Non-constant discounting and Ak-type growth models

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Abstract

This paper analyzes an Ak-type endogenous growth model under non-constant discounting, assuming both naïve and sophisticated consumers. For both type of consumers an isoelastic utility with an intertemporal elasticity below one guarantees observational equivalence under exponential and non-constant discounting, but rejects strong equivalence (identical overall impatience does not leads to identical growth rates). Further, polices aimed at increasing productivity of the economy are less growth-enhancing than typically predicted by the literature with exponential discounting.

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1 Introduction

Recent literature on non-constant discounting argues that the degree of impatience of individuals decreases with the time distance from the present, see Laibson (1997). Consumers

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are highly impatient when confronted with consumption between today or tomorrow but much more patient if the one-day delay takes place in one year from now. This idea has generated a great debate on the appropriateness of the standard hypothesis of a constant rate of time preference, versus the alternative assumption of a declining rate of time preference (see, for example, Frederick *et al.* (2002)). When consumers present a time-varying discount rate and they cannot pre-commit their future behaviour, the literature distinguishes two type of agents: naïve and sophisticated. Naïve consumers mistakenly believe that their future selves will stick to the present consumption plan, and need to revise their consumption plans at any instant in time. The optimal decisions of naïve consumers are time inconsistent, contrary to those of sophisticated agents who play a game against their future selves knowing that these will be more impatient than perceived at the current time.

Recently there has been a growing interest inn the consequences for the economic growth of moving from exponential discounting to non-constant discounting. Barro (1999) is the first author to deal with this question for a neoclassical growth model. For a log-utility function and sophisticated agents, he concludes observational equivalence between exponential discounting and quasi-hyperbolic discounting.[1] As stated recently in Farzin and Wendner (2014), this conclusion is not generally true. For a general class of hyperbolic discount functions, naïve consumers, and a short planning horizon, they prove non-equivalence when the intertemporal elasticity of substitution is different from one.

This same question is analyzed in Strulik (2015) for Ak-type endogenous growth models. Considering naïve consumers with log-utility, the author first concludes observational equivalence between hyperbolic and exponential discounting. A second finding is that the assumption of an identical overall impatience under hyperbolic and exponential discounting leads to exactly the same growth rate under both discounting methods (denoted as strong equivalence).

Focusing, like Strulik (2015), on Ak-type growth models, our first research question is whether observational equivalence and strong equivalence still hold true if sophisticated consumers are assumed instead. Secondly, inspired in the reject result obtained by Farzin and Wendner (2014) for the neoclassical growth model, we study the robustness of both the observational equivalence and the strong equivalence to changes in the utility function, specifically assuming a constant intertemporal elasticity of substitution different from one.

Our first finding is that both the observational and the strong equivalence highlighted by Strulik (2015) remain valid when consumers behave sophisticatedly under log-utility. The second main result establishes that regardless of the type of consumers, an intertemporal elasticity of substitution lower than one[2] preserves observational equivalence although it rejects strong equivalence. Thus observational equivalence is robust to changes in either the

[1]This result has been extended by Krusell *et al.* (2002), and Findley and Caliendo (2014) among others.
[2]This seems to be the empirically relevant and a generally used assumption.

type of consumers or the elasticity of substitution, although strong equivalence fades away for a non-unitary elasticity of substitution.

Even when the two discount methods are observationally equivalent, Krusell *et al.* (2002) highlight that differences in welfare properties arise. In the same line, we focus on the fact that identical policies do not have the same implications under both discounting methods. Specifically any policy aimed at increasing the productivity of the economy is less growth-enhancing than typically predicted by the literature with exponential discounting. All the results are obtained for a general discount function with a non-constant but decreasing instantaneous rate of time preference. No specific functional form is required.

