Financial Constraints and Investment: An Alternative Empirical Framework

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Abstract In the empirical literature on financial constraints, firms are usually assigned to distinct groups, according to whether or not they are supposed to be financially constrained (FC). Several recent empirical papers studying the relationship between firms’ cash flows and investment, have found mixed results regarding whether or not more FC firms show higher or lower sensitivity of investment to cash flows (CFSI). We show that these mixed results may be attributable to the use of different “ex-ante” sample-splitting schemes. In response to these issues, we develop a novel research strategy that allows for estimating firm-specific investment-cash flow sensitivities and identifying the whole distribution of sensitivities across firms. Specifically, we apply entropy-based estimation methods, which outperform classical estimation methods under various general conditions. Our empirical results, based on a large panel-data set of U.S. firms, demonstrate some of the limitations of prevailing classification schemes. The approach we propose offers new and interesting prospects for re-examining the existing empirical evidence on financial constraints.

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1 Introduction

The impact of capital market frictions on corporate financing policies has been the focus of a large number of theoretical and empirical studies. Both theory and empirics highlight the existence of financial constraints (FC) that potentially limit the capacity of the firm to grow over time. While theoretical arguments, related to asymmetric information and agency problems, seem compelling and fully in line with the pecking order corporate finance paradigm (Jensen and Meckling (1977), Myers and Majluf (1984)), results from the empirical literature are mixed and are often conflicting, showing a lack of consensus about an adequate and reliable measure to capture constraints.

The metric most widely believed to be informative about the level of constraints, is the cash flow sensitivity of investment (CFSI). This parameter measures the investment response as a result of an increase in cash flow and was first proposed in the seminal study by Fazzari et al. (1988). More recent research has revealed some theoretical inconsistencies as well as conflicting empirical evidence. On the theoretical side, Kaplan and Zingales (2000) argue that investment-cash flow sensitivities might not increase monotonically with the level of FC, making an aggregate sensitivity difficult to interpret. Additionally, Cooper and Ejarque (2003) and Alti (2003) show that under certain conditions, firms might invest according to retained earnings, even in the absence of market frictions. On the empirical side, Kaplan and Zingales (1997), Cleary (1999), and Kadapakkam et al. (1998) find that investment-cash flow sensitivities may actually be lower for firms ex-ante identified as being constrained, thereby contradicting most of the previous empirical evidence. Thus, while the literature recognizes that cash flow is a significant determinant of corporate investments, it remains unclear to what
extent a significant sensitivity truly reflects the existence of financial constraints (Pawlina and Renneboog (2005)).

One reason why the CFSI metric is difficult to evaluate lies within its econometrical operationalization. Methodologically, estimation of the CFSI for a certain sample proceeds along the following line of reasoning: Firstly, the sample under study is split up in sub-samples according to a criterion that is chosen to reflect differences in “susceptibility” to capital market imperfections such as size, payout policy, debt rating, etc. Then, for each sub-sample, an aggregate sensitivity is estimated by regressing corporate investment on operational cash flow and a number of controls using standard panel data regression techniques. The “traditional” (i.e., Fazzari et al. (1988)) finding is that the aggregate sensitivity is higher for the sub-sample most susceptible to capital market imperfections, which is considered evidence in favour of the CFSI metric. Conversely, the finding that the aggregate sensitivity is lower for the sub-sample most susceptible to capital market imperfections is considered evidence against the CFSI metric. This practice, common to all empirical studies in this literature, can be labelled “ex-ante sample splitting”.

This methodological operationalization is limited and can be misleading because different splits might produce conflicting estimation results for the sample under study. In effect, such an inconsistency might reflect two things: (a) the splitting criterion is unable to distinguish between firms with a different susceptibility to capital market imperfections (i.e., badly chosen splitting criterion); and/or, (b) the estimated sensitivity is not indicative of the level of financial constraints (i.e., badly chosen metric). The fact that we cannot distinguish between (a) and (b) makes it very difficult to evaluate the CFSI metric in terms of its ability to capture financial constraints. Therefore, the results are only as strong as the choice of the ex-ante sample splitting scheme. Despite the recognition of this drawback by several authors (Schiantarelli (1995) and Hubbard (1998)), and despite the limitations of its application, the ex-ante sample splitting remains
standard practice even in the most recent literature on investment-cash flow sensitivities (Islam and Mozumdar (2007)), and in the related literature on cash-cash flow sensitivities (Almeida et al. (2004) and Lin (2007)).

The purpose of this paper is to explore the potential of a new methodological operationalization that allows for taking into account firm-specific effects. We show that the GME estimator developed by Golan et al. (1996) provides interesting opportunities to model heterogeneous slopes without having to rely on the assumptions of the classical linear regression model (CLRM). As a result, firm-specific sensitivities can be estimated making ex-ante sample splitting unnecessary. Besides the clear benefit of circumventing the ex-ante sample splitting, the GME estimator has some interesting econometrical properties that make the estimation results more reliable than traditional GMM estimates (Golan et al. (1996) and Peeters (2004)).

The remainder of this paper is organized as follows: the next section discusses existing empirical evidence on financial constraints and on the problems related to the traditional econometrical operationalization. The third section introduces the GME estimator and shows how this approach can be applied for estimation of firm-specific investment-cash flow sensitivities. Section four applies the GME estimator to a large unbalanced sample of U.S. firms. In section five we conclude and discuss the benefits and research opportunities this new method provides.

