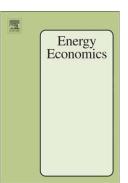
## Accepted Manuscript

Leverage effect in energy futures revisited

M. Angeles Carnero, Ana Pérez

PII:	S0140-9883(18)30002-1
DOI:	doi:10.1016/j.eneco.2017.12.029
Reference:	ENEECO 3868
To appear in:	Energy Economics
Received date:	27 July 2017
Revised date:	21 December 2017
Accepted date:	25 December 2017



Please cite this article as: Carnero, M. Angeles, Pérez, Ana, Leverage effect in energy futures revisited, *Energy Economics* (2018), doi:10.1016/j.eneco.2017.12.029

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Leverage effect in energy futures revisited

M. Angeles Carnero\*

Dpto. Fundamentos del Análisis Económico, Universidad de Alicante.

Ana Pérez $^{\dagger}$ 

Dpto. Economía Aplicada and IMUVA, Universidad de Valladolid.

#### Abstract

The objective of this paper is to replicate the results in Kristoufek (2014) on the leverage effect in energy futures and to analyze its robustness to both the methodology and the type of returns used. We first apply correlation-based tools for detecting both conditional heteroscedasticity and leverage effect. Then, we estimate asymmetric and long memory GARCH-type models using the data provided by Kristoufek (2014) by considering different software and the possibility that innovations follow a non-Gaussian distribution. Our findings confirm most of the results in the replicated paper. In particular, we can strongly confirm there is a significant leverage effect in the return series of WTI (West Texas Intermediate) and Brent crude oils. For the heating oil and the natural gas series, the statistical significance of the leverage effect depends on both the methodology and the type of returns used.

JEL Classification: C22, G10, Q40

Keywords: Conditional heteroscedasticity, Quasi Maximum Likelihood, Robust estimators, TGARCH, EGARCH, FIEGARCH.

<sup>\*</sup>Dpto. Fundamentos del Análisis Económico, Universidad de Alicante. E-mail: acarnero@ua.es

<sup>&</sup>lt;sup>†</sup>Corresponding Author. Dpto. Economía Aplicada. Universidad de Valladolid. C/ Avda. Valle Esgueva 6, 47011 Valladolid. Spain. Tel: 34 983423317, E-mail: perezesp@eaee.uva.es

#### 1 Introduction

It is already well known that time series of financial returns are conditionally heteroscedastic with volatilities responding asymmetrically to negative and positive past returns. In particular, the volatility increases tend to be higher in response to past negative shocks ('bad' news) than to positive shocks ('good' news) of the same magnitude. Following Black (1976) this feature is usually referred to as *leverage effect*.

Whether or not the leverage effect is present in energy commodities markets is an open question which has attracted the interest of researchers during the last decade. Most of the empirical literature on the topic have used the same methodology, based on estimating several asymmetric GARCH-type models to the financial returns and testing the statistical significance of the coefficient capturing the leverage effect. Kristoufek (2014) points out that by doing so, the leverage effect is assumed ex ante and the volatility process is estimated as a part of a model under various assumptions and restrictions. Hence, it can occur that the coefficient capturing the leverage effect is statistically significant, not only because the effect is actually present, but also because the model is missespecified. To overcome this problem, Kristoufek (2014) proposes first, to estimate the volatility outside the returns model. Then, taking into account the possibility that the volatility is a long-memory process on the edge of stationarity, he proposes to compute the correlation between returns and volatility using two detrended correlation coefficients to deal with potential non-stationary series.

Table 1 contains a brief summary of the empirical results in those papers reviewed in Kristoufek (2014) that analyze the same four energy commodities as he does, namely WTI and Brent crude oils, heating oil and natural gas. We are aware that there are many other papers dealing with the problem, however, as we can see in the last column of the table, the selected articles are good examples of the mixed results found.

Regarding the WTI crude oil returns, Kristoufek (2014) finds what he calls the standard leverage effect (significant negative correlation between returns and volatility). This result agrees with Reboredo (2011), Nomikos and Andriosopoulos (2012) and Chkili et al. (2014). However, Agnolucci (2009), Cheong (2009), Chang (2012) and Wu et al. (2012) find that the leverage effect in different asymmetric GARCH models is not statistically significant. On the other hand, Fan et al. (2008) and Zhang et al. (2008)

find an inverse leverage effect (positive shocks result in larger increases of the volatility than negative shocks) by estimating different asymmetric GARCH models. Zhang et al. (2008) justify this effect by arguing that when oil price increases, the expectation is that oil supply will decrease, which makes traders to buy oil as soon as possible increasing even more the price of oil and also its volatility.

With respect to the Brent crude oil, Kristoufek (2014), as well as Cheong (2009) and Reboredo (2011), also find the standard leverage effect. However, this result differs from Fan et al. (2008) who find that there is not leverage effect in the Brent returns. Furthermore, Wei et al. (2010) find mixed results for Brent but also for WTI returns.

Finally, Kristoufek (2014) finds the standard leverage effect for heating oil, although weaker than for WTI and Brent crude oils, and the inverse leverage effect for natural gas. These results are in line with Nomikos and Andriosopoulos (2012) but differ from Chkili et al. (2014) who find the standard leverage effect when asymmetric and long-memory models are estimated to both the spot and future returns of the natural gas.

The different and sometimes contradictory results summarized above can be explained by several reasons. First, as shown in the second column of Table 1, the data used in the different articles are not the same. Some authors consider spot prices while others consider future contracts with different maturities and, also, the data frequency is different: some prices are observed daily while others are observed weekly. Second, the sample period analyzed also varies among the different papers, as we can see in the third column of Table 1. And finally, the methodology is not always the same. Even though many authors use asymmetric GARCH-type models, the large number of alternative models (with different parametrizations) that are able to cope with the leverage effect, as well as the possibility of using different software with several estimators implemented, make the comparison very difficult (a further discussion on this topic is included in the online Appendix A). As an illustration, Table 2 reviews, for the models we will consider in this paper, different software that can be used for estimation purposes. It is worth noticing that one should be very careful when comparing the results obtained from different software packages as the parametrization of the same model can change from one to another. For example, assuming a TGARCH model (to be described in Section 2), the leverage coefficient estimated using the Oxford MFE Toolbox is equal in magnitude, but with opposite sign, to the leverage coefficient estimated using Stata. Moreover, even

when the parametrization of the model is the same, different software will start the estimation procedure with different initial values, leading to possibly different estimates; see Brooks et al. (2001) for a detailed discussion on this topic.

On top of the reasons given above to explain the mixed results, we agree with Kristoufek (2014) in the possibility of obtaining misleading results when estimating asymmetric GARCH-type models if the underlying assumptions are not satisfied. In particular, if the assumed model is not flexible enough to capture the empirical characteristics of the data, then it will be very likely that the Gaussian Quasi Maximum Likelihood estimators, usually implemented in the more commonly used software, lead us to incorrect conclusions. In this sense, the assumed distribution for the innovations of the model, as well as the estimator used, are essential to obtain reliable results.

For instance, it is already well known that in the presence of outliers, maximum likelihood based methods do not have good properties in symmetric GARCH models and can lead to biased estimators. Therefore, the extreme observations in the returns of energy commodities could be partly responsible for the mixed results found in the literature. In this context, robust estimators of both the parameters and the volatility are needed; see, for example, the proposals in Carnero et al. (2007, 2012), Muler and Yohai (2008) and Hill (2015), among many others. Alternatively, some authors deal with this problem by applying methodologies based on detecting and correcting outliers; see, for example, Doornik and Ooms (2005), Charles and Darné (2014a, 2014b), Behmiri and Manera (2015) and Laurent et al. (2016).

With respect to the effect of outliers in detecting the leverage effect, Carnero et al. (2016) show that outliers bias the sample cross-correlations between past and squared returns, which are often used to identify this effect. In particular, they show that one isolated big outlier biases the sample cross-correlations towards zero and hence could hide true leverage effect, whereas the presence of two or more big consecutive outliers could lead to detecting spurious asymmetries or asymmetries of the wrong sign. To overcome this problem they propose robust cross-correlations which are shown to outperform other measures in identifying asymmetric conditionally heteroscedastic models. Moreover, biased estimators of the parameters and volatilities are also expected in asymmetric GARCH models if robust methods are not used.

Taking all this into account, the objective of this paper is to replicate the results

found in Kristoufek (2014) by working with the supplementary data on daily prices of future contracts provided in Appendix A of that paper. To face this goal, we first take the same returns and the same methodology used by Kristoufek(2014) and show how we reach the same results and conclusions using different software. Then, we analyze whether employing a different methodology and another type of returns lead to the same conclusions. In particular, we compute both the sample and robust crosscorrelations to detect possible leverage effects, and then we estimate three popular asymmetric GARCH-type models (TGARCH, EGARCH and FIEGARCH) assuming different distributions for the innovations (Gaussian, GED and Student). The TGARCH and EGARCH models have been chosen because, as Rodríguez and Ruiz (2012) show, they are more flexible than their competitors to cope simultaneously with the restrictions for positivity of the conditional variance and stationarity and the features observed in financial returns, namely, excess kurtosis, positive and persistent autocorrelations of squares and negative cross-correlations between squared and lagged returns. The FIE-GARCH model is chosen as a natural generalization of the EGARCH model that is able to capture long-memory in the volatility (as it is claimed to be present in the replicated paper).

Our results show that, when the three asymmetric GARCH-type models are estimated using the Brent and WTI crude oil returns, the leverage effect is statistically significant at 5% significance level, regardless of the model, the estimator and the type of returns used. This confirms the results in Kristoufek (2014) who also finds the standard leverage effect for these two series. When the previous models are fitted to the returns of heating oil, the estimated coefficient capturing the leverage effect is negative in most cases, in line with the results in Kristoufek (2014), but its statistical significance depends on both the model and the type of returns used. A similar result is found when the asymmetric GARCH models are applied to the returns of natural gas. In general, the positive sign of the estimated leverage coefficient (indicating an inverse leverage effect) agrees with the results in Kristoufek (2014), however its statistical significance depends on the fitted model. These results support our findings from applying cross-correlation based methods to detect possible leverage in these four series.

The rest of the paper is organized as follows. Section 2 describes the methodology we employ to identify and estimate the leverage effect, as an alternative to the methods used

in the replicated paper. Section 3 contains the empirical results obtained by applying both methodologies to the same data analyzed in Kristoufek (2014) and discusses the differences and similarities found. The robustness of such results to the return definition is discussed in Section 4. Finally, Section 5 concludes the paper with a summary of the main results.

## 2 Methodology

Kristoufek (2014) claims that the leverage effect can be seen as a correlation between returns and volatility. To measure this effect and deal with the potential non-stationarity of the volatility, he utilizes two detrended cross-correlation coefficients, namely DCCA and DCMA, between contemporaneous returns and volatility. However, we wonder whether computing such correlation is appropriate to capture the leverage effect, since this effect is commonly understood as the asymmetric response of volatility to negative and positive past returns; see the seminal paper by Black (1976) as well as Nelson (1991), Zakoian (1994), Engle (2011), Hibbert et al. (2008) and the references therein. Therefore, in Section 2.1, we focus on the dynamic relationship between lagged returns and current volatility. Alternatively, the presence of leverage effect can be detected by estimating asymmetric GARCH-type models and testing the statistical significance of the leverage coefficient. This is the approach discussed in Section 2.2.

