by hand.

With this curvature, the following closed, invariant form can be written.

$$\begin{aligned} H &= Tr(\Omega^6) + \alpha Tr(\Omega^4)Tr(\Omega^2) + \beta\left(Tr(\Omega^2)\right)^3 + \gamma Tr(\Omega^4)\mathscr{F} \\ &+ \delta\left(Tr(\Omega^2)\right)^2\mathscr{F} + \epsilon Tr(\Omega^2)\mathscr{F}^2 + \sigma\mathscr{F}^3 \end{aligned} \tag{1.5}$$

where $\alpha - \sigma$ are constants and $Tr(\Omega^2)$, $Tr(\Omega^4)$ are other closed-invariant forms not considered before. Note that we do not add $Tr(\Omega^3)$ nor $Tr(\Omega^5)$ and $Tr(\Omega)$ because they are identically zero.

Note that the second expression of equation(1.4) tells us that the H closed invariant form is gauge invariant under transformations $\mathscr{A} \to \mathscr{A} + d\Lambda$ of the three-form field $\mathscr{A}$.

3.2 Scale factor

All quantities f, Ω, $\mathscr{A}$, $\mathscr{F}$ are dimensionless. We consider dimensional quantities by introducin a scale factor, λ, with dimensions $[\lambda] = L^{-1}$ (in geometrized units, for which c = 1 = G, and all the quantities have dimensions expressed in terms of powers of L).

Now we set:

$$f = \lambda\, e_a\gamma^a + \frac{1}{4}w_{ab}\gamma^{ab} + \frac{1}{5!}w_{a_1\ldots a_5}\gamma^{a_1\ldots a_5} \quad , \tag{2.6}$$

where,

$$\begin{aligned} f_a &= \lambda e_a \ , & [e_a] &= L \ , \\ f_{ab} &= w_{ab} \ , & [w_{ab}] &= [w_{a_1\ldots a_5}] = L^0 \ , \\ f_{a_1\ldots a_5} &= w_{a_1\ldots a_5} \ , \end{aligned} \tag{2.7}$$

and,

$$\mathscr{A} = \lambda^3 A \ , \quad [A] = L^3 \quad . \tag{2.8}$$

A physical action describing gravity wuld have dimensions of an action, which in geometric units is L^9 (in $D = 11$). So we start with I_{CS} and expand in λ as a result of the change of scale,

$$I_{CS} = I_{CS}|_0 + \lambda\, I_{CS}|_1 + ... \tag{2.9}$$

We keep the power λ^9, since $\lambda^9\, I_{CS}|_9$ is dimensionless and $[\lambda^9] = L^{-9}$, so $[I_{CS}|_9] = L^9$.

Also, $I_{CS}|_9 = \int_{\mathscr{M}''} B|_9$ and $dB|_9 = H|_9$, where:

$$\begin{aligned} B &= B|_0 + \lambda\, B|_1 + ... \\ H &= H|_0 + \lambda\, H|_1 + ... \end{aligned} \tag{2.10}$$

This expansion in λ is used in the method of Lie algrebra expansions first used in [10] and studied in general in [11].

3.3 Expansion of H

We are interested in $H|_9$. Since H contains the curvatures Ω_a, Ω_{ab}, $\Omega_{a_1 \ldots a_5}$, we need to know their expressions in terms of e_a, w_{ab}, $w_{a_1 \ldots a_5}$.

To simplify the calculations, we write

$$\Omega = df + f^2 \ , \tag{3.11}$$

with,

$$f = \lambda e_a \gamma^a + \frac{1}{4} w_{ab} \gamma^{ab} + \frac{1}{5!} w_{a_1 \ldots a_5} \gamma^{a_1 \ldots a_5} \tag{3.12}$$

$$\equiv \lambda e + w_L + w_5 \ , \tag{3.13}$$

so:

$$\Omega = \lambda de + dw + (\lambda e + w)(\lambda e + w) \tag{3.14}$$

$$= \lambda(de + ew + we) + dw + w^2 + \lambda^2 e^2 \tag{3.15}$$

$$\equiv \lambda T + R + \lambda^2 \Omega_2 \ . \tag{3.16}$$

One has to bear in mind that T contains a piece proportional to γ^a, but also a piece proportional to $\gamma^{a_1 \cdots a_5}$. The previous equations tells us that, in order to obtain the piece $H|_9$ that comes *e.g.* from $Tr(\Omega^6)$, one has to consider all the contributions containing n_2 factors Ω_2 , n_0 factors R and n_1 factors T in which a way that,

1. $n_0 + n_1 + n_2 = 6$ (there are 6 curvatures)
2. $n_1 + 2n_2 = 9$

The only two solutions for this case are:

- $n_2 = 4$, $n_1 = 1$, $n_0 = 1$, or
- $n_2 = 3$, $n_1 = 3$, $n_0 = 0$

But R, T, Ω_2 contributons come from any of the 6 Ω in $Tr(\Omega^6)$, so we finally have:

$$Tr(\Omega^6)|_9 = Tr(\mathcal{W}(\Omega_2^4, T, R)) + Tr(\mathcal{W}(\Omega_2^3, T^3, R^0)) \ , \tag{3.17}$$

where *e.g.* $\mathcal{W}(\Omega_2^4, T, R)$ is the sum of all "words" that can be formed with four Ω_2, one T nd one R.

It is even easier to consider directly the field equations.

Chapter 4

Field equations

It is easy to see (see, for example [11]) that the field equations for I_{CS} can be obtained directly from H. Let us call these equations $E(f)=0$ and $E(\mathscr{A})=0$ respectively. Then:

$$\begin{aligned} E(f) &= 6\Omega^5 + 4\alpha Tr(\Omega^2)\Omega^3 + 2\alpha Tr(\Omega^4)\Omega + 6\beta Tr(\Omega^2)^2\Omega \\ &+ 4\gamma\mathscr{F}\Omega^3 + 4\delta\mathscr{F}Tr(\Omega^2)\Omega + 2\epsilon\mathscr{F}^2\Omega \ , \end{aligned} \tag{0.1}$$

and,

$$E(\mathscr{A}) = \gamma Tr(\Omega^4) + \delta(Tr(\Omega^2))^2 + 2\epsilon\mathscr{F}Tr(\Omega^2) + 3\sigma\mathscr{F}^2 \ . \tag{0.2}$$

We need these equations for e and w (w_L and w_5), and $\mathscr{A}$ for the action $I|_9$, which comes from $H|_9$. It is easy o see that:

$$\begin{aligned} E(e) &= E(f)|_{9-1=8} \ , \\ E(w) &= E(f)|_9 \ , \\ E(A) &= E(\mathscr{A})|_{9-3=6} \ . \end{aligned} \tag{0.3}$$

So we just have to find $E(f)|_8$, $E(H)|_9$ and $E(A)|_6$ by tking into account that:

$$\begin{aligned} \Omega &= \lambda T + R + \lambda^2\Omega_2 \ , \\ \mathscr{F} &= \lambda^3 F = \lambda^3 dA \ . \end{aligned} \tag{0.4}$$

4.1 Field equation for w

We need to know the contributions of all terms in equation (0.1). Here we have to consider again all the contributions containing n_2 factors Ω_2 , n_0 factors R and n_1 factors T in which a way that:

- $6\,\Omega^5|_9$ corresponds to n_0, n_1, n_2 such that:

$$n_2 = 4, n_1 = 1, n_0 = 0 \ ,$$

so

$$6\,\Omega^5|_9 = 6\mathcal{W}(\Omega_4^2, T) \ .$$

- $4\ \alpha Tr(\Omega^2)\,\Omega^3|_9$. also contains $4\Omega_2$ and $1\ T$ in all possible combinations. If T is one of the $3\Omega's$ in Ω^3, then $Tr(\Omega^2) = Tr(\Omega_2^2) = Tr(e^4) = 0$ because $e^4 \propto \gamma^{a_1 \cdots a_4}$ and $Tr(\gamma^{a_1 \cdots a_k}) = 0$; only $Tr(\mathbb{I}) = 32$.

  So T has to be one of the two $\Omega's$ inside the trace. Now, $T = de + ew + we$, and $Tr(e^2(de + ew + we)) = Tr(e^2 de) + Tr(e^3 w + e^2 we) = Tr(e^2 de)$, but $e^2 \propto \gamma^{a_1 a_2}$ and $de \propto \gamma^{a_1}$. And there is no way this can give something proportional to the unit matrix, so:

$$4\alpha Tr(\Omega^2)\,\Omega^3|_9 = 0 \ .$$

- $2\ \alpha Tr(\Omega^4)\,\Omega|_9$. Here there are also $4\Omega^2$ and $1\ T$. The contribution $Tr(\Omega_2^4)T$ vanishes because $Tr(\Omega_2^4) = Tr(e^8) \propto Tr(\gamma^{a_1 \cdots a_8}) = 0$.

  When T is inside the trace, we have:

$$Tr(T\Omega^3) = Tr(de\, e^6 + we^7 + ewe^6) = 0 \ .$$

- $6\beta Tr(\Omega^2)^2\,\Omega|_9$. This is zero for the same reasons as above.

- $4\gamma\mathscr{F}\,\Omega^3|_9$. Since $\mathscr{F} = \lambda^3 F$, we have to find $\Omega^3|_6$. In this case,n_0, n_1, n_2 have to satisfy:

$$n_2 = 3, n_1 = 1, n_0 = 0 \ .$$

  Hence,

$$4\gamma\mathscr{F}\,\Omega^3|_9 = 4\gamma F e^6 \ .$$

- $4\delta\mathscr{F}Tr(\Omega^2)\,\Omega|_9$. Here, as before, $n_1 = 0, n_0 = 0$, but $Tr(\Omega_2^2) = 0$
  So,

$$4\delta\mathscr{F}Tr(\Omega^2)\,\Omega|_9 = 0 \ .$$

- $2\epsilon\mathscr{F}^2\,\Omega|_9$. Since $\mathscr{F}^2 = \lambda^6 F^2$, we need $\Omega|_3$, but there is no such contribution:

$$2\epsilon\mathscr{F}^2\,\Omega|_9 = 0 \ .$$

So the w equation is simply:

$$6\,\mathcal{W}(\Omega_4^2, T) + 4\gamma F e^6 = 0 \ . \tag{1.5}$$

Now the equation actually have different contributions; one proportional to $\gamma^{a_1a_2}$, which gives the equation for w_L, and another proportional to $\gamma^{a_1\ldots a_5}$, which gives the equation for w_5.

