

Optimal design of motion-compensated diffusion gradient waveforms

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Synopsis (100/100 words): Diffusion-Weighted MRI (DW-MRI) often suffers from motion-related artifacts in organs that experience physiological motion. Importantly, organ motion during the application of diffusion gradients results in signal losses, which complicate image interpretation and bias quantitative measures. Motion-compensated gradient designs have been proposed, however they typically result in substantially lower b-values or severe concomitant gradient effects. In this work, we develop an approach for design of first- and second-order motion-compensated gradient waveforms based on a b-value maximization formulation including concomitant gradient nulling, and we compare it to existing techniques. The proposed design provides optimized b-values with motion compensation and concomitant gradient nulling.

Purpose(718/750 words): Diffusion-Weighted MRI (DW-MRI) is widely used in brain and body imaging. However, DW-MRI often suffers from motion-related artifacts, particularly in organs that experience substantial physiological motion¹ (eg: heart and liver). In these organs, the presence of macroscopic tissue motion during the application of diffusion gradients can result in signal voids, which complicate the interpretation of DW images and introduce bias and variability in the quantification of diffusion parameters². For these reasons, there is significant interest in the development of motion-compensated (eg: nth order moment nulled) diffusion gradient waveforms. A recent method for the design of first and second order moment nulled diffusion gradient waveforms based on a constrained optimization formulation, Convex Optimized Diffusion Encoding³ (CODE), has demonstrated excellent promise for DW-MRI in heart and liver. However, it is unclear whether CODE results in optimal waveforms (ie: maximum b-value for a given echo time). Furthermore, due to the asymmetry of CODE waveforms, they strongly suffer from concomitant gradient⁴ (CG) effects, which may introduce severe artifacts. Therefore, the purpose of this work is to formulate a technique for direct b-value maximization for the design of optimal diffusion gradient waveforms. Further, the proposed formulation also includes correction for CG effects.

Methods: Several motion-compensated symmetric diffusion gradient waveforms have been designed to null the first (M_1) or second order (M_2) gradient moments^{3,5,6}. However, these traditional designs do not maximize the b-value for a given TE, and therefore result in severely reduced b-values if motion compensation is desired. Recently, a formulation of the gradient waveform design as a constrained optimization (CODE) was introduced. CODE seeks to provide the shortest waveform that achieves a given b-value by formulating the diffusion gradient waveform

design as a constrained maximization problem. In principle, CODE seeks to maximize the b-value, defined as
$$b = \gamma^2 \int_0^{TE} F(t)^2 dt,$$
 where

$$F(t) = \int_0^t G(\tau) d\tau,$$

and $G(t)$ is the gradient waveform. However, in order to facilitate the constrained optimization problem CODE reformulates the problem to maximize an alternative function, which may lead to suboptimal gradient waveforms. In order to overcome this limitation, we propose a new approach ('Direct b-value Maximization') that directly optimizes the b-value based on the same formulation and constraints as CODE, as shown in Table 1.

Further, we also consider the effect of CGs on the gradient waveform. CGs appear every time we generate a magnetic field gradient as described by Maxwell's equations⁴, and result in a

spatially-varying dephasing along the three axes which depends on the applied gradients,
$$\Phi(x, y, z) = \gamma \int B_c(x, y, z, t) dt,$$
 where B_c is described in Ref². To correct for the presence of CGs, we incorporate $\Phi(x, y, z) = 0$ as a constraint in the proposed formulation.

In this work, the b-values achievable over a range of TE with the proposed Direct b-value Maximization (with and without CG correction) are compared to previously proposed gradient waveform designs for three different types of constraints: zero moment nulling ($M_0=0$, as required for any diffusion encoding waveform), first order moment nulling ($M_0=M_1=0$), and second order moment nulling ($M_0=M_1=M_2=0$).