#### 2.1 Detection of leverage

The identification of the leverage effect is often based on the sample cross-correlations between past returns,  $y_{t-h}$ , and squared returns,  $y_t^2$ , the latter regarded as a proxy for the underlying volatility; see, for instance, Bollerslev et al. (2006), Ruiz and Veiga (2008), Zivot (2009), Rodríguez and Ruiz (2012) and Tauchen et al. (2012). If the volatility increase is larger (smaller) in response to negative than positive past returns of the same magnitude, then the cross-correlations between  $y_{t-h}$  and  $y_t^2$  are negative (positive). Hence, negative values of these cross-correlations indicate potential leverage effect. However, as Carnero et al. (2016) show, the sample cross-correlations are not robust to the presence of extreme observations and could convey misleading results. In such cases, robust measures are more appropriate.

In this paper, we will first compute the usual sample cross-correlations between  $y_{t-h}$ and  $y_t^2$ , defined as

$$r_{12}(h) = \frac{\sum_{t=h+1}^{T} (y_{t-h} - \overline{y}) (y_t^2 - \overline{y^2})}{\sqrt{\sum_{t=1}^{T} (y_t - \overline{y})^2} \sqrt{\sum_{t=1}^{T} (y_t^2 - \overline{y^2})}},$$
(1)

for h = 1, 2, ..., where T is the sample size,  $\overline{y} = \frac{1}{T} \sum_{t=1}^{T} y_t$  and  $\overline{y^2} = \frac{1}{T} \sum_{t=1}^{T} y_t^2$ . For comparison purposes, we will also compute the robust cross-correlation introduced by Carnero et al. (2016), which is based on applying Ramsay's weights to the sample variances and cross-covariances, and is defined as follows

$$\widetilde{r}_{12,W}(h) = \frac{\widetilde{\gamma}_{12}(h)}{\sqrt{\widetilde{\gamma}_1(0)\widetilde{\gamma}_2(0)}}$$
(2)

where

$$\widetilde{\gamma}_{12}(h) = \frac{\sum_{t=h+1}^{T} w_{t-h} \left( y_{t-h} - \overline{Y}_w \right) w_t^2 (y_t^2 - \overline{Y}_w^2)}{\sum_{t=h+1}^{T} w_{t-h} w_t^2},$$

$$\widetilde{\gamma}_1(0) = \frac{\sum_{t=1}^{T} w_t \left( y_t - \overline{Y}_w \right)^2}{\sum_{t=1}^{T} w_t}, \widetilde{\gamma}_2(0) = \frac{\sum_{t=1}^{T} w_t^2 (y_t^2 - \overline{Y}_w^2)^2}{\sum_{t=1}^{T} w_t^2}$$

with

$$\overline{Y}_{w} = \frac{\sum_{t=1}^{T} w_{t} y_{t}}{\sum_{t=1}^{T} w_{t}}, \ \overline{Y}_{w}^{2} = \frac{\sum_{t=1}^{T} w_{t}^{2} y_{t}^{2}}{\sum_{t=1}^{T} w_{t}^{2}}, \ w_{t} = \exp\left(-a\frac{|y_{t} - \overline{y}|}{\widehat{\sigma}_{y}}\right), \ \widehat{\sigma}_{y} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (y_{t} - \overline{y})^{2}}.$$

Following Teräsvirta and Zhao (2011), we use a = 0.3. Notice that when applying the weights  $w_t$  to the series in levels, every observation will be downweighted except those equal to the sample mean, and when applying squared weights,  $w_t^2$ , to the squared observations, bigger observations in squares are more downwards weighted than their corresponding observations in levels.

The cross-correlations in (1) and (2), when computed using daily data, are capturing the dynamic relationship between the past return observed h days ago (i.e., yesterday if

h = 1) and the volatility today. However, the estimated correlation coefficients between returns and volatility computed by Kristoufek (2014) are contemporaneous (h = 0), only capturing the relationship between the return and the volatility at the same day.

#### 2.2 Asymmetric GARCH-type models

GARCH-type models are the most widely used to represent the dynamic evolution of the volatility of financial returns. Incorporating the leverage effect into such models is important to better capture the dynamic behavior of financial returns and improve the forecasts of future volatility. Among the pleiad of alternative GARCH models that are able to cope with the leverage effect, we focus on the TGARCH model proposed by Zakoian (1994) and the EGARCH model proposed by Nelson (1991). These two models are flexible specifications for representing the evolution of asymmetric variances, as compared to other asymmetric GARCH models; see Rodríguez and Ruiz (2012). We also consider a GARCH-type model that represents both leverage and long-memory in the volatility, namely the FIEGARCH model introduced by Bollerslev and Mikkelsen (1996). In all cases, we only consider the simplest parametrizations.

The TGARCH model accommodates the asymmetric relationship between past returns and volatility, by making the latter, denoted as  $\sigma_t$ , be a function of both the magnitude and the sign of past returns. In particular, if  $y_t$  denotes the series of demeaned returns, the basic TGARCH model is given by the following equations:<sup>1</sup>

$$y_t = \sigma_t \, \varepsilon_t \tag{3}$$

$$\sigma_t = \omega + \alpha |y_{t-1}| + \beta \sigma_{t-1} + \delta y_{t-1} \tag{4}$$

where  $\sigma_t$  is the volatility and  $\varepsilon_t$  is a sequence of independent identically distributed (i.i.d.) random variables with zero mean and unit variance. When  $y_{t-1}$  is positive, the volatility response is linear in  $y_{t-1}$  with slope  $(\delta + \alpha)$  but if  $y_{t-1}$  is negative, the slope of the response is  $(\delta - \alpha)$ . Thus, the volatility can respond asymmetrically to rises and falls in stock prices and the value of  $\delta$  is expected to be negative. Under the constraints

<sup>&</sup>lt;sup>1</sup>This is the parametrization used in Rodríguez and Ruiz (2012), but other equivalent reparametrizations are possible; see, for instance, the original one in Zakoian (1994) or those in He and Teräsvirta (1999) and He et al. (2002).

 $\omega > 0, \beta \ge 0$  and  $\alpha \ge |\delta|, \sigma_t$  is always positive and represents the conditional standard deviation of  $y_t$ . Moreover, the model is covariance stationary if  $\delta^2 < 1 - \alpha^2 - \beta^2 - 2\alpha\beta\mu_{|\varepsilon|}$ , where  $\mu_{|\varepsilon|} = E|\varepsilon_t|$ . The conditions for existence of higher-order moments, as well as the analytical expressions for unconditional moments and cross-moments can be found in He and Teräsvirta (1999) and He et al. (2008).

Alternatively, the EGARCH model specifies the log-squared volatility as a function of both the magnitude and the sign of lagged returns innovations. In particular, the basic EGARCH model is given by equation (3) and the following equation for the volatility:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left( |\varepsilon_{t-1}| - \mu_{|\varepsilon|} \right) + \delta \varepsilon_{t-1}.$$
(5)

When  $\varepsilon_{t-1}$  is positive, the log-volatility response is linear in  $\varepsilon_{t-1}$  with slope  $(\delta + \alpha)$  but if  $\varepsilon_{t-1}$  is negative, the slope of the response is  $(\delta - \alpha)$ . Thus, as in the TGARCH model, the value of  $\delta$  is expected to be negative for the model to capture the leverage effect.<sup>2</sup> Since the volatility equation (5) is specified in terms of logarithms, there are no inequality constraints on the parameters to ensure the positivity of  $\sigma_t$ . Moreover, the model is covariance stationary under certain conditions on both the parameters and the innovation distribution. For instance, if  $|\beta| < 1$  and  $\varepsilon_t$  is N(0,1) or GED with thickness parameter v > 1, the model is covariance stationary and possesses finite moments of any order, but this is not the case for some Student-t distributions; see Theorem 2.2 and Theorem A1.2 in Nelson (1991). The analytical expressions for unconditional moments and cross-moments can be found in Demos (2002), He et al. (2002) and Karanasos and Kim (2003). Moreover, under the assumption that  $\varepsilon_t$  is Gaussian,  $\mu_{|\varepsilon|} = \sqrt{2/\pi}$ , whereas for a GED distribution with parameter v, we have  $\mu_{|\varepsilon|} = \Gamma(2/v)/\sqrt{\Gamma(3/v)\Gamma(1/v)}$  and we have  $\mu_{|\varepsilon|} = \sqrt{(v-2)/\pi}\Gamma((v-1)/2)/\Gamma(v/2)$ , for a Student-t with v > 2 degrees of freedom, where  $\Gamma(\cdot)$  is the Gamma function.

The FIEGARCH model is an extension of the EGARCH model that allows for longmemory in the volatility by introducing a fractional operator in equation (5). In partic-

<sup>&</sup>lt;sup>2</sup>Some authors make a distinction between asymmetry, referred to as the different impacts on conditional volatility of positive and negative shocks of equal magnitude, and leverage effect, regarded as the negative correlation between returns shocks and subsequent shocks to volatility; see Chang and McAleer (2017) for the regularity conditions that an EGARCH(1,1) model obtained from a random coefficient complex nonlinear moving average process, should fulfill to capture asymmetry and/or leverage effects.

ular, the equation for the volatility in the basic FIEGARCH model is the following:

$$(1 - \beta L)(1 - L)^d \log(\sigma_t^2) = \omega + \alpha \left( |\varepsilon_{t-1}| - \mu_{|\varepsilon|} \right) + \delta \varepsilon_{t-1},$$

where L is the lag operator such that  $Lx_t = x_{t-1}$  and  $(1-L)^d$  is the fractional operator defined as

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} L^k.$$

When d = 0, the EGARCH model in (5) is obtained. As for the EGARCH model, no restrictions on the coefficients are required for the conditional variance to be positive. Moreover, if  $\varepsilon_t$  is N(0,1) or GED with parameter v > 1 and  $|\beta| < 1$  and d < 0.5, the model is covariance stationary. For further theoretical results on the main properties of the FIEGARCH model, see Ruiz and Veiga (2008) and Lopes and Prass (2014).

#### 2.3 Estimation methods

The three models introduced above are usually estimated by maximizing the conditional log-likelihood function, given by

$$L(\theta) = \sum_{t=1}^{T} l_t(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \log \sigma_t^2 + \sum_{t=1}^{T} \log f\left(\frac{y_t}{\sigma_t}\right),\tag{6}$$

where  $\theta$  denotes the parameter vector to be estimated and  $f(\cdot)$  is the probability density of  $\varepsilon_t$ . In particular, if  $\varepsilon_t$  is assumed to be N(0, 1), the corresponding Gaussian loglikelihood function, that will be denoted as  $L_N$ , comes up, namely:

$$L_N(\theta) = -\frac{T}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^T \left(\log \sigma_t^2 + \frac{y_t^2}{\sigma_t^2}\right).$$

The resultant estimator is the Gaussian Quasi-Maximum Likelihood estimator (QML here onwards) which is the most commonly used one for GARCH-type models, in general, and in particular, for the asymmetric GARCH models introduced in Section 2.2; see, for instance, Bollerslev and Mikkelsen (1996) and Ruiz and Veiga (2008) for an empirical application using QML with FIEGARCH models and Zivot (2009) for an application of QML with both TGARCH and EGARCH models.

The lack of robustness of the QML estimator in symmetric GARCH models is already well known; see, for instance, Carnero et al. (2007) and the references therein.