2 Investment-cash flow sensitives and ex-ante sample splitting

The empirical debate on the relationship between financial constraints and firm investment decisions has been heated. The seminal work of Fazzari et al. (1988) proposed that investment-cash flow sensitivities are positively related to the existence of financial constraints. The microeconomic rationale behind this metric is that financially constrained firms, having only limited access to external funds, depend mainly on their internally generated funds to finance corporate investments. As a result, they should display positive investment-cash flow sensitivities,
in contrast with unconstrained firms, who should display no sensitivity to cash availability. Fazzari et al. (1988) provide empirical evidence supporting their assertion, based on the behaviour of firms exhibiting low payout ratios, which they argue will be more susceptible to capital market imperfections and, thus, be more likely to be financially constrained. Many subsequent studies have followed their approach and have found that investment-cash flow sensitivities are indeed positively related to the existence of financial constraints, using alternative samples, investment models or ex-ante sample splitting schemes (Hoshi et al. (1991), Gilchrist and Himmelberg (1995), Carpenter and Petersen (2002), Bond et al. (2003), Alayannis and Mozumdar (2004) and Islam and Mozumdar (2007)).

A number of studies have criticized the approach on empirical as well as theoretical grounds. Empirical counterexamples have been provided by Kaplan and Zingales (1997), Kadapakkam et al. (1998) and Cleary (1999), who find that investment-cash flow sensitivities are lower for firms ex-ante defined as being constrained. On the theoretical level, some studies have raised questions about the underlying econometrical design. For example, Bond et al. (2003) suggest using GMM to account for potential endogeneity of the cash flow parameter, while Alayannis and Mozumdar (2004) indicate that negative cash flow observations and influential outliers may have a disproportionately large effect on the point estimates. Although these studies emphasize a careful econometrical operationalization of the metric, they do not challenge its usefulness on a fundamental theoretical level.

Other studies have provided criticism on a more fundamental theoretical level. Kaplan and Zingales (2007) question the assumed “monotonicity” between investment-cash flow sensitivities and financial constraints. They also question the ability of the market-to-book ratio to control for investment opportunities. Alti (2003) and Cooper and Ejarque (2003) go one step further and show that under certain conditions, a significant CFSI might occur even in the absence of capital market imperfections. Finally, Moyen (2004) shows that financially un-
constrained firms are inclined to raise more debt in times of high cash flow, which would be a driving factor for a significant CFSI for unconstrained firms.

While these theoretical considerations cast serious doubt upon the ability of the CFSI metric to capture financial constraints, it is difficult to assess how strong this impact is when measuring financial constraints in a “real-world” sample. For example, Pawlina and Reneboog (2005) and Lin (2007) suggest that the literature recognizes that investments are driven by internal funds for a large number of firms, although it remains unclear to what extent these investment-cash flow sensitivities reflect limited access to external funds as a result of financial constraints.

We surmise that the econometric operationalization of “ex-ante sample splitting” may contribute to much of the debate in the literature since most previous studies have split up their sample a priori, using a criterion that is likely to reflect different susceptibility to capital market imperfections. Popular ex-ante splitting schemes include: dividend-payout (Fazzari, et al. (1988)); size (Gilchrist and Himmelberg (1995) and Carpenter and Petersen (2002)); tangibility-ratio (Bhagat et al. (2005)); debt ratings (Kashyap et al. (1994) and Almeida (2004)); dividend-investment correlation (Lin (2007)); strength of banking relationships (Hoshi et al. (1991) and Deloof (1998)); and, cross-country comparison (Kadapakkam et al. (1998), Bond et al. (2003) and Islam and Mozumdar (2007)). Other studies have used a statistic that captures a firm’s financial health according to a number of financial variables (Kaplan and Zingales (1997) and Cleary (1999)).

The main problem with “ex-ante sample splitting” is that an aggregate coefficient in a certain sub-sample could be informative about an unobservable economic phenomenon besides financial constraints. There could be factors related to the splitting scheme that are not related to financial constraints. To illustrate this point, consider for instance the commonly used splitting scheme that uses size as measured by the natural logarithm of total assets. Suppose the available firm-years are divided into a small group versus a large group, and
the regression analysis reveals a higher aggregate CFSI estimate for the small sub-sample (as in Gilchrest and Himmelberg (1995), for example). This could reflect financial constraints, but could equally well reflect another unobservable \(^1\) economic phenomenon. For instance, it could reflect that smaller firms invest mainly in smaller investment projects for which no external capital is required. In this case, the difference in CFSI reflects a difference in demand for external funding sources. Alternatively, it could reflect that smaller firms are more active in industries \(^2\) where investments have a shorter time-span but occur more regularly (i.e. industries that require more capital replacement versus new capital expenditures). In this case, the difference in CFSI reflects differences in the nature of the investment projects, rather than financial constraints. Finally, the higher aggregate CFSI in the smaller sub-sample could simply reflect the presence of a few influential outliers in the smaller sub-sample that bias the point estimate upwards.

The conclusion is that estimating an aggregate CFSI can be considered an indication of financial constraints, but is by no means undisputable empirical evidence of it. It could be that a researcher is picking up financial constraints, but it could equally well be that he/she is picking up another unobservable factor inherent to the splitting criterion. In other words, using ex-ante sample splitting, a researcher can never be sure whether or not he/she captures constraints. The results are only as strong as the ex-ante sample splitting criterion is at adequately reflecting differences in capital market imperfections, which ultimately remains a matter of assumption (Schiantarelli (1995)). And even if the splitting scheme does

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\(^1\) Not controlling for unobserved factors is in fact a violation of the “ceteris paribus” principle which might lead to omitted variable bias. Examples of this bias can be found in standard econometrics textbook such as in Stock and Watson (2007), pg 186-192.

\(^2\) Most empirical specifications control for time and industry dummies within each sub-group, but do not account for industry-differences between the ex-ante defined groups. Note that using time and industry dummies in each sub-sample separately does not solve for differences between the classes.
accurately reflect differences in capital market imperfections, the point estimates might be biased when firm-heterogeneity is ignored.

As Hansen et al. (2004) point out\(^3\): “when an economic theory is not a theory of averages, the econometrical operationalization should account for individual firm-differences and should not be based on means, which statistically neutralize firm-differences”. Essentially, financial constraints happen at the *firm* level, not at an *aggregate* level. As such, it seems logical that an econometrical operationalization should account for firm differences. Following this line of reasoning, we explore the possibilities of using the GME estimator in the context of financial constraints. As we will argue in the next section, this approach allows for firm-heterogeneity, thereby avoiding some of the pitfalls of using an ex-ante sample splitting approach.

