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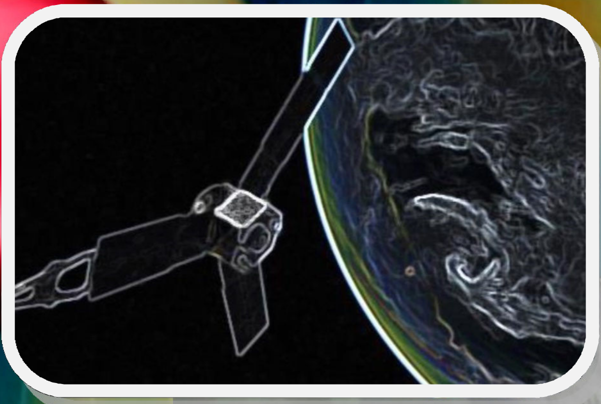

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CONTENTS

Valuation of commodity derivatives under jump-diffusion processes*

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1 Introduction

The estimation of the market prices of risk is an open question in the jump-diffusion derivative literature when a closed-form solution for the future pricing problem is not known. In this paper, we obtain some results that relate the drifts and jump intensities of the risk-neutral processes with future and spot prices. These results provide an original procedure to estimate the risk-neutral drifts and jump intensities. These functions are not observable but their estimation is necessary for pricing commodity derivatives. Moreover, this new approach avoids the estimation of the physical drift as well as the market prices of risk in order to price commodity futures. Finally, an application to NYMEX (New York Mercantile Exchange) data is illustrated.

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2 The valuation model

In this section, we present a two-factor jump-diffusion model of commodity future prices. The first factor is the spot price S, and the second factor is the instantaneous convenience yield δ. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space equipped with a filtration $\mathcal{F}$ satisfying the usual conditions, [6]. For simplicity and tractability and as usual in the literature, we assume that the distribution of the jump size under $\mathcal{Q}$ measure, risk-neutral probability measure, is known and equal to the distribution under $\mathcal{P}$ measure. That is, we assume that all risk premium related to jump is artificially absorbed by the change in the intensity of jump under the physical measure to $\lambda_S^{\mathcal{Q}}$ under risk-neutral measure, see [4]. The factors of the model are assumed to follow these joint jump-diffusion stochastic processes:

$$dS = \left(\mu_S - \sigma_S \theta^{W_S}\right) dt + \sigma_S dW_S^{\mathcal{Q}} + Y d\tilde{N}_S^{\mathcal{Q}}, \tag{1}$$

$$d\delta = \left(\mu_\delta - \sigma_\delta \theta^{W_\delta}\right) dt + \sigma_\delta dW_\delta^{\mathcal{Q}}, \tag{2}$$

where μ_S and μ_δ are the drifts under $\mathcal{P}$ measure, σ_S and σ_δ the volatilities, Y is the jump amplitude and it is a random variable which follows a normal distribution $N(0, \sigma_Y)$. Moreover, $W_S^{\mathcal{Q}}$ and $W_\delta^{\mathcal{Q}}$ are the Wiener processes. The market prices of risk of Wiener processes are $\theta^{W_S}(S, \delta)$ and $\theta^{W_\delta}(S, \delta)$ and $\tilde{N}_S^{\mathcal{Q}}$ represents the compensated Poisson process, under $\mathcal{Q}$ measure, the risk-neutral measure, with intensity $\lambda_S^{\mathcal{Q}}(S, \delta) = \lambda_S(S, \delta)\theta^{N_S}(S, \delta)$. We assume that the increments to standard Brownian motions are correlated with:

$$dW_S^{\mathcal{Q}} dW_\delta^{\mathcal{Q}} = \rho dt,$$

and $dW_S^{\mathcal{Q}}$ is assumed to be independent of $d\tilde{N}_S^{\mathcal{Q}}$, which means that the diffusion and jump components are independent of each other. We suppose that all functions satisfy enough technical regularity conditions: see [5]. Under the above assumptions, a commodity future price at time t with maturity at time T, $t \leq T$, can be expressed as $F(t, S, \delta; T)$ and at maturity is

$$F(T, S, \delta; T) = S. \tag{3}$$

We also assume that the price of a future can be expressed by

$$F(t, S, \delta; T) = E^{\mathcal{Q}}[S(T)|\mathcal{F}_t], \tag{4}$$

where $E^{\mathcal{Q}}$ denotes the conditional expectation under the $\mathcal{Q}$ measure. The future price (4) is the solution of the following partial integro-differential equation

