Universidad de Valladolid

Escuela de Doctorado Universidad de Valladolid

PROGRAMA DE DOCTORADO EN ECONOMÍA

TESIS DOCTORAL:

New Estimation Techniques in Commodity Derivative Models under the Risk-neutral Measure

Nuevas Técnicas de Estimación en Modelos de Derivados de Materias Primas bajo la Medida Neutral al Riesgo

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Doctora por la Universidad de Valladolid

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Acknowledgements

First and foremost, I want to thank my advisors, Dr. Lourdes Gómez del Valle and Dr. Julia Martínez Rodríguez. It has been an honor to be their Ph.D. student. Their guidance helped me in all the time of research and writing of this thesis. I could not have imagined having better advisors and mentors for my Ph.D. study.

I would like to express my special appreciation and thanks to Dr. José Luis García Lapresta, you have been a tremendous mentor for me. I would like to thank you for encouraging my research and for allowing me to grow as a research scientist. Your advice on my career has been priceless.

I would especially like to thank Dr. Pierre Six at Neoma Business School, Rouen, France who provided me an opportunity to join his team as a researcher, and who gave me access to the research facilities. Without his precious support, it would not be possible to conduct this research. I met amazing people in Rouen. Thanks to everyone, especially Jungsun Cho, Peiran Guo, and Lei Yin at the Neoma Business School who supported me through good and bad times. Thanks to professors from Neoma and their organization who worked with me over those three months. I appreciate all their contributions of time and ideas to make my Ph.D. experience productive and stimulating. The joy and enthusiasm they have for their research was contagious and motivational for me, even during tough times in the Ph.D. pursuit.

Special thanks to all professors in the Faculty of Economics and Business at UVa have contributed immensely to my personal and professional time. This group has been a source of friendships as well as good advice and collaboration.

Nobody has been more important to me in the pursuit of this project than the members of my family. I would like to thank them for all their love and encouragement. For my parents who raised me with a love of science and supported me in all my pursuits. Words cannot express how grateful I am to my mother, father and my brother for all of the sacrifices that they have made on my behalf. This work would not have been possible without the financial support of my parents.

A special acknowledgment goes to my office mate of many years: Veronica Arredondo. She was a real friend ever since we began to share an office in 2015. Veronica is an amazing person in too many ways. I also thank Raquel González, my other great office mate who has been supportive in every way.

I would like to express my very great appreciation to my friends. Their unconditional support has been essential all these years. I am not sure I would continue at UVa without their encouragement. I must express my very profound gratitude to my Spanish friends and families which our friendship began in Valladolid and became ripe throughout my years at UVa. Thank you for providing me with unfailing support and constructive recommendation throughout my years of study. This accomplishment would not have been possible without you.

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University of Valladolid
April 2018

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Chapter 1

Introduction

Commodity markets are very important for different industries. On the one hand, they are fundamental to production companies which look for hedging unwanted commodity exposure. On the other hand, they are very interesting for investors who use commodities as investments, Back and Prokopczuk [4]. Moreover, the importance of commodity markets has increased considerably since the start of this century. As a result, they are of great interest for researchers.

Recent developments in commodity prices have been atypical in many ways. From the mid-2000s a rising trend of commodity prices was observed in the markets. In fact, the boom between 2002 and mid-2008 was the most important one in several decades, both in magnitude and duration. After the beginning of the current global crisis, the prices started to decline and this fact affected a large number of commodities. However, on the whole, since 2009, and especially since the summer of 2010, global commodity prices started to increase again. During this period of time, commodity prices also had a high volatility.

These developments also coincide with important changes in commodity market basics. For example, changes in fundamental supply and demand relationships, especially in emerging economies which are experiencing a fast growth. In addition, the aim to reduce the use of fossil fuels in energy consumption, the debate about global climate change and its link with agricultural production has had an important impact on recent commodity price evolution. However, these factors are not enough to explain the behaviour of commodity prices. In fact, the greater presence of financial investors in these markets looking to diversify their portfolios beyond traditional securities, is an additional factor. This phenomenon is called financialization of commodity markets and its importance has increased considerably since 2004, as can be perceived by the rising volume of financial investments in commodity derivative markets. This situation is important because the activity of these financial participants can move commodity prices away from the levels fixed by supply and demand relationships. Financial investors make trading decisions based on factors not totally related to the corresponding commodity, such as portfolio considerations or following the market trend. Moreover, this fact can have negative effects on both the consumers and producers of commodities.

1.1 Commodity markets

Trading in commodity spot markets is quite limited because most market participants look for a financial exposure to the movements of the underlying commodity price instead of the underlying commodity itself. Therefore, as Back and Prokopczuk [4] state: "*trading and price discovery take place in the futures markets*".

Commodity derivatives are traded on organized exchanges as well as on over the counter (OTC), usually with a financial institution. Futures exchanges where trade standardized clearly defined products, offer higher liquidity, price transparency, and reduce default risk. However, the variety of these exchange-traded standardized contracts is quite limited and they do not always provide a perfect hedge. In the OTC markets, traders can go beyond standardized futures products and customize them in terms of the contracts they trade. Although they offer great freedom and potentially lower trading costs, these markets may leave both parties at risk, if they are not using the services of a clearing house. In fact, currently, OTC markets are still rather opaque in all parts of the world.

In the US, the most popular exchanges are those run by the CME Group, which originated after the Chicago Mercantile Exchange (CME) and Chicago Board of Trade (CBT) merged in 2006. The New York Mercantile Exchange (NYMEX) is among its operations.

Nowadays, tradable commodities are divided into four categories:

- Metals. They are probably the most traded commodities. They can also be divided into various categories, for instance, precious metals (gold, silver, etc.), base metals (copper, aluminum, etc.), carbon, steel and so on.
- Energy. It is one of the most influential commodities. It includes crude oil, natural gas, coal and so on.
- Agricultural. These are goods such as cotton, cocoa, sugar, orange juice, etc.
- Livestock and meat. These commodities include live cattle, feeder cattle and lean hogs and they are more volatile than other traded commodities.

A well-known way to invest in commodities is through a futures contract, which is an agreement between two parties to buy (or sell) a specific quantity of a commodity at a fixed specific price and at a predetermined future date on an organized exchange. Another way is through option contracts. Options provide greater flexibility to market participants because they are designed to offer the right (not the obligation) to buy (or sell) a specific commodity or contract at a predetermined specific price and at a given date in the future, see Cummins and Murphy [26]. In financial commodity markets, there are mainly two types of investors:

- Hedgers buy and sell contracts to protect themselves from possible commodity price movements. They try to avoid risks.
- Speculators try to make some profit analyzing the commodity markets, forecasting derivative price movements and trading.

1.2 Spot and futures prices

In commodity markets, it is interesting to know the term structure of commodity futures prices, which is the relationship between the futures prices and the maturity for any delivery rate Lautier [54]. In fact, changes in the slope of the commodity futures curves have very important consequences for investment decisions and risk management and have been the focus of numerous studies, see Acharya et al. [1], Bessembinder and Lemmon [13], and Geman and Ohana [35].