As an additional benefit, the GME methodology is very much suited to tackle “ill-posed” or “ill-conditioned” problems, in contrast with the traditional Classical Linear Regression Model (CLRM) that requires many assumptions about error terms and unobserved heterogeneity. Based upon some experimental designs, Golan et al. (1996) were able to show that the GME estimator tends to “outperform” the traditional GMM (2SLS) estimator. Therefore, a second aim of this paper, besides circumventing ex-ante sample splitting, is to come up with more reliable estimation results.

### 3 Heterogeneous slopes using entropy econometrics

#### 3.1 An introduction to GME

The fundamentals that underlie the GME estimation procedure are based upon the concepts of Information Theory. In this context, Entropy is a measure of

\(^3\) The context of Hansen et al. (2004) is a Bayesian operationalization of the Resource Based View, a predominant paradigm in the strategic management literature. Although this context is very different from the context of capital market imperfections, we believe there are many similarities in terms of econometrical operationalization.
uncertainty or missing information. Let us consider an event $X$ with $K$ discrete possible outcomes described in a discrete probability distribution $(p_1, p_2, ..., p_K)$. If an outcome has a small probability of occurring, we would be very surprised if the outcome did occur, and therefore ascribe a large amount of information content to that outcome. Vice-versa, for an outcome with a high probability, occurrence would be anything but surprising and hence occurrence is less informative. Generally the information content of a single outcome is inversely proportional to its probability and can be written as $I_k = -\log(p_k)$.

Fraser (2000) provides an intuitive example of the information measure using the analogy of a cricket match between England and Australia, which we transform to a soccer match between Belgium and the Netherlands. Suppose we define the soccer match as a Bernoulli trial with success defined as a win for Belgium. Unfortunately, anyone who has witnessed previous encounters must admit that success has only a small chance of occurring. Therefore we would be surprised if Belgium won and this outcome would be very informative about the quality of Belgian soccer. We would not be surprised if Belgium lost, and the information content of that outcome provides little information since it was expected with near certainty. Figure 1 gives a graphical representation of the inverse relation between probability and uncertainty.

The Entropy or Expected Information of an event is the average information content of each possible outcome and can be written as:

$$H(p) = -\sum_{k=1}^{K} p_k \ln p_k. \quad (1)$$

The Entropy measure reaches a minimum value of zero when $p_k = 0$, for $k = 1, \ldots, K$. In this case, all possible outcomes have a zero chance of occurring and missing information is zero. The measure reaches a maximum when $p_1 = p_2 = \ldots = p_K = 1/K$. In this special case all outcomes are uniformly distributed and have an equal chance of occurring. In that case missing information is maximal,
Figure 1: **Negative relation between probability and informational content.** This figure represents the negative relationship between the probability of an outcome and the information content of that outcome. The horizontal axis represents the prior probability assigned to the outcome and lies between 0 and 1. The vertical axis gives the information content that can be described by the $-\log(.)$ following Fraser (2000).

since we cannot distinguish between possible outcomes. Following the latter line of reasoning the Entropy measure can be viewed as a distance metric between the discrete uniform distribution and the distribution generating $p_k$.

Judge and Golan (1992) apply these principles from Information Theory to the standard econometrical regression model $y = X\beta + e$ where $y$ is a $T \times 1$ matrix of the dependent variable. $X$ is the $T \times K$ matrix of independent variables and $e$ is a $T \times 1$ matrix of disturbances. To retrieve the unknown parameters $\beta$, they suggest, following the results of Jaynes (1957), to maximize the Entropy measure subject to a data consistency constraint and normalization constraints. The solution obtained by working through this maximization problem is consistent with the data, while expressing “maximum uncertainty” of “least information”. In other words, maximizing the Entropy amounts to estimating the unknown probabilities “in the greatest number of ways, but still consistent with the data at hand”.

Additionally, Golan et al. (1996) suggest using non-sample or prior information to improve the accuracy of the estimates (i.e., posterior information)\footnote{Using prior information in conjunction with data to obtain a posterior distribution about the parameters to be estimated is fully in line with the Bayesian econometrical}. If
the prior information is fully consistent with the data, zero information is gained from the data and the posterior distribution will be identical to the prior. When data do not correspond with prior information, additional information is gained from the data when calculating the posterior distribution. Since it is necessary for the support space for $\beta$ to contain the true value of the parameter, Fraser (2000) suggests using a discrete support interval around a spike prior with symmetrical or asymmetrical endpoint depending upon the problem under consideration. The range should be wide enough to ensure the true value for $\beta$ to lie within the interval.

More specifically, following Golan et al. (1996), the GME formulation can be written as follows:

$$y = X\beta + e = XZp + Vw.$$  \hfill (2)

In this expression, $Z$ is a $K \times KM$ matrix of known discrete support values for $\beta$ and $p$ is the $K \times M$ matrix of probabilities to be estimated with $M$ the number of support points. Similarly, $V$ is a $T \times TJ$ matrix of known support values for $e$ and $w$ is a vector of probabilities to be estimated with $J$ the number of support points. Using this reparametrization it is possible to write the standard estimation problem as a general GME problem:

$$\max_p H(p) = -\sum_{k=1}^{K} p_k \ln p_k$$  \hfill (3)

s.t. \hspace{1cm} \hspace{1cm} \hspace{1cm} y = X\beta + e = XZp + Vw \hspace{1cm} (4)

$$\sum_{k=1}^{K} p_{km} = 1 \hspace{1cm} m = 1, ..., M$$  \hfill (5)

framework that can be found in Lancaster (2004). That is why refer to GME as a quasi-Bayesian methodology as in Fraser (2000).

