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Máster en Ingeniería Industrial

Development of a model-assisted approach for the requirement-driven design and optimization of a quadrotor unmanned aerial vehicle

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TÍTULO: Development of a model-assisted approach for the requirement-driven design and optimization of a quadrotor unmanned aerial vehicle

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Resumen

Abstract

En este trabajo se ha llevado a cabo un estudio sobre las distintas áreas de aplicación de los cuadricópteros, identificando en cada caso los requerimientos técnicos que deben cumplir. Se ha estudiado la relación entre sus distintos parámetros técnicos y el comportamiento final del cuadricóptero, obteniendo un modelo matemático del sistema. A partir de estas ecuaciones, se ha construido un modelo de simulación en MATLAB/Simulink, que permite medir el impacto que cada uno de los componentes tiene sobre el comportamiento final del sistema. Este modelo se ha aplicado a varios casos de ejemplo para comprobar su validez.
Como aplicación práctica, se ha diseñado y construido un cuadricóptero real. Comparando su comportamiento con los resultados obtenidos de la simulación, se ha demostrado que el modelo matemático desarrollado proporciona una aproximación realista que puede servir de ayuda en el proceso de diseño de este tipo de cuadricópteros.

In this thesis, a research on the different application areas of quadcopters has been made, identifying for each case the set of requirements that must be fitted. The relationships between the flight behavior and the technical parameters of the components have been studied, obtaining as a result the mathematical model of the system. These equations have been used to build a MATLAB/Simulink simulation model, which allows measuring the impact that each parameter has on the final flight behavior. This model has been applied to some example cases, in order to prove its validity.
As a practical approach, a real quadcopter has been designed and built. By comparing its flight behavior with the simulation results, it has been proved that the virtual model provides a realistic approach for the flying features of this kind of drones.

Palabras clave

Keywords

Cuadricóptero, drone, UAV, simulación, Simulink

Quadrotor, drone, UAV, simulation, Simulink

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Master thesis by
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Development of a model-assisted approach for the requirement-driven design and optimization of a quadrotor unmanned aerial vehicle

Unmanned aerial vehicles (UAV) are built in various different designs, one of them being the quadcopter design where the vehicle is driven by four horizontally aligned propellers. Recent technological advances in electronics and the accompanying decline in price of the necessary components opened up new markets for the application of these vehicles. Each of these applications comes with an individual set of requirements. To design a quadcopter suitable for a specific application, its properties and parameters need to be tuned accordingly. However, requirements are often conflicting and the relationship between the parameters and the behaviour of the system are complex. A systematic approach for tuning the parameters is therefore necessary.

The purpose of this thesis is to develop an approach for optimizing a quadcopter for a specific set of requirements. The approach should utilize a mathematical description of the flight characteristics as a function of geometrical and technical properties. The system equations will be transferred to a simulation-model, which can then be used to assess the influence of each individual property on the complience of the requirements

In the course of this project, the following steps should be completed:

- Depicting possible applications for quadcopters, combining them to application clusters and identifying critical requirements
- Researching the technical aspects of building a quadcopter and identifying system parameters that can be influenced to change the flight characteristics of the system
- Creating a Mathematical modeling of the flight characteristics that includes all relevant technical and geometrical parameters
- Transfering the model to MATLAB/Simulink and using simulation to assess the relationship between the parameters and its flight behavior
- Demonstrating the optimization process on selected sample applications
- Practical implementation of an optimized system

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Abstract

Due to the recent technological advances and costs reductions, the use of drones in many different commercial and private fields nowadays is growing exponentially. As the complexity of this kind of aircrafts increases, the necessity of developing new methods and tools to support the designing process arises. This thesis is focused on a particular drone configuration called quadcopter, and its purpose is to obtain a model that allows optimizing the hardware parameters for each particular scenario of use.

A research on the different application areas of quadcopters has been made, identifying for each case the set of requirements that must be fitted. The relationships between the flight behavior and the technical parameters of the components have been studied, obtaining as a result the mathematical model of the system. These equations have been used to build a MATLAB/Simulink simulation model, which allows measuring the impact that each parameter has on the final flight behavior. This model has been applied to some example cases, in order to prove its validity.

As a practical approach, a real quadcopter has been designed and built. By comparing its flight behavior with the simulation results, it has been proved that the virtual model provides a realistic approach for the flying features of this kind of drones.

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Declaration of originality

I hereby declare that the topic of this thesis is not identical to one previously submitted by me for a different examination. Furthermore I declare that neither any part nor the whole of the thesis has been submitted for a degree to any other University or Institution.

I certify that I am the sole author of this thesis and did not use any other than the specified sources. Wherever other people's work has been used, it has properly and accordingly been acknowledged and referenced. This includes text, as well as drawings, sketches, figures and the like.

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Signature

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I. Nomenclature

Variable	**Unit**	**Description**
F_n	N	Thrust force created by each propeller
F_{GRAV}	N	Gravity force
F_{DRAG}	n	Air drag force
m_{QUAD}	kg	Quadcopter total mass
m_{FRAME}	kg	Frame mass
m_{BAT}	kg	Battery mass
$m_{MOT\text{-}PROP}$	kg	Motor-propeller system mass
m_{ELEC}	kg	Electronics mass
m_{EXTRA}	kg	Extra equipment mass
n	rpm	Rotational speed of the motor
Kv	rpm/Volt	Speed constant of a brushless motor
U_{BAT}	Volts	Battery voltage
x_{ESC}	%	ESC regulation percentage
P_{MOTOR}	Watts	Electric power consumed by each motor
I_{MOTOR}	Amps	Current consumed by each motor
ρ_{AIR}	kg/m^3	Air density
ρ_{FRAME}	kg/m^3	Frame material density
A	m^2	Propeller area
A_{eff}	m^2	Quadcopter effective area
v_E	m/s	Speed of the air accelerated by the propeller
P_{AERO}	Watts	Aerodynamic force produced by the propeller
d	m	Propeller diameter
p	inch	Propeller pitch
S	-	Number of cells of a LiPo battery
C	-	Discharge rate of a LiPo battery
$I_{BAT,\ MAX}$	Amps	Maximum current that the battery is able to provide
t_F	minutes	Flight time
D	m	Distance between each motor and the center of the quadcopter
I	kg· m^2	Inertia momentum
α	º	Inclination angle
y_{DISP}	m	Arm vertical displacement
a	m	Arm height
b	m	Arm width
h	m	Altitude over the sea level
T	K	Ambient temperature

II. Figures

III. Tables

IV. Formulas

1. Introduction and objectives

In the early twentieth century the first unmanned aerial vehicles, or UAVs, appeared. These pilotless planes were designed only with military purposes, such dropping explosives autonomously on the target or exploring hostile terrains. Despite the fact that this technology was very expensive and not completely efficient, these aircrafts shown the potential of using this kind of machines to perform tasks that resulted too difficult or risky for the conventional manned planes. (Sifton, 2012)

In the recent years, some technological improvements, such the miniaturization of microelectronics devices and more efficient control programs, have led to the rebirth of the drones industry. In addition, as the production costs have fallen, drones have become more affordable and their popularity is increasing exponentially. Not in vain, its use is spreading in many commercial and private fields, which have nothing in common with the army.

As the use of drones increases, they tend to be more complex and the necessity of optimize their parameters appears. Depending on the task that it is intended to develop, each drone has different behavioral requirements that must fit. Therefore, in order to achieve the optimum performance, the technical and geometrical parameters of every component have to be carefully tuned for every particular situation. The necessity of finding a relationship between the drone parameters and its flying features arises.

To study this problem, it is necessary to set a starting point, as there are several drone configurations and each one has different characteristics. One of the possible drone designs is the quadcopter or quadrotor, which obtain its thrust force from the movement of four propellers vertically aligned to four motors. This paper is focused on the study of this kind of drones.

The purpose of this thesis is to provide a realistic approach that relates the technical and geometrical parameters of the quadcopter components with its behavior and flight features. A mathematical description is made in order to build a MATLAB/Simulink model, which will allow simulating the quadcopter behavior and measuring how the variation of each parameter affects to the system response. In addition, to prove the validity of the model, it is used to support the designing process of a real quadcopter that will be built.

This project is divided into six chapters, beginning with “Introduction and objectives” as the first one.

The second chapter, “State of the art”, introduces a research on the quadcopters situation nowadays. This research includes the most important quadcopters manufacturing companies and the most popular quadcopters on the market, in terms of sales. In addition, the main scenarios of use where quadcopters are used are listed and explained. The set of requirements matched to each scenario is also described.

On the third chapter, “Quadcopter basic fundaments”, a theoretical introduction of the quadcopter technical aspects is done. These aspects include the movement equations, the basic components and the most significant features that are considered as behavioral indicators.

On the fourth chapter, “Mathematical approach and virtual model”, the relationships between the quadcopter behavior and the features of its components are obtained. These relationships are used to build a MATLAB/Simulink model, which is applied to some example cases in order to check the validity of the provided results.

On the fifth chapter, “Design and construction of a real quadcopter”, the development process of the quadcopter built is explained. Its technical parameters, such as the components features, the electric schematic and the 3D-printed pieces blueprints are presented and justified. The control loop is also implemented into a MATLAB/Simulink model in order to simulate it and check its proper performance.

On the sixth chapter, “Conclusions”, the obtained results and the conclusions drawn in each chapter are presented as a final summary.

The seventh chapter, “Bibliography”, is dedicated to collect all the bibliography used on this thesis.

On the eighth chapter, “Annexes”, additional information of some technical aspects of the thesis can be found.

2. State of the art

In the 20th Century, military research led to the apparition of the firsts Unmanned Aerial Vehicles (UAVs). These aircrafts had only military purposes and their technology was not very efficient, but they showed high potential for situations where the use of man-piloted planes was too difficult or risky.

Nowadays, due to some technological advances, like the miniaturization of electronic components and the improvements in programming and autopilot systems, those primitive pilotless vehicles have evolved into autonomous flying robots, able to execute successfully more complex tasks. In addition, as the production costs have fallen, they have become more affordable and their popularity is growing exponentially, not only for professional purposes but also for particular users. (Murphy, 2016)

As a result, in the recent years, some companies that have noticed the potential of this fast-growing industry have emerged, and more resources are being invested to research this topic and develop new techniques and models. According the IFI CLAIMS database, the number of patents related to drones within the last years presents the evolution shown on the Figure 1. (Carrascosa, 2014)

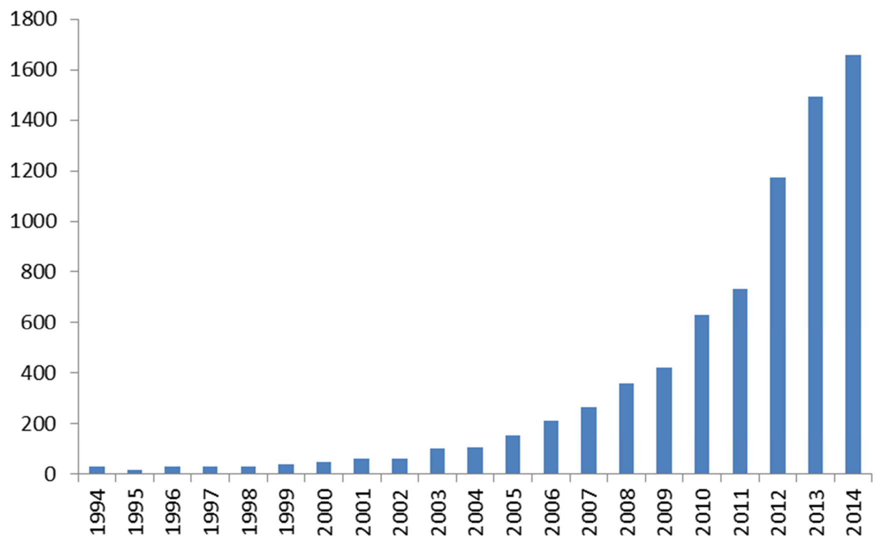


Figure 1: Evolution of the number of patents in the drones industry

The three biggest drones manufacturing companies, in terms of annual revenues in the year 2015, are DJI Innovations, Parrot and 3D Robotics. A brief introduction of these companies and their highlighted products is done on the following paragraphs. (www.airdronecraze.com)

<u>DJI Innovations</u>

Founded in 2006 by Frank Wang, this Chinese company claims to be the sales leader in the recreational drones market, with revenues close to $1.0 Billion in 2015.

Its popularity comes mainly from the quadcopter series “Phantom”. It was released in 2013 with the model Phantom 1, and gathers a group of quadcopters created for high quality aerial photography and cinematography. Despite the good performance of these drones, the Phantom is not only targeted toward commercial purposes, but also for amateur and domestic use. The last Phantom quadcopter, that was launch on March 2016, is the Phantom 4, which can be seen on the Figure 2.

Figure 2: DIJ Phantom 4 quadcopter

This company not only develops drones, but also accessories such as flying platforms, flight controllers for multi-rotors, aerial and handheld gimbals and ground stations. Their products are suitable for industrial, professional or amateur purposes. (www.dji.com)

Parrot

This company was founded in 1994 in France. Traditionally, it has been focused on the design and production of wireless devices for mobile phones and automobiles, such as Bluetooth hand free kits.

In 2010 Parrot launched its first quadcopter, the Parrot AR-Drone, a mid-range hobby drone with integrated FPV (first person view) system and controlled through a smartphone app. After around half a million units were sold, in 2012 it was substituted by its improved version, the Parrot AR-Drone2.0, which can be seen on the Figure 3.

Figure 3: Parrot AR-Drone 2.0

Despite the lack of previous experience that Parrot had on the drones industry, these two quadcopters have seized a large part of the consumer drones market. (www.parrot.com)

<u>3D Robotics</u>

It was founded in 2009 in Berkeley, USA. They design and manufacture commercial and recreational drones. In May 2015 they released a quadcopter called SOLO, which is supposed to be the world's first "smart drone" because of its use simplicity. This quadcopter is aimed on the aerial photography market, and can be used either for professional or amateur purposes. Its appearance can be seen on the Figure 4. (www.3dr.com)

Figure 4: 3DR SOLO Quadcopter

Some market researches and sales projections made by the International Association for Unmanned Vehicle Systems, suggest that the consumer drone market will reach $4 Billion revenue by the year 2020, being this amount divided as it is shown on the Figure 5. (www.picopter.org)

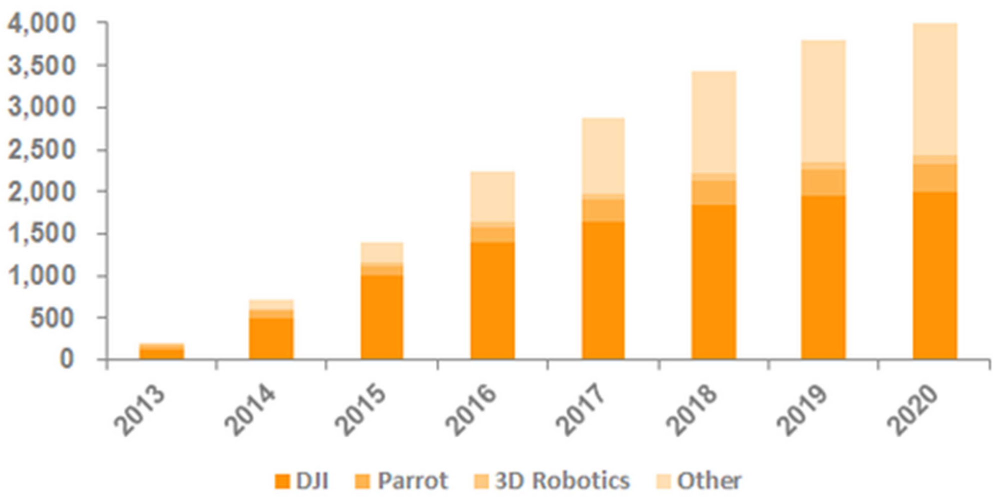


Figure 5: Annual sales revenues, in millions of dollars

In addition, this industry is expected to have a great impact on the economy, by creating more than 100.000 new jobs in the decade to come. (DroneCompanies, 2016)

2.1 Scenarios of use

As the industry grows and the drones become more reliable and affordable, the application of these flying robots increases rapidly in numerous fields. (Craigi, 2015)

Some of the scenarios that have already implemented this technology are listed and described below:

Commercial photography and video

The use of drones allows capturing quickly and inexpensively aerial images or videos that could not be easily taken in other ways. There is a vast range of drones on the market aimed on this use, sorted by the quality of the filming equipment and the flying features.

This kind of quadcopters must be easy to use, as they are not targeted exclusively to the professional market. A long flight time is desirable, mainly if the quadcopter is intended to film video. A good stability is also mandatory, as it has to deal with the wind effects without changing excessively its position.

Logistics

Drones can also be used to deliver packages. Given the destination coordinates, it can reach fixed point thanks to its GPS assisted flight controller. The delivering area depends on the quadcopter speed and on the battery capacity, which must be high in order to allow the quadcopter to cover long distances.

As an example, the largest Internet-based retailer in the world, Amazon, has recently announced that is working on a new project called Amazon Prime Air, a future service that will deliver packages with a weight of up to three kilograms in 30 minutes or less using drones. This service is still in the testing phase, but it is expected to be ready soon. (www.amazon.com)

Apart from the flight time, other requirement that this kind of quadcopters must fit is the capability to lift the packages weight. The correct motors must be chosen in order to provide thrust force enough.

Agriculture

Drones have also a great potential in the agriculture fields. If implemented properly, farmers are able to use equipped cameras to get a better glimpse of their yields or even to spray their crops.

Depending on the intended use, the required features of this kind of quadcopters may change. For example, the requirements for a drone used to take pictures of the crops match with the requirements of a commercial photography quadcopter, while for spraying the crops additional equipment and more strict features may be necessary. (www.iuavs.com)

Entertainment

As a result of the cheaper prices and the increasing ease of operation, the number of non-professional customers that use the drones only with leisure purposes has increased in the recent years. There is a wide range of products aimed on this market sector, being the price one of the most valuable characteristics. These quadcopters tend to be small-sized, light and not very powerful, as they can be used even by kids.

Communication

Researchers from the University of North Texas have recently presented an UAV capable of supplying Wi-Fi signal to disaster-struck areas with a range of up to 5 km. The damage to the communications infrastructure that often accompanies the destruction of buildings in storm-ravaged areas makes it difficult for response teams to communicate efficiently, and to keep disaster victims informed. This new kind of drones could help to avoid this problem. A mandatory requirement of this kind of drones is a long flight time. (Lavars, 2014)

Emergencies and natural disasters

Search and rescue in rough terrain or a disaster area is always a challenging task. Using a drone makes finding victims faster and easier than in the conventional way, which can mean the difference between life and death. Drones can provide real-time high definition video footage in areas hard hit by natural disasters like flooding, hurricanes, tornadoes, avalanches and earthquakes that cause large areas to be almost impossible and too dangerous to reach by land.

In addition, in an emergency situation, a drone can deliver lifesaving first aid while waiting for the ambulance, such as defibrillator for a heart attack. Larger UAVs can also be used to safely deliver medicine, food and water to otherwise unreachable victims.

In this case, the most important requirements are long flight times and a fast speeds, as well as an accurate GPS system, in order to reach precisely the prefixed points.

Security and surveillance

UAVs offer law enforcement agencies a bird's eye view of crime and disaster scenes that they may not otherwise be able to get. Police drones make possible for instance to track a fugitive on the run in areas where a police helicopter cannot operate, such as low altitudes, or situations where the police want the drone to keep unnoticed.

The requirements in this case are a long flight time, capability to be remote controlled from long distances and speed enough to take part in cars chases.

Hostile environments research

One of the most significant benefits of unmanned aerial vehicles is their ability to easily get to places where it would be costly or dangerous for humans to go. Active volcanoes research or radioactivity-contaminated areas can be examples of these harmful scenarios.

However, these quadcopters need to be properly enhanced in order to resist extreme conditions. Apart from this, depending on the purpose of the researches, its requirements may vary.

Industrial facilities inspections

Inspecting radio and telecommunications towers or wind turbines is a very dangerous work. From 2004 to 2012, 95 workers died climbing these towers and 17 manned aircraft performing aerial photography crashed, killing 19 people. Using drones for this task will completely erase this kind of accidents. (Calderone, 2016)

The equipped cameras of these quadcopters must be able to provide good quality images; otherwise its work could be useless. The stability is also an important point to have into account in order to avoid possible crashes into the towers.

Prevention of forest fires

In highly risky areas, like forests in summer, drones equipped with thermal cameras can detect fire risk even before it starts, making the firefighters' reaction much faster. (www.firerescue1.com)

This kind of drones must be built with temperature resistant materials and allow long flight times in order to cover big areas.

Sports

In August 2016, the first drone racing championship was held in New York. The pilots fly their quadcopters through a three-dimensional course at speeds up to 190 km/h. Pilots steer from the point of view of the drone by wearing First Person View (FPV) goggles that display a live image transmitted by an onboard camera. (www.droneworlds.com)

These drones must have very powerful motors that allow them to reach speeds of up to 150 km/h.

3. Quadcopter basic fundaments

A quadcopter is a particular design of UAV that obtains the propulsion force from the rotation speed of four propellers. These propellers are driven by four electric motors, which are horizontally aligned directly below or above them. The motors are mounted on a platform called frame, which have four arms settled in a cross shape. Each propeller generates a single thrust force that depends on its angular speed, and the combination of these forces gives as a result the total force vector and the momentums experimented by the system. Therefore, by changing the speed that each motor transmits to its propeller, the control of the system can be achieved. In order to compensate the momentums on the vertical axis, there are two motors spinning clockwise and two spinning counterclockwise.

One of the most important advantages that the quadcopters have, comparing to other UAVs configurations, is the capability of execute vertical take-offs and landings. Once it is in the air, it can stay hovering or change its position in any of the three dimensions. This feature makes quadcopters very flexible aircrafts, able to flight in challenging environments or situations.

Depending on the control system, it is possible to differ between two types of quadcopters. The first group includes the remotely controlled quadcopters. In this case a pilot on the ground controls the quadcopter movements through a wireless transmitter. The quadcopter must have a receiver device to collect these control signals. The second group refers to the quadcopters able to fly autonomously. Working in conjunction with GPS systems, this kind of aircrafts have modern flight controllers able to reach prefixed points marked in a map and perform routine tasks like taking-off or landing without human interaction.

3.1 Movement equations

The purpose of this section is to calculate the relationships between the thrust forces/momentums created by each propeller and the quadcopter different movements.

The spatial orientation can be described as the combination of three angles: roll, pitch and yaw. The roll angle (φ) represents the rotation around the front-to-back axis, the pitch angle (θ) represents the rotation around the side-to-side axis and the yaw angle (ψ) represents the rotation around the vertical axis. To measure these angles it is necessary to define the axes of the reference system, according to one of the two possible approaches.

Despite the fact that every quadcopter has four motors and similar shape, depending on the chosen reference system, it is possible to differ between two configurations: “X” Quadcopters and “+” Quadcopters. The difference between these two groups is the main direction of displacement relative to the frame geometry, as can be seen on the Figure 6.

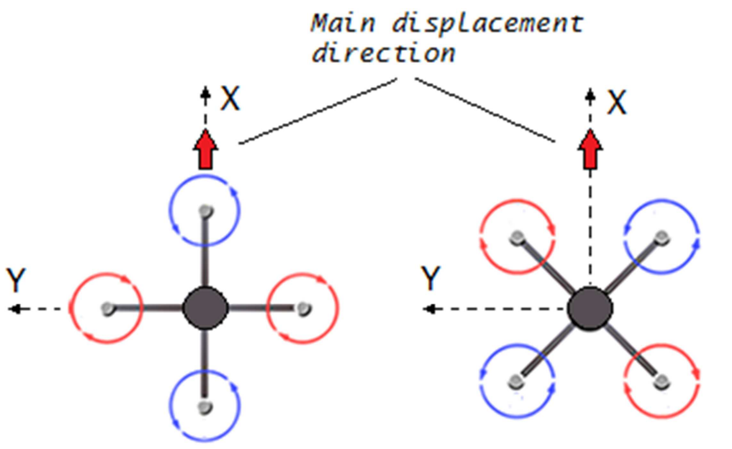


Figure 6: Left: "+" configuration; Right: "X" configuration

Both configurations present the same perform features, but the movement equations for each system must be differently calculated.

<u>"+" Configuration</u>

The forces and momentums produced by each motor and the geometrical parameters involved in this situation are represented on the Figure 7.

