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PROPOSAL AND COMPARISON OF DIFFERENT OPTIMAL TUNING CRITERIA FOR TMDS

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Abstract. Lively structures subjected to excitations such as wind, earthquake or human-induced forces can undergo relevant stresses and accelerations that can compromise some limit state criteria. Passive, semi-active and active vibration control devices have been proposed as possible solutions to reduce the level of vibration in civil structures such as bridges, multi-storey buildings or thin floor structures, among others [1]. Tuned Mass Dampers (TMD) are usually installed in order to mitigate structural vibrations by increasing its damping through the application of inertial forces generated in response to the movement of the structure [2]. It is known that different algorithms can be used to improve their tuning. When more than one TMD is advisable, in the case of structures with spatially distributed and closely spaced natural frequencies, the TMDs design may not be obvious because simplified tuning concepts [3] cannot consider the coupling effects that appear [4]. In such cases, different criteria lead to different tuning parameters.

In this work, different scalar values (called "indicators", evaluated according to different response considerations) are proposed to find the optimum tuning of two TMDs installed in a two-storey building model. Some of these indicators are based on the frequency response of the structure but others are evaluated according to the time response under a reference excitation. In all cases the tuning parameters are obtained by minimizing the selected indicator. The obtained solutions are compared in order to show the importance of making a good choice of the indicator when designing vibration control devices. Some practical limitations are also discussed through an experimental setup (two-storey building model) that provides the means to test some of the solutions obtained.

1 INTRODUCTION

During the past decades, enormous improvements have taken place in the structural and construction industries, resulting in lighter and more slender structures. Consequently, higher displacements and accelerations appear when they are subjected to a dynamic load such as wind, earthquake or human-induced forces. Many devices have been already designed to cope with this problem including passive, semi-active and active ones. In this work, only the passive control device known as Tuned Mass Damper (TMD) is considered for mitigating the vibrations induced by a quake-like acceleration applied on the basement of a two-storey building model.

The concept of TMD is already a century old, presented for the first time by the naval engineer Hermann Frahm in 1911 [5]. Since then, much effort has been made to understand the coupling effects between TMDs and structures leading to more efficient designs. One of the most important works in this area is the one conducted by J.P. Den Hartog in 1928 which ended by establishing the basics to understanding the behavior of TMDs [3]. His research was entirely based on the Frequency Response Function of a SDOF undamped model subjected to multiple kinds of loads. With the approach of Den Hartog [3], some decades later G.B. Warburton [6] developed a set of closed-form formulae to calculate the optimum properties of a single TMD.

Since then, and following the path opened by Den Hartog and Warburton, FRFs of more complex structural models have been developed to carry out different types of optimizations of one or multiple TMDs properties [7][8][9]. However, the time domain behaviour has had very little impact on the further development of the state of the art. In this work, two simple time domain indicators are proposed and compared with the traditional optimization approach: the maximum of the acceleration responses $\ddot{x}(t)$ and the area under $\ddot{x}(t)$ curve. Both indicators are obtained by simulating the response of the two-storey building model subjected to the El Centro (1940) earthquake accelerogram (Figure 1). Another two timed-based indicators are evaluated in order to compare the solutions: the total amount of energy dissipated by the TMDs and the Maximum Transient Vibration Value of the acceleration response of the model. Some conclusions will be obtained concerning the benefits of using one approach or the other.

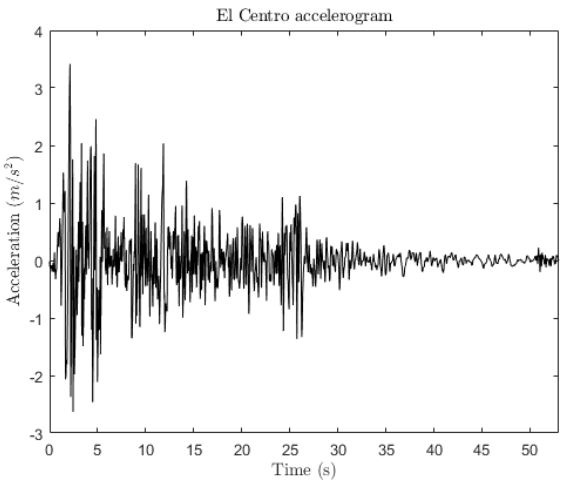


Figure 1: El Centro accelerogram.