To overcome this drawback, some authors propose estimation methods resistant to outliers which consist of maximizing the log-likelihood based on heavy tailed distributions; see, for instance, Sakata and White (1998). Actually, in the seminal paper of Nelson (1991), he estimates the EGARCH model by maximizing the loglikelihood function in (6) assuming that  $\varepsilon_t$  followed a GED distribution normalized to have zero mean and unit variance. In such a case, the density of  $\varepsilon_t$  will be

$$f(\varepsilon) = \frac{\upsilon}{\lambda 2^{(1+1/\upsilon)} \Gamma(1/\upsilon)} \exp\left(-\frac{1}{2} \left|\frac{\varepsilon}{\lambda}\right|^{\upsilon}\right),\tag{7}$$

where  $\lambda = \sqrt{2^{-2/\nu}\Gamma(1/\nu)/\Gamma(3/\nu)}$  and  $\nu > 0$  is the tail-thickness parameter. When  $\nu = 2$ , the GED collapses to the N(0,1) but it provides thicker (thinner) tails than the Normal when  $\nu < 2$  ( $\nu > 2$ ). Putting back (7) into (6), the corresponding GED log-likelihood function, that will be denoted as  $L_{GED}$ , is obtained, namely:

$$L_{GED}(\theta) = T \left[ \log \left(\frac{\upsilon}{\lambda}\right) - \left(1 + \frac{1}{\upsilon}\right) \log 2 - \log \Gamma \left(\frac{1}{\upsilon}\right) \right] - \frac{1}{2} \sum_{t=1}^{T} \left( \log \sigma_t^2 + \left|\frac{y_t}{\lambda \sigma_t}\right|^{\upsilon} \right).$$
(8)

Finally, we also consider an estimator based on maximizing the Student-likelihood, i.e., assuming that  $\varepsilon_t$  follows a Student-t distribution with v degrees of freedom normalized to have zero mean and unit variance. In such a case, the log-likelihood function, that will be denoted as  $L_{Stu}$ , will be:

$$L_{Stu}(\theta) = T \log \left( \frac{\Gamma((\upsilon+1)/2)}{\sqrt{\pi(\upsilon-2)}\Gamma(\upsilon/2)} \right) - \frac{1}{2} \sum_{t=1}^{T} \left[ \log \sigma_t^2 + (\upsilon+1) \log \left( 1 + \frac{1}{\upsilon-2} \frac{y_t^2}{\sigma_t^2} \right) \right].$$
(9)

The resultant estimators obtained by maximizing (8) and (9) will be denoted as QML-GED and QML-t, respectively; see, for example, Rodríguez and Ruiz (2012) for an empirical application of QML-t in TGARCH and EGARCH models.

The asymptotic properties of these three estimators for asymmetric GARCH-type models are not well known. Pan et al. (2008) show that QML is consistent and asymptotically Normal for a general asymmetric GARCH model that includes as a particular case the TGARCH model. In particular, they show that, provided that  $\varepsilon_t$  is symmetrically distributed with  $E\varepsilon_t^2 = 1$  and  $E\varepsilon_t^4 < \infty$  and some regularity assumptions hold, it follows that

$$\sqrt{T}(\widehat{\theta}_{QML} - \theta) \xrightarrow{\mathcal{L}} N(0, \Lambda^{-1}\Omega\Lambda^{-1}),$$
(10)

where

$$\Lambda(\theta) = E\left(\frac{\partial^2 l_t(\theta)}{\partial \theta \partial \theta'}\right) \text{ and } \Omega(\theta) = E\left(\frac{\partial l_t(\theta)}{\partial \theta}\frac{\partial l_t(\theta)}{\partial \theta'}\right),$$

where  $l_t(\theta)$  is given in (6).

The asymptotic distribution of the QML estimator in EGARCH and FIEGARCH models is still unknown. Its finite sample properties are studied and compared to Whittle estimators in Pérez and Zaffaroni (2008). In EGARCH models, Straumann and Mikosch (2006) prove the consistency of the QML estimator in a very particular case and Wintenberger (2013) extends this result under less restrictive conditions and proves the consistency and asymptotic Normality of a new estimator, called stable QML. With respect to QML-GED and QML-t, no asymptotic theory exists in the context of asymmetric GARCH models. Hence, we will assume the usual practice of researchers using GARCH models that the asymptotic distribution of the three QML estimators discussed above is that in (10) and we will approximate their asymptotic variance by the so-called "sandwich" estimator

$$Var(\widehat{\theta}_{QML}) \approx H(\widehat{\theta}_{QML})^{-1}B(\widehat{\theta}_{QML})H(\widehat{\theta}_{QML})^{-1},$$

where  $H(\theta)$  denotes the Hessian matrix of the log-likelihood and  $B(\theta)$  is the inner product of the gradient (or score) of the log-likelihood, namely

$$H(\theta) = \sum_{t=1}^{T} \frac{\partial^2 l_t(\theta)}{\partial \theta \partial \theta'}, \quad B(\theta) = \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} \frac{\partial l_t(\theta)}{\partial \theta'}.$$

When estimating the TGARCH and EGARCH models by QML, the vector parameter is  $\theta = (\omega, \alpha, \beta, \delta)'$  but it becomes  $\theta = (\omega, \alpha, \beta, \delta, \upsilon)'$  when either QML-GED or QML-t are applied. In the FIEGARCH model, there is an additional parameter d and so,  $\theta = (\omega, \alpha, \beta, \delta, d)'$  for QML and  $\theta = (\omega, \alpha, \beta, \delta, d, \upsilon)'$  for QML-GED or QML-t.

## 3 Empirical results based on open-close returns

Kristoufek (2014) faces the treatment of the leverage effect focusing on four energy commodities futures, namely Brent and WTI crude oils, heating oil and natural gas, observed from 4 January 2000 to 28 June 2013. In particular, the paper works with

the open-close returns, defined as  $r_t = \log(C_t) - \log(O_t)$ , and the Garman-Klass (GK) estimator of the squared volatility, defined as

$$\widehat{\sigma_{GK,t}^2} = \frac{\left(\log\left(H_t/L_t\right)\right)^2}{2} - \left(2\log 2 - 1\right)\left(\log\left(C_t/O_t\right)\right),\tag{11}$$

where  $H_t$  and  $L_t$  are daily highs and lows, respectively, and  $C_t$  and  $O_t$  are daily closing and opening prices, respectively. However, in the data downloaded from the Appendix A of Kristoufek (2014), these four prices are only available for three out of the four series, namely Brent and WTI crude oils and heating oil. For natural gas, no closing prices ( $C_t$ ) are provided. Hence, for this series, we cannot compute either the open-close returns or the GK estimator of the squared volatility, defined in (11). Therefore, in this section, we focus our analysis on Brent and WTI crude oils and heating oil.

First, we try to replicate the results in Kristoufek (2014) by applying the same methodology as he does for the treatment of the leverage effect in energy futures, using the same data (open-close returns) but different statistical packages. Second, we apply a different methodology to the same data for both detecting and estimating the leverage effect. In particular, we apply robust correlation-based methods for the detection of leverage and then we estimate this effect in the context of asymmetric GARCH-type models using robust methods. In doing so, we try to find out whether analyzing the same data with different methodologies could lead to different conclusions. Furthermore, in Section 4, we will analyze the robustness of the results to the return definition used, by considering other type of returns that are commonly used in the literature and can be computed, with the available data, for the four commodities.

#### **3.1** Descriptive statistics and correlation analysis

Figure 1 displays, in its first two rows, the open-close returns,  $r_t$ , and the estimated volatility,  $\widehat{\sigma_{GK,t}}$ , respectively, for Brent and WTI crude oils and heating oil. The sample sizes of these series are: T = 3453 (Brent), T = 3367 (WTI) and T = 3370 (Heating oil). These plots should replicate Figures 1 and 2 in Kristoufek (2014) for the three series mentioned above. We can see that the estimated volatilities are exactly the same as those plotted in Figure 2 of the replicated paper. However, the returns plotted in Figure 1 of the replicated paper, although very similar, are not exactly the same as ours,

since they have a different scale. This fact becomes more clear by looking at Table 3, where we display the same descriptive statistics and tests as in Table 1 of the replicated paper, computed using Stata<sup>3</sup> (the version used along this paper is Stata/SE13.1). By comparing the top panel of both tables, which describes the raw returns, we realize that we are not able to find the same means and standard deviations. However, we are finding exactly the same skewness and excess kurtosis coefficients, as well as the same values of all the Jarque-Bera, Ljung-Box Q(30), ADF and KPSS test statistics. As expected, all series exhibit excess kurtosis and the Jarque-Bera test for Normality always rejects the null. Moreover, using the Q(30) test statistic, the null hypothesis of uncorrelated returns will be rejected in all cases, suggesting that a model for the conditional mean (an autorregresive or moving average model) should be estimated. Nevertheless, when a more suitable test statistic is considered, for example  $CH(30)^4$ , the null is never rejected at 5% significance level. In particular, the values (and p-values) of CH(30)for the three series considered are 30.704 (0.4301), 38.355 (0.1408) and 32.905 (0.3267), respectively. We have also computed the heteroscedastic-corrected Q-test proposed by Diebold (1988), obtaining the same conclusion. Hence, the returns will be assumed to be uncorrelated, as expected. The middle and bottom panels of Table 3 describe the standardized returns and logarithmic volatility, computed as  $\frac{r_t}{\widehat{\sigma_{GK,t}}}$  and  $\log(\widehat{\sigma_{GK,t}})$ , respectively. In this case we are able to replicate everything in the corresponding panels of Table 1 of the replicated paper, except the mean and the standard deviation of the standardized returns.

Table 4 displays the results from two long-memory tests, namely the modified rescaled range test  $V_T$  (Lo, 1991) and the rescaled variance test  $M_T$  (Giraitis et al., 2003), as well as the estimated Hurst exponent, H, for the logarithmic volatility, using GPH estimator (Geweke and Porter-Hudak, 1983), computed with our own codes in Matlab. These results should replicate Table 2 and the corresponding results on GPH in Table 3 of the replicated paper. As we can see, we replicate all the values of  $V_T$  and  $M_T$ , except those for the returns of heating oil, as well as all the optimal lags  $q^*$ . With respect to GPH

 $<sup>^{3}</sup>$ The lags used to compute the ADF and KPSS test statistics were chosen following the default value given in Gretl, as this was the software used in the replicated paper. Along this paper we have used Gretl 2017d.

<sup>&</sup>lt;sup>4</sup>This is a test statistic proposed by Cumby and Huizinga (1992) which is robust to conditional heteroscedasticity and it is implemented in Stata.

estimates of the Hurst exponent, our values are around 1 and nearly the same as those in the replicated paper for Brent and WTI crude oils, and slightly different for heating oil. However, our conclusions are the same as those in Kristoufek (2014), that is, returns and standardized returns are not long-term dependent, as expected, while logarithmic volatility exhibits significant and possibly nonstationary long memory.

Regarding the leverage effect, Kristoufek (2014) measures this effect by computing the correlation coefficients DCCA and DCMA, between contemporaneous standardized returns and logarithmic volatility. However, as discussed in Section 2, we wonder whether computing the contemporaneous correlation between standardized returns (instead of past returns) and logarithmic volatility (rather than volatility) is appropriate to capture the leverage effect. Instead, we should be looking at the dynamic relationship between past returns and current volatility, as explained in Section 2.1. Then, it seems to be more interesting to compute the DCCA coefficient between lagged returns and current volatility. The bottom panels of Figure 1 display both coefficients. In particular, the third row plots the values of the DCCA coefficient for Brent and WTI crude oils and heating oil computed as in Kristoufek (2014). These graphs should replicate the black lines in Figure 3 of the replicated paper but they actually mimic the gray lines in such a figure, which are supposed to be the other correlation coefficient, DMCA, considered by Kristoufek (2014). The last row of Figure 1 plots DCCA coefficients between current returns, as well as lagged 1 and 2 returns, and volatility. Noticeably, these coefficients are negative, suggesting the presence of leverage effect.