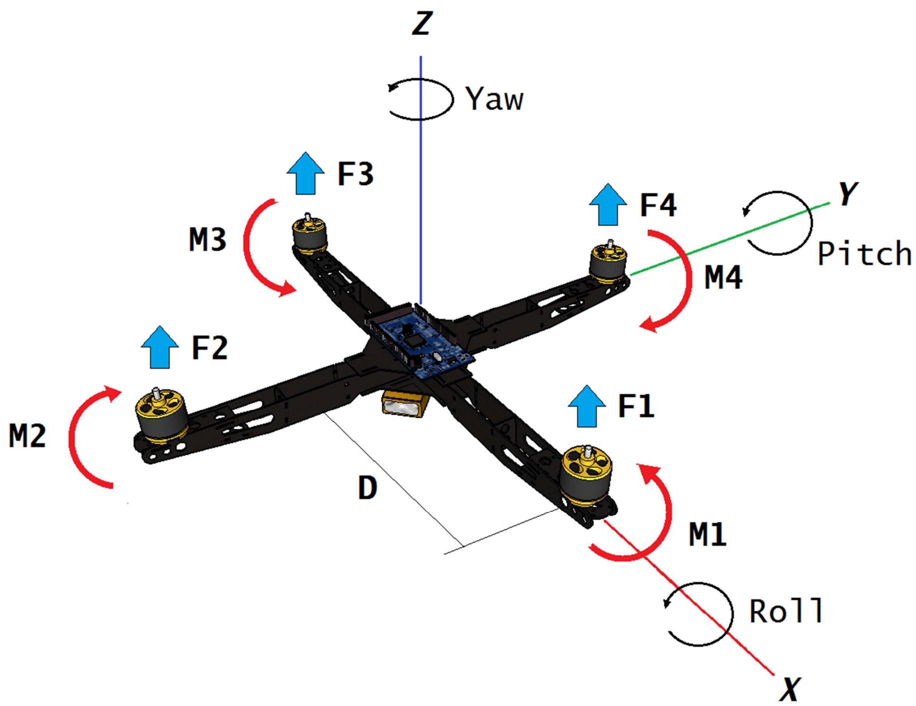


Figure 7: Forces and momentums in a "+" quadcopter

According to the angular momentum conservation law, it is possible to obtain the following equations:

$$\frac{d\bar{L}}{dt} = I \cdot \frac{dw}{dt} = M_{ext} \tag{3.1}$$

$$\frac{dw}{dt} = \frac{M_{ext}}{I} \tag{3.2}$$

where “w” is the angular speed, “M_{ext}” is the sum of every exterior momentum acting on the system and “I” is the inertia momentum in the rotation axis

By applying these expressions to the “+” quadcopter system, the equations that relate each orientation angle with the forces and momentums created by the motors are obtained:

Roll:

$$\left(\frac{dw}{dt}\right)_{Axis\ X} = \ddot{\phi} = \frac{F_4 \cdot D - F_2 \cdot D}{I_{xx}} = \frac{(F_4 - F_2) \cdot D}{I_{xx}} \tag{3.3}$$

Pitch:

$$\left(\frac{dw}{dt}\right)_{Axis\ Y} = \ddot{\theta} = \frac{F_1 \cdot D - F_3 \cdot D}{I_{yy}} = \frac{(F_1 - F_3) \cdot D}{I_{yy}} \tag{3.4}$$

Yaw:

$$\left(\frac{dw}{dt}\right)_{Axis\ Z} = \ddot{\psi} = \frac{M_1 + M_3 - M_2 - M_4}{I_{zz}} \tag{3.5}$$

<u>“X” Configuration</u>

The forces and momentums produced by each motor and the geometrical parameters involved in this situation are represented on the Figure 8.

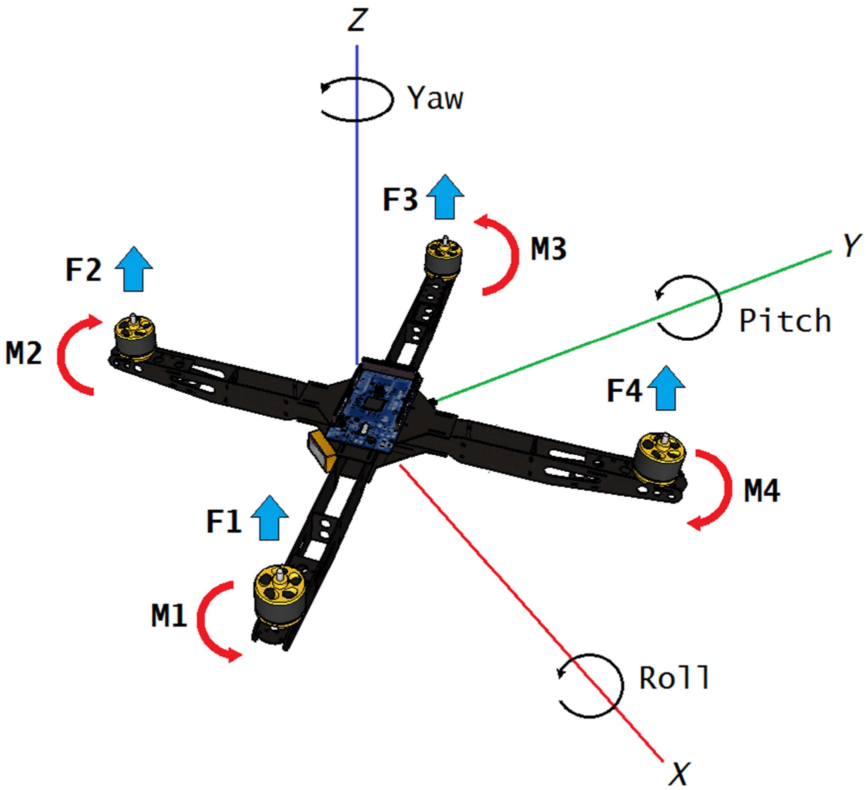


Figure 8: Forces and momentums in a "X" quadcopter

In this case, it has to be taken into consideration that the distances between each motor and the axes X and Y, are the distances of the arm "D" multiplied by two compensation factors. As the angle between each arm is 90º, these compensation factors are equal to cos45º and sen45º.

By applying the angular momentum conservation law equations, the expressions for the angles result in:

Roll:

$$\left(\frac{dw}{dt}\right)_{Axis\ X} = \ddot{\phi} = \frac{(F_4 + F_3 - F_1 - F_2) \cdot D \cdot cos45º}{I_{xx}} \tag{3.6}$$

Pitch:

$$\left(\frac{dw}{dt}\right)_{Axis\ Y} = \ddot{\theta} = \frac{(F_4 + F_1 - F_3 - F_2) \cdot D \cdot sen45º}{I_{yy}} \tag{3.7}$$

Yaw:

$$\left(\frac{dw}{dt}\right)_{Axis\ Z} = \ddot{\psi} = \frac{M_1 + M_3 - M_2 - M_4}{I_{zz}} \tag{3.8}$$

About the displacement movements, on the Figure 9 the equilibrium of forces of a quadcopter is represented. The combined forces that the propellers generate, give as a result the total thrust force of the system. This force, depending on the inclination angle, can be discomposed into two terms: vertical thrust and horizontal thrust. The vertical thrust must compensate the gravity force, while the horizontal thrust has to deal with the drag force caused by the air resistance.

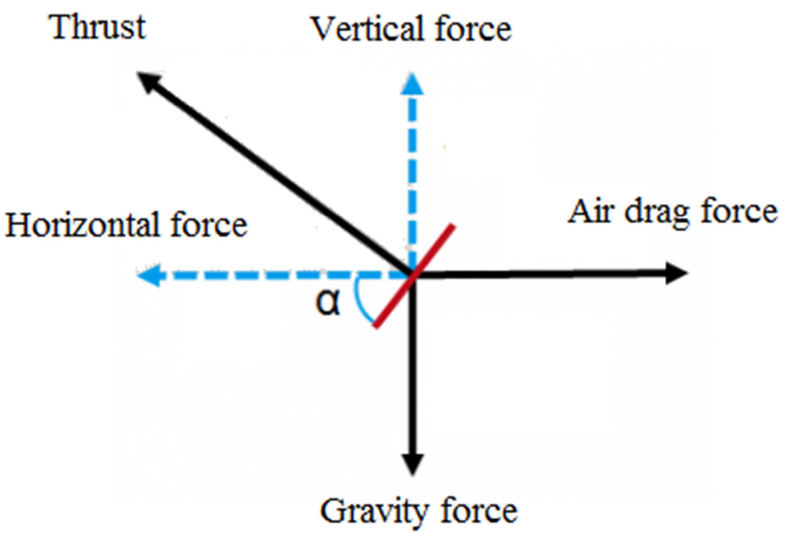


Figure 9: Forces involved on the quadcopter equilibrium

There are two particular situations where the quadcopter is only moving in one dimension: quadcopter gaining altitude and quadcopter moving forward. These particular situations are studied on the following paragraphs.

Quadcopter gaining altitude without lateral movements

In this situation all the generated thrust is applied on the vertical direction. The opposing forces to the movement are the gravity and the air resistance. According to Newton's second law:

$$T - F_{GRAV} - F_{DRAG} = m \cdot a \tag{3.9}$$

Being the opposing forces:

$$F_{GRAV} = m \cdot g \tag{3.10}$$

$$F_{DRAG} = \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A \cdot v^2 \tag{3.11}$$

where "m" is the quadcopter mass, "g" the earth gravity acceleration, "ρ_{AIR}" is the air density, "C_D" is the drag coefficient, "A" is the effective area of the quadcopter and "v" is the quadcopter vertical speed

By combining these expressions, the formula that describes this situation is obtained:

$$T - m \cdot g - \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A \cdot v^2 = m \cdot \dot{v} \tag{3.12}$$

Quadcopter flying forward without altitude variation

In this case the vertical thrust must compensate the gravity force in order to keep the altitude constant. The horizontal force makes the quadcopter to move forward. According to Newton's second law:

$$T_H - F_{DRAG} = m \cdot a \tag{3.13}$$

$$F \cdot sen\,\alpha - \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A_{eff} \cdot v^2 = m \cdot \dot{v} \tag{3.14}$$

The effective area in this case is a function of the pitch angle α and must be calculated as the vertical projection of the quadcopter top area.

$$A_{eff} = A \cdot sen\,\alpha \tag{3.15}$$

The vertical thrust must compensate the gravity force, then:

$$F \cdot cos\,\alpha = m \cdot g \tag{3.16}$$

By combining these expressions, the formula that describes this situation is obtained:

$$m \cdot g \cdot tan\,\alpha - \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A \cdot sen\,\alpha \cdot v^2 = m \cdot \dot{v} \tag{3.17}$$

3.2 Basic components

The features and equipment that a quadcopter requires depend mainly on the tasks that it is intended to perform. Quadcopters designed for more complex tasks may need more sophisticated equipment or additional devices. However, there are some components that are essential for the proper perform of every quadcopter. These components are listed and briefly explained below. (Benson, 2014)

Frame

It is the quadcopter chassis and the base where all the other components are assembled. It must be strong enough to resist the impacts and vibrations that the quadcopter may suffer, but also as light as possible in order to reduce the weight of the system.

There is a huge range of sizes and weights, but most of them have the same “X” appearance. For quadcopters with additional requirements, such big cameras or voluminous equipment, other configurations can be considered, but that is an exception.

On the Table 1, some materials commonly used to build quadcopter frames are listed and analyzed. The designer must consider the quadcopter requirements and choose the most appropriate for each particular case.

Material	Advantages	Disadvantages
Wood	Inexpensive material Easy parts replacement	Doesn't resist high temperatures Weak woods may twist and bend
Plastic	Suitable for 3D printing Lightness	In large sizes tends to flex Not very rigid
Aluminum	Strength Long durability	Heavyweight
Carbon fiber	Lightness Strength	Long mounting process Expensive material

Table 1: Advantages and disadvantages of different frame materials

Motors

They are fixed to the frame and its only purpose is to spin the propellers to generate thrust.

Brushless motors are commonly used for quadcopters and multirotors, as they are more powerful and effective than other conventional DC motors. These brushless motors contain several electromagnets, or coils, connected together into three main sections. By activating and deactivating these sections at very specific times, it is possible to use the magnetic force created by the stator to spin the rotor, which contains permanent magnets. (Zhao, 2011) A basic scheme can be seen on the Figure 10.

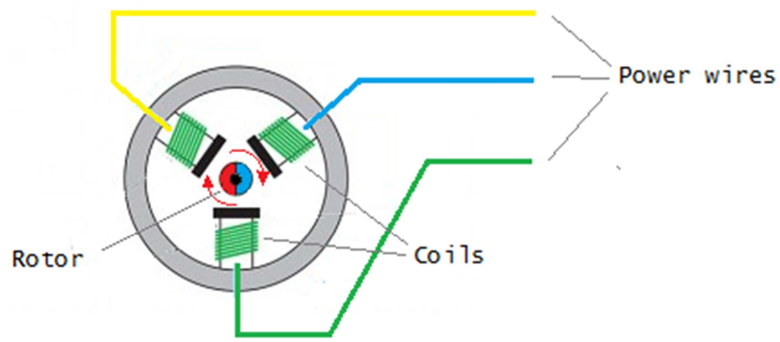


Figure 10: Brushless motor schematic

Generating the necessary pulses to control the electromagnets precisely used to be a challenging task a few years ago. Nowadays, because of the improvements on microelectronics, little devices called Electronics Speed Controllers (ESCs) that perform this control have surged.

The rotational speed of a brushless motor is proportional to its operation voltage. The constant of proportionality "Kv" describes the relationship between these parameters. It is usually provided by the manufacturer in rpm/V. Higher Kv ratings allow the motor to reach faster speeds, but this also lead to more power consumption and therefore a faster exhaustion of the battery.

The motor maximum power suggests the maximum torque that it is able to generate. More powerful motors allow creating stronger thrust forces and lifting heavier loads, but their weight and size increase as well.

Electronic speed controllers (ESCs)

These devices are used to vary the speed of electric motors. Its design varies depending on the motors that are intended to control: brushed or brushless. The ESCs for brushless motors contain a circuit that, from the battery DC power, creates a high frequency tri-phase AC power output. The voltage applied to the motor is adjusted by pulse width modulation, according to the percentage that the flight controller requests. The ESC must be able to handle the maximum current that the motor might consume. (Benson, 2014)

It is also important to notice that the quadcopter motors need a very fast response that cannot be provided by every ESC. Different applications of brushless motors may need different controller features. It is important to ensure that the controller chosen for the quadcopter is able to work at the necessary frequency.

Propellers

A propeller is a type of fan that converts the rotational motion of the motor into propulsion force. The airflow is accelerated behind the propeller because of the jump of pressure produced between the forward and rear surfaces of the blades. For quadcopters, the tendency is to use 2-blades propellers, but it is also possible to find exceptions that use 3-blades propellers.

The produced thrust force depends on the motor rotational speed and on the propeller geometry. The parameters that define the propeller geometry are the diameter and a characteristic distance called pitch, which describes the travelling distance of the propeller after a single revolution. (Benson, 2014)

There is a huge variety of diameters and pitches, as well as materials such as plastic, reinforced plastic, carbon fiber and wood. Different propellers provide different flight parameters and the designer must choose them according to the quadcopter requirements.

Battery

In quadcopters, and in every multi-rotor drone, lithium polymer (LiPo) batteries are almost exclusively used. The features that make this kind of batteries appropriates for this application are their large capacity, that allows them to hold lots of energy in a small package, their light weight and its flexibility in terms of size and shape and their capability of provide high discharging rates.

A LiPo battery is the result of some individual cells connected in series, being the nomenclature 2S for two-cell batteries, 3S for three-cell batteries and so on. Each single cell has a nominal voltage of 3,7V, so the battery nominal voltage is calculated as 3,7 times the number of cells. In a quadcopter, this voltage determines how fast the motors are able to spin. The charging process has to be done through devices specially designed for this purpose, called LiPo chargers. These chargers ensure that every cell is charged at the same time and with the same voltage, which is essential for the correct performance of the battery. A basic scheme of a 3S LiPo battery can be seen on the Figure 11.

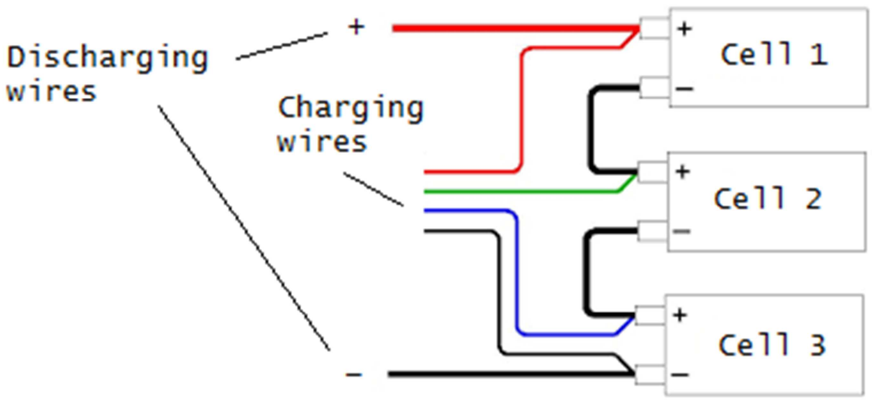


Figure 11: LiPo battery cells schematic

Because of its charging necessities, LiPo batteries have two different connectors, one for charging and one for discharging. Its exterior appearance can be seen on the Figure 12.

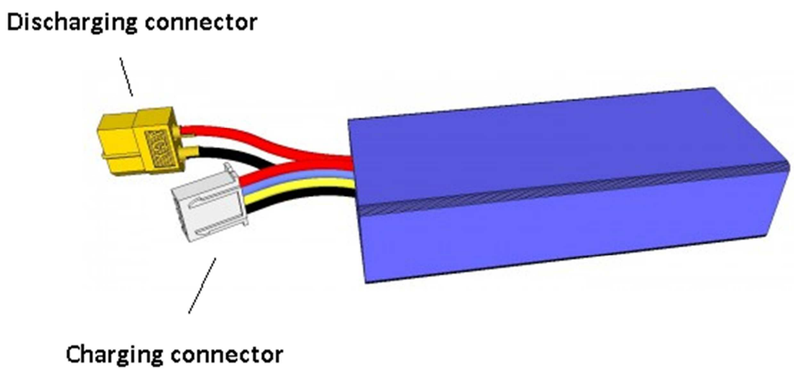


Figure 12: LiPo battery exterior appearance

Other significant feature is the capacity. This parameter measures the amount of power that the battery is able to hold. Higher capacities allow longer times of use until the battery exhaustion, but also increase its weight. However, there is also a maximum amount of power that each battery is able to provide per second in safety conditions. The discharge rating measures how fast it is possible to discharge the battery without harming it and determines the maximum current that must be requested by the motors.

It can be appreciated that the battery parameters are strongly interconnected with the motors performance and the quadcopter behavior, so it is essential to choose the correct battery depending on the flight necessities.

Concerning safety, LiPo batteries contain pressurized hydrogen gas and have a tendency to burn or explode when a failure occurs. That is why it is suggested to keep them in a LiPo safe bag when charging and protect them from possible impacts on the quadcopter frame. (Benson, 2014)

Flight Controller

The flight controller is composed by a programmable microcontroller and a control program. Its task is to collect all the data provided by the sensors, make the necessary decisions considering the quadcopter aim and execute the necessary adjustments in the system parameters in order to obtain the desired response. It is necessary to configure the flight controller depending on the quadcopter physical features and its intended purpose.

In remote controlled quadcopters, the microcontroller set the necessary movements to rid the difference between the target signals received by the pilot and the quadcopter real orientation received by the sensors. In autonomous quadcopters it is necessary to program prior to the flight the tasks to develop and the coordinates to reach. This kind of quadcopters tend to need more equipment, such GPS systems, and more complex control programs able to calculate trajectories between different spatial points.

Sensors

These devices provide the flight controller the information about the quadcopter environment, position or orientation. Depending on the intended use and the requirements of each quadcopter, the combination of sensors that it needs may vary. The most common sensors used in quadcopters nowadays are listed and explained on the following paragraphs. (Hobden, 2015)

Accelerometer: This sensor measures linear accelerations in up to three axes. As it can detect the gravity force direction, it allows knowing where the ground is and keeping the horizontal stability.

Gyroscope: This sensor measures the angular acceleration in up to three axes. It does not measure the absolute orientation angles but it is possible to obtain them indirectly. This device allows the quadcopter flying stable.

Inertia Measurement Unit (IMU): This device integrates an accelerometer sensor and a gyroscope sensor in a single board.

Magnetometer: This sensor, also known as electronic magnetic compass, measures the earth magnetic field and determines the north direction.

Barometer: This sensor measures the quadcopter altitude. It is based on the relationship between the height level over the sea and the air pressure.

GPS: The Global Positioning System use the signals sent by some satellites orbiting around the Earth to determine precisely the quadcopter geographic position. This system allows the quadcopter to reach specific coordinates previously fixed or to go back autonomously to the starting point once the flight has finished.

Altitude sensor: Sometimes barometer sensors are not enough to measure precisely the quadcopter altitude, obstacles like mountains or buildings cannot be detected with them. To avoid this problem other altitude sensors, based on ultrasonic, laser or infrared technology, are used.

Other equipment

Beside the components above mentioned, the quadcopter equipment may include also other electronic devices such different types of cameras, first person view systems, antennas to transmit data in real time or even claws or hooks to pick up objects.

3.3 Behavioral features

To define precisely the flight features of a quadcopter, it is necessary to identify significant indicators that can be objectively measured. These parameters provide the set of requirements that must be fixed in the designing process of every quadcopter depending on its intended use. In this section, these indicators are identified and briefly explained.

Stability: This parameter measures the capability of the quadcopter to resist the external disturbances such as the wind force without changing its position or suffering vibrations. The stability is important in tasks where motion accuracy is needed, such as aerial photography.

Maximum Speed: This parameter defines how fast the quadcopter is able to fly. It is necessary to differ between vertical speed and horizontal speed, as they may not be the same. Higher speeds contribute also to faster responses of the quadcopter, being this parameter essential when the quadcopter is intended to perform acrobatic movements.

Maximum load to lift: This parameter defines the maximum weight that is possible to add to the quadcopter without affecting its flying capability. This value is very important when the quadcopter is intended to perform delivering tasks.

Flight time: This parameter provides the limit time that the quadcopter can fly without recharging its battery. The longer the flight time, the largest distances the quadcopter is able to cover.

Control mode: This parameter defines if the quadcopter is able to fly autonomously, or if contrary, it needs to be constantly controlled by a pilot. Autonomous quadcopters tend to be more complex but also are able to develop more challenging tasks.

Resistance to hostile environments: Quadcopters designed to fly in harmful environments where their components may be deteriorated, need special treatments or protecting material in order to enhance their resistance against the adverse conditions. As an example, quadcopters used in the prevention and fight against fire must be able to resist high temperatures.

All these parameters are strongly interconnected with the technical and geometrical features of the quadcopter components. In the next chapter, these relationships are studied.

4. Mathematical approach and virtual model

The quadcopter flight behavior is determined by the technical parameters of the chosen components and the frame geometry. For each application, it is necessary to optimize these parameters in order to obtain the most appropriate response that fits the set of requirements. In this chapter, the relationships between these parameters and the quadcopter flight characteristics are studied. As the obtained equations are strongly interconnected, a MATLAB/Simulink model is built to simplify the optimization process. The resulting model is applied to some example cases in order to check its validity.

4.1 Calculation of the flight parameters

In this section, the features of the different components of the quadcopter are studied in order to obtain the equations that relate their technical parameters with the flight behavior. Assuming a hovering situation, the force that each motor must transmit to its propeller, as well as the power consumed, is calculated. In this situation, the maximum flight time can be obtained. The maximum vertical and horizontal speeds that the quadcopter is able to reach are also calculated. Finally, the stability is described in terms of inertia momentums.

4.1.1 Force that each propeller must generate in a hovering situation

As starting point, it is assumed a simplified shape of the frame, which dimensions are defined by the distances "D", "a" and "b". This structure must hold the effects of the thrust forces and momentums created by each propeller and the total weight of the system. It has been assumed that motors 1 and 3 spin counterclockwise, therefore their momentums have opposite sense to motors 2 and 4. This situation is represented on the Figure 13.