Note that $e^6 \propto \gamma^{a_1\ldots a_6}$ is also proportional to $\gamma^{a_1\ldots a_5}$ because $\gamma^{a_1\ldots a_6} \propto \epsilon^{\,a_1\ldots a_6 b_1\ldots b_5}\gamma_{a_1\ldots a_5}$.

4.1.1 Equation for w_L ($i.e.$ for w_{ab})

We consider (1.5) in more detail:

$$\mathcal{W}(\Omega_4^2, T) = e^8T + e^6Te^2 + e^4Te^4 + e^2Te^6 + Te^8 \quad , \tag{1.6}$$

and T is given by:

$$\begin{aligned} T &= de + ew + we = de + ew_L + w_Le + ew_5 + w_5e \\ &= T_L + ew_5 + w_5e \quad , \end{aligned} \tag{1.7}$$

so:

$$\begin{aligned} \mathcal{W}(\Omega_4^2, T) &= e^8T + e^6Te^2 + e^4T_Le^4 + e^2T_Le^6 + T_Le^8 \\ &+ e^9w_5 + e^8w_5e + e^7w_5e^2 + e^6w_5e^3 + e^5w_5e^4 \\ &+ e^4w_5e^5 + e^3w_5e^6 + e^2w_5e^7 + ew_5e^8 + w_5e^9 \\ &= \mathcal{W}((e^2)^2, T_L) + \mathcal{W}(e^9, w_5) \quad . \end{aligned} \tag{1.8}$$

To see how the $\gamma_{a_1 a_2}$ and $\gamma_{a_1 \ldots a_5}$ contributions come out, note the identity:

$$\gamma_a \gamma_{a_1 \ldots a_k} = \sum_{i=1}^{k} (-1)^{i-1} \, \eta_{a a_i} \gamma_{a_1 \ldots \hat{a}_i \ldots a_k} + \gamma_{a a_1 \ldots a_k} \quad . \tag{1.9}$$

When contracted with the two-form, say e_a and $B^{a_1 \ldots a_k}$, one gets:

$$e^a \gamma_a B^{a_1 \ldots a_k} \gamma_{a_1 \ldots a_k} = k \, e^a B_{a a_2 \ldots a_k} + \gamma_{a a_1 \ldots a_k} e^a B^{a_1 \ldots a_k} \quad ,$$

i.e., all terms in the sum (1.8) add up, and the first term appears k times. The same pattern exists when two matrices $\gamma_{a_1 \ldots a_k}$, $\gamma_{a_1 \ldots a_6}$ are multiplied, but now there are contributions with al possible number of contractions: the first term of (1.8) is the one-contraction contribution with no contractions. The $e^8 T_L$ terms have the structure $\gamma^{(8)} \cdot \gamma^{(1)}$. This gives:

$$\gamma^{(8)} \cdot \gamma^{(1)} = \gamma^{(9)} + \gamma^{(7)} \tag{1.10}$$

where $\gamma^{(9)}$ have no contractions and $\gamma^{(7)}$ one contraction.

The $\gamma^{(7)}$ contraction will cancel because only the atrices that are symmetric with all indices does contribute. So the $e^8 T_L$ terms only appear in the w_L equation, because:

$$\gamma^{a_1 \ldots a_9} \propto \epsilon^{a_1 \ldots a_9 ab} \gamma_{ab} \quad . \tag{1.11}$$

In general, since:

$$\gamma^{a_1 \ldots a_{11}} \propto \epsilon^{a_1 \ldots a_{11}} \quad , \tag{1.12}$$

we have:

$$\gamma^{a_1 \ldots a_k} = \frac{(-1)^k}{(11-k)!!} \epsilon^{a_1 \ldots a_k a_{k+1} \ldots a_{11}} \gamma_{a_{k+1} \ldots a_{11}} \quad , \tag{1.13}$$

On the other hand, the $e^9 w_5$ terms of (1.8) are of the form:

$$\gamma^{(9)} \cdot \gamma^{(5)} = \gamma^{(14)} + \gamma^{(12)} + \gamma^{(10)} + \gamma^{(8)} + \gamma^{(6)} + \gamma^{(4)} \quad , \tag{1.14}$$

The $\gamma^{(10)}$ contribution vanishes because there is no w_a. The only symmetric γ is γ_6. So the $e^9 w_5$ terms only appear in the w_5 equation.

The w_L equations are:

$$e_{a_1} \ldots e_{a_8} T_{L a_9} \gamma a_1 \ldots a_9 + e_{a_1} \ldots e_{a_6} T_{L a_9} \gamma a_1 \ldots a_9 + \ldots \tag{1.15}$$

It is clear that all five terms in (1.8) give the same contribution, so we finally get:

$$5\, e_{a_1}...e_{a_8}T_{La_9}\gamma a_1...a_9 = 0 \quad . \tag{1.16}$$

It is a well known fact that this equation implies $T_L = 0$, which can be used to express $w_{ab\mu}$ in terms of $e^a{}_\mu$ and its derivatives, $w_{ab} = -w_{ab\mu}\, dx^\mu$, $e_a = -e^a{}_\mu dx^\mu$.

4.1.2 Equation for w_5 ($i.e.$ for $w_{a_1...a_5}$)

This equation has two contributions. One is given by $4\gamma F e^6$ in (1.5), which is proportional to $\gamma^{(6)}$, and the contribution with four contractions from the terms with 9 $e's$ and one w_5 in (1.8), which is also proportional to $\gamma^{(6)}$.

We compute the contribution with four contractions in Appendix 6.1. After a long process we obtain:

$$\mathcal{W}(e^9, w_5) \to 2 \cdot \frac{9!}{4!}\, e_{a_1}...e_{a_5}e^{b_1}...e^{b_4}w_{b_4...b_1a_6}\gamma^{a_1...a_6} \quad . \tag{1.17}$$

Taking into account both terms in (1.8), we see that the w_5 equation is:

$$12 \cdot \frac{9!}{4!}\, e_{a_1}...e_{a_5}e^{b_1}...e^{b_4}w_{b_4...b_1a_6}\gamma^{a_1...a_6} + 4\gamma F e_{a_1}...e_{a_6}\gamma^{a_1...a_6} = 0 \quad . \tag{1.18}$$

Let us see what this equation leads to:

First we write:

$$F = F_{b_1...b_4}e^{b_1}...e^{b_4} \quad , \tag{1.19}$$

($i.e.$, $F_{b_1...b_4}$ are the components of the four form F in terms of the e^b)

So we have:

$$\frac{9!}{2}\, e_{a_1}...e_{a_5}e_{b_1}...e_{b_4}e_c w^{b_4...b_1}{}_{a_6}{}^{c}\gamma^{a_1...a_6} + 4\gamma F^{b_1...b_4}e_{a_1}...e_{a_6}e_{b_1}...e_{b_4}\gamma^{a_1...a_6} = 0 \quad . \tag{1.20}$$

Where:

$$w^{b_4...b_1}{}_{a_6} = w^{b_4...b_1}{}_{a_6}{}^{c}e_c \quad .$$

We now write the products of ten $e's$ as:

$$e_{a_1}...e_{a_5}e_{b_1}...e_{b_4}e_c = \epsilon_{a_1...a_5b_1...b_4cd}E^d \ ,$$

for some ten-form E^d. Factorizing out this form and $\gamma^{a_1...a_6}$, we have:

$$\frac{9!}{2}\epsilon_{[a_1...a_5|b_1...b_4cd|}\, w^{b_4...b_1}{}_{a_6]}{}^{c} + 4\gamma\epsilon_{a_1...a_6b_1...b_4d}F^{b_1...b_4} = 0 \ , \tag{1.21}$$

where the bracket [ ] indicates antisymmetrization in $a_1...a_6$ with weight 1.

We refer to the Appendix 6.2, where it is shown that the solution is given by:

$$w^{d_1...d_5}{}_d = -\frac{40}{9!}\gamma\, F^{[d_1...d_4}\,\delta_d^{d_5]} \ . \tag{1.22}$$

This equation relates the components of the gauge field one-form $w^{d_1...d_5}$ to the components of the four-form $F = dA$. It can be written as:

$$w^{d_1...d_5} = -\frac{40}{9!}\gamma\, F^{d_1...d_4}e^{d_5} \ . \tag{1.23}$$

Let us check this result substituting (1.23) in the first term of (1.19), we get:

$$\frac{9!}{2}e_{a_1}...e_{a_5}e^{b_1}...e^{b_4}\left(-\frac{40}{9!}\gamma\, F_{[b_1...b_4}\, e_{a_6]}\gamma^{a_1...a_6}\right) = -4\,\gamma\, Fe_{a_1}...e_{a_6}\gamma^{a_1...a_6} \ .$$

4.2 Field equation for $\mathscr{A}$

We need to know the contributions of all terms in equation (0.2). We will expand the calculation in Appendix 6.3. Now we show the result:

We know that the addition of all these contributions give us the final equation result:

$$4\,\gamma\, 32e_{a_1}...e_{a_6}\, D\, w_{a_7...a_{11}}\epsilon^{a_1...a_{11}} + 3\sigma F^2 = 0 \ . \tag{2.24}$$

Now we have to write this final result in a more convenient form:

$$4\,\gamma\, 32e_{a_1}...e_{a_6}\, D\, w_{a_7...a_{11}}\epsilon^{a_1...a_{11}} = -3\sigma F^2 \ ,$$

$$4\,\gamma\,32 e_{a_1}...e_{a_6}\,D_{b_1} w_{a_7...a_{11} b_2} e^{b_1} e^{b_2} \epsilon^{a_1...a_{11}} = -3\,\sigma F_{b_1...b_4} F_{c_1...c_4} e^{b_1}...e^{b_4} e^{c_1}...e^{c_4} \quad .$$

Now we can use $E_{d_1...d_3} = \epsilon_{d_1...d_3 b_1...b_8} e^{b_1}...e^{b_8}$, then:

$$4\,\gamma\,32\,\epsilon_{a_1...a_6 b_1 b_2 d_1...d_3}\,D^{b_1}\,w_{a_7...a_{11}}{}^{b_2}\,\epsilon^{a_1...a_{11}} = -3\,\sigma \epsilon_{b_1...b_4 c_1...c_4 d_1...d_3} F^{b_1...b_4} F^{c_1...c_4} \quad ,$$

so we obtain:

$$4\,\gamma\,32\,(-1)^{\eta}\cdot 6!\,\delta^{a_7...a_{11}}_{b_1 b_2 d_1...d_3}\,D^{b_1}\,w_{a_7...a_{11}}{}^{b_2} = -3\,\sigma \epsilon_{b_1...b_4 c_1...c_4 d_1...d_3} F^{b_1...b_4} F^{c_1...c_4} \quad .$$