A futures curve is upward sloping when futures prices are higher than the spot price and this situation is called contango. On the contrary, when futures prices are below the current spot price the futures curve is downward sloping and this situation is called backwardation. The futures curve can also show a humped shape. Moreover, the shape of the term structure of futures prices changes over time, see for example Hambur and Stenner [42].

The shape of the term structure of the commodity futures curve has been traditionally explained by two broad strands: the theory of storage, which started with Brennan [16], and Kaldor [50] among others, and the hedging pressure literature, which was pioneered by Hicks [45], and Keynes [51].

The theory of storage focuses on the overall benefits of holding the physical commodity and some aspects related to inventories. Owning a commodity provides some benefits, but also some costs. In consequence, the (net) convenience yield is defined as the difference between the gross convenience yield and the costs of holding the physical commodity, such as storage, transportation and so on. According to this theory, there is a relation between the spot and futures prices. If the benefits of holding the commodity (convenience yield) are higher than the financial costs (interest rates), the futures curve is in backwardation. However, if the interest rate is higher than the convenience yield, the futures curve is in contango, see Back and Prokopczuk [4] for more detail.

Moreover, holding the physical commodity allows producers to fulfill unexpected changes in demand, avoid temporary shortages of supply and the insurance of the production process, see Kwok [52]. Then, following supply and demand arguments, there is a negative relationship between inventory levels and price, see Back and Prokopczuk [4]. That is, if inventory levels are high, the futures curve is in contango and if inventory levels are low the futures curve is in backwardation.

In contrast, the hedging pressure literature focuses on the risk premium. This strand considers that the futures price is the sum of the expected future spot price and risk premium. The theory of normal backwardation proposed by Keynes [51] is based on the general assumption that commodity producers usually want to enter into a short position, because they wish to guarantee a certain price level for the delivery. This contract provides a form of insurance against any decline in the spot price to the producers. Thereby, producers are willing to pay a risk premium in order to hedge their exposure. Commodity consumers may also want to protect themselves against increases in the spot price. Then, they will enter into a long position in the futures contract. If the hedging activity of producers of a particular commodity is greater than that of consumers, the futures price will be lower than the expected future spot price and the futures price will be a downward biased estimator of future spot prices, see Back and Prokopczuk [4]. Moreover, this fact will induce speculators to balance the market taking the opposite long position. However, this assumption of hedgers usually being on average in a net short position is not always true. If the hedging activity of consumers is greater than that of producers, there will be an excess of commercial market participants looking to enter a long position, and then, the future price will be above the expected future spot price. In this case, speculators should be compensated for taking a short position in the commodity. Therefore, the futures price may carry either a positive or a negative risk premium depending on the net position of hedgers for each commodity at each moment, see Cootner [25], and Hambur and Stenner [42]. In this sense, Hambur and Stenner [42] define the risk premium as the return that speculators expect to obtain as compensation for buying or selling some commodity futures contracts. The role of risk premia for different markets have been widely analyzed, see for example Acharya et al. [1], Basu and Miffre [8], and Ronn and Wimschulte [69].

1.3 Valuation models

As far as commodity derivatives pricing is concerned, several models have arisen over recent years. First, the state variables of these models as well as their dynamics must be determined. Then, there must be a balance between a tractable and easy implement model and they should also be able to capture the commodity price properties.

The spot price is, obviously, a variable that we should consider for pricing commodity futures. Many models specify a stochastic process for the spot price dynamics and, then, arbitrage arguments are used for valuation, see Schwartz [73].

Due to the interaction of demand and supply, a mean-reverting behaviour usually arises in the commodity dynamics literature. For example, Bessembinder et al. [12] show strong evidence of mean-reversion in commodity markets. However, Brooks and Prokopczuk [18] show that mean-reversion is not supported when estimating a stochastic volatility model with jumps. In fact, in the literature, different approaches are found. Schwartz [73] considers a constant mean-reversion while Schwartz and Smith [74] assume a long-term stochastic mean.

Brennan and Schwartz [17] propose a one-factor model when the convenience yield is modelled by means of a deterministic function of the spot. However, one-factor models are not consistent with empirical observations, since they give futures prices which are perfectly correlated. Then, they seem not to be able to capture the real dynamics of futures prices. Gibson and Schwartz [36] introduce a model with a joint diffusion process for the spot price and the convenience yield.

More precisely, they consider a geometric Brownian motion for the spot price dynamics and the convenience yield is described through an Ornstein-Ulhenbeck process. As the convenience yield is unobservable in the market, Gibson and Schwartz [36] propose a proxy for this variable. Schwartz [73] considers a three-factor model with an additional factor, the interest rate, which follows a diffusion process. Later, Miltersen and Schwartz [59] also consider a three-factor model in order to price commodity futures and futures options. Recently, Schöne and Spinler [72] propose an affine diffusion model with stochastic volatility to price commodity futures and options.

Many commodity markets are characterized by exceptional abrupt changes in the prices as a consequence of failures in production or transportation, weather conditions affecting agricultural commodities, or unanticipated macroeconomic events among other reasons. In the literature, so as to take into account these abrupt changes of the commodity prices, a jump term in the stochastic process is added. For example, Hilliard and Reis [46] propose a model that allows for jumps in the spot price process. In particular, they consider that the spot price follows a jump-diffusion process. Yan [86] proposes a model for pricing commodity derivatives with jumps in the spot price and volatility. Schmitz et al. [71] assume a stochastic volatility model with jumps for agricultural commodities and its effect on option prices is analyzed. Hilliard and Hilliard [47] use a standard geometric Brownian motion augmented by jumps to describe the underlying spot and mean-reverting diffusion for the interest rate and convenience yield state variables for gold and copper prices.

Finally, it is widely shown that supply and demand of most commodities follow seasonal cycles. In particular, agricultural commodities and a vast majority of energy commodities present a seasonal pattern in their prices. On the one hand, the supply of agricultural commodities depends on the weather which affects the production. On the other hand, the demand for energy is higher during the cold season. Therefore, prices are higher during this season than during the hot season. For example, Sørensen [75] shows seasonal patterns for agricultural products such as soybean, corn, and wheat markets, and Manoliu and Tompaidis [57] and Paschke and Prokopczuk [63] find strong seasonal effects in some energy commodities such as natural gas, heating oil or gasoline. Cartea and Figueroa [20], Li et al. [55] and Lucia and Schwartz [56], consider the seasonality in electricity markets, García Mirantes et al. [34] in the natural gas markets, Kyriakou et al. [53] in petroleum commodities and Back and Prokopczuk [4] in the soybean, corn heating oil and natural gas markets. Arismendi et al. [3] analyze the importance of the seasonal behaviour in the volatility of price commodity options. Mu [60] analyzes how weather affects natural gas price volatility and shows a strong seasonal pattern. Geman and Ohana [35] and Suenaga et al. [79] find that the natural gas price volatility is higher during the winter than during the summer.

Sørensen [75] includes a deterministic seasonal component in the model to describe the seasonal variations in the commodity price. More precisely, he assumes that the log spot price is the sum of a deterministic trigonometric function and two latent factors. In fact, he finds strong evidence for the inclusion of his proposed seasonality component in soybeans, corn and wheat. Considering trigonometric polynomials to model the seasonal component has some advantages such as functions are continuous in time and few parameters are needed. In other cases, seasonal dummy variables are considered as a different modeling approach, see for example Benth et al. [11], and Lucia and Schwartz [56]. This approach is very flexible, but a larger number of parameters are usually needed. García Mirantes et al. [34] assume trigonometric components generated by stochastic processes as seasonal factors.