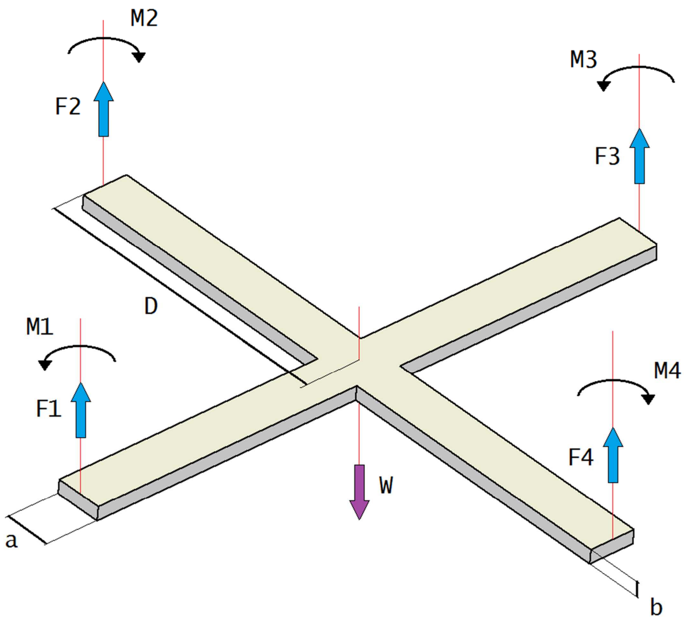


Figure 13: Forces and momentums held by the frame

According to Newton's second law, the quadcopter acceleration in the vertical direction can be described as:

$$a = \frac{\sum F}{m_{QUAD}} = \frac{F_1 + F_2 + F_3 + F_4 - W}{m_{QUAD}} \tag{4.1}$$

In a hovering situation the quadcopter acceleration is equal to zero and the thrust created by the propellers must compensate the gravitational force. Assuming that each rotor creates the same vertical thrust, this force can be described as:

$$F = \frac{W}{4} = \frac{m_{QUAD} \cdot g}{4} \tag{4.2}$$

where "g" is the gravity acceleration

It can be appreciated that, in order to calculate the single force that each propeller must generate to compensate the gravity effect, it is necessary to calculate first the total mass of the system.

The total mass of a quadcopter is obtained by summing the masses of all the components that integrate it. These components are the frame, the motors and propellers, the LiPo battery, the electronic installation and other possible devices that could be equipped.

$$m_{QUAD} = m_{FRAME} + 4 \cdot m_{MOTOR} + m_{BATTERY} + m_{ELEC} + m_{EXTRA} \tag{4.3}$$

The calculation of these masses is explained on the following paragraphs.

Frame mass

Assuming the volume of the simplified shape, its mass can be calculated as the sum of the four arms masses:

$$m_{ARM} = \rho_{FRAME} \cdot V_{ARM} = \rho_{FRAME} \cdot (D \cdot a \cdot b) \tag{4.4}$$

$$m_{FRAME} = 4 \cdot m_{ARM} = 4 \cdot \rho_{FRAME} \cdot (D \cdot a \cdot b) \tag{4.5}$$

In order to set the distance "D" as the only geometrical variable, it is necessary to find a relationship between this distance and the parameters "a" and "b". As the distance between each motor and the quadcopter center increases, the width and height of the arms needs to be adjusted in order to avoid vibrations or excessive bending.

When a thrust force is applied by the propeller, the extreme is slightly displaced in the vertical direction, as can be seen on the Figure 14.

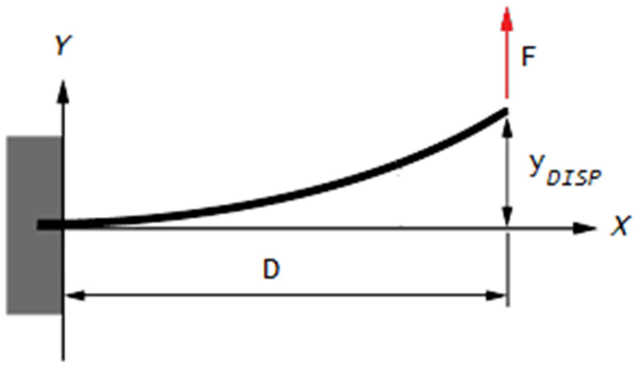


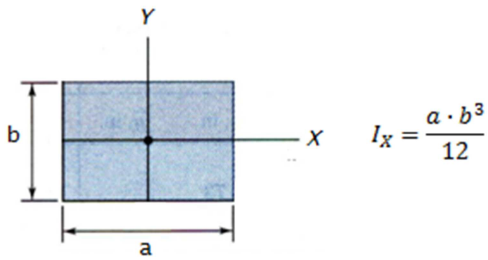
Figure 14: Vertical displacement of the quadcopter arms

In order to relate the parameters "a" and "b" with the distance "D", it is necessary to set a maximum vertical displacement that can be considered acceptable. Assuming that each arm acts as a fixed beam, this displacement can be calculated through the Euler-Bernoulli equation. (www.sc.ehu.es)

$$y_{DISP} = \frac{D^3 \cdot F}{3 \cdot I \cdot Y} \tag{4.6}$$

where "I" is the inertia momentum in the transverse axis and "Y" is the Young modulus of the frame material

As the section of the arm is rectangular, the formula used to obtain the inertia momentum is:



It is necessary to estimate a proportion between the height and width of the rectangle. This relationship has been assumed to be 3:1.

The Young modulus is the mechanical property that relates the stress made and the resulting deformation of solid materials. The values of this parameter for the most common materials used in quadcopter frames can be seen on the Figure 15.

Material	Young modulus [GPa]
Plastic	3,5
Aluminum	10
Carbon fiber	230

Figure 15: Young modulus of common materials in quadcopter frames

In order to obtain the relationship between the arm distance and its section parameters, the following assumption is done: for a thrust force of 5N, the maximum vertical displacement acceptable is a 5% of the total arm longitude. The calculus is done for the more pessimistic case, which means considering a plastic frame, as it is the material with the lowest Young modulus.

By combining all these equations and assumptions, the expressions that relate the distance and the geometrical parameters of the arm results in:

$$y_{DISP} = D \cdot 0{,}05 = \frac{D^3 \cdot 5}{3 \cdot \frac{3 \cdot b^4}{12} \cdot 3{,}5 \cdot 10^9} \tag{4.7}$$

$$b = \sqrt[4]{1{,}1428 \cdot 10^{-7} \cdot D^2} \tag{4.8}$$

$$a = 3 \cdot \sqrt[4]{1{,}1428 \cdot 10^{-7} \cdot D^2} \tag{4.9}$$

<u>Motors and propellers mass</u>

There is not a clear relationship between the mass and the power of a brushless motor. In most of the studied motors, this relationship is around 3-4 watts per gram, but this rule is not always true. Therefore, the value of the mass provided by the manufacturer for each motor must be directly introduced in the equation.

In the propellers mass calculation, the same problem appears. As the mass depends on its material and on its dimensions, there is not a proportional relationship applicable to every propeller. Therefore, its mass has to be added to the motor mass in each case.

<u>LiPo battery mass</u>

The battery capacity measures the amount of power that the battery is able to hold. The bigger the capacity is, the bigger the physical size and weight of the battery. In order to find a relationship between the capacity and the weight, the data from 47 commercial batteries have been collected and sorted by their number of cells, as can be seen on the Table 2. (www.hobbyking.com)

Number of cells (S)									
2		3		4		5		6	
Capacity [mAh]	*Mass [g]*	*Capacity [mAh]*	*Mass [g]*	*Capacity [mAh]*	*Mass [g]*	*Capacity [mAh]*	*Mass [g]*	*Capacity [mAh]*	*Mass [g]*
360	16	700	80	1000	124	2200	305	1550	247
950	47	1300	119	1300	155	2700	404	3000	566
1300	79	1400	115	1800	207	3000	414	5000	765
1700	92	1800	152	2200	257	3300	480	5400	821
2200	134	2200	188	4000	320	4000	537	5800	843
4200	220	2700	198	5200	433	4500	581	6600	794
6600	340	4000	244	6000	633	4500	574	8000	1110
7500	350	6000	478	8000	643	4900	625	10000	1370
		8000	644	10000	804	5800	776	12000	1525
				16000	1290	8000	1054	16000	1920

Table 2: Capacity and mass of the researched batteries

By representing these data graphically, it is possible to extrapolate the following functions:

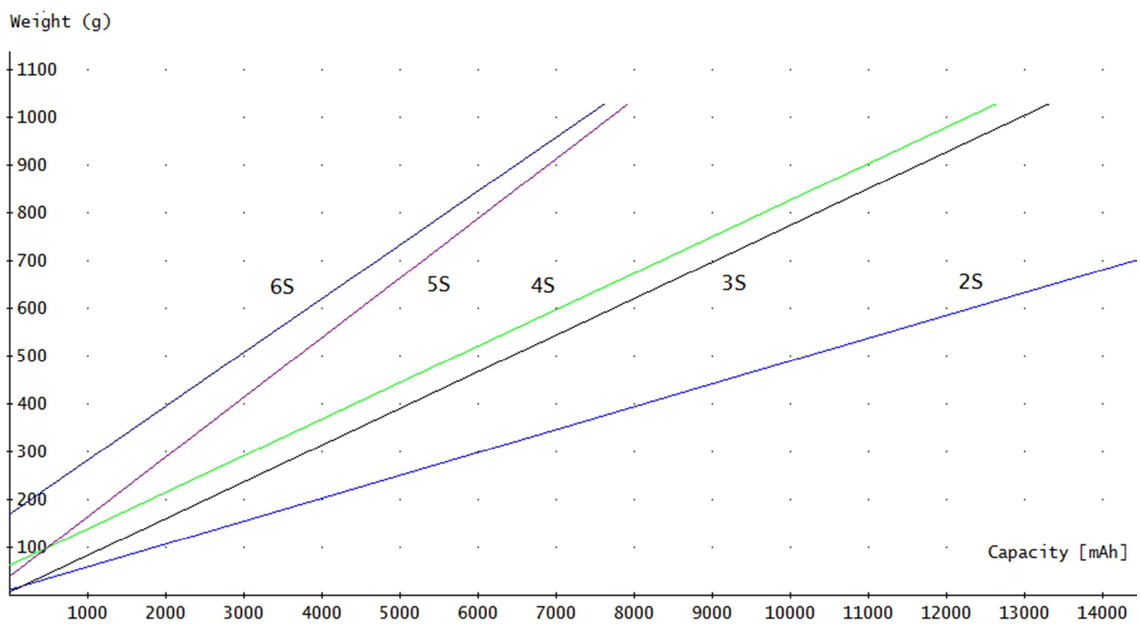


Figure 16: Relationship between capacity and mass, depending on the number of cells

For this calculation, it is therefore possible to assume that the mass of a LiPo battery increases lineally with the capacity, obeying to the following equation:

$$m_{BAT} = A \cdot Capacity + B \qquad (4.10)$$

The parameters A and B depend on the number of cells, and their values can be seen on the Table 3.

S	A [g/mAh]	B [g]
2	0,0478	11,631
3	0,0767	6,9683
4	0,0764	62,733
5	0,1248	39,575
6	0,1126	169,96

Table 3: Parameters in the mass equation of a LiPo battery

Electronic installation mass

The basic electronic installation of a quadcopter includes the controller, the ESCs and the wires.

About the controller, the size and weight of this component are very similar for every quadcopter. Thanks to the improvements in microelectronics, microchips have been miniaturized and have small encapsulations that do not depend on its complexity or sophistication level. A research on the most common flight controllers and their weights has been made, as can be seen on the Table 4. (www.hobbyking.com)

Flight controller	Mass [g]
Arduino UNO	25
Flip32	29
AfroFlight	18
OpenPilot	62
KingKong	38
LUX	26
TBS Colibri	18

Table 4: Masses of commonly used flight controlers

Considering the big scale difference between these values and the quadcopter total weight, the controller mass can be assumed as the average of these masses, approximately 30 grams, for every quadcopter.

The ESC modules are also very light compared with the system mass. Depending on the different manufacturers, its mass varies between 10 and 20 grams. As the effect of these devices is not very significant in the system total mass calculation, they are considered to be 15 grams for every quadcopter.

The wires mass calculation depend on the frame size, as the wires must cover the distance between each ESC and the battery. In order to improve the quadcopter stability, the battery is assumed to be always placed in the center of the frame. Assuming that the wires are made of cooper, which has a linear density of 7,037 g/m, and that there are two wires per arm, their mass is calculated as:

$$m_{WIRES} = 4 \cdot 2 \cdot D \cdot 7{,}037 = D \cdot 56{,}29 \tag{4.11}$$

where “D” is expressed in meters

The complete expression for the electronic installation mass results in:

$$m_{ELEC} = 30 + 4 \cdot 15 + 56{,}29 \cdot D = 90 + 56{,}29 \cdot D \quad (4.12)$$

Other possible equipment mass

Each quadcopter, depending on its features, may have extra equipment in addition to the basic components. Cameras, GPS systems or antennas are some examples. The weight of these devices must be taken into consideration in order to obtain an accurate approximation of the quadcopter total mass. For each case, this value must be estimated and introduced into the equation.

4.1.2 Power consumed and torque created by the motors in a hovering situation

Once the required thrust that each propeller must create to compensate the gravitational force is known, it is possible to calculate the necessary angular speed that each motor must provide.

The pressure and speed of the air before and after crossing the propellers change. In fluid dynamics, the momentum theory or disk actuator theory, states that the thrust force created by a propeller disk can be calculated as the product of the propeller disk area and the jump of pressure that the air experiences. (NASA, 2015)

$$F = A \cdot \Delta P = A \cdot (P_e - P_0) \quad (4.13)$$

where “A” is the area of the propeller disk (a circle with the same diameter as the propeller), “P_e” is the total pressure of the exit air after crossing the propeller and “P_0” is the total pressure of the input air before crossing the propeller

This situation is represented on the Figure 17.

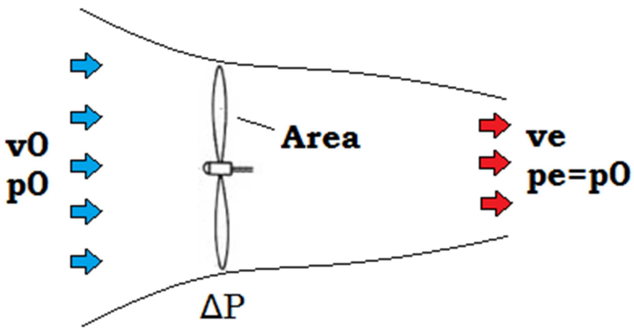


Figure 17: Air conditions before and after the propeller

The air total pressure is composed by the static pressure and the dynamic pressure. The static pressure only depends on the altitude, as it is caused by the weight of the air column above it.

The dynamic pressure depends on the air velocity. Applying the Bernoulli's equation, the expression for the total air pressure depending on the velocity results in:

$$P = p + \frac{1}{2}\rho_{AIR} \cdot v^2 \tag{4.14}$$

where “P” is the total pressure, “p” is the static pressure and “v” is the air speed

The static pressure of the air before the propeller is equal to the static pressure after the propeller, as the altitude is the same in both points. The jump of pressure can be therefore simplified to:

$$\Delta P = \left[p_e + \frac{1}{2}\rho_{AIR} \cdot v_e^2\right] - \left[p_0 + \frac{1}{2}\rho_{AIR} \cdot v_0^2\right] = \frac{1}{2}\rho_{AIR} \cdot (v_e^2 - v_0^2) \tag{4.15}$$

For the particular situation when the quadcopter is hovering and the speed of the air before entering into the propeller is zero, the thrust force created is:

$$F = A \cdot \frac{1}{2}\rho_{AIR} \cdot v_E^2 = \frac{\pi d^2}{4} \cdot \frac{1}{2}\rho_{AIR} \cdot v_E^2 \tag{4.16}$$

Where “d” is the propeller diameter

In order to calculate the speed of the airflow at the exit of the propeller, the parameter “pitch” is used. This parameter is defined as the distance that the propeller would move in one revolution if it were moving through a soft solid, like a screw through the wood, and is represented on the Figure 18. (Benson, 2014)

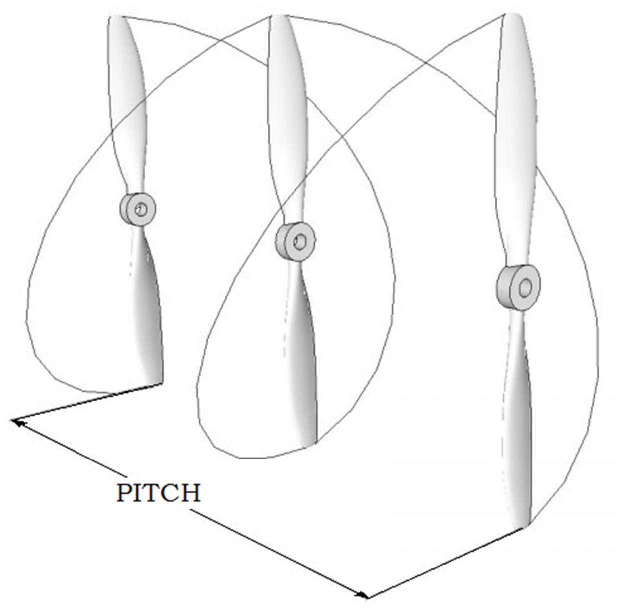


Figure 18: Propeller pitch distance

The exit velocity of the airflow through the propeller can be calculated with the following equation:

$$v_E = n \cdot p \cdot \left(\frac{0{,}0254}{60}\right) \tag{4.17}$$

where “n” is the speed given in rpm, “p” is the pitch expressed in inches and a conversion factor is applied in order to transform the result into m/s

It is also necessary to calculate the air density depending on the ambient conditions, the altitude over the sea level and the temperature. As a simplification, it is assumed that air is an ideal gas. Therefore, the ideal gas law can be applied to describe its state.

$$P \cdot V = n \cdot R_G \cdot T \tag{4.18}$$

where “P” is the pressure, “V” is the volume, “n” is the number of moles, “Rg” is the universal gas constant and “T” is the temperature

Density is the combined mass of the molecules of an ideal gas in relation to their occupied volume:

$$\rho = \frac{m}{V} = \frac{n \cdot M}{V} = \frac{P \cdot M}{R_G \cdot T} \tag{4.19}$$

where “M” is the molar mass

The air pressure in a particular point depends on its altitude over the sea level, as it is affected by the weight of the air column above it. The equation that relates these parameters is known as the barometric formula. (Berberan-Santos, 1996)

$$P_h = P_0 \cdot e^{\frac{-M \cdot g \cdot h}{k \cdot T}} \tag{4.20}$$

where "P_h" is the air pressure at a particular altitude, "P_0" is the air pressure at sea level, "M" is the molar mass, "g" is the gravity acceleration, "k" is the Boltzmann's constant and "T" is the temperature

By combining there equations, the expression to calculate the air density is obtained:

$$\rho_{AIR} = \frac{101325(1 - 2{,}5577 \cdot 10^{-5} \cdot h)^{5{,}25588}}{287{,}05 \cdot T} \tag{4.21}$$

The aerodynamic power generated by the propeller while produces a concrete thrust can be also calculated applying the disk actuator theory. (NASA, 2015) This expression results in:

$$P_{AERO} = \sqrt{\frac{F^3}{2 \cdot \rho_{AIR} \cdot A}} = \frac{1}{2} \cdot F \cdot v_e \tag{4.22}$$

Assuming that the power transfer between the motor and the propeller is done in ideal conditions, and it is no affected by frictions or energy losses, the electric power that each motor must generate matches with the aerodynamic power. The electric power of a motor is defined as:

$$P_{MOTOR} = U \cdot I_{MOTOR} \tag{4.23}$$

where "U" is the voltage applied to the motor and "I" the consumed current

The ESC uses PWM regulation to vary the applied voltage to the motor, being the voltage of the battery the highest value when the PWM percentage is at 100%.

$$P_{MOTOR} = U_{BAT} \cdot x_{ESC} \cdot I_{MOTOR} \tag{4.24}$$

At this point it is also possible to calculate the torque produced on the axis by each motor. It depends on the power consumed and on the angular speed, according to the following equation:

$$M = \frac{P_{MOTOR}}{\omega} \tag{4.25}$$

where “w” is expressed in rad/s

The rotational speed of a brushless motor is proportional to its operation voltage. Being “Kv” the constant of proportionality.

$$n = Kv \cdot V = Kv \cdot V_{BAT} \cdot x_{ESC} \tag{4.26}$$

where “n” is expressed in rad/s

By applying the convenient transformation coefficient, it is possible to define the motor torque as:

$$M = \frac{P_{MOTOR}}{n \cdot \left(\frac{2\pi}{60}\right)} = \frac{P_{MOTOR}}{Kv \cdot V_{BAT} \cdot x_{ESC} \cdot \left(\frac{2\pi}{60}\right)} \tag{4.27}$$

4.1.3 Maximum vertical and horizontal speeds

Because of the air resistance force, there is a maximum speed that the quadcopter is able to reach. This limit speed is calculated based on the movement equations described in the chapter “Quadcopter basic fundaments”. It is possible to differ between the maximum speed in the vertical direction and the maximum speed in the horizontal direction.

Maximum vertical speed

Assuming that the quadcopter is only moving on the vertical direction, the expression that defines this situation is:

$$T - F_{DRAG} = T - \ m \cdot g - \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A \cdot v^2 = m \cdot a \tag{4.28}$$

The maximum thrust force is calculated as the combination of the maximum force that the propellers can create.

$$T_{MAX} = 4 \cdot F_{MAX} \tag{4.29}$$

The drag coefficient depends on the shape of the quadcopter, but generally this kind of aircrafts is not very aerodynamic. On the Figure 19 the drag coefficients of some geometric figures empirically measured are presented. (www.wikipedia.com) It is assumed that for these speeds calculations a drag coefficient of 1,3 is used.

Figure 19: Drag coefficients of some geometric figures

The area of the quadcopter is calculated assuming the geometrical dimensions shown on the Figure 20.

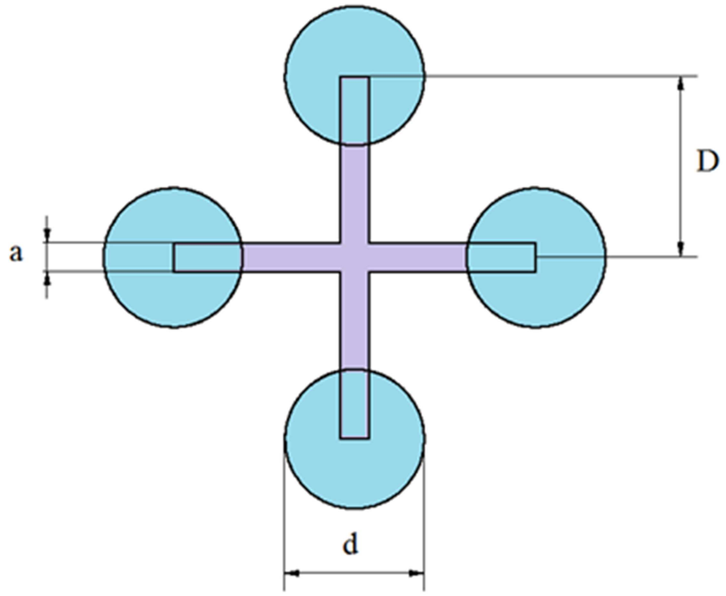


Figure 20: Quadcopter area approach

Considering the area of the propellers and the area of the frame that is not covered by them, the expression of the quadcopter area results in:

$$A = 4 \cdot \pi \cdot \frac{d^2}{2} + 4 \cdot a \cdot \left(D - \frac{d}{2}\right) \tag{4.30}$$

When the limit is speed is reached the acceleration is equal to zero and the maximum speed equation can be obtained. By applying in this expression the maximum thrust force that the propellers are able to create, the maximum vertical speed equation results in:

$$v_V = \sqrt{\frac{2(T_{MAX} - m \cdot g)}{\rho_{AIR} \cdot C_D \cdot A}} \tag{4.31}$$

where "T_{MAX}" is the maximum thrust force, "m" is the quadcopter mass, "g" is the earth gravity acceleration, "ρ_{AIR}" is the air density, "C_D" is the air resistance coefficient and "A" is the quadcopter area

Maximum horizontal speed

It is assumed that the quadcopter is only moving on the horizontal direction, keeping its altitude constant. In this case, the quadcopter is flying with an inclination angle, which can be calculated taking into account that the vertical thrust must compensate the gravity force when the propellers are providing the maximum thrust. This situation is shown on the following image.

