



UNIVERSIDAD DE VALLADOLID ESCUELA DE INGENIERIAS INDUSTRIALES

Grado en Ingeniería Química

Implementing a Subsea Oil-Water Separation System in Modelica

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TÍTULO: Implementing a Subsea Oil-Water Separation System in Modelica

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Resumen

Los separadores compactos de agua-petróleo son esenciales para el tratamiento de las corrientes extraídas en yacimientos de petróleo en aguas profundas.

El propósito de este proyecto ha sido implementar un modelo en el lenguaje Modélica para estos separadores.

Se implementaron tres bloques que incluyen un separador de gravedad para la principal etapa de separación y dos separadores centrífugos para conseguir purificación de las corrientes de salida. Un nuevo caso evitando el uso del deoiler (trazas de petróleo en una fase continua de agua) fue considerado.

La ventaja de desarrollar estos bloques en Modelica es que permite un amplio abanico de configuraciones del sistema, encontrando la mejor configuración global, así como formar parte de mayores sistemas.

Este trabajo pertenece a un proyecto más amplio en el marco del grupo SUBPRO cuyo objetivo es crear una librería para la tecnología Subsea en un programa empleado cada más por las empresas del ámbito ingenieril.

Palabras claves

Subsea, modelado, separado compacto, modelica, petróleo, líquido-líquido

Abstract

Compact oil-water separators are essential for treatment of the production streams of oil fields at deep water.

The aim of this work has been to implement a model for a compact oil-water system control and optimization purposes in Modelica code.

We have implemented three blocks that include a gravity separator for the bulk separation of water and oil and two inline swirl separators for purification.

A new case avoiding the use of deoiler (oil in continuous a water phase) was considered.

The advantage of developing these units in Modelica is that it allows wide variations in the system, finding the best overall configuration or simulating as part of bigger models.

This work belongs to a larger project whose aim is to create a library with all the subsea equipment.

The model was added to the SUBPRO Modelica library.

Keywords:

Subsea, modelling, compact separator, modelica, oil, liquid-liquid



SPECIALIZATION PROJECT 2015

TKP 4550

PROJECT TITLE:

Implementing a Subsea Oil -Water Separation System in Modelica

 $\mathbf{B}\mathbf{y}$

Víctor Fernández Iniesta

Supervisor for the project:
Associate Professor Johannes Jäschke

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Abstract

The aim of the project was to obtain a Modelica model of a subsea liquid-liquid separator.

A subsea water-oil separation system consisting in one horizontal gravity separator and two swirl separators has been modeled in the equation based, object-oriented language Modelica. The units can be simulated separately or the overall system. The system can be connected in bigger systems. The modeling methodology, how the final model of the cycle is carried out, is described in the report. A new combination of separation was proposed and simulated.

A steady-state solution of the Modelica model was obtained, an alternative system was simulated.

Preface

This report is reflecting the work done as part of the specialization project (TKP4550) in the fifth year of the study program of MSc in Chemical Engineering at the Norwegian University of Science and Technology.

I would like to thank my supervisor Professor Johannes Jäschke, for their suggestions, guidance and support through the project. I would also like to thank Christoph Backi for his help regarding Modelica.

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	ii	

Contents

A	Abstract	i
F	Preface	ii
L	List of figures	v
L	ist of tables	vi
1	Introduction	1
2	Process Description	3
	2.1 Model inputs	3
	2.2 Fluid properties	6
3	Computer tools (Dymola)	8
4	Modeling methodology	9
	4.1 Gravity separator	9
	4.2 Deoiler	11
	4.3 Dewaterer	15
	4.4 Nodes	16
5	Results	17
	5.1 The Modelica model construction	17
	5.2 Validation of the results	19
	5.2.1 Gravity separator	19
	5.2.2 Swirl separator for Oil-in-Water Emulsions (Deoiler)	19
	5.2.3 Swirl separator for Water-in-Oil Emulsions (Dewaterer)	21
	5.2.4 Optimal values in the system	22
	5.2.5 New system Gravity separator and Dewaterer	23
6	Discussion	26
L	List of symbols and abbreviations	27
R	References	29
A	Appendix A: Modelica Code	30
	A.1 Connector	30
	A.2 Units used in the models	30
	A.3 Deoiler (Water continuous phase)	31
	A.3.1 swirl sep2	31

A.3.2 heaviside	33
A.3.3 DeOiler	33
A.3.4 swirl_func3	34
A.3.5 Block_Deoiler	36
A.4 Dewaterer (Oil continuous phase)	37
A.4.1 swirl_sep2_o	37
A.4.2 DeWaterer	39
A.4.3 swirl_func3_o	40
A.4.4 Block_Dewaterer	41
A.5 Gravity separator	42
A.6 Node	44
A.7 Numerical solvers	44
A.7.1 Runge-Kutta	44
A.7.2 Shooting method	45

List of figures

2.1	Flow diagram for the separation system. The subscript of each	
	stream corresponds to the values in the code	3
4.1	Horizontal gravity separator	10
4.2	Deoiler. The swirl element introduces a rotation in the liquid. The	
	inlet flow is separated by centrifugal buoyancy forces	12
4.3	Dewaterer. The water droplets are pushed outwards by the	
	centrifugal buoyancy	15
5.1	Purity of product streams versus flow split of the gravity	
	separator. $q_{in}=20m^3/h \alpha_{in}=0.4$	19
5.2	Dispersed performance versus flow rate simulated for the three	
	swirl elements. FS = $\alpha_{in DO} = 0.135$	20
5.3	Dispersed performance versus flow rate simulated for the three	
	swirl elements. FS = $\alpha_{in DO} = 0.135$	21
5.4	Dispersed performance versus flow rate in dewaterer. $\alpha_{in} = FS$	
	=0.9 and long swirl element were used	21
5.5	Purity of products versus flow split in dewaterer. $q_{in DW} =$	
	$7.2 \mathrm{m}^3 \cdot \mathrm{h}^{\text{-}1} \propto \! \mathrm{in} = \! 0.9$, large swirl element was used	22
5.6	System consisting in gravity separator and deoiler with recycle of	
	heavy phase stream	24

List of tables

2.1	Empirical parameters for swirl separators	4
2.2	The design conditions for the refrigeration cycle	4
2.3	Optimal operation for fixed inlet flow rate optimized by Preben	5
2.4	Properties of the fluids used in the separators	
	model	6
2.5	Fitted values for the coefficients in the polynomial for the viscosity of	
	the emulsion	7
5.1	Optimal values found in Modelica model were $q_{IN}=\!20m^3\cdot h^{1}$, $\alpha_{in}=0.4$	
	and large swirl element	23
5.2	Optimal values found in Modelica for model gravity separator and	
	dewaterer where q_{IN} =20m³ \cdot h⁻¹ , $\alpha_{\text{in}}=$ 0.4 $$ and large swirl element	25

1 Introduction

The use of subsea technology is nowadays increasing allowing operate reservoirs in remote places and in extreme conditions, being a cost effective alternative [1].

The goal is maximize the production and lay the ground for further growth, to achieve this it is needed to find compact and environmental friendly solutions, improve recovery from the reservoirs, permit long tie-backs to land, and developing equipment for ultradeep water.

Subsea separators have stricter limitations in comparison with topside separators, for existing facilities the space is minimal and the degree of compactness is a must.

The expected life of subsea equipment can have a huge impact on project economics. Because of high intervention costs and potential loss of production, reliability and availability of the system must be kept high.

Deepwater intervention will be costly because of the need for specialized equipment and support infrastructure. Keeping the subsea equipment light and compact will allow the use of less costly and more abundant dive-support vessels rather than having to use heavy-lift vessels.

It is needed to choose the best compromise for cost, function, operability and maintainability.

Subsea liquid-liquid separation is still a challenging issue. Technical problems have to be faced in terms of interface control, water quality, outlet water cut or solid removal.[2]

This technology offers some advantages as accelerates and increases production and recovery, prolongs the economic life of the fields, provides a flexible sollution for changing operating conditions and debottlenecks flowlines [3].

Due to these reasons an accurate control and optimization is key for the subsea separators.

This project was motivated by first of all reproducing parts of the work P. FürstTyvold [4] did in his Master Thesis, which was done in MATLAB, and implement a close to similar model in Modelica.

The intention of using Modelica was to avoid the causality which dominates many programming languages and take advantage of the benefits Modelica provides allowing with the use of connectors the simulation in bigger systems.

Once a model is written, the Modelica engine compiles the code into C-code or XML whereby almost anything can be further analyzed (like optimization).

Several books, articles and internet forums have been frequently employed to gain knowledge about Modelica usage, but the book by P.Fritzson [6][7] have been used as base material to build a model.

2 Process Description

2.1 Model inputs

The flow diagram for the separation system is depicted in Figure 2.1 and physical process dimensions¹ are announced in Table 2.2 empirical parameters for swirl separators are given in Table 2.1, finally the estimated optimal operation made by FürstTyvold is showed in Table 2.3

The gravity separator does a bulk separation while dewaterer and deoiler purifiy the water-rich and the oil-rich products, respectively.

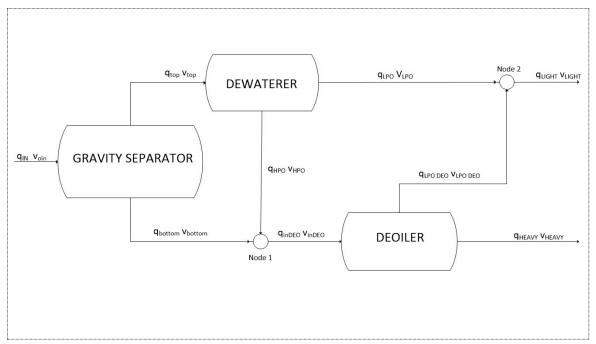


Figure 2.1: Flow diagram for the separation system. The subscript of each stream corresponds to the values in the code

¹ The variables presented are inherited from the exact same separation system defended by P.FürstTyvold's Master Thesis [4].