2 MODEL DESCRIPTION AND IDENTIFICATION

A two-storey building model has been manufactured (Figure 2, right) in the laboratory to perform some experimental tests. It comprises of two units joint together, one of 0.5 m and the other 0.75 m height. The assembly is placed in a moving table to perform the modal analysis and to obtain its physical and modal properties. The test is carried out by applying an impact to the base and measuring both input and output accelerations. The correlation between basement and 1st floor accelerations and basement and 2nd floor accelerations gives the two FRFs of the model used to update the computational model.

To obtain the physical properties of the model a curve fitting procedure to the experimental FRFs must be done. The analytical FRFs are obtained through the state-space model of the system, which is schematically represented in Figure 1, left. Its mass and stiffness matrices are shown in Equation 1 and, with them, the eigenproblem $(-\omega_i^2 M + K)\phi_i = 0$ can be solved to obtain the natural modes and frequencies, ϕ_i and ω_i respectively. The damping matrix C is defined in this work (without loss of generality) upon the modal damping ratios ξ_i as stated in Equation 2, where $\tilde{C}$ modal damping matrix containing the products $2\xi_i\omega_i$ in its diagonal and V is the modal matrix whose columns are the natural modes.

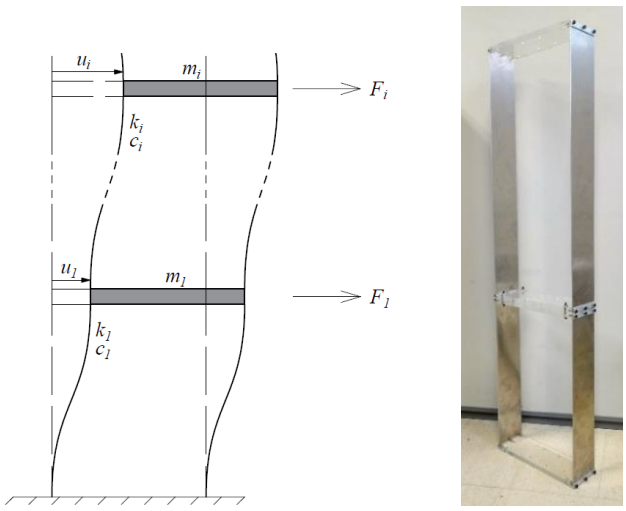


Figure 2: Scale structure: left, the 2-floor schematic model; right, actual 2-floor building.

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \qquad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{1}$$

$$C = V^{-T}\,\tilde{C}\,V^{-1} \tag{2}$$

The four matrices of the state-space model can be obtained by rewriting the equations of motion system (Equation 3) properly. In this equation, r is a vector of ones which distribute the base excitation through the structure and a_b is the basement acceleration. Considering the accelerations of both floors of the model as the output and being q_i the displacement of the i-th floor relative to the ground, the four matrices A to D of the state-space equations can be written as shown in Equation 4, where x is the state vector containing displacements and accelerations of the model ($x = [q\ \dot{q}]^T$), u is the input vector ($u = a_b$), y is the output vector ($y = [\ddot{q}_1\ \ddot{q}_2]^T$), I_{nxn} is the n by n identity matrix and $\emptyset_{nxn}$ is the n by n zeros matrix.

$$M\ddot{q} + C\dot{q} + Kq = -Mr\, a_b \tag{3}$$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{4}$$

$$A = \begin{bmatrix} \emptyset_{2x2} & I_{2x2} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B = \begin{bmatrix} \emptyset_{2x1} \\ r \end{bmatrix} \qquad C = \begin{bmatrix} -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad D = \emptyset_{2x1}$$

Programming this approach in Matlab and making use of its state-space functions *tf()* and *lsim()*, both frequency and time responses can be calculated. The six properties of the model $(m_1, m_2, k_1, k_2, \xi_1, \xi_2)$ can be obtained by minimizing the error between the analytical state-space model and the experimental FRFs. The result is shown in Figure 3, leading to the set of properties in Table 1