5 As for the number of discrete support values, Fraser (2000) suggests that $M$ and $J$ should be determined by computational time, rather than accuracy of estimation.
\[ \sum_{t=1}^{T} w_{tj} = 1 \quad j = 1, \ldots, J. \] (6)

Equation (3) is the Entropy measure serving as an objective function that needs to be maximized. Equation (4) is the data-consistency constraint, which is the parametrical version of the regression model that incorporates the prior non-sample information. Equations (5) and (6) are normalization constraints that ensure that for each parameter, the estimated probabilities add up to one.

3.2 GME and firm-specific investment-cash flow sensitivities

In order to estimate the CFSI metric, we use the commonly used equation that relates corporate investment to the market-to-book ratio and cash flow:

\[ \frac{I}{K_{i,t}} = \alpha + \beta \frac{CF}{K}(CF/K)_{i,t} + \gamma \frac{M}{B}(M/B)_{i,t} + \mu_t + \gamma_s + u_{i,t}. \] (7)

Where \( I_{i,t} \) is investments in PP&E during year \( t \), \( K_{i,t} \) is beginning-of-year book value of PP&E (measured as PP&E ending-balance in the previous year), \( M/B_{i,t} \) is beginning-of-year market value of common equity divided by the book value of common equity and \( CF_{i,t} \) is net income before extraordinary items + depreciation and amortization. Time dummies (\( \mu_t \)) and industry dummies (\( \gamma_s \)) are added to control for heterogeneity in investment policy over time and between industries. This equation has been estimated by Kaplan and Zingales (1997), and Cleary (1999, 2006), among others, and uses the market-to-book ratio as a proxy for investment opportunities.

The CFSI is the investment response as a result of change in cash flow and is given by the cash flow regression coefficient as indicated in (8):

\[ CFSI = \hat{\beta}_{CF/K}. \] (8)

The corresponding GME formulation of equation (7) can be written as follows:
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\[
\max_p H(.) = -\sum_m p_{\alpha_m} \ln p_{\alpha_m} - \sum_m p_{\beta_m} \ln p_{\beta_m} - \sum_i \sum_m p_{\nu_{i,m}} \ln p_{\nu_{i,m}} \quad (9)
\]

s.t.

\[
\frac{I}{K_{i,t}} = \sum_m p_{\alpha_m} s_{\alpha_m} + \left( \sum_m p_{\beta_m} s_{\beta_m} + \sum_m p_{\nu_{i,m}} s_{\nu_{i,m}} \right) \frac{CF}{K_{i,t}} \quad (10)
\]

\[
\sum_m p_{\alpha_m} = \sum_m p_{\beta_m} = \sum_m p_{\nu_{i,m}} = \sum_m p_{\gamma_m} = \sum_m p_{\mu_{i,m}} = 1. \quad (11)
\]

In this formulation equation (9) is the objective function, which is the Entropy-measure that needs to be maximized with respect to \( p \) in order to estimate the unknown probabilities. Equation (10) is the data-consistency constraint which is the parametrical version of the CFSI model given in equation (7). In this equation, each parameter is defined as a linear combination of the predetermined support vector \( s \) and a vector of probabilities \( p \). The support vector \( s \) summarizes the non-sample or prior information in a discrete interval around the spike value \( \psi \). We model this interval as follows:

\[
s_{\beta_m} = [\psi_{\beta} - 3\sigma_{\beta}; \psi_{\beta}; \psi_{\beta} + 3\sigma_{\beta}]. \quad (12)
\]

Where \( \psi_{\beta} \) is the spike value for \( \beta \) given by \( \hat{\beta}_{CF/K} \), the estimated cash flow coefficient from running a GMM regression analysis of equation (7). The second set of constraints in (11) summarizes the normalization constraints, which ensure that for each parameter, the estimated probabilities sum up to one.

Maximizing equation (9) with respect to \( p \) subject to the constraints in (10) and (11) yields parameter estimates for all unknown parameters; that is:

\[
\hat{\alpha} = \sum_m \hat{p}_{\alpha_m} s_{\alpha_m}, \quad \hat{\beta} = \sum_m \hat{p}_{\beta_m} s_{\beta_m}, \quad \nu = \sum_m \hat{p}_{\nu_{i,m}} s_{\nu_{i,m}}, \quad \gamma = \sum_m \hat{p}_{\gamma_m} s_{\nu_{i,m}}.
\]

Hence we have model that allows for firm-specific slopes as follows:
\[ I/K_{i,t} = \alpha + (\beta + \nu_i)CF/K_{i,t} + \gamma_{M/B}(M/B)_{i,t} + u_{i,t}, \]  

Equation (13)

\[ CFSI_i = \hat{\beta} + \hat{\nu}_i. \]  

Equation (14)

Equation (14) represents the firm-specific investment-cash flow sensitivities estimated with GME as opposed to a single aggregate CFSI estimated in the traditional regression framework. Hence, it is no longer needed to rely on ex-ante sample splitting. Instead, we can focus directly on the metric and its ability to capture constraints. In the discussion section we explore conceptually how these individual sensitivities can be used to evaluate the metric in its ability to capture constraints.

Besides the conceptual benefit of circumventing the ex-ante sample splitting there are two additional econometrical benefits worth mentioning. First, the GME estimator tackles a number of problems that might bias the estimates of the CLRM, such as under-determinacy and ill-conditioning. A problem is under-determined when the number of parameters exceeds the number of data points. Ill-conditioning occurs when the data are generated non-experimentally or linear dependencies exist among the set of explanatory variables. Golan et al. (1996) point out that these problems, which are likely to occur in any practical application in economics, could lead to unstable parameter estimates and unreliable hypothesis testing. Secondly, econometrical research by Pesaran and Smith (1995) has shown that pooled regression estimates can be highly misleading when estimating relationships in heterogeneous panels. In fact, this might be the econometrical rationale behind the finding of Allayannis and Mozumdar (2004) that a few influential outliers could heavily bias an aggregate CFSI estimate. The GME estimator does not suffer from this potential bias, because slope heterogeneity is allowed for. Moreover, studying the distribution of firm-specific sensitivities could give further insights in the nature of the bias of an aggregate coefficient in different sub-samples.
4 Sensitivity analysis