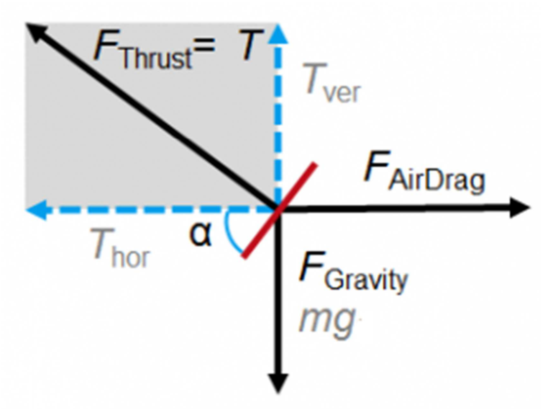


Figure 21: Quadcopter forces equilibrium

The inclination angle is calculated as:

$$\alpha = \arccos(\frac{m_{QUAD} \cdot g}{T_{MAX}}) \tag{4.32}$$

The expression that defines this equilibrium situation is:

$$T \cdot \cos\alpha - F_{DRAG} = m \cdot g \cdot tan\,\alpha - \frac{1}{2}\rho_{AIR} \cdot C_D \cdot A_{effective} \cdot v^2 = m \cdot \dot{v} \tag{4.33}$$

The effective area in this case is calculated as the vertical projection of the quadcopter top area.

$$A_{effective} = A \cdot sen\,\alpha \tag{4.34}$$

When the speed limit is reached the acceleration is zero and the horizontal speed equation can be obtained.

$$v_H = \sqrt{\frac{2 \cdot m \cdot g \cdot \tan \alpha}{\rho_{AIR} \cdot C_D \cdot A \cdot sen\ \alpha}} \tag{4.35}$$

4.1.4 Maximum flight time

The flight time is defined as the time while the battery is able to provide enough energy to the system. It depends on the battery capacity and on the current drawn by the motors and other electronic components. The current consumed by the motors depend on the required throttle that they have. Therefore, the flight time is variable and depends on the pilot requirements. However, it is possible to calculate the longest flight time that a battery can provide. The minimum current consumption is obtained in the situation that the quadcopter is hovering, as its motors only have to compensate the system weight force. For this situation the current drawn by each motor can be calculated.

$$t_F = \frac{C}{4 \cdot I_{MOTOR} + I_{OTHERS}} \tag{4.36}$$

where “C” is the capacity expressed in Amps·hour and “t_F” the flight time expressed in hours

The component that consumes more current apart from the motors is the controller. Taking as example the researched flight controllers, the current consumed by these devices is around 0,045A.

Given the big scale difference, this current is assumed to be zero, resulting the simplified flight time in:

$$t_F \simeq \frac{C}{4 \cdot I_{MOTOR}} \tag{4.37}$$

4.1.5 Stability

Despite the fact that the stability of a quadcopter not only depends on its components, but also on the control software, the size and weight of the frame are involved in the quadcopter behavior. According to the movements equations developed on the chapter 3, bigger inertia momentums lead to decrease the influence that external parameters have on the system, as it is necessary to apply more force to obtain the same response. It is possible to conclude that bigger inertia momentums contribute to a system stability improvement, but taking into account that this assertion is only applicable when the control software is properly configured.

In order to calculate these momentums, a geometrical approach of the quadcopter is assumed. The motors are modeled as solid cylinders and the frame arms as four thin rectangular sheets. The weight of the battery is assumed to act as a single mass in the gravity center, so it has no effect on the system inertia. Depending on the quadcopter configuration, "x" or "+", the inertia momentums are referred to different axes, according to the scheme that can be seen on the Figure 22.

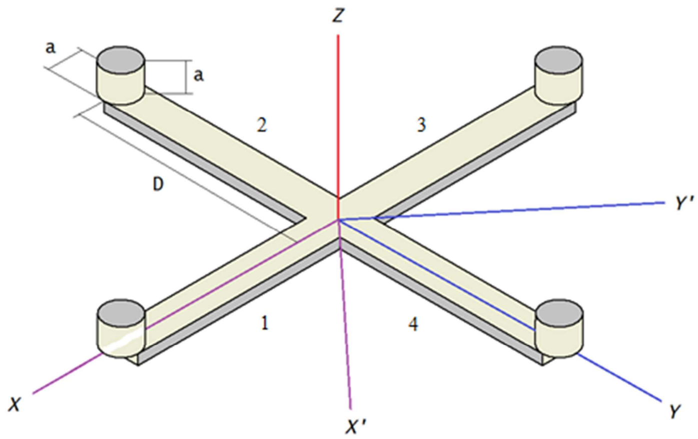


Figure 22: Quadcopter geometrical approach to calculate the inertia momentums

The expressions to calculate the inertia momentums on different rotational axes of the figures involved on the geometrical model, a thin rectangular sheet and a solid cylinder, can be seen on the Figure 23. (www.wikipedia.com)

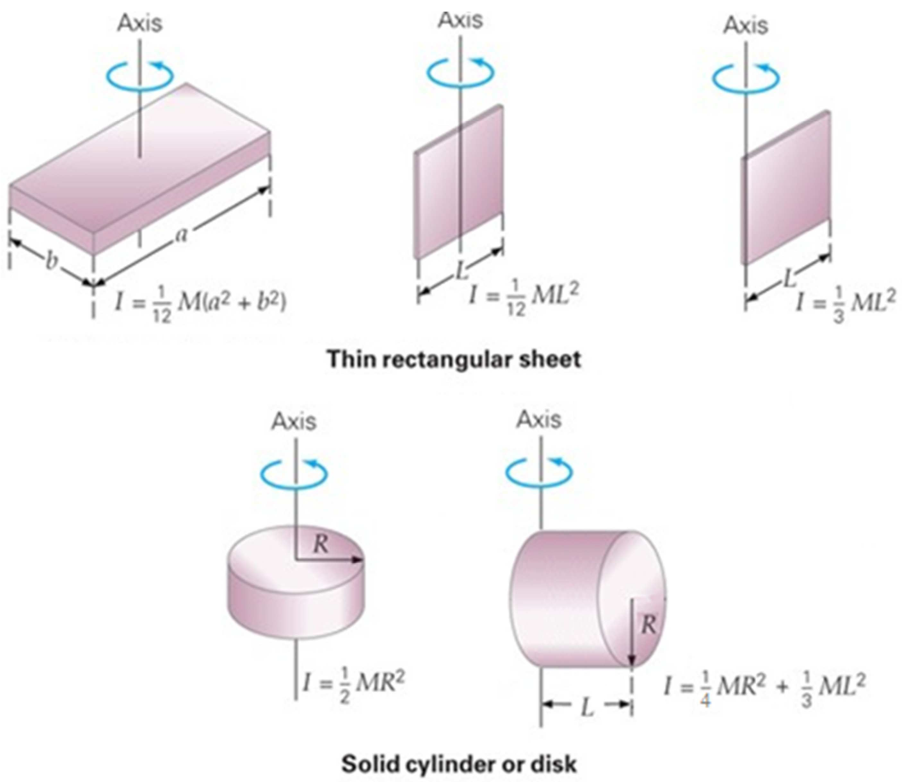


Figure 23: Inertia momentum expressions of the model figures

It is possible to obtain the inertia momentum about a different axis, if both axes are parallel, by applying the Steiner theorem. This theorem provides an expression to determine the inertia momentum about any axis if the inertia momentum about the parallel axis through the gravity center is known.

$$I = I_{GC} + m \cdot d^2 \tag{4.38}$$

where "I" is the resulting inertia momentum, "I_{GC}" is the inertia momentum about a parallel axis through the gravity center, "m" is the object mass and "d" is the distance between both axes

From the thin rectangular sheet and the solid cylinder expressions, and applying the Steiner theorem when it is needed, the inertia momentums about the different rotational axes of the quadcopter approach result in:

<u>Axis X</u>

$$I_X = I_{ARM\ 1,3} + I_{ARM\ 2,4} + I_{MOTOR\ 1,3} + I_{MOTOR\ 2,4}$$

$$I_X = \frac{1}{12}(2 \cdot m_{ARM}) \cdot a^2 + 2 \cdot \frac{1}{3} m_{ARM} \cdot D^2 + 2 \cdot \left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + 2 \cdot \left[\left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + m_{MOTOR} \cdot D^2\right] \tag{4.39}$$

<u>Axis Y</u>

Due to the symmetry of the system, the inertia momentum about the axis Y is the same as about the axis X.

$$I_Y = \frac{1}{12}(2 \cdot m_{ARM}) \cdot a^2 + 2 \cdot \frac{1}{3} m_{ARM} \cdot L^2 + 2 \cdot \left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + 2 \cdot \left[\left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + m_{MOTOR} \cdot D^2\right] \tag{4.40}$$

<u>Axis X'</u>

$$I_{X'} = I_{ARM\ 1,2,3,4} + I_{MOTOR\ 1,2,3,4} \tag{4.41}$$

$$I_{X'} = 4 \cdot \left[\frac{1}{3} m_{ARM} \cdot (D \cdot cos45)^2\right] + 4 \cdot \left[\left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + m_{MOTOR} \cdot (D \cdot cos45)^2\right] \tag{4.42}$$

Axis Y'

Due to the symmetry of the system, the inertia momentum about the axis Y' is the same as about the axis X'.

$$I_{Y'} = 4 \cdot \left[\frac{1}{3} m_{ARM} \cdot (D \cdot cos45)^2\right] + 4 \cdot \left[\left(\frac{1}{4} m_{MOTOR} \cdot \frac{a^2}{4} + \frac{1}{3} m_{MOTOR} \cdot a^2\right) + m_{MOTOR} \cdot (D \cdot cos45)^2\right] \tag{4.43}$$

Axis Z

$$I_Z = I_{ARM\ 1,3} + I_{ARM\ 2,4} + I_{MOTOR\ 1,2,3,4}$$

$$I_Z = 2 \cdot \left[\frac{1}{12}(2 \cdot m_{ARM}) \cdot (4D^2 + a^2)\right] + 4 \cdot \left(\frac{1}{2} m_{MOTOR} \cdot \frac{a^2}{4} + m_{MOTOR} \cdot D^2\right) \tag{4.44}$$

4.2 MATLAB/Simulink model

In this chapter, all the correlations previously obtained are combined to build a virtual model of the system. The purpose of this model is to provide realistic estimations for the flight parameters of a quadcopter depending on the charracteristics of its components. It can be used to support the designing process of a quadcopter, as it allows to optimice the technical parameters depending on the specific requremets. In addition, it detects incompatibilities between components.

The model input parameters can be seen on the Table 5, and are composed by the technical features of each component and the external conditions.

Model inputs	
Component	***Parameters***
Motor	Constant Kv
	Maximum power the motor can provide
	Mass
Battery	Number of cells (2-6)
	Capacity
	Discharge rate
Propeller	Diameter
	Pitch
Frame	Distance between motor and system center
	Density
	Other equipment mass
External conditions	Altitude over the sea level
	Temperature

Table 5: Input parameters of the Simulink model

The model output provides information about different aspects of the system:

Static equilibrium

For this situation, when the quadcopter is hovering in the air, with neither vertical speed nor acceleration, the following parameters are calculated:

- Angular speed of the motors
- Input voltage of the motors
- ESC percentage of regulation
- Current consumed by the motors
- Power consumed by the motors

Physical propierties

The model calculates the total mass of the quadcopter and the inertia momentums about each rotational axis.

Operation limits

At maximum throttle, the maximum load that the quadcopter is able to lift, the maximum vertical speed and the horizontal maximum speed, as well as the inclination anlge of the quadcopter in this particular situation, are calculated.

Design errors

The model output includes three variables that detect if the system has design failures that would make the quadcopter unable to flight either properly or safety. When any of these errors appears, it means that the programmed configuration is incompatible.

- Error 1: Changes from 0 to 1 when the total mass of the quadcopter is higher than the maximum weight that it is able to lift.
- Error 2: Changes from 0 to 1 when the current consumed by the motors can be higher than the maximun current that que battery can provide safely.
- Error 3: Changes from 0 to 1 when the power requested by each motor can be higher than the maximum power that a motor is able to produce.

Maximum flight time

The flight time is defined as the period while the battery is able to provide the energy requested by the system. This time is inversely proportional to the motors current consumption, and therefore it depends on the movements that the quadcopter is forced to perform. In the hovering situation, as the only force that the quadcopter must compensate is its own weight, the motors are consuming the minimum necessary current. In this situation is possible to calculate the maximum time that the quadcopter is able to stay in the air.

The interface of the model is shown on the Figure 24. The input parameters are introduced on the left side and the results appear on the digital displays of the right side.

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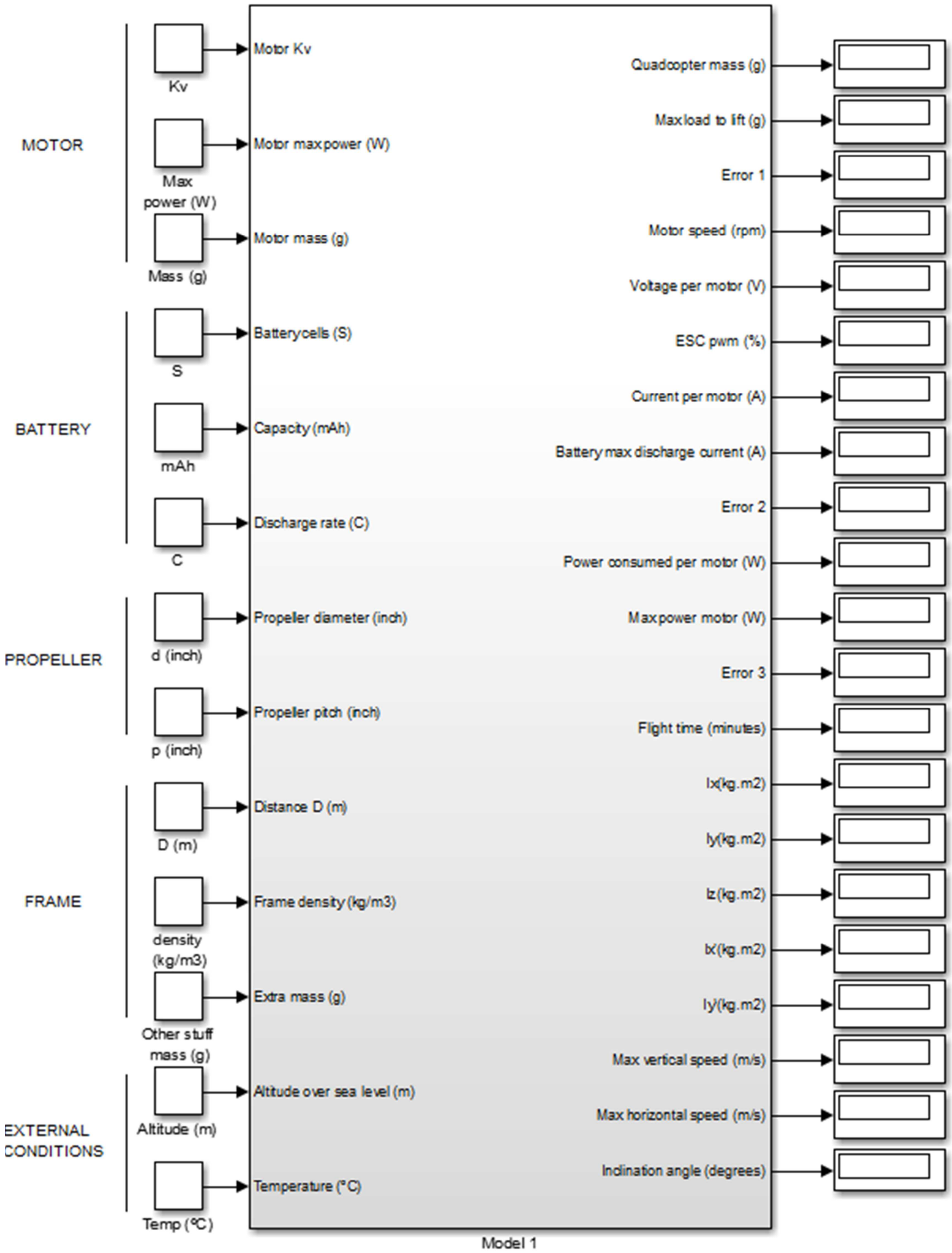

Figure 24: MATLAB/Simulink model interface

The model configuration that remains under the interface mask can be seen on the Figure 25.

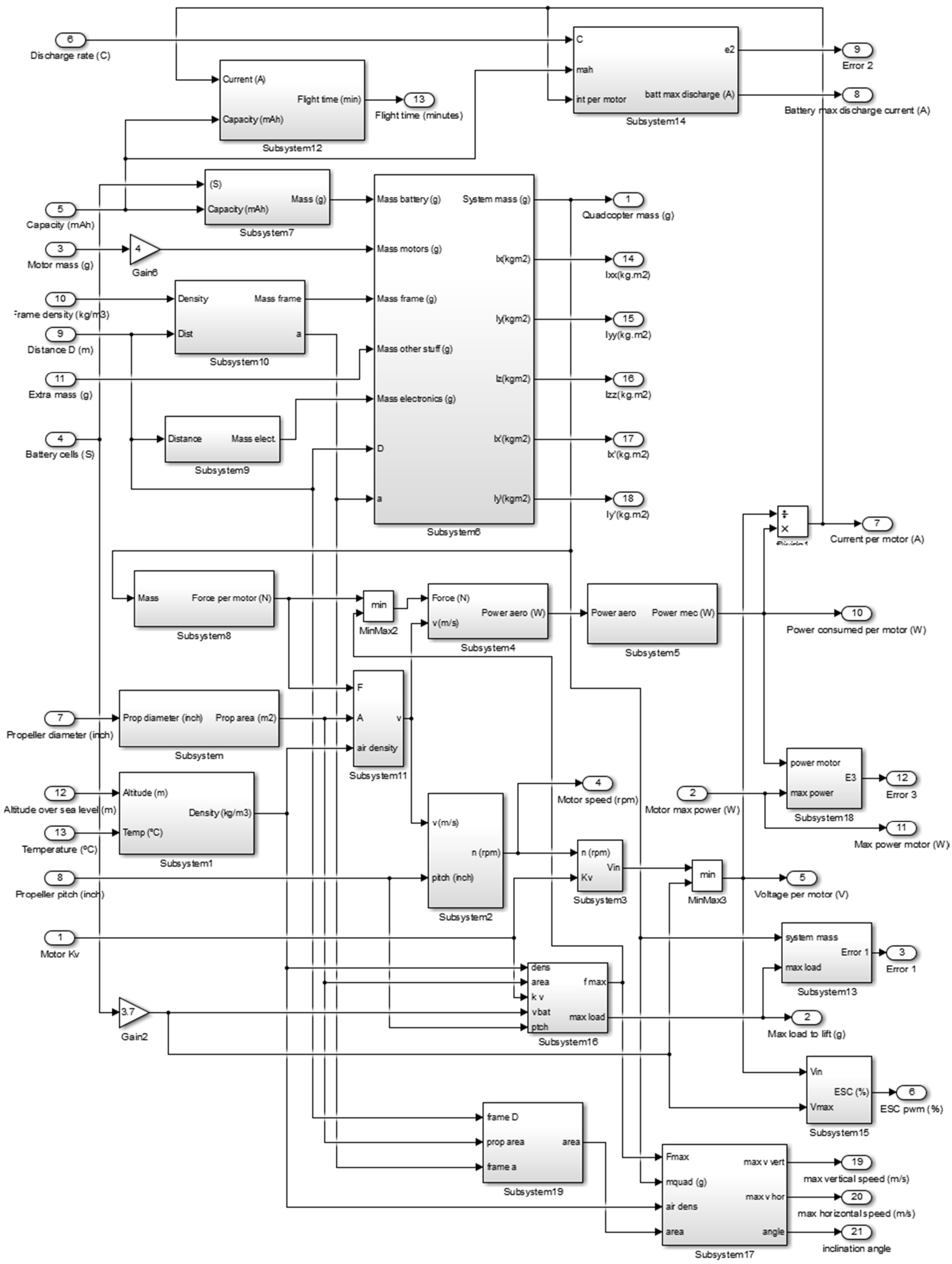

Figure 25: MATLAB/Simulink model operations

It can be appreciated that some subsystems have been used in order to simplify the model. The operations that compose each subsystem can be seen in detail in the annex A, at the end of the document.

4.3 Application to some scenarios

In this section, the created model is applied to different scenarios in order to demonstrate its validity in the designing process of a quadcopter. From the combination of the technical parameters introduced by the user, the model provides the variables that define the quadcopter flight behavior. By adjusting the input parameters, it is possible to visualize how each parameter affects to the final response, and depending on the particular situation, to obtain the most proper configuration. For each scenario, there may be many different alternatives that give as a result a suitable quadcopter that fits the set of requirements. The designer can chose in these cases the optimum solution, based on other aspects such as the price of the components.

About the ambient conditions, every scenario has been supposed to be placed in Bochum, being its altitude 104m over the sea level and its average temperature 15ºC.

The chosen scenarios to perform these tests are photography and video, package delivering, entertainment, surveillance and racing drones. On the following pages, the quadcopter requirements in each case are analyzed and the model is applied to obtain a satisfactory hardware configuration.

The input technical parameters are based on real components, according to the characteristics provided by the manufacturers.

Photography and video

In order to take good quality pictures or videos, the most important requirement to fit is the stability, as the quadcopter must be able to flight steadily and avoid tremors caused by the wind. Despite the fact that the stability depends more on the control software than on the hardware, the inertia momentums also affect to the vulnerability of the quadcopter to suffer disturbances. Bigger frames make the inertia momentums higher and therefore the system more resistant to wind effects.

The sophisticated equipment that this kind of quadcopters carry can be heavy, which force to ensure that the motors are powerful enough to lift the total weight.

It may be also desired disposing long flight times, mainly if the quadcopter is intended to film videos. In this case, a battery with high capacity is essential.

As an example case, a set of requirements is fixed. It is assumed that the quadcopter frame must have a distance "D" of 0,4 meters, the weight of the camera is 800 grams, and the flight time in a hovering situation must be over 20 minutes.

For this particular situation, a possible configuration that fit all the requested requirements is obtained using the model. The results can be seen on the Table 6.

Inputs			**Outputs**	
Motor	*Kv*	1200	*Total mass*	1,469 kg
	Max. Power	50 W	*Max. extra weight that is possible to lift*	1,652 kg
	Mass	52 g	*Motor speed*	12100 rpm
Battery	*S*	4	*Voltage per motor*	10,15 V
	Capacity	4000 mAh	*% ESC*	68,6%
	C	5	*Current consumed per motor*	2,7 A
Propeller	*Diameter*	7 inch	*Max. Discharge current*	20 A
	Pitch	3 inch	*Power consumed per motor*	27,83 W
Frame	*Distance D*	0,4 m	*Max. flight time*	21,8 min
	Density	1200 kg/m^3	*Max. vertical speed*	12,01 m/s
	Extra mass	0,8 kg	*Max. horizontal speed*	16,5 m/s
			Max. inclination angle	61º
			Ix	0,03755 kg·m^2
			Iy	0,03755 kg·m^2
			Ix'	0,0375 kg·m^2
			Iy'	0,0375 kg·m^2
			Iz	0,07493 kg·m^2

Table 6: Inputs and results of the "Photography and video" scenario

Package delivering

The critical parameters are the strength to lift heavy loads and the capability of covering long distances, which means long flight times. The frame material must be resistant, as it will have to deal with heavy loads. The use of 3d printed frames is discarded, in benefit of aluminum or carbon fiber frames.

As an example, it is assumed a scenario where the quadcopter is used to deliver autonomously packages with maximum weight of 3kg and across a 5km-radius area. Assuming an horizontal average speed of 5 m/s, the quadcopter could reach the delivery area limit and return in 33,3 minutes.

For this particular situation, a possible solution obtained from the model can be seen on the Table 7.