Table 1: Properties of the identified model

$M_1 = 2.17\ kg$	$K_1 = 1210\ N/m$	$\xi_1 = 0.41\ \%$
$M_2 = 1.76\ kg$	$K_2 = 341.4\ N/m$	$\xi_2 = 0.11\ \%$

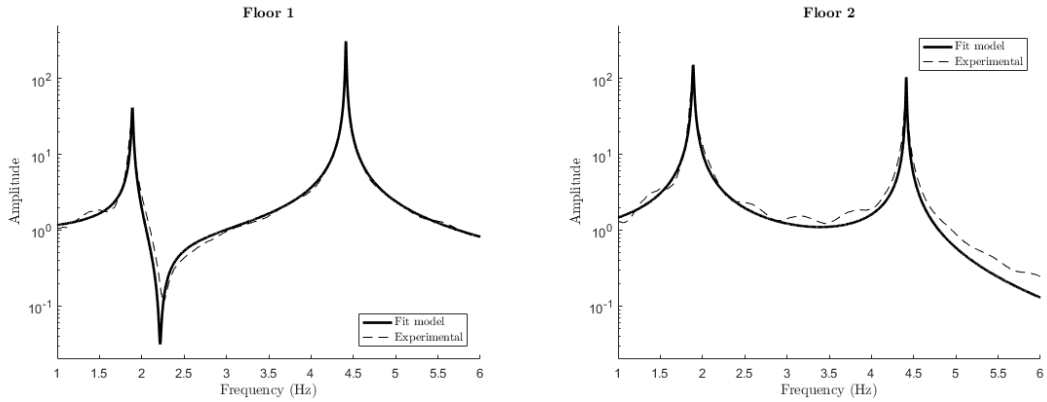


Figure 3: Result of the curve-fitting of the experimental FRF

These properties lead to the natural frequencies $f_1 = 1.89\ Hz$ and $f_2 = 4.41\ Hz$ and the natural modes shown in Figure 4.

2.1. Model of the TMD

A TMD is modelled as shown in Figure 5 and is included directly in the mathematical model presented above. When adding a single TMD, the three matrices M, C and K grow by a row and a column. The mass m_j of the TMD is put in the new diagonal position of the mass matrix. The stiffness k_j is put in the new diagonal position of the stiffness matrix, summed up to the corresponding diagonal position of the floor to which the TMD is attached and two more negative terms $(-k_j)$ are placed in the intersections of the rows and columns affected. Finally, the TMD damper constant c_j is added in a similar way. In order to add a second TMD, the same procedure must be followed.

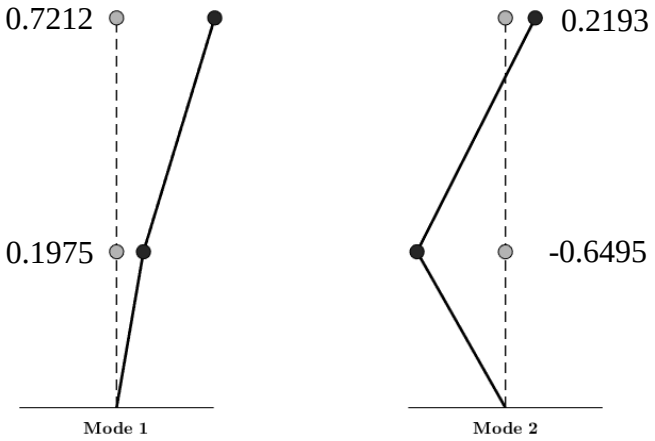


Figure 4: Natural modes of the model

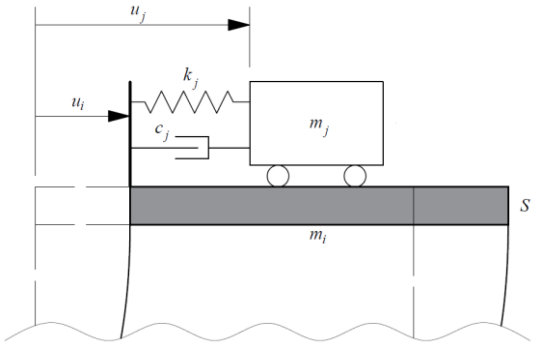


Figure 5: TMD number j attached to the floor number i

The model presented here is used through the entire paper to perform the corresponding optimizations and it is programmed in all the cost functions to obtain the value of the indicator for a set of properties of the TMDs.