As we have demonstrated, the GME estimator combines both prior information and data to estimate the parameter of interest. This is very much in line with the Bayesian econometrical framework where a prior distribution is confronted with data in order to form a posterior parameter distribution. For this reason, Fraser (2000) refers to the GME estimator as a Quasi-Bayesian methodology. An objection often raised by researchers not adhering to the Bayesian econometrical framework is the sensitivity of the estimator to alternative prior beliefs. In other words: what if I don’t believe the prior belief? While it is true that the prior information plays a role in the estimation process its effect should not be overstated. In the Bayesian framework it is generally accepted that prior information might complement the data and that data information usually predominates the posterior distribution especially in large samples (Lancaster, 2004). With enough data points, the effect of prior information is often overshadowed by the effect of the data. Additionally, when prior information is carefully constructed and operationalized in a prior distribution, its use is a strength that complements the analysis rather than a weakness. Therefore, incorporating relevant prior information in the analysis is a natural thing to do for the Bayesian researcher. However, for the apprehensive reader, a Bayesian analysis is often complemented with a sensitivity-analysis that reports estimation results for a variety of prior beliefs. As a result, a reader with slightly different prior beliefs can assert how much estimation results deviate due to his/her beliefs. The logic behind this analysis is that the reader can choose which prior distribution fits best with his expectations about the parameter.

The same logic applies to the GME estimator. While it is true that the results are partially driven by the prior non-sample information, the sensitivity of the results should be relatively low for a large enough sample and for relevant supports. Golan et al. (1996) argue that a necessary condition for reliable estimation is that the support interval should include the true value of the coefficient. In
equation (12) we have expressed that non-sample prior information is bounded by a wide interval around its GMM estimate. Using a support interval around a “reasonable” reference value limits the chance that the true value of the parameter exceeds these support bounds. Additionally our sample is sufficiently large for the data-information to play the dominant role in the estimation process.

However, for that apprehensive reader we construct a sensitivity algorithm for the GME estimator using the cross-entropy formulation developed in Golan et al. (1996). They show that the GME maximization problem described in equations (3) to (6) might be transformed into a dual minimization problem where the objective function is the cross-entropy between the parameter \( p \) en prior information \( q \). More specifically the equivalent formulation is:

\[
\min_{p,q} H(p, q) = -\sum_{k=1}^{K} p_k \frac{\ln p_k}{\ln q_k}
\]

s.t. \( y = X\beta + e \) \hspace{1cm} (16)
\[
\sum_{k=1}^{K} p_{km} = 1 \quad m = 1, ..., M \hspace{1cm} (17)
\]
\[
\sum_{k=1}^{K} q_k = 1. \hspace{1cm} (18)
\]

In this formulation, the objective function (15) is the entropy distance between the data in the form of \( p \) and the prior \( q \) that needs to be minimized. Equation (16) is the data-consistency constraint and (17) and (18) are normalization constraints that ensure that the estimated probabilities for both parameters and supports add up to one. This formulation provides a powerful tool for analyzing differences in parameter estimates due to a change in prior belief. It allows us to construct and implement a mathematical algorithm that repeats the estimation process \( n \) times for different supports randomly drawn out of a distribution of choice. The \( n \) i.i.d. drawings can be viewed as the sampling distribution of the firm-specific estimates for various discrete supports. As a result, we get an indication of the sensitivity of the estimation results to alternative prior information, very much in
line with the sensitivity-analysis that usually complements a Bayesian estimation problem. The next section includes a table that summarizes a sensitivity-analysis where the posterior means CFSI is presented for a variety of prior distributions.

5 Results

5.1 Data and summary statistics

Annual data of US-based firms were extracted from the COMPUSTAT database over the seven year time period from 1998 to 2004. Regulated and Financial industries according to their two-digit SIC code were deleted (i.e., 43XX, 48XX, 49XX, 6XXX, 9XXX). The only requirement for inclusion in the dataset is that firms report no negative values for $M/B$, $TA$ or $K$ during the sample period. In order to remove outliers, the top and lowest 1% of observations was deleted for every variable used. As an additional check, all variables required for estimation of the regression equation (7) were winsorized\(^6\) in order to reduce impact of extreme observations, while keeping as much information in the dataset. These procedures leave us with a large unbalanced dataset of 25,879 firm-year observations.

Table 1 presents descriptive statistics for a selection of financial variables. The mean value of total assets is around $2.3 billion. Median sales growth, defined as the percentage growth of annual sales, is around 8%. The average debt ratio is about 26% and the mean payout ratio is 12.4%. Mean values for cash flow to net fixed assets, total capital expenditures to net fixed assets and the market-to-book ratio are 11.4%, 17.0% and 2.62 respectively. Finally, the averages for the tangibility-ratio (measured as PP&E divided by net fixed assets), the current ratio and for ROE (defined as income before extraordinary items divided by common equity) are 64.1%, 2.8 and 1.2% respectively.

\(^6\) The following winsorizing rules were used, as in Cleary (1999): $I/K_{i,t} = 2$, if $I/K_{i,t} > 2$; $I/K_{i,t} = -2$, if $I/K_{i,t} < -2$; $MB_{i,t} = 10$, if $MB_{i,t} > 10$; $CF/K_{i,t} = 5$, if $CF/K_{i,t} > 5$; $CF/K_{i,t} = -5$, if $CF/K_{i,t} < -5$. 
Table 1: **Descriptive Statistics.** This table presents means and Quartiles for a number of financial and growth variables. Payout-ratio is defined as total dividends and stock repurchases divided by net income. Debt-ratio is total debt divided by total assets. Slack is defined as (cash holdings + short-term investments + 0.5 inventories + 0.7 accounts receivables – short term loans). The sample consists of 25,879 observations over a 7 year time-period from 1998-2004.

5.2 **Pooled estimation using GMM**

Following the traditional econometrical operationalization, equation (7) was estimated using GMM\(^7\) for various sub-samples split up according to a priori measures of financing frictions. We experiment with 5 different splitting schemes that have been used extensively in the literature: payoutratio, size, tangibility ratio, debt rating and KZ index.