If we use the expression (2.19):

$$\begin{aligned}
\delta^{a_7...a_{11}}_{b_1 b_2 d_1...d_3}\,D^{b_1}\,w_{a_7...a_{11}}{}^{b_2} &= -\frac{40}{9!}\,\gamma\,\delta^{a_7...a_{11}}_{b_1 b_2 d_1...d_3}\delta^{b_2}_{a_{11}}\,D^{b_1}\,F_{a_7...a_{10}} \\
&= -\frac{40}{9!}\,\gamma\,\delta^{a_7...a_{10} b_2}_{b_1 b_2 d_1...d_3}\,D^{b_1}\,F_{a_7...a_{10}} \\
&= -(-7)\,\frac{40}{9!}\,\gamma\,\delta^{a_7...a_{10}}_{b_1 d_1...d_3}\,D^{b_1}\,F_{a_7...a_{10}} \\
&= 4!\cdot 7\,\frac{40}{9!}\,\gamma\,D^{b_1}\,F_{b_1 d_1...d_3} = \frac{1}{54}\,\gamma\,D^{b_1}\,F_{b_1 d_1...d_3} \quad ,
\end{aligned}$$

so:

$$(-1)^{\eta}\,\frac{128\cdot 6!}{54}\,\gamma^2\,D^{b_1}\,F_{b_1 d_1...d_3} = -3\,\sigma \epsilon_{b_1...b_4 c_1...c_4 d_1...d_3} F^{b_1...b_4} F^{c_1...c_4} \quad ,$$

and we will use $\eta = 1$:

$$D^{b_1}\,F_{b_1 d_1...d_3} = \left(\frac{9\,\sigma}{5120\,\gamma^2}\right)\epsilon_{b_1...b_4 c_1...c_4 d_1...d_3} F^{b_1...b_4} F^{c_1...c_4} \quad . \tag{2.25}$$

Which is the expression we were looking for.

4.3 Field equation for e

We need to know the contributions of all terms in equation (0.1) again, but now we have to find $E(f)|_8$ instead of $E(f)|_9$. We have performed the calculations in Appendix 6.4, so we show that the contributions that give us our action are:

$$6\mathcal{W}(\Omega_2^3, T^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4, R)|_{\gamma'} + 4\gamma F(\Omega_2^2 T + \Omega_2 T \Omega_2 + T\Omega_2^2)|_{\gamma'} = 0 \tag{3.26}$$

Where $|_{\gamma'}$ selects the contribution proportional to a γ matrix with a single index γ^a, or equivalently, a γ matrix with ten indices, $\gamma^{a_1 \cdots a_{10}}$

As we can see, whe should expand the contributions given by $6\Omega^5|_8$, which are $6\mathcal{W}(\Omega_2^3, T^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4, R)|_{\gamma'}$, but its calculation is a very tedious procedure, so we will try another way which is to analyze all terms in the stress energy tensor, which are of two possible types: $F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}\, g_{\alpha\rho}$ and $F^{\mu\nu\rho}{}_{\alpha} F_{\mu\nu\rho\beta}$ (in the dx^μ basis). This means that Einsteins equations can be written as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = P\, F^{\alpha\rho\gamma}{}_{\mu} F_{\alpha\rho\gamma\nu} + Q\, F^{\alpha\rho\gamma\delta} F_{\alpha\rho\gamma\delta} g_{\mu\nu} \ . \tag{3.27}$$

Where P and Q are two constants. We know that the covariant derivative $\nabla^\mu \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)$ of the e.h.s vanishes, so the covariant derivative of the r.h.s must also vanish. On the other hand we know that:

$$\partial_{[\mu}\, F_{\nu\rho\gamma\tau]} = 0 \ , \tag{3.28}$$

$$\nabla^\mu F_{\mu\nu\rho\sigma} = \epsilon_{\nu\rho\sigma\lambda_1 \ldots \lambda_4 \tau_1 \ldots \tau_4} F^{\lambda_1 \ldots \lambda_4} F^{\tau_1 \ldots \tau_4} \ . \tag{3.29}$$

There is a factor before $\epsilon_{\nu\rho\sigma\lambda_1 \ldots \lambda_4 \tau_1 \ldots \tau_4}$ which is not important for the argument.

Then, $\nabla^\mu \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)$ has to vanish when we use the two equations above. This fixes the relative factor between the two terms in (3.27):

$$\begin{aligned} \nabla^\mu \left[P\, F^{\alpha\beta\gamma}{}_{\mu} F_{\alpha\rho\gamma\nu} + Q\, F^{\alpha\beta\gamma\delta} F_{\alpha\rho\gamma\delta} g_{\mu\nu}\right] &= P\, \nabla^\mu F^{\alpha\beta\gamma}{}_{\mu}\, F_{\alpha\rho\gamma\nu} + P\, F^{\alpha\beta\gamma}{}_{\mu}\, \nabla^\mu\, F_{\alpha\beta\gamma\nu} \\ &+ 2Q\, \nabla^\mu F^{\alpha\beta\gamma\delta} F_{\alpha\rho\gamma\delta}\, g_{\mu\nu} \ . \end{aligned}$$

Where we have used $\nabla^\mu g_{\mu\nu} = 0$.

Now, using (3.28), the third term in the r.h.s is:

$$2Q\nabla^{\alpha}F^{\mu\nu\gamma\delta}F_{\alpha\rho\gamma\delta}g_{\mu\nu} \quad .$$

Using (3.29), the first term is:

$$-P\epsilon^{\alpha\beta\gamma}{}_{\lambda_1\ldots\lambda_4\tau_1\ldots\tau_4}F^{\lambda_1\ldots\lambda_4}F^{\tau_1\ldots\tau_4}F_{\alpha\rho\gamma\nu} \quad ,$$

but this can be easily shown to vanish by using the Schouten identity.

So we are left with:

$$F^{\alpha\beta\gamma}{}_{\mu}\nabla^{\mu}F_{\alpha\beta\gamma\nu}\left[P+8Q\right] \quad .$$

Since this has to vanish, P = - 8Q.

So we don't have to worry about the relative factor in the stress-energy tensor. We only need to fix the overall factor.

Taking the trace of equation (3.27), we get:

$$R-\frac{11}{2}R = P\,F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\gamma} + Q(11)F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\gamma} \quad ,$$

$$-\frac{9}{2}R = P\left(1-\frac{11}{8}\right)F_{\mu\nu\rho\gamma}F^{\mu\nu\rho\sigma} \quad ,$$

i.e.,

$$R = P\,\frac{3}{8}\cdot\frac{2}{9}\,F_{\mu\nu\rho\gamma}\,F^{\mu\nu\rho\sigma} = \frac{1}{12}\,PF_{\mu\nu\rho\gamma}\,F^{\mu\nu\rho\sigma} \quad . \tag{3.30}$$

So, we can get the required value of P by requiring that this equation is the one that corresponds to supergravity. This value has to coincide with the one obtained from our action, which is:

$$6\mathcal{W}(\Omega_2^3,T^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4,R)|_{\gamma'} + 4\gamma F(\Omega_2^2T+\Omega_2T\Omega_2+T\Omega_2^2)|_{\gamma'} = 0 \quad .$$

We refer to the Appendix 6.5 to writte the equation in a more explicit form:

$$\begin{aligned} 0 \;&=\; 6\mathcal{W}(e^8,w_5^2)|_{\gamma'} \;+6\mathcal{W}(\Omega_2^4,R_L)|_{\gamma'} \\ &+\; 4\gamma F(e^5w_5+e^4w_5e+e^3w_5e^2+e^2w_5e^3+ew_5e^4+w_5e^5)|_{\gamma'} \quad . \end{aligned}$$

Let us now compute the trace of this equation times $e^a\gamma_a$:

$$\begin{aligned} 0 &= 6\,Tr(9w_5ew_5e^8+9w_5^2e^9+9w_5e^2w_5e^7+9w_5e^3w_5e^6+9w_5e^4w_5e^5) \\ &+ 6Tr(4R_Le^9)+4\gamma F\,6Tr(w_5e^6)\ . \end{aligned} \tag{3.31}$$

We use this expression to obtain one equation of the form $(R = ...)$, where R is de Ricci scalar.

First, we use the w_5 equation, which gives:

$$w_5 = k(\hat{F}e+e\hat{F})\ , \qquad \hat{F} = F_{a_1...a_4}\gamma^{a_1...a_4}\ ,$$

Of course, we know the value of k, we will substitute it later. In terms of $\hat{F}$, the equation is:

$$\begin{aligned} 0 &= 54k^2\,Tr(\hat{F}e^2\hat{F}e^9+\hat{F}e^3\hat{F}e^8+e\hat{F}e\hat{F}e^9+e\hat{F}e^2\hat{F}e^8) \\ &+ \hat{F}e\hat{F}e^10+\hat{F}e^2\hat{F}e^9+e\hat{F}^2e^{10}+e\hat{F}e\hat{F}e^9+\hat{F}e^3\hat{F}e^8 \\ &+ \hat{F}e^4\hat{F}e^7+e\hat{F}e^2\hat{F}e^8+e\hat{F}e^3\hat{F}e^7+\hat{F}e^4\hat{F}e^7+\hat{F}e^5\hat{F}e^6 \\ &+ e\hat{F}e^3\hat{F}e^7+e\hat{F}e^4\hat{F}e^6+\hat{F}e^5\hat{F}e^6+\hat{F}e^6\hat{F}e^5+e\hat{F}e^4\hat{F}e^6 \\ &+ +e\hat{F}e^5\hat{F}e^5)+24Tr(R_Le^9)+48\gamma\,k\,F\,Tr(\hat{F}e^7)\ . \end{aligned} \tag{3.32}$$

That is:

$$\begin{aligned} 0 &= 54k^2\,Tr(\hat{F}^2e^{11}+3\hat{F}e\hat{F}e^{10}+4\hat{F}e^2\hat{F}e^9+4\hat{F}e^3\hat{F}e^8+4\hat{F}e^4\hat{F}e^7+4\hat{F}e^5\hat{F}e^6) \\ &+ 24Tr(R_Le^9)+48\gamma\,k\,F\,Tr(\hat{F}e^7)\ . \end{aligned}$$

Let us compute the term containing R_L:

$$24\,Tr(R_Le^9) = 24\cdot 32\cdot R_{a_1a_2}e_{a_3}...e_{a_11}\epsilon^{a_1...a_{11}}\ .$$

Now, we write:

$$R_{a_1a_2} = R_{a_1a_2,b_1b_2}e^{b_1}e^{b_2}\ ,$$

so the term is:

$$24\,Tr(R_Le^9) = 24\cdot 32\cdot R_{a_1a_2,}{}^{b_1b_2}e^{b_1}e^{b_2}\,e_{a_3}...e_{a_{11}}\epsilon^{a_1...a_{11}}\ .$$

Now, calling $E = e_{a_1}...e_{a_{11}}\epsilon^{a_1...a_{11}}$, we can prove that:

$$e_{b_1}e_{b_2}e_{a_3}...e_{a_{11}} = \frac{\eta}{11!}\epsilon_{b_1b_2a_3...a_{11}}E \ ,$$

where $\eta = -1$, $(+1)$ if the metric is mostly plus (mostly minus).