Further correlation analysis is performed in Figure 2. This figure displays, in its first row, the correlograms of the returns with the hereroscedastic-corrected 95% confidence bands proposed by Diebold (1988), given by

$$\pm \frac{1.96}{\sqrt{T}} \left[ 1 + \frac{\widehat{\gamma}_2(h)}{\left(\widehat{\gamma}(0)\right)^2} \right],$$

for lags h = 1, 2, ...30, where  $\hat{\gamma}_2(h)$  and  $\hat{\gamma}(0)$  are the h - th sample autocovariance of the squared returns and the sample variance of the returns, respectively. As  $\hat{\gamma}_2(h) > 0$ , in our case, these bands are wider than the usual 95% Barlett bands  $(\pm 1.96/\sqrt{T})$  and show no evidence of autocorrelated returns, confirming our previous result on the robust test CH(30).

The correlograms of squared returns, displayed in the 2nd row of Figure 2, show

that there is significant and persistent correlation in the squares, indicating conditional heteroscedasticity and possible long-memory in the volatility, as suggested in the replicated paper. In fact, when we test for conditional heteroscedasticity, by applying the Q and CH statistics to the squared returns, the values of both statistics for lag 30, are 2609.6 and 81.4 for Brent, 4789.2 and 73.0 for WTI and 1378.7 and 116.1 for heating oil, respectively, being all of them significant at 1% significance level. Figure 2 also displays, in its 3rd row, the robust autocorrelations of squares proposed by Teräsvirta and Zhao (2011), which are resistant to outliers. In this case, the difference between robust and non-robust correlations are not remarkable. However, as we will see in Section 4, the use of robust correlations will be essential when dealing with outlying observations. Finally, the last two rows of Figure 2 display the sample and robust cross-correlations between past returns and current squared returns, as given in (1) and (2), respectively. In general, these cross-correlations are negative, suggesting leverage effect in the three series. This feature will be further investigated in the next subsection, where asymmetric GARCH-type models will be estimated for the open-close returns.

#### **3.2** Estimation results

The TGARCH, EGARCH and FIEGARCH models, described in Section 2.2, have been fitted to the series of open-close returns. Tables 5, 6 and 7 report the estimation results for these three models, respectively. For each estimation method considered (QML, QML-GED and QML-t), we also report, at the bottom rows of each panel, some diagnostics based on the residuals,  $\hat{\varepsilon}_t = y_t/\hat{\sigma}_t$ , where  $\hat{\sigma}_t$  is the estimated volatility for each model. The estimation has been mainly performed by using the Oxford MFE Toolbox for Matlab, though in some particular cases, we have written our own codes. When carrying out the estimation (which is done by minimizing the minus log-likelihood function), we keep the default options for the optimization in the MFE Toolbox. Moreover, the variance-covariance matrix estimator used is the so-called "sandwich" estimator (also known as robust covariance matrix estimator or heteroscedasticity-consistent covariance matrix estimator), as described in Section 2.3. As pointed out by Brooks et al. (2001) we are aware that different software could give different results. Therefore, to check for the robustness of our results we have repeated the estimation of some models in Stata

and Gretl, obtaining similar results<sup>5</sup>. In all cases, special care has been taken with the reparametrization used in each package in order to compute the estimated parameter values and the standard errors. This issue is particularly important in the TGARCH model for two reasons. First, there are several (equivalent) parametrizations available in the literature; see, for instance, Zakoian (1994), He and Teräsvirta (1999) and Rodríguez and Ruiz (2012). Second, the GJR model, proposed by Glosten et al. (1993), is sometimes referred to as TGARCH, as in Zivot (2009). Moreover, we should also be cautious with the results obtained for the EGARCH and FIEGARCH models, because we compute standard errors assuming the usual asymptotic distribution for QML estimators in (10). This is a common practice among practitioners using GARCH models since almost all of the major software packages do the same. However, as discussed in Section 2.3, there is no theoretical results supporting such a practice. Actually, when estimating FIEGARCH models, we have faced some numerical problems. This could be related to the point made by Wintenberger (2013) regarding the unreliability of QML methods for non-invertible EGARCH models.

Our discussion on the estimation results will be mainly focused on the estimated parameter  $\delta$ , since we are interested in the leverage effect. In general, the results obtained for each commodity are quite similar for the three models and the three estimation methods considered, but there are remarkable differences between the series. For the Brent and WTI crude oil series, the estimated  $\delta$  is always negative and statistically significant at 1% significance level, regardless of the model and estimator used. This provides strong evidence of leverage effect in both series, in agreement with the features of the cross-correlograms in Figure 2. The heating oil series is characterized, in all cases, by a negative estimated  $\delta$  but this is closer to zero than in both crude oil series, becoming no longer significant when the FIEGARCH model is fitted to this series. These findings confirm the results of the replicated paper using a different methodology.

On the other hand, the fractional parameter d in FIEGARCH models is always estimated larger than 0.5, suggesting a nonstationary long-memory behavior of the volatility, in agreement with the estimated GPH Hurst exponents, H, displayed in Table 4. Notice that, since H = d + 0.5, nonstationary long memory is found for  $H \ge 1$  ( $d \ge 0.5$ ).

<sup>&</sup>lt;sup>5</sup>The results are not displayed here to save space but they are available in the online Appendix A.

Actually, based on the reported standard errors, we never reject the null hypothesis  $H_0: d \ge 0.5$  at any reasonable significance level. It is also worth mentioning that the thickness parameter v in QML-GED is always estimated smaller than 2, suggesting fat tails. In fact, for the TGARCH model, the corresponding values of the test statistic to test Normality ( $H_0: v = 2$ ) are -7.3484, -5.9776, -5.9484 for Brent, WTI and heating oil, respectively, which clearly rejects  $H_0: v = 2$  against  $H_1: v < 2$  (thick tails). The corresponding values for the EGARCH model are -7.3419, -5.9836, -5.7746 and for the FIEGARCH model are -7.1855, -6.0701, -6.1016, leading to the same conclusions. Accordingly, the estimated values of the parameter v of the Student error distribution in QML-t, also indicate fat tails.

When looking at residuals diagnostics, as expected, the values of the test statistics  $Q_2(30)$  and  $CH_2(30)$  for remaining autocorrelation in the squared residuals, have been reduced remarkably in all estimated models, as compared to their values for the squared returns. Only for the Brent crude oil, the values of  $CH_2(30)$  remain significant at 10% in all TGARCH and EGARCH models, indicating possible long memory in the volatility. In all other cases, both statistics are no longer significant, indicating that the estimated models have been able to properly capture the dynamics in the conditional variance of the returns<sup>6</sup>.

Finally, Figure 3 compares, for each series considered, the QML estimated volatilities from the TGARCH, EGARCH and FIEGARCH models versus the GK estimated volatilities computed from (11). For example, the graph in the 1st row and 1st column is the scatter plot of the Brent volatility  $\widehat{\sigma}_{GK,t}$  (in x-axis) against the Brent volatility  $\widehat{\sigma}_t$  from the estimated TGARCH model (in y-axis), the latter being computed from (4) using as parameter values the QML estimated parameters in the first panel of Table 5. As expected, the volatilities estimated by both methods are around the diagonal, but in some cases the GK estimated volatilities tend to be more extreme than asymmetric GARCH volatilities, which seem to be smoother. When volatilities are estimated by QML-GED and QML-t, similar graphs are obtained<sup>7</sup>.

<sup>&</sup>lt;sup>6</sup>As discussed in Li and Mak (1994), the test statistic  $Q_2$  applied to a residual series from a conditional heteroscedastic model becomes conservative. This could explain why  $Q_2$  rejects less frequently than  $CH_2$ .

<sup>&</sup>lt;sup>7</sup>They are not displayed here to save space but they are available upon request.

### 4 Sensitivity analysis to the return definition

In this section, we analyze whether the results previously discussed are robust to the return definition by considering other type of returns commonly used in the literature. In particular, we consider the open-open returns, defined as  $ro_t = \log(O_t) - \log(O_{t-1})$ , as these can be computed with the available data for the four commodities analyzed in the replicated paper<sup>8</sup>. The sample sizes of these four series have one observation less than the open-close returns, i.e. T = 3452 (Brent), T = 3366 (WTI), T = 3369 (Heating oil) and T = 3368 (Natural gas). When comparing these series, plotted in Figure 4, with the open-close returns displayed in Figure 1 of the replicated paper, we observe that both share similar patterns. However, the open-open returns seem to be affected by large outliers, especially the natural gas, which exhibits two very extreme consecutive observations around the middle of the sample period. These two observations are due to the high open price of the natural gas on 14 April 2006, which was 11.26 dollars, whereas the open prices on the previous and posterior days were 6.78 and 7.15 dollars, respectively. These large changes (first increase and decrease later) in the open price yield two consecutive outliers in the returns, the first one positive and the second one negative, of magnitudes about 12 times the standard deviation of the series. These extreme observations, that are not present in the open-close gas returns in the replicated paper, are expected to affect dramatically our results, as we will confirm next.

#### 4.1 Descriptive statistics and correlation analysis

Table 8 contains descriptive statistics of the open-open returns introduced above,  $ro_t$ , computed using Stata. As we can see, these series share similar properties to the openclose returns described in Table 3. However, it is worth mentioning the difference in the excess kurtosis coefficient for heating oil, being 1.6150 for the open-close returns and 3.5041 for the open-open returns. This is due to the presence of some outliers at the beginning of the sample (compare Figures 1 and 4). For the natural gas, the difference is even more remarkable, with the excess kurtosis coefficient of the open-open returns

<sup>&</sup>lt;sup>8</sup>We have also analyzed close-close returns for the three series we could compute them and we have checked that they share similar properties to the open-close returns. The results are not displayed here to save space but are available upon request.

being 18.765 as compared to the corresponding one for the open-close returns in the replicated paper, which is 1.6458.

The differences between the behavior of open-open and open-close returns are further illustrated when comparing Figure 5 to Figure 2. In both pictures, the sample autocorrelations of squared returns are persistent and highly significant, for Brent, WTI and heating oil, indicating possible long-memory in their volatilities. However, this is not the case for the natural gas, that exhibits a very high positive and significant 1st-order autocorrelation with the correlations for higher lags being pushed downwards towards zero. Noticeably, this is the typical pattern of the correlogram of the squared observations in the presence of consecutive outliers (see Carnero et al. (2007)), and this could be the case here, since, as commented above, the natural gas returns exhibit two big consecutive outliers (see Figure 4). However, when we look at the robust autocorrelations of squared returns of natural gas, the picture completely changes and the correlations become significantly different from zero even for long lags, resembling the patterns of the other three commodities.