R _c /R [-]	0.25	-
C _{decay} [-]	0.04	-
$ m K_{re ext{-en}} \ [m^2]$	$2\cdot 10^{\text{-4}}$	-
$D_d(v^{ heta}_{max})$ [μm]	$(-107 \cdot v^{\theta}_{max} + 600) \cdot 10$ -6	$v_{\text{max}}^{\theta} \le 4.45$
Dd(V max) [μπη]	$(-8v^{\theta}_{max} \cdot +160) \cdot 10-6$	$v^{\theta}_{max} > 4.45$

 ${\bf Table~2.1~Empirical~parameters~for~swirl~separators}$

Process unit	Condition description	n	Symbol	Value
	Length		L[m]	7
Gravity	Outer pipe		R[m]	1.7
separator	Weir height		$H_{\rm w}[{ m m}]$	2.55
	Droplet diameter		$D_d[\mu m]$	120
	Length		L[m]	1.7
Deoiler (Swirl	Outer pipe		R[m]	0.05
separator for	Inner pipe		$R_{i}[m] \\$	0.025
continuous	Swirl	${\rm number}$		
water phase)	large/strong/weak		Ω [-]	7.0/5.0/3.0
Dewaterer	Length		L[m]	1.7
(Swirl	Outer pipe		R[m]	0.05
separator for	Inner pipe		$R_{i}[m]$	0.043
continuous oil	Swirl	${\rm number}$		
phase)	large/strong/weak		Ω [-]	7.0/5.0/3.0

Table 2.2: The design conditions for the refrigeration cycle

The inputs used in oil-in water separator were based in experimental data of Van Campen[5]. No experimental data available were found for the gravity separator and water-in-oil separator. Therefore a set of dimensions estimated by P.FürstTyvold[4] were taken.

	$\mathrm{q_{in}} =$
Variable	$20 \mathrm{m}^3 \cdot \mathrm{h}^{\text{-}1}$
$\mathrm{FS}_{\mathrm{G}}\left[ext{-} ight]$	0.33
α_{t} [-]	0.94
$ m q_t \ [m^3 \cdot h^{1}]$	6.7
α _b [-]	0.13
$ m q_b \ [m^3 \cdot h^{-1}]$	13.3
FS _{DW} [-]	0.91
α _{LPO DEW} [-]	0.98
QLPO DEW	
$[\mathrm{m}^3\cdot\mathrm{h}^{\text{-}1}]$	6.1
$\alpha_{ m HPO\ DEW}$ [-]	0.5
QHPO DEW	
$[\mathrm{m}^3\cdot\mathrm{h}^{\text{-}1}]$	0.6
$\alpha_{ m in\ DEO}$ [-]	0.15
$q_{ ext{IN DEO}} \left[ext{m}^3 \cdot ext{h}^{ ext{-}} ight.$	
1]	13.9
FS _{DEO} [-]	0.15
αlpo deo [-]	0.83
$ m q_{LPO\; DEO} \;\; [m^3 \cdot h^2]$	
1]	2
α _{HEAVY} [-]	0.03
$ m q_{HEAVY} [m^3 \cdot h^2]$	
1]	11.9
$\alpha_{ m LIGHT}$ [-]	0.94
$q_{LIGHT} \ [m^3 \cdot h^{-1}]$	8,1

 $\begin{array}{cccc} \textbf{Table} & \textbf{2.3} & \text{Optimal} \\ \text{operation for fixed inlet flow} \\ \text{rate} & \text{optimized} & \text{by} \\ \text{P.FürstTyvold} \\ \end{array}$

2.2 Fluid properties

The fluid properties were chosen following the FürstTyvold Thesis which in turn, he uses the experiments of Van Campen [5].

The water phase is brine made of tap water and sodium chloride (NaCl). The oil phase is a refined mineral oil blended with zinc free additives [5]. The densities and viscosities of the two phases are shown in Table 2.4.

Liquid	Density $[\mathrm{kg}\cdot\mathrm{m}^{\text{-3}}]$	$\begin{array}{c} \text{Viscosity} \\ [\text{mPa} \cdot \text{s}] \end{array}$
Oil	881	8.8
Brine	1064	1.0

Table 2.4 Properties of the fluids used in the separators model

The models require the viscosity of the emulsion as a function of the oil cut. The viscosity is expressed by a third order polynomial function (eq. 2.1), which is fitted to experimentally measured values by Van Campen [5].

$$\mu = \mu_c (1 + \alpha \varphi + b \varphi^2 + c \varphi^3) \tag{2.1}$$

Where μ_c is the viscosity of the continuous phase, ϕ is the volume fraction of the dispersed phase and a, b, c are the fitted values for the coefficients in the polynomial.

The phase inversion point of this particular emulsion is measured to be at the oil volume fraction of 0.66. This means that the viscosity function must be divided into two different regions with different polynomial coefficients. One for oil-in-water emulsions ($\alpha \le 0.66$) and one for water-in-oil emulsions ($\alpha > 0.66$).

The polynomial coefficients for the two viscosity equations are listed in Table 2.5.

Emulsion	Range in α [-]	a [-]	b [-]	c [-]
Oil in water	0 - 0.66	110	-400	470
Water in oil	0.66 - 1	-1.6	27	23

 ${\bf Table~2.5~Fitted~values~for~the~coefficients~in~the~polynomial~for~the~viscosity~of~the~emulsion}$

3 Computer tools (Dymola)

The modeling and simulations are performed in the environment Dymola by use of the GCC compiler. The user friendly environment is suited for both modeling, simulation and plotting of results. The GCC compiler handles most of the features present in the Modelica language.

They were defined units of each magnitude as well as the inputs as parameters to facilitate handling in the simulation environment without having to enter in the code, and making easy the understanding of every variable.

Two codes were developed, one for the simulation of each block individually and other by the use of connectors allowing simulate the complete separation system.

Due to the use of connectors the system can be implemented easily in a bigger system simply connecting the inputs and outputs of the system.

4 Modeling methodology

The system in Figure 2.1 is considered when discussing the modeling approach below. The Modelica model code is attached in Appendix A.

This model is based in the MATLAB code by FürstTyvold [4] with small variations in the model, but using the advantages of Modelica and simulating the complete system at the same time.

The initial values of the system model are the flow rate in the gravity separator, inlet oil volume fraction, flow split of each separator and swirl separators swirl number.

The outputs are the heavy outlet flow rate, heavy phase volume oil fraction, light outlet flow rate and light volume oil fraction.

4.1 Gravity separator

The model described below estimate the oil volume fractions in the outlets (Set of equations 4.1) given the inlet composition, the flow rate and the flow split of the gravity separator. Also the physical dimensions of the separator and the properties of the emulsion must be known.

The model is based on two independent plug flows flowing through the separator and exiting through two respective outlets. The oil droplets are pushed upwards by gravitational buoyancy forces and passes from the lower to the upper plug flow while an equal amount of water moves in the opposite direction. The vertical movement of the droplets were estimated assuming uniform droplet size distribution.

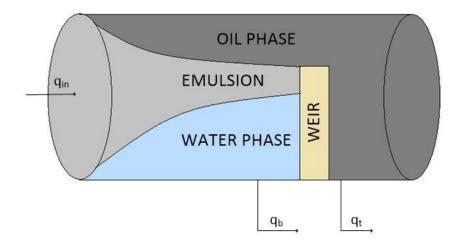


Figure 4.1 Horizontal gravity separator.

$$v_h = \frac{q_b}{A_b} \tag{4.1a}$$

$$A_b = \frac{R^2}{2} \cdot \left[2\cos^{-1}\left(\frac{R - H_w}{R}\right) - \sin\left(2\cos^{-1}\left(\frac{R - H_w}{R}\right)\right) \right]$$
(4.1b)

$$v_v = \frac{2r_d^2(\rho_d - \rho)g}{9\mu(\alpha)} \tag{4.1c}$$

$$\Delta h = \frac{v_v}{v_h} L \tag{4.1d}$$

$$A_e = \frac{R^2}{2} \cdot \left[2\cos^{-1}\left(\frac{R-d}{R}\right) - \sin\left(2\cos^{-1}\left(\frac{R-d}{R}\right)\right) \right] \tag{4.1e}$$

$$\alpha_b = \alpha_{in} \frac{A_e}{A_b} \tag{4.1f}$$

$$\alpha_t = \frac{1}{q_t} [\alpha_{in} q_{in} - \alpha_b q_b]$$
 (4.1g)

Where:

v_h = Horizontal velocity of a drop moving under the weir

 $H_w = Weir height$

 $q_b = Volumetric flow rate of the bottom outlet stream$

 $A_b = Cross section area of the lower part of the separator$

R = Radius of the separator

 $v_v = Vertical velocities of the droplets$

 $r_d = Radius of the droplet$

 ρ_d = Density of the droplet

 ρ = Density of the continuous phase

g = Gravitational acceleration

 $\mu(\alpha)$ = Viscosity of the emulsion function of the oil cut (α)

 Δh = Vertical distance of a droplet entering the separator at the bottom of the tank travel during the residence time of the separator

L = Horizontal distance from the inlet to the weir

 A_e = Area of circular segment that determines the amount of oil left in the bottom part of the tank at the end of the separator

d = Segment who determines the smallest distance of the circular area A_e

 $\alpha_b = \mathrm{Oil}$ volume fraction of the bottom outlet

 $\alpha_{in} = \text{Oil volume fraction of the inlet}$

 $\alpha_t = \text{Oil volume fraction in the top outlet}$

 $q_t = Flow rate in the top outlet$

 $q_{in} =$ Flow rate in the inlet

 $q_b = Flow rate in the bottom outlet$

4.2 Deoiler

The model described below (Set of equations 4.2) provides the necessary equations to estimate the oil volume fraction in the outlet (heavy phase and light phase) given the inlet composition, flow rate and the flow split of the separator.