So:

$$24\,Tr(R_Le^9) = 24 \cdot 32 \cdot \frac{\eta}{11!}R_{a_1a_2,}{}^{b_1b_2}\epsilon_{b_1b_2a_3...a_{11}}\epsilon^{a_1...a_{11}}E \ ,$$

where $\epsilon_{b_1b_2a_3...a_{11}}\epsilon^{a_1...a_{11}} = \eta\,9!\,\delta^{a_1a_2}_{b_1b_2}$, so:

$$24\,Tr(R_Le^9) = \frac{24 \cdot 32}{110} \cdot 2\,R_{a_1a_2,}{}^{a_1a_2}E = \frac{48 \cdot 32}{110}\,R\,E \ .$$

Now we refer to Appendix 6.6 where we perform the calculations of the remaining terms.

We can obtain the value of the constant P of equation (3.30) comparing with the result of (6.21). We also substitute the value of k, so:

$$\begin{aligned} R &= -\frac{110}{48 \cdot 32} \cdot \left(\gamma\,k\frac{256}{55} + k^2 \cdot \frac{193536}{11}\right)F^2 \\ &= -\left(\frac{1}{3}\,\gamma\,k + 1260\,k^2\right)\,F^2 \ . \end{aligned} \quad (3.33)$$

Now we compute the value of k using (1.23):

$$\begin{aligned} w_5 &= \frac{1}{5!}\,w_{a_1...a_5}\gamma^{a_1...a_5} = -\frac{1}{5!}\frac{40}{9!}\gamma\,F^{d_1...d_4}e^{d_5}\gamma_{d_1...d_5} \\ &= k(\hat{F}e + e\hat{F}) = k(F^{a_1...a_4}\gamma_{a_1...a_4}e^{a_5}\gamma_{a_5} + e^{a_5}\gamma_{a_5}F^{a_1...a_4}\gamma_{a_1...a_4}) \\ &= 2k\,F^{a_1...a_4}e^{a_5}\gamma_{a_1...a_5} \ , \end{aligned} \quad (3.34)$$

so:

$$k = -\frac{20}{5! \cdot 9!}\gamma \ .$$

We can calculate the value of the constant P:

$$P = 12 \cdot \gamma^2 \left(\frac{1}{3} \cdot \frac{20}{5! \cdot 9!} - \frac{1260 \cdot 20^2}{(5! \cdot 9!)^2} \right) \quad . \tag{3.35}$$

Chapter 5

Conexion with supergravity in $D = 11$

The objective of this chapter is to analyze the possible connexion between supergravity in D = 11 with the CS theory we have developed. Then, we will compare the field equations we have obtained with the field equations which come from supergravity in $D = 11$. As we have found this conexion, it means that probably there is a relation between supergravity in $D = 11$ and a CS theory.

We have to compare both field equations for $\mathscr{A}$ and e with the equations of supergravity in $D = 11$. That will allow us to determine the value of the constant σ that we introduced by hand:

From equation (2.25) we can obtain the relation between σ and γ. The equation that supergravity provides (see, for example [12]) for the $\mathscr{A}$ field is:

$$D^{b_1}\, F_{b_1 d_1 \ldots d_3} = \left(\frac{1}{3^2 \cdot 2^7}\right) \epsilon_{b_1 \ldots b_4 c_1 \ldots c_4 d_1 \ldots d_3} F^{b_1 \ldots b_4} F^{c_1 \ldots c_4} \quad . \tag{0.1}$$

Which means that:

$$\left(\frac{9\,\sigma}{5120\,\gamma^2}\right) = \left(\frac{1}{3^2 \cdot 2^7}\right) \quad ,$$

so:

$$\sigma = \gamma^2 \left(\frac{40}{81}\right) \quad . \tag{0.2}$$

Now, we have to extract the value of γ from the field equation for e in order to determine σ. The equation that supergravity provides (see, for example [12]) for the e

field is:

$$R = \left(\frac{1}{12}\right)^2 F^2 \ . \tag{0.3}$$

We have from (3.33) that:

$$\gamma^2 = \frac{1}{12^2 \cdot \left(\frac{1}{3} \cdot \frac{20}{5! \cdot 9!} - \frac{1260 \cdot 20^2}{(5! \cdot 9!)^2}\right)} \ , \tag{0.4}$$

so:

$$\sigma = \frac{5}{1458} \cdot \frac{1}{\left(\frac{1}{3} \cdot \frac{20}{5! \cdot 9!} - \frac{1260 \cdot 20^2}{(5! \cdot 9!)^2}\right)} \ . \tag{0.5}$$

Then, we have obtained the constant which goes with the four form $\mathscr{F}$ in the closed invariant form (1.5).

Chapter 6

Appendices

In this chapter we will show the calculations performed along the different chapters and sections in order to make more clear the reading of this work.

6.1 Contribution $\mathcal{W}(e^9, w_5)$ of the field equation for w_5

In this section we compute the contribution with four contractions $\mathcal{W}(e^9, w_5)$ to obtain the equation for w_5.

- First term:

$$e^9 w_5 \to \binom{9}{4}\binom{5}{4} \cdot 4!\, e_{a_1} ... e_{a_5} e^{b_1} ... e^{b_4} w_{b_4 ... b_1 a_6} \gamma^{a_1 ... a_6} \quad .$$

- Second term:

$$\begin{aligned} e^8 w_5 e \quad &\to \quad \binom{8}{4}\binom{5}{4} \cdot 4!\, e_{a_1} ... e_{a_4} e^{b_1} ... e^{b_4} w_{b_4 ... b_1 a_5} e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad 5 \cdot \binom{8}{3}\binom{4}{3} \cdot 3!\, e_{a_1} ... e_{a_5} e^{b_1} ... e^{b_3} w_{b_3 ... b_1 a_6 b_4} e^{b^4} \gamma^{a_1 ... a_6} \quad , \end{aligned}$$

we can check that:

$$\binom{8}{4}\binom{5}{4} \cdot 4! + 5 \cdot \binom{8}{3}\binom{4}{3} \cdot 3! = \binom{9}{4}\binom{5}{4} \cdot 4! \quad .$$

- Third term:

$$\begin{aligned} e^7 w_5 e^2 \quad &\to \quad \binom{7}{4}\binom{5}{4} \cdot 4!\, e_{a_1} ... e_{a_3} e^{b_1} ... e^{b_4} w_{b_4 ... b_1 a_4} e_{a_5} e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad 5 \cdot 2 \cdot \binom{7}{3}\binom{4}{3} \cdot 3!\, e_{a_1} ... e_{a_4} e^{b_1} ... e^{b_3} w_{b_3 ... b_1 a_5 b_4} e^{b^4} e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad \binom{5}{2} 2! \binom{7}{2}\binom{3}{2} \cdot 2!\, e_{a_1} ... e_{a_5} e^{b_1} e^{b_2} w_{b_2 b_1 a_6 b_3 b_4} e^{b^4} e^{b_3} \gamma^{a_1 ... a_6} \quad , \end{aligned}$$

we can check that:

$$\binom{7}{4}\binom{5}{4} \cdot 4! + 5 \cdot 2 \cdot \binom{7}{3}\binom{4}{3} \cdot 3! + \binom{5}{2} 2! \binom{7}{2}\binom{3}{2} \cdot 2! = \binom{9}{4}\binom{5}{4} \cdot 4! \quad .$$

- Fourth term:

$$\begin{aligned} e^6 w_5 e^3 \quad &\to \quad \binom{6}{4}\binom{5}{4} \cdot 4!\, e_{a_1} e_{a_2} e^{b_1} ... e^{b_4} w_{b_4 ... b_1 a_3} e_{a_4} ... e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad 5 \cdot 3 \cdot \binom{6}{3}\binom{4}{3} \cdot 3!\, e_{a_1} ... e_{a_3} e^{b_1} ... e^{b_3} w_{b_3 ... b_1 a_4 b_4} e^{b^4} e_{a_5} e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad \binom{5}{2}\binom{3}{2} 2! \binom{6}{2}\binom{3}{2} \cdot 2!\, e_{a_1} ... e_{a_4} e^{b_1} e^{b_2} w_{b_2 b_1 a_5 b_3 b_4} e^{b^4} e^{b_3} e_{a_6} \gamma^{a_1 ... a_6} \\ &+ \quad \binom{5}{3}\binom{3}{3} 3! \binom{6}{1}\binom{2}{1} e_{a_1} ... e_{a_5} e^{b_1} w_{b_1 a_6 b_2 b_3 b_4} e^{b^4} ... e^{b_1} e_{a_6} \gamma^{a_1 ... a_6} \quad , \end{aligned}$$

we can check that:

$$\begin{aligned}\binom{9}{4}\binom{5}{4}\cdot 4! &= \binom{6}{4}\binom{5}{4}\cdot 4! + 5\cdot 3\cdot \binom{6}{3}\binom{4}{3}\cdot 3! \\ &+ \binom{5}{2}\binom{3}{2}2!\binom{6}{2}\binom{3}{2}\cdot 2! + \binom{5}{3}\binom{3}{3}3!\binom{6}{1}\binom{2}{1} \ .\end{aligned}$$