Inputs			**Outputs**	
Motor	*Kv*	530	*Total mass*	5,508 kg
	Max. Power	250 W	*Max. extra weight that is possible to lift*	3,8 kg
	Mass	108 g	*Motor speed*	9000 rpm
Battery	*S*	6	*Voltage per motor*	17 V
	Capacity	20000 mAh	*% ESC*	76,56%
	C	5	*Current consumed per motor*	7,56 A
Propeller	*Diameter*	11 inch	*Max. Discharge current*	100 A
	Pitch	5 inch	*Power consumed per motor*	128,6 W
Frame	*Distance D*	0,5 m	*Max. flight time*	39,64 min
	Density	2500 kg/m^3	*Max. vertical speed*	12,67 m/s
	Extra mass	0 kg	*Max. horizontal speed*	19,7 m/s
			Max. inclination angle	61°
			Ix	0,1601 kg·m^2
			Iy	0,1601 kg·m^2
			Ix'	0,1599 kg·m^2
			Iy'	0,1599 kg·m^2
			Iz	0,3197 kg·m^2

Table 7: Inputs and results of the "Package delivering" scenario

Entertainment

There are not strict conditions for this scenario of use. Sometimes this kind of quadcopters are designed to develop aerial acrobatics, which requires reduced dimensions and powerful motors. Others, used with leisure purposes, have a FPV (first person view) camera that allows the pilot to see the quadcopter view at real time. These quadcopters need bigger dimensions to improve the stability. There are also very basic quadcopters designed for beginners, with the only purpose to serve as a tool to improve their piloting skills.

As an example case, a very simple quadcopter targeted to novel pilots is designed. The hardware requirements in this case are reduced dimensions and weight in order to reduce the possible damage in case of crash. The distance between the motors and the frame must be 15 cm and the total weight less than 0,5 kg. In addition, in order to reduce costs and make the quadcopter more affordable, a plastic frame is used.

For this particular situation, a possible solution obtained from the simulation can be seen on the Table 8.

Inputs			**Outputs**	
Motor	*Kv*	2000	*Total mass*	0,414 kg
	Max. Power	84 W	*Max. extra weight that is possible to lift*	3,07 kg
	Mass	33 g	*Motor speed*	5400 rpm
Battery	*S*	2	*Voltage per motor*	2,72 V
	Capacity	1700 mAh	*% ESC*	36,74%
	C	20	*Current consumed per motor*	2,15 A
Propeller	*Diameter*	5 inch	*Max. Discharge current*	34 A
	Pitch	5 inch	*Power consumed per motor*	5,85 W
Frame	*Distance D*	0,15 m	*Max. flight time*	11,86 min
	Density	1000 kg/m3	*Max. vertical speed*	23,87 m/s
	Extra mass	0 kg	*Max. horizontal speed*	25,67 m/s
			Max. inclination angle	82º
			Ix	0,0018 $kg \cdot m^2$
			Iy	0,0018 $kg \cdot m^2$
			Ix'	0,0017 $kg \cdot m^2$
			Iy'	0,0017 $kg \cdot m^2$
			Iz	0,0036 $kg \cdot m^2$

Table 8: Inputs and results of the "Entertainment" scenario

<u>Surveillance</u>

This scenario is just a particular application of the photography and video quadcopters, mainly used by the police and security forces. The basic premises of this kind of quadcopters are still valid, but there are also some aspects that can be improved. In order to take part in spying missions, it is necessary to keep unnoticed, so reduced dimensions are desired. If it is intended to take part in chases, fast speeds are also required.

As an example case, it is designed a police quadcopter to surveillance stadiums surroundings in sport events. This quadcopter is assumed to have the following requirements: frame distance between the motors and the center of 20 cm, capability to carry a video camera and the necessary components to transmit the images, with a total weight of 800g, a maximum flight time over 30 minutes and a maximum horizontal speed over 20m/s.

For this particular situation, a possible solution obtained from the simulation can be seen on the Table 9.

Inputs			**Outputs**	
Motor	*Kv*	1500	*Total mass*	2,127 kg
	Max. Power	100 W	*Max. extra weight that is possible to lift*	1,456 kg
	Mass	40 g	*Motor speed*	17000 rpm
Battery	*S*	4	*Voltage per motor*	11,4 V
	Capacity	11000 mAh	*% ESC*	77,05%
	C	20	*Current consumed per motor*	4,96 A
Propeller	*Diameter*	6 inch	*Max. Discharge current*	220 A
	Pitch	3 inch	*Power consumed per motor*	56,59 W
Frame	*Distance D*	0,2 m	*Max. flight time*	33,25 min
	Density	1000 kg/m3	*Max. vertical speed*	14,59 m/s
	Extra mass	800 kg	*Max. horizontal speed*	22,89 m/s
			Max. inclination angle	53,5º
			Ix	0,0043 kg·m^2
			Iy	0,0043 kg·m^2
			Ix'	0,0042 kg·m^2
			Iy'	0,0042 kg·m^2
			Iz	0,0085 kg·m^2

Table 9: Inputs and results of the "Surveillance" scenario

Racing drones

In this case, the most important aspect is the maximum speed. The motors must be powerfull and the frame small and light. As an example, a quadcopter with the following requirements is designed: frame distance between center and motors of 15 cm, total weight of less than 0,5kg and maximum horizontal speed up to 40 m/s.

For this particular situation, a possible solution obtained from the simulation can be seen on the Table 10.

Inputs			**Outputs**	
Motor	*Kv*	2300	*Total mass*	0,466 kg
	Max. Power	84 W	*Max. extra weight that is possible to lift*	5,3 kg
	Mass	26 g	*Motor speed*	7800 rpm
Battery	*S*	3	*Voltage per motor*	3,39 V
	Capacity	3200 mAh	*% ESC*	30,56%
	C	30	*Current consumed per motor*	3,25 A
Propeller	*Diameter*	4 inch	*Max. Discharge current*	96 A
	Pitch	5 inch	*Power consumed per motor*	11,05 W
Frame	*Distance D*	0,15 m	*Max. flight time*	14,74 min
	Density	1000 kg/m3	*Max. vertical speed*	40,18 m/s
	Extra mass	0 kg	*Max. horizontal speed*	42,4 m/s
			Max. inclination angle	84,64º
			Ix	0,0015 $kg\cdot m^2$
			Iy	0,0015 $kg\cdot m^2$
			Ix’	0,0014 $kg\cdot m^2$
			Iy’	0,0014 $kg\cdot m^2$
			Iz	0,0030 $kg\cdot m^2$

Table 10: Inputs and results of the "Racing drones" scenario

From these cases of study, it is possible to conclude that the model can be a valuable tool for the designing process of a quadcopter, as it saves a lot of calculation time to the designer and ensures the chosen configuration compatibility.

In addition, it allows representing the effects that each parameter has on the final system behavior. In this part, dimensionless graphics are used to show the tendency of the different correlations. The parameters that cause more impact on the system are the constant Kv of the motor, the capacity and number of cells of the battery, the diameter and pitch of the propeller and the frame dimensions.

Increasing the Kv constant of the motors, gives as a result more strength to lift heavier loads, but the motors consume more current and the maximum flight time is reduced. The Figure 26 shows the evolution of these parameters.

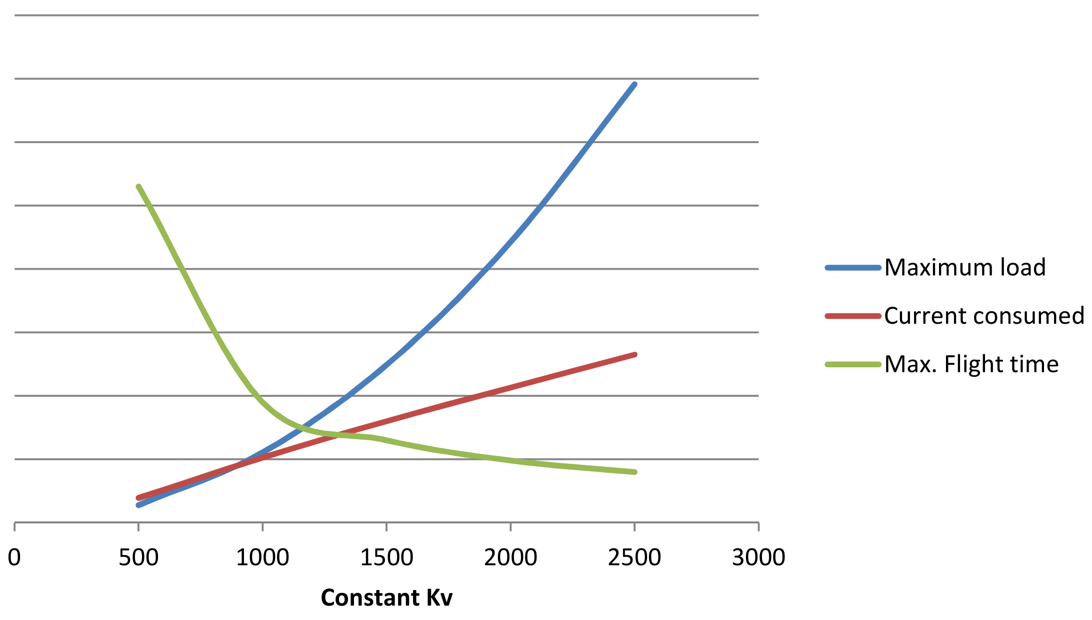

Figure 26: Evolution of the parameters affected by the constant Kv

The battery capacity is related to its weight and to the maximum flight time, as can be seen on the Figure 27.

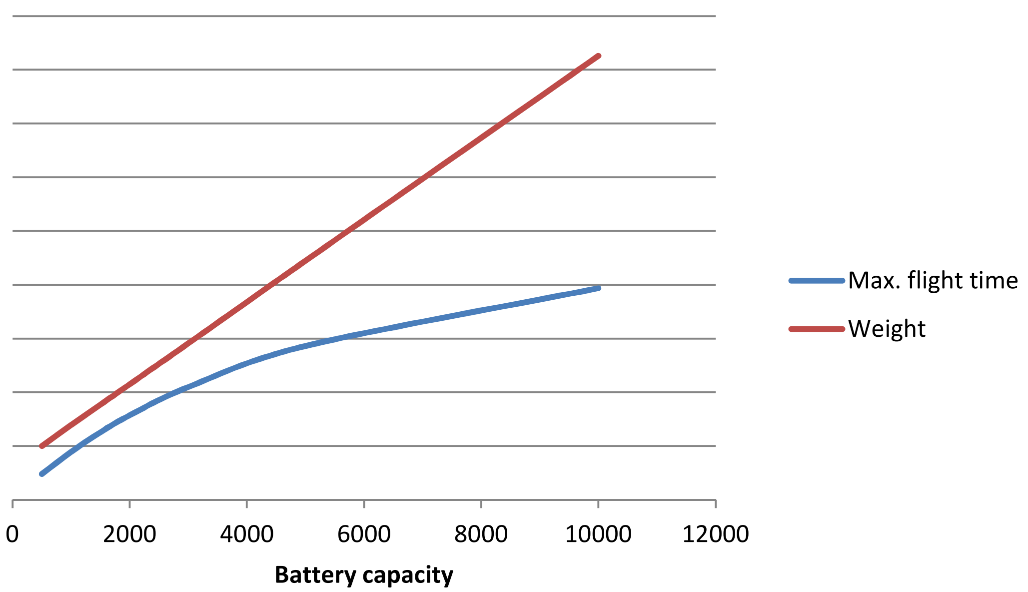

Figure 27: Evolution of the parameters affected by the battery capacity

The number of cells of the battery affects to its weight and to the maximum motor speed, but mainly to the maximum load that the quadcopter is able to lift. The evolution of these variables can be seen on the Figure 28.

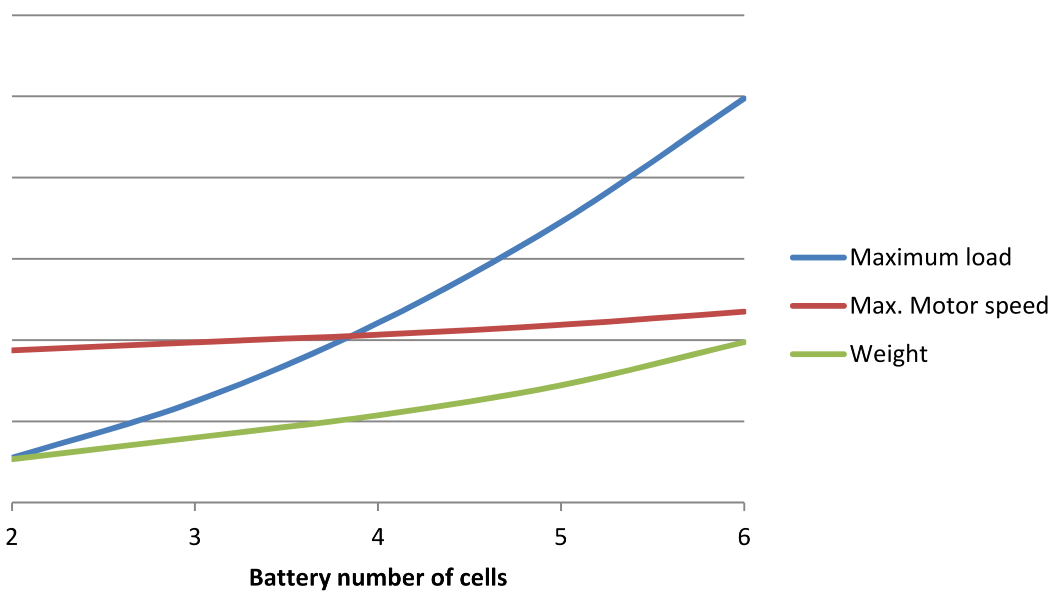

Figure 28: Evolution of the parameters affected by the battery number of cells

Bigger propellers allow lifting heavier loads, but it also leads to more power consumption. These parameters are represented on the Figure 29.

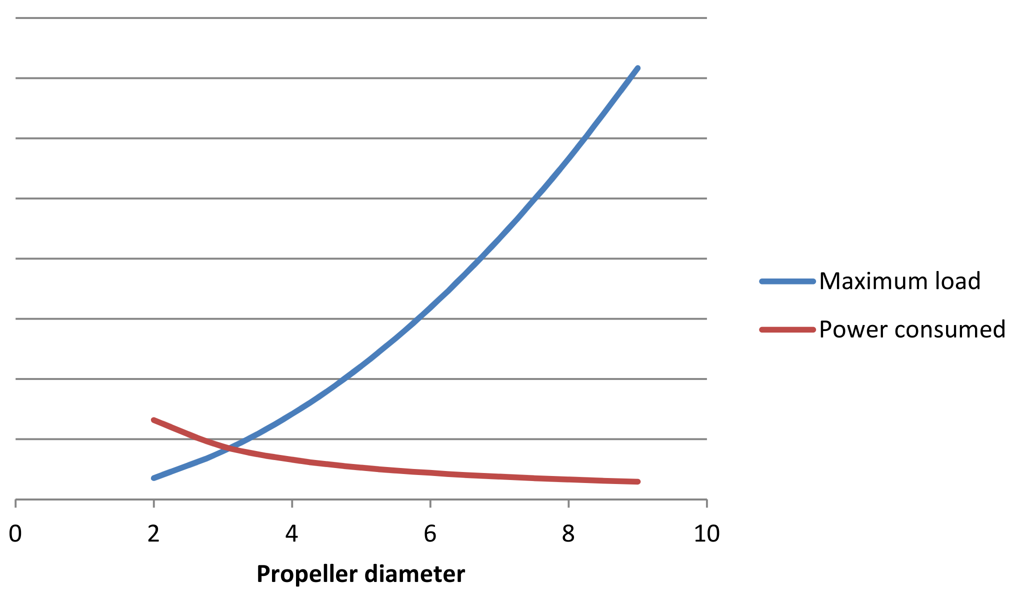

Figure 29: Evolution of the parameters affected by the propeller diameter

The propeller pitch is also involved on the maximum load that the quadcopter is able to lift. However, increasing the pitch leads to shorter flight times. This can be seen on the Figure 30.

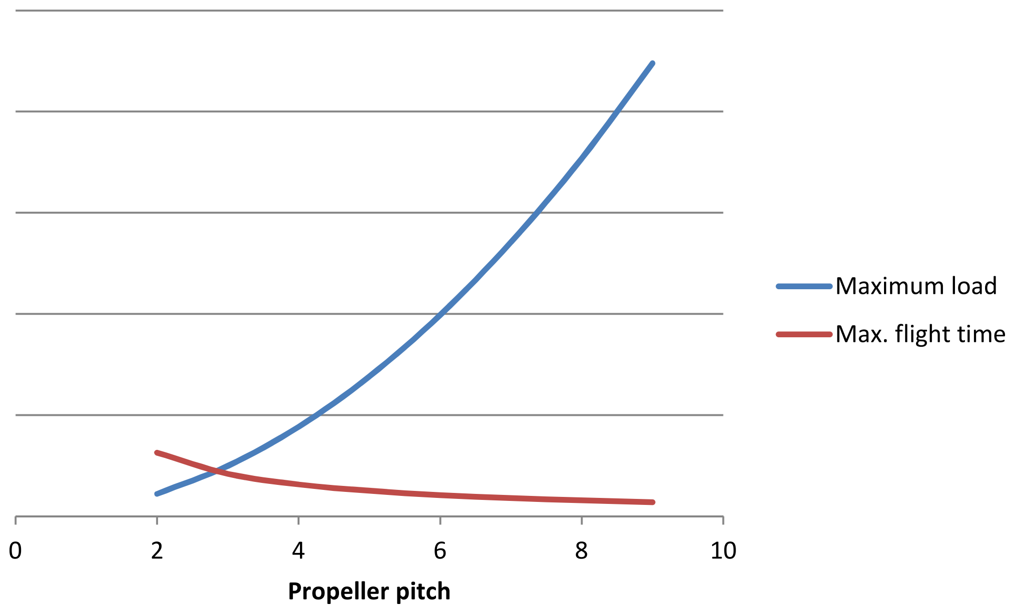

Figure 30: Evolution of the parameters affected by the propeller pitch

The dimensions of the frame have a direct effect on the weight and on the inertia momentums of the system, as can be seen on the Figure 31.

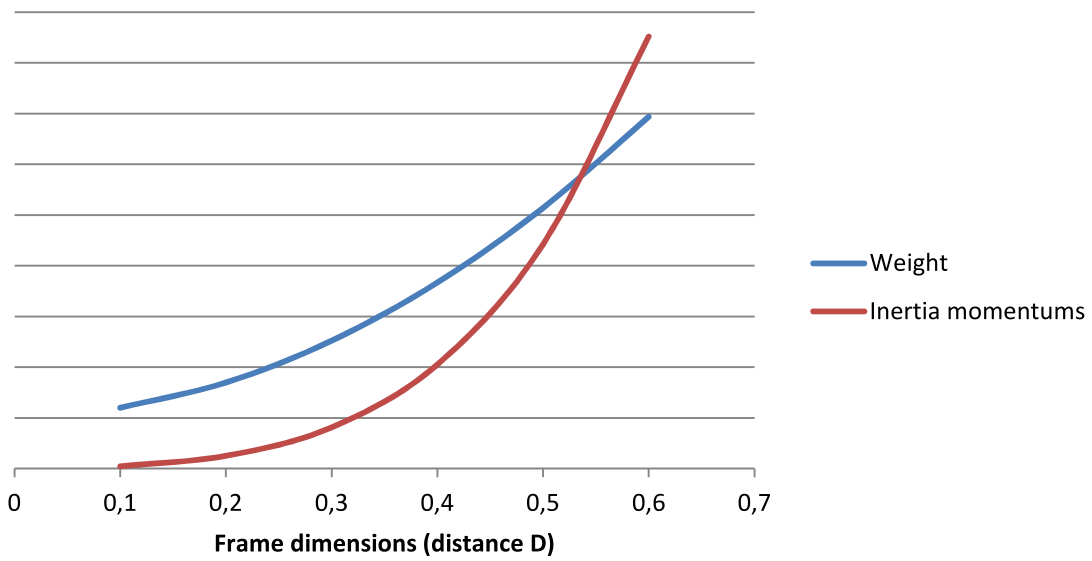

Figure 31: Evolution of the parameters affected by the frame dimensions

5. Design and construction of a real quadcopter

In order to prove the validity of the model in the designing process of a quadcopter, in this chapter it is applied to a real prototype, comparing its real flight parameters with the the simulation results.

As it is possible to face this task in many different ways, it is necessary to define first the features and the set of requirements that the quadcopter must fit. In order to simplify the process, this quadcopter must be as simple as possible, which means that it only has the basic components needed to fly.

These components are: Four brushless motors, four propellers, four Electronic Speed Controllers, a LiPo battery, a flight controller, a gyro sensor and a frame. More complex quadcopters would need more equipment but these components are enough for this prototype.

The control of the system is made through a radio transmitter-receiver system, which allows the pilot to change the quadcopter vertical thrust and the orientation angles by moving two levers. These levers are configured to transmit the signals represented on the Figure 32.

Figure 32: Control signals of the quadcopter

5.1 Chosen components

As the quadcopter does not have to fit any requirement but the simplicity, there is not a clear criterion about how to choose the components. In order to ensure a good price-quality relationship, a research on the most popular quadcopter configurations, in terms of sales in the recent years, has been done, taking into account different component suppliers. From the results, the list of components that have been considered appropriate for this prototype has been obtained, and is shown below. (www.hobbyking.com)

Motors: DYS BE1806-2300kv Brushless Motor

Features and dimensions:

- Kv: 2300 rpm/V
- Voltage: 3S-4S (11.1v to 14.8v)
- Weight: 24g
- Max. power: 89W
- Max. Current: 8A
- Motor Mount Holes: M2 x 12mm /16mm

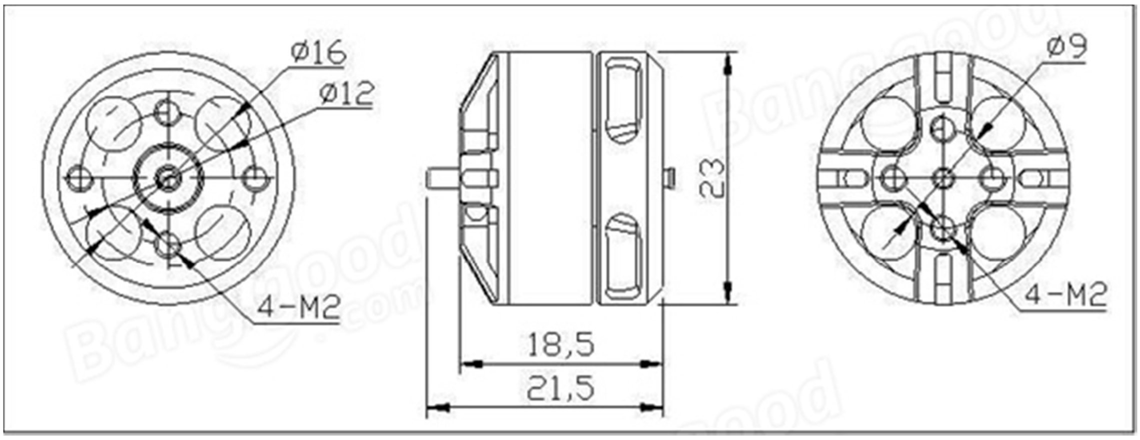

Figure 33: Brushless motor DYS BE1806-2300kv dimensions

Figure 34: Brushless motor DYS BE1806-2300kv appearance

Electronic Speed Controllers: Afro 20 Amp OPTO ESC

It must be able to deal at least with the maximum current consumed by the motors, in this case 8A. This ESC is able to provide 20A, which gives a big margin to avoid lacks of energy at peak requests or overheating problems.

Features:

- Max. current: 20A
- Voltage range: 2S-6S (7,4V to 22,2V)
- Battery elimination circuit (BEC): No. The controller cannot be fed via the ESC.
- Input frequency: 1kHz
- Weight: 26,5g
- Dimensions: 45x25x11 mm

Figure 35: ESC Afro 20 Amp appearance

Propellers: 5x4 inches (diameter and pitch)

These dimensions provide a balanced behavior between fast response, stability and power consumption. They are made of Glass Reinforced Plastic.

Figure 36: 5x4 Propellers appearance

LiPo Battery: MultiStar Racer Series 1400mAh 3S 40-80C

Features:

- Capacity: 1400mAh
- Configuration: 3S (11.1V)
- Discharge rate: 40C constant (peak 80C)
- Weight: 115g
- Dimensions: 86x34x30 mm

The battery must be able to provide the maximum current requested by the motors. In this case:

$$I_{REQUESTED} = 8 \cdot 4 = 32\ A$$

With 1400 mAh of capacity and discharge rate of 40C, the maximum current is:

$$I_{PROVIDED} = 1400 \cdot 0{,}001 \cdot 40 = 56\ A$$

$$I_{PROVIDED}\ > \ I_{REQUESTED}$$

This battery can be used to supply the chosen motors.