2 The model

Following Strulik (2015), we analyze the endogenous growth model in Romer (1986) or any other model which can be reduced to an Ak-type endogenous growth model. The representative consumer maximizes, at each t, his lifetime utility subject to the budget constraint:

$$\max_{c_t(s)} \int_t^{\infty} u\left[c_t(s)\right]\, \theta(s-t)\, ds, \tag{1}$$

$$\text{s.t.:} \qquad \dot{k}_t(s) = rk_t(s) + w_t(s) - c_t(s), \quad k_t(t) = k_t, \tag{2}$$

where t is the current date, $j = s - t$ measures the time distance from the present and $\theta(j) \geq 0$ is the discount function which measures the time preference. Here $c_t(s)$, $k_t(s)$ and $w_t(s)$ denote consumption, capital and the wage rate. As Strulik (2015) we assume a constant interest rate r (this will be true for any Ak-type endogenous growth model).

We assume a general discount function $\theta(j)$ satisfying: $\theta(j) > 0$, $\dot{\theta}(j) < 0$, $\forall j \geq 0$ and $\theta(0) = 1$. Moreover, the instantaneous discount rate, $\rho(j) := -\dot{\theta}(j)/\theta(j)$ satisfies $\rho(j) > 0$, $\dot{\rho}(j) < 0, \forall j \geq 0$. Finally, $\lim_{t\to+\infty} \rho(j)$ can be strictly positive (quasi-hyperbolic discounting) or null (hyperbolic discounting).

We assume an isoelastic utility function $u(c)$. The resulting solutions for consumption and capital depend on the consumers' behavior, i.e. whether they behave naïvely or sophisticatedly.

2.1 Naïve consumers

Following the same reasoning as in Strulik (2015) for naïve consumers with an isoelastic utility function, and not necessarily logarithmic, the consumption at time t reads:[3]

$$c_{\mathrm{N}}(t) = \frac{k_t + \int_t^{\infty} w_t(s) e^{-r(s-t)} ds}{\int_t^{\infty} [\theta(s-t)]^{\frac{1}{\sigma}} e^{-\frac{\sigma-1}{\sigma} r(s-t)} ds}, \tag{3}$$

[3]Henceforth, subscripts N and S denote naïve and sophisticated consumers.

where $1/\sigma$ is the intertemporal elasticity of substitution, equal to 1 when a logarithmic utility function is used.

Log-differentiating (3), and taking into account (2), it follows that

$$\frac{\dot{c}_{\mathrm{N}}(t)}{c_{\mathrm{N}}(t)} = r - \frac{c_{\mathrm{N}}(t)}{k_t + \int_t^\infty w_t(s) e^{-r(s-t)} ds} - \left[\frac{-1 - \frac{1}{\sigma}\int_t^\infty \frac{\dot{\theta}(s-t)}{\theta(s-t)} [\theta(s-t)]^{\frac{1}{\sigma}} e^{-\frac{\sigma-1}{\sigma} r(s-t)} ds}{\int_t^\infty [\theta(s-t)]^{\frac{1}{\sigma}} e^{-\frac{\sigma-1}{\sigma} r(s-t)} ds} + \frac{\sigma-1}{\sigma} r \right].$$

Then, the modified Ramsey rule for naïve consumers is obtained,

$$\gamma_{\mathrm{N}} := \frac{\dot{c}_{\mathrm{N}}}{c_{\mathrm{N}}} = \frac{1}{\sigma}\left(r - \lambda_{\mathrm{N}}\right) \tag{4}$$

with

$$\lambda_{\mathrm{N}} = \int_0^\infty \rho(j)\omega_{\mathrm{N}}(j) dj, \qquad \omega_{\mathrm{N}}(j) = \frac{[\theta(j)]^{\frac{1}{\sigma}} e^{-\frac{\sigma-1}{\sigma} rj}}{\int_0^\infty [\theta(i)]^{\frac{1}{\sigma}} e^{-\frac{\sigma-1}{\sigma} ri} di} \in (0,1). \tag{5}$$

The effective rate of time preference, λ_{N}, is constant and can be interpreted as a weighted mean of the instantaneous discount rates, $\rho(j)$, with weights $\omega_{\mathrm{N}}(j)$, with $\int_0^\infty \omega_{\mathrm{N}}(j) dj = 1$.