- Fifth term:

$$\begin{aligned}e^5 w_5 e^4 \ \rightarrow\ & \binom{5}{4}\binom{5}{4}\cdot 4!\, e_{a_1}e^{b_1}...e^{b_4}w_{b_4...b_1a_2}e_{a_3}...e_{a_6}\gamma^{a_1...a_6} \\ + \ & 5\cdot 4\cdot \binom{5}{3}\binom{4}{3}\cdot 3!\, e_{a_1}e_{a_2}e^{b_1}...e^{b_3}w_{b_3...b_1a_3b_4}e^{b^4}e_{a_4}...e_{a_6}\gamma^{a_1...a_6} \\ + \ & \binom{5}{2}\binom{4}{2}2!\binom{5}{2}\binom{3}{2}\cdot 2!\, e_{a_1}...e_{a_3}e^{b_1}e^{b_2}w_{b_2b_1a_4b_3b_4}e^{b^4}e^{b_3}e_{a_5}e_{a_6}\gamma^{a_1...a_6} \\ + \ & \binom{5}{3}\binom{4}{3}3!\binom{5}{1}\binom{2}{1}\, e_{a_1}...e_{a_4}e^{b_1}w_{b_1a_5b_2...b_4}e^{b^4}...e^{b_2}e_{a_6}\gamma^{a_1...a_6} \\ + \ & \binom{5}{4}4!\, e_{a_1}...e_{a_5}w_{a_6b_1...b_4}e^{b^4}...e^{b_1}\gamma^{a_1...a_6} \ ,\end{aligned}$$

we can check that:

$$\begin{aligned}\binom{9}{4}\binom{5}{4}\cdot 4! &= \binom{5}{4}\binom{5}{4}\cdot 4! + 5\cdot 4\cdot \binom{5}{3}\binom{4}{3}\cdot 3! + \binom{5}{2}\binom{4}{2}2!\binom{5}{2}\binom{3}{2}\cdot 2! \\ &+ \binom{5}{2}\binom{4}{2}2!\binom{5}{2}\binom{3}{2}\cdot 2! + \binom{5}{3}\binom{4}{3}3!\binom{5}{1}\binom{2}{1} + \binom{5}{4}4! \ .\end{aligned}$$

Now we need the terms $e^4 w_5 e^5$ and so on, which have the same structure of the five terms analyzed so far, with the same factors, but changing right and left. Tipically, we will have:

$$e_{a_1}...e_{a_k}e^{b_1}...e^{b_l}w_{b_l...b_1a_{k+1}b_{l+1}...b_4}e^{b_4}...e^{b_{l+1}}e_{a_{k+2}}...e_{a_6}\gamma^{a_1...a_6} \ ,$$

and:

$$e_{a_k+2}...e_{a_6}e^{b_4}...e^{b_{l+1}}w_{b_{l+1}...b_4a_{k+1}b_l...b_1}e^{b_1}...e^{b_l}e_{a_1}...e_{a_k}\gamma^{a_{k+2}...a_6a_{k+1}a_1...a_k} \ .$$

If we reorder the indices $a_{k+1}...a_6$ with $a_1...a_k$ in the second expression, we will get the same sign as the one that appears by interchanging $e_{a_{k+2}...e_6}$ and $e_{a_1}...e_{a_6}$ (because they are different forms). The same can be said of the b's and $e_{b's}$. So both terms are the same. The contribution of $e^5 w_5 e^4$ will be the same of that of $e^4 w_5 e^4$, and so on.

So we only need to sum the contributions obtained above, taking care of reorganizing the indices to have a common factor:

$$
\begin{aligned}
\mathcal{W}(e^9, w_5) \quad \rightarrow \quad & 2\Bigg[\binom{9}{4}\binom{5}{4}4! + \binom{8}{4}\binom{5}{4}4! - 5\binom{8}{3}\binom{4}{3}3! \\
+ \quad & \binom{7}{4}\binom{5}{4}4! - 5\cdot 2\binom{7}{3}\binom{4}{3}3! + \binom{5}{2}2!\binom{7}{2}\binom{3}{2}2! \\
+ \quad & \binom{6}{4}\binom{5}{4}4! - 5\cdot 3\binom{6}{3}\binom{4}{3}3! + \binom{5}{2}\binom{3}{2}2!\binom{6}{2}\binom{3}{2}2! \\
- \quad & \binom{5}{3}\binom{3}{3}3!\binom{6}{1}\binom{2}{1} + \binom{5}{4}\binom{5}{4}4! - 5\cdot 4\binom{4}{3}\binom{5}{3}3! \\
+ \quad & \binom{5}{2}\binom{4}{2}2!\binom{3}{2}\binom{5}{2} - \binom{5}{3}\binom{4}{3}3!\binom{2}{1}\binom{5}{1} + \binom{5}{4}4!\Bigg] \\
\cdot \quad & e_{a_1}...e_{a_5}e^{b_1}...e^{b_4}w_{b_4...b_1a_6}\gamma^{a_1...a_6} \quad .
\end{aligned}
$$

It turns out that the contributions of e^8we and e^7we^2 cancel each other, and this is also the same case of e^6we^3 and e^5we^4. So we obtaine:

$$
\mathcal{W}(e^9, w_5) \rightarrow 2\cdot\frac{9!}{4!}\, e_{a_1}...e_{a_5}e^{b_1}...e^{b_4}w_{b_4...b_1a_6}\gamma^{a_1...a_6} \quad .
$$

6.2 Derivation of the equation for w_5

In this section we will analyze the expresson (1.21) contracting it with $\epsilon^{a_1...a_6d_1...d_5}$,

$$\frac{9!}{2}\epsilon^{a_1...a_5d_1...d_5}\,\epsilon_{a_1...a_5b_1...b_4cd}w^{b_4...b_1}{}_{a_6}{}^{c}+4\gamma\,\epsilon_{a_1...a_6b_1...b_4d}\,\epsilon^{a_1...a_6d_1...d_5}F^{b_1...b_4}=0\ .$$

Now, we use the property:

$$\epsilon^{a_1...a_kb_1...b_{11-k}}\epsilon_{a_1...a_kc_1...c_{11-k}}=(-1)^{\eta}k!\,\delta^{b_1...b_{11-k}}_{c_1...c_{11-k}}\ , \tag{2.1}$$

where η depends on the signature of the metric *,

$$(+-...-)\rightarrow\eta=0,\qquad(-+...+)\rightarrow\eta=1,$$

and,

$$\delta^{b_1...b_k}_{a_1...a_k}=\sum_{\sigma\in s_k}\delta^{b_1}_{a_{\sigma(1)}}...\delta^{b_k}_{a_{\sigma(k)}}\ . \tag{2.2}$$

We have, irrespective of the metric,

$$\frac{9!}{2}5!\,\delta^{a_6d_1...d_5}_{b_1...b_4cd}w^{b_4...b_1}{}_{a_6}{}^{c}+4\gamma\,6!\,\delta^{d_1...d_5}_{b_1...b_4d}F^{b_1...b_4}=0\ . \tag{2.3}$$

Now, we use the property:

$$\delta^{aa_1...a_{k-1}}_{b_1...b_k}=\sum_{l=1}^{k}\delta^{a}_{b_l}\delta^{a_1...a_k}_{b_1...\hat{b}_l...b_k}\ , \tag{2.4}$$

in the first term, with $a=a_6$. The contributions $\delta^{a_6}_{b_1}...\delta^{a_6}_{b_4}$ vanish because $w^{b_1...b_4}{}_{a_6}$ is antisymmetric. So we have:

$$\frac{9!}{2}5!\,\delta^{a_6}_{c}\delta^{d_1...d_5}_{b_1...b_4d}w^{b_4...b_1}{}_{a_6}{}^{c}-\frac{9!}{2}5!\,\delta^{a_6}_{d}\delta^{d_1...d_5}_{b_1...b_4c}w^{b_4...b_1}{}_{a_6}{}^{c}4\gamma\,6!\,\delta^{d_1...d_5}_{b_1...b_4d}F^{b_1...b_4}=0\ , \tag{2.5}$$

$$\left(\frac{9!}{2}\,5!\,w^{b_4...b_1}{}_{c}{}^{c}+4\gamma\,6!\,F^{b_1...b_4}\right)\delta^{d_1...d_5}_{b_1...b_4d}-\frac{9!}{2}\,5!\,\delta^{d_1...d_5}_{b_1...b_4c}w^{b_4...b_1}{}_{d}{}^{c}=0\ . \tag{2.6}$$

*We will use the signature $(-+...+)$ in the end of this work, when we compare our final equations with that of the supergravity model.

Contracting d_5 and d in (2.6), we get:

$$\left(\frac{9!}{2}\,5!\,w^{b_4\ldots b_1}{}_c{}^c + 4\gamma\,6!\,F^{b_1\ldots b_4}\right)(11-4)\delta^{d_1\ldots d_4}_{b_1\ldots b_4} - \frac{9!}{2}\,5!\,w^{b_4\ldots b_1}{}_c{}^c\delta^{d_1\ldots d_4}_{b_1\ldots b_4} = 0 \ , \tag{2.7}$$

so:

$$\frac{9!}{2}\,5!\,6w^{b_4\ldots b_1}{}_c{}^c = -4\cdot 7!\gamma F^{b_1\ldots b_4}$$

$$\begin{aligned} w^{b_4\ldots b_1}{}_c{}^c &= -\frac{4\cdot 7!}{9!\,6!}\gamma\,2F^{b_1\ldots b_4} && (2.8)\\ &= -\frac{56}{9!}\gamma\,F^{b_1\ldots b_4} \ . && (2.9)\end{aligned}$$

Substituting this in (2.6), we have:

$$\begin{aligned} \frac{9!}{2}\,5!\,5!w^{d_1\ldots d_4}{}_d{}^{d_5} &= \left(\frac{56\gamma}{9!}\frac{9!}{2}\,5! + 4\gamma 6!\right)F^{b_1\ldots b_4}\delta^{d_1\ldots d_5}_{b_1\ldots b_4 d} && (2.10)\\ &= (-28\cdot 5!\gamma + 24\cdot 5!\gamma)\,F^{b_1\ldots b_4}\delta^{d_1\ldots d_5}_{b_1\ldots b_4 d} && (2.11)\\ &= -4\cdot 5!\,\gamma\,F^{b_1\ldots b_4}\delta^{d_1\ldots d_5}_{b_1\ldots b_4 d} && (2.12)\\ &= -4\cdot 5!\,\gamma\,5!F^{b_1\ldots b_4}\delta^{[d_1\ldots d_4}_{b_1\ldots b_4}\,\delta^{d_5]}_d && (2.13)\\ &= -4\cdot 5!\,\gamma\,5!F^{[d_1\ldots d_4}\,\delta^{d_5]}_d \ . && (2.14)\end{aligned}$$

Then:

$$\begin{aligned} w^{[d_1\ldots d_4}{}_d{}^{d_5]} &= \frac{4\cdot 5!\cdot 5!\,\gamma\,2}{9!\cdot 5!\cdot 5!}\,F^{[d_1\ldots d_4}\,\delta^{d_5]}_d && (2.15)\\ &= \frac{4\cdot 2}{9!}\,F^{[d_1\ldots d_4}\,\delta^{d_5]}_d \ . && (2.16)\end{aligned}$$