3 FREQUENCY DOMAIN INDICATORS

In this section, only the standard frequency domain indicator in is considered, that is, the maximum peak value of a FRF. Regarding only the accelerances of the system, different combinations can be considered: maximum of a single FRF in a limited range of frequency, maximum of every FRF in a limited range, maximum of a single FRF in the whole range and maximum of all the FRFs in the whole range of frequencies. In addition, when multiple FRFs are taken into consideration the sum of all maxima may be also minimized. That makes a lot of different scenarios as the optimizations can be carried out for one or two TMDs. To simplify the task, the cases to discuss will be named after the number of FRFs taken into account (‘Si’ for a single, i-th floor, and ‘A’ for both floors) and the range of frequency considered (‘Lj’ for limited range around the j-th natural frequency and ‘W’ for the whole range), preceded by an ‘F’ (from Frequency domain).

The limited ranges are established around the original natural frequencies of the model and

including the possible double peaks after splitting by the addition of the TMDs. As there are only two natural frequencies, only two limited ranges will be considered: L1 will cover from 0.5 to 3 Hz and L2 will cover from 3 to 6 Hz.

The case in which one TMD is tuned to reduce the FRF peak in a limited range of frequencies is the problem solved by Den Hartog and Warburton [3][6]. For this, the next terminology will be used: DH_{ij}, where i means where the TMD is placed and j which mode is to be reduced.

4 TIME DOMAIN INDICATORS

Regarding the time domain indicators, two different sets will be proposed: the maximum of the response acceleration of some floor (TM) and the area under its curve (TA).

4.1. Peak acceleration response

The first indicator follows the same philosophy than the frequency indicators: minimizing the peak of the response but, this time, in the time domain. An appreciable reduction of the response is expected, especially in the regions of higher accelerations, but, globally, the response may not be minimal. This indicator is named here TM and can be computed for a single response (‘S1’,’S2’) or for all of them (‘A’). In the latter situation, the indicator may contain the maximum of all the maximum values of each floor (‘AM’) or, alternatively, the sum of them all (‘AS’). Equation 5 provides the definitions for these concepts:

$$TMSj = \max(\ddot{x}_j) \quad ; \quad TMAM = \max\left(\max_{j}(\ddot{x}_j)\right) \quad ; \quad TMAS = \max\left(\sum_j \max(\ddot{x}_j)\right) \tag{5}$$

4.2. Area under the acceleration response

To ensure a global reduction of the response, the areas under the time histories are also calculated and minimized. With this procedure, the maximum values of the responses may not be minimal but rather the total amount of movement during the whole simulation. The indicator is named TA and, once again, it can be computed for a single floor (‘S1’,’S2’) or the sum of them all (‘AS’).

$$TASj = \int_0^T \ddot{x}_j(t)dt \quad ; \quad TAAS = \sum_j \int_0^T \ddot{x}_j(t)dt \tag{6}$$

Once these four criteria (F, DH, TM, TA) are used to tune the TMDs, their performance is compared by evaluating two standard quality indicators: the dissipated energy and the maximum transient vibration value.

4.3. Dissipated energy

It is well known that one of the aims of a TMD is to absorb part of the energy of the structure where it is attached. It also helps to redistribute the energy flow in the system, increasing the quantity of energy transmitted through the spring (k_j) up to the mass (m_j). As a way for evaluating the performance of each optimization the total amount or energy dissipated by the

damper (c_j) of the TMD can be computed with Equation 7, where $\dot{x}_j$ is its velocity relative to the ground, $\dot{x}_i$ is the velocity of the floor where the TMD is attached to (also relative to the ground) and T is the total simulation time.

$$W_{Cj} = \int_0^T c_j \left(\dot{x}_j(t) - \dot{x}_i(t)\right)^2 dt \tag{7}$$

The equation shows that the energy dissipated by the TMD increases with its damping constant. However, it depends also on the relative velocity between the TMD and its floor. If the damper constant is too high it is expectable to obtain little relative velocities and, thus, small values for the dissipated energy.