Finally, comparing the last two rows of Figures 2 and 5, we can see that the crosscorrelations between past returns and current squared returns for Brent and WTI crude oils, are very similar and mainly negative, suggesting possible leverage effect. However, for the heating oil series, the leverage effect seems to be more clear when looking at the cross-correlations computed with open-close returns (Figure 2) than those computed with the open-open returns (Figure 5), which could become non significant. Again, the behavior of the gas series is rather different, possibly due to the effect of consecutive outliers: the 1st sample cross-correlation is typically pushed upwards to a positive value while the others become close to zero (see Carnero et al. (2016) for a theoretical discussion on this feature). However, when the robust cross-correlations are computed, another picture comes up, with all the cross-correlations being around zero. Hence, the possible inverse leverage effect (positive correlation between volatility and past returns) found in the natural gas by some authors, included the replicated paper, could be an artifact due to the misleading effect of outliers.

#### 4.2 Estimation results

Tables 9, 10 and 11 report the estimation results obtained when the TGARCH, EGARCH and FIEGARCH models, respectively, are fitted to the open-open returns. In order to check whether results change due to the definition of the return, we will compare these tables with Tables 5, 6 and 7, focussing mainly in the estimated leverage parameter  $\delta$ .

For the Brent and WTI crude oil series, there are no remarkable differences between the results from open-open and open-close returns. Again, regardless of the model and estimator used, the estimated  $\delta$  is always negative and statistically significant at 5% significance level, providing strong evidence of leverage effect in both series. This finding confirms the results of the replicated paper and highlights their robustness to both the methodology and the type of returns used.

However, for the heating oil series, some differences arise. In particular, when openopen returns are used, the point estimates of  $\delta$  are nearly always negative but, in general, they are not significant at 5% level, indicating no leverage effect, as suggested by the cross-correlograms in Figure 5. This result slightly differs from previous results with open-close returns, for which both the methodology in the replicated paper and the one discussed in this paper, find statistically significant leverage effect. Besides, we also find that, regardless of the model, estimator and type of returns used, the heating oil series is characterized by a weaker leverage effect (estimated  $\delta$  closer to zero) than in the two crude oil series (Brent and WTI), as pointed out in the replicated paper.

Regarding the fractional differencing parameter d in the FIEGARCH model, the point estimates for both crude oils and heating oil are smaller for the open-open than for the open-close returns but they still suggest a nonstationary long-memory behavior of the volatility. Moreover, for these three series, the estimated thickness parameter v in QML-GED and QML-t, are also the same for both type of returns, indicating fat tails.

The results for natural gas are quite different from the other three series. In most cases, the parameter  $\delta$  is estimated positive (suggesting inverse leverage effect) but it is never significant at 1%, although it becomes significant at 5% and 10% in some cases. Hence, we partly agree with the replicated paper, who finds inverse leverage effect for the natural gas using open-close returns, but unlike him, we cannot confirm that  $\delta$  is statistically significant when using open-open returns. With respect to the estimated

parameter d, QML provides a rather different value than those provided by QML-GED and QML-t, the latter being more reliable, since these two estimators are expected to be more robust to outliers. In fact, the estimated values of d for natural gas are smaller than those for Brent and WTI crude oils and heating oil, in agreement with the estimated Hurst exponent H in the replicated paper. Also notice that, for natural gas, the thickness parameter v in QML-GED is estimated closer to one (indicating very heavy tails) than in the other series. The same happens with the parameter v of the Student error distribution in QML-t, which becomes much lower (heavier tails) for the natural gas. Again, these results could be due to the pernicious effect of consecutive outliers which render QML methods unreliable.

When looking at the residuals, no remarkable differences show up with respect to our results in Section 3.2, and for the natural gas, the values of the test statistics Q and CH indicate no remaining serial correlation. Only the value of  $Q_2(30)$  is statistically significant at 1% when Gaussian QML is applied.

Finally, Figure 6 compares the volatilities estimated from our three parametric asymmetric GARCH models by using the two types of returns, for the series of Brent, WTI and heating oil, for which both open-open and open-close returns can be computed with the available data. For each commodity and each model, we represent the scatter plot of the estimated volatilities using open-open returns versus the estimated volatilities using the open-close returns. For example, the graph in the 1st row and 1st column is the scatter plot of the Brent volatility of the open-open returns (in x-axis) obtained from the estimated TGARCH model in (4) using as parameter values the QML estimated parameters in the first panel of Table 9 versus of the Brent volatility of the open-close returns (in y-axis) computed from (4) using as parameter values the QML estimated parameters in the first panel of Table 5. As expected, the estimated volatilities of both type of returns are around the diagonal, but the open-open returns of WTI and heating oil seem to be more volatile than the corresponding open-close returns.

## 5 Conclusions

In this paper we have replicated the results in Kristoufek (2014) on the leverage effect in energy futures by working with the same open-close returns and methodology as he

does, obtaining the same results and conclusions but using different software. We have also analyzed the robustness of the results to both the methodology and the type of returns used. In particular, using both, open-close and open-open returns, we compute the sample and robust cross-correlations between past returns and squared returns to detect leverage effect and then, we estimate three popular asymmetric GARCH-type models (TGARCH, EGARCH and FIEGARCH) with different distributions for the innovations (Gaussian, GED and Student). Our findings strongly confirm the results in Kristoufek (2014) for two out of the four series analyzed, namely WTI and Brent crude oils, where the standard leverage effect (negative correlation between past returns and current volatility) is found. However, for the heating oil and the natural gas series, we cannot totally confirm his results since the statistical significance of the leverage effect depends on both the methodology and the type of returns used. The presence of consecutive outliers in the natural gas and its possible effect on both the correlationbased tools and the QML estimators is also discussed, stressing the need for robust methods to be applied in this setting.

## Acknowledgments

We thank the editor and the referees for their helpful comments and suggestions that have improved the paper. Financial support from the Spanish Government under projects ECO2014-58434-P and ECO2016-77900-P is gratefully acknowledged by the first and second authors, respectively. As usual, we are responsible for any remaining errors.

## References

- [1] Agnolucci, P. (2009) Volatility in crude oil futures: A comparison of the predictive ability of GARCH and implied volatility models. *Energy Economics* 31, 316–321.
- [2] Behmiri, N.B. and Manera, M. (2015) The role of outliers and oil price shocks on volatility of metal prices. *Resources Policy* 46, 139–150.
- Black, R. (1976) Studies in stock price volatility changes, Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Sections, American Statistical Association, 177–181.
- [4] Bollerslev, T. and Mikkelsen, H.O. (1996) Modeling and pricing long-memory in stock market volatility. *Journal of Econometrics* 73, 151–184.
- [5] Bollerslev T., Litvinova J. and Tauchen, G. (2006) Leverage and volatility feedback effects in high-frequency data. *Journal of Financial Econometrics* 4, 353–384.
- [6] Brooks, C., Burke, S.P. and Persand, G. (2001) Benchmarks and the accuracy of GARCH model estimation. *International Journal of Forecasting* 17, 45–56.
- [7] Carnero, M.A., Peña, D. and Ruiz, E. (2007) Effects of outliers on the identification and estimation of GARCH models. *Journal of Time Series Analysis* 28, 471–497.
- [8] Carnero, M.A., Peña, D. and Ruiz, E. (2012) Estimating GARCH volatility in the presence of outliers. *Economics Letters* 114, 86–90.
- [9] Carnero, M.A., Pérez, A. and Ruiz, E. (2016) Identification of asymmetric conditional heteroscedasticity in the presence of outliers. SERIEs: Journal of the Spanish Economic Association 7, 179–201.
- [10] Chang, C. L. and McAleer, M. (2017) The correct regularity condition and interpretation of asymmetry in EGARCH. *Economics Letters* 161, 52–55.
- [11] Chang, K. (2012) Volatility regimes, asymmetric basis effects and forecasting performance: An empirical investigation of the WTI crude oil futures market. *Energy Economics* 34, 294–306.

- [12] Charles, A. and Darné, O. (2014a) Volatility persistence in crude oil markets. Energy Policy 65, 729–742.
- [13] Charles, A. and Darné, O. (2014b) Large shocks in the volatility of the Dow Jones Industrial Average index: 1928–2013. *Journal of Banking and Finance* 43, 188–199.
- [14] Cheong, C. (2009) Modeling and forecasting crude oil markets using ARCH-type models. *Energy Policy* 37, 2346 –2355.
- [15] Chkili, W., Hammoudeh, S. and Nguyen, D. (2014) Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory. *Energy Economics* 41, 1–18.
- [16] Cumby, R. and Huizinga, J. (1992) Testing the autocorrelation structure of disturbances in ordinary least squares and instrumental variables regressions. *Econometrica* 60, 185–195.
- [17] Demos, A. (2002) Moments and dynamic structure of a time-varying-parameter stochastic volatility in mean model. *The Econometrics Journal* 5, 345–357.
- [18] Diebold, F.X. (1988) Empirical Modeling of Exchange Rate Dynamics, Springer-Verlag.
- [19] Doornik, J. and Ooms, M. (2005) Outlier detection in GARCH models. TI 2005-092/4 Tinbergen Institute Discussion Paper
- [20] Engle, R.F. (2011) Long term skewness and systematic risk. Journal of Financial Econometrics 9, 437–468.
- [21] Fan, Y., Zhang, Y., Tsai, H. and Wei, Y. (2008) Estimating 'Value at Risk ' of crude oil price and its spillover effect using the GED-GARCH approach. *Energy Economics* 30, 3156–3171.
- [22] Geweke, J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models. *Journal of Time Series Analysis* 4, 221–238.

- [23] Giraitis, L., Kokoszka, P., Leipus, R. and Teyssiere, G. (2003) Rescaled variance and related tests for long memory in volatility and levels. *Journal of Econometrics* 112, 265–294.
- [24] Glosten, L.R., Jagannathan, R. and Runkle, D.E. (1993) On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance* 48, 1779–1801.
- [25] He, C. and Teräsvirta, T. (1999) Properties of moments of a family of GARCH processes. *Journal of Econometrics* 92, 173–192.
- [26] He, C., Teräsvirta T. and Malmsten H. (2002) Moment structure of a family of first-order exponential GARCH models. *Econometric Theory* 18, 868–885.
- [27] He, C., Silvennoinen, A. and Teräsvirta T. (2008) Parameterizing unconditional skewness in models for financial time series. *Journal of Financial Econometrics* 6, 208–230.
- [28] Hibbert, A.M., Daigler, R.T. and Dupoyet, B. (2008) A behavioral explanation for the negative asymmetric return-volatility relation. *Journal of Banking and Finance* 32, 2254–2266.
- [29] Hill, J.B. (2015) Robust estimation and inference for heavy tailed GARCH. Bernoulli 21, 1629–1669.
- [30] Karanasos, M. and Kim J. (2003) Moments of the ARMA-EGARCH model. The Econometrics Journal 6, 146–166.
- [31] Kristoufek, L. (2014) Leverage effect in energy futures. *Energy Economics* 45, 1–9.
- [32] Laurent, S., Lecourt, C. and Palm, F.C. (2016) Testing for jumps in conditionally Gaussian ARMA-GARCH models, a robust approach. *Computational Statistics & Data Analysis*, 100, 383-400.
- [33] Li, W.K. and Mak, T.K. (1994) On the squared residual autocorrelations in nonlinear time series with conditional heteroskedasticity. *Journal of Time Series Analy*sis 15, 627–636.

