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# Primary Study on the Stochastic Spiral Optimization Algorithm

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***Abstract*—Authors propose a simple, but powerful, modification of the Spiral Optimisation Algorithm, which includes stochastic disturbances in its searching trajectories. As a primary study, several tests were carried out implementing proposed and original algorithm to solve different benchmarking problems. Results were compared with some reported in literature. It was noticed the proposed modification enhances the spiral–inspired method in terms of convergence, specifically at high–dimensional optimisation problems.**

## INTRODUCTION

Innumerable optimisation techniques have emerged aiming to tackle specific or general economic and/or engineering problems since centuries ago. Many of them have been inspired by natural phenomena, *e.g.* prehistoric hunting strategies, and collective behaviour of several entities like animals or celestial bodies, as anyone can easily find in literature. A very specific case of breathtaking inspirational source is the logarithmic spiral, which is observable on a vast quantity of scenarios in nature [1], [2]. Hence, equiangular spiral patterns have been employed in numerical methods aimed to solve optimisation problems. In 1970, Jones presented a deterministic algorithm which implements Newton-steps following spiral trajectories [3]. This method overcame the performance of Powell and Marquardt algorithms, by solving various benchmarking functions and parameter estimation problems. Four decades later, Tamura and Yasuda proposed the Deterministic Spiral Optimisation Algorithm (DSOA) which is entirely based on spiral trajectories [4], [5], [6]. It is a population–based direct–solving method with proved stability [7]. DSOA has a simple deterministic procedure kernel. This makes its implementation easy but renders its structure excessively rigid. Thus, it has been extensively modified and employed on diverse areas of knowledge, for example: designing digital filters [8], multi-layer electromagnetic absorbers [9], and microchannel heat sinks [10]; and solving multiple problems [11].

This work presents a primary study about a modification of the DSOA, based on the disturbed or noisy motion concept. This randomness inclusion idea was also insinuated by Tamura and Yasuda in [12]. Hence, the modified method is named Stochastic Spiral Optimisation Algorithm (SSOA), emphasising the slight difference with the original methodology, *i.e.* DSOA. SSOA has been stated as an alternative to sophisticated optimisation arrangements which involve complex searching strategies, hybrid algorithms, hyper–heuristic methods, and so on. The proposed technique is tested using typical benchmarking functions and compared against DSOA by employing several values for tuning parameters. It was noticed that DSOA is enhanced on accuracy and convergence speed through the proposed stochastic variation, *i.e.*, SSOA.

The current manuscript is organised as follows: Section 1 introduces both algorithms and important foundations. Next section details the methodology carried out to obtain results, which are discussed in Section 3. Lastly, the most important remarks are displayed in the conclusions section.

## I. FOUNDATIONS

In this section some important concepts are presented. At first, three general definitions are displayed, subsequently, the base version and the proposed modification for the spiral optimisation algorithm are described.

**Definition 1.** *Let $\mathfrak{X}^n = \{\vec{x}_1^n, \vec{x}_2^n, \ldots, \vec{x}_M^n\}$ be a finite set of candidate solutions for any optimisation problem in $\mathbb{R}^D$, with an objective function given by $f : \mathbb{R}^D \to \mathbb{R}$. $D$ is the dimensionality of the problem, and $M$ is the number of candidate solutions. Thus, $\vec{x}_m^n = (x_{m,1}^n, x_{m,2}^n, \ldots, x_{m,D}^n)^\intercal$ denotes the $m$–th candidate in $\mathbb{R}^D$ at the time $n$ of an iterative procedure, with a maximum number of iterations $N$.*

**Definition 2.** *Let $\vec{x}_*^n \in \mathfrak{X}^n$ be the best solution found at the $n$-th iteration,* i.e. $\vec{x}_*^n = \arg\min\left(\{f(\mathfrak{X}^n)\} \cup f(\vec{x}_*^{n-1})\right)$*, with $\{f(\mathfrak{X}^n)\} = \{f(\vec{x}_1^n), f(\vec{x}_2^n), \ldots, f(\vec{x}_M^n)\}$.*

**Definition 3.** *Let $\mathfrak{X}^{n+1}$ represent the finite set of new candidate solutions. Each new candidate $\vec{x}_m^{n+1}$ is obtained through an iterative procedure, namely, an optimisation algorithm.*

### A. Deterministic Spiral Optimisation Algorithm (DSOA)

DSOA is a deterministic direct–solving metaheuristic procedure based on the logarithmic spiral dynamic [4]. It is also known as SOA, SPO or SO according to [7], [12], [13]. The basic idea consists on the rotation behaviour of a set of points