We now use this equation to solve for $w^{d_1\ldots d_4}{}_d{}^{d_5}$ (not antisymmetrization). To this end, we use the following trick; first we make equation (2.15) more explicit,

$$\begin{aligned} w^{d_1d_2d_3d_4d_5}{}_d &- w^{d_1d_2d_3}{}_d{}^{d_5d_4}\\ -w^{d_1d_2}{}_d{}^{d_5d_4d_3} &- w^{d_1}{}_d{}^{d_3d_4d_5d_2}\\ -w_d{}^{d_1d_2d_3d_4d_5} &= \frac{4\cdot 2}{9!}\gamma\left(F^{d_1d_2d_3d_4}\delta^{d_5}_d\right. && (2.17)\\ -F_d{}^{d_2d_3d_4}{}_\eta{}^{d_5d_1} &- F^{d_1}{}_d{}^{d_3d_4}{}_\eta{}^{d_5d_2}\\ -F^{d_1d_2}{}_d{}^{d_4}{}_\eta{}^{d_5d_3} &- \left.F^{d_1d_2d_3}{}_{d\,\eta}{}^{d_5d_4}\right) \ .\end{aligned}$$

Now we antisymmetrize in the indices $d_1 ... d_5$, with weight one,

$$w^{d_1 \dots d_5}{}_d - 4 \cdot w^{[d_1 \dots d_4}{}_d{}^{d_5]} = -\frac{4 \cdot 2}{9!} \gamma F^{d_1 d_2 d_3 d_4} \delta_d^{d_5} \ , \tag{2.18}$$

and, substituting (2.15) in (2.18),

$$w^{d_1 \dots d_5}{}_d = -\frac{40}{9!} \gamma F^{[d_1 \dots d_4} \delta_d^{d_5]} \ . \tag{2.19}$$

6.3 Contributions of the field equation for $\mathscr{A}$

In this section we compute all the contributions to obtain the field equation for $\mathscr{A}$:

- $\gamma Tr(\Omega^4)|_6$ corresponds to n_0, n_1, n_2 such that:

$$n_2 = 2, n_1 = 2, n_0 = 0 \ , or \quad n_2 = 3, n_1 = 0, n_0 = 1 \ ,$$

so:

$$\gamma Tr(\Omega^4)|_6 = \gamma\, Tr(\mathcal{W}(\Omega_2^2, T^2, R^0)) + \gamma\, Tr(\mathcal{W}(\Omega_2^3, T^0, R^1)) \ .$$

The first contribution can be calculated as follows:

$$\gamma\, Tr(\mathcal{W}(\Omega_2^2, T^2, R^0)) = 4\gamma\, Tr(\Omega_2^2 T^2) + 2\gamma\, Tr(\Omega_2 T\, \Omega_2 T) \ ,$$

using the expression $T_L + w_5 e + e w_5$, where $T_L = 0$ from equation (1.16) and $w_5 \propto \gamma^{a_1 \ldots a_5}$, we can forget about T_L, then:

$$\begin{aligned} Tr(\Omega_2^2 T^2) &= Tr(e^4 (w_5 e + e w^5)^2) \\ &= Tr(e^5 w_5 e w_5 + e^5 w_5^2 + e^4 w_5 e^2 w_5 + e^4 w_5 e w_5 e) \\ &= Tr(e^5 w_5^2 + e^4 w_5 e^2 w_5) \ , \end{aligned}$$

$$\begin{aligned} Tr(\Omega_2 T\, \Omega_2 T) &= Tr(e^2 (w_5 e + e w^5) e^2 (w_5 e + e w^5)) \\ &= Tr(e^3 w_5 e^3 w_5 + e^3 w_5 e^2 w_5 e + e^2 w_5 e^4 w_5 + e^2 w_5 e^3 w_5 e) \\ &= Tr(e^3 w_5 e^2 w_5 e + e^2 w_5 e^4 w_5) \ . \end{aligned}$$

These final tems $e^3 w_5 e^2 w_5 e + e^2 w_5 e^4 w_5$ add up, so we have:

$$\begin{aligned} 4\gamma\, Tr(\Omega_2^2 T^2) + 2\gamma\, Tr(\Omega_2 T\, \Omega_2 T) &= 4\gamma Tr(e^6 w_5^2) + 4\gamma Tr(e^4 w_5 e^2 w_5) + 4\gamma Tr(e^2 w_5 e^4 w_5) \\ &= -4\gamma Tr(e^6 w_5^2) \ . \end{aligned}$$

The second contribution can be calculated as follows:

$$\gamma\, Tr(\mathcal{W}(\Omega_2^3, T^0, R^1)) = 4\gamma\, Tr(\Omega_2^3\, R) \ ,$$

We know that $R = dw + w \wedge w$, but $w = w_L + w_5$, where $w_L \propto \gamma^2$ and $w_5 \propto \gamma^5$ so we have:

$$R = dw_L + dw_5 + w_L^2 + w_L \wedge w_5 + w_5 \wedge w_L + w_5^2 \ .$$

We can analyze eacht term in order to see the γ which is related:

$$\begin{aligned} R_L &= dw_L + w_L^2 \propto \gamma^2 \ , \\ Dw_5 &= dw_5 + w_5 \wedge w_L + w_L \wedge w_5 \propto \gamma^5 \ , \end{aligned}$$

then:

$$4\gamma \, Tr(e^6 \, R) = 4\gamma \, Tr(e^6 \{R_L + Dw_5 + w_5 \wedge w_5\}) \ ,$$

and we consider only the terms which go with γ^{11}, so:

$$4\gamma \, Tr(e^6 \, R) = 4\gamma \, Tr(e^6 \, Dw_5) + 4\gamma \, Tr(e^6 \, w_5^2) \ .$$

The second term of this expression vanishes with the result of the first contribution $-4\gamma Tr(e^6 w_5^2)$, so finally:

$$\begin{aligned} 4\gamma \, Tr(e^6 \, R) &= 4\gamma \, Tr(e^6 \, Dw_5) = 4\gamma \, Tr(e_{a_1}...e_{a_6} \, D \, w_{a_7}...w_{a_{11}} \gamma^{a_1...a_{11}}) \\ &= 4\gamma 32 e_{a_1}...e_{a_6} \, D \, w_{a_7...a_{11}} \epsilon^{a_1...a_{11}} \ . \end{aligned}$$

- $\delta \left(Tr(\Omega^2)\right)^2 |_6$

The first contribution can be calculated as follows:

$$\delta \left(Tr(\Omega_2^2)\right)^2 = \delta \left(Tr(T^2)Tr(\Omega_2^2) + Tr(\Omega_2 T + T\Omega_2)Tr(\Omega_2 T + T\Omega_2) + Tr(\Omega_2^2)Tr(T^2)\right)$$

We know:

$$Tr(\Omega_2^2) = Tr(e^4) = 0 \ ,$$

and it is easy to demonstrate that

$$Tr(\Omega_2 T + T\Omega_2) = 0 \ .$$

For the second case we do not have contributions, so:

$$\delta\left(Tr(\Omega^2)\right)^2|_6 = 0 \ .$$

- $2\epsilon\mathscr{F}Tr(\Omega^2)|_6 = 2\epsilon F Tr(\Omega^2)|_3$ which corresponds to n_0, n_1, n_2 such that:

$$n_2 = 1, n_1 = 1, n_0 = 0 \ , or$$

  we obtain:

$$2\epsilon F Tr(\Omega^2)|_3 = 2\epsilon F Tr(\Omega_2 T + T\Omega_2) = 0 \ .$$

- $3\sigma\mathscr{F}^2|_6 = 3\sigma F^2|_0 = 3\sigma F^2$

The addition of all this contributions give us the final equation result:

$$4\,\gamma\,32 e_{a_1}...e_{a_6}\,D\,w_{a_7...a_{11}}\epsilon^{a_1...a_{11}} + 3\sigma F^2 = 0 \ .$$

6.4 Contributions of the field equation for e

In this section we compute all the contributions to obtain the field equation for e:

- $6\,\Omega^5|_8$ corresponds to n_0, n_1, n_2 such that:

$$n_2 = 3, n_1 = 2, n_0 = 0 \ \ , or \quad n_2 = 4, n_1 = 0, n_0 = 1 \ \ ,$$

so we have this two contributions that we will examine later:

$$6\,\Omega^5|_8 = 6\mathcal{W}(\Omega_2^3, T^2, R^0) + 6\mathcal{W}(\Omega_2^4, T^0, R^1) \ \ .$$

- $4\,\alpha\, Tr(\Omega^2)\Omega^3|_8$

The first possibility corresponds to $n_2 = 3, n_1 = 2, n_0 = 0$:

- $Tr(\Omega_2^2)\,\Omega_2\, T^2 = 0$, that is because $Tr(\Omega_2^2) \propto Tr(e^4) = 0$,
- $Tr(\Omega_2 T)\,\Omega_2^2 T = 0$, the trace $Tr(e^2(T_L + ew_5 + w_5 e)) = Tr(e^3 w_5 + e^2 w_5 e) = 0$,
- $Tr(T^2)\Omega_2^3 = 2Tr(ew_5^2 e)\Omega_2^3$, that is because:

  $Tr(T^2) = Tr((ew_5 + w_5 e)^2) = Tr(ew_5 ew_5 + ew_5^2 e + w_5 e^2 w_5 + w_5 ew_5 e) = 2Tr(ew_5^2 e)$,

  so as, we have the total contribution:

$$4\,\alpha\, Tr(\Omega^2)\Omega^3 = 4\alpha Tr(T^2)\Omega_2^3 = 8\alpha Tr(ew_5^2 e)e^6 \ \ .$$

  Which is propotional to γ^6. That means that we have not to consider it because we are looking for contributions which go with γ^{a_1}.

The other option, $n_2 = 4, n_1 = 0, n_0 = 1$:

- $Tr(\Omega_2^2)\, R\, \Omega_2^2 = 0$,
- $Tr(\Omega_2 R)\,\Omega_2^3 = Tr(\Omega_2 R + R\Omega_2)\Omega_2^3 = 2Tr(e^2(R_L + Dw_5 + w_5 \wedge w_5))e^6 = 2Tr(e^2 Dw_5 + e^2 w_5^2)e^6$.

  Then we have:

$$4\,\alpha\, Tr(\Omega^2)\Omega^3 = 8\,\alpha Tr(e^2 w_5^2)e^6 \ \ .$$

  Which does not contribute.