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Results: Figure 1 shows waveforms generated by the proposed Direct b-value Maximization method (with and without CG nulling), as well as CODE waveforms. These examples illustrate the higher b-value achievable with Direct b-value Maximization compared to CODE. Figure 2 demonstrates higher b-values with the proposed method for $M_0=M_1=0$ and $M_0=M_1=M_2=0$, while the waveforms are identical to CODE designed waveforms for $M_0=0$. Although CG correction results in a b-value reduction, the proposed CG-corrected method still provides higher b-values than traditional designs, like MONO, BIPOLAR, and motion-compensated (MOCO⁶).

Discussion: These preliminary results illustrate the potential of the proposed formulation to maximize the b-value achievable with motion-compensated diffusion gradient waveforms subject to hardware and timing constraints. Further, inclusion of CG correction also improves the traditional designs, likely by eliminating the dead time between radiofrequency pulses. This formulation provides optimized motion-compensated gradient waveforms with potential application of DW-MRI in organs that experience substantial physiological motion^{2,3}. However, validation of these results in phantom and in-vivo studies is still required.

Conclusion: We have developed an optimized approach for the design of motion-compensated diffusion gradient waveforms. The proposed method maximizes the achievable b-value while nulling the first and second order moments, as well as nulling the first order of CGs. This approach may have application for motion-compensated DW-MRI in the heart, liver, and other organs that experience substantial physiological motion.

References:

1. Taouli B, et al. Diffusion-weighted MR Imaging of the Liver1. *Radiology*. 2009;254(1):47-66.
2. Murphy P, et al. Error Model for reduction of cardiac and respiratory motion effects in quantitative liver DW-MRI. *Magn Reson. Med*. 2013;70(5):1460-1469.
3. Aliotta E, et al. Convex Optimized Diffusion Encoding (CODE) Gradient Waveforms for Minimum Echo Time and Bulk Motion-Compensated Diffusion Weighted MRI. *Magn Reson Med*. 2016;00:00-00.
4. Bernstein MA, et al. Concomitant Gradient Terms in Phase Contrast MR: Analysis and Correction. *Magn Reson Med*. 1998;39(2):300-308.
5. Simonetti OP, et al. Significance of the Point of Expansion in the Interpretation of Gradient Moments and Motion Sensitivity. *J Magn Reson Imaging*. 1991;1(5):569-577.
6. Stoeck CT, et al. Second-Order Motion-Compensated Spin Echo Diffusion Tensor Imaging of the Human Heart. *Magn Reson Med*. 2016;75:1669-1676.

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Table 1 CODE and Direct b-value Maximization Formulation. For the particular case under study, $RF_{90} = 5.3$ ms, $RF_{180} = 4.3$ ms, $T_{EPI} = 52.8$ ms, $G_{max} = 74$ mT/m, and $SR_{max} = 50$ T/m/s. TE varies between 70 and 120 ms.

Formulation	CODE	Direct b-value Maximization
Pulse Sequence Constraints	$G(FR_{90}) = 0; G(RF_{180}) = 0; G(T_{EPI}) = 0$	
Hardware Constraints	$G(t) = G_{MAX}; G(t) = SR_{MAX}$	
Moment Constraints	$M_0 = \int_0^{TE} G(t) dt = 0; M_1 = \int_0^{TE} tG(t) dt = 0; M_2 = \int_0^{TE} t^2 G(t) dt = 0$	
Concomitant Gradients Correction Constraint	—	$\phi(x, y, z) = \gamma \int G(t) dt = 0$
b-value Formulation	$b = \gamma^2 \int_0^{T_{Diff}} F(t)^2 dt; F(t) = \int_0^t G(\tau) d\tau$	
Objective Function	$b = \int_0^{T_{Diff}} F(t)^2 dt; G(t) = \operatorname{argmax}_G \beta(G)$	$G(t) = \operatorname{argmax}_G b(G)$