$$\begin{aligned} F_t &+ \left(\mu_S - \sigma_S \theta^{W_S}\right) F_S + \left(\mu_\delta - \sigma_\delta \theta^{W_\delta}\right) F_\delta + \frac{1}{2}\sigma_S^2 F_{SS} + \frac{1}{2}\sigma_\delta^2 F_{\delta\delta} + \rho\sigma_S\sigma_\delta F_{S\delta} \\ &+ \lambda_S^{\mathcal{Q}} E_{Y_S}^{\mathcal{Q}}[F(t, S + J_S, \delta) - F(t, S, \delta)] = 0 \qquad (5) \end{aligned}$$

with terminal condition (3), see the version of Feynman-Kac Dynkin lemma for jump-diffusion processes in [3].

3 Exact results and approximations

In this section, we propose a new approach for estimating the functions of the risk-neutral jump-diffusion stochastic factors directly from data in the markets. Then, as we know all the coefficients of the partial integro-differential equation (5), we can price the future prices. Therefore, it is not necessary to estimate the market prices of risk for pricing commodity futures.

Theorem 1 *Let $F(t, S, \delta; T)$ be a solution to (5) subject to (3), and S follows a jump-diffusion process given by (1) and δ follows a diffusion process given by (2), then:*

$$\begin{aligned} \frac{\partial F}{\partial T}|_{T=t} &= \left(\mu_S - \sigma_S \theta^{W_S}\right)(t), \qquad (6) \\ \frac{\partial (SF)}{\partial T}|_{T=t} &= \left(2S\frac{\partial F}{\partial T}|_{T=t} + \sigma_S^2 + \lambda_S^{\mathcal{Q}} E_{Y_S}^{\mathcal{Q}}[J_S^2]\right)(t), \qquad (7) \\ \frac{\partial (\delta F)}{\partial T}|_{T=t} &= \left(\delta\frac{\partial F}{\partial T}|_{T=t} + S(\mu_\delta - \sigma_\delta \theta^{W_\delta} + \rho\sigma_S\sigma_\delta\right)(t). \qquad (8) \end{aligned}$$

Parallel results for jump-diffusion interest rate models can be found in [2].

4 Empirical application

In order to show how the approach in Section 3 can be implemented, we will price natural gas futures with data from the NYMEX. Natural gas spot and future prices were obtained from the Energy Information Administration

Table 1: Measures of error, MAE, RMSE and PRMSE, for the out of sample period of time, January 2015-April 2015.

	RMSE	PRMSE	MAE
F1	1.929×10^{-1}	6.919	1.448×10^{-1}
F6	2.567×10^{-1}	8.815	2.271×10^{-1}
F9	1.666×10^{-1}	5.375	1.290×10^{-1}
F12	1.858×10^{-1}	5.678	1.509×10^{-1}
F18	1.711×10^{-1}	5.323	1.399×10^{-1}
F24	3.218×10^{-1}	9.001	2.892×10^{-1}
F36	4.229×10^{-1}	11.261	3.865×10^{-1}
F44	1.933×10^{-1}	5.334	1.711×10^{-1}

of the U.S. Department of Energy (E.I.A. database) and Quandl platform. The sample period covers from January 1997 to April 2015. We also consider future prices with maturities equal to $1, 2, 3, 4, 6, 9, 12, 18, 24, 36$ and 44 months. We use data from January 1997 to December 2014 for estimating the risk-neutral functions and we keep data from January to April 2015 in order to evaluate the results of our approach.

As it is well known in the literature, the convenience yield is not observed in the markets. Therefore, we approximate it as in (2), as usual in the literature.

In order to analyse the behaviour of our model, we use the root mean square error (RMSE), the percentage root mean square error (PRMSE) and the mean square error (MAE) for the out of sample period of time as measures of error:

$$\begin{aligned} RMSE &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(F_t-\hat{F}_t\right)^2}, \\ PRMSE &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\frac{F_t-\hat{F}_t}{F_t}\right)^2}, \\ MAE &= \frac{1}{n}\sum_{t=1}^{n}|F_t-\hat{F}_t|, \end{aligned}$$

where n is the number of observations, F_t is the market future price, and $\hat{F}_t$

is the predicted future price by our jump-diffusion model.

Table 1 shows these three measures of error for the future prices along the out of sample period of time for different maturities. Overall, the three measures get the highest level for futures with maturities of 24 and 36 months, i.e., for long maturities.

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