1.4 Methodology

The commodity pricing models proposed in this dissertation consider diffusion and jump-diffusion stochastic processes. Moreover, we add a seasonal component in the spot price process. In this regard, the role of the jumps and seasonality is studied. So as to develop this research, we use the following methodology.

The fundamental tool for working with financial pricing models in continuous time is stochastic calculus. In this respect, we assume all tools in a complete filtered probability space where we describe the state variable dynamics as stochastic processes. More precisely, we use diffusion and jump-diffusion processes by means of Brownian processes and a jump term. This term is modelled by a Poisson process with its corresponding jump intensity. Jump sizes are assumed to be random variables with a known distribution (normal or exponential). Moreover, we use the Ito product rule and Ito's Lemma of the stochastic calculus, see Applebaum [2], Cont and Tankov [23], Øksendal [62], Protter [67], and Shreve [76] for more detail. In this context, we utilize the differential and integral notations of the jump-diffusion stochastic processes.

In order to price commodity derivatives, we consider arbitrage-free valuation, that is, we make a change from the physical measure to a risk-neutral measure, by means of Girsanov-type measure transformation, see Bremaud [15], and Shreve [76]. Then, we express the processes under this equivalent martingale measure. When we consider jump-diffusion processes, we have to take into account that the risk-neutral measure is not unique, that is, the market is incomplete.

For pricing commodity derivatives, the estimation of the process functions is a necessary step. We approximate the seasonal component of the commodity as a Fourier trigonometric polynomial. Then, we estimate all the parameters simultaneously by means of a nonlinear least square method, see Benth et al. [11], Lucia and Schwartz [56], and Stoer and Bulirsch [78].

Concerning the rest of the functions of the risk-neutral stochastic processes, we estimate them by means of nonparametric techniques, in order to avoid imposing arbitrary functions in the model. In particular, we use the Nadaraya-Watson estimator with Gaussian kernels as weight functions, see Figà-Talamanca and Roncoroni [32], Fusai and Roncoroni [33], Härdle [43], and Härdle and Muller [44], because this procedure is sufficiently flexible to allow for potential nonlinearities in the drift, the diffusive volatility and the intensity of the discontinuous jump component.

So as to illustrate the implementation process and the behaviour of our approach, we use an energy commodity: natural gas. This commodity and its corresponding contracts (futures, options) are actively traded and also very liquid in the markets, see Cummins and Murphy [26]. Henry Hub natural gas spot price was obtained from the US Energy Information Administration of the US Department of Energy (EIA database). The daily natural gas futures prices with short-maturities (till 4 months) were also obtained from the EIA database and for higher maturities (till 44 months) from the Quandl platform. All the data are divided into two different groups: the in-sample data, which we use for estimating all the functions and parameters, and the most recent data, which is included in the out-of-sample data. This most recent data is used to evaluate the proposed approach.

Finally, as we consider a nonparametric approach in this research, a closed-form solution cannot be obtained. An approximated commodity derivative price can be computed in two ways: solving numerically the corresponding partial integro-differential equation and Monte Carlo simulation approach, which provides the commodity derivative price as the conditional expectation of the final payoff of the derivative. Both techniques are related by means of Feynman-Kac Theorem, see Pascucci [64], and Shreve [76]. In this research, we use the Monte Carlo method which generates a great number of paths of the underlying under the risk-neutral measure. This method is widely used by researchers and practitioners in the markets, especially for multiple-factor models because of its simplicity and efficiency, see Glasserman [37], and Wilmott [84].

1.5 Contributions

In the commodity literature, it is very common to use affine models because of their simplicity and tractability. Simple parametric functions are chosen to obtain a closed-form solution for the derivative price. In fact, in most of the cases, the market prices of risk are considered constant. Then the functions of the risk-neutral processes are estimated using the corresponding derivative prices. For example, Gibson and Schwartz [36], Schwartz [73], and Cortazar et al. [24] consider affine models with one, two or three factors and obtain a closed-form solution for some commodity derivatives. However, they do not discuss jumps or seasonality in their models.

In the literature, the empirical evidence does not show that affine models are the best ones to price commodity derivatives. If we consider other dynamics for the state variables which could be realistic, a closed-form solution is not usually known. Then, the market prices of risk or the functions of the risk-neutral processes cannot be estimated, because they are unobservable. In fact, this problem is one of the open questions in the commodity derivative pricing literature, and the main goal of this thesis.

In order to solve this problem, we propose a new approach to estimate the whole functions of the risk-neutral processes of the model directly from market data, even if a closed-form solution is not known. That is, we design new estimation techniques for the different behaviour of the commodities in the market. Moreover, so as to illustrate how to implement these techniques, we use natural gas contracts actively traded on the NYMEX.

In the different chapters of this thesis, we show our main contributions. In Chapter 3, we consider a two-factor jump-diffusion commodity model, whose factors are the spot price and the convenience yield. We assume that the spot price follows a jump-diffusion process because of the abrupt changes that happen in the commodity market (see Deng [27] among others) and the convenience yield follows a diffusion process, as commonly found in the literature; see Schwartz [73], and Yan [86]. The main contribution of this chapter is twofold. First, we obtain some results that allow us to estimate the functions of a two-factor risk-neutral jump-diffusion commodity model directly from the spot and futures prices in the market. Finally, we show the effect of considering jumps in the commodity spot price.

In Chapter 4, we also consider a jump-diffusion process for the spot price, but we assume different jump size distributions. In this case, our goal is to understand the role of the jump and its distribution. Then, we price natural gas futures and futures options with the different jump size distributions. As futures prices are considered an important information source of expected spot prices, financial investors use them to hedge against the risk of commodity price. Therefore, we also analyze the natural gas futures risk premium.

In the literature, empirical evidence of seasonality has been widely shown for different commodity markets such as electricity, natural gas, soybean and so on; see Arismendi et al. [3], Back and Prokopczuk [4], Cartea and Figueroa [20], Kyriakou et al. [53], Li et al. [55], and Lucia and Schwartz [56]. In order to take into account this fact, in Chapter 5, we add a multiplicative seasonal component in the model. In particular, we assume a seasonal deterministic function which is a trigonometric polynomial whose parameters are approximated by means of the nonlinear least square method. As in this case, a closed-form solution is not known, we prove some results to estimate the risk-neutral functions of the model using data from the markets. Finally, we show how to implement this approach using natural gas data and we analyze the role of the seasonality in the futures, futures options and futures risk premia.

Chapter 2

Introducción

En la actualidad los mercados de materias primas son muy relevantes por diferentes motivos. Por un lado, son fundamentales para las empresas que buscan cubrirse del riesgo de los posibles cambios de los precios de las materias primas. Por otro lado, son muy atractivos, también, para los inversores financieros que los utilizan para diversificar sus inversiones, véase Back y Prokopczuk [4]. Además, la importancia de estos mercados ha aumentado considerablemente desde el comienzo de este siglo, convirtiéndose en un tema de investigación muy actual.