Note that $\lambda_{\mathrm{N}} < \rho_0$ with $\rho_0 = \rho(0)$. Therefore, if $\rho_0 < r$ then γ_{N} is a positive constant. Further, since $\theta(j)$ is not exponential, expression (5) is well defined either if $\sigma > 1$ or if $\sigma = 1$ and $\int_0^\infty \theta(i) di$ is convergent. The alternative scenario with $\sigma < 1$ would require additional conditions on the discount function, $\theta(j)$, to guarantee the convergence of the integral which defines weights $\omega_{\mathrm{N}}(j)$ in (5). In what follows we restrict the analysis to the case $\sigma \geq 1$, both to guarantee convergence and because it has usually been suggested as the relevant case by the literature.

2.2 Sophisticated consumers

As proved in Barro (1999), in the absence of any commitment, the usual Ramsey rule for the growth rate of consumption is modified to

$$\gamma_{\mathrm{S}}(t) := \frac{\dot{c}_{\mathrm{S}}(t)}{c_{\mathrm{S}}(t)} = \frac{1}{\sigma}\left(r - \lambda_{\mathrm{S}}(t)\right), \tag{6}$$

where

$$\lambda_{\mathrm{S}}(t) = \int_0^\infty \rho(j)\omega_{\mathrm{S}}(t,j)\, dj, \text{ with } \omega_{\mathrm{S}}(t,j) = \frac{\theta(j) e^{-\frac{\sigma-1}{\sigma}\int_t^{t+j}(r-\lambda_{\mathrm{S}}(s))\, ds}}{\int_0^\infty \theta(i) e^{-\frac{\sigma-1}{\sigma}\int_t^{t+i}(r-\lambda_{\mathrm{S}}(s))\, ds} di} \in (0,1). \tag{7}$$

The function $\lambda_{\mathrm{S}}(t) > 0$ can again be interpreted as a weighted mean of the instantaneous discount rates, $\rho(j)$, with weights $\omega_{\mathrm{S}}(t,j)$. Contrary to the case with naïve consumers, equation (7) now defines $\lambda_{\mathrm{S}}(t)$ implicitly. This expression is well defined for $\sigma \geq 1$ under the same conditions as in the case with naïve consumers. Again the case $\sigma < 1$ would require additional conditions, similar to the case with naïve consumers.

Remark 1 *For a log-utility function ($\sigma = 1$), we have that*[4] $\lambda_S(t) = \lambda_S = \lambda_N = \left[\int_0^\infty \theta(j)\, dj\right]^{-1}$. *The effective rate of time preference is the same constant for naïve and sophisticated consumers, equal to the propensity to consume out of wealth.*

The following proposition shows that the effective rate of time preference for sophisticated consumers is also constant for $\sigma > 1$.

Proposition 2 *Under condition $\sigma > 1$ and $\rho_0 < r$ there always exists a unique positive constant λ_S which satisfies equation (7) and such that $\lambda_S(t) = \lambda_S > 0$ and $\gamma_S(t) = \gamma_S > 0$ for all $t \geq 0$.*

Proof. For $\sigma > 1$, and $\rho_0 < r$ it follows that $\lambda_{\rm S} < \rho_0$ and integrating by parts, we get:

$$\int_0^\infty \rho(j)\theta(j)e^{-\frac{\sigma-1}{\sigma}(r-\lambda_{\rm S})j}\, dj = \theta(0) - \frac{\sigma-1}{\sigma}(r-\lambda_{\rm S})\int_0^\infty \theta(j)e^{-\frac{\sigma-1}{\sigma}(r-\lambda_{\rm S})j}\, dj,$$

and then

$$\lambda_{\rm S} = \frac{\sigma}{\int_0^\infty \theta(j)e^{-\frac{\sigma-1}{\sigma}(r-\lambda_{\rm S})j} dj} - (\sigma-1)r. \tag{8}$$