- [34] Lo, A. (1991) Long-term memory in stock market prices. *Econometrica* 59, 1279– 1313.
- [35] Lopes, S.R.C. and Prass, T.S. (2014) Theoretical results on fractionally integrated exponential autoregressive conditional heteroskedastic processes. *Physica A* 401, 278–307.
- [36] Muler, N. and Yohai, V. (2008) Robust estimates for GARCH models. Journal of Statistical Planning and Inference 138, 2918–2940.
- [37] Nelson, D.B. (1991) Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* 59, 347–370.
- [38] Nomikos, N. and Andriosopoulos, K. (2012) Modelling energy spot prices: Empirical evidence from NYMEX. *Energy Economics* 34, 1153–1169.
- [39] Pan, J., Wang H. and Tong H. (2008) Estimation and tests for power-transformed and threshold GARCH models, *Journal of Econometrics* 142, 352–378.
- [40] Pérez, A. and Zaffaroni, P. (2008) Finite sample properties of maximum likelihood and Whittle estimators in EGARCH and FIEGARCH models. *Quantitative and Qualitative Analysis in Social Sciences* 2, 78–97
- [41] Reboredo, J. (2011) How do crude oil prices co-move? A copula approach. Energy Economics 33, 948 –955.
- [42] Rodríguez, M.J. and Ruiz E. (2012) GARCH models with leverage effect: differences and similarities. *Journal of Financial Econometrics* 10, 637–668.
- [43] Ruiz, E. and Veiga, H. (2008) Modelling long-memory volatilities with leverage effect: A-LMSV versus FIEGARCH. Computational Statistics & Data Analysis 52, 2846–2862.
- [44] Sakata, S. and White, H. (1998) High breakdown point conditional dispersion estimation with application to S&P 500 daily returns volatility. *Econometrica* 66, 529–567.

- [45] Straumann, D. and Mikosch T. (2006) Quasi-maximum-likelihood estimation in heteroskedastic time series: A stochastic recurrence equations approach. Annals of Statistics 34, 2449–2495.
- [46] Tauchen, G. Bollerslev, T. and Sizova, N. (2012) Volatility in equilibrium: asymmetries and dynamic dependencies. *Review of Finance* 16, 31–80.
- [47] Teräsvirta, T. and Zhao Z (2011) Stylized facts of return series, robust estimates and three popular models of volatility. *Applied Financial Economics* 21, 67–94.
- [48] Wei, Y., Wang, Y. and Huang D. (2010) Forecasting crude oil market volatility: Further evidence using GARCH-class models. *Energy Economics* 32, 1477 –1484.
- [49] Wintenberger, O. (2013) Continuous invertibility and stable QML estimation of the EGARCH(1,1) model. Scandinavian Journal of Statistics 40, 846–867.
- [50] Wu, C., Chung, H. and Chang, Y. (2012) The economic value of co-movement between oil price and exchange rate using copula-based GARCH models. *Energy Economics* 34, 270 –282.
- [51] Zakoian, J.M. (1994) Threshold heteroskedastic models. Journal of Economic Dynamics and Control 18, 931–955.
- [52] Zhang, Y., Fan, Y., Tsai, H. and Wei, Y. (2008) Spillover effect of US dollar exchange rate on oil prices. *Journal of Policy Modeling* 30, 973–991.
- [53] Zivot, E. (2009) Practical issues in the analysis of univariate GARCH models. In: Mikosch T, Kreiß J-P, Davis RA, Andersen TG (eds) Handbook of Financial Time Series. Springer-Verlag, Berlin, pp 113–155.

and/or Natural gas	Results	Inverse leverage for WTI No leverage for Brent	Inverse leverage for WTI	No leverage for WTI	No leverage for WTI Leverage for Brent	Mixed results for WTI Mixed results for Brent	Leverage for WTI Leverage for Brent	No leverage for WTI	No leverage for WTI	Leverage for WTI Leverage for Heating oil Inverse leverage for Natural gas	Leverage for WTI Leverage for Natural gas	Leverage for WTI Leverage for Brent Leverage for Heating oil Inverse leverage for Natural gas	Leverage for WTI Leverage for Brent Mixed results for Heating oil Mixed results for Natural gas
Table 1: Articles in which the leverage effect is estimated using prices of WTI, Brent, Heating oil and/or Natural gas	Methodology	Asymmetric GARCH models	Asymmetric GARCH models	Asymmetric GARCH models	Asymmetric and long-memory GARCH models	Asymmetric and long-memory GARCH models	Asymmetric GARCH models	Asymmetric GARCH models	Asymmetric GARCH models	Asymmetric GARCH models	Asymmetric and long-memory GARCH models	Detrended Cross Correlation coefficient	Robust Cross-Correlation Asymmetric and Long-memory GARCH models
estimated using pric	Sample considered	From $20/05/1987$ to $01/08/2006$	From $04/01/2000$ to $31/05/2005$	From $31/12/1991$ to $02/05/2005$	From 04/01/1993 to 31/12/2008	From 06/01/1992 to 31/12/2009	From $03/01/1997$ to $04/06/2010$	From $02/01/1990$ to $28/12/2009$	From $01/01/1990$ to $28/07/2010$	From $12/09/2000$ to $01/02/2010$	From $07/01/1997$ to $31/03/2011$	From 04/01/2000 to 28/06/2013	From $04/01/2000$ to $28/06/2013$
hich the leverage effect is	$\operatorname{Data}$	Daily spot prices of WTI and Brent	Daily spot prices of WTI	Daily future prices of WTI	Daily spot prices of WTI and Brent	Daily spot prices of WTI and Brent	Weekly spot prices of WTI and Brent	Weekly future prices of WTI	Weekly future prices of WTI	Daily spot prices of WTI, Heating oil and Natural gas	Daily spot and future prices of WTI and Natural gas	Daily future prices of WTI, Brent, Heating oil and Natural gas	Daily future prices of WTI, Brent, Heating oil and Natural gas
Table 1: Articles in w <sup>†</sup>	Paper	Fan et al. (2008)	Zhang et al. (2008)	Agnolucci (2009)	Cheong (2009)	Wei et al. (2010)	Reboredo (2011)	Wu et al. (2012)	Chang (2012)	Nomikos and Andriosopoulos (2012)	Chkili et al. (2014)	Kristoufek (2014)	This paper

29



 Table 2: Software available to estimate different asymmetric GARCH models

 Model
 Assumed error distribution

Model	Assu	umed error distribu	ition
	Gaussian	Student-t	GED
TGARCH	EVIEWS MFE-Toolbox G@RCH4.0 Stata Gretl Splus R	EVIEWS MFE-Toolbox G@RCH4.0 Stata Gretl R	EVIEWS MFE-Toolbox G@RCH4.0 Stata Gretl R
EGARCH	EVIEWS Matlab MFE-Toolbox G@RCH4.0 Stata Gretl Splus R	EVIEWS Matlab MFE-Toolbox G@RCH4.0 Stata Gretl R	EVIEWS MFE-Toolbox G@RCH4.0 Stata Gretl R
FIEGARCH	G@RCH4.0 Splus	G@RCH4.0	G@RCH4.0

٢

	Statistic	Brent crude oil	WTI crude oil	Heating oil
Returns	Mean	0.0000	0.0003	-0.0002
	Std.Dev.	0.0206	0.0217	0.0211
	Skewness	-0.2045	-0.1523	-0.0618
	Excess Kurtosis	3.0454	3.5389	1.6150
	Jarque-Bera	1358***	1770***	$368.4^{***}$
	Q(30)	56.578***	93.258***	48.808**
	ADF	$-9.3723^{***}$	$-8.984^{***}$	$-9.064^{***}$
	KPSS	0.0729	0.2090	0.0936
Standardized returns	Mean	0.0435	0.0721	0.0008
	Std.Dev.	1.0430	1.0037	1.0521
	Skewness	0.0051	0.0032	0.0046
	Excess Kurtosis	-0.3861	-0.5599	-0.4994
	Jarque-Bera	$21.47^{***}$	43.98***	$35.04^{***}$
	Q(30)	38.880	49.038**	37.539
C	ADF	$-13.472^{***}$	$-9.9800^{***}$	$-8.9790^{***}$
	KPSS	0.1990	0.1130	0.0604
Logarithmic volatility	Mean	-4.1419	-4.0664	-4.0993
	Std.Dev.	0.4612	0.4348	0.4422
	Skewness	0.0974	0.5431	0.1736
	Excess Kurtosis	2.0910	0.9238	0.2861
	Jarque-Bera	$634.5^{***}$	285.3***	28.43***
	Q(30)	12000***	16000***	15000***
	ADF	$-4.117^{***}$	$-4.135^{***}$	$-3.777^{***}$
	KPSS	$2.0700^{***}$	$1.1000^{***}$	$5.2100^{***}$

Table 3: Descriptive statistics for open-close returns (Table 1 of the replicated paper)

\*\*, \*\*\*: statistically significant at 5% and 1% respectively

Service

Table 4: Long-term memory tests and GPH estimates of the Hurst exponent for logarithmic volatility (Tables 2 and partially 3 of the replicated paper)

			/	
	Statistic	Brent crude oil	WTI crude oil	Heating oil
	$V_T$	1.4605	1.5972	1.3539
Returns	$M_T$	0.0742	0.1141	0.0655
	$q^*$	2	1	0
	$V_T$	1.5400	1.5731	1.1522
Standardized returns	$M_T$	0.1058	0.0970	0.0663
	$q^*$	2	2	1
	$V_T$	2.6777***	2.8427***	3.2813***
Logarithmic volatility	$M_T$	$0.6969^{***}$	$0.5707^{***}$	$0.8269^{***}$
	$q^*$	18	20	19
$\bigcirc$	GPH	1.0354***	1.0979***	1.0966***
	St. error	0.0576	0.0607	0.0663
I	C 1		10 1 1	

\*\*\*: evidence of long-memory at 1% significance level

open-close returns						
Estimator	Parameter	Brent crude oil	WTI crude oil	Heating oil		
QML	ω	$0.0003^{***}$ $(0.000)$	0.0002***	$0.0002^{**}$		
	lpha	$0.0557^{***}$ (0.010)	$0.0473^{***}$	0.0445*** (0.007)		
	$\beta$	$0.9441^{***}$	$0.9516^{***}$	$0.9553^{***}$		
	$\delta$	$-0.0187^{***}$	$-0.0168^{***}$	$-0.0100^{***}$		
	Log-Likelihood	8829.8	8460.5	8457.9		
Deciduala	Q(30)	29.342	38.644	31.264		
Residuals	CH(30)	29.084	37.852	31.360		
	$Q_2(30)$	39.876	23.595	32.587		
	$CH_{2}(30)$	44.709**	31.404	34.988		
QML-GED	ω	0.0002*** (0.000)	0.0002*** (0.000)	0.0002* (0.000)		
	$\alpha$	$0.0549^{***}$	$0.0474^{***}$	$0.0441^{***}$		
	eta	$0.9449^{***}$ (0.016)	$0.9520^{***}$	$0.9557^{***}$ $(0.012)$		
	δ	$-0.0179^{***}$	$-0.0162^{***}$	$-0.0102^{**}$		
	v	1.5444	1.5995	1.6312		
	Log-Likelihood	8859.4	8482.2	8475.5		
Residuals	Q(30)	29.425	38.517	31.282		
nesiduais	CH(30)	29.149	37.755	31.379		
	$Q_2(30)$	39.943	23.593	32.754		
	$CH_{2}(30)$	44.656**	31.662	34.991		
QML-t	ω	0.0002* (0.000)	0.0002** (0.000)	0.0002 (0.000)		
	lpha	$0.0543^{***}$	$0.0479^{***}$ (0.008)	$0.0439^{***}$		
	eta	$0.9455^{***}$ $(0.016)$	$0.9519^{***}$ (0.009)	$0.9559^{***}$ (0.013)		
	$\delta$	$-0.0176^{***}$	$-0.0156^{***}$	$-0.0104^{**}$		
	v	10.379	11.559	12.613		
	Log-Likelihood	8861.6	8487.8	8477.2		
Residuals	Q(30)	29.484	38.378	31.295		
Residuals	CH(30)	29.191	37.666	31.395		
	$Q_2(30)$	40.043	23.544	32.838		
	$CH_{2}(30)$	44.608**	31.952	34.974		