Figure 37: LiPo battery MultiStar Racer Series 1400mAh 3S 40-80C appearance

Transmitter and receiver system: Flysky FS T-6 Channels

The most important feature of a RX-TX system is its number of channels. This number defines how many movements or functions can be modified by the pilot. The necessary number of channels depends on the complexity of the quadcopter intended to be controlled. In this case a minimum of 4 channels are needed to control the throttle, pitch, yaw and roll. However, having more channels is desirable, in case some other functions could be added in the future. The 6-channel transmitter Flysky FS T-6 has been chosen.

Figure 38: Flysky FS T-6 Transmitter and receiver appearance

<u>Flight controller: Arduino UNO</u>

The main advantages of Arduino over other controllers are the simplicity and flexibility that it offers. The Arduino platform provides a lot of free code libraries that allow the designer to concentrate on testing the prototype, instead of spending the time building supporting circuitry or writing low level code.

Features:

- Microcontroller: ATmega328P
- Operating voltage: 5V
- Input Voltage (recommended): 7-12V
- Digital I/O pins: 14
- PWM digital I/O pins: 6
- Analog input pins: 6
- DC current per I/O pin: 20mA
- Clock speed 16 MHz
- Dimensions: 68,6x53,4 mm
- Weight: 25g

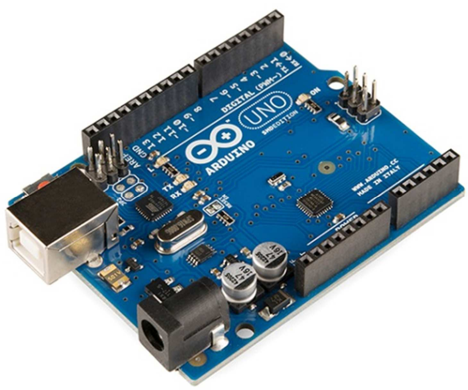

Figure 39: Arduino UNO board appearance

Gyro sensor: Adafruit L3GD20H

A gyroscope is a type of sensor that can sense twisting and turning motions. This breakout board is based around the latest gyro technology, with three full axes of sensing. The chip can be set to ±250, ±500, or ±2000 degree-per-second scale for a large range of sensitivity. There is also built in high and low pass sensing to make data processing easier. The chip supports both I2C and SPI so it is possible to interface with any microcontroller easily.

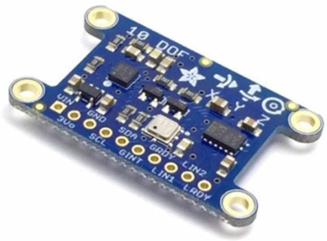

Figure 40: Gyro sensor Adafruit L3GD20H

5.2 Electric schematic

The electric schematic of the system can be seen on the Figure 41. It can be appreciated that the ESCs of motors 2 and 4 are connected differently than in motors 1 and 3, as they must spin counterclockwise.

All the Arduino pins connections can be seen on the table 12.

Pin		Connection
Analog	*A0*	-
	A1	-
	A2	-
	A3	-
	A4	SDA pin of Gyro
	A5	SCL pin of Gyro
Digital	*D0*	-
	D1	-
	D2	-
	D3	-
	D4	Data pin of ESC 1
	D5	Data pin of ESC 2
	D6	Data pin of ESC 3
	D7	Data pin of ESC 4
	D8	Channel 1 of receiver
	D9	Channel 2 of receiver
	D10	Channel 3 of receiver
	D11	Channel 4 of receiver
	D12	-
	D13	-
	GND	• GND pin of ESC 1 • GND pin of ESC 2 • GND pin of ESC 3 • GND pin of ESC 4
Power	*IOREF*	-
	RESET	-
	3,3V	-
	5V	• Vcc pin of Gyro • Vcc pin of receiver
	GND	• Terminal "-" of battery • GND of Gyro • GND of receiver
	Vin	Terminal "+" of battery

Table 11: Arduino pins connections in the quadcopter prototype

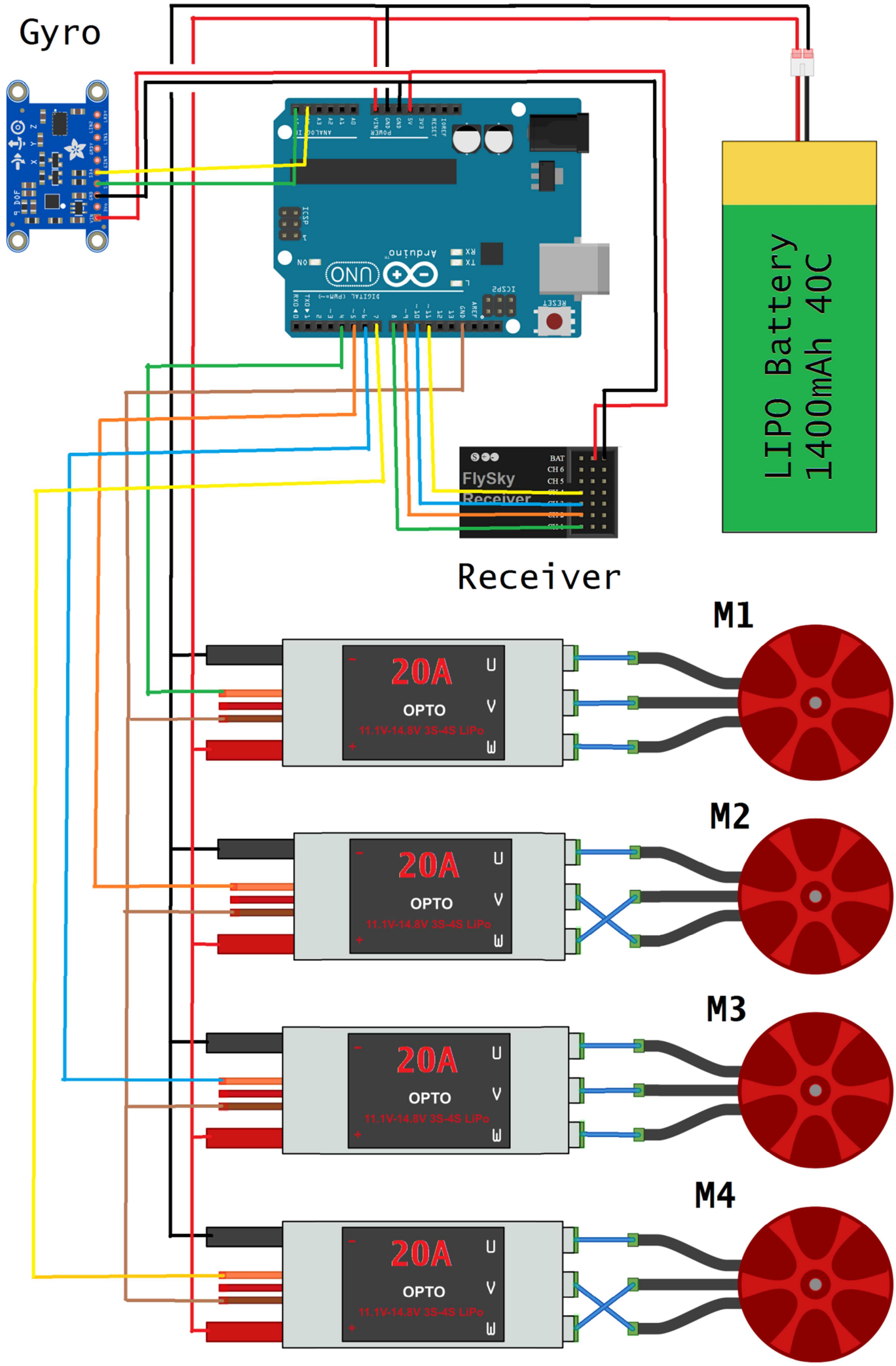

Figure 41: Electric installation of the quadcopter schematics

5.3 3-D printed frame

The Quadcopter frame is designed using the computer program *Autodesk Inventor*. From this virtual model, every part of the frame can be obtained through a 3d printer.

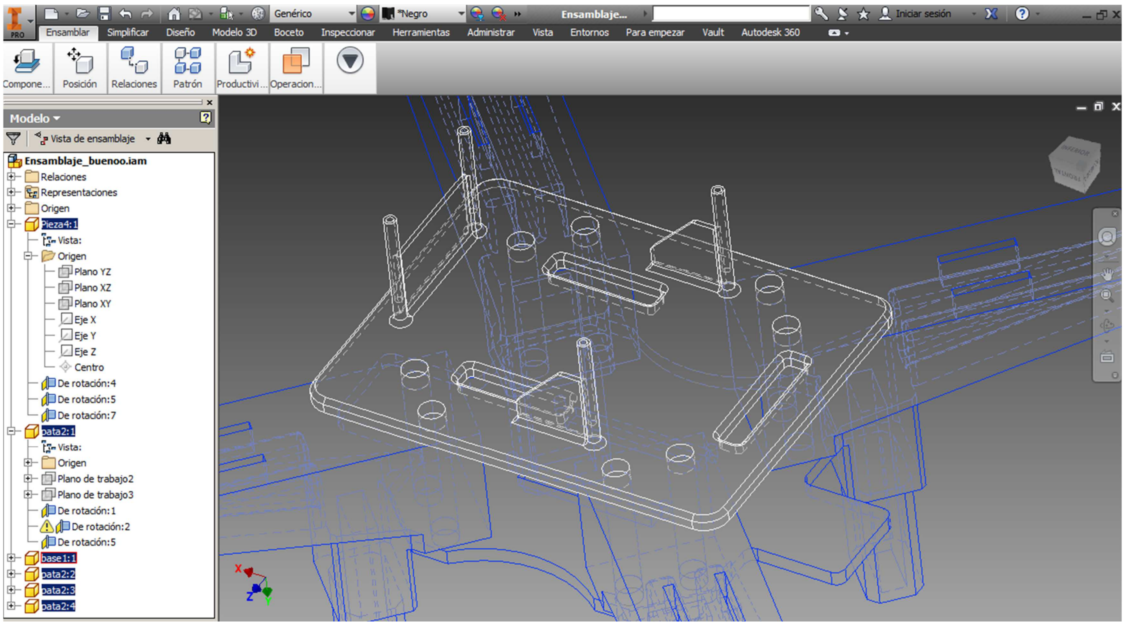

Figure 42: Autodesk Inventor screenshot

A middle-sized frame has been considered to be the most appropriate size for the quadcopter, as more little frames may be weaker and bigger frames are heavier. A distance of 15 cm between each motor and the system center has been assumed.

The designed frame is composed by six pieces: Four arms, one top base and one bottom base. Their appearance and main dimensions can be seen on the figures 43, 44 and 46. The assembly of all the parts together is done with eight M5x60 bolts.

Arms

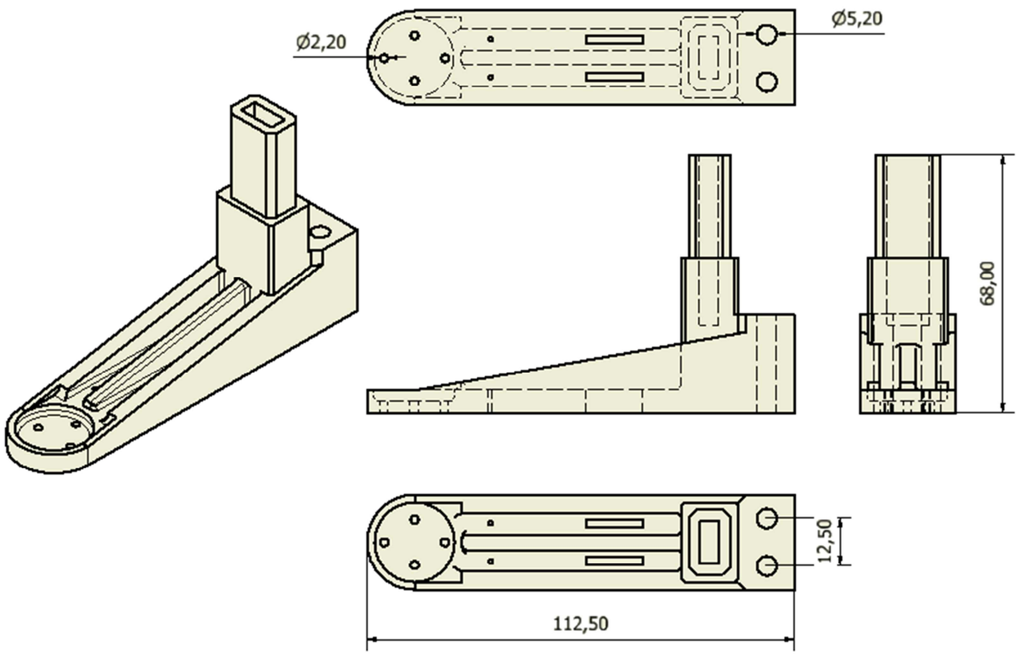

Figure 43: Quadcopter arm dimensions

Each motor is assembled to the arm with four M2x5 bolts, which fit into the four small drills. The two big drills are used to fix each arm between the bases, using M5x60 bolts.

Top base

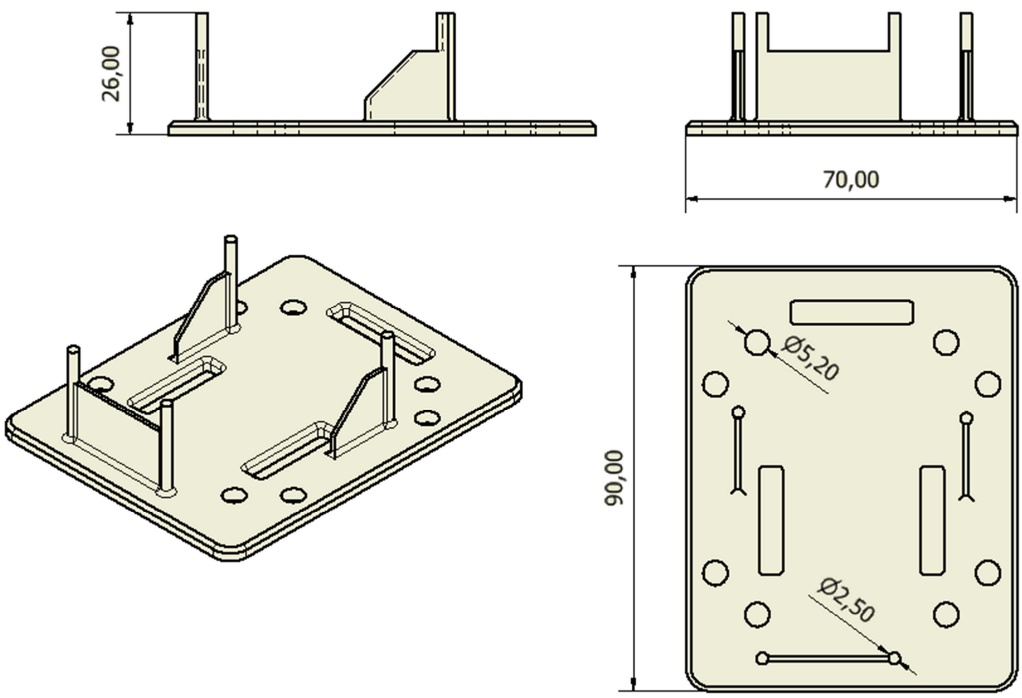

Figure 44: Quadcopter top base dimensions

The four elevated cylinders fit into the Arduino UNO board holes, in order to fix it to the frame. The Figure 45 shows the localization of these holes.

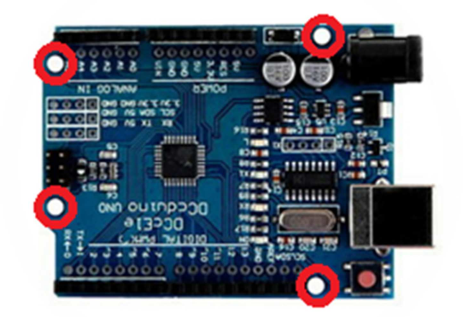

Figure 45: Drills in the Arduino UNO board

Bottom base

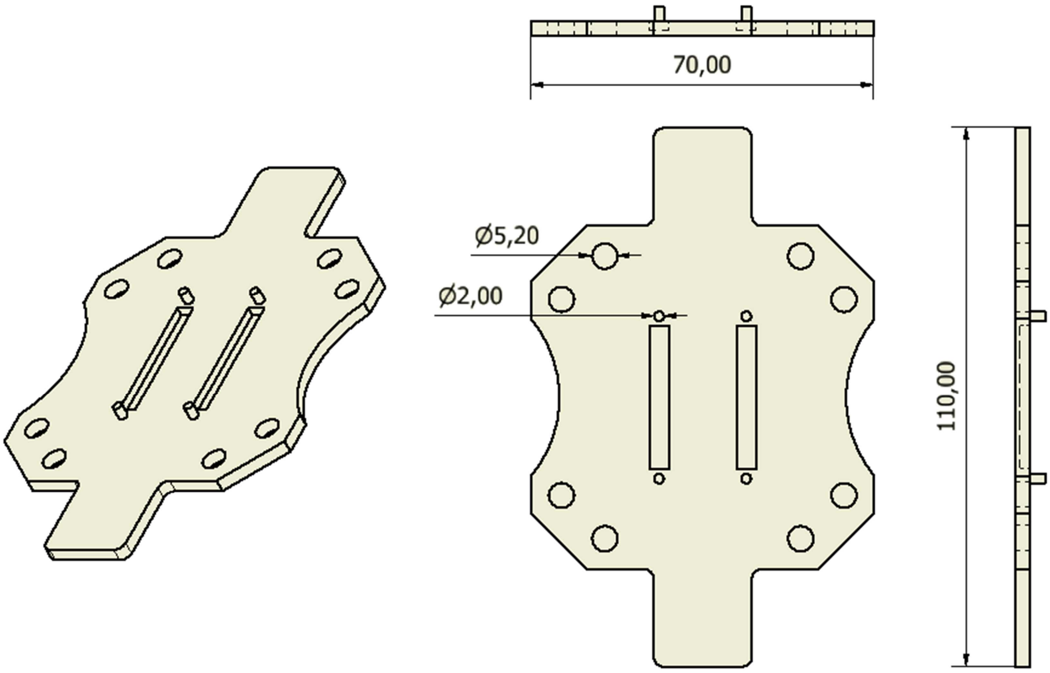

Figure 46: Quadcopter bottom base dimensions

In this case, the four little cylinders fit into the gyro sensor board holes, which can be seen on the Figure 47.

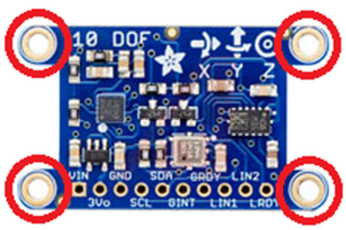

Figure 47: Drills in the gyro sensor board

Assembled frame

Once all the pieces are assembled together, the final result appearance can be seen on the Figures 48 and 49.

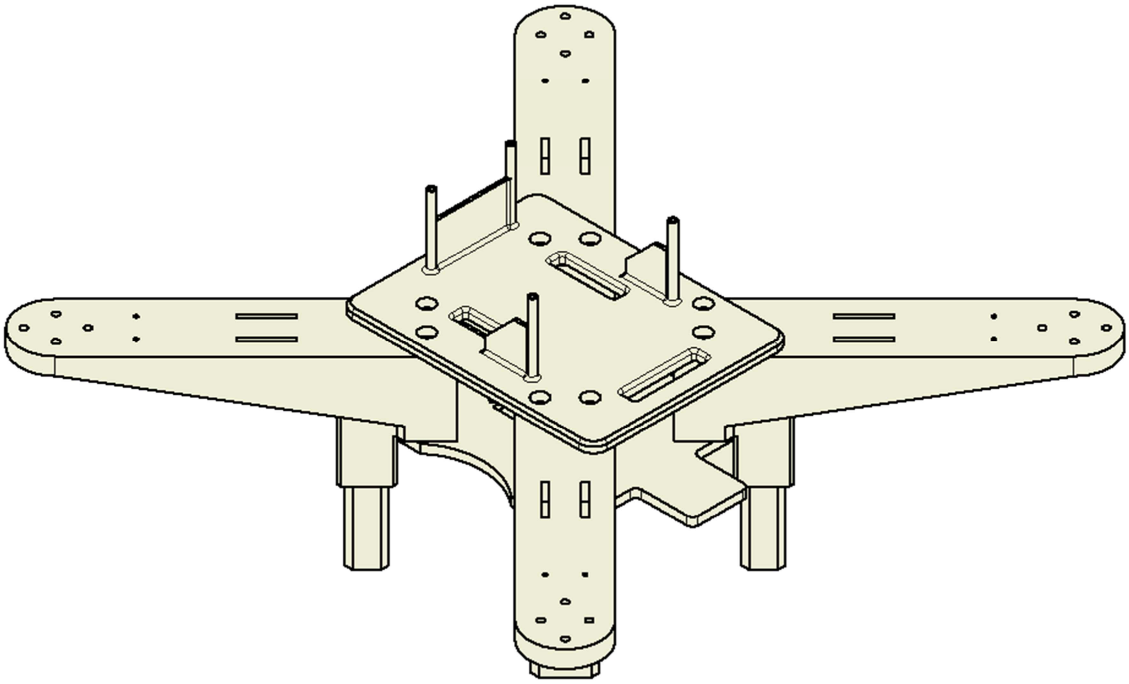

Figure 48: Top view of the assembled frame

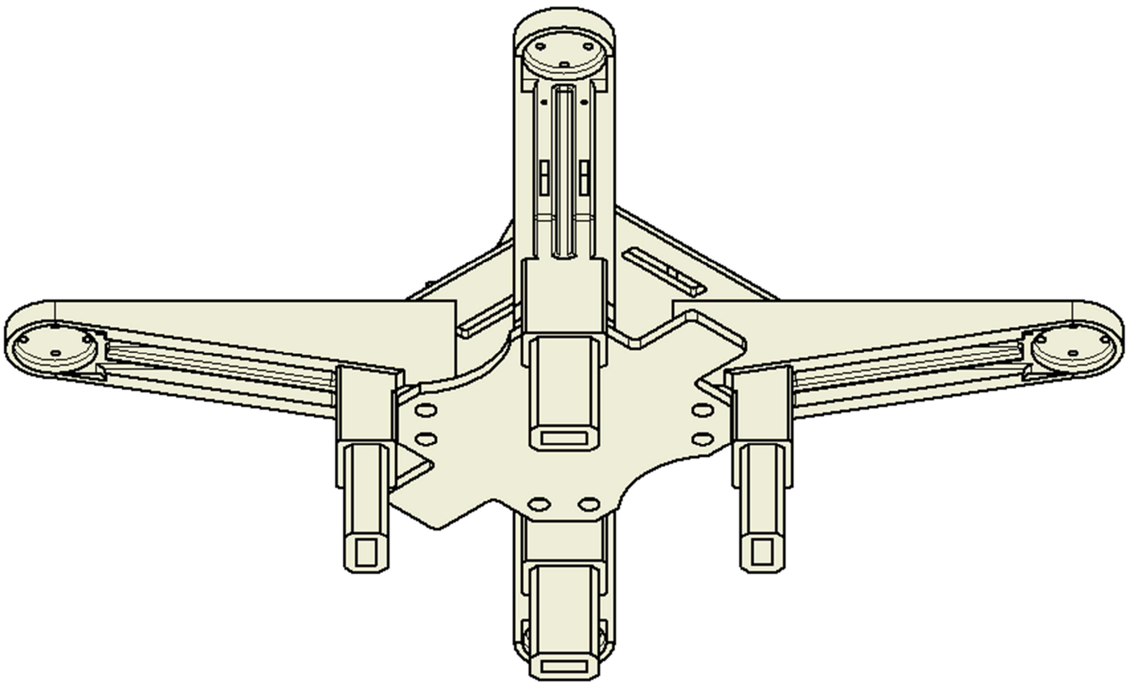

Figure 49: Bottom view of the assembled frame

5.4 Control loop

To control the system, three PID controllers are used, one per each angle of movement. The quadcopter movement equations, that relate these angles with the forces and momentums made by each motor, were already deduced in the chapter 3.

$$\ddot{\phi} = \frac{(F_4 + F_3 - F_1 - F_2) \cdot d}{J_x}$$

$$\ddot{\theta} = \frac{(F_4 + F_1 - F_3 - F_2) \cdot d}{J_y}$$

$$\ddot{\psi} = \frac{M_1 + M_3 - M_2 - M_4}{J_z}$$

A PID controller continuously calculates the error between a desired set point and the real state of a measured variable. It attempts to minimize this error over the time by adjusting a control variable. In this case, the desired set point for each angle is given by the pilot through the wireless transmitter, while the gyro signal provides the current orientation angles. A scheme of the PID signals can be seen on the Figure 50. (Ramos, 2007)

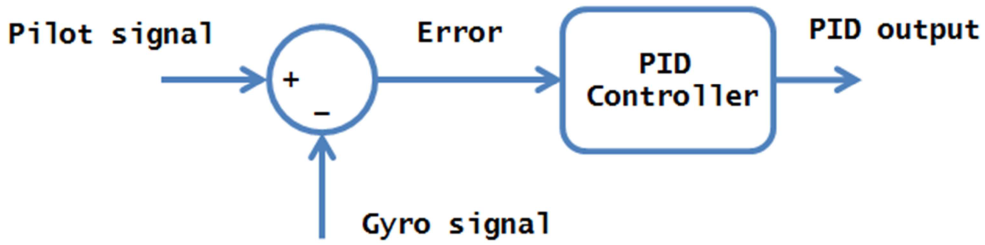

Figure 50: PID controller signals

The system variable that can be adjusted by the controllers is the input voltage received by each motor. These voltages do not depend only on the PID's output signals but also on the throttle. According to the movement equations, and in order to compensate the effect that each rotor has on the orientation angles, the four signals that control the voltage of each motor result in:

$$Voltage\ M1 = THROTTLE - PID_{ROLL_{OUT}} + PID_{PITCH_{OUT}} + PID_{YAW_{OUT}}$$

$$Voltage\ M2 = THROTTLE - PID_{ROLL_{OUT}} - PID_{PITCH_{OUT}} - PID_{YAW_{OUT}}$$

$$Voltage\ M3 = THROTTLE + PID_{ROLL_{OUT}} - PID_{PITCH_{OUT}} + PID_{YAW_{OUT}}$$

$$Voltage\ M4 = THROTTLE + PID_{ROLL_{OUT}} + PID_{PITCH_{OUT}} - PID_{YAW_{OUT}}$$

To check that these expressions are correct, and the quadcopter responses accord with the expected behavior, a virtual model of the system movement is built in order to perform some simulations.