The RHS in this equation can be regarded as a decreasing function of $\lambda_{\rm S}$. Since $\int_0^\infty \theta(j)e^{-\frac{\sigma-1}{\sigma}rj} dj < \sigma/(r(\sigma-1))$ the RHS runs from a positive value towards $-(\sigma-1)r < 0$. Then there always exists a unique $\lambda_{\rm S} > 0$ satisfying equation (8). Since $\lambda_{\rm S} < \rho_0 < r$, then $\gamma_{\rm S}(t) = \gamma_{\rm S} = (r-\lambda_{\rm S})/\sigma > 0$ for all $t \geq 0$. ■

Henceforth, we will consider the effective rate of time preference of sophisticated consumers as the value $\lambda_{\rm S}$ independent of time, and correspondingly, $\gamma_{\rm S}$ the constant growth rate of consumption.

Summarizing previous results and the new ones in this paper, we can say that, depending on the discount function (exponential discounting (E) or time-varying discounting) and on the naïve (N) or sophisticated (S) behaviour of consumers, the growth rate for consumption and the dynamics of the capital arising from problem (1)-(2) are given by:

$$\gamma_i = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(r - d_i), \text{ with } d_i = \begin{cases} \bar{\rho} & \text{if } i = E \\ \lambda_{\rm N} & \text{if } i = N \\ \lambda_{\rm S} & \text{if } i = S \end{cases} \tag{9}$$

$$\dot{k}(t) = Ak(t) - c(t) - \delta k(t), \quad k(0) = k_0 \tag{10}$$

where d_i is the effective rate of time preference for each model $i \in \{E, N, S\}$ and $\delta \geq 0$ is the depreciation rate of capital.

Equation (10) arises from (2) once the expressions of r and $w(t)$ are replaced. In the model by Romer (1986), $r = \alpha A - \delta$ and $w(t) = (1-\alpha)Ak(t)$, where α and $1-\alpha$ are the

[4] This is immediately obvious by integrating expressions (5) and (7), since we are considering $\lim_{j\to\infty} \theta(j) = 0$, in the scenario with $\sigma = 1$.

capital and labor elasticities of a Cobb-Douglas production function with learning-by-doing externalities and a unit-mass population. A similar equation would be obtained for other Ak-type endogenous growth models.

The system of two differential equations (9)-(10) has a unique unstable balanced path (there is no transitional dynamics). Along the balanced path consumption is proportional to capital and both grow at the constant rate γ_i, $i \in \{E, N, S\}$.

It is worth highlighting that under non-constant discounting, removing the hypothesis of log-utility implies that the effective rate of time preference becomes dependent on the rate of return of the economy (both for naïve and sophisticated consumers). This is in contrast to the case with constant discounting. The next proposition shows a positive relationship. In consequence a policy aimed to increase the productivity of the economy does not have such a strong effect on the growth rate of the economy as in the solution under exponential discounting.

Proposition 3 *Under assumption*[5] $\sigma > 1$ *it follows that* $d\lambda_N/dr > 0$ *and* $0 < d\lambda_S/dr < 1$. *Consequently,* $0 < d\gamma_S/dr < d\gamma_E/dr$ *and* $d\gamma_N/dr < d\gamma_E/dr$.

Proof. See the appendix. ■

3 Observational equivalence versus strong equivalence

A non-constant discounting method is said to be *observationally equivalent* to exponential discounting if for every non-constant discount function there exists an exponential discount function, so that the observed consumption paths are the same under both discounting methods (see, for example, Farzin and Wendner (2014)).