Los precios de las materias primas han experimentado variaciones recientemente, que han sido bastante anómalas por diferentes motivos. Desde la primera década del año 2000, estos precios han reflejado, en general, una tendencia creciente. De hecho, entre el año 2002 y mediados del año 2008 este crecimiento, tanto en magnitud como en duración, ha sido el más importante en décadas. Posteriormente, con el comienzo de la reciente crisis global, los precios comenzaron a disminuir y este fenómeno afectó a un gran número de materias primas. Sin embargo, desde el año 2009, y especialmente desde el verano de 2010, los precios en general comenzaron de nuevo a aumentar. Durante estos años los precios de las materias primas han experimentado también una gran volatilidad.

Este fenómeno coincide, también, con una serie de cambios importantes que han tenido lugar en las bases de los mercados. Por ejemplo, tanto la oferta como la demanda de los países emergentes han crecido de manera considerable en poco tiempo. Por otro lado, el objetivo general de reducir el uso de combustibles fósiles como fuente de energía y el debate sobre el cambio climático y su relación con la agricultura han tenido también un importante efecto sobre la evolución de los precios de las materias primas. Sin embargo, estos factores no son suficientes para explicar el comportamiento más reciente de estos precios. Así, un factor adicional es la mayor presencia de inversores financieros en estos mercados, los cuales buscan diversificar sus inversiones más allá de los tradicionales activos financieros. Este fenómeno se conoce como *financialization* de los mercados y su relevancia ha aumentado considerablemente desde el año 2004, como se puede deducir a partir del elevado crecimiento experimentado por el número de inversiones financieras realizadas en los mercados de materias primas desde dicho año. Este factor es muy importante porque la actividad de estos inversores financieros puede mover los precios de las materias primas lejos de los niveles que establecen las leyes de la oferta y la demanda. Los inversores financieros toman decisiones de negocio que no se basan totalmente en las características propias de las materias primas, sino más bien en aspectos relacionados con la composición de sus carteras y la tendencia seguida por los mercados en un determinado momento. Estos comportamientos pueden, incluso, tener efectos negativos tanto sobre los consumidores como sobre los productores de las materias primas.

2.1 Los mercados de materias primas

La negociación de las materias primas en los mercados es bastante limitada ya que sus participantes buscan, normalmente, una exposición financiera a los movimientos de los precios de la materia prima subyacente más que a la propia materia prima. Por tanto, tal y como señalan Back y Prokopczuk [4], la negociación y la fijación de los precios de las materias primas tiene lugar en los mercados de futuros de dichos productos.

Los derivados sobre materias primas se negocian tanto en mercados organizados como en mercados no organizados u *over the counter*, utilizando normalmente una institución financiera como intermediario. En los mercados de futuros se negocian productos estandarizados, claramente definidos, que ofrecen una gran liquidez, transparencia y reducen el riesgo de posible impago. Sin embargo, el número y variedad de este tipo de contratos estandarizados es bastante limitado y no siempre proporcionan una cobertura perfecta a sus participantes. En los mercados no organizados los agentes pueden negociar contratos más amplios que los futuros estandarizados existentes y adaptarlos a sus preferencias y necesidades. Sin embargo, a pesar de que estos contratos ofrecen mayor libertad, y en general menores costes potenciales de negociación, pueden incurrir en mayor riesgo de impago para ambas partes, pues no suelen utilizar los servicios de las cámaras de compensación. De hecho, todavía actualmente, los mercados no organizados suelen ser bastante opacos en todas las partes del mundo.

En Estados Unidos los mercados más conocidos son aquellos pertenecientes al CME Group, el cual surgió después de la fusión del Chicago Mercantil Exchange (CME) y el Chicago Board of Trade en 2006. De hecho, el New York Mercantil Exchange (NYMEX) también pertenece a este grupo y es uno de los mercados más activos a nivel mundial.

Las materias primas negociadas en los mercados son muy diversas pero pueden agruparse en cuatro grandes categorías:

- Metales. Este tipo de materias primas son probablemente las más negociadas y pueden, a su vez, dividirse en otras categorías, como por ejemplo metales preciosos (oro, plata,...), metales no preciosos (cobre, aluminio,...), acero, etc

- Energía. Estas materias primas son las más influyentes e incluyen petróleo, gas natural, carbón, etc.

- Productos agrícolas. En este grupo se incluye el algodón, el cacao, el azúcar, el zumo de naranja, etc.

- Ganadería y carne, como por ejemplo el ganado vacuno, porcino, etc.

Los futuros son una forma muy conocida de invertir en materias primas. Estos contratos se realizan en un mercado organizado entre dos partes con el objetivo de comprar (o vender) una cantidad establecida de una materia prima determinada a un precio fijo y en una fecha estipulada. Otra posible forma de invertir en materias primas es a través de opciones. Las opciones proporcionan una mayor flexibilidad a los inversores que los futuros, ya que están diseñadas para ofrecer el derecho (no la obligación) de comprar (o vender) una determinada materia prima o contrato a un determinado precio y en una fecha futura establecida de antemano, véase Cummins y Murphy [26]. En los mercados financieros de materias primas participan, fundamentalmente, dos tipos de participantes:

- Los inversores, que compran y venden contratos para protegerse de los movimientos que pueden experimentar los precios de las materias primas. Por tanto, buscan evitar riesgos.

- Los especuladores, que tratan de enriquecerse analizando el comportamiento de los mercados de materias primas, adelantándose a los posibles movimientos en sus precios y comprando y vendiendo los diferentes contratos en circulación.

2.2 Los precios al contado y los futuros

En los mercados de materias primas es interesante conocer la estructura temporal de los precios de los futuros sobre materias primas o curva de futuros, la cual es la relación existente entre los precios de los futuros para diferentes vencimientos en un determinado momento, véase Lautier [54]. De hecho, los cambios en la pendiente de la curva de futuros tienen importantes consecuencias sobre las decisiones de inversión y gestión de riesgo, y han sido el centro de numerosos trabajos de investigación, véanse Acharya et al. [1], Bessembinder y Lemmon [13] y Geman y Ohana [35].

Una curva de futuros se dice que tiene pendiente creciente cuando los precios de los futuros son mayores que los precios al contado. En esta situación se considera que el mercado está en *contango*. Por el contario, cuando el precio de los futuros está por debajo del precio al contado, la curva de futuros tiene pendiente decreciente y se dice que el mercado está en *backwardation*. Sin embargo, lo más habitual es que estas curvas presenten tramos crecientes y decrecientes, incluso su forma suele variar a lo largo del tiempo, véase por ejemplo Hambur y Stenner [42].

La forma de la estructura temporal de los precios de los futuros ha venido explicada tradicionalmente por dos corrientes: la teoría de inventarios, la cual comenzó con Brennan [16] y Kaldor [50] entre otros, y la literatura sobre la presión de cobertura o *hedging pressure*, de la cual fueron pioneros Hicks [45] y Keynes [51].