Table 5: Estimation of the TGARCH model with QML, QML-GED and QML-t using open-close returns

\*,\*\*,\*\*\*: statistically significant at 10%, 5% and 1% respectively

open-close returns							
Estimator	Parameter	Brent crude oil	WTI crude oil	Heating oil			
QML	ω	$-0.0864^{***}$	$-0.0705^{**}$ (0.024)	$-0.0494^{**}$ (0.020)			
	lpha	$0.1068^{***}$ (0.021)	$0.0900^{***}$ (0.013)	$0.0863^{***}$			
	$\beta$	$0.9888^{***}$ $(0.004)$	$0.9908^{***}$ (0.003)	$0.9936^{***}$			
	$\delta$	$-0.0349^{***}$	$-0.0307^{***}$	$-0.0203^{***}$			
	Log-Likelihood	8830.7	8460.1	8463.0			
Residuals	Q(30)	29.085	38.351	31.056			
nesiduais	CH(30)	28.919	37.455	31.629			
	$Q_2(30)$	38.441	22.512	30.883			
	$CH_{2}(30)$	45.810**	30.787	37.083			
QML-GED	ω	$-0.0825^{**}$ $_{(0.034)}$	$-0.0676^{***}$ (0.020)	$-0.0463^{**}$ (0.019)			
	lpha	$0.1049^{***}$	$0.0902^{***}$ $(0.013)$	$0.0858^{***}$ (0.012)			
	β	$0.9893^{***}$ (0.004)	$0.9912^{***}$	$0.9940^{***}$ (0.002)			
	δ	$-0.0333^{***}$ (0.008)	$-0.0297^{***}$	$-0.0202^{***}$ (0.008)			
	v	1.5448	1.5991	1.6362			
	Log-Likelihood	8860.0	8481.9	8479.8			
Residuals	Q(30)	29.216	38.240	31.046			
Residuals	CH(30)	29.027	37.373	31.640			
	$Q_2(30)$	38.615	22.564	31.051			
	$CH_{2}(30)$	45.918**	31.052	37.268			
QML-t	ω	$-0.0773^{**}$	$-0.0633^{***}$	$-0.0422^{**}$ (0.018)			
	lpha	$0.1035^{***}$	$0.0910^{***}$ $(0.013)$	$0.0858^{***}$ (0.013)			
	eta	$0.9899^{***}$ (0.004)	$\substack{0.9917^{***}\(0.002)}$	$0.9945^{***}$			
	$\delta$	$-0.0324^{***}$	$-0.0284^{***}$	$-0.0202^{***}$ (0.006)			
	v	10.321	11.541	12.630			
	Log-Likelihood	8862.2	8487.4	8481.7			
Desiduals	Q(30)	29.334	38.073	31.018			
Residuals	CH(30)	29.133	37.258	31.648			
	$Q_2(30)$	38.833	22.583	31.169			
	$CH_{2}(30)$	46.278**	31.444	37.528			

Table 6: Estimation of the EGARCH model with QML, QML-GED and QML-t using open-close returns

Estimator	Parameter	Brent crude oil	WTI crude oi	l Heating oil
QML	ω	-0.0142 (0.010)	$-0.0044^{*}$ (0.003)	-0.0042 (0.005)
	lpha	$0.1909^{***}$ $(0.031)$	$0.1541^{***}$ (0.025)	$0.1105^{**}$ $(0.052)$
	eta	$\underset{(0.140)}{0.1245}$	$\underset{(0.170)}{0.0526}$	$\underset{(0.362)}{0.3123}$
	$\delta$	$-0.0665^{***}$	$-0.0502^{***}$	-0.0231 (0.017)
	d	$0.7016^{***}$	$0.7915^{***}$ $(0.046)$	$0.7879^{***}$
	Log-Likelihood	8834.6	8456.3	8457.0
Residuals	Q(30)	29.803	38.887	32.565
Residuais	CH(30)	29.874	37.651	32.382
	$Q_2(30)$	29.816	21.906	32.526
	$CH_{2}(30)$	35.848	29.262	37.789
QML-GED	ω	$-0.0163^{*}_{(0.010)}$	-0.0074 (0.005)	-0.0062 (0.006)
	α	$0.1797^{***} \\ \scriptstyle (0.044)$	$0.1482^{***}$ (0.020)	$0.1002^{**}$
	β	$\underset{(0.2956)}{0.1473}$	$\underset{(0.074)}{0.0707}$	$\underset{(0.3864)}{0.3864)}$
	δ	$-0.0606^{***}$	$-0.0472^{***}$ (0.012)	$-0.0210$ $_{(0.016)}$
	d	$0.7127^{***}$ $(0.079)$	$0.7988^{***}$ (0.063)	$0.7816^{***}$ $_{(0.109)}$
	v	1.5545	1.5933	1.6217
	Log-Likelihood	8862.4	8479.0	8475.7
Residuals	Q(30)	29.888	38.765	32.530
residuals	CH(30)	29.935	37.562	32.376
	$Q_2(30)$	30.044	21.768	32.028
	$CH_{2}(30)$	36.516	29.486	37.411
QML-t	ω	-0.0147 (0.010)	-0.0066 (0.006)	$-0.0053$ $_{(0.014)}$
	lpha	$0.1728^{***}$	$0.1435^{***}$ (0.022)	$\underset{(0.116)}{0.0900}$
	eta	$\underset{(0.2556)}{0.1695}$	$\underset{(0.0916)}{0.0967}$	$\underset{(0.7095)}{0.4584}$
	$\delta$	$-0.0572^{***}$	$-0.0439^{***}$ (0.013)	$\underset{(0.0456)}{-0.0186}$
	d	$0.7198^{***}$ (0.067)	$0.8031^{***}_{(0.083)}$	$0.7821^{***}_{(0.186)}$
	$\upsilon$	10.600	11.331	11.977
	Log-Likelihood	8862.2	8485.1	8478.2
Residuals	Q(30)	29.898	38.628	32.416
rusiquais	CH(30)	29.952	37.483	32.348
	$Q_2(30)$	30.225 $35$	21.731	31.432
	$CH_{2}(30)$	37.185	29.881	37.173

Table 7: Estimation of the FIEGARCH model with QML, QML-GED and QML-t using open-close returns

r	Table 8: Descripti	ve statistics for o	open-open ret	urns
Statistic	Brent crude oil	WTI crude oil	Heating oil	Natural gas
Mean	0.0004	0.0004	0.0004	0.0001
Std.Dev.	0.0219	0.0240	0.0231	0.0377
Skewness	-0.2476	-0.1592	-0.2604	0.7462
Excess Kurtosis	2.0974	3.5503	3.5041	18.765
Jarque-Bera	668***	1782***	$1762^{***}$	4973***
Q(30)	$61.701^{***}$	64.918***	$40.726^{*}$	$64.271^{***}$
CH(30)	36.530	34.461	29.527	31.966
$Q_2(30)$	3480.0***	3003.8***	724.7***	$633.1^{***}$
$CH_2(30)$	80.258***	72.321***	117.71***	80.661***

: statistically significant at 10%, 5% and 1% respectively

Estimator	Parameter	Brent crude oil	WTI crude oil	Heating oil	Natural gas
$\operatorname{QML}$	ω	$0.0003^{**}$	0.0005**	$0.0003^{*}$	$0.0010^{*}_{(0.001)}$
	lpha	$0.0613^{***}$	$0.0723^{***}$ $(0.016)$	$0.0566^{***}$	$0.0749^{**}$ $(0.029)$
	eta	$0.9385^{***}$ $(0.015)$	$0.9234^{***}$	$0.9432^{***}$ $(0.016)$	$0.9172^{***}$ $(0.035)$
	$\delta$	$-0.0168^{***}$	$-0.0270^{***}$	$\underset{(0.001)}{0.0006}$	$\underset{(0.012)}{0.0077}$
	Log-Likelihood	8606.9	8122.1	8186.4	6481.5
Residuals	Q(30)	33.673	37.403	29.593	31.888
Residuals	CH(30)	33.220	33.891	28.043	28.714
	$Q_2(30)$	36.781	22.558	36.554	51.920***
	$CH_{2}(30)$	$40.584^{*}$	24.334	26.848	27.695
QML-GED	ω	0.0003*** (0.000)	0.0004*** (0.000)	0.0002* (0.000)	0.0009*** (0.000)
	$\alpha$	$0.0612^{***}$	$0.0683^{***}$	$0.0532^{***}$	$0.0809^{***}$
	eta	$0.9386^{***}$ $(0.013)$	$0.9287^{***}$ $_{(0.015)}$	$0.9466^{***}$	$0.9147^{***}$ $(0.014)$
	δ	$-0.0166^{***}$	$-0.0258^{***}$	$\underset{\scriptscriptstyle(0.002)}{-0.0018}$	$0.0156^{st}_{(0.008)}$
	v	1.7429	1.6066	1.6201	1.2847
	Log-Likelihood	8614.8	8145.0	8207.4	6670.2
Residuals	Q(30)	33.641	37.317	29.570	30.807
residuais	CH(30)	33.191	33.655	28.010	29.134
	$Q_2(30)$	36.758	25.116	$40.559^{*}$	30.705
	$CH_{2}(30)$	$40.575^{*}$	25.100	27.074	29.061
QML-t	ω	0.0003*** (0.000)	0.0003*** (0.000)	0.0002 (0.000)	0.0008*** (0.000)
	$\alpha$	$0.0606^{***}$	$0.0627^{***}$	$0.0503^{***}$	$0.0806^{***}$
	eta	$0.9392^{***}$ $(0.013)$	$0.9358^{***}$ $(0.018)$	$0.9495^{***}$ $(0.016)$	$0.9144^{***}$
	$\delta$	$-0.0165^{***}$	$-0.0242^{***}$	$-0.0049^{*}_{(0.003)}$	$0.0185^{**}$
	$\upsilon$	17.242	10.639	11.109	6.7493
	Log-Likelihood	8618.2	8157.2	8216.9	6730.4
Residuals	Q(30)	33.667	37.238	29.602	30.697
residuals	CH(30)	33.199	33.323	28.032	29.330
	$Q_2(30)$	36.919	29.265	$45.589^{**}$	26.532
	$CH_{2}(30)$	$40.625^{*}$	25.799	27.162	29.186

Table 9: Estimation of the TGARCH model with QML, QML-GED and QML-t using open-open returns