**Scheme 1: Payout ratio**

Following Fazzari *et al.* (1988), Almeida *et al.* (2004) and others, firms were ranked annually according to their payout ratio, defined as total dividends and stock repurchases divided by net income. Firms in the bottom (top) three deciles of the payout distribution were labelled LOW-PAYOUT (HIGH-PAYOUT). The annual ranking is to ensure that firms are able to change ex-ante constraints status over the sample period, which makes the sample splitting less rigid, as pointed out by Schiantarelli (1997).

\(^7\) The GMM estimator developed by Arellano and Bond (1991) was used in order to control for potential endogeneity of the cash flow parameter (Bond *et al.*, 2003).
Scheme 2: Size
Following Gilchrest and Himmelberg (1995) and Kadapakkam et al. (1998) among others, firms were ranked annually according to their size measured as the natural log of total assets. Firms in the bottom (top) three deciles of this size-distribution were labelled SMALL (LARGE).

Scheme 3: Tangibility ratio
Following Bhagat et al. (2005), firms were ranked annually according to their tangibility ratio measured as PP&E divided by net fixed-assets. Again, firms in the bottom (top) three deciles of the tangibility-distribution were labelled LOW TANG (HIGH TANG).

Scheme 4: Debt rating
Following Kashyap et al. (1994) among others, firms were labelled RATED if their debt was rated during that year, and labelled UNRATED if their debt was not rated. Again, this classification was verified annually in order for firms to “jump” constraints status during the sample period.

Scheme 5: KZ index
Following Lamont et al. (2001) and Almeida et al. (2004), we construct a linear index of firm financial constraints that can be written as follows:

\[
KZ = -1.002 \times CF + 0.283 \times Q + 3.139 \times Leverage - 39.368 \times Dividends - 1.315 \times Cash \ holdings.
\]

This index is believed to be positively related to the level of financial constraints according to a factoring analysis as described in the Kaplan and Zangales (1997) study. We rank firms annually according to this index and label firms in the bottom (top) three deciles of the distribution as LOW KZ (HIGH KZ).
Table 2 summarizes the number of firm-years for each classification scheme and the association between the schemes in terms of percentage overlap between the schemes. For instance, the first column indicates that 8,634 firm-years were classified as having a LOW PAYOUT-RATIO. Of these 8,634 observations, 38.8% were labelled SMALL according to the second classification scheme, 34.2% were labelled LOW TANG according to the third classification scheme, 80.8% were UNRATED and 32.2% had a HIGH KZ index. As such, this table indicates how consistently different classification schemes capture observations.

In general, we see that there is a somewhat positive association between the schemes. However, this positive association is rather low for most of the schemes. For instance, of the 7,741 firm-years classified as SMALL, 32% had a LOW TANG RATIO. However, an almost equal share of 27.3% actually received a classification HIGH TANG RATIO. This means that a substantial share of observations defined as constrained according to one splitting scheme, received a classification as being unconstrained according to a second splitting scheme. This is not an exception as can be seen by going through the table. For instance, of the 9,652 observations with a HIGH PAYOUT RATIO, 57.5% are labelled UNRATED according to the debt rating scheme. This means that the majority of observations classified unconstrained by the first classification scheme, were classified constrained by the fourth classification scheme.

Generally, the schemes payout, size and debt rating tend to be most positively correlated, whereas the tangibility ratio and the KZ index produce less clear positive correlations. But even when the schemes are positively associated, overlaps between the schemes rarely exceed 50%, which would mean at least half of the observations would receive the same ex-ante classification according to different schemes.

We believe that Table 2 indicates a potential problem with the ex-ante sample splitting framework, namely the inability to capture the same observations. The schemes are not able to identify undisputably those firms that are more likely
Table 2: **Association across ex-ante splitting schemes.** This table presents number of firm-years and association across the different ex-ante sample splitting schemes. The diagonal elements represent the number of firm-years within each sub-sample of the various splitting schemes. This number of firm-years is further split up in the various splitting schemes in the off-diagonal elements.

<table>
<thead>
<tr>
<th></th>
<th>Payout ratio</th>
<th>Firm size</th>
<th>Tangibility ratio</th>
<th>Debt rating</th>
<th>KZ index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Unrated</td>
</tr>
<tr>
<td>Payout ratio</td>
<td>Low</td>
<td>8,634</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>9,652</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm size</td>
<td>Low</td>
<td>38.8%</td>
<td>15.5%</td>
<td>7,741</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>48.2%</td>
<td></td>
<td>7,797</td>
<td></td>
</tr>
<tr>
<td>Tangibility ratio</td>
<td>Low</td>
<td>34.2%</td>
<td>21.6%</td>
<td>32.0%</td>
<td>23.0%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>26.0%</td>
<td>37.1%</td>
<td>27.3%</td>
<td>38.9%</td>
</tr>
<tr>
<td>Debt rating</td>
<td>Unrated</td>
<td>80.8%</td>
<td>57.5%</td>
<td>99.8%</td>
<td>25.1%</td>
</tr>
<tr>
<td></td>
<td>Rated</td>
<td>19.2%</td>
<td>42.5%</td>
<td>0.2%</td>
<td>74.9%</td>
</tr>
<tr>
<td>KZ Index</td>
<td>Low</td>
<td>32.2%</td>
<td>21.9%</td>
<td>27.5%</td>
<td>22.3%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>27.0%</td>
<td>30.7%</td>
<td>32.1%</td>
<td>31.5%</td>
</tr>
</tbody>
</table>

Financial Constraints and Investment: An Alternative Empirical Framework
Table 3: **Pooled estimation results.** This table reports GMM estimates of regression equation (7) for the various sub-samples. T-statistics are given in parenthesis and *, ** and *** indicate significance at the 10%, 5% and 1% significance level. The $\chi^2$-statistic gives the joint significance of the model.