La teoría de inventarios se centra en el beneficio global que reporta poseer la materia prima físicamente así como de algunos aspectos relacionados con su almacenamiento. Poseer la materia prima proporciona ciertos beneficios pero también conlleva ciertos costes. Por tanto, el rendimiento de conveniencia (neto) se define como la diferencia entre el rendimiento de conveniencia bruto menos el coste de poseer físicamente el bien, como por ejemplo los costes de almacenamiento, transporte, etc. En línea con esta teoría, existe una relación entre el precio al contado y el precio de los correspondientes futuros. Si los beneficios de poseer las materias primas (rendimiento de conveniencia) son mayores que sus costes financieros (tipo de interés), la curva de futuros estará en *backwardation*, sin embargo, si los costes financieros son superiores al rendimiento de conveniencia, entonces la curva estará en *contango*, véase Back y Prokopczuk [4] para más información.

La posesión de la materia prima permite también a los productores cubrir cambios inesperados de la demanda, periodos temporales de escasez de oferta y asegurarse el proceso de producción véase Kwok [52]. Por tanto, a partir de argumentos relacionados con la oferta y la demanda se establece también una relación negativa entre el nivel de los inventarios y el precio, véase Back y Prokopczuk [4]. Así, si el nivel de los inventarios es elevado, la curva de futuros estará en *contango* y, si el nivel de los inventarios es bajo, la curva de futuros estará en *backwardation*.

Por el contrario, la teoría de la *hedging pressure* se centra en la prima de riesgo. Esta corriente considera que el precio de los futuros es la suma del precio al contado esperado más una prima de riesgo. La teoría de la *normal backwardation* propuesta por Keynes [51] se basa en la hipótesis general de que los productores, normalmente, toman posiciones cortas en los contratos de futuros porque desean asegurarse cierto precio en el momento de la entrega de la materia prima. Este contrato proporciona a los productores una forma de cubrirse del riesgo de descenso del precio al contado. Por tanto, los productores están dispuestos a pagar una prima de riesgo que les cubra su exposición a variaciones del precio de la materia prima. Por otro lado, los consumidores también pueden estar interesados en protegerse de posibles subidas del precio. Por tanto, tomarán una posición larga en los correspondientes contratos de futuros. Si la actividad de cobertura de los productores de una determinada materia prima es mayor que la actividad de los consumidores, el precio de los futuros será menor que el precio al contado esperado, y el precio de los futuros será un estimador sesgado por defecto del precio al contado esperado, véase Back y Prokopczuk [4]. Esta situación inducirá a los especuladores a equilibrar el mercado tomando la posición contraria, es decir, una posición larga. Sin embargo, esta situación no siempre es válida. Si la actividad de cobertura de los consumidores es mayor que la de los productores, habrá un exceso de participantes en el mercado buscando entrar en una posición larga y, por tanto, el precio del futuro será menor que el precio al contado esperado en un momento futuro. En este caso, los especuladores deberán ser de alguna manera recompensados por tomar una posición corta en el futuro y así equilibrar el mercado. Por tanto, los precios de los futuros pueden incluir una prima positiva o negativa dependiendo de la posición neta de los coberturistas para cada materia prima y del instante de tiempo considerado, véanse Cootner [25] y Hambur y Stenner [42]. En este sentido, Hambur y Stenner [42] definen la prima de riesgo como el rendimiento que los especuladores esperan obtener como compensación por comprar y/o vender contratos de futuros sobre materias primas. El papel que desempeña la prima de riesgo en los diferentes mercados de materias primas ha sido ampliamente analizado, véanse por ejemplo Acharya et al. [1], Basu y Miffre [8] y Ronn y Wimschulte [69].

2.3 Modelos de valoración

En los últimos años se han propuesto diferentes modelos relacionados con la valoración de derivados de materias primas. En este tipo de modelos, en primer lugar, se deben establecer las variables de estado y su dinámica, todo ello buscando un equilibrio entre que el modelo sea sencillo de implementar y que sea capaz de capturar adecuadamente las propiedades de los precios.

Evidentemente, el precio al contado de la materia prima debería ser una de las variables a tener en cuenta para valorar futuros. Muchos modelos describen la dinámica de esta variable mediante un proceso estocástico y utilizan argumentos de no arbitraje para la valoración del derivado, véase Schwartz [73].

Debido a la relación entre la demanda y la oferta se suele considerar que el precio al contado tiene un comportamiento de reversión a la media. Por ejemplo, Bessembinder et al. [12] muestran una fuerte evidencia de reversión a la media en los mercados de materias primas, sin embargo, Brooks y Prokopczuk [18] afirman que la reversión a la media no se mantiene cuando el modelo presenta saltos y volatilidad estocástica. De hecho, en la literatura se han propuesto diferentes tipos de dinámicas: Schwartz [73] supone que la reversión a la media es constante mientras que Schwartz y Smith [74] consideran una media a largo plazo estocástica.

Brennan y Schwartz [17] proponen un modelo de un factor donde el rendimiento de conveniencia se describe mediante una función determinista del precio al contado. Sin embargo, los modelos unifactoriales no son consistentes con la observación empírica, ya que proporcionan precios de futuros que están perfectamente correlacionados. Por tanto, parece que estos modelos no son capaces de reflejar la dinámica de los precios de los futuros.

Gibson y Schwartz [36] introducen un modelo con procesos de difusión conjuntos del precio al contado y el rendimiento de conveniencia; más concretamente, consideran que el precio al contado sigue un movimiento Browniano geométrico y el rendimiento de conveniencia está descrito mediante un proceso Ornstein-Ulhenbeck. Como el rendimiento de conveniencia no es observable en el mercado, Gibson y Schwartz [36] proponen una proxy para esta variable. Schwartz [73] considera un modelo de tres factores con un factor adicional, el tipo de interés, que sigue un proceso de difusión. Posteriormente, Miltersen y Schwartz [59] también proponen un modelo de tres factores para valorar futuros de materias primas y opciones de futuros. Recientemente, Schöne y Spinler [72] consideran un modelo de difusión afín con volatilidad estocástica para valorar también futuros y opciones.

Muchas materias primas se caracterizan por presentar excepcionalmente cambios bruscos en los precios en el mercado, como consecuencia de fallos en la producción o el transporte, de las condiciones climáticas, en el caso de materias primas agrícolas, o de sucesos macroeconómicos inesperados entre otras razones. En la literatura, con el fin de tener en cuenta estos cambios bruscos en el precio de las materias primas, se añade un término de salto en el proceso estocástico. Por ejemplo, Hilliard y Reis [46] proponen un modelo que permite saltos en el proceso del precio al contado, en particular, suponen que este precio sigue un proceso de difusión con saltos. Yan [86] considera un modelo de valoración de derivados de materias primas con saltos en el precio al contado y la volatilidad. Schmitz et al. [71], en el caso de las materias primas agrícolas, suponen que la volatilidad es un proceso estocástico con saltos, y analizan su efecto en los precios de las opciones. Hilliard y Hilliard [47] usan, para describir el precio al contado, un movimiento Browniano geométrico con un término de salto y, para el tipo de interés y el rendimiento de conveniencia, procesos de difusión con reversión a la media, todo ello para valorar precios de opciones del oro y el cobre.

