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$SU(2)$, Associated Laguerre Polynomials and Rigged Hilbert Spaces

Enrico Celeghini, Manuel Gadella and Mariano A del Olmo

Abstract We present a family of unitary irreducible representations of $SU(2)$ realized in the plane, in terms of the Laguerre polynomials. These functions are similar to the spherical harmonics defined on the sphere. Relations with an space of square integrable functions defined on the plane, $L^2(\mathrm{R}^2)$, are analyzed. We have also enlarged this study using rigged Hilbert spaces that allow to work with iscrete and continuous bases like is the case here.

1 Introduction

The representations of a Lie algebra are usually considered as ancillary to the algebra and developed starting from the algebra, i.e. from the generators and their commutation relations. The universal enveloping algebra (UEA) is constructed and a complete set of commuting observables selected, choosing between the invariant operators of the algebra and of a chain of its subalgebras. The common eigenvectors of this complete set of operators are a basis of a vector space where the Lie algebra generators are realized as operators.

We propose here an alternative construction that allows to add to the representations obtained following the reported recipe, new ones not achievable following the previous approach. Starting from a concrete vector space

E. Celeghini
Dpto di Fisica, Università di Firenze and INFN–Sezione di Firenze, Firenze, Italy,
Dpto. de Física Teórica, Universidad de Valladolid, E-47005, Valladolid, Spain
e-mail: celeghini@fi.infn.it

M. Gadella and M.A. del Olmo
Dpto de Física Teórica and IMUVA, Univ. de Valladolid, E-47005, Valladolid, Spain
e-mail: manuelgadella1@gmail.com , e-mail: marianoantonio.olmo@uva.es

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of functions with discrete labels and continuous variables, we consider the recurrence relations that allow to connect functions with different values of the labels. These recurrence relations are not operators but allow us to introduce, for each label and for each continuous variable, an operator that reads its value. In this way, recurrence relations are rewritten in terms of rising and lowering operators built by means of the above defined operators.These rising and lowering operators are often genuine generators of the Lie algebras considered by Miller [1] and the procedure gives simply the representations of the algebras in a well defined function space [2, 3]. However it can happen that the commutators, besides the values required by the algebra, have additional contributions. The essential point of this paper is that these additional contributions (as exhibited here) can be proportional to the null identity that defines the starting vector space. As this identity is zero on the whole representation, the Lie algebra is well defined and a new representation in a space of functions has been found.

We do not discuss here the general approach, but we limit ourselves to a simple example where all aspects are better understandable. We start thus from the associated Laguerre functions (ALF) and, following the proposed construction, we realize the algebra $su(2)$ in terms of the appropriate rising and lowering operators. The ALF support in reality a larger algebra [4] but we prefer to consider here only the subalgebra $su(2)$. The reasons for this choice are twofold: first in this way the technicalities are reduced at the minimum and second it has been very nice for us to discover that not all representations of a so elementary group like $SU(2)$ where known.

As discussed in [5, 6, 7] the presence of operators with spectrum of different cardinality implies that, as considered for the first time in Lie algebras in [8], the space of the group representation is not a Hilbert space but a rigged Hilbert space (RHS) [9]. Thus, we introduce the above setting within the context of RHS since the RHS is the perfect framework where discrete and continuous bases coexist. In addition, the same RHS serves as a support for a representation on it of a Lie algebra as continuous operators as well as for its UEA. Therefore, the connection between discrete and continuous bases and Lie algebras with RHS is well established.

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