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around a reference centre point, following a spiral–shaped trajectory. The centre is iteratively updated via a fitness criterion, *i.e.*, $\vec{x}_*^n$, which is given by an objective function $f(\vec{y})$, $\vec{y} \in \mathbb{R}^D$ from an optimisation problem. Spiral dynamics have shown to strengthen diversification and intensification strategies, both common in metaheuristic methods [7]. The above mentioned idea is mathematically formulated with Definitions 1 to 3 as,

$$\vec{x}_m^{n+1} = r\mathbf{R}_D(\theta)\vec{x}_m^n - (r\mathbf{R}_D(\theta) - \mathbf{I}_D)\vec{x}_*^n, \tag{1}$$

where $\mathbf{I}_D \in \mathbb{R}^{D\times D}$ is the identity matrix and, $r \in (0, 1)$ and $\theta \in (0, 2\pi)$ are the control parameters of spiral dynamics, which represent the convergence rate and the rotation angle, between a $m$–th point ($\vec{x}_m^n$) and the centre point ($\vec{x}_*^n$), respectively. Likewise, $\mathbf{R}_D(\theta) \in \mathbb{R}^{D\times D}$ is the rotation matrix defined as the product of all possible combinations of 2–D rotation matrices $\mathbf{R}_{d,k}(\theta_{d,k}) \in \mathbb{R}^{D\times D}$, per plane $d, k$ in the search space, as is shown,

$$\mathbf{R}_D(\theta) \triangleq \prod_{d=1}^{D}\prod_{k=1}^{d} \mathbf{R}_{d,k}(\theta_{D-d,D+1-k}), \quad \forall\, d,k \in \{1,2,\ldots,D\} \wedge d \neq k. \tag{2}$$

From equation (2), an element $\rho_{p,q}$ with $p, q \in \{1, 2, \ldots, D\}$, $\mathbf{R}_{d,k}(\theta_{d,k}) = (\rho_{p,q}) \in \mathbb{R}^{D\times D}$, is defined as,

$$\rho_{p,q} \triangleq \begin{cases} 1, & \text{if } p = q, \\ \cos(\theta_{d,k}), & \text{if } p = d \wedge q = d, \\ \sin(\theta_{d,k}), & \text{if } p = k \wedge q = d, \\ -\sin(\theta_{d,k}), & \text{if } p = d \wedge q = k, \\ \cos(\theta_{d,k}), & \text{if } p = k \wedge q = k, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Subsequently, DSOA is summarised in Pseudocode 1.

**Pseudocode 1.** Deterministic Spiral Optimisation Algorithm

**Input:** $f : \mathbb{R}^D \to \mathbb{R}$, $M > 2$, $\theta \in (0, 2\pi)$, and $r \in (0, 1)$
Stopping criteria: $N \gg 1$, and others (if they exist)
**Output:** $\vec{x}_*^n$
Determine $\mathbf{R}_D(\theta)$ using (2)
Initialise $\mathfrak{X}^0$
Find $\vec{x}_*^0$ using Definition 2
$n \leftarrow 0$
**while** ($n \leq N$) & (any stopping criteria is not reached) **do**
    Update $\mathfrak{X}^{n+1}$ with (1) ▷ Deterministic spiral dynamic
    Find $\vec{x}_*^{n+1}$ using Definition 2
    $n \leftarrow n + 1$
**end while**

### B. Stochastic Spiral Optimisation Algorithm (SSOA)

SSOA is a modification of DSOA presented in this work, which aims to tackle the major known drawbacks of the original strategy, *i.e.* its slow convergence. For that reason, some random disturbances are included in the spiral dynamics of each searching point. It could be considered as an approach to natural behaviour. This idea is formalised by remodelling the DSOA kernel equation in (1) such as,

$$\vec{x}_m^{n+1} = \tilde{r}\mathbf{R}_D(\theta)\vec{x}_m^n - \vec{\delta_x}(n) \odot (\tilde{r}\mathbf{R}_D(\theta) - \mathbf{I}_D)\vec{x}_*^n, \tag{4}$$

where $\tilde{r}$ is the stochastic convergence rate, defined as a uniformly distributed random value between $r_l$ and $r_u$, *i.e.*, $\tilde{r} \sim \mathcal{U}(r_l, r_u)$; $\vec{\delta_x}(n) \in \mathbb{R}^D$ is the radius scale vector which depends of the current step ($n$), and other additional metrics (if they are previously defined); and $\odot$ is the Hadamard–Schur's product. Other parameters remain unchanged and follow definitions given in DSOA. In this work, for the sake of simplicity, parameters $\tilde{r}$ and $\vec{\delta_x}(n)$ are chosen as,

$$\tilde{r} \sim \mathcal{U}(r_l, 1.0), \tag{5}$$

$$\vec{\delta_x}(n) = \vec{u}, \tag{6}$$

since $0 < r_l < 1.0$ is the lower limit of $\tilde{r}$, and $\vec{u} \in \mathbb{R}^D$ is a vector of i.i.d. random variables with $\mathcal{U}(0.0, 1.0)$.

As with DSOA, the basic scheme of SSOA is presented in Pseudocode 2.

**Pseudocode 2.** Stochastic Spiral Optimisation Algorithm

**Input:** $f : \mathbb{R}^D \to \mathbb{R}$, $M > 2$, $\theta \in (0, 2\pi)$, and $r_l \in (0, 1)$
Stopping criteria: $N \gg 1$, and others (if they exist)
**Output:** $\vec{x}_*^n$
Determine $\mathbf{R}_D(\theta)$ using (2)
Initialise $\mathfrak{X}^0$
Find $\vec{x}_*^0$ using Definition 2
$n \leftarrow 0$
**repeat**
    Update $\mathfrak{X}^{n+1}$ with (4) ▷ Stochastic spiral dynamic
    Find $\vec{x}_*^{n+1}$ using Definition 2
    $n \leftarrow n + 1$
**until** ($n < N$) & (any stopping criteria is not reached)

In addition, an illustrative example of the difference between both described spiral dynamics, *i.e.* deterministic and stochastic from (1) and (4), respectively, is presented in Fig. 1. In that figure, the diversity of paths which a point can follow in a randomly disturbed spiral behaviour is noticed.