5.4.1 Simulation

A MATLAB/Simulink model of the quadcopter movements is built. By changing the set point of the orientation angles, it is possible to visualize how the system reacts and asses if the result matches with the expectations.

In order to make the simulation as realistic as possible, it is necessary to introduce as well some system variables, such as the components technical parameters and the external conditions.

The interface appearance of this model can be seen on the Figure 51.

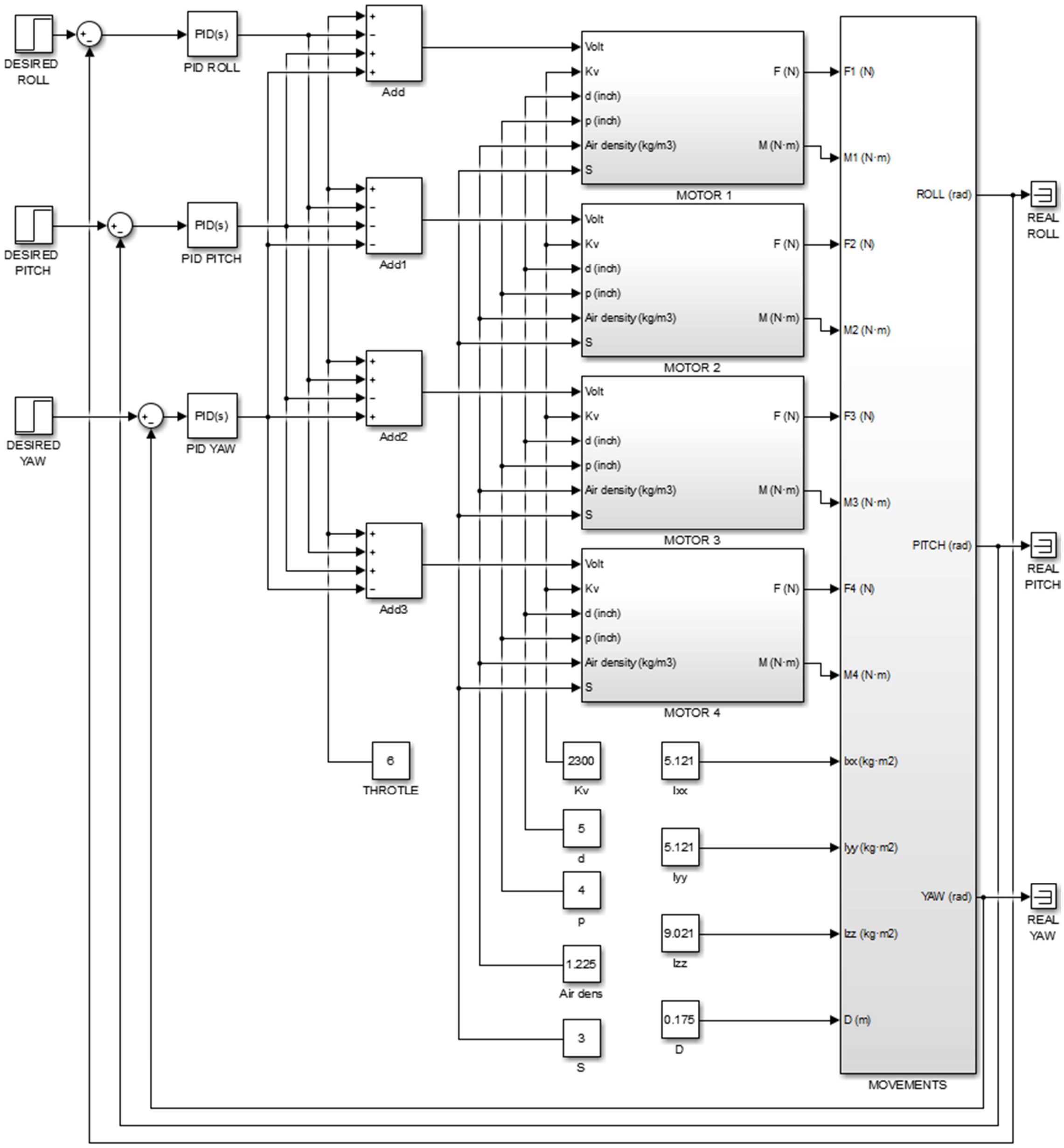

Figure 51: Simulink model for the control loop simulation

The operations under the interface mask of each subsystem can be seen on the Annex B, at the end of the document.

It is not necessary to set manually the PID parameters, as the program includes a tool that tunes them automatically to provide the best response for each model. is obtained.

Once all the variables are fixed, it is possible to visualize how the quadcopter orientation angles change when the set points of the desired angels are modified. The results of this simulation can be seen on the Figure 52.

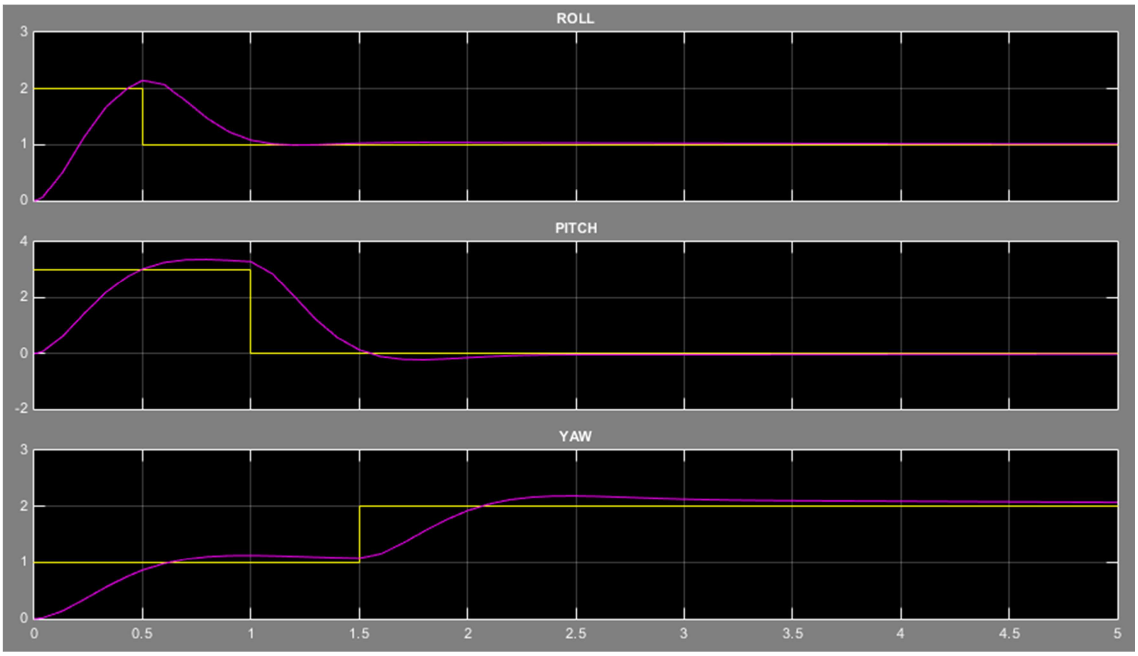

Figure 52: Control loop simulation results

Being the yellow line the desired set point and the purple one the real angle, it can be appreciated that the quadcopter acts as it is expected and shows a fast reaction without excessive overshoot or oscillations. It can be concluded that the designed control loop works properly and is suitable to be implemented in the real prototype.

5.4.2 Implementation

The PID output is composed by three signals: Proportional, integral and derivative. These expressions are obtained as can be seen on the Figure 53, being "K_P", "K_I" and "K_D" the proportional, integral and derivative constants.

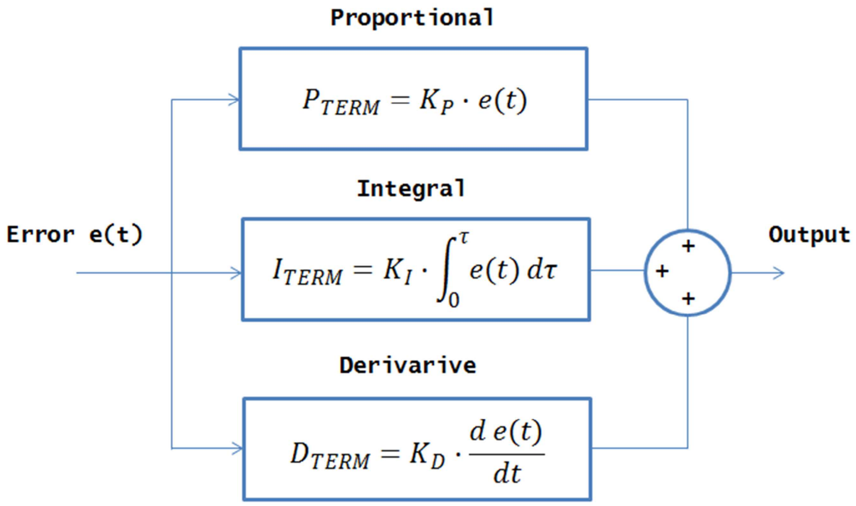

Figure 53: PID controller proportional, integral and derivative terms

In the controller, these values must be updated every clock cycle. An approximation of these terms is done in order to implement them in the program code.

$$Proportional\ term = (Gyro\ signal - Receiver\ signal) \cdot K_P$$

$$Integral\ term = (Int.term)_{Previous\ cycle} + [(Gyro\ signal - Receiver\ signal) \cdot K_I]$$

$$Derivative\ term = \left[(Gyro\ signal - Receiver\ signal) - (Error)_{Previous\ cycle}\right] \cdot K_D$$

The P, I and D terms have different effects on the final system performance, and need to be tuned by adjusting the constants K_P, K_I and K_D. Finding the balance between these three parameters is essential to obtain an effective system response, but it is not easy.

In this case the tuning process has been made empirically. The following steps have been followed.

1. To obtain K_D, being K_P and K_I equal to zero, it is increased until the point that the quadcopter starts oscillating at constant amplitude.
2. To obtain K_P, being K_D the value previously obtained and K_I still equal to zero, it is increased until the point that the quadcopter starts overcompensating its movements.
3. To obtain K_I, it is increased until the point that the quadcopter starts oscillating.

The obtained parameters can be seen on the Table 13.

	K_P	K_I	K_D
PID Roll	1,5	0,05	15
PID Pitch	1,5	0,05	15
PID Yaw	4	0,02	0,1

Table 12: Quadcopter PID's parameters

5.5 Assembly and results

Once the frame is assembled, all the components are mounted and the control program is implemented into the controller, the quadcopter is ready to fly. The appearance of the assembled frame and the final quadcopter can be seen on the figures 54 and 55.

Figure 54: Appearance of the assembled frame

Figure 55: Appearance of the built quadcopter

For the chosen components, the flight behavior provided by the Simulink model can be seen on the table 14.

Inputs			**Outputs**	
Motor	*Kv*	2300	*Total mass*	0,512 kg
	Max. Power	89 W	*Max. extra weight that is possible to lift*	5,6 kg
	Mass	52 g	*Motor speed*	7500 rpm
Battery	*S*	3	*Voltage per motor*	3,3 V
	Capacity	1400 mAh	*% ESC*	29,59%
	C	40	*Current consumed per motor*	2,43 A
Propeller	*Diameter*	5 inch	*Max. Discharge current*	56 A
	Pitch	4 inch	*Power consumed per motor*	8,02 W
Frame	*Distance D*	0,15 m	*Max. flight time*	8,59 min
	Density	1200 kg/m^3	*Max. vertical speed*	33,83 m/s
	Extra mass	0 kg	*Max. horizontal speed*	35,42 m/s
			Max. inclination angle	84,98º
			Ix	0,00272 $kg \cdot m^2$
			Iy	0,00272 $kg \cdot m^2$
			Ix'	0,00271 $kg \cdot m^2$
			Iy'	0,00271 $kg \cdot m^2$
			Iz	0,00538 $kg \cdot m^2$

Table 13: Real quadcopter Simulink model results

After some flight tests, the correct performance of the quadcopter is proved and some flight parameters are measured. The average flight time, assuming that the battery is completely charged when presents a voltage of 12,6V and completely discharged when presents a voltage of 9,3V, is 7,8 minutes. As it was expected, this value is under the maximum time predicted by the model. The total system weight is 578 grams, which is quite close to the estimated mass.

It has been impossible to measure other parameters such as the power consumed by the motors, the maximum speeds or the inertia momentums.

At this point, it is possible to conclude that the Simulink model provides realistic estimations for the flight parameters of a quadcopter.

6. Conclusions

On the first part of the thesis, the state of the art of the quadcopters industry nowadays has been studied. The principal application scenarios have been identified and the set of requirements in each case has been defined, providing real commercial examples. The importance of this industry and its potential growth in the next years has resulted evident, and the necessity of developing new techniques and models to improve this technology has been justified.

The second part of the thesis focuses on the creation of a virtual model to support the designing process of a quadcopter and help to optimize its parameters depending on its particular scenario of use. A mathematical model of the system has been obtained by studying the different relationships between the components features and the flight behavior. From this mathematical approach, a MATLAB/Simulink model has been built and applied to some example cases, in order to prove its validity. In addition, the effects that the variations of each technical parameter produce on the system performance have been explained and graphically represented through dimensionless curves.

Finally, in order to compare the simulation results with real-world measures, a simple quadcopter prototype has been built. On the third part of the thesis, the designing process of this quadcopter, as well as its main characteristics, are explained. Another virtual model has been created to simulate the control program of the quadcopter, providing satisfactory results. After some test flights, the quadcopter performance has proved to be correct and match the expectations. Despite the fact that it has been impossible to compare all the system parameters due to the lack of proper measuring devices, the results of the simulation has been considered realistic and therefore the goal of this thesis has been reached.

On future study lines, this work could be continued by improving the model accuracy with more detailed mathematical descriptions of the components. It would be also interesting to implement a 3-D animation to visualize the quadcopter movements on the simulation.

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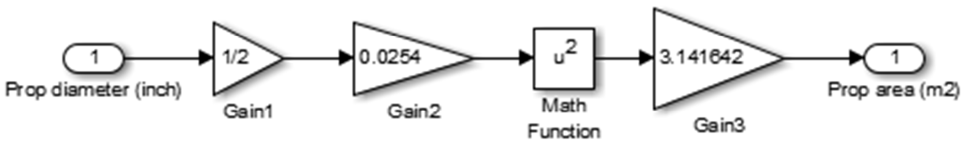
8. Annexes

A. Subsystems of the flight behavior Simulink model

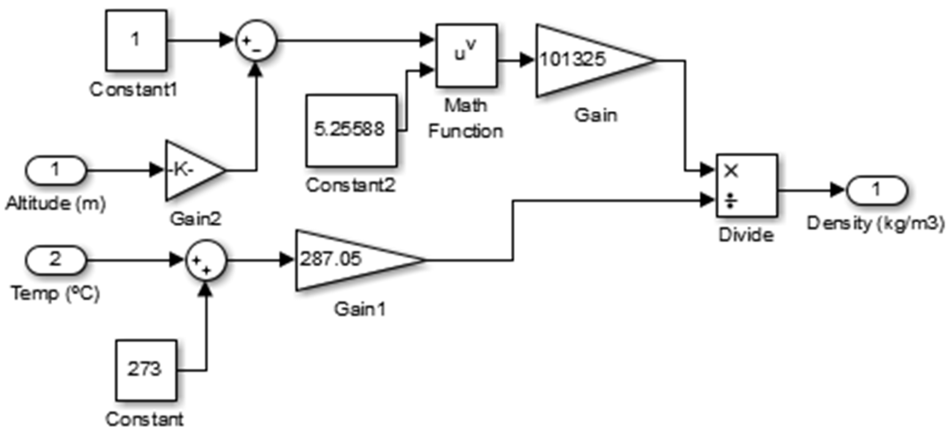
Subsystem 0

Inputs	Outputs
• *Propeller diameter*	• *Propeller area*



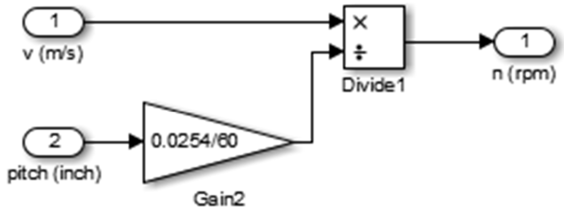
Subsystem 1

Inputs	Outputs
• *Altitude* • *Temperature*	• *Air density*



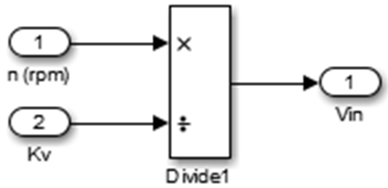
Subsystem 2

Inputs	Outputs
• *Airflow velocity* • *Propeller pitch*	• *Motor angular speed*



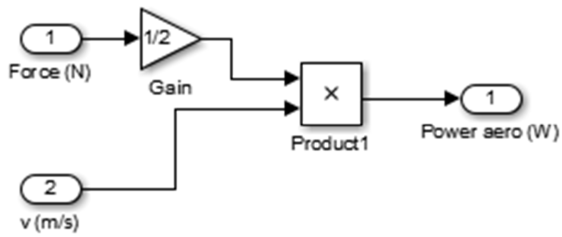
Subsystem 3

Inputs	Outputs
• *Motor angular speed* • *Constant Kv*	• *Motor input voltage*



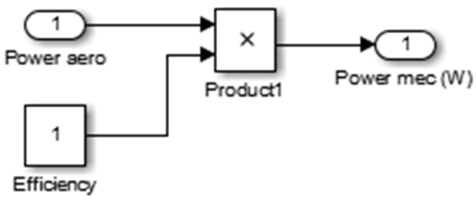
Subsystem 4

Inputs	Outputs
• *Thrust force made by each propeller* • *Airflow velocity*	• *Aerodynamic power*



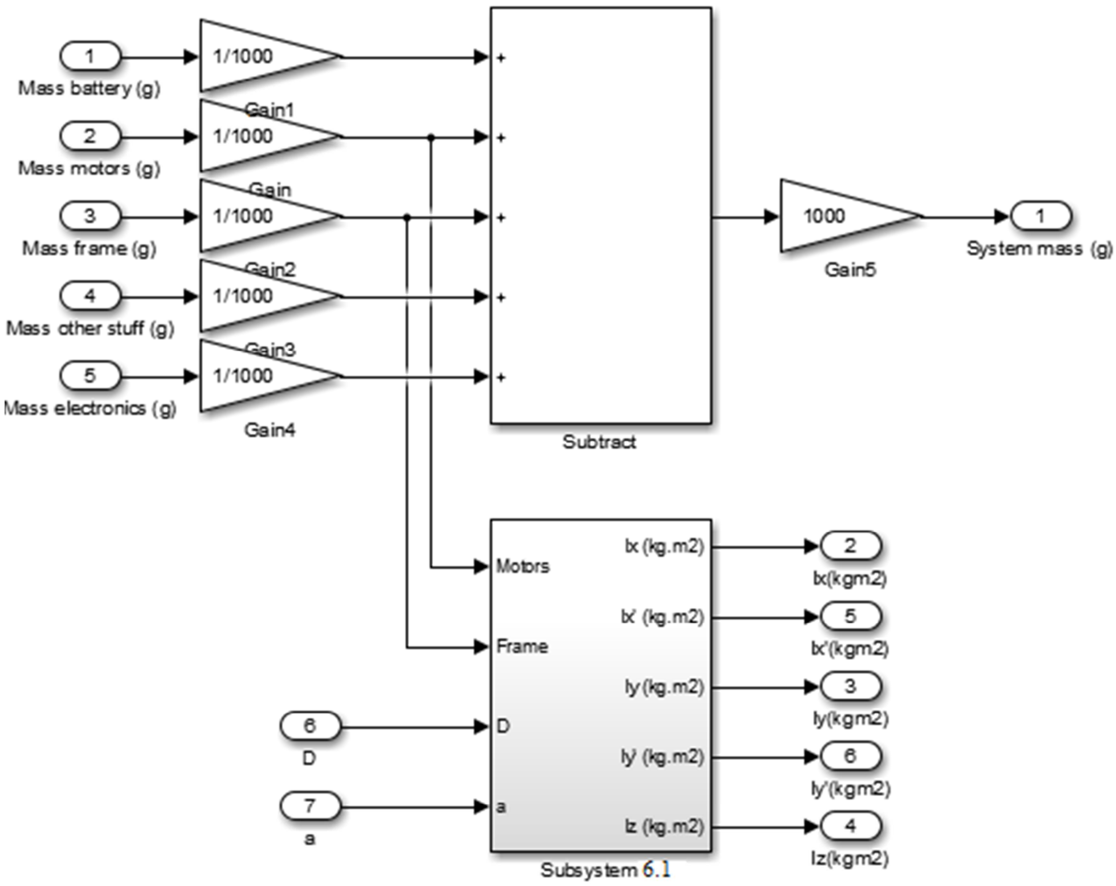
Subsystem 5

Inputs	Outputs
• *Aerodynamic power*	• *Mechanic power*



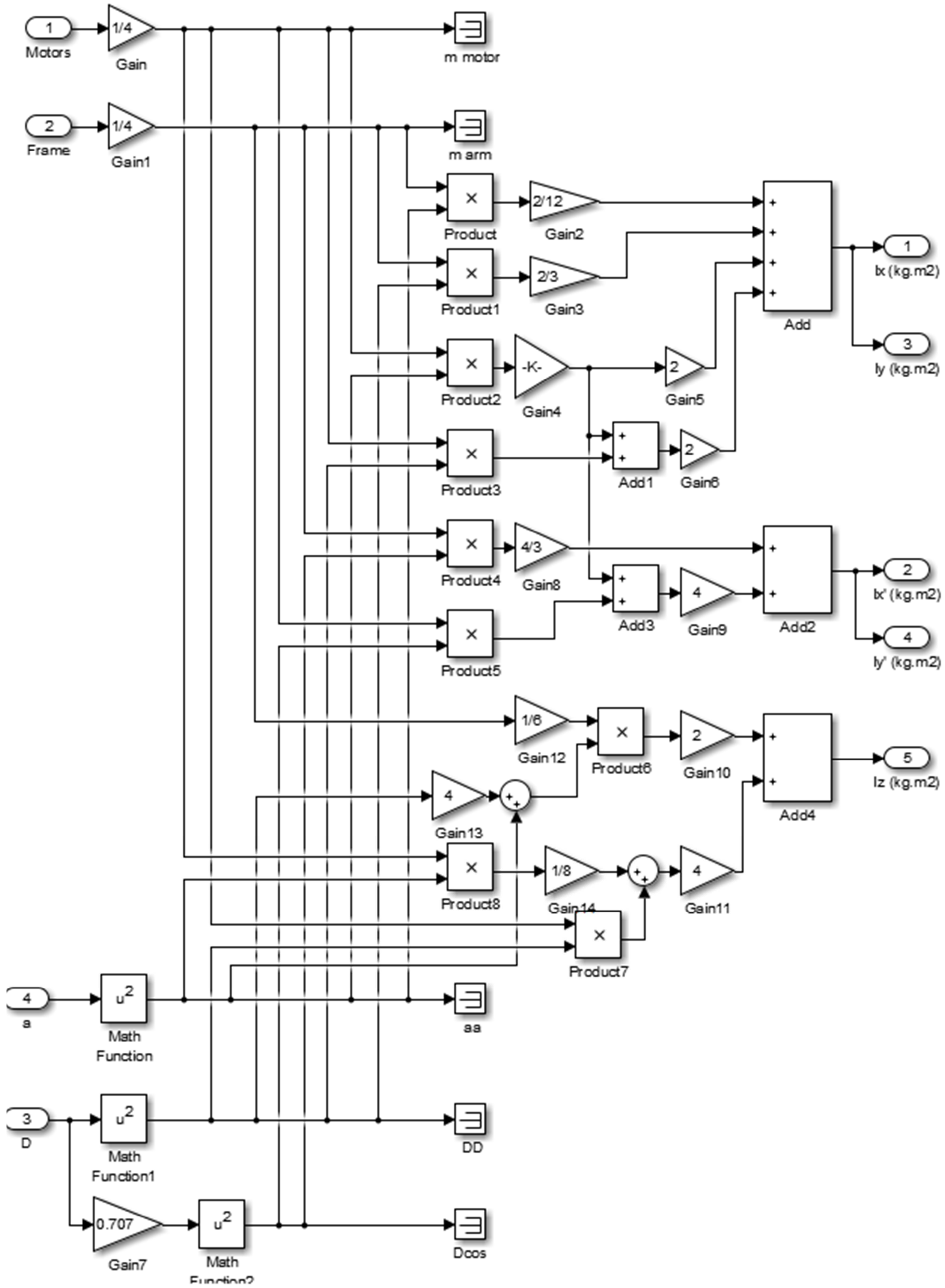
<u>Subsystem 6</u>

Inputs	Outputs
• *Battery mass* • *Motor mass* • *Frame mass* • *Other stuff mass* • *Electronics mass* • *Distance D* • *Distance a*	• *Total mass* • *Inertia momentums*