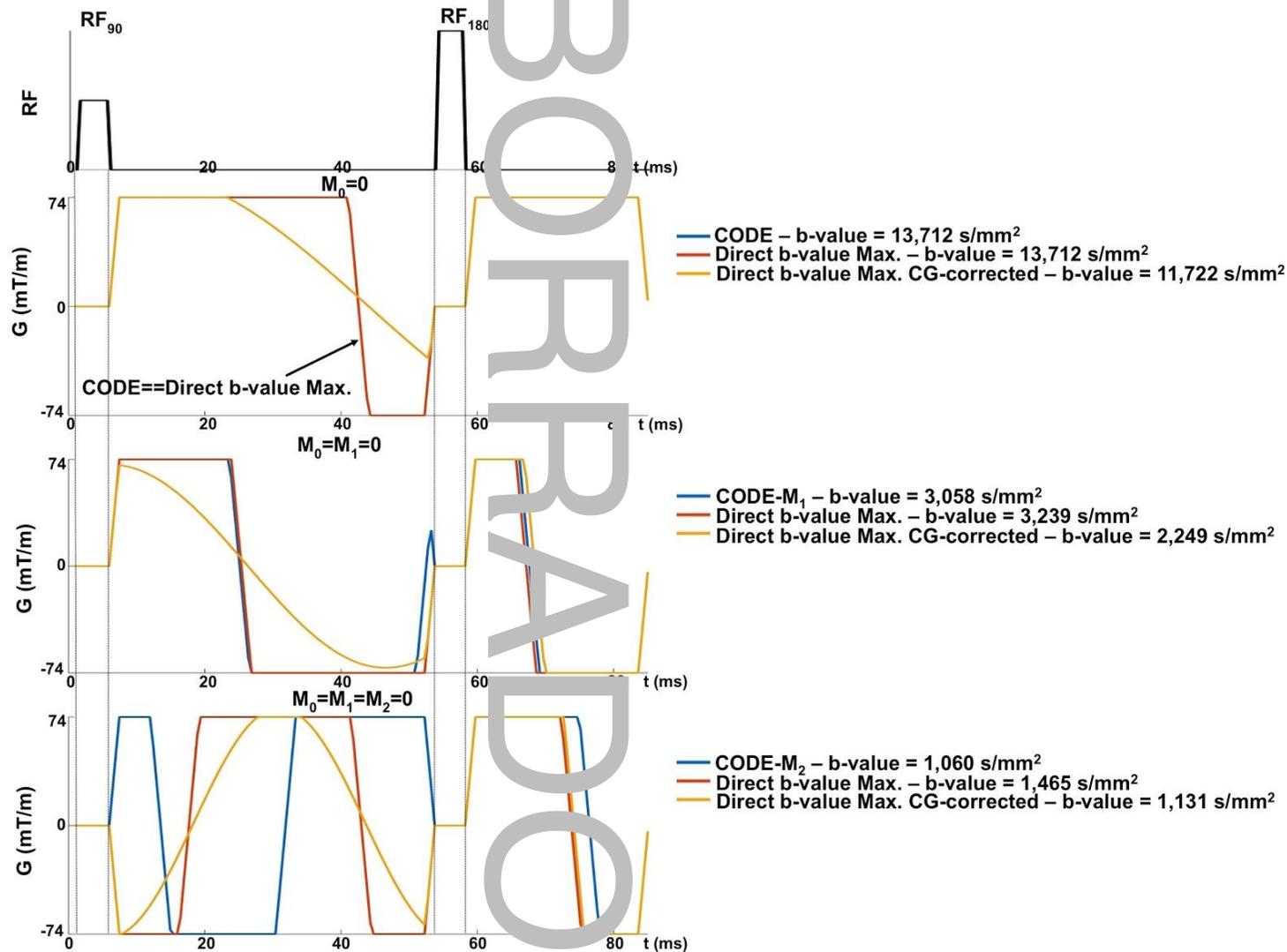


Fig 1. (66/100 words) Waveforms designed by CODE, and Direct b-value Maximization (with and without CG correction) for each of the nth order moment nulling for $TE=111.5ms$. (Top) Waveforms including zero moment nulling ($M_0=0$), (Center) first order moment nulling ($M_0=M_1=0$), and (Bottom) second order moment nulling ($M_0=M_1=M_2=0$). For clarity, the traditional waveforms (MONO, BIPOLAR, and MOCO) are not shown, and the radio frequency pulses are ideally represented on the upper timeline.