## II. Methodology

All experiments were performed in a numerical computing platform, running on an iMac model 15.1, with an Intel Core i5 CPU at 1.6–2.7 GHz, 8 GB RAM, and macOS Sierra v10.12.1 as operating system. Each case of study was repeated a hundred times for statistical purposes. Moreover, the initial random distribution of candidate solutions ($\mathfrak{X}^0$) was the same for all simulations.

In this work, the spiral–based optimisation method enhancement was explored according three cases of study and using a set of nine benchmarking functions, displayed in Table I and (7)–(15), from [14]. The first one dealt with the minimisation of a single 2D benchmarking function, *i.e.*, Rastrigin function in (7), employing both deterministic (DSOA) and stochastic (SSOA) spiral optimisation algorithms. For that, $\theta = \pi/8$,

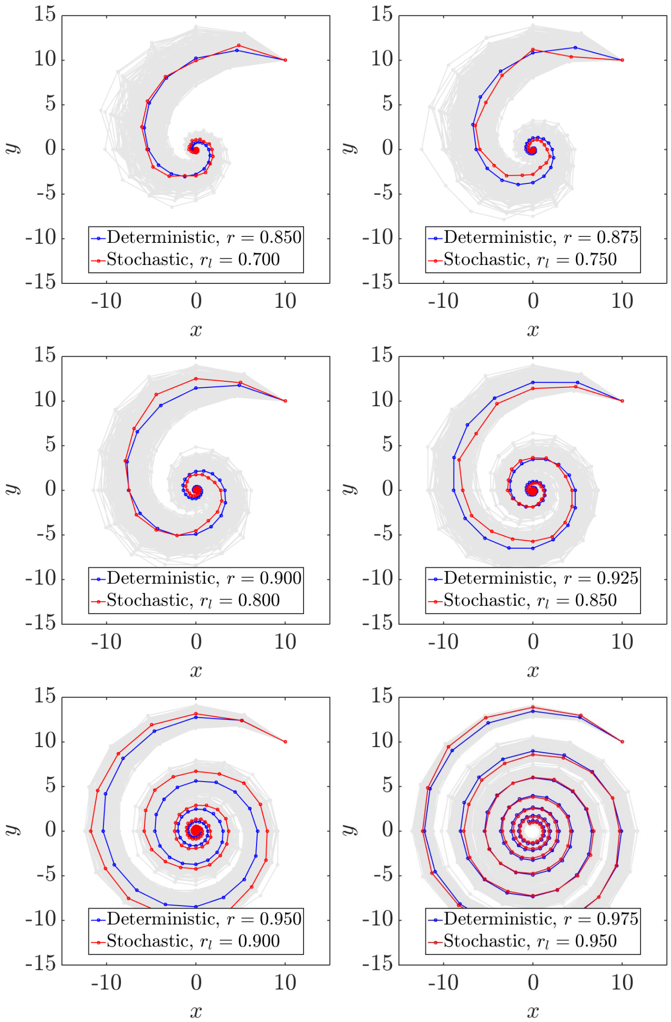


Fig. 1. Illustrative example for spiral trajectories in 2D of a moving point, which starts at $(10, 10)$, and rotates with an angle of $\theta = \pi/8$ around a centre point at $(0, 0)$. Several trajectories are described by a deterministic (in blue colour) and stochastic (in red and grey colours) spiral dynamics, using $r_l = 0.70, 0.75, \ldots, 0.95$, and $r = (1 + r_l)/2$.

$N = 1000$, $M = 20$, $r = 0.95$, and $r_l = 0.90$ were employed as parameters values, and each technique was ran one hundred times. Fig. 2 shows a glance at this function.

In the second case, results from these methods were compared against published literature (results) [5], [6], both implemented in several multidimensional problems from Table I, with dimensions equal to 3, 10, 30, 50 and 100. Common parameter values were set as $N = 100$, and $M = 20$, and three groups per each method were established according to data supplied in Table II. In the last case of study, other benchmarking functions were tackled as an extension of the previous case (Table I) using different dimensions values, $D = 2, 5, 10, \ldots, 30, 50$, and 100.

$$f_1(\vec{x}) = ||\vec{x}||^2 + 10\left(D - ||\cos(2\pi\vec{x})||^2\right), \tag{7}$$

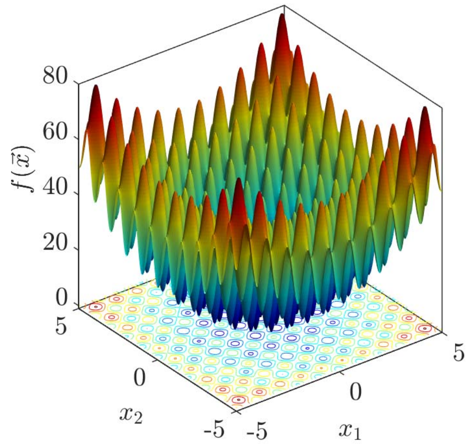


Fig. 2. Representation of the Rastrigin function in a bidimensional searching space, $\vec{x} \in [-5, 5]^2$.