We need to know previously the physical dimensions of the separator, including the swirl number furthermore the model contains three parameters that must be determined (C_{decay} , k_{re-en} and R_c).

Other inputs required are the physical properties of the emulsion, the densities of both phases and the correlation between the viscosity and the oil volume fraction.

The model is based on the average droplet size, providing an estimation of separation performance. The break-up of droplets will increase the swirl flow and a correlation between average droplet size and maximum tangential velocity is also required.

The density difference between the oil droplets and the continuous phase forces toward the centre of the separator the light droplets.

Re-entrainment flow rate was included in the model to compensate the simplification of the velocity profiles, to correct the existing net flux crossing the boundary between regions, this flow rate assumes that the amount of liquid reentrained will increase when the difference in the axial velocities increase.

The centrifugal acceleration is the driving force of the separation and can be described as a Rankine vortex. A Rankine vortex has two zones, the inner region with a solid body rotation and an outer region that was simplified assuming constant velocity in this region. The separation radio between regions is at R_c . For the dewaterer separator considered in the experiment of Van Campen [5], R_i is greater than R_c , only constant tangential velocity is used in the model from $(r_{in} > R_i)$ to R_i .

The swirling flow is expected to lose momentum caused by stress from the pipe wall, to count with it was introduced a parameter C_{decay} that can be found experimentally.

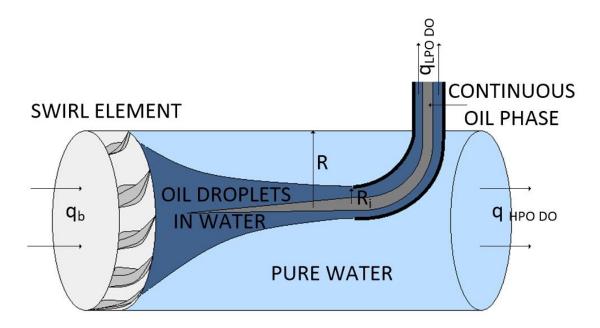


Figure 4.2 Deoiler. The swirl element introduces a rotation in the liquid. The inlet flow is separated by centrifugal buoyancy forces

$$FS = \frac{q_{LPO}}{q_{in}} \tag{4.2a}$$

$$v_{z}(r) = \begin{cases} \frac{q_{LPO}}{\pi R_{i}^{2}}, & 0 \le r \le R_{i} \\ \frac{q_{hPO}}{\pi (R^{2} - R_{i}^{2})}, & R_{i} < r \le R \end{cases}$$
(4.2b)

$$a_c(r) = \frac{v_\theta(r)^2}{r} \tag{4.2c}$$

$$v_{\theta}^{0}(r) = \begin{cases} v_{\theta}^{\max \frac{r}{R_{c}}}, & 0 \leq r \leq R_{c} \\ v_{\theta}^{\max}, & R_{c} < r \leq R \end{cases}$$
 (4.2d)

$$v_{\theta}^{max} = \Omega v_{z,b} \tag{4.2e}$$

$$v_{\theta}(r,z) = v_{\theta}^{0}(r)e^{\frac{-C_{decay,z}}{2R}}$$
(4.2f)

$$v_r(r,z) = \frac{2r_d^2(\rho_d - \rho)v_\theta^2(r,z)}{9\mu(r,z)}$$
(4.2g)

$$\alpha_c(r,z) = \alpha_{in} \frac{FS(R^2 - R_i^2) + (1 - FS)(r_{in}^2 - R_i^2)}{(1 - FS)(r^2 - R_i^2) + FS(R^2 - R_i^2)}$$
(4.2h)

$$\tau = \frac{\pi (R^2 - R_i^2)L}{(1 - FS)q_{in}} \tag{4.2i}$$

The goal for the model is to find the radial inlet position (r) of the droplet that exits the separator at $r = R_i$ and it is found by integrating the radial velocity (4.2g) from t = 0 to $t = \tau$ using the second order explicit Runge-Kutta integrator dividing r into 10 equal steps.

$$k_1 = v_r(t_n, r_n) \Delta t \tag{4.2j}$$

$$k_2 = v_r(t_n + \Delta t, r_n + k_1) \Delta t \tag{4.2k}$$

$$k_2 = v_r(t_n + \Delta t, r_n + k_1)\Delta t$$
 (4.2k)
 $r_{n+1} = r_n + \frac{k_1 + k_2}{2}$ (4.2l)

Viscosity and radial velocity (4.2h and 4.2g), are function of the radial inlet position of the droplet and the radial outlet position is fixed, the governing differential equation is a boundary value problem. The inlet position that lead to the droplet exiting at $(r = R_i)$ is founded using the shooting method (Newton-Raphson method) to solve the value boundary problem.

Assuming that the distribution of oil droplets in the radial and tangential direction is uniform at the inlet:

$$\alpha'_{LPO} = \min \left[1, \alpha_{in} \frac{FS(R^2 - R_i^2) + (1 - FS)(r_{in}^2 - R_i^2)}{FS(R^2 - R_i^2)} \right]$$
(4.2m)

$$\alpha'_{HPO} = \frac{\alpha_{in} - \alpha'_{LPO}FS}{1 - FS} \tag{4.2n}$$

$$q_{re-en} = k_{re-en} \Delta v \tag{4.20}$$

$$\alpha_{LPO} = \frac{1}{q_{LPO}} \left[\alpha'_{LPO} (q_{LPO} - q_{re-en}) + \alpha'_{HPO} q_{re-en} \right] \quad (4.2p)$$

$$\alpha_{HPO} = \frac{\alpha_{in} - \alpha_{LPO} FS}{1 - FS} \quad (4.2q)$$

Where:

FS = Flow Split

 $q_{in} = Volumetric flow rate at inlet$

 $q_{LPO} = Volumetric flow rate at light phase outlet$

q_{HPO} = Volumetric flow rate at heavy phase outlet

 $R_i = Radius$ of the light phase outlet

R = Radius of the deoiler

 $a_c = Centrifugal acceleration$

 $v_{\theta} = \text{Tangential velocity of the fluid}$

 v^0_θ = Tangential velocity right downstream of the swirl element

 $R_c = Rankine radius$

 $v^{max}_{\theta} = Maximum tangential velocity$

 $v_{z,b} = Bulk \text{ velocity in the axial direction}$

 $\Omega = \text{Proportionality constant}$, swirl number

 $C_{decay} = Experimental decaying factor$

 $v_r = Radial \ velocity \ in \ Stokes \ regime$

 $r_d = Droplet radius$

 $\mu = Viscosity$ of the emulsion

 $\rho_{\rm d} = \text{Dispersed phase density}$

 $\rho = \text{Continuous phase density}$

 $\tau = \text{Residence time of the droplet}$

L = Distance between the swirl element and the inner pipe

 $r_n = Radial position of the droplet$

 $\Delta = \text{Runge-Kutta time step}$

 $V_r = Radial \text{ velocity of the droplet}$

 α'_{LPO} = Oil volume fraction in the LPO assuming distribution of oil droplets in the radial and tangential direction is uniform at inlet

 α'_{HPO} = Oil volume fraction in the HPO assuming distribution of oil droplets in the radial and tangential direction is uniform at inlet

 α'_{in} = Oil volume fraction in the inlet assuming distribution of oil droplets in the radial and tangential direction is uniform at inlet

 $R_i = Radius of the inner pipe$

 $r_{in} = Inlet position of the droplet that exits at <math>r = R_i$

 $q_{re-en} = Re-entrainment rate$

 $k_{re-en} = Proportionality constant of re-entrainment$

 $\Delta v = Velocity$ difference between the inner and the outer region

 α_{LPO} = Final oil volume fraction in the LPO

4.3 Dewaterer

The model described below (Set of equations 4.3) predicts the oil volume fraction in the outlet streams govern the inlet composition, flow rate and the flow split, for water in an oil phase. The model is based on the same physical laws as the Deoiler (Oil in water phase).

The main difference is that density of the droplets is greater than the density of the continuous phase being forced the droplets by the centrifugal buoyancy forces outwards.

The equations are the same than 4.2 set except for the equation 4.2d in Deoiler the tangential velocity didn't operate changing in the discontinuous region but for Dewaterer are expected that droplets travel from the inner region to the outer. To avoid problems with the numerical solvers it is employed the smoothing approximation of Balakrishna and Biegler resulting in the next equation.

$$v_{\theta}^{0} = v_{\theta}^{max} - max\left(v_{\theta}^{max}\left[1 - \frac{r}{R_{c}}\right], 0\right)$$

$$(4.3a)$$

Where:

 $v_{\theta} = Tangential velocity of the fluid$

 v^0_θ = Tangential velocity right downstream of the swirl element

 $v^{\max}_{\theta} = Maximum tangential velocity$

 $R_c = Rankine \ radius$

SWIRL ELEMENT

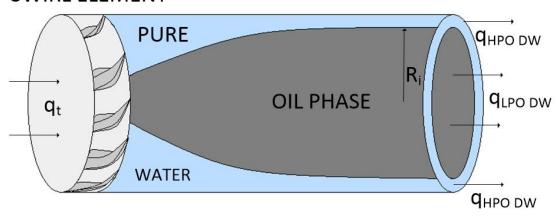


Figure 4.3 Dewaterer. The water droplets are pushed outwards by the centrifugal buoyancy forces

4.4 Nodes

Two nodes were used as mixers with two inputs and one output for the Deoiler inlet and for the light phase system outlet (Rich in oil).