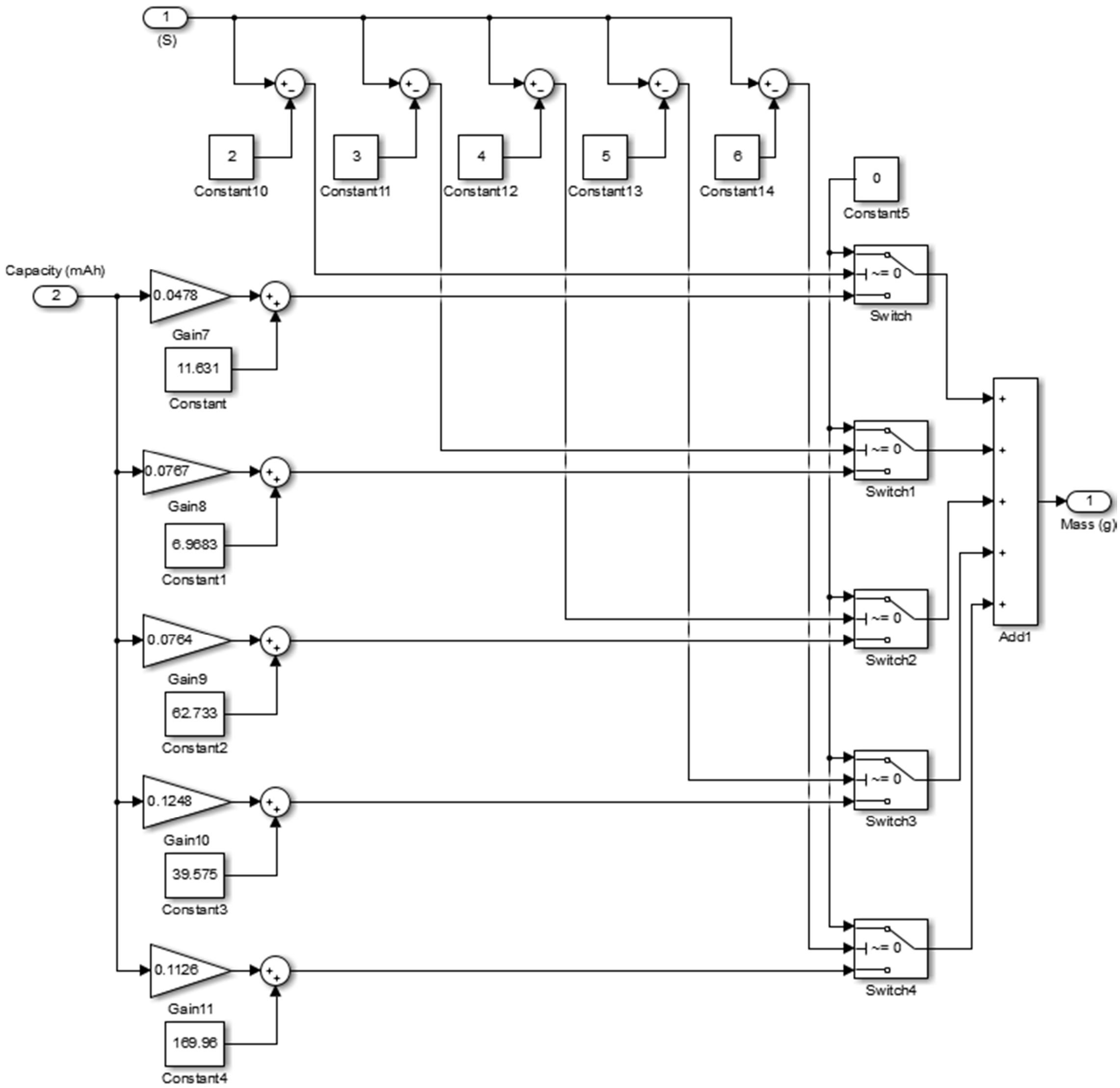
<u>Subsystem 6.1</u>

Inputs	Outputs
• *Motor mass* • *Frame mass* • *Distance D* • *Distance a*	• *Inertia momentums*


1
Motors
1/4
Gain
2
Frame
1/4
Gain1
m motor
m arm
Product
2/12
Gain2
Product1
2/3
Gain3
Product2
-K-
Gain4
2
Gain5
Product3
Add1
2
Gain6
Product4
4/3
Gain8
Add3
4
Gain9
Product5
Add
1
Ix (kg.m2)
3
Iy (kg.m2)
Add2
2
Ix' (kg.m2)
4
Iy' (kg.m2)
1/6
Gain12
Product6
2
Gain10
4
Gain13
Add4
5
Iz (kg.m2)
Product8
1/8
Gain14
4
Gain11
Product7
4
a
Math
Function
aa
3
D
Math
Function1
DD
0.707
Gain7
Math
Dcos

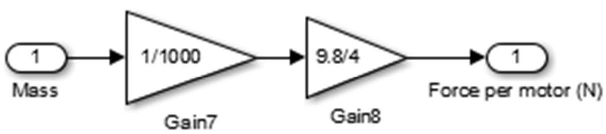
Subsystem 7

Inputs	Outputs
• *Battery cells number (S)* • *Battery capacity*	• *Battery mass*



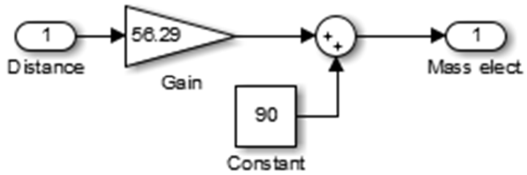
Subsystem 8

Inputs	Outputs
• *Total mass*	• *Thrust force made by each propeller*



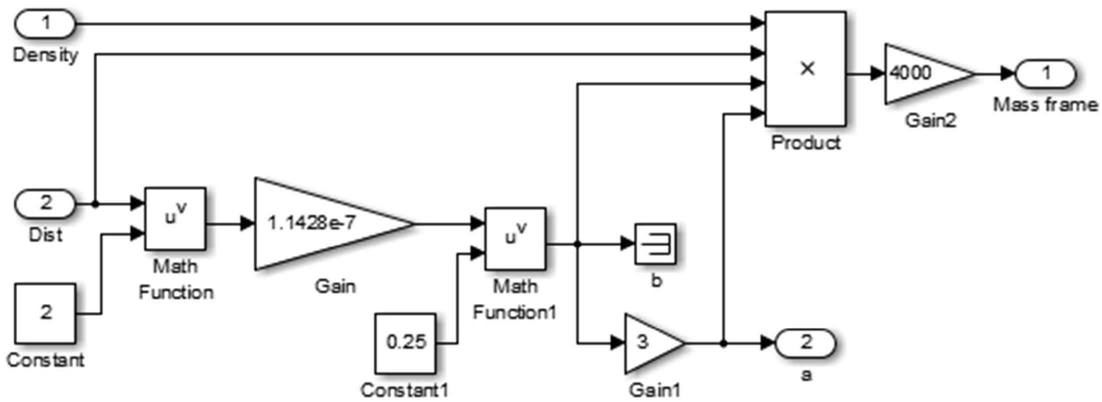
Subsystem 9

Inputs	Outputs
• *Distance D*	• *Electronics mass*



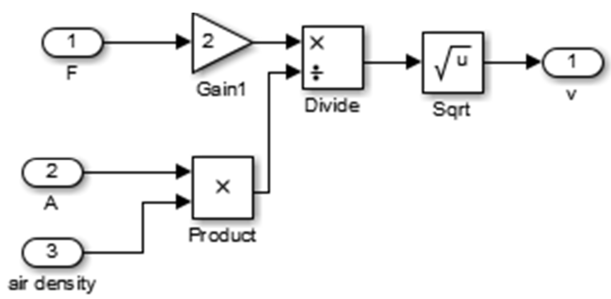
Subsystem 10

Inputs	Outputs
• *Frame density* • *Distance D*	• *Frame mass*



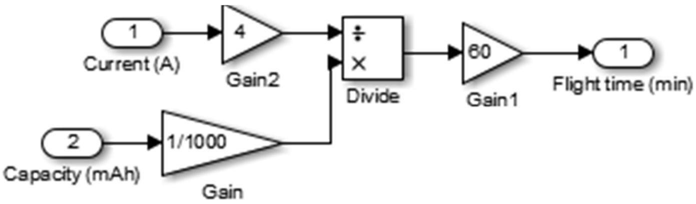
Subsystem 11

Inputs	Outputs
• *Thrust force made by each propeller* • *Propeller area* • *Air density*	• *Airflow velocity*



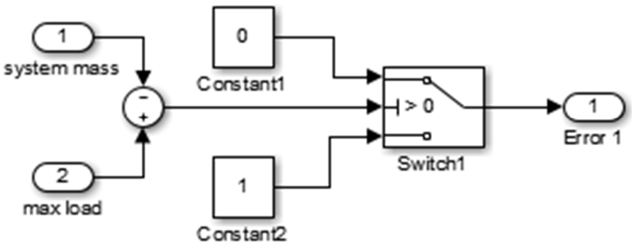
Subsystem 12

Inputs	Outputs
• *Motor input current* • *Battery capacity*	• *Flight time*



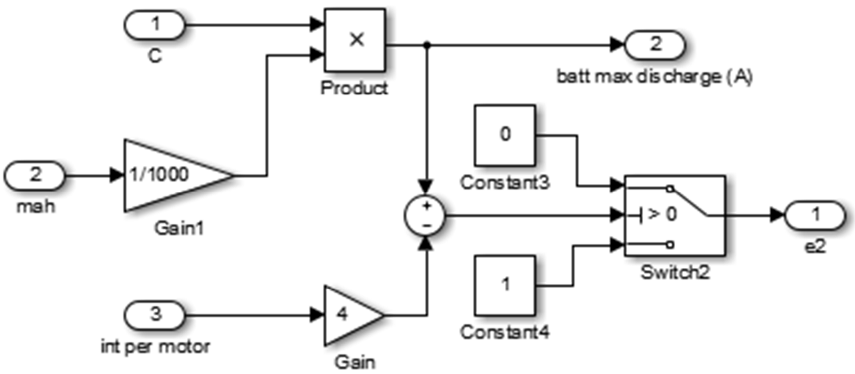
Subsystem 13

Inputs	Outputs
• *Total mass* • *Max. load to lift*	• *Error 1*



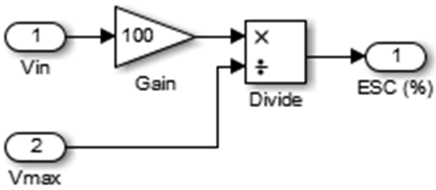
Subsystem 14

Inputs	Outputs
• *Dischrage rate (C)* • *Battery capacity* • *Motor input current*	• *Error 2* • *Battery max. Discharge current*



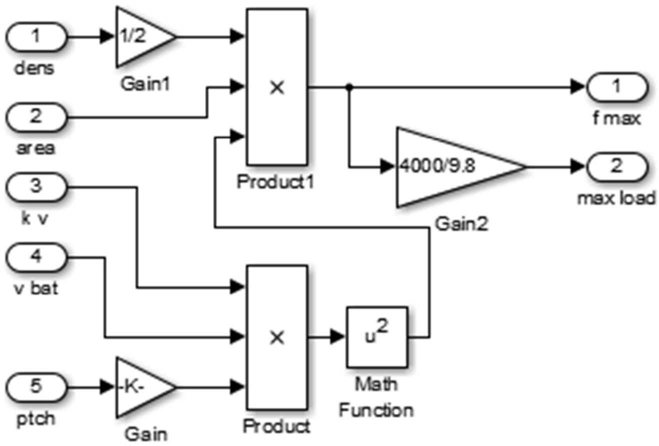
<u>Subsystem 15</u>

Inputs	Outputs
• *Motor input voltage* • *Battery voltage*	• *ESC pwm percentage*



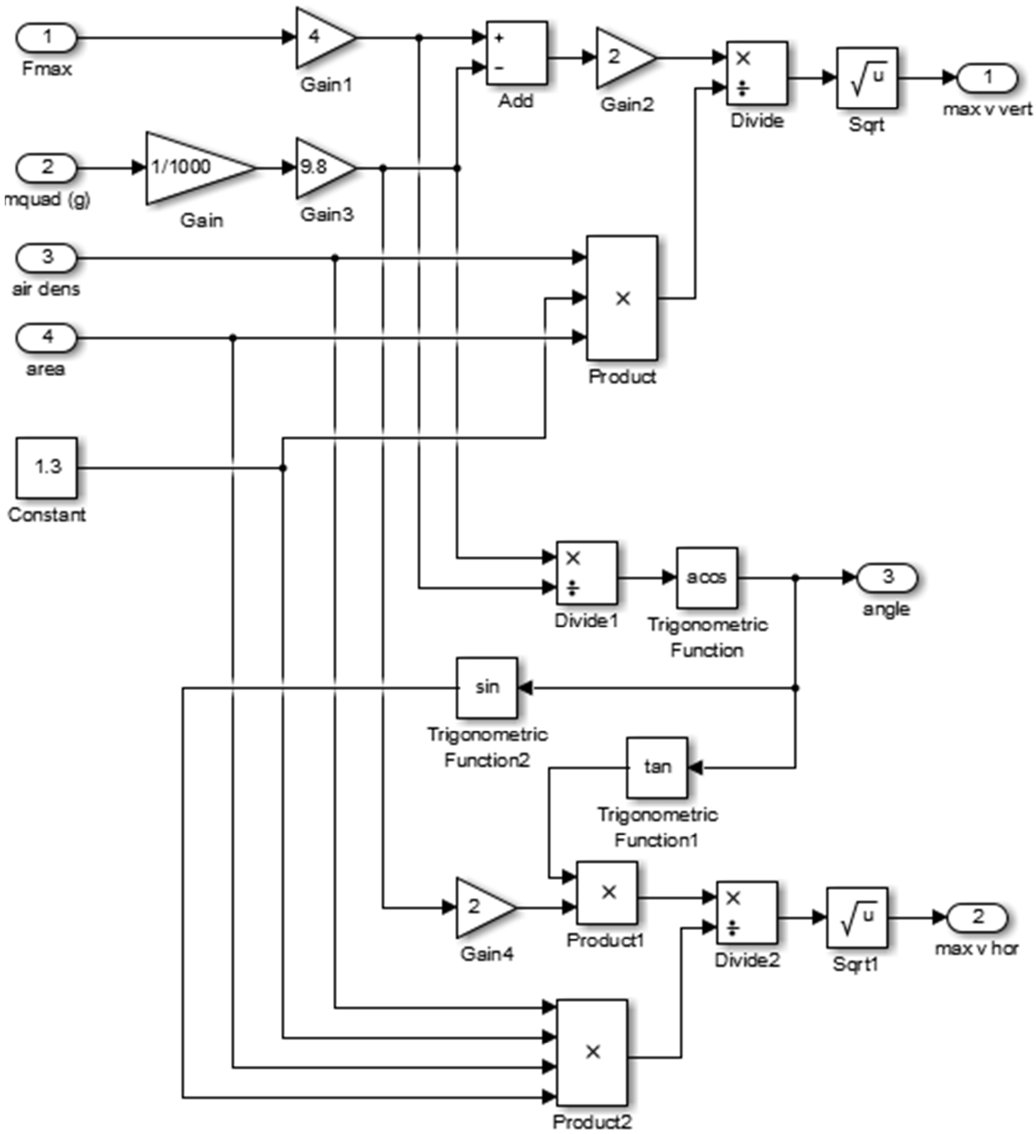
<u>Subsystem 16</u>

Inputs	Outputs
• *Air density* • *Propeller area* • *Constant Kv* • *Battery voltage* • *Propeller pitch*	• *Max. thrust force per rotor* • *Max. load to lift*



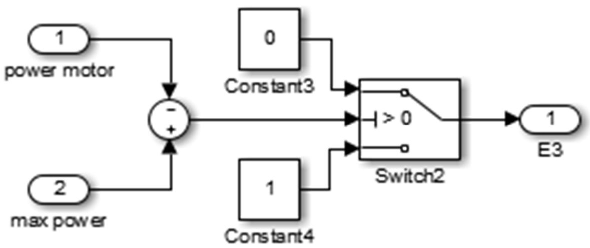
<u>Subsystem 17</u>

Inputs	Outputs
• *Max. thrust force per rotor* • *Quadcopter total mass* • *Air density* • *Quadcopter area*	• *Max. vertical speed* • *Max. horizontal speed* • *Inclination angle at max. horizontal speed*



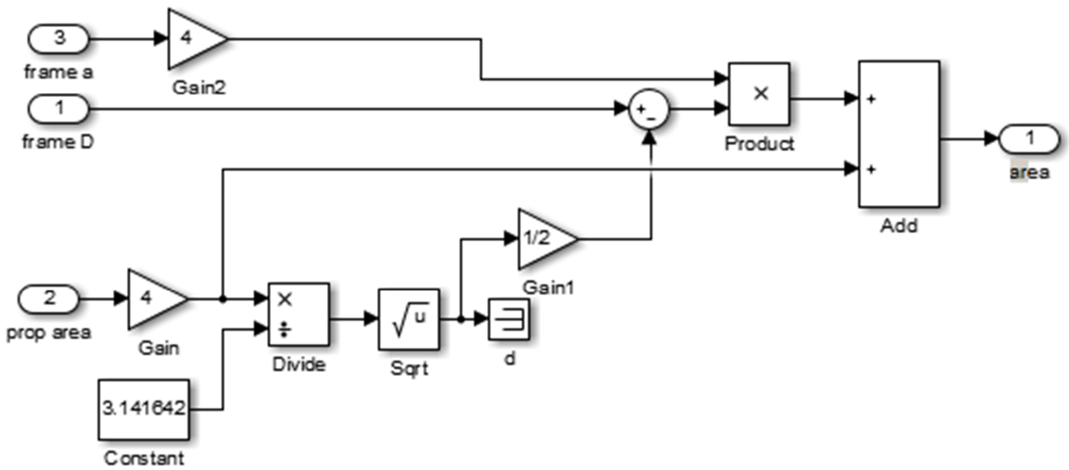
Subsystem 18

Inputs	Outputs
• *Power consumed by a motor* • *Motor max. power*	• *Error 3*



Subsystem 19

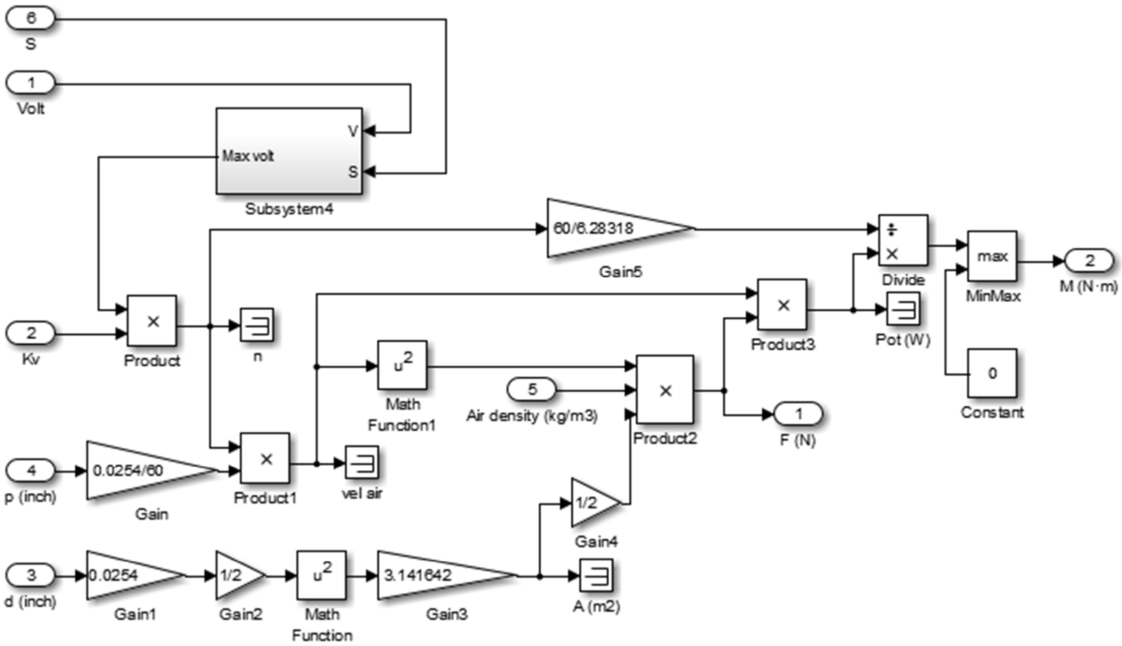
Inputs	Outputs
• *Frame distance a* • *Frame distance D* • *Propeller area*	• *Quadcopter area*



B. Subsystems of the control loop Simulink model

<u>Brushless motor subsystem</u>

Inputs	Outputs
• *Battery configuration* • *Battery voltage* • *Contstant Kv* • *Propeller diameter* • *Propeller pitch* • *Air density*	• *Motor torque* • *Motor thrust force*



<u>Quadcopter angles subsystem</u>

Inputs	Outputs
• *Motor 1 thrust force* • *Motor 2 thrust force* • *Motor 3 thrust force* • *Motor 4 thrust force* • *Frame distance D* • *Motor 1 torque* • *Motor 2 torque* • *Motor 3 torque* • *Motor 4 torque*	• *Resulting Roll* • *Resulting Pitch* • *Resulting Yaw*