Moreover, Strulik (2015) states that non-constant discounting and exponential discounting are *strongly equivalent* if the constant discount rate $\bar{\rho}$ which guarantees identical consumption paths also ensures that both discounting methods are controlled to show identical overall impatience. That is:

$$\int_0^\infty e^{-\bar{\rho}j}\, dj = \int_0^\infty \theta(j)\, dj, \quad \Longleftrightarrow \quad \bar{\rho} = \left[\int_0^\infty \theta(j)\, dj\right]^{-1}. \tag{11}$$

In the next proposition we prove that the results on observational and strong equivalence obtained by Strulik (2015) are valid regardless of whether consumers are or are not time-consistent.

[5]The alternative assumption $\sigma < 1$ can be analyzed under the conditions on the discount function which guarantee convergence. Following the same reasoning as in the proof of this Proposition, it follows that $d\gamma_{\mathrm{N}}/dr > d\gamma_{\mathrm{E}}/dr > 0$. Thus for naïve consumers a policy that aims to increase productivity is less or more growth-enhancing than in the case of exponential discounting, depending on whether σ is greater or lower than one. However, for sophisticated consumers nothing can be said about the comparison between $d\gamma_{\mathrm{S}}/dr$ and $d\gamma_{\mathrm{E}}/dr$.

Proposition 4 *For sophisticated consumers with log-utility, non-constant discounting is observationally and strongly equivalent to exponential discounting.*

Proof. From Remark 1 λ_{S} is a positive constant. Then from equation (9) observationally equivalence is proved by setting $\bar{\rho} = \lambda_{\text{S}}$. Moreover, when $\sigma = 1$, $\lambda_{\text{S}} = \left[\int_0^\infty \theta(j)\, dj\right]^{-1}$ and strong equivalence follows. ■

Moving away from the log-utility specification, the results on observational and strong equivalence are analyzed next.

Proposition 5 *For an isoelastic utility with $\sigma > 1$, non-constant discounting is observationally equivalent to exponential discounting regardless of whether consumers are naïve or sophisticated.*

Proof. Straightforward from equation (9) taking into account that λ_{N} and λ_{S} are constant. ■

Ak-type endogenous growth models show no transitional dynamics, the rate of return and the effective rate of time preference remain always constant. Thus, contrary to the finding by Farzin and Wendner (2014) for the neoclassical growth model, the observational equivalence is robust to changes in the intertemporal elasticity of substitution. This result holds true for naïve and sophisticated consumers.

Proposition 6 *For an isoelastic utility with $\sigma > 1$ non-constant discounting is not strongly equivalent to exponential discounting regardless of whether consumers are naïve or sophisticated.*

Proof. The strong equivalence condition for either naïve or sophisticated consumers can be stated as $((11) \Rightarrow \gamma_i = \gamma_E,\ i \in \{N, S\})$. From (5) and (7) this condition can be rewritten as:

$$\left[\int_0^\infty \theta(j)\, dj\right]^{-1} = \lambda_i,\ i \in \{N, S\}. \tag{12}$$

From Proposition 3, λ_{N} and λ_{S} are strictly monotonous increasing functions in r. Therefore, condition (12) is never satisfied except for a specific value of r. ■

When we move apart from log-utility to an intertemporal constant elasticity of substitution lower than one observational equivalence is preserved. However, the strong equivalence result does not hold true in general.

4 Conclusions

The paper analyzes to what extent the conclusions of an Ak-type endogenous growth model with a constant discount rate can be generalized to the case of non-constant discounting.

When checking for *observational and strong equivalence*, we find that it is relatively unimportant whether individuals are naïve or sophisticated. What matters most is consumers' intertemporal elasticity of substitution. Under log-utility, substitution and income effects exactly cancel out, and the propensity to consume out of wealth is independent of the rate of return of the economy. At any moment in time, when individuals settle the ratio of consumption per unit of wealth, they are only concerned about their time preferences from the current time on. Since non-constant discounting depends only on the time distance from the present then, as stated in Remark 1, the propensity to consume out of wealth is the constant defined by the inverse of the overall impatience. It is, then, always possible to find a constant discount rate at which exponential discounting shows the same propensity to consume out of wealth than non-constant discounting. Therefore the two scenarios would show identical consumption paths (*observational equivalence*), and identical overall impatience (*strong equivalence*).