Each node consists in the mass balance equations.

Node A:

$$q_{in DO} = q_{HPO DW} + q_b \tag{4.4a}$$

$$\alpha_{in DO} = \frac{q_{HPO DW}\alpha_{HPO DW} + q_b\alpha_b}{q_{in DO}}$$
 (4.4b)

Node B:

$$q_{LIGHT} = q_{LPO\ DW} + q_{LPO\ DO} \tag{4.4c}$$

$$\alpha_{LIGHT} = \frac{q_{LPO\ DW}\alpha_{LPO\ DW} + q_{LPO\ DO}\alpha_{LPO\ DO}}{q_{LIGHT}} \tag{4.4d}$$

Where:

 $q_{\text{IN DO}} = \text{Inlet flow rate in Deoiler}$

q_{HPO DW} = Outlet flow rate in Dewaterer

 $q_b =$ Outlet flow rate of the bottom in the gravity separator

q_{LIGHT} = Outlet flow rate of the light outlet in the system

q_{LPO DW} = Outlet flow rate of the LPO in Dewaterer

 $q_{LPO\ DO} = Outlet$ flow rate of the LPO in Deoiler

 $\alpha_{\text{IN DO}} = \text{Oil volume fraction in Deoiler}$

 $\alpha_{HPO\ DW}$ = Oil volume fraction in Dewaterer

 $\alpha_b = \text{Oil volume fraction of the bottom in the gravity separator}$

 $\alpha_{\text{LIGHT}} = \text{Oil volume fraction}$ of the light outlet in the system

 $\alpha_{LPO\ DW} = Oil\ volume\ fraction\ of\ the\ LPO\ in\ Dewaterer$

 $\alpha_{LPO\ DO} = Oil\ volume\ fraction\ of\ the\ LPO\ in\ Deoiler$

5 Results

5.1 The Modelica model construction

The most time spent was in making the model from scratch. We can simulate the model block by block running the model of each block individually the parameters and inputs can be modified in the simulation environment easily.

Otherwise the connected code can be simulated, the main difference is that we can modify only the inlet flow rate in the system, inlet oil cut and flow splits of each separator while the rest parameters are determined by them.

The gravity separator model was based by equations being a non-causal model however, Deoiler and Dewaterer were modeled by the use of functions.

The main function in Deoiler is swirl_func3 who determine the outlet streams of Deoiler. This function call at the solver RK2 and, RK2 call at swirl_sep2 giving the radial velocity of a droplet afterwards Deoiler function is called estimating the outlet composition having as input the greatest inlet radial position of all the droplets that exits in LPO, the shooting function also solve the boundary problem.

Dewaterer block is modeled as the same way than Deoiler.

The connectors link the blocks sharing the volume flow rate and oil volume fraction between outlets and inlets in each block, in addition they allow connect the complete system with others models facilitating the simulations with bigger system or change the system link configuration. They were defined as:

```
connector Flow_port

import Oil_Water_Separation_System;
import SI = Oil_Water_Separation_System.Units;
SI.OilVolumeFraction volfracc "Volume fraction in the connection point";
SI.VolumeFlowRate volflow "Volume flow rate in the connection point";
end Flow_port;
```

The complete system simulation code is shown below, the complete code can be found in the Appendix, initial values are the inlets found as optimal.

```
1
    model SimulationOilWaterSystem
2
3
    Oil_Water_Separation_System.BLOCK_GRAVITY_SEPARATOR.Block_GravitySeparator GS;
     Oil_Water_Separation_System.BLOCK_DEWATERER.Block_Dewaterer DEW;
4
     Oil_Water_Separation_System.Node_2inlets_1outlet NODE1;
 6
     Oil_Water_Separation_System.BLOCK_DEOILER.Block_Deoiler DEO;
7
     Oil Water Separation System.Node 2inlets 1outlet NODE2;
8
9
    equation
     connect(GS.flowporttop,DEW.flowportindew);
10
     connect(DEW.flowportoutdewH,NODE1.flowportinleta);
11
     connect(GS.flowportbottom,NODE1.flowportinletb);
12
13
     connect(NODE1.flowportoutlet,DEO.flowportindeo);
     connect(DEO.flowportoutdeoL,NODE2.flowportinleta);
14
15
     connect(DEW.flowportoutdewL,NODE2.flowportinletb);
16
    end SimulationOilWaterSystem;
17
```

The code for the system consisting in gravity separator and Dewaterer with recycle of the heavy phase outlet is showed below, explanation of the system in section 5.2.5.

```
model SimulationgravityDewaterer
1
2
      Oil_Water_Separation_System.BLOCK_GRAVITY_SEPARATOR.InputStream IN(qin=0.005
3
     5,vo in=0.4);
      Oil_Water_Separation_System.BLOCK_GRAVITY_SEPARATOR.Block_GravitySeparator G
4
5
     S(FS=0.49);
      Oil_Water_Separation_System.BLOCK_DEWATERER.Block_Dewaterer DEW(FS=0.85);
6
7
      Oil_Water_Separation_System.Node_2inlets_1outlet NODE1;
8
9
     equation
10
      connect(IN.flowport,NODE1.flowportinleta);
      connect(DEW.flowportoutdewH,NODE1.flowportinletb);
11
      connect(NODE1.flowportoutlet,GS.flowportinlet);
12
13
      connect(GS.flowporttop,DEW.flowportindew);
14
15
     end SimulationgravityDewaterer;
```

5.2 Validation of the results

5.2.1 Gravity separator

The model was made as an exact copy of the model made by P. FürstTyvold. The results showed of the gravity separator in the thesis differ from the MATLAB code thus, this difference was the same in the Modelica model. It is showed in the figure 5.1. It affects the inlet conditions of the swirl separators.

Note that is not expected that the model predicts accurate values for purities of the outputs in the gravity separator.

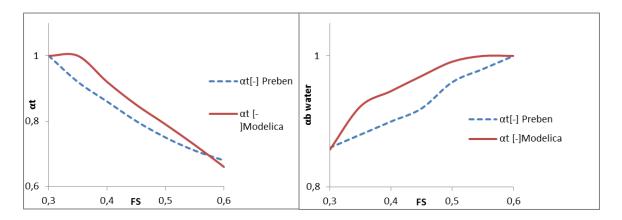


Figure 5.1 Purity of product streams versus flow split of the gravity separator. $q_{in}=20m^3/h \alpha_{in}=0.4$

It is expected that the separation efficiency decreases with increases in the flow rate. The gravitational acceleration is constant in this separator.

The simulation (Figure 5.1) shows that increasing the flow split the purity of oil in the top stream decreases while the purity of water in the bottom outlet increases.

5.2.2 Swirl separator for Oil-in-Water Emulsions (Deoiler)

In order to facilitate the study of the swirl separator was defined dispersed performance as:

Dispersed performance η_{dis} is a measure about how much desired liquid exits in the expected output. It is one if all the pure oil exits in LPO and all the pure water exits in HPO.

$$\eta_{dis} = 1 - \frac{(1 - \alpha_{LPO})q_{LPO} + \alpha_{HPO}q_{HPO}}{q_{in}}$$

$$(5.1)$$

The figure 5.2 shows the effect of the inlet flow rate on the efficiency, it is analysed in the three swirl elements large is the greater swirl number and strong the lower.

The behaviour is influenced by the empirical droplet size correlation. The local minimum correspond with the maximum tangential velocity of 4.45 m \cdot s⁻¹. This figure shows clearly that q_{in} should be necessary control at the maximum dispersed efficiency value.

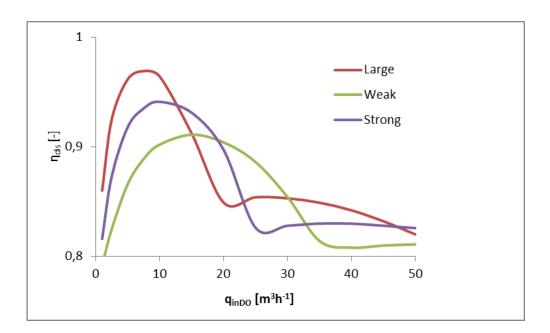


Figure 5.2 Dispersed performance versus flow rate simulated for the three swirl elements. $FS = \alpha_{in DO} = 0.135$

Flow split is a parameter that can be modified affecting the quality of the product streams (Figure 5.3), when the purity of the HPO increases the purity of the HPO decreases, the optimal value of the flow split depends on the main objective of the separator.

Note that the results presented by the Modelica model are same as in MATLAB model.

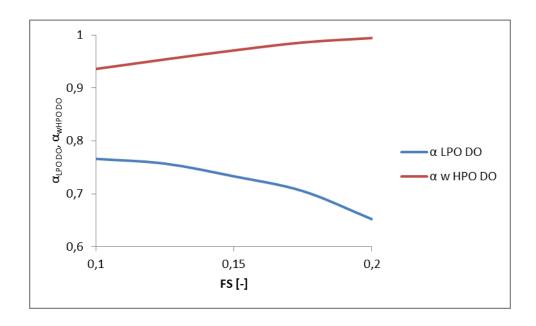


Figure 5.3 Purity of the output flow streams versus flow split inlet flow rate = 13.65m³h⁻¹ and inlet oil cut = 0.135.