TABLE I. Multidimensional benchmarking functions and, some of their features [6], [14]. All functions have zero global minima, $f_k(\vec{x}^*) = 0$, $k = 1, \ldots, 9$, except for the $2^n$ *minima function* with $f_3(\vec{x}^*) \approx -78.3323D$.

| Case | $k$ | Name | Domain | $\vec{x}^{*\intercal} \in \mathbb{R}^D$ | Eq. |
|---|---|---|---|---|---|
| 1 | 1 | Rastrigin | $[-5, 5]$ | $(0, \ldots, 0)$ | (7) |
| 2 | 1 | Rastrigin | $[-5, 5]$ | $(0, \ldots, 0)$ | (7) |
| | 2 | Schwefel | $[-5, 5]$ | $(0, \ldots, 0)$ | (8) |
| | 3 | $2^n$ minima | $[-5, 5]$ | $(-2.9, \ldots, -2.9)$ | (9) |
| | 4 | Griewank | $[-50, 50]$ | $(0, \ldots, 0)$ | (10) |
| 3 | 5 | Sphere | $[-10, 10]$ | $(0, \ldots, 0)$ | (11) |
| | 6 | Step 2 | $[-100, 100]$ | $(0, \ldots, 0)$ | (12) |
| | 7 | Salomon | $[-100, 100]$ | $(0, \ldots, 0)$ | (13) |
| | 9 | Rosenbrock | $[-30, 30]$ | $(1, \ldots, 1)$ | (15) |

$$f_2(\vec{x}) = \sum_{d=1}^{D} \left( \sum_{i=1}^{d} x_i \right)^2, \tag{8}$$

$$f_3(\vec{x}) = \sum_{d=1}^{D} (x_d^4 - 16x_d^2 + 5x_d), \tag{9}$$

$$f_4(\vec{x}) = \frac{||\vec{x}||^2}{4000} + \prod_{k=1}^{D} \cos\left(\frac{x_k}{\sqrt{k}}\right), \tag{10}$$

$$f_5(\vec{x}) = ||\vec{x}||^2, \tag{11}$$

$$f_6(\vec{x}) = ||\lfloor \vec{x} + 0.5 \rfloor||^2, \tag{12}$$

$$f_7(\vec{x}) = 1 - \cos\left(2\pi||\vec{x}||\right) + 0.1||\vec{x}||, \tag{13}$$

$$f_8(\vec{x}) = 20\left(1 - e^{-0.02||\vec{x}||/\sqrt{D}}\right) + e - e^{||\cos(2\pi\vec{x})||^2/D}, \tag{14}$$

$$f_9(\vec{x}) = \sum_{d=1}^{D-1} \left[100(x_{d+1} - x_d^2)^2 + (x_d - 1)^2\right], \tag{15}$$

TABLE II. Parameter values used in DSOA and SSOA implementations.

| Method | $r$ | $\theta$ | Method | $r_l$ | $\theta$ |
|---|---|---|---|---|---|
| DSOA 1 | | $\pi/2$ | SSOA 1 | | $\pi/2$ |
| DSOA 2 | 0.95 | $\pi/4$ | SSOA 2 | 0.90 | $\pi/4$ |
| DSOA 3 | | $\pi/8$ | SSOA 3 | | $\pi/8$ |

## III. Results and Discussion

Fig. 3 presents the iterative evolution of the fitness value per spiral–based optimisation method, *i.e.* DSOA and SSOA, whilst they were solving the 2D Rastrigin minimisation problem. This figure also shows mean performance from the one hundred repetitions carried out per method. There is an evident and steady fitness improvement in all SSOA procedures when compared to DSOA. Only 6% of these executions reaches a fitness value which approximates zero before 500 iterations, and the other part showed an stationary behaviour around different non–optimum values. It is a common issue observed when the Rastrigin function is used, as anyone shall perceive from Fig. 2. This performance can be unmistakable classified as a stagnation state in a local optimum, narrowly related to a slow convergence. Hence, it is shown that the main drawback of DSOA is corrected by using a disturbed dynamic, at least in this case of study. Now, it is interesting to explore the main features of the proposed algorithm.

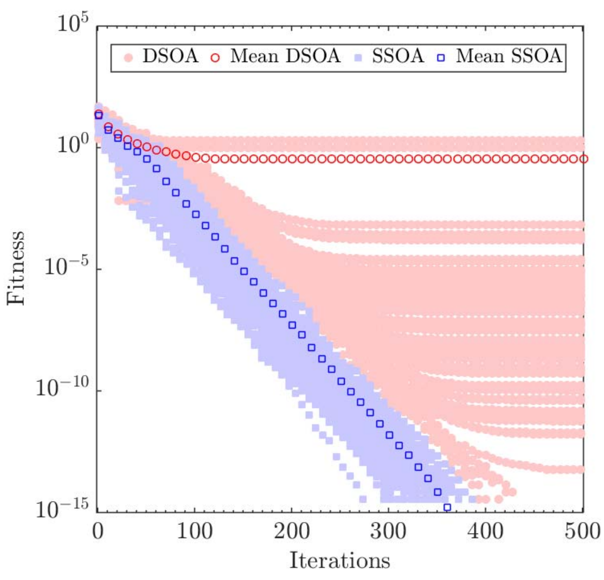


Fig. 3. Iterative procedure performed by all the one hundred executions of both DSOA and SSOA, solving the 2D Rastrigin function.