When the intertemporal elasticity of substitution is different from one (here assumed to be less than one), the sensitivity of future consumption to current wealth depends on the way individuals discount the future, but also on the rate of return of the economy. At any given time, consumption per unit of wealth does not depend on the overall impatience, but on the interaction of future values of the instantaneous discount rate with the constant rate of return. Since this rate of return is constant, there also exists a constant rate of discount that equates the propensity to consume out of wealth under the two scenarios (exponential and non-constant discounting). This rate guarantees identical consumption paths and hence *observational equivalence*. However, this constant discount rate is typically different from the rate that leads to identical overall impatience, so rejecting *strong equivalence*.

We also analyze the effect of a policy aimed to boost total factor productivity when consumers show a declining rate of time preference, and the intertemporal elasticity of substitution is less than one. When the income effect dominates the substitution effect, a policy that raises the rate of return also increases the effective rate of time preference (either for naïve or sophisticated consumers). In consequence, its impact is less growth-enhancing than predicted by the standard literature.

Appendix

Proof of Proposition 3

Proof. From equation (5) it follows that:

$$\frac{d\lambda_{\mathrm{N}}}{dr} = -\frac{\sigma - 1}{\sigma} \int_0^\infty \rho(j)\omega_{\mathrm{N}}(j)[j - \mu_{\mathrm{N}}]\, dj, \quad \text{with} \quad \mu_{\mathrm{N}} = \int_0^\infty j\omega_{\mathrm{N}}(j)\, dj \in (0, \infty).$$

Weights $\omega_{\mathrm{N}}(j)$ can be interpreted as a probability density function with mean μ_{N}. Moreover, since $\rho(j) > 0$ is a strictly decreasing function, it immediately follows that: $\rho(j) >$

$\rho(\mu_{\rm N})$, $\forall j \in [0, \mu_{\rm N})$, and $\rho(j) < \rho(\mu_{\rm N})$, $\forall j \in (\mu_{\rm N}, +\infty)$, then

$$\int_0^\infty \rho(j)\omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj = \int_0^{\mu_{\rm N}} \rho(j)\omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj + \int_{\mu_{\rm N}}^\infty \rho(j)\omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj$$

$$< \quad \rho(\mu_{\rm N})\int_0^{\mu_{\rm N}} \omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj + \rho(\mu_{\rm N})\int_{\mu_{\rm N}}^\infty \omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj = \rho(\mu_{\rm N})\int_0^\infty \omega_{\rm N}(j)[j-\mu_{\rm N}]\,dj = 0.$$

The last integral is null because it is the sum of the deviations from the mean weighted by the probability density function $\omega_{\rm N}(j)$. Since $\sigma > 1$, it immediately follows that $d\lambda_{\rm N}/dr > 0$ and $d\gamma_{\rm N}/dr < d\gamma_{\rm E}/dr$.

Implicit differentiation of (8) leads to:

$$\frac{d\lambda_{\rm S}}{dr} = \frac{\int_0^\infty \rho(j)\frac{d\omega_{\rm S}}{dr}(j)\,dj}{1+\int_0^\infty \rho(j)\frac{d\omega_{\rm S}}{dr}(j)\,dj} = \frac{(1-\sigma)\int_0^\infty \rho(j)\omega_{\rm S}(j)[j-\mu_{\rm S}]dj}{\sigma + (1-\sigma)\int_0^\infty \rho(j)\omega_{\rm S}(j)[j-\mu_{\rm S}]\,dj},$$

with $\mu_{\rm S} = \int_0^\infty j\omega_{\rm S}(j)dj \in (0, \infty)$.

Following the reasoning above, since $\sigma > 1$, it follows that $0 < d\lambda_{\rm S}/dr < 1$ and $0 < d\gamma_{\rm S}/dr < d\gamma_{\rm E}/dr$. ■

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