5.2.3 Swirl separator for Water-in-Oil Emulsions (Dewaterer)

The performance estimated in Deoiler has the same behaviour than Dewaterer this is caused by having the same dimensions and the same droplet size equation.

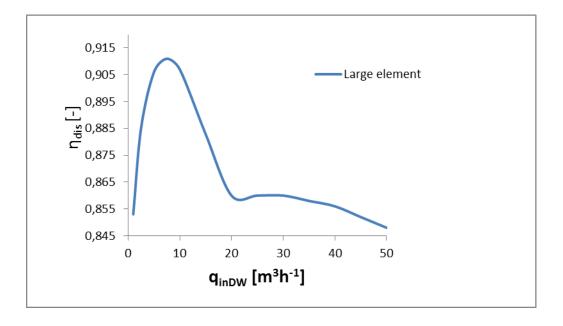


Figure 5.4 Dispersed performance versus flow rate in Dewaterer. $\alpha_{in} = FS = 0.9$ and long swirl element were used.

When the flow split increases, the HPO water volume fraction increases while the LPO oil volume rate decreases this is the same conclusion than in Deoiler. Depending on the purpose of the separator we can manipulate the flow split.

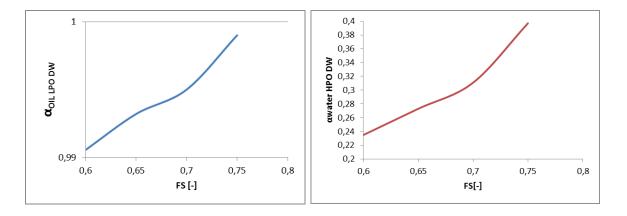


Figure 5.5 Purity of products versus flow split in Dewaterer. $q_{in DW} = 7.2 \text{m}^3 \cdot \text{h}^{-1}$ ain =0.9, large swirl element was used.

5.2.4 Optimal values in the system

One optimal solution was found in the system showed in Figure 2.1 manipulating flow splits with water volume fraction in outlet heavy phase < 0.3 and maximizing the oil volume fraction in the light phase while the inputs were $q_{\rm IN} = 20 {\rm m}^3 \cdot {\rm h}^{-1} \; \alpha_{\rm in} = 0.4$. It was found simulating the global system. Global dispersed efficiency for this system is $\eta_{\rm DIS~G}$ 0.95.

Variable	$q_{in}=20 \mathrm{m}^3\cdot\mathrm{h}^{\text{-}1}$
$\mathrm{FS}_{\mathrm{G}}\left[ext{-} ight]$	0.36
α_{t} [-]	0.98
$\mathrm{q_t}$ $\mathrm{[m^3\cdot h^{\text{-}1}]}$	7.2
$\alpha_{\mathrm{b}}\left[- ight]$	0.07
$\mathrm{q_b} \;\; \mathrm{[m^3 \cdot h^{ ext{-}1}]}$	12.8
FS _{DW} [-]	0.88
$\alpha_{ m LPO\ DEW}$ [-]	0.995
$ m q_{LPO\; DEW} \;\; [m^3 \cdot h^{ ext{-}1}]$	6.33
$\alpha_{ m HPO\ DEW}$ [-]	0.9
$ m q_{HPO\ DEW}\ [m^3\cdot h^{-1}]$	0.87
$\alpha_{ m in\ DEO}$ [-]	0.135
$ m q_{IN~DEO}~[m^3\cdot h^{-1}]$	13.65
FS _{DEO} [-]	0.135
α _{LPO DEO} [-]	0.73
$ m q_{LPO\ DEO}\ [m^3\cdot h^{-1}]$	1.84
$\alpha_{\mathrm{HEAVY}}\left[- ight]$	0.03
${ m q}_{ m HEAVY} { m [m^3 \cdot h^{ ext{-}1}]}$	11.81
$\alpha_{ m LIGHT}$ [-]	0.935
$q_{ ext{LIGHT}} \ \ [ext{m}^3 \cdot ext{h}^{ ext{-}1}]$	8.19

Table 5.1 Optimal values found in Modelica for complete system model were q_{IN} =20m³· h¹¹ , $\alpha_{in}=0.4$ and large swirl element.

5.2.5 New system Gravity separator and Dewaterer

A new system distribution was simulated avoiding the use of Deoiler showing good results, the difference is that the Light Phase Output lose 7% of purity, a cost-analysis should be made.

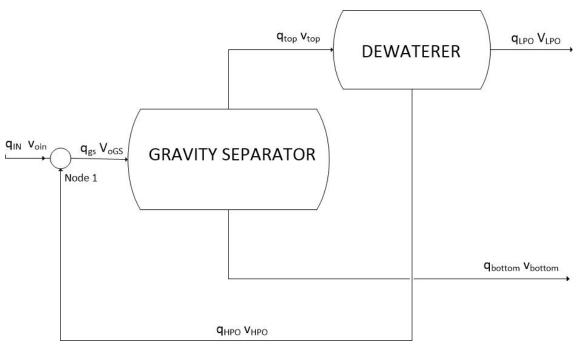


Figure 5.6 System consisting in gravity separator and deoiler with recycle of heavy phase stream.

The optimal results maintaining the water phase output with 0.03 volume fraction in oil and maximizing the oil volume fraction in light output. The $\eta_{DIS G}$ (dispersed efficiency) is 0.92

Variable	$q_{in} = 20m^3 \cdot h^{-1}$
FS _G [-]	0.49
$\alpha_{\text{in GS}}$ [-]	0.39
$q_{in GS} [m^3 \cdot h^{-1}]$	21.57
α_t [-]	0,78
$q_t [m^3 \cdot h^{-1}]$	10,47
α _b [-]	0.03
q _b [m ³ ·h ⁻¹]	11
FS _{DW} [-]	0.85
α _{LPO DEW} [-]	0.86
$q_{LPO\ DEW}\ [m^3\cdot h^{-1}]$	9
α _{HPO DEW} [-]	0.39
$q_{HPO\ DEW}\ [m^3\cdot h^{-1}]$	1.57

6 Discussion

The model shows the same result that the model that was based except for gravity separator discussed in (Section 5.2.1).

Another proposal was simulated avoiding the use of Deoiler losing 2% of performance.

The advantage of use the Modelica model developed is the versatility using the connectors, allowing create quickly modifications as seen in 5.2.5. If it is needed reoptimization with tools as JM compiler a complete equation based model should be made.

The gravity separator model should be improved with experimental values and the behaviour of the interaction between the two phases.

Future Work:

- Developing a model 100% conciliable with Subpro library, especially attention should be paid to the physical properties of the fluid, basing in the media used for this library.
- Applying a control structure and optimization using self-optimizing control.
- Integrating the liquid-liquid system in the complete subsea module and re-optimize.

List of symbols and abbreviations

Symbol^2	Dimension	Description
Latin letters		
a	•	Polinomial coefficient
$A_{\rm b}$	m^2	Cross section area of the lower part of the gravity separator
a_{c}	$ ext{m}\cdot ext{s}^{ ext{-}2}$	Centrifugal acceleration
$ m A_e$	m^2	Area of circular segment that determines the amount of oil left in the bottom part of the tank at the end of the gravity separator
$C_{ m decay}$	-	Experimental decaying factor
d	m	Segment who determines the smallest distance of the circular area $A_{\rm e}$
$\mathrm{D_{d}}$	μm	Droplet diameter
FS	-	Flow split
g	$\mathrm{m}\cdot\mathrm{s}^{\text{-}2}$	Gravitational acceleration
$ m H_w$	m	Weir height
${ m L}$		Length
\mathbf{R}	m	Radius
$q_{ m re-en}$	m^2	Re-entrainment flowrate
$ m V_{h}$	$ ext{m}\cdot ext{s}^{ ext{-}1}$	Horizontal velocity of a drop moving under the weir
$ m V_{ m V}$	$\mathrm{m}\cdot\mathrm{s}^{\text{-}1}$	Vertical velocities of the droplets
$ m V_{ m Z},$	$\mathrm{m}\cdot\mathrm{s}^{\text{-}1}$	Bulk velocity in the axial direction
Greek		
letters	_	
α	-	Oil volume fraction of the bottom outlet
Δ	-	Runge-Kutta time step
$\eta_{ m DIL}$	-	Diluted efficiency
$\eta_{ m DIS}$	-	Dispersed efficiency
μ	$\mathrm{kg}\cdot\mathrm{s}^{\text{-1}}\cdot\mathrm{m}^{\text{-}}$	Viscosity
	1	

² Continued on next page

Symbol^3	Dimension	Description
ρ	$ ext{kg}\cdot ext{m}^{ ext{-}3}$	Density
τ	-	Residence time of the droplet
Ω	-	Swirl number
Superscript	_	
4	_	uniform distribution at inlet
О	-	Right downstream of the swirl element
max	-	Maximum
Subscript	_	
b	_	Bottom
d	-	Droplet
DE, DEO	-	Deoiler
DW, DEW	-	Dewaterer
GS	-	Gravity separator
HPO	-	Heavy phase outlet
i	-	Input
in	-	Inlet
$_{ m LPO}$	-	Light phase outlet
t	-	Top
Θ	-	Tagential

³ Continued from previous page

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Appendix A: Modelica Code

This appendix contains the Modelica code created during the project.