Subsequently, Table III displays the obtained results from the second case of study, via DSOA and SSOA with several values for their angles according to Table II. These data are tabulated together with the reported results from Tamura and Yasuda in [5], [6] for comparison purposes. It is easy to notice, from Table III, that SSOA implementations gave better results in high–dimensional problems, *i.e.*, $N \geq 30$. However, deterministic based methods barely overcame stochastic ones in the particular case of the $2^n$ minima function. This function has recognised properties like multimodality and associated high hardness, which make it an evident challenge.

The last case from Table I represents an extension of previously performed simulations. The best implementations of DSOA and SSOA were selected from Table III, such as DSOA 2 and SSOA 3 (Table II), respectively. Therefore, Table IV exhibits the performances obtained using both spiral-based methods solving five minimisation problems with several values of degrees of freedom. Albeit DSOA and SSOA produced accurate solutions for low dimensionality problems, SSOA surmounted DSOA when optimisation problems have more than two design variables. A good example of that is observed on results for the Rosenbrock function (the last column of Table IV), where the deterministic dynamics is strongly affected by dimensionality, which increases the difficulty of finding the minimum, located at the flat valley. This effect is observed on SSOA as well, but in a lower scale. The last row of Table IV shows the overall stats of carried out simulations per method and problem. It is easy to recognise the improvement of implementing the disturbed spiral dynamics to solve an optimisation problem. Furthermore, it is noticed that the stochastic modification reduces the chance to get trapped in a local solution.

## IV. Conclusions

A modification for the Deterministic Spiral Optimisation Algorithm (DSOA) was proposed. This method is based on the concept of disturbed trajectories or noisy motion, and it is labelled as Stochastic Spiral Optimisation Algorithm (SSOA). Three cases of study were carried out with DSOA and SSOA solving nine standard benchmarking functions, and three sets of tuning parameters, as a primary study. Moreover, reported results from literature were employed for comparative purposes. From results, it was shown how SSOA corrects the main drawback of DSOA, which is related to local convergence. Furthermore, SSOA enhances DSOA capabilities to deal with high–dimensional problems. Therefore, it is possible to say that the performance of the spiral dynamics inspired algorithm is improved by including disturbances in its searching trajectories.

Authors continue looking for practical applications of this algorithm and extensively testing its performance against other optimisation methods.


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TABLE III. Statistics from the second case of study: data reported in [5], [6], and results obtained via DSOA and SSOA.