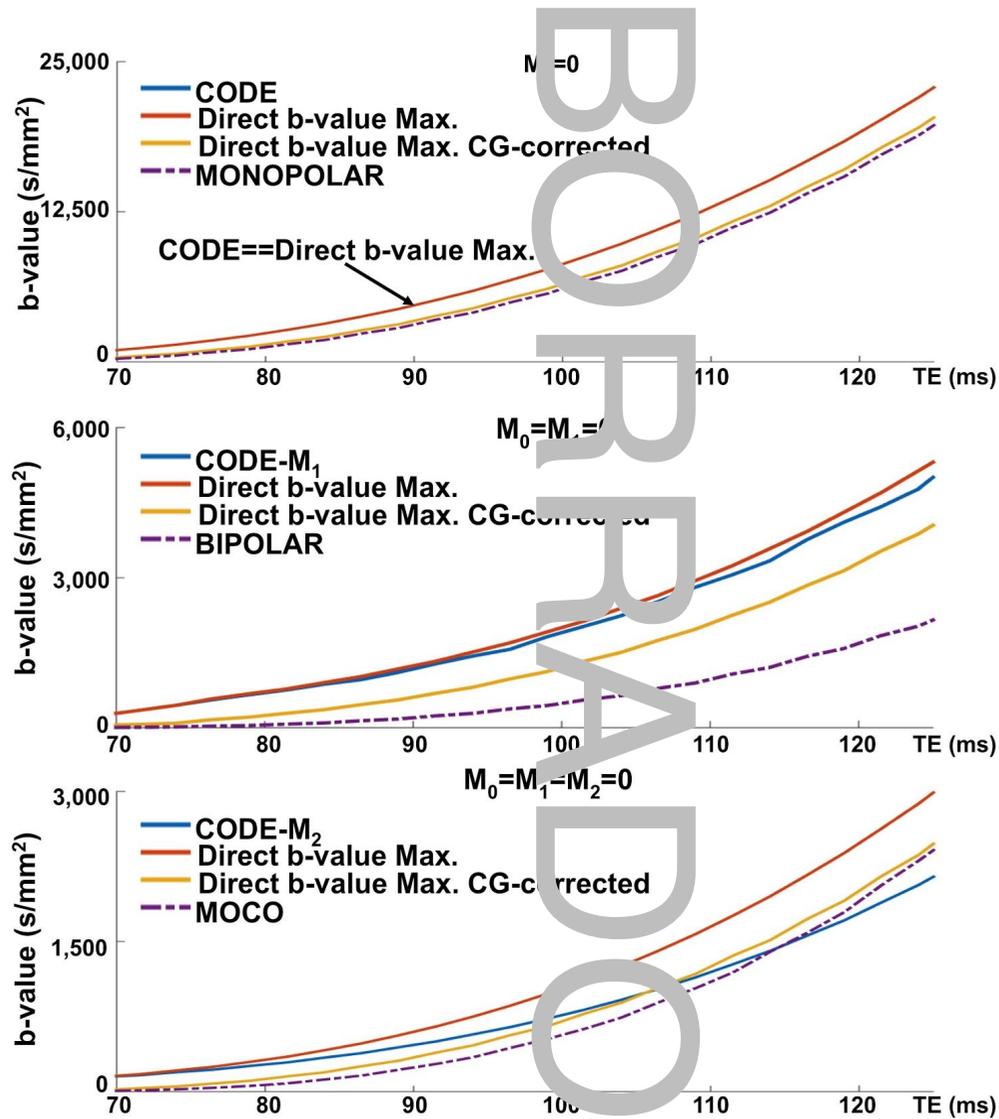


Fig 2. (53/100 words) Comparison of achievable b-value versus TE for CODE, Direct b-value Maximization (with and without CG correction), and the traditional gradient waveforms. (Top) Achievable b-values including zero moment nulling ($M_0=0$), (Center) first order moment nulling ($M_0=M_1=0$), and (Bottom) second order moment nulling ($M_0=M_1=M_2=0$).