Finalmente, es bien conocido que la oferta y la demanda de muchas materias primas presentan ciclos estacionales. En particular, las agrícolas y la mayoría de las energéticas tienen un patrón estacional en sus precios. Por un lado, la oferta de las materias primas agrícolas depende del clima el cual afecta a la producción. Por otro lado, la demanda de energía es mayor durante las estaciones frías. Por tanto, los precios son más altos durante estas estaciones que en el periodo cálido. Por ejemplo, Sørensen [75] muestra el patrón estacional que presentan productos agrícolas como la soja, el maíz y el trigo, y Manoliu y Tompaidis [57] y Paschke y Prokopczuk [63] encuentran un fuerte efecto estacional en algunas materias primas de la energía como el gas natural, el petróleo o la gasolina. Cartea y Figueroa [20], Li et al. [55] y Lucia y Schwartz [56] consideran estacionalidad en mercados de la electricidad, García Mirantes et al. [34] en los del gas natural, Kyriakou et al. [53] en los del petróleo y Back y Prokopczuk [4] en mercados de la soja, el maíz, el petróleo y el gas natural. Arismendi et al. [3] analizan la importancia de la estacionalidad en la volatilidad a la hora de valorar opciones. Mu [60] muestra cómo afecta el clima a la volatilidad del precio del gas natural y que presenta un fuerte patrón estacional. Geman y Ohana [35] y Suenaga et al. [79] encuentran que la volatilidad del precio de gas natural es más alta durante el invierno que durante el verano.

Sørensen [75] incluye una componente estacional determinista en el modelo para describir las variaciones estacionales en el precio de la materia prima. Más concretamente, supone que el logaritmo del precio al contado es la suma de una función trigonométrica determinista y dos factores latentes. De hecho, encuentra una fuerte evidencia de estacionalidad en el precio de la soja, el maíz y el trigo. Considerar polinomios trigonométricos en la componente estacional del modelo tiene algunas ventajas, como que las funciones son continuas en el tiempo y que para su descripción son necesarios pocos parámetros. En otros casos, se consideran variables estacionales dummy para su modelización, véanse por ejemplo Benth et al. [11] y Lucia y Schwartz [56]. Este enfoque es muy flexible, pero generalmente requiere de un gran número de parámetros. García Mirantes et al. [34] proponen como factores estacionales componentes trigonométricos generados por procesos estocásticos.

2.4 Metodología

En los modelos de valoración propuestos en este trabajo se utilizan procesos de difusión y de difusión con saltos. Además, añadimos una componente estacional en el precio al contado de la materia prima, y analizamos el papel que tienen los saltos y la componente estacional a la hora de valorar futuros y opciones. Para desarrollar esta investigación, utilizamos la siguiente metodología.

La herramienta fundamental en los modelos de valoración financiera es el cálculo estocástico. En este contexto, consideramos un espacio de probabilidad filtrado completo donde describimos la dinámica de las variables de estado mediante procesos estocásticos. Más concretamente, utilizamos procesos de difusión y difusión con saltos mediante un proceso Browniano y un término de salto. Este término es modelizado con un proceso de Poisson con su correspondiente intensidad de salto y los tamaños de salto vienen determinados mediante variables aleatorias que siguen una distribución de probabilidad concreta (normal o exponencial). Además, utilizamos la regla del producto de Ito, así como el Lema de Ito del cálculo estocástico, véanse Applebaum [2], Cont y Tankov [23], Øksendal [62], Protter [67] y Shreve [76] para más detalle. A lo largo del trabajo utilizamos las notaciones diferencial e integral de los procesos estocásticos de difusión y saltos.

Para valorar derivados de materias primas consideramos el argumento de no arbitraje, es decir, hacemos una transformación de la medida física a la neutral al riesgo, mediante el cambio de medida de tipo Girsanov, véanse Bremaud [15] y Shreve [76]. Entonces, expresamos los procesos estocásticos bajo esta medida martingala equivalente. Cuando consideramos procesos de difusión con saltos, debemos tener en cuenta que la medida neutral al riesgo no es única, es decir, el mercado no es completo.

Un paso previo a la valoración de derivados de materias primas es la estimación. En este trabajo, utilizamos la estimación paramétrica con el método de mínimos cuadrados no lineales, véanse Benth et al. [11], Lucia y Schwartz [56] y Stoer y Bulirsch [78], para obtener simultáneamente todos los parámetros del polinomio trigonométrico de Fourier que aproxima la componente estacional del precio de la materia prima. En lo que respecta al resto de las funciones de los procesos neutrales al riesgo, las estimamos utilizando técnicas no paramétricas, evitando de esta forma restricciones arbitrarias en las funciones. En particular, utilizamos el estimador de Nadaraya-Watson con el núcleo Gaussiano para las funciones peso, véanse Figà-Talamanca y Roncoroni [32], Fusai y Roncoroni [33], Härdle [43] y Härdle et al. [44], porque este procedimiento es flexible y permite no linealidades en la tendencia, la volatilidad y la intensidad del salto.

Con el propósito de ilustrar el proceso de implementación del enfoque propuesto, utilizamos una materia prima energética: el gas natural. Esta materia prima y sus correspondientes contratos (futuros y opciones) son muy líquidos y se negocian activamente en el mercado, véase Cummins y Murphy [26]. El precio al contado del gas natural Henry Hub fue obtenido de la US Energy Information Administration del US Department of Energy (EIA database). Los precios diarios de los futuros del gas natural con vencimientos cortos (hasta 4 meses) fueron también obtenidos de la base de datos EIA, y los precios con vencimientos más largos (hasta 44 meses) de la plataforma

Quandl. Los datos los hemos dividido en dos grupos: el periodo de estimación, que se utiliza precisamente para realizar la estimación de las funciones y parámetros, y un periodo de tiempo de predicción, que es una muestra más reciente y que es donde se valoran los derivados.

Finalmente, como en este trabajo utilizamos estimación no paramétrica, no es posible obtener una forma cerrada de la solución. En estos casos, se puede calcular el precio del derivados mediante dos procedimientos: resolviendo numéricamente la ecuación en derivadas parciales integro-diferencial o con el método de Monte Carlo, el cual calcula una aproximación al precio del derivado mediante la esperanza condicionada de la condición final del derivado. Ambas técnicas están relacionadas por medio del Teorema de Feynman-Kac, véanse Pascucci [64] y Shreve [76]. En esta investigación, usamos el método de Monte Carlo generando un gran número de simulaciones del precio al contado subyacente bajo la medida neutral al riesgo. Este método es ampliamente utilizado por investigadores y profesionales de los mercados, especialmente para modelos multifactoriales, debido a su sencillez, véanse Glasserman [37] y Wilmott [84].

2.5 Contribuciones

En la literatura es muy habitual utilizar modelos afines para valorar derivados de materias primas, por su sencillez y adaptabilidad. Para poder obtener una forma cerrada del precio del derivado, se suelen considerar funciones paramétricas sencillas. De hecho, en la mayoría de los casos, los precios de riesgo del mercado se consideran constantes. Entonces, las funciones de los procesos bajo la medida neutral al riesgo se estiman a partir de las observaciones de los precios del derivado. Por ejemplo, Gibson y Schwartz [36], Schwartz [73] y Cortazar et al. [24] consideran modelos afines con uno, dos o tres factores, y obtienen una forma cerrada de la solución para el precio del derivado. Sin embargo, estos autores no introducen saltos ni estacionalidad en sus modelos. Además, no hay ninguna evidencia empírica de que los modelos afines sean mejores para valorar derivados de materias primas. Si consideramos otras dinámicas más realistas para las variables de estado, no se suele conocer una forma cerrada de la solución. Entonces, los precios de riesgo del mercado o las funciones de los procesos neutrales al riesgo no se pueden estimar, porque no son observables. De hecho, este problema es una de las cuestiones abiertas en la valoración de derivados de materias primas, y es el principal objetivo de esta tesis.