<table>
<thead>
<tr>
<th>Dep var: INV/K</th>
<th>Independent variables</th>
<th>ex-ante splitting scheme</th>
<th>CF/K</th>
<th>MB</th>
<th>$\chi^2$-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CF/K</td>
<td>MB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. payout-ratio</td>
<td></td>
<td><strong>Low</strong></td>
<td>0.07</td>
<td>0.007</td>
<td>57.46***</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>High</strong></td>
<td>0.23</td>
<td>0.005</td>
<td>85.91***</td>
</tr>
<tr>
<td>2. size</td>
<td></td>
<td><strong>Small</strong></td>
<td>0.06</td>
<td>0.009</td>
<td>173.18***</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Large</strong></td>
<td>0.15</td>
<td>0.006</td>
<td>745.35***</td>
</tr>
<tr>
<td>3. tangibility-ratio</td>
<td></td>
<td><strong>Low</strong></td>
<td>0.11</td>
<td>0.004</td>
<td>272.61***</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>High</strong></td>
<td>0.1</td>
<td>0.007</td>
<td>338.05***</td>
</tr>
<tr>
<td>4. debrating</td>
<td></td>
<td><strong>Unrated</strong></td>
<td>0.16</td>
<td>0.003</td>
<td>251.91***</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Rated</strong></td>
<td>0.11</td>
<td>0.009</td>
<td>890.76***</td>
</tr>
<tr>
<td>5. KZ index</td>
<td></td>
<td><strong>High</strong></td>
<td>0.05</td>
<td>0.004</td>
<td>94.69***</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Low</strong></td>
<td>0.09</td>
<td>0.004</td>
<td>191.94***</td>
</tr>
</tbody>
</table>
outlays for most firms. However, in terms of magnitude, the CFSI estimates differ largely across different splitting schemes. For three classification schemes (payout, size, KZ index), a higher CFSI is found in the unconstrained sub-sample. HIGH PAYOUT, LARGE and LOW KZ observations have an estimated sensitivity of 0.23, 0.15 and 0.09 respectively. Their constrained counterparts (LOW PAYOUT, SMALL, and HIGH KZ) have a lower CFSI with values 0.07, 0.06 and 0.05, respectively. These results support the Kaplan and Zingales (1997), Cleary (1999), and Kapadakkam (1998) results, in the sense that ex-ante defined constrained firms exhibit lower investment-cash flow sensitivities.

However, using the debt rating and the tangibility scheme, we find opposite results. The LOW TANGIBILITY and UNRATED observations have a higher estimated CFSI with values 0.11 and 0.16, respectively. These results are in line with Fazzari et al. (1988), Carpenter and Petersen (2002), among others who find that investment-cash flow sensitivities are higher in firms ex-ante defined as being constrained. As we have argued before, all this is not very surprising given the low overlap between the different schemes.

5.3 Firm-heterogeneity using GME

In Table 4 we report the firm-specific sensitivities estimated by the GME procedure described in equations (9)- (13). The mean sensitivity is 0.19, which comes from a wide variation in firm-specific estimates ranging from a minimum of -0.3 to a maximum of 1.3. These are consistent with estimates for the full samples provided by previous studies. Some 13% of the firms show a negative sensitivity over the observed sample period, which means positive investments despite negative cash flows, or vice versa, negative investments (divestitures) despite positive cash flows. The kernel density graph indicates that the vast majority of the firms have a positive CFSI with a peak around 0.20. However, closer inspection reveals that there is also a small bump in the density function around 0.6 indicating some firms have much higher CFSI estimates.
Table 4: GME estimation results. This table reports mean, median, minimum, maximum and deciles of the distribution of firm-specific CFSI estimates estimated with the GME estimator described in equations (9)- (13). The table includes the graph of the kernel density function of the estimated distribution. In this graph, the vertical axis represents the percentage of firms and the horizontal axis represents the CFSI.

We have identified that an additional benefit of GME is its potential in dealing with ill-conditioned and ill-posed inference problems. As a result, GME would produce more accurate and consistent estimation results by reducing the potential bias from violation of traditional CLRM assumptions. Figure 2 contrasts the traditional GMM point estimator with the GME distribution for the different sub-samples resulting from the different ex-ante sample splitting schemes. This gives us insight into the nature of the bias of an aggregate GMM estimate.

In general, the GMM estimator seems to be influenced to a certain extent by the observations in the tails of the distribution. For example, the kernel distribution of high-payout firms is skewed to the right, indicating a small number of firm-years with rather high investment-cash flow sensitivities. As a result, the GMM estimate (0.23) lies very much at the right-hand side of the kernel density. This finding is observable in most sub-samples and is most noticeable in the last graph of Figure 2 (KZ index). As can be seen, the GME distribution of high KZ-observations lies further to the right than the distribution of low KZ-observations. However, the GMM estimate for high KZ-observations (0.05)
is actually lower than the GMM estimate for the low KZ-observations (0.09). Again, this can be attributed to the fat right tail in the skewed kernel density function for low KZ-observations.
These findings suggest that a few extreme observations might have a disproportionately large effect on an aggregate sensitivity, which is consistent with the findings of Allayannis and Mozumdar (2004). The econometrical rationale behind this finding has been reported by Pesaran and Smith (1994) who argue that pooled regression estimates can be highly misleading when estimating relationships in heterogeneous panels. The GME estimator does not suffer from this potential bias, because slope heterogeneity is allowed for.

5.4 Sensitivity to alternative priors

In the previous section, we described a GCE algorithm for investigating the responsiveness of the estimation results to the use of alternative prior information. The idea behind the algorithm is to estimate a posterior mean sensitivity for n different support values drawn i.i.d. from a prior distribution of choice. Table 5 presents the posterior mean for \( n = 100 \) draws from two uniform and a normal prior distribution. The uniform prior distribution \( U(-10,10) \) represents an uninformed researcher who has no prior knowledge of the CFSI parameter. The uniform prior \( U(-3,3) \) represents an equally uninformed researcher, although he/she has a better clue about the range of the CFSI parameter. The normal prior \( N(0.5, 0.3) \) represents an informed researcher who expects the CFSI parameter to lie somewhere around 0.5 with a reasonable standard deviation.