- Appendix A.1 Connector.
- Appendix A.2 Units used in the models.
- Appendix A.3 Deoiler (Water continuous phase).
- Appendix A.4 Dewaterer (Oil continuous phase).
- Appendix A.5 Gravity Separator
- Appendix A.6 Node.
- Appendix A.7 Numerical solvers.

A.1 Connector

```
connector Flow_port

import Oil_Water_Separation_System;
import SI = Oil_Water_Separation_System.Units;

SI.OilVolumeFraction volfracc "Volume fraction in the connection point";
SI.VolumeFlowRate volflow "Volume flow rate in the connection point";

end Flow_port;
```

A.2 Units used in the models

```
package Units
1
2
3
       type Area = Real(final quantity = "Area", final unit = "m2", min = 0);
       type DynamicViscosity = Real(final quantity = "DynamicViscosity", final unit = "Pa.s",
4
5
                               min = 0);
       type Density = Real(final quantity = "Density", final unit = "kg/m3", displayUnit =
6
                     "g/cm3", min = 0.0);
7
       type Length = Real(final quantity = "Length", final unit = "m");
8
       type Velocity = Real(final quantity = "Velocity", final unit = "m/s");
9
10
       type VolumeFlowRate = Real(final quantity = "VolumeFlowRate", final unit = "m3/s");
11
       type OilVolumeFraction = Real(final quantity = "OilVolumeFraction", final unit = "1", min
12
                                = 0, \max = 1);
       type FlowSplit = Real(final quantity = "FlowSplit", final unit = "1", min = 0, max = 1);
13
14
       type Efficiency = Real(final quantity = "FlowSplit", final unit = "1", min = 0, max = 1);
15
       type SwirlElement = String(final quantity = "Swirl element weak, strong or large",
16
                          final unit = "adim");
```

```
17
18 end Units;
```

A.3 Deoiler (Water continuous phase)

A.3.1 swirl_sep2

This function contains the most part of the Deoiler model equations, returns the radial velocity of a droplet.

```
1
    function swirl_sep2
2
3
     input Real t;
4
     input Real x;
     input Real ini[:];
5
     //Array size defined at function call
6
7
     input String element;
8
9
     output Real DXDT;
10
     protected
     constant Real rhoo = 881 "872 kg/m3";
11
     constant Real rhow = 1064;
12
13
     constant Real pi = Modelica.Constants.pi;
14
     Real qin;
15
     Real Ro;
16
     Real Ri;
17
     Real Vo_in;
18
     Real ta;
19
     Real rin;
     Real FS;
20
21
     Real r;
22
     Real Rc;
23
     Real vaaux "va used before apply the vancampen scaling factor";
     Real va;
24
25
     Real k;
26
     Real vt;
27
     Real rd;
28
     Real Vo_c;
29
     Real mum;
30
     Real vr;
31
32
    algorithm
     qin := ini[1];
```

```
34
      Ro := ini[2];
35
      Ri := ini[3];
36
      Vo_in := ini[4];
37
      ta := ini[5];
38
      rin := ini[6];
39
      FS := ini[7];
40
      r := x;
41
      Rc := 0.25 * Ro;
42
      vaaux := qin / (pi * Ro ^ 2);
43
      //Van Campen thesis scaling factor Weak, Strong or Large
44
      if element == "w" then
45
       k := 3.5;
      elseif element == "s" then
46
47
       k := 5;
48
      elseif element == "I" then
49
       k := 7;
50
      end if:
51
52
      vt := k * vaaux;
53
      //Experimental correlation between rd and vt
54
      if vt > 4.45 then
      rd := ((-8 * vt) + 160) / 2 * 1e-6;
55
56
57
       rd := ((-107 * vt) + 600) / 2 * 1e-6;
58
      end if;
59
60
      va := qin * (1 - FS) / (pi * (Ro ^ 2 - Ri ^ 2));
61
              Vo_c := Vo_in * (FS * (Ro ^ 2 - Ri ^ 2) * (Ro ^ 2 - r ^ 2) + (rin ^ 2 - Ri ^ 2) * (Ro ^ 2 -
62
     r^2)*(1-FS))/((r^2-Ri^2)*(Ro^2-r^2)*(1-FS)+FS*(Ro^2-
     Ri ^ 2) * (Ro ^ 2 - r ^ 2));
63
      mum := 0.47 * Vo_c ^ 3 - 0.4 * Vo_c ^ 2 + 0.11 * Vo_c + 0.001;
64
      vr := (2 / 9 * (rhoo - rhow) * rd ^ 2 / mum * (vt * exp(-
65
     0.04 * va * t / (2 * Ro))) ^ 2 / r * heaviside(r - Rc) + 2 / 9 * (rhoo -
66
     rhow) * rd ^ 2 / mum * r * (vt * exp(-0.04 * va * t / (2 * Ro)) / Rc) ^ 2 * heaviside(Rc -
67
     r)) * heaviside(r - 0.9 * Ri);
68
69
      DXDT := vr;
70
71 end swirl_sep2;
```

A.3.2 heaviside

This function was created to complete the code due to it is not implemented in the Modelica standard library.

```
function heaviside
1
2
3
     input Real heaviin;
     output Real heaviout;
4
5
    algorithm
6
7
     if heaviin < 0 then
       heaviout := 0;
8
9
     elseif heaviin > 0 then
10
       heaviout := 1;
     else
11
12
       heaviout := 0.5;
13
     end if;
14
    end heaviside;
15
```

A.3.3 DeOiler

The function estimates the outlet composition.

```
1
    function DeOiler
2
3
     input Real qin;
4
     input Real Vo_in;
     input Real FS;
5
6
     input Real rin;
7
     input Real p2[3];
8
     output Real Vo_LPO;
9
     output Real Vo_HPO;
10
     output Real qen;
11
    protected
     constant Real pi = Modelica.Constants.pi;
12
     constant Real k = 2e-4;
13
14
     Real Ro;
15
     Real Ri;
16
     Real qi;
17
     Real qo;
18
     Real u_HPO;
```

```
Real u_LPO;
19
20
     Real du;
21
22
    algorithm
23
     Ro := p2[2];
24
     Ri := p2[3];
25
     qi := FS * qin;
     qo := qin - qi;
26
27
     Vo_LPO := Vo_in * (FS * (Ro ^ 2 - Ri ^ 2) + (rin ^ 2 - Ri ^ 2) * (1 - FS)) / (FS * (Ro ^ 2 -
28
     Ri ^ 2));
     Vo_HPO := (Vo_in * qin - Vo_LPO * qi) / qo;
29
30
     u_HPO := qo / (pi * (Ro ^ 2 - Ri ^ 2));
     u_LPO := qi / (pi * Ri ^ 2);
31
32
     du := u_LPO - u_HPO;
33
     qen := k * du;
     if du >= 0 then
34
35
      Vo_LPO := (Vo_LPO * (qi - qen) + Vo_HPO * qen) / qi;
36
     else
37
      Vo_{HPO} := (Vo_{HPO} * (qo + qen) - Vo_{LPO} * qen) / qo;
38
      Vo_LPO := (Vo_in * qin - Vo_HPO * qo) / qi;
39
     end if;
40
     if Vo LPO > 1 then
41
42
      Vo_LPO := 1;
43
     elseif Vo_LPO < Vo_in then
44
      Vo_LPO := Vo_in;
     end if;
45
46
47
     Vo_HPO := (Vo_in * qin - Vo_LPO * qi) / qo;
48
49
    end DeOiler;
```

A.3.4 swirl func3

This is the main function of Deoiler, the function estimates the outlet streams given the inlet conditions.

```
function swirl_func3

import Oil_Water_Separation_System;
input Real Vo_in;
input Real qin;
input Real FS;
input String element;
```

```
8
      output Real Vo_LPO;
9
      output Real Vo_HPO;
10
      output Real Edil;
11
      output Real Edis;
12
      output Real qen;
     protected
13
14
      constant Real pi = Modelica.Constants.pi;
15
      //Parameters
16
      parameter Real Lsw = 1.7;
17
      parameter Real Ro = 0.05;
18
      parameter Real Ri = 0.025;
19
      //Local variables
20
      Real qi;
21
      Real qo;
22
      Real ta;
23
      Real rin;
24
      Real x0;
25
      Real ini[7];
26
      Real h;
27
      Real T[11];
28
      Real X[11];
29
      Real TotSep;
30
      Real rin_0;
31
      Real rho;
32
      Real tol;
33
      Real p2[3];
34
35
    algorithm
36
      qi := FS * qin;
37
      qo := qin - qi;
      ta := pi * (Ro ^ 2 - Ri ^ 2) * Lsw / qo;
38
39
      rin := 0.999 * Ro;
40
      x0 := rin;
41
      ini := {qin, Ro, Ri, Vo_in, ta, rin, FS};
42
      h := ta / 10;
43
      (T,X) := RK2 (\{0.0,ta\}, x0, h, ini,element);
44
45
      if X[end] > Ri and X[end] < Ro then
46
       TotSep := 0;
47
       rin_0 := 0.99 * Ro;
48
       rho := 1;
49
       tol := 10 ^ (-10) * Ri;
       (T,X) := shooting( {0.0,ta},h,Ri,rin_0,rho,tol,ini, element);
50
51
       rin := X[1];
52
      else
```

```
53
       TotSep := 1;
54
       rin := Ro;
55
     end if;
56
57
     p2 := {Lsw, Ro, Ri};
     (Vo_LPO,Vo_HPO,qen) := DeOiler(qin, Vo_in, FS, rin, p2);
58
     Edil := Vo LPO * qi / (Vo in * qin);
59
     Edis := 1 - ((1 - Vo_LPO) * qi + Vo_HPO * qo) / qin;
60
61
62
    end swirl func3;
```