Así pues, para resolver este problema, proponemos un nuevo enfoque para estimar las funciones de los procesos neutrales al riesgo del modelo a partir de los datos del mercado, incluso cuando no se conoce una expresión de la solución. Es decir, diseñamos nuevas técnicas de estimación para diferentes comportamientos del precio al contado de la materia prima en el mercado. Además, para ilustrar cómo implementar estas técnicas, utilizamos futuros del gas natural negociados en el NYMEX.

En los diferentes capítulos de esta tesis mostramos nuestras principales contribuciones. En el Capítulo 3 consideramos un modelo de dos factores de difusión con saltos cuyas variables son el precio al contado y el rendimiento de conveniencia. Suponemos que el precio al contado sigue un proceso de difusión con saltos, para que recoja los cambios bruscos que se producen en los mercados de materias primas (véase Deng [27], entre otros), y el rendimiento de conveniencia sigue un proceso de difusión, como es habitual en la literatura, véanse Schwartz [73] y Yan [86]. La principal contribución de este capítulo es doble. En primer lugar, obtenemos resultados que nos permiten estimar las funciones de un modelo de dos factores de difusión con saltos directamente a partir de los precios de los futuros. Finalmente, mostramos el efecto de considerar saltos en el precio al contado sobre los precios de futuros.

En el Capítulo 4 también consideramos un proceso de difusión con saltos para el precio al contado, pero suponemos diferentes distribuciones para los tamaños de salto. En este caso, nuestro objetivo es analizar el papel que juega dicha distribución en el modelo y, para ello, valoramos futuros del gas natural y opciones de futuros, con diferentes distribuciones del salto. Los inversores financieros consideran los precios de los futuros como una fuente de información sobre el precio esperado de la materia prima, y lo utilizan para diseñar sus estrategias de cobertura del riesgo. En este capítulo analizamos la prima de riesgo de los futuros de materias primas.

Diferentes investigadores han mostrado evidencia empírica de estacionalidad en diversos mercados de materias primas como la electricidad, el gas natural, la soja, etc, véanse Arismendi et al. [3], Back y Prokopczuk [4], Cartea y Figueroa [20], Kyriakou et al. [53], Li et al. [55] y Lucia y Schwartz [56]. Con el fin de tener en cuenta este hecho, en el Capítulo 5 añadimos una componente estacional multiplicativa en el modelo previo. En particular, consideramos una función estacional determinista mediante un polinomio trigonométrico, cuyos parámetros son aproximados mediante el método de mínimos cuadrados no lineal. Como en este caso no existe una forma cerrada de la solución, probamos resultados para estimar las funciones de los procesos neutrales al riesgo usando datos del mercado. Finalmente, mostramos cómo implementar estas técnicas utilizando datos de los futuros del gas natural y analizamos el papel de la estacionalidad en los precios de los futuros, las opciones sobre futuros y las primas de riesgo de los futuros.

Chapter 3

A new technique to estimate the risk-neutral processes in jump-diffusion commodity futures models

[This chapter has been previously published (jointly with Lourdes Gómez-Valle and Julia Martínez-Rodríguez) in *Journal of Computational and Applied Mathematics*, vol. 309, pp. 435-441, 2017.]

Journal of Computational and Applied Mathematics 309 (2017) 435–441

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A new technique to estimate the risk-neutral processes in jump–diffusion commodity futures models

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ARTICLE INFO

Article history:
Received 19 October 2015
Received in revised form 18 December 2015

Keywords:
Commodity futures
Jump–diffusion stochastic processes
Risk-neutral measure
Numerical differentiation
Nonparametric estimation

ABSTRACT

In order to price commodity derivatives, it is necessary to estimate the market prices of risk as well as the functions of the stochastic processes of the factors in the model. However, the estimation of the market prices of risk is an open question in the jump–diffusion derivative literature when a closed-form solution is not known. In this paper, we propose a novel approach for estimating the functions of the risk-neutral processes directly from market data. Moreover, this new approach avoids the estimation of the physical drift as well as the market prices of risk in order to price commodity futures. More precisely, we obtain some results that relate the risk-neutral drifts, volatilities and parameters of the jump amplitude distributions with market data. Finally, we examine the accuracy of the proposed method with NYMEX (New York Mercantile Exchange) data and we show the benefits of using jump processes for modelling the commodity price dynamics in commodity futures models.

1. Introduction

The behaviour of many commodity futures has become highly unusual over the past decades. Prices have experienced significant run-ups, and the nature of their fluctuations has changed considerably. This is partly due to financial firms with no inherent exposure to the commodities have adopted strategies of portfolio diversification into commodity futures markets as an asset class, see [1]. However, energy commodities are different from financial assets such as equity and fixed-income securities. For example, changes in market expectations, or even unanticipated macroeconomic developments may cause sudden jumps in energy prices, see [2]. Therefore, traditional modelling techniques are not directly applicable.

In order to price commodity derivatives, the empirical features of the commodity prices need to be considered. First, the spot price and other factors were assumed to follow diffusion processes. For example, Gibson and Schwartz [3] assumed that the spot price and the convenience yield were mean-reverting diffusion processes. Then, Schwartz [4] reviewed one and two-factor models and developed a three-factor mean-reverting diffusion model. Later, Miltersen and Schwartz [5] also considered a three-factor model in order to price commodity futures and futures options. More recently, in the literature, jump–diffusion models have been considered because there are numerous empirical studies which show that commodity prices exhibit jumps, [6,7] and so on. Hilliard and Reis [8] considered a three-factor model where the spot price follows a jump–diffusion stochastic process. Yan [9] extended existing commodity valuation models to allow for stochastic volatility and simultaneous jumps in the spot price and volatility. Hilliard and Hilliard [10] used the standard geometric Brownian

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http://dx.doi.org/10.1016/j.cam.2015.12.028
0377-0427/

motion augmented by jumps to describe the underlying spot and mean-reverting diffusions for the interest rate and convenience yield state variables for gold and copper prices.

In this paper, we consider a two-factor jump–diffusion commodity model, where one of the factors is the commodity spot price. In the commodity literature, it is very common to use affine models for its simplicity and tractability. They select the simple parametric functions for the model in order to obtain a closed-form solution for the pricing problem. This is mainly important for the market prices of risk, which are assumed to be constant in most of the cases. Then, all the functions can be easily estimated and the commodity derivatives priced. However, there is not any empirical evidence either consensus about affine models are the best models to price commodity futures. Furthermore, the market prices of risk are not observed in the markets. If we considered other more realistic functions for the state variables or the market prices of risk or even a nonparametric approach, then, the model would not be affine anymore, a closed-form solution could not be obtained and therefore, the estimation of the market prices of risk would not be possible. In fact, this last problem is an open question in the jump–diffusion commodity literature.