Table 5 shows that the mean estimated CFSI is not sensitive to the use of alternative information. The sensitivity averaged over the different draws is 0.21, 0.21 and 0.19 for the \( U(-10,10), U(-3,3) \) and \( N(0.5,0.3) \) distribution respectively. Additionally, for each prior distribution, the standard deviation of the mean sensitivities is very low (0.0001), indicating a very small variance over the 100 draws. This means that even if we start from very uninformative prior beliefs, the estimation results will not differ much. In other words, no matter from which prior information a researcher starts, he/she will reach similar conclusions using
<table>
<thead>
<tr>
<th>prior distribution</th>
<th>posterior mean CFSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform (-10, 10)</td>
<td>mean 0.21, std. dev. 0.0001</td>
</tr>
<tr>
<td>Uniform (-3, 3)</td>
<td>mean 0.21, std. dev. 0.0001</td>
</tr>
<tr>
<td>Normal (-0.5, 0.3)</td>
<td>mean 0.20, std. dev. 0.0001</td>
</tr>
</tbody>
</table>

Table 5: Sensitivity Analysis. This table analyzes the responsiveness of the results to the use of alternative prior information using the Quasi-Bayesian sensitivity framework discussed in section three. The mean and standard deviation of the posterior mean CFSI using $n=100$ draws from three different distributions as prior information are presented. For each distribution the 100 CFSI estimates were estimated using the GCE-algorithm described in equations (15)-(18).

As indicated previously, this finding can be attributed to the large sample, in which the data tend to dominate the prior information.
6 Conclusions and discussions

This paper uses recent advances in econometrics to address a key problem in the empirical literature on financial constraints. All existing studies compare aggregate investment-cash flow sensitivities between ex-ante defined sub-samples using standard regression techniques. Such an ex-ante sample splitting framework, however, comes with a number of limitations. Firstly, the results are only as strong as the splitting scheme used to reflect different degrees in capital market imperfections, which is subjective in nature. We have demonstrated that some “popular” splitting schemes do not necessarily capture the same firm-year observations and classification overlaps across various schemes are rather low. As a result, the estimates are sensitive to the splitting scheme that is used. Using five schemes that have been used extensively in the literature, we find that two schemes reflect the view of Fazzari et al. (1988), and three schemes contradict this view. Secondly, a point estimate in a certain sub-sample summarizes all available firm-year information of that sub-sample into a single aggregate coefficient, thereby ignoring firm heterogeneity. As a result, much relevant firm-specific information is ignored and statistically neutralized (Hansen, 2004). Finally, an aggregate coefficient might be influenced heavily by a few influential outliers as indicated by Allayannis and Mozumdar (2004) and Pesaran and Smith (1995).

These limitations pose difficulties for the traditional econometrical operationalization of estimating regressions in ex-ante defined sub-samples. Our approach allows for firm-specific sensitivities by introducing slope heterogeneity into the regression equation. Consequently, ex-ante sample splitting is no longer necessary, and the usefulness of the CFSI metric can be evaluated directly. We show that the GME estimator developed by Golan et al. (1996) is very much suited for introducing slope heterogeneity. We show the calculations for a large unbalanced sample of US listed firms and find a wide sensitivity ranging from -0.3 through 1.3.
Moreover, allowing for firm differences enables us to study the entire distribution of firm-specific sensitivities, which provides insights into the nature of the bias of an aggregate point estimate. We find that GMM point estimates tend to be biased substantially by a few influential outliers. An additional benefit of the GME estimator is that it does not require any assumptions of the error term as the CLRM does. As a result, the estimation results should be more accurate and reliable because there is no bias resulting from violations of traditional assumptions.

It is not our aim in this paper to enter the debate on the relationship between investment-cash flow sensitivities and financial constraints. Rather, our focus is on the development of an econometrical operationalization that makes ex-ante sample splitting unnecessary by taking into account firm-differences. In light of this, we believe the GME estimator is an exiting new methodology that provides a number of interesting research opportunities to re-examine the evidence on financial constraints.

Firstly, as we have indicated before, the firm-specific sensitivities could be used to make a direct evaluation of the CFSI metric in capturing financial constraints. This could be done by isolating different sensitivity classes and comparing these classes in terms of financial variables that would be informative about financial constraints. Building up profiles of different sensitivity classes would provide valuable insights in the ability if the CFSI metric to capture financial constraints. This could be considered an “ex-post” evaluation of financial constraints in contrast with the ex-ante sample splitting approach.

Secondly, such an evaluative framework of financial constraints could be established for alternative metrics of financial constraints as well. Recent research has witnessed a shift in focus towards an alternative metric, namely the cash flow sensitivity of cash, believed to be positively related to financial constraints (Almeida et al. (2004); Khurana et al. (2006); Han and Qiu (2007); Lin (2007)). Like the CFSI, this metric is also estimated through regression analysis in distinct
ex-ante defined sub-samples, and therefore the objections raised in this paper apply equally well to the literature on cash-cash flow sensitivities. Future research could aim at evaluating both metrics in their ability to capture financial constraints and clear out which metric would be more suitable in different samples (metric selection).

Finally, in this literature, models differ in several dimensions such as the time-variation aspect (static versus dynamic models); investment paradigm (Q models versus neoclassical models versus accelerator models); dependent variable (investments in fixed assets versus total growth versus R&D assets) and different controls for the investment opportunities bias. These different dimensions yield many possible combinations for model building and results may be sensitive to the specification used. The literature should come to terms as to which model is best suited to measure constraints for certain populations, by objectively evaluating different models in their ability to capture financial constraints. Our approach of estimating firm-specific sensitivities and evaluating models on their discriminating power between ex post defined sensitivity classes provides interesting research opportunities in this area (model selection).

References