A.3.5 Block Deoiler

This is the model that enables the simulation of Deoiler giving the inlet conditions to the rest of the functions.

```
1
     model Block_Deoiler
2
3
      import SI = Oil_Water_Separation_System.Units;
4
5
      parameter SI.FlowSplit FS = 0.15 "Deoiler Flow split qLPO respect qinlet";
      parameter String element = "I";
6
7
8
       SI.Efficiency Edil
9
       "Dilute efficiency, Fraction of oil that is kept in LPO, '1' if we don't have oil in HPO, '0' if
10
      we don't have oil in LPO";
11
       SI.Efficiency Edis
       "Dispersed efficiency, how much of the liquid that exits through the desired outlet; '1'
12
     means HPO is pure water and LPO is pure oil";
13
       SI.OilVolumeFraction gen
14
15
       "Amount of reentrainment, Some water will be re-
     entrained in the light phase product and viceversa for low Flow Split, caused for the diffe
16
17
     rence of axial velocities between LPO and HPO";
18
19
      Oil Water Separation System.Connector.Flow port flowportindeo;
      Oil_Water_Separation_System.Connector.Flow_port flowportoutdeoH;
20
      Oil_Water_Separation_System.Connector.Flow_port flowportoutdeoL;
21
22
23
     equation
       (flowportoutdeoL.volfracc,flowportoutdeoH.volfracc, Edil, Edis, gen) = swirl func3( flo
24
25
     wportindeo.volfracc, flowportindeo.volflow, FS, element);
26
       flowportoutdeoL.volflow = flowportindeo.volflow*FS;
       flowportoutdeoH.volflow = flowportindeo.volflow*(1-FS);
27
```

```
28
29 end Block_Deoiler;
```

A.4 Dewaterer (Oil continuous phase)

A.4.1 swirl_sep2_o

This function contains the most part of the Deoiler model equations, returns the radial velocity of a droplet.

```
function swirl_sep2_o
1
2
3
      input Real t;
4
      input Real x;
5
      input Real ini[:];
      //Array size defined at function call time
6
7
      input String element;
8
      output Real vr;
9
      protected
10
      constant Real rhoo = 881 "872 kg/m3";
11
      constant Real rhow = 1064;
12
      constant Real beta = 10;
13
       constant Real pi = Modelica.Constants.pi;
14
       Real r;
15
       Real qin;
16
       Real Ro;
17
       Real Ri;
18
       Real Vo_in;
19
       Real ta;
20
       Real rin;
21
       Real FS;
22
       Real Rc;
23
       Real va;
24
       Real Vw_in;
25
       Real vt0;
26
       Real Vw;
27
       Real mum;
28
       Real f2;
29
       Real f1;
30
       Real f;
31
       Real ac;
32
       Real k;
33
       Real rd;
```

```
34
      algorithm
35
       qin := ini[1];
36
       Ro := ini[2];
37
       Ri := ini[3];
38
       Vo in := ini[4];
39
       ta := ini[5];
40
       rin := ini[6];
41
       FS := ini[7];
42
       Vw_in := 1 - Vo_in;
43
       r := x;
44
       Rc := 0.25 * Ro;
45
       va := qin / (pi * Ro ^ 2);
46
47
       if element == "w" then
48
        k := 3.5;
49
       elseif element == "s" then
        k := 5;
50
       elseif element == "I" then
51
52
        k := 7;
53
       end if;
54
55
       vt0 := k * va;
56
       //Experimental correlation between rd and vt
57
58
       if vt0 > 4.45 then
        rd := ((-8 * vt0) + 160) / 2 * 1e-6;
59
60
       else
61
        rd := ((-107 * vt0) + 600) / 2 * 1e-6;
62
       end if:
63
       Vw := Vw_in * (((1 - FS) * Ri ^ 2 + FS * (Ri ^ 2 - rin ^ 2)) / ((1 - FS) * Ri ^ 2 + FS * (Ri ^ 2 -
64
65
      r ^ 2)));
       mum := 0.203 * Vw ^ 3 + 0.237 * Vw ^ 2 - 0.014 * Vw + 0.0088;
66
67
       f2 := (vt0 * exp(-0.04 * va * t / (2 * Ro))) ^ 2 / r;
68
       f1 := (vt0 * exp(-0.04 * va * t / (2 * Ro)) / Rc) ^ 2 * r;
69
       f := f2 - f1;
70
       ac := f2 - 0.5 * ((f ^ 2 + beta ^ 2) ^ 0.5 + f);
71
       vr := 2 / 9 * (rhow - rhoo) * rd ^ 2 / mum * ac;
72
73
      end swirl_sep2_o;
```

A.4.2 DeWaterer

The function estimates the outlet composition.

```
1
     function DeWaterer
2
3
      input Real qin;
      input Real Vo_in;
4
5
      input Real FS;
6
      input Real rin;
7
      input Real p3[3];
8
      output Real Vo_LPO;
9
      output Real Vo_HPO;
10
      output Real qen;
11
     protected
12
      constant Real pi = Modelica.Constants.pi;
13
      constant Real k = 2e-4;
14
      Real Ro;
15
      Real Ri;
16
      Real qi;
17
      Real qo;
18
      Real Vw_LPO;
19
      Real Vw_HPO;
20
       Real u_HPO;
21
      Real u_LPO;
22
       Real du;
23
24
     algorithm
25
      Ro := p3[2];
26
      Ri := p3[3];
27
      qi := FS * qin;
28
      qo := qin - qi;
29
      Vw_{HPO} := (1 - Vo_{in}) * (((1 - FS) * Ri ^ 2 + (Ri ^ 2 - rin ^ 2) * FS) / ((1 - FS) * Ri ^ 2));
      Vw_LPO := ((1 - Vo_in) * qin - Vw_HPO * qo) / qi;
30
31
      u_HPO := qo / (pi * (Ro ^ 2 - Ri ^ 2));
32
      u_LPO := qi / (pi * Ri ^ 2);
33
      du := u LPO - u HPO;
34
      qen := k * du;
35
36
      if du >= 0 then
        Vw_LPO := (Vw_LPO * (qi - qen) + Vw_HPO * qen) / qi;
37
        Vw_HPO := ((1 - Vo_in) * qin - Vw_LPO * qi) / qo;
38
39
40
        Vw HPO := (Vw HPO * (qi + qen) - Vw LPO * qen) / qi;
```

```
41
      end if;
42
      if Vw HPO > 1 then
43
44
       Vw_HPO := 1;
45
      elseif Vw HPO < 1 - Vo in then
46
       Vw_HPO := 1 - Vo_in;
47
      end if;
48
49
      Vw_LPO := ((1 - Vo_in) * qin - Vw_HPO * qo) / qi;
50
      Vo_LPO := 1 - Vw_LPO;
      Vo_HPO := 1 - Vw_HPO;
51
52
53
     end DeWaterer;
```

A.4.3 swirl_func3_o

This is the main function of Dewaterer, the function estimates the outlet streams given the inlet conditions.

```
function swirl_func3_o
1
2
3
      import Oil_Water_Separation_System;
4
      input Real Vo_in;
5
      input Real qin;
6
      input Real FS;
7
      input String element;
8
      output Real Vo LPO;
9
      output Real Vo_HPO;
10
      output Real Edil;
11
      output Real Edis;
12
      output Real gen;
13
     protected
14
      constant Real pi = Modelica.Constants.pi;
15
      //Parameters
16
      parameter Real Lsw = 1.7;
17
      parameter Real Ro = 0.05;
18
      parameter Real Ri = 0.043;
19
      //Local variables
20
      Real qi;
21
      Real qo;
22
       Real ta;
23
       Real rin;
24
       Real ini[7];
25
       Real h;
```

```
26
       Real T[11];
27
       Real X[11];
28
       Real Vw LPO;
29
       Real Vw_HPO;
30
       Real rin 0;
       Real rho;
31
32
       Real tol;
33
       Real p3[3];
34
35
      algorithm
36
       qi := FS * qin;
37
       qo := qin - qi;
38
       ta := pi * Ri ^ 2 * Lsw / qi;
39
       rin_0 := 0.8 * Ri;
40
       ini := {qin, Ro, Ri, Vo_in, ta, rin_0, FS};
41
       h := ta / 10;
       rho := 1;
42
43
       tol := 10 ^ (-10) * Ri;
44
       (T, X) := Shooting ({0.0, ta}, h, Ri, rin_0, rho, tol, ini, element);
45
       rin := X[1];
46
       p3 := {Lsw, Ro, Ri};
47
       (Vo_LPO,Vo_HPO,qen) := DeWaterer(qin,Vo_in,FS,rin,p3);
48
       Vw LPO := 1 - Vo LPO;
49
       Vw_HPO := 1 - Vo_HPO;
       Edil := Vo LPO * qi / (Vo in * qin);
50
51
       Edis := 1 - ((1 - Vo_LPO) * qi + Vo_HPO * qo) / qin;
      end swirl func3 o;
52
```