- $2\,\alpha Tr(\Omega^4)\Omega|_8$

  Where we have the first contribution corresponding to $n_2 = 3, n_1 = 2, n_0 = 0$:

  - $Tr(\mathcal{W}(\Omega_2^3, T))T = Tr(\Omega_2^3 T + \Omega_2^2 T\Omega_2 + \Omega_2 T\Omega_2^2 + T\Omega_2^3)T = 4Tr(\Omega_2^3 T)T = 4Tr(e^6 w_5 e e^7 w_5)T = 0$,
  - $Tr(\mathcal{W}(\Omega_2^2, T^2))\,\Omega_2 = -4\gamma Tr(e^6 w_5^2)e^2$, which we know from equation for $\mathscr{A}$,

    so:

    $$2\alpha Tr(\Omega^4)\,\Omega = -8Tr(e^6 w_5^2)\,e^2 \ ,$$

    which is proportional to $\gamma^{a_1 a_2}$ and will not conribute.

  The other option, $n_2 = 4, n_1 = 0, n_0 = 1$:

  - $Tr(\mathcal{W}(\Omega_2^3, R))\,\Omega_2 = 4Tr(\Omega_2^3 R)\Omega_2 = 4Tr(e^6 R_L + e^6 Dw_5 + e^6 w_5^2)e^2 = 4Tr(e^6 Dw_5 + e^6 w_5^2)\,e^2$,
  - $Tr(\mathcal{W}(\Omega^4))\,R = 0$,

    so:

    $$2\alpha Tr(\Omega^4)\Omega = 8\alpha Tr(e^6 Dw_5 + e^6 w_5^2)\,e^2 \ ,$$

    which does not contribute because it goes with $\gamma^{a_1 a_2}$.

- $6\beta Tr(\Omega^2)^2\,\Omega|_8$

  The first contribution:

  - $Tr(T^2)\,Tr(\Omega_2^2)\Omega_2 = 0$,
  - $Tr(\Omega_2 T)^2\Omega_2 = 0$, as we have seen previously.

  The second contribution:

  - $Tr(\Omega_2^2)^2 R = 0$,
  - $Tr(\Omega_2 R)\,Tr(\Omega_2^2)\Omega_2 = 0$,

  So there are not contributions.

- $4\gamma\mathscr{F}\Omega 3|_8 = 4\gamma\lambda^3 F\Omega^3|_8 = 4\gamma F\Omega^3|_5$

  Then we have the possibilities:

  $$n_2 = 2, n_1 = 1, n_0 = 0 \ ,$$

  so:

  $$4\gamma F\Omega^3|_5 = 4\gamma F(\Omega_2^2 T + \Omega_2 T\Omega_2 + T\Omega_2^2) \ .$$

- $4\delta\mathscr{F}\, Tr(\Omega^2)\Omega|_8 = 4\delta F\, Tr(\Omega^2)\Omega|_5$

  We have the possibilities:

  - $Tr(\Omega_2^2)T = 0$,
  - $Tr(\Omega_2 T + T\Omega_2) = 0$,

  so there are not contributions:

  $$4\delta\mathscr{F}\, Tr(\Omega^2)\Omega|_8 = 0 \ .$$

- $2\epsilon\mathscr{F}^2\Omega|_8 = 2\epsilon F^2\Omega|_2 = 2\epsilon F^2\Omega_2$

  That is because the only possibility s $n_2 = 1, n_1 = 0, n_0 = 0$. This term does not contribute because it goes with $\gamma^{a_1 a_2}$.

6.5 Explicit form of equation for e

In this section we will write the equation (3.31) in a more explicit form.

That is:

$$\begin{aligned} 0 &= 6\,(T^2e^6 + Te^2Te^4 + Te^4Te^2 + Te^6T + e^2T^2e^4 + e^2Te^2Te^2 + e^2Te^4T) + e^4T^2e^2 \\ &+ e^4Te^2T + e^6T^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4, R_L)|_{\gamma'} + 6(e^8w_5^2 + e^6w_5^2e^2 + e^4w_5^2e^4 + e^2w_5^2e^6 \\ &+ w_5^2e^8)|_{\gamma'} + 4\gamma F(e^4T + e^2Te^2 + Te^4)|_{\gamma'} \;, \end{aligned}$$

$$\begin{aligned} 0 &= 6\,(T^2e^6 + Te^2Te^4 + Te^4Te^2 + Te^6T + e^2T^2e^4 + e^2Te^2Te^2 + e^2Te^4T) + e^4T^2e^2 \\ &+ e^4Te^2T + e^6T^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4, R_L)|_{\gamma'} + 6(e^8w_5^2 + e^6w_5^2e^2 + e^4w_5^2e^4 + e^2w_5^2e^6 \\ &+ w_5^2e^8)|_{\gamma'} + 4\gamma F(e^4T + e^2Te^2 + Te^4)|_{\gamma'} \;, \end{aligned}$$

so:

$$\begin{aligned} 0 &= 6\mathcal{W}(e^8, w_5^2)|_{\gamma'} + 6\mathcal{W}(\Omega_2^4, R_L)|_{\gamma'} \\ &+ 4\gamma F(e^5w_5 + e^4w_5e + e^3w_5e^2 + e^2w_5e^3 + ew_5e^4 + w_5e^5)|_{\gamma'} \;. \end{aligned}$$

Let us now compute the trace of this equation times $e^a\gamma_a$:

$$\begin{aligned} 0 &= Tr\,\big(6\mathcal{W}(e^8, w_5^2)|_{\gamma'}\, e + 6\mathcal{W}(\Omega_2^4, R_L)|_{\gamma'}\, e + 4\gamma F(e^5w_5e \\ &+ e^4w_5e^2 + e^3w_5e^3 + e^4w_5e^2 + ew_5e^5 + w_5e^6)|_{\gamma'}\,) \;. \end{aligned} \tag{5.20}$$

Using the cyclic property of the trace,

$$\begin{aligned} 0 &= 6\,Tr(9w_5ew_5e^8 + 9w_5^2e^9 + 9w_5e^2w_5e^7 + 9w_5e^3w_5e^6 + 9w_5e^4w_5e^5) \\ &+ 6Tr(4R_Le^9) + 4\gamma F\,6Tr(w_5e^6) \;. \end{aligned}$$

6.6 Calculation of the remaining terms of equation (3.32)

In this section we will compute the remaining terms of equation (3.32):

Now we compute theterm:

$$
\begin{aligned}
48\gamma\, k\, F\, Tr(\hat{F}e^7) &= 48\gamma\, k\, F_{b_1\ldots b_4} e^{b_1}\ldots e^{b_4}\cdot 32\,\epsilon_{a_1\ldots a_{11}} F^{a_1\ldots a_4} e^{a_5}\ldots e^{a_{11}} \\
&= \frac{48\cdot 32\cdot\gamma\cdot k\cdot\eta}{11!}\, F_{b_1\ldots b_4} F^{a_1\ldots a_4}\epsilon_{a_1\ldots a_{11}}\epsilon^{b_1\ldots b_4 a_5\ldots a_{11}} E \\
&= \frac{48\cdot 32\cdot\gamma\cdot k\cdot\eta}{11!}\cdot 7!\, F_{b_1\ldots b_4}\, F^{a_1\ldots a_4}\,\delta^{b_1\ldots b_4}_{a_1\ldots a_4}\, E \\
&= \frac{48\cdot 32\cdot 7!\cdot 4!\cdot\gamma\cdot k\cdot\eta}{11!}\, F_{a_1\ldots a_4}\, F^{a_1\ldots a_4} E \quad .
\end{aligned}
$$

The calculation of the remaining terms is a little bit trickier:

- The first term:

$$
\begin{aligned}
54k^2\, Tr(\hat{F}^2 e^{11}) &= 54\, k^2\, Tr(\hat{F}^2 e_{a_1}\ldots e_{a_{11}}\gamma^{a_1\ldots a_{11}}) \\
&= 54\, k^2\, Tr(\hat{F}^2)E = 54\, k^2\, 4!\, 32\, F_{a_1\ldots a_4} F^{a_1\ldots a_4} E \quad .
\end{aligned}
$$

- The second term:

$$
\begin{aligned}
54\cdot 3k^2\, Tr(\hat{F}e\hat{F}e^{10}) &= 54\cdot 3\, k^2\, Tr(\hat{F}\gamma^{a_1}\hat{F}\gamma^{a_2\ldots a_{11}}\,\frac{\eta}{11!}\,\epsilon_{a_1\ldots a_{11}} E) \\
&= \frac{54\cdot 3k^2\eta}{11!}\, Tr(\hat{F}\gamma^{a_1}\hat{F}\epsilon_{a_1\ldots a_{11}}\gamma^{a_2\ldots a_{11}})\, E \\
&= \frac{54\cdot 3k^2\eta}{11!}\cdot 10!\eta\, Tr(\hat{F}\gamma^{a_1}\hat{F}\gamma_{a_1})E \quad .
\end{aligned}
$$

Now, we need $\gamma^a\hat{F}\gamma_a$:

$$
\begin{aligned}
\gamma^a\hat{F}\gamma_a &= \gamma^{ab_1\ldots b_4} F_{b_1\ldots b_4}\gamma_a + 4\, F^{ab_1\ldots b_3}\gamma_{b_1\ldots b_3}\gamma_a \\
&= 4\gamma^{b_4 b_1\ldots b_3} F_{b_1\ldots b_4} + 4F^{ab_1\ldots b_3}\gamma_{b_1\ldots b_3 a} + 11\gamma^{b_1\ldots b_4} F_{b_1} \\
&= 3\hat{F} \quad .
\end{aligned}
$$

So this term is:

$$
\frac{54\cdot 3k^2}{11!}\, 3\cdot 32\cdot 4!\, F^{a_1\ldots a_4} F_{a_1\ldots a_4} E \quad .
$$

- The third term:

$$
\begin{aligned}
54 \cdot k^2\, 4\, Tr(\hat{F}e^2 \hat{F}e^9) &= \frac{54\, k^2\, 4\, \eta}{11!}\, Tr(\hat{F}\gamma^{a_1 a_2}\hat{F}\gamma^{a_3 \ldots a_{11}}\epsilon_{a_1 \ldots a_{11}} E) \\
&= \frac{-1}{11!} \cdot 54\, k^2 \cdot 4 \cdot 9!\, Tr(\hat{F}\gamma^{a_1 a_2}\hat{F}\gamma_{a_1 a_2}) \ \ .
\end{aligned}
$$