This indicator will be named W_C and followed by a number indicating the number of the TMD to which it refers in the considered scenario.

4.4. Maximum Transient Vibration Value

Finally, the Maximum Transient Vibration Value (MTVV) will also be used to compare performances. This indicator is defined in the ISO 2631 in the context of human perception of structural vibrations. It is evaluated as the maximum of the RMS trend curve of the response in terms of accelerations. RMS trend curve is obtained, point by point, as the root mean square (RMS) of a one second window of the acceleration response with an overlapping of 50% (0,5 s) between each pair of points (Equation 8). As an example, Figure 6 shows the acceleration (grey) and its RMS trend curve (black), being its maximum the MTVV value sought.

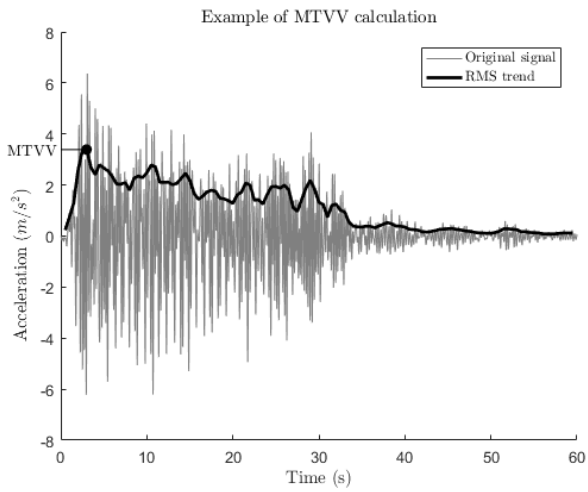


Figure 6: Example of RMS trend and MTVV calculation

$$MTVV = \max_{j} \sqrt{\frac{1}{N}\sum_i \ddot{x}\left(\Delta t_j\right)^2} \qquad with\ \Delta t_j = [t_1 \quad \dots \quad t_i \quad \dots \quad t_N] \tag{8}$$

This indicator represents in some way the amplitude of the maximum of the response and so it is closely related to the TM indicator. Low values of MTVV are expected when the TM criteria will be used.

5 RESULTS

The whole optimizations are carried out using the genetic algorithm included in Matlab (*ga()* function) with each cost function programmed separately. Two types of situations are considered: the installation of one TMD (denoted by '_1T') or two TMDs (denoted by '_2T'). In all cases, the algorithm itself puts the TMDs in the best locations. Thus, the optimization contains, apart from the 6 properties of both TMDs, 2 more integer values indicating the floor p_j to which they are attached to.

In the case of frequency domain, one TMD is first tuned to minimize the FRF of the second floor (S2) around first mode (L1) (FS2L1_1T) and another TMD for the second mode (L2) in the first floor (S1) (FS1L2_1T). Both solutions can be compared with those obtained via Den Hartog formula (DH12 and DH21). Then, two TMDs are considered to minimize at the same time the first mode in the second floor and the second mode in the first floor (FS2L1_S1L2_2T). Then, two TMDs are designed for minimizing the whole second floor FRF (FS2W_2T) and another two TMDs for reduce the first mode in all FRFs as much as possible (FAL1_2T). Finally, two TMDs are obtained to minimize the maximum of all the FRFs in the whole range (FAWM_2T) and another two to minimize the sum of all those maxima (FAWS_2T). Solutions are shown in Table 2.