A.4.4 Block Dewaterer

This is the model that enables the simulation of Dewaterer giving the inlet conditions to the rest of the functions.

```
1
      model Block_Dewaterer
2
       import Oil_Water_Separation_System;
3
       import SI = Oil_Water_Separation_System.Units;
4
       parameter SI.FlowSplit FS = 0.91 "Dewaterer Flow split qLPO respect qinlet";
5
       parameter String element = "I";
6
7
       SI.Efficiency Edil
8
        "Dilute efficiency, Fraction of oil that is kept in LPO, '1' if we don't have oil in HPO, '0' if
9
      we don't have oil in LPO";
10
       SI.Efficiency Edis
        "Dispersed efficiency, how much of the liquid that exits through the desired outlet; '1'
11
```

```
12
     means HPO is pure water and LPO is pure oil";
13
       SI.OilVolumeFraction qen
14
       "Amount of reentrainment, Some water will be re-
     entrained in the light phase product and viceversa for low Flow Split, caused for the diffe
15
     rence of axial velocities between LPO and HPO";
16
17
18
      Oil Water Separation System.Connector.Flow port flowportindew;
19
      Oil_Water_Separation_System.Connector.Flow_port flowportoutdewH;
20
      Oil Water Separation System. Connector. Flow port flowportoutdewL;
     algorithm
21
22
      (flowportoutdewL.volfracc,flowportoutdewH.volfracc,Edil,Edis,gen) := swirl_func3_o(
23
       flowportindew.volfracc, flowportindew.volflow, FS,element);
24
       flowportoutdewL.volflow := flowportindew.volflow*FS;
25
26
       flowportoutdewH.volflow := flowportindew.volflow*(1-FS);
27
28
     end Block Dewaterer;
```

A.5 Gravity separator

The function returns the bottom and top outlet flow rates and oil volume fractions given the inlet conditions and flow split of the separator.

```
model Block GravitySeparator
1
2
3
      import SI = Oil_Water_Separation_System.Units;
4
      //Constants
5
       constant Real pi = Modelica.Constants.pi;
6
      constant Real g = Modelica.Constants.g n;
7
      //Parameters
8
       parameter SI.Length Lsep = 7.0 "Gravity separator length";
9
       parameter SI.Length R = 1.7 "Separator radio";
10
      parameter SI.Density rhoo = 881 "Oil Density";
       parameter SI.Density rhow = 1064 "Water Density";
11
       parameter SI.Length rd = 60e-6 "Droplet Radio";
12
13
       parameter SI.FlowSplit FS = 0.33 "Flow split of the gravity separator";
14
       parameter SI.VolumeFlowRate qin = 0.006944 "Inlet flow rate";
15
      parameter SI.OilVolumeFraction vo_in = 0.4 "Inlet oil volume fraction";
16
      //Variables
17
      SI.VolumeFlowRate qt "Flow to dewatered";
      SI.VolumeFlowRate qb "Flow to deoiler";
18
```

```
19
       SI.Length Hw "Wall heigth";
20
       SI.DynamicViscosity mum "Emmulsion dynamic viscosity";
21
      SI.Area AHw "cross section area of the lower part of the separator(the circular
22
      segment limited by wall height)";
23
       SI. Velocity vh
        "Horizontal velocity of a droplet moving through the separator under wall height";
24
25
       SI. Velocity vy "Vertical velocities of the droplets caused by the gravitational buoyancy
         forces";
26
27
       SI.Length h "vertical distance, of a droplet entering the separator at the bottom
28
      of the tank, travels during its residence time in the separator";
29
       SI.Length d "Distance between wall and h";
30
      SI.Area Ad "Area of this circular segment";
       SI.OilVolumeFraction vw b "Water volume fraction of the bottom outlet";
31
      SI.OilVolumeFraction vo_b "Oil volume fraction of the bottom outlet";
32
33
       SI.OilVolumeFraction vo t "Oil volume fraction of the top outlet";
      SI.OilVolumeFraction vo_bn "Auxiliar value";
34
35
      //Ports
      Oil_Water_Separation_System.Connector.Flow_port flowporttop "Outlet top part";
36
      Oil Water Separation System. Connector. Flow port flowportbottom
37
38
        "Outlet bottom part";
39
       //equations
40
41
      equation
42
       Hw = 0.75 * 2 * R;
43
      qt = FS * qin;
44
       qb = qin - qt;
45
       mum = 0.47 * vo in ^3 - 0.4 * vo in ^2 + 0.11 * vo in + 0.001;
46
       AHw = R ^2 / 2 * (2 * acos((R - Hw) / R) - sin(2 * acos((R - Hw) / R)));
47
       vh = qb / AHw;
       vv = 2 * (-g) * (rhoo - rhow) * rd ^ 2 / (9 * mum);
48
49
       h = Lsep * (vv / vh);
       d = max(Hw - h, 0) "%if h>Hw -> vo_b=0";
50
51
       Ad = R ^2 / 2 * (2 * acos((R - d) / R) - sin(2 * acos((R - d) / R)));
       vw_b = (AHw - Ad) / AHw + Ad / AHw * (1 - vo_in);
52
53
       vo bn = 1 - vw b;
       vo_t = min((vo_in - vo_bn * (1 - FS)) / FS, 1);
54
55
56
       if vo_t >= 1 then
57
       vo_b = (vo_in - vo_t * FS) / (1 - FS);
58
       else
59
        vo b = vo bn;
60
       end if;
61
62
       //Set the ports
63
       flowporttop.volflow = qt;
```

```
flowportbottom.volflow = qb;
flowporttop.volfracc = vo_t;
flowportbottom.volfracc = vo_b;
end Block_GravitySeparator;
```

A.6 Node

The node connect two inputs mixing them and given the outlet flow rate and oil volumetric fraction.

```
1
     model Node_2inlets_1outlet
2
3
     Oil_Water_Separation_System.Connector.Flow_port flowportinleta;
4
     Oil Water Separation System. Connector. Flow port flowportinletb;
5
     Oil_Water_Separation_System.Connector.Flow_port flowportoutlet;
6
7
     equation
8
      flowportoutlet.volflow = flowportinleta.volflow + flowportinletb.volflow;
9
      flowportoutlet.volfracc = ((flowportinleta.volflow*flowportinleta.volfracc) + (flowportinleta.volfracc)
10
     nletb.volflow*flowportinletb.volfracc))/(flowportinleta.volflow + flowportinletb.volflow);
11
12
     end Node_2inlets_1outlet;
```

A.7 Numerical solvers

A.7.1 Runge-Kutta

The function is a second order Runge-Kutta solver, is called by swirl_func3 and swirl_func3_o to solve the radial inlet position of the droplet.

```
1
      function RK2
2
3
       input Real tspan[:];
4
       input Real yi;
5
       input Real h;
       input Real ini[:];
6
7
       input String element;
8
       output Real t[11];
9
       output Real y[11];
```

```
protected
10
11
       Real d;
12
       Real k1;
13
       Real k2;
14
       Integer i;
15
       //Real aux;
16
17
      algorithm
18
       t := tspan[1]:h:tspan[2];
19
       while t[end] < tspan[2] loop
20
21
        t[end + 1] := tspan[2];
22
       end while;
23
24
       //aux := size(t, 1) - 1;
25
       d := t[2] - t[1];
26
       y[1] := yi;
27
       for i in 1:10 loop
28
        k1 := d * swirl_sep2_o(t[i], y[i], ini, element);
29
        k2 := d * swirl_sep2_o(t[i + 1], y[i] + k1, ini, element);
       // k1 := d*swirl_sep2(t[i],y[i],ini,element) change in dewaterer;
30
31
       // k2 := d*swirl_sep2(t[i + 1],y[i] + k1,ini,element) change in dewaterer;
32
33
        y[i + 1] := y[i] + (k1 + k2) / 2;
34
       end for;
35
36
     end RK2;
```

A.7.2 Shooting method

The shooting method (Newton-Raphson method) solve the value boundary problem finding the droplet inlet position.

```
1
     function shooting
2
3
      import Oil_Water_Separation_System;
4
      input Real tspan[:];
5
      input Real h;
6
      input Real yf;
      input Real gamma0;
7
8
      input Real rho;
9
      input Real tol;
10
      // Ini and element are the parameters needed to call swirl_sep2
11
      input Real ini[:];
```

```
input String element;
12
13
      output Real t[11];
14
      output Real y[11];
15
     protected
16
       Real maxiter;
17
       Real gammanew;
18
       Real iter;
19
       Real fnk;
20
       Real gamma1;
21
       Real dgamma;
22
       Real a;
23
       Real fnka;
24
       Real jacob;
25
       Real init[7];
26
       Real ta[11];
27
       Real ya[11];
28
29
     algorithm
30
      init[:] := ini;
31
      gammanew := gamma0;
32
      iter := 0;
33
      maxiter := 100;
34
      fnk := 10 * yf;
35
36
       while abs(yf - fnk) > tol and iter < maxiter loop
37
        iter := iter + 1;
38
        gamma1 := gammanew;
39
        init[6] := gamma1;
40
        (t,y) := RK2( tspan, gamma1,h,init,element);
41
        fnk := y[end];
42
43
        if gamma1 <> 0 then
         dgamma := -gamma1 / 100;
44
45
        else
46
         dgamma := -0.01;
47
        end if;
48
49
        a := gamma1 + dgamma;
50
        init[6] := a;
51
        (ta,ya) := RK2(tspan, a,h, init,element);
52
        fnka := ya[end];
53
        jacob := (fnka - fnk) / dgamma;
54
        a := gamma1 - dgamma;
55
56
        if jacob == 0 then
```

```
57     gammanew := gamma1 + max(abs(dgamma), abs(1.1 * tol));
58     else
59     gammanew := gamma1 - rho * (1 / jacob) * (fnk - yf);
60     end if;
61
62     end while;
63
64     end shooting;
```