| Problem | $N$ | | DSOA [5] | DSOA [6] | DSOA 1 | DSOA 2 | DSOA 3 | SSOA 1 | SSOA 2 | SSOA 3 |
|---|---|---|---|---|---|---|---|---|---|---|
| Rastrigin | 3 | Mean | 1.46 | - | 1.99 | 1.74 | **2.66** | 0.27 | 0.72 | 0.83 |
| | | Std. Dev. | 1.15 | - | 1.71 | 1.36 | 3.73 | **0.55** | 1.02 | 1.26 |
| | | Best | **0.00** | - | 0.01 | **0.00** | **0.00** | **0.00** | **0.00** | **0.00** |
| | | Worst | 6.00 | - | 9.11 | 6.20 | 25.00 | **2.68** | 3.60 | 5.29 |
| | 10 | Mean | - | 24.00 | 51.49 | 50.61 | 57.45 | 44.80 | 44.93 | **15.48** |
| | | Std. Dev. | - | 13.00 | 28.29 | 23.46 | 22.18 | **8.93** | 10.57 | 21.17 |
| | | Best | - | 4.90 | 14.01 | 15.32 | 8.43 | 1.48 | 0.29 | **0.00** |
| | | Worst | - | 72.00 | 122.47 | 125.80 | 119.84 | 64.56 | 66.04 | **55.27** |
| | 30 | Mean | 98.00 | - | 247.47 | 253.66 | 264.59 | 236.54 | 229.71 | **10.85** |
| | | Std. Dev. | **26.00** | - | 57.30 | 50.84 | 55.65 | 31.02 | 60.24 | 47.67 |
| | | Best | 49.00 | - | 128.87 | 117.63 | 134.44 | 173.37 | 0.03 | **0.00** |
| | | Worst | **189.00** | - | 371.79 | 358.67 | 435.77 | 313.24 | 311.81 | 242.61 |
| | 50 | Mean | - | 230.00 | 508.17 | 524.87 | 506.53 | 423.55 | 437.28 | **0.00** |
| | | Std. Dev. | - | 30.00 | 89.99 | 91.59 | 82.49 | 97.53 | 63.41 | **0.00** |
| | | Best | - | 177.00 | 314.50 | 341.62 | 326.09 | 2.26 | 10.61 | **0.00** |
| | | Worst | - | 314.00 | 730.01 | 771.03 | 723.39 | 537.70 | 540.86 | **0.00** |
| | 100 | Mean | 628.00 | 628.00 | 1239.12 | 1209.07 | 1171.04 | 845.82 | 831.47 | **0.00** |
| | | Std. Dev. | 48.00 | 48.00 | 114.12 | 111.52 | 112.92 | 234.62 | 245.92 | **0.00** |
| | | Best | 516.00 | 516.00 | 1034.27 | 983.28 | 963.97 | 14.47 | 8.88 | **0.00** |
| | | Worst | 729.00 | 729.00 | 1692.62 | 1583.71 | 1479.29 | 1043.46 | 1106.66 | **0.00** |
| Schwefel | 3 | Mean | **0.00** | - | **0.00** | **0.00** | 0.01 | **0.00** | **0.00** | **0.00** |
| | | Std. Dev. | **0.00** | - | **0.00** | **0.00** | 0.03 | **0.00** | **0.00** | **0.00** |
| | | Best | **0.00** | - | **0.00** | **0.00** | **0.00** | **0.00** | **0.00** | **0.00** |
| | | Worst | **0.00** | - | 0.03 | 0.04 | 0.29 | **0.00** | **0.00** | **0.00** |
| | 10 | Mean | - | 0.10 | 2.45 | 3.00 | 16.27 | **0.00** | 0.24 | 0.01 |
| | | Std. Dev. | - | 0.10 | 3.28 | 5.09 | 15.34 | **0.01** | 0.67 | 0.05 |
| | | Best | - | **0.00** | 0.01 | **0.00** | 0.01 | **0.00** | **0.00** | **0.00** |
| | | Worst | - | 0.80 | 20.63 | 24.80 | 63.33 | **0.10** | 4.16 | 0.49 |
| | 30 | Mean | 20.00 | - | 65.66 | 71.10 | 144.97 | 55.16 | 69.26 | **18.61** |
| | | Std. Dev. | **16.00** | - | 24.30 | 27.75 | 73.37 | 34.22 | 34.54 | 30.43 |
| | | Best | 4.10 | - | 19.14 | 26.98 | 48.79 | 0.15 | 0.06 | **0.00** |
| | | Worst | **91.00** | - | 159.77 | 191.73 | 480.89 | 145.46 | 140.81 | 133.90 |
| | 50 | Mean | - | **59.00** | 218.94 | 187.01 | 400.16 | 196.77 | 219.50 | 89.32 |
| | | Std. Dev. | - | **30.00** | 78.82 | 49.79 | 163.72 | 81.90 | 77.90 | 97.38 |
| | | Best | - | 15.00 | 90.11 | 98.97 | 119.02 | 5.30 | 1.81 | **0.00** |
| | | Worst | - | **250.00** | 529.73 | 334.92 | 878.30 | 373.27 | 380.90 | 300.01 |
| | 100 | Mean | **255.00** | **255.00** | 790.08 | 825.92 | 1313.51 | 884.89 | 958.76 | 446.88 |
| | | Std. Dev. | **147.00** | **147.00** | 215.53 | 229.27 | 569.67 | 256.35 | 267.30 | 389.29 |
| | | Best | 88.00 | 88.00 | 444.61 | 437.95 | 514.76 | 317.47 | 165.47 | **0.00** |
| | | Worst | **964.00** | **964.00** | 1607.96 | 1639.15 | 3371.35 | 1410.87 | 1508.24 | 1273.46 |