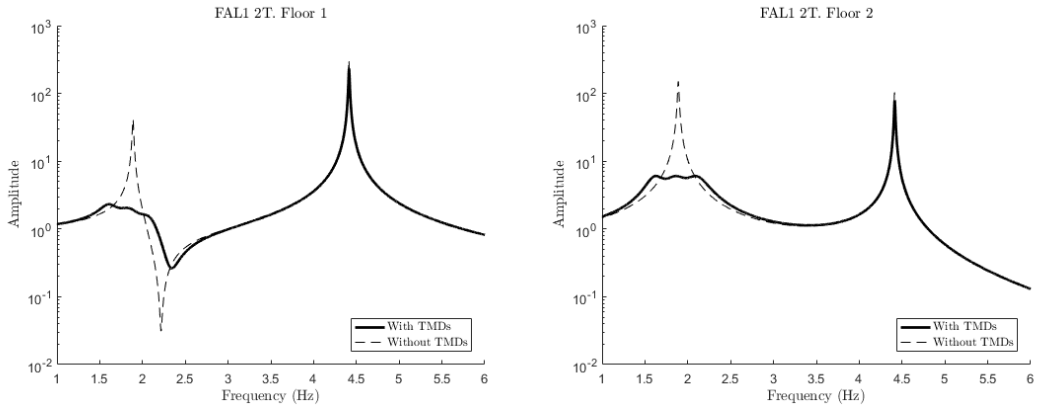
Table 2: Solutions from optimizations in the frequency domain

Case	m_j (kg)	f_j (Hz)	ξ_j (%)	p_j	W_{Cj} (J)	$MTVV_j$ (m/s^2)
FS2L1_1T	0.119	1.79	14.63	2	2.51	8.08 ; 6.07
FS1L2_1T	0.119	4.21	13.04	1	0.31	3.97 ; 12.64
DH12	0.119	4.15	13.55	1	0.87	5.39 ; 13.44
DH21	0.119	1.75	15.01	2	2.50	8.10 ; 6.10
FS2L1_ S1L2_2T	0.064 0.055	1.83 4.31	10.79 9.26	2 1	2.50 0.22	3.66 ; 6.80
FS2W_2T	0.115 0.004	1.79 4.40	14.7 2.66	2 1	2.45 0.22	4.10 ; 6.13
FAL1_2T	0.038 0.081	1.98 1.71	9.83 10.93	2 2	0.82 1.70	8.25 ; 5.94
FAWM_2T	0.087 0.032	1.81 4.36	12.66 7.18	2 1	2.46 0.21	3.72 ; 6.47
FAWS_2T	0.069 0.050	1.83 4.32	11.66 8.67	2 1	2.50 0.22	3.68 ; 6.73

Table 3: Solutions from optimizations in the time domain

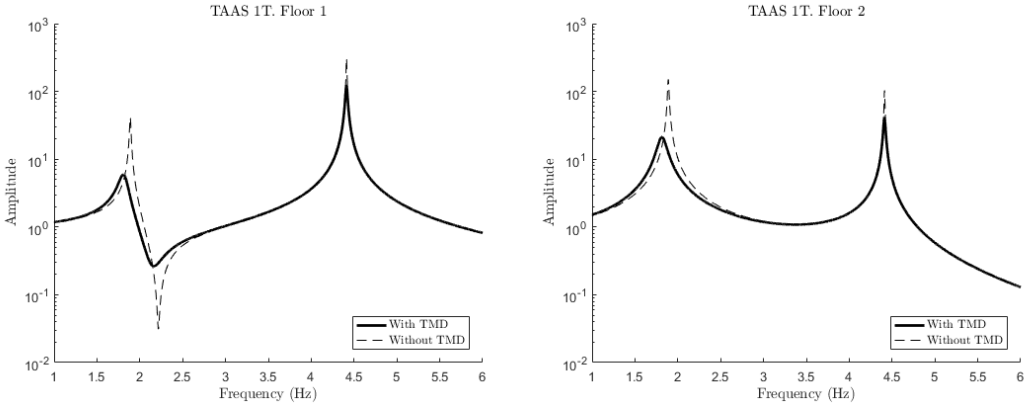
Case	m_j (kg)	f_j (Hz)	ξ_j (%)	p_j	W_{Cj} (J)	$MTVV_j$ (m/s^2)
TMS2_1T	0.119	1.85	2.87	2	1.94	8.59 ; 5.42
TMS2_2T	0.011 0.108	4.39 1.81	0.0 11.99	1 2	0.0 1.53	7.04 ; 5.91
TMAM_1T	0.119	1.86	29.45	2	2.75	7.54 ; 6.53
TMAM_2T	0.107 0.012	1.81 4.40	1.34 0.07	2 1	1.51 0.07	5.92 ; 5.97
TMAS_2T	0.105 0.014	1.80 4.24	2.02 4.82	2 1	0.19 1.65	3.46 ; 5.54
TAS2_1T	0.119	1.81	23.87	2	2.69	7.75 ; 6.38
TAS2_2T	0.112 0.007	1.83 4.36	12.22 2.56	2 1	2.41 0.20	3.77 ; 6.05
TAAS_1T	0.119	2.08	47.42	2	2.78	7.01 ; 8.61
TAAS_2T	0.091 0.028	1.83 4.36	11.04 5.37	2 1	2.42 0.21	3.71 ; 6.33