Estimator	Parameter	Brent crude oil	WTI crude oil	Heating oil	Natural gas
QML	ω	$-0.0890^{**}$ $_{(0.035)}$	$-0.1380^{**}$ (0.054)	$-0.0484^{**}$ (0.022)	$-0.1857^{**}$ (0.082)
	lpha	$0.1185^{***}$ (0.023)	$0.1386^{***}$ (0.030)	$0.0977^{***}$	$0.1437^{***}$ $(0.042)$
	eta	$0.9885^{***}$ $(0.005)$	$0.9816^{***}$	$0.9936^{***}$	$0.9708^{***}$
	$\delta$	$-0.0312^{***}$	$-0.0476^{***}$	$-0.0053$ $_{(0.004)}$	$\underset{(0.011)}{0.0093}$
	Log-Likelihood	8608.5	8121.0	8197.7	6496.8
Residuals	Q(30)	33.531	37.151	29.005	28.154
nesiduais	CH(30)	33.253	33.695	27.849	29.731
	$Q_2(30)$	36.065	22.611	27.560	11.388
	$CH_{2}(30)$	41.016*	24.762	30.408	26.658
QML-GED	ω	$-0.0853^{***}$ $_{(0.031)}$	$-0.1209^{**}$ (0.050)	$-0.0483$ $_{(0.052)}$	$-0.1778^{***}$ (0.050)
	lpha	$0.1181^{***}$	$0.1299^{***}$ (0.028)	$0.0960^{***}$ $(0.026)$	$0.1517^{***}$
	β	$0.9889^{***}$ (0.004)	$0.9839^{***}$ (0.007)	$0.9936^{***} \\ \scriptstyle (0.007)$	$0.9726^{***}$
	δ	$-0.0305^{***}$ (0.009)	$-0.0458^{***}$	$-0.0077$ $_{(0.058)}$	$\underset{(0.020)}{0.0214}$
	v	1.7448	1.6050	1.6420	1.2964
	Log-Likelihood	8616.2	8144.1	8215.2	6674.5
Residuals	Q(30)	33.473	37.082	29.034	26.850
residuais	CH(30)	33.199	33.443	27.881	30.176
	$Q_2(30)$	36.212	25.529	27.870	4.861
	$CH_{2}(30)$	41.191*	25.671	30.388	27.786
QML-t	ω	$-0.0816^{***}$ (0.028)	$-0.1003^{**}$ $_{(0.040)}$	$-0.0493^{**}$ (0.021)	$-0.1802^{***}$ (0.046)
	$\alpha$	$0.1170^{***}$	$0.1183^{***}$	$0.0966^{***}$	$0.1524^{***}$
	eta	$0.9894^{***}$ $(0.004)$	$0.9867^{***}$ $(0.005)$	$0.9935^{***}$ $(0.003)$	$0.9728^{***}$
	$\delta$	$-0.0303^{***}$	$-0.0435^{***}$	$-0.0109$ $_{(0.009)}$	$0.0269^{*}_{(0.016)}$
	v	17.206	10.584	11.688	6.9229
	Log-Likelihood	8619.6	8156.5	8222.5	6731.8
Residuals	Q(30)	33.476	37.032	29.125	26.533
nesiduais	CH(30)	33.187	33.097	28.000	30.410
	$Q_2(30)$	36.470	30.149	27.810	3.620
	$CH_{2}(30)$	$41.362^{*}$	26.453	30.187	28.063

Table 10: Estimation of the EGARCH model with QML, QML-GED and QML-t using open-open returns

Estimator	Parameter	Brent crude oil	WTI crude oil	Heating oil	Natural gas
QML	ω	-0.0200 (0.017)	-0.0167 (0.011)	-0.0114 (0.012)	0.0014 (0.010)
	lpha	$0.2060^{***}$ (0.039)	0.1841*** (0.037)	$0.1012^{***}$ (0.030)	$\underset{(0.037)}{0.2149^{***}}$
	eta	$\underset{(0.157)}{0.1486}$	$0.2631^{*}_{(0.154)}$	$0.7042^{***}$ (0.129)	$\underset{\scriptscriptstyle(0.078)}{-0.0473}$
	$\delta$	$-0.0677^{**}$ (0.028)	$-0.0756^{***}$	$\underset{(0.037)}{0.0027}$	$\underset{(0.0636)}{-0.0116}$
	d	0.6846*** (0.077)	$0.6768^{***}$ (0.059)	$0.6299^{***}$ (0.123)	$0.7402^{***}$
	Log-Likelihood	8614.7	8124.2	8188.4	6495.4
Residuals	Q(30)	33.433	37.054	29.693	26.485
nesiduais	CH(30)	33.147	33.054	28.406	30.159
	$Q_2(30)$	30.989	14.530	29.117	3.999
	$CH_{2}(30)$	31.985	18.641	26.933	26.891
QML-GED	ω	$-0.0207^{**}$	-0.0196 (0.013)	$-0.0127^{*}_{(0.007)}$	-0.0263 (0.036)
	lpha	$0.2030^{***}$ (0.045)	$0.1800^{***}$ $(0.043)$	$0.0972^{***}$ $(0.024)$	$\underset{(0.052)}{0.1603^{***}}$
	β	$\underset{(0.257)}{0.1387}$	$\underset{(0.170)}{0.2488}$	$0.6851^{***}_{(0.136)}$	$\underset{(0.461)}{0.5669}$
	δ	$-0.0658^{**}$	$-0.0758^{**}$ $_{(0.032)}$	$-0.0014^{***}$ (0.000)	$\underset{(0.038)}{0.0189}$
	d	$0.6946^{***}$	$0.6862^{***}$ (0.058)	$0.6435^{***}$	$0.5813^{**}$ $_{(0.260)}$
	v	1.7591	1.6056	1.6135	1.2918
	Log-Likelihood	8621.4	8147.7	8210.1	8479.8
Residuals	Q(30)	33.384	37.092	29.759	26.815
Itesiduais	CH(30)	33.099	33.020	28.451	30.217
	$Q_2(30)$	30.868	14.938	33.356	4.223
	$CH_{2}(30)$	32.109	18.542	26.841	26.071
QML-t	ω	$-0.0190^{**}$ (0.008)	$-0.0183^{**}$ (0.009)	$-0.0116^{**}$	$-0.0315$ $_{(0.026)}$
	$\alpha$	$0.2002^{***}$ (0.030)	$0.1734^{***}$	$0.0937^{***}$ (0.025)	$0.1577^{***}$ (0.055)
	eta	$\underset{(0.104)}{0.1194}$	$\underset{(0.206)}{0.2357}$	$0.6512^{***}$	$0.6124^{*}_{(0.335)}$
	$\delta$	$-0.0650^{***}$	$-0.0756^{**}$	$-0.0076$ $_{(0.013)}$	$\underset{(0.028)}{0.0263}$
	d	$0.7057^{***}$ $(0.044)$	$0.6970^{***}$	$0.6640^{***}$	$0.5530^{***}$
	$\upsilon$	17.927	10.654	10.635	6.767
	Log-Likelihood	8624.6	8161.1	8220.7	6725.1
Residuals	Q(30)	33.382	37.162	29.884	26.709
nesiduais	CH(30)	33.091	32.964	28.535	30.515
	$Q_2(30)$	30.916 $39$	15.756	$41.257^{*}$	3.200
	$CH_{2}(30)$	32.260	18.407	26.562	26.559

Table 11: Estimation of the FIEGARCH model with QML, QML-GED and QML-t using open-open returns

Figure 1: Daily open-close returns and estimated Garman-Klass volatility of energy futures together with the Detrended Cross Correlation - DCCA - coefficient between standardized returns and log-volatility (Partially Fig. 1, Fig. 2 and Fig. 3 of the replicated paper) and DCCA coefficient between returns and volatility

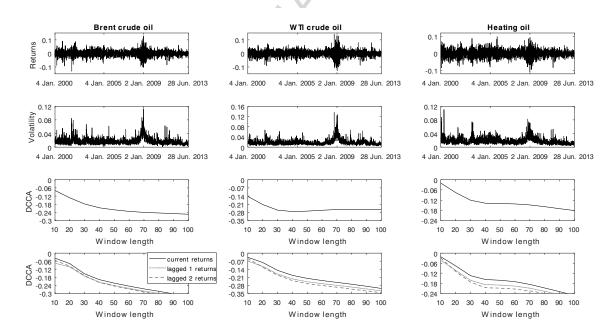


Figure 2: Correlograms of returns (open-close) and squared returns and crosscorrelograms between lagged returns and squared returns

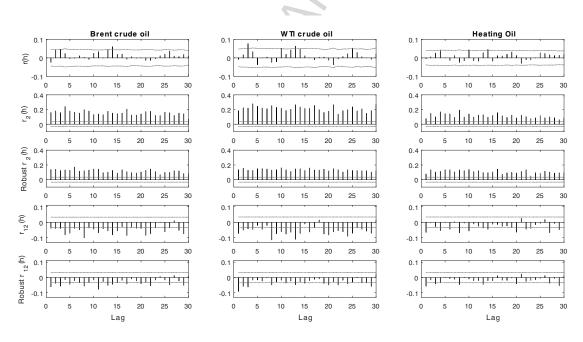


Figure 3: Scatter plots of volatilities estimated by Garman-Klass and TGARCH, EGARCH and FIEGARCH models using open-close returns and assuming Gaussian innovations

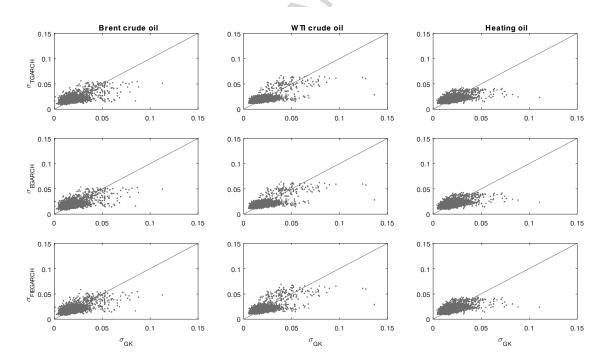
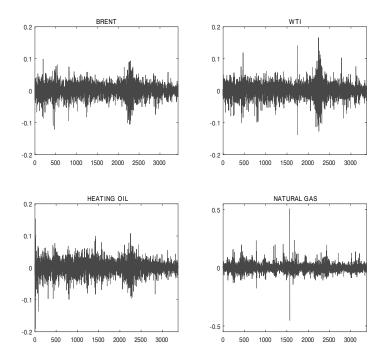




Figure 4: Daily open-open returns of energy futures



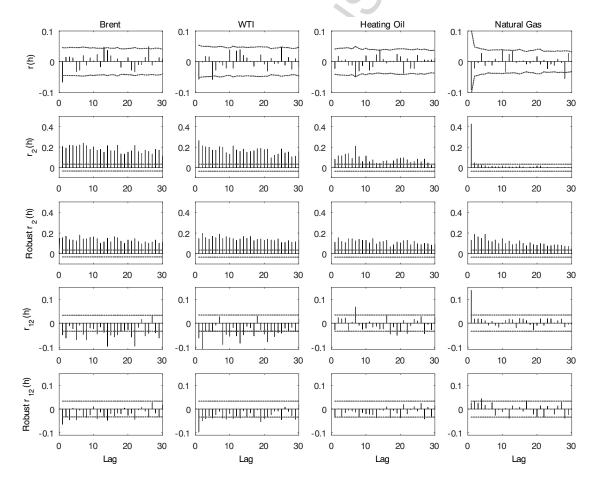


Figure 5: Correlograms of returns (open-open) and squared returns and cross-correlograms between lagged returns and squared returns

Figure 6: Scatter plots of volatilities estimated by TGARCH, EGARCH and FIE-GARCH models of open-open vs open-close returns assuming Gaussian innovations