We need:

$$
\begin{aligned}
\gamma^{a_1 a_2}\hat{F}\gamma_{a_1 a_2} &= \gamma^{a_1}\gamma^{a_2}\hat{F}\gamma_{a_1 a_2} = -\gamma^{a_1}\gamma^{a_2}\hat{F}\gamma_{a_2 a_1} \\
&= -\gamma^{a_1}\gamma^{a_2}\hat{F}\gamma_{a_2}\gamma_{a_1} + 11\hat{F} = -\gamma^{a_1}\,(3\hat{F})\gamma_{a_1} + 11\hat{F} \\
&= -9\hat{F} + 11\hat{F} = 2\hat{F} \ \ .
\end{aligned}
$$

So the term is:

$$
-\frac{54\, k^2\, 4}{110} \cdot 2 \cdot 32 \cdot 4!\, F^{a_1 \ldots a_4} F_{a_1 \ldots a_4} E \ \ .
$$

- The fourth term:

$$
\begin{aligned}
54 \cdot k^2\, 4\, Tr(\hat{F}e^3 \hat{F}e^8) &= \frac{\eta}{11!} \cdot 54\, k^2\, 4\, Tr(\hat{F}\gamma_{a_1 \ldots a_3}\hat{F}\gamma_{a_4 \ldots a_8}\epsilon^{a_1 \ldots a_{11}} E) \\
&= -\frac{8!}{11!} \cdot 54\, k^2 \cdot 4\, Tr(\hat{F}\gamma_{a_1 \ldots a_3}\hat{F}\gamma^{a_1 \ldots a_3}) E \ \ .
\end{aligned}
$$

We need:

$$
\begin{aligned}
\gamma_{a_1 \ldots a_3}\hat{F}\gamma^{a_1 \ldots a_3} &= (\gamma_{a_1 a_2}\gamma_{a_3} - 2\gamma_{a_1}\eta_{a_2 a_3})\hat{F}\gamma^{a_1 \ldots a_3} = \gamma_{a_1 a_2}\gamma_{a_3}\hat{F}\gamma^{a_3 a_1 a_2} \\
&= \gamma_{a_1 a_2}\gamma_{a_3}\hat{F}(\gamma^{a_3}\gamma^{a_1 a_2} - \eta^{a_3 a_1}\gamma^{a_2} + \eta^{a_3 a_2}\gamma^{a_1}) \\
&= \gamma_{a_1 a_2}\gamma_{a_3}\hat{F}(\gamma^{a_3}\gamma^{a_1 a_2} - 2\,\gamma_{a_1 a_2}\gamma^{a_1}\hat{F}\gamma^{a_2} \\
&= 66\hat{F} \ \ ,
\end{aligned}
$$

where $\gamma_{a_1 a_2}\gamma^{a_1}\hat{F}\gamma^{a_2} = \gamma_{a_1}\hat{F}\gamma^{a_1} + 22\gamma_{a_2}\hat{F}\gamma^{a_2} = 60\hat{F}$.

So the term is:

$$
-\frac{66 \cdot 54\, k^2\, 4 \cdot 32 \cdot 4!}{110 \cdot 9}\, F^{a_1 \ldots a_4} F_{a_1 \ldots a_4} E \ \ .
$$

- The fifth term:

$$54 \cdot k^2\, 4\, Tr(\hat{F} e^4 \hat{F} e^7) \;=\; \frac{54}{11!} \cdot 7!\, k^2\, 4\, Tr(\hat{F}\gamma_{a_1\ldots a_4}\hat{F}\gamma^{a_1\ldots a_4})E \ .$$

Wee need:

$$\begin{aligned}
\gamma_{a_1\ldots a_4}\hat{F}\gamma^{a_1\ldots a_4} &= \gamma_{a_1\ldots a_3}\gamma_{a_4}\hat{F}\gamma^{a_1\ldots a_4} = -\gamma_{a_1\ldots a_3}\gamma_{a_4}\hat{F}\gamma^{a_4 a_1\ldots a_3} \\
&= -\gamma_{a_1\ldots a_3}\gamma_{a_4}\hat{F}(\gamma^{a_4}\gamma^{a_1\ldots a_3} - 3\,\eta^{a_4 a_1}\gamma^{a_2 a_3}) \\
&= -\gamma_{a_1\ldots a_3}\gamma_{a_4}\hat{F}\gamma^{a_4}\gamma^{a_1\ldots a_3} + 3\gamma_{a_1\ldots a_3}\gamma^{a_1}\hat{F}\gamma^{a_2 a_3} \ ,
\end{aligned}$$

where $3\gamma_{a_1\ldots a_3}\gamma^{a_1}\hat{F}\gamma^{a_2 a_3} = 6\gamma_{a_3 a_2}\hat{F}\gamma^{a_2 a_3} + 33\gamma_{a_2 a_3}$, so:

$$\gamma_{a_1\ldots a_4}\hat{F}\gamma^{a_1\ldots a_4} \;=\; -3 \cdot 66\,\hat{F} + 27 \cdot (2)\hat{F} = -144\,\hat{F} \ .$$

So the term is:

$$-\frac{54}{11!} \cdot 7!\, k^2\, 4 \cdot 32 \cdot 4!\, 144\, F^{a_1\ldots a_4}F_{a_1\ldots a_4}E \ .$$

- The last term:

$$54 \cdot k^2\, 4\, Tr(\hat{F} e^5 \hat{F} e^6) \;=\; \frac{54\, k^2 \cdot 4 \cdot 6!}{11!}\, Tr(\hat{F}\gamma_{a_1\ldots a_5}\hat{F}\gamma^{a_1\ldots a_5}) \ .$$

We need:

$$\begin{aligned}
\gamma_{a_1\ldots a_5}\hat{F}\gamma^{a_1\ldots a_5} &= \gamma_{a_1\ldots a_4}\gamma_{a_5}\hat{F}\gamma^{a_5 a_1\ldots a_4} \\
&= \gamma_{a_1\ldots a_4}\gamma_{a_5}\hat{F}\,(\gamma^{a_5}\gamma^{a_1\ldots a_4} - 4\eta^{a_5 a_1}\gamma^{a_2 a_3 a_4}) \\
&= -3 \cdot 144\hat{F} - 4\gamma_{a_1\ldots a_4}\gamma^{a_1}\hat{F}\gamma^{a_2 a_3 a_4} \ ,
\end{aligned}$$

where, $4\gamma_{a_1\ldots a_4}\gamma^{a_1}\hat{F}\gamma^{a_2 a_3 a_4} \;= 4{\cdot}3\,\gamma_{a_4 a_3 a_2}\hat{F}\gamma^{a_2 a_3 a_4} + 44\,\gamma_{a_2 a_3 a_4}\hat{F}\gamma^{a_4 a_3 a_2} = 32\gamma_{a_1\ldots a_3}\hat{F}\gamma^{a_1\ldots a_3}$, so:

$$\gamma_{a_1\ldots a_5}\hat{F}\gamma^{a_1\ldots a_5} = -432\hat{F} + 32 \cdot 66\hat{F} = 1680\hat{F} \ .$$

So the last term is:

$$\frac{54\, k^2}{11!} \cdot 4 \cdot 6! \cdot 1680 \cdot 4! \cdot 32 F_{a_1\ldots a_4}F^{a_1\ldots a_4}E \ .$$

Putting all terms together, equation (5.20) can be written as:

$$\begin{aligned} 0 &= \frac{48\cdot 32}{110}\, R + 48\cdot 32\cdot\gamma\cdot k\cdot\frac{7!\cdot 4!}{11!}\, F_{a_1\ldots a_4}F^{a_1\ldots a_4} + \\ &= 54\, k^2\, 4\cdot 32\left(1-\frac{9}{11}-\frac{8}{110}+\frac{4\cdot 66}{110\cdot 9}-\frac{7!\cdot 4\cdot 144}{11!}+\frac{1680\cdot 4\cdot 6!}{11!}\right) F_{a_1\ldots a_4}F^{a_1\ldots a_4} \quad , \end{aligned}$$

which can be written as:

$$\begin{aligned} 0 &= \frac{48\cdot 32}{110}\, R + 48\cdot 32\cdot\gamma\cdot k\cdot\frac{7!\cdot 4!}{11!}\, F_{a_1\ldots a_4}F^{a_1\ldots a_4} \\ &+ 54\, k^2\, 32\cdot\frac{4!\cdot 8!}{11!}\cdot 420\, F_{a_1\ldots a_4}F^{a_1\ldots a_4} \quad . \end{aligned} \tag{6.21}$$

Chapter 7

Conclusions

We have succeded in formulating the bosonic part of $D = 11$ supergravity as a limit of a CS-like theory based on the group $SP(32)$, where the three-form field has been added. Along the way we have shown that the auxiliary field that appears in the first-order version of $D = 11$ supergravity [13] can be replaced by some gauge fields corresponding to the generators of $sp(32)$

It is true that we have added the field $\mathscr{A}$ “by hand" at this stage. Of course, when adding fermions, the requirement of supersymmetry will leed to the presence of $\mathscr{A}$, which has to be present, in $D = 11$ by counting of degrees of freedom of supermultiplets containing spins $s \leq 2$ and a graviton ($s = 2$). Also, the $\mathscr{A}$ field is the one that couples naturally to membranes via the term:

$$A_{\mu\nu\rho}\,\frac{\partial x^{\mu}}{\partial\xi^{i}}\,\frac{\partial x^{\nu}}{\partial\xi^{j}}\,\frac{\partial x^{\rho}}{\partial\xi^{k}}\,\epsilon^{ijk}$$

Moreover, we have founded some of the values of the constants present in the original CS-like action, so that, the resulting equation coincide with that of $D = 11$ supergravity. In fact, only two of them were relevant, the other do not appear in the bosonic equations of the λ^{a} term of the expansion. They however appear when the spin field ψ is restored. We can not provide an argument giving the values of the adjusted constants because they are fixed to supersymmetry, which is not considered here.

To conclude, this result implies that, at least in the bosonic case, there is a connection between the CS theory and supergravity, as conjectured by Horava [7]. Wether this connection remains when the ψ field is restored was not the topic of the present work. However, it can be shown that the same procedure, applied to $Osp(1|32)$ does not lead to the full supergravity in $D = 4$. Therefore, if there is a connection, it will be subtle, and certainly not the kind of connection conjectured by Horava. In any case, the result presented her provides evidence on favour of a connection between supergravity and $Osp(32)$, which by the way was conjectured by Cremmer, Julia and Scherk in their work in $D = 11$ supergravity.

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