| $2^n$ minima | 3 | Mean | **-229.00** | - | -216.26 | -213.54 | -214.22 | -228.69 | -225.80 | -220.65 |
| | | Std. Dev. | 14.30 | - | 19.20 | 20.28 | 16.10 | **10.32** | 11.88 | 10.37 |
| | | Best | **-235.00** | - | **-235.00** | **-235.00** | **-235.00** | -234.62 | -234.88 | -234.31 |
| | | Worst | -174.00 | - | -174.26 | -147.95 | -178.44 | -182.07 | **-191.08** | -186.91 |
| | 10 | Mean | - | **-684.00** | -662.19 | -677.00 | -669.47 | -559.91 | -559.43 | -557.46 |
| | | Std. Dev. | - | 42.00 | 46.25 | 46.33 | 49.26 | **26.36** | 30.09 | 30.24 |
| | | Best | - | -783.00 | -768.30 | **-783.29** | -755.03 | -632.89 | -646.29 | -637.60 |
| | | Worst | - | **-585.00** | -538.89 | -556.65 | -414.10 | -505.27 | -503.58 | -478.08 |
| | 30 | Mean | -1846.00 | - | -1801.60 | **-1850.29** | -1757.60 | -1246.53 | -1247.52 | -1211.30 |
| | | Std. Dev. | 81.00 | - | 89.72 | 91.80 | 130.65 | **62.63** | 74.02 | 70.05 |
| | | Best | -2038.00 | - | -1996.34 | -2040.20 | **-2078.83** | -1424.80 | -1468.78 | -1403.61 |
| | | Worst | **-1649.00** | - | -1602.14 | -1573.35 | -1368.55 | -1141.69 | -1080.05 | -1037.14 |
| | 50 | Mean | - | -2757.00 | -2778.42 | **-2786.31** | -2538.11 | -1837.33 | -1813.99 | -1778.72 |
| | | Std. Dev. | - | 132.00 | 136.00 | 163.64 | 183.97 | **83.79** | 92.20 | 94.81 |
| | | Best | - | -3048.00 | -3094.26 | **-3139.19** | -2874.71 | -2076.63 | -2058.07 | -2098.44 |
| | | Worst | - | **-2438.00** | -2371.74 | -2318.11 | -2011.52 | -1643.05 | -1454.79 | -1546.05 |
| | 100 | Mean | -4737.00 | -4763.00 | -4879.04 | **-4900.40** | -4170.46 | -3127.95 | -3139.32 | -3161.63 |
| | | Std. Dev. | 267.00 | 268.00 | 237.59 | 258.61 | 320.18 | 185.73 | 190.38 | **166.40** |
| | | Best | -5417.00 | -5438.00 | -5376.06 | **-5558.55** | -5017.86 | -3588.16 | -3655.47 | -3698.13 |
| | | Worst | -4055.00 | -4053.00 | -4119.56 | **-4179.72** | -3366.30 | -2241.27 | -2659.90 | -2824.63 |
| Griewank | 3 | Mean | 0.06 | - | 0.07 | 0.08 | 0.08 | **0.03** | 0.05 | 0.05 |
| | | Std. Dev. | 0.04 | - | 0.06 | 0.08 | 0.11 | **0.02** | 0.03 | 0.04 |
| | | Best | 0.01 | - | **0.00** | 0.01 | **0.00** | **0.00** | **0.00** | **0.00** |
| | | Worst | 0.20 | - | 0.28 | 0.51 | 0.74 | **0.08** | 0.14 | 0.14 |
| | 10 | Mean | - | 0.30 | 0.35 | 0.29 | 0.46 | 0.69 | 0.50 | **0.09** |
| | | Std. Dev. | - | **0.14** | 0.23 | 0.19 | 0.45 | 0.28 | 0.40 | 0.23 |
| | | Best | - | 0.08 | 0.07 | 0.06 | 0.05 | **0.00** | **0.00** | **0.00** |
| | | Worst | - | 1.00 | 1.45 | 1.13 | 2.16 | 1.04 | 1.02 | **0.92** |
| | 30 | Mean | 1.00 | - | 1.40 | 1.51 | 2.31 | 0.19 | 0.01 | **0.00** |
| | | Std. Dev. | 0.10 | - | 0.33 | 0.38 | 0.56 | 0.22 | **0.01** | 0.03 |
| | | Best | 0.70 | - | 1.04 | 1.04 | 1.10 | **0.00** | **0.00** | **0.00** |
| | | Worst | 1.10 | - | 2.50 | 2.78 | 4.23 | 1.01 | **0.04** | 0.26 |
| | 50 | Mean | - | 1.20 | 2.59 | 2.30 | 4.28 | 0.32 | 0.47 | **0.02** |
| | | Std. Dev. | - | **0.06** | 0.63 | 0.56 | 0.78 | 0.22 | 0.19 | 0.09 |
| | | Best | - | 1.10 | 1.54 | 1.35 | 2.45 | 0.03 | 0.08 | **0.00** |
| | | Worst | - | 1.40 | 4.64 | 4.37 | 6.67 | 0.99 | 0.94 | **0.74** |
| | 100 | Mean | 1.80 | 1.70 | 7.59 | 7.66 | 9.90 | 0.49 | 0.48 | **0.00** |
| | | Std. Dev. | 0.10 | 0.10 | 1.12 | 1.19 | 1.08 | 0.18 | 0.18 | **0.00** |
| | | Best | 1.50 | 1.50 | 4.47 | 5.16 | 7.00 | 0.14 | 0.18 | **0.00** |
| | | Worst | 2.00 | 2.00 | 10.28 | 11.36 | 13.18 | 0.92 | 0.95 | **0.02** |