Concerning the solutions in the time domain, one and two TMDs have been sought to reduce the maximum of the second floor's response (TMS2_1T and TMS2_2T). In the case of two TMDs, also the stroke of the TMDs has been minimized because the optimization led to zero damping and, consequently, very high strokes. After that, one and two TMDs are designed to reduce the maximum of both responses (1st and 2nd floors simultaneously, TMAM_1T and TMAM_2T). The sum of all of them is also minimized but only with two TMDs (TMAS_2T). Then, one and two TMDs are considered to reduce the area under the curve response of the top floor (TAS2_1T and TAS2_2T) and finally one and two TMDs have been put on the structure to reduce the total area of all responses (TAAS_1T and TAAS_2T).



(a) (b)

Figure 7: FRFs of the FAL1_2T case. (a) floor 1; (b) floor 2



(a) (b)

Figure 8: FRFs of the TAAS_1T case. (a) floor 1; (b) floor 2

As it can be stated by observing the whole set of results is that all TMDs frequencies are focused to one of the two natural frequencies of the structure, no matter what the objective to minimize was. In fact, TMDs placed on the top floor have their natural frequency a little below 1.89 Hz and the others, placed on the first floor have their frequency close to 4.41 Hz. There is only two exceptions. The first one is FAL1_2T case, that has two TMDs placed on the second floor and tuned to the first mode, one of them over 1.89 Hz. This is because of the *Multi TMD* effect: instead of resulting in a couple of peaks, this optimization leads to three perfectly levelled maxima in the proximity of the first natural frequency (Figure 7). The other exception is TAAS_1T, with the only TMD tuned at 2.08 Hz. (Figure 8) and with a very high damping ratio (0.4742).

6 CONCLUSIONS

To show the time responses, TAAS_2T case has been taking as reference (Figure 9). Figure 9a and 9b show (black line) the accelerations in the first and second floor (respectively), compared with the response of the building without TMDs. Figure 9c shows the energy dissipated by both TMDs.

According to frequency domain indicators, the solution for the FS2L1_S1L2_2T case is the one that dissipates more (2.72 J), together with FAWS_2T case, being both cases very similar in tuning parameters. FS2W_2T case is the design with less maximum MTVV (6.13 m/s2). Note that in this case, although two TMDs are found, most of the mass is placed in the one tuned to the first mode.

FS1L2_1T and DH12 cases (TMD tuned to mode 2) are the worst, according not only to their low dissipation of energy (less than 0.88 J) but also to the high values for the MTVV (more than 12.6 m/s2). From this point of view, both indicators seem to be correlated. Also, almost the rest of the “F” cases dissipate the same amount of energy (around 2.6 J) and the maximum MTVV value is around 6 m/s2

Regarding time domain indicators, TMAS_2T case is the one with less maximum MTVV (5.54 m/s2) and, unexpectedly, TAAS_1T is the design that dissipates the most (2.78 J). However, TMAS_2T dissipates only 1.89 J and maximum MTVV value is the corresponding

to TAAS_1T case (8.61 m/s2), which is contradictory and shows that both indicators are not correlated.

For both types of indicators, table 2 and 3 show that several designs dissipate the same amount of energy (around 2.7 J). According to Figure 9c, where W_{Cj} curves became asymptotic, that seem to be the total energy available in the structure after the earthquake. This result reveal that W_{Cj} indicator should be replaced by another one accounting not for the dissipation in the whole time (60 s) but for the dissipation ratio, being preferable to dissipate energy the more quickly the better.