TABLE IV. Statistics from the third case of study via DSOA and SSOA.

| | | Sphere | | Step | | Salomon | | Ackely 1 | | Rosenbrock | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Dim. | | DSOA | SSOA | DSOA | SSOA | DSOA | SSOA | DSOA | SSOA | DSOA | SSOA |
| 2 | Mean | 3.05E-03 | **1.84E-05** | **0.00** | **0.00** | 6.38E-02 | **2.66E-02** | 7.75E-02 | **2.24E-03** | 34.86 | **8.12E-02** |
| | Std. Dev. | 2.15E-02 | **4.48E-05** | **0.00** | **0.00** | **3.54E-02** | 4.04E-02 | 0.11 | **1.96E-02** | 136.52 | **0.55** |
| | Best | 5.39E-07 | **1.33E-36** | **0.00** | **0.00** | 4.75E-04 | **1.08E-16** | 1.50E-04 | **8.88E-16** | **4.87E-06** | 3.01E-04 |
| | Worst | 0.26 | **3.07E-04** | **0.00** | **0.00** | **0.10** | **0.10** | 0.56 | **0.34** | 1186.19 | **7.91** |
| 5 | Mean | 4.67E-02 | **1.90E-03** | 8.53 | **0.13** | 0.26 | **0.11** | 0.98 | **0.57** | 178002.50 | **80.93** |
| | Std. Dev. | 0.15 | **3.03E-03** | 81.25 | **0.44** | 0.27 | **4.21E-02** | 0.50 | **0.29** | 874982.30 | **948.09** |
| | Best | 1.33E-04 | **4.75E-09** | **0.00** | **0.00** | **0.10** | **0.10** | 0.18 | **1.12E-04** | **1.40** | 2.51 |
| | Worst | 1.03 | **3.48E-02** | 1.40E+03 | **4.00** | 2.70 | **0.66** | 5.04 | **1.30** | 11100000.00 | **16043.10** |
| 10 | Mean | 7.46 | **4.96E-03** | 6.20E+02 | **0.25** | 2.08 | **0.19** | 2.27 | **0.80** | 488316.40 | **1183.94** |
| | Std. Dev. | 22.92 | **1.00E-02** | 2.02E+03 | **0.82** | 2.68 | **0.10** | 1.15 | **0.62** | 1515044.00 | **14648.44** |
| | Best | 1.87E-03 | **2.45E-27** | **0.00** | **0.00** | 0.20 | **0.10** | 0.79 | **7.99E-15** | 9.19 | **8.62** |
| | Worst | 139.91 | **0.10** | 1.14E+04 | **6.00** | 11.60 | **0.60** | 6.12 | **2.13** | 12200000.00 | **235992.00** |
| 15 | Mean | 4.99 | **5.36E-02** | 439.61 | **4.72** | 2.81 | **0.48** | 2.63 | **1.57** | 759946.20 | **2092.32** |
| | Std. Dev. | 12.16 | **5.56E-02** | 1053.67 | **4.80** | 1.90 | **0.23** | 0.82 | **0.39** | 4435832.00 | **35088.97** |
| | Best | 1.24E-02 | **5.46E-04** | 2.00 | **0.00** | 0.40 | **0.10** | 1.21 | **2.35E-02** | 61.60 | **14.31** |
| | Worst | 109.21 | **0.38** | 14548.00 | **32.00** | 11.60 | **1.81** | 5.89 | **3.02** | 43300000.00 | **607822.00** |
| 20 | Mean | 42.59 | **4.20E-02** | 4400.25 | **4.48** | 7.07 | **0.40** | 4.02 | **1.10** | 5844834.00 | **53.70** |
| | Std. Dev. | 59.56 | **6.23E-02** | 6488.07 | **7.19** | 3.94 | **0.28** | 1.17 | **0.81** | 16435640.00 | **57.55** |
| | Best | 5.68E-02 | **1.59E-24** | 22.00 | **0.00** | 0.90 | **0.10** | 1.66 | **2.89E-13** | 223.09 | **18.76** |
| | Worst | 366.68 | **0.36** | 33796.00 | **57.00** | 18.40 | **1.83** | 7.15 | **2.72** | 101000000.00 | **458.94** |
| 25 | Mean | 33.38 | **9.63E-02** | 3266.79 | **9.62** | 7.48 | **0.66** | 3.90 | **1.62** | 2487773.00 | **134.33** |
| | Std. Dev. | 32.67 | **7.14E-02** | 3588.55 | **7.95** | 2.56 | **0.32** | 0.86 | **0.46** | 5691636.00 | **246.11** |
| | Best | 0.95 | **2.90E-03** | 91.00 | **0.00** | 2.30 | **0.20** | 2.15 | **3.92E-02** | 3304.93 | **28.16** |
| | Worst | 268.97 | **0.40** | 24613.00 | **42.00** | 16.00 | **2.14** | 6.85 | **2.49** | 53700000.00 | **4057.22** |
| 30 | Mean | 121.43 | **4.98E-02** | 12252.18 | **4.73** | 12.83 | **0.47** | 5.06 | **0.81** | 16924720.00 | **76.95** |
| | Std. Dev. | 98.58 | **0.11** | 9864.04 | **11.89** | 4.16 | **0.36** | 0.87 | **0.82** | 24729690.00 | **108.69** |
| | Best | 4.53 | **1.22E-23** | 617.00 | **0.00** | 3.80 | **0.10** | 2.98 | **4.34E-13** | 34135.10 | **28.82** |
| | Worst | 502.76 | **1.02** | 55345.00 | **70.00** | 23.00 | **2.80** | 7.78 | **2.08** | 161000000.00 | **987.27** |
| 50 | Mean | 324.55 | **0.39** | 33406.81 | **36.91** | 21.96 | **0.80** | 5.88 | **1.23** | 63132200.00 | **481.50** |
| | Std. Dev. | 178.10 | **0.41** | 17075.24 | **38.86** | 3.98 | **0.54** | **0.58** | 0.90 | 58844030.00 | **539.39** |
| | Best | 49.45 | **2.90E-22** | 5829.00 | **0.00** | 11.70 | **0.10** | 4.19 | **2.65E-12** | 2440000.00 | **48.81** |
| | Worst | 904.50 | **1.94** | 81452.00 | **175.00** | 33.80 | **2.95** | 7.45 | **2.82** | 304000000.00 | **3225.70** |
| 100 | Mean | 1145.35 | **0.82** | 118019.70 | **76.79** | 39.57 | **1.11** | 6.67 | **1.24** | 287253300.00 | **961.86** |
| | Std. Dev. | 237.31 | **0.81** | 23586.24 | **68.75** | 2.97 | **0.72** | **0.32** | 0.90 | 113793400.00 | **828.52** |
| | Best | 594.29 | **4.62E-21** | 66280.00 | **0.00** | 29.80 | **0.10** | 5.79 | **7.45E-12** | 100000000.00 | **98.91** |
| | Worst | 1771.62 | **5.18** | 184973.00 | **324.00** | 47.00 | **4.07** | 7.65 | **2.31** | 696000000.00 | **3918.04** |
| Stats | Mean | 186.64 | **0.16** | 19157.14 | **15.29** | 10.46 | **0.47** | 3.50 | **0.99** | 41896570.00 | **562.85** |
| | Std. Dev. | 368.86 | **0.40** | 37902.24 | **36.23** | 12.57 | **0.49** | 2.22 | **0.81** | 99080870.00 | **12682.42** |
| | Best | 5.39E-07 | **1.33E-36** | **0.00** | **0.00** | 4.75E-04 | **1.08E-16** | 1.50E-04 | **8.88E-16** | **4.87E-06** | 3.01E-04 |
| | Worst | 1771.62 | **5.18** | 184973.00 | **324.00** | 47.00 | **4.07** | 7.78 | **3.02** | 696000000.00 | **607822.00** |


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