Finally, TMAS_2T case is the one with less maximum MTVV (5.54 m/s2) so, for the El Centro loading, it is the best design, but it remains to be check if it is also the best solution for other types of earthquakes. There are 9 cases (FS2L1_S1L2_2T, FS2W_2T, FAWM_2T, FAWS_2T, TMS2_2T, TMAM_2T, TMAS_2T, TAS2_2T and TAAS_2T) where the maximum MTVV values are under 7.1 m/s2 and all of them resolve to 2 TMDs, being the one with more mass tuned to mode 1 and placed in the second floor and the other tuned at mode 2 and placed in the first floor. This is a reasonable strategy, having in mind that the second floor has the maximum modal coordinate for mode 1, 0.7212 and the first floor the maximum for made 2, 0.6495.

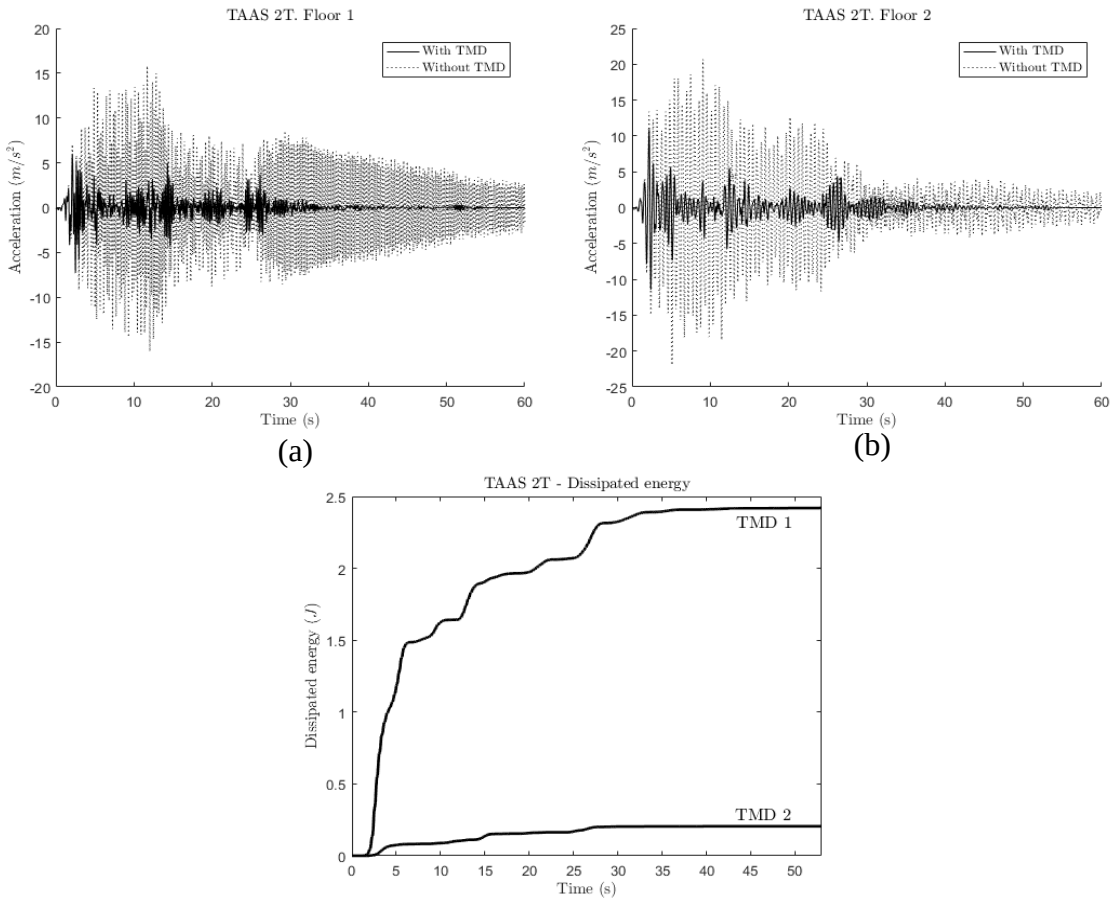


Figure 9: Example for time responses in terms of acceleration (m/s2) and dissipation of energy (J)

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