The main contribution of this paper is twofold. First, we obtain some results that allow us to estimate the risk-neutral functions of a two-factor jump–diffusion commodity model directly from commodity spot and futures data on the markets. Therefore, we can obtain a closed-form solution or a numerical approximation for the pricing problem without estimating the market prices of risk, which are not observed and possible to estimate when a closed-form solution is not known. Second, we show the effect of considering jumps in the commodity spot price over the futures prices. We use NYMEX data and a nonparametric approach to estimate the whole functions of our two-factor model. We think that using a nonparametric approach is more realistic than using an affine model.

The remaining of the paper is arranged as follows. In Section 2, we present a two-factor jump–diffusion model to price commodity futures. In Section 3, we prove some results which allow us to estimate the risk-neutral drift, jump intensity and parameters of the distribution of the jump amplitude from spot commodity price and futures data. In Section 4, we estimate our two-factor jump–diffusion model with NYMEX data by means of a nonparametric approach, we price commodity futures and we show its supremacy over a diffusion model. Section 5 concludes.

2. The valuation model

In this section, we present a two-factor commodity futures model. The first factor is the spot price S, and the second factor is δ, which could be, for example, the instantaneous convenience yield or the volatility among other possible variables. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space equipped with a filtration $\mathcal{F}$ satisfying the usual conditions, see [11,12] or [13]. The factors of the model are assumed to follow this joint jump–diffusion stochastic process:

$$dS(t) = \mu_S(S(t), \delta(t))dt + \sigma_S(S(t), \delta(t))dW_S(t) + J(S(t), \delta(t), Y(t))dN(t), \tag{1}$$

$$d\delta(t) = \mu_\delta(S(t), \delta(t))dt + \sigma_\delta(S(t), \delta(t))dW_\delta(t), \tag{2}$$

where μ_S and μ_δ are the drifts, σ_S and σ_δ the volatilities. The jump amplitude J is a function of the two factors and Y which is a random variable with probability distribution Π. Moreover, W_S and W_δ are Wiener processes and N represents a Poisson process with intensity λ. We assume that the standard Brownian motions are correlated with:

$$Cov(W_S, W_\delta) = \rho t.$$

However, W_S and W_δ are assumed to be independent of N. We also assume that the jump magnitude and the jump arrival time are uncorrelated with the diffusion parts of the processes. We suppose that the functions $\mu_S, \mu_\delta, \sigma_S, \sigma_\delta, J, \lambda$ and Π satisfy suitable regularity conditions: see [11,14]. Under the above assumptions, a commodity futures price at time t with maturity at time T, $t \le T$, can be expressed as $F(t, S, \delta; T)$ and at maturity it is

$$F(T, S, \delta; T) = S.$$

Finally, we assume that there exists a replicating portfolio for the futures price and then, the futures price can be expressed by

$$F(t, S, \delta; T) = E^{\mathcal{Q}}[S(T)|S(t) = S, \delta(t) = \delta], \tag{3}$$

where $E^{\mathcal{Q}}$ denotes the conditional expectation under the $\mathcal{Q}$ measure which is known as the risk-neutral probability measure. The two-factor model (1)–(2) under $\mathcal{Q}$ measure is as follows:

$$dS = \left(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J]\right) dt + \sigma_S dW_S^{\mathcal{Q}} + Jd\tilde{N}^{\mathcal{Q}}, \tag{4}$$

$$d\delta = \left(\mu_\delta - \sigma_\delta\theta^{W_\delta}\right) dt + \sigma_\delta dW_\delta^{\mathcal{Q}}, \tag{5}$$

where $W_S^{\mathcal{Q}}$ and $W_\delta^{\mathcal{Q}}$ are the Wiener processes under $\mathcal{Q}$ and $Cov(W_S^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}) = \rho t$. The market prices of risk of Wiener processes are $\theta^{W_S}(S, \delta)$ and $\theta^{W_\delta}(S, \delta)$, and $\tilde{N}^{\mathcal{Q}}$ represents the compensated Poisson process, under $\mathcal{Q}$ measure, with intensity $\lambda^{\mathcal{Q}}(S, \delta) = \lambda(S, \delta)\theta^N(S, \delta)$.

3. Exact results and approximations

In the literature, researchers have devoted the greatest attention to affine models such as [4,8–10]. One of the main reasons is that a closed-form solution for the commodity futures price is found. Moreover, this fact allows the application of different estimation techniques, like the Kalman Filter or Maximum Likelihood. However, there is neither evidence nor consensus that affine models are the most suitable for pricing futures contracts.

In the literature, to the best of our knowledge, there is no approach for estimating the market prices of risk for pricing commodity derivatives with jumps, unless a closed-form solution is known. Bandi and Nguyen [15] and Johannes [14] showed how to estimate the functions of a jump–diffusion process by means of their moment equations for interest rate models. However, this approach does not allow us to estimate the market prices of risk, which are necessary to price commodity derivatives but not observable.

In this section, we propose a new approach for estimating the functions of the risk-neutral jump–diffusion stochastic factors of a commodity model directly from market data. Then, we can price futures and the estimation of the market prices of risk can be avoided.

Theorem 1. *Let $F(t, S, \delta; T)$ be the price of the future (3), and S and δ follow the stochastic processes given by (4) and (5), respectively, then:*

$$\frac{\partial F}{\partial T}(t, S, \delta; T) = \left(\mu_S - \sigma_S \theta^{W_S} + \lambda^{\mathbb{Q}} E_Y^{\mathbb{Q}}[J]\right)(T), \tag{6}$$

$$\frac{\partial (SF)}{\partial T}(t, S, \delta; T) = \left(2S\frac{\partial F}{\partial T} + \sigma_S^2 + \lambda^{\mathbb{Q}} E_Y^{\mathbb{Q}}[J^2]\right)(T), \tag{7}$$

$$\frac{\partial (\delta F)}{\partial T}(t, S, \delta; T) = \left(\delta\frac{\partial F}{\partial T} + S(\mu_\delta - \sigma_\delta \theta^{W_\delta}) + \rho\sigma_S\sigma_\delta\right)(T). \tag{8}$$

We prove these results by means of (3). The detailed proof of this theorem can be found in the Appendix. Analogous results, but for diffusion processes, are also shown in [16]. Parallel results for one-factor jump–diffusion interest rate models can be found in [17,18].

In order to implement Theorem 1 we rely on numerical differentiation. We use futures prices at a point that is inside the time interval to approximate the slopes at the boundary of the time domain. This fact allows us to consider futures prices with a high spectrum of maturities for the estimation of the functions of the model. More precisely, we obtain a fourth order approximation to the slopes by the well-known difference formula:

$$\frac{\partial g}{\partial T}\Big|_{T=t} = \frac{-25g(t) + 48g(t+\Delta) - 36g(t+2\Delta) + 16g(t+3\Delta) - 3g(t+4\Delta)}{12\Delta} + O(\Delta^4). \tag{9}$$

Finally, it is important to remark that after approximating the slopes in Theorem 1, any parametric or nonparametric technique can be applied to estimate them. In this paper, we use the Nadaraya–Watson nonparametric estimator in order to avoid imposing arbitrary restrictions to the different functions of the model. Suppose a data set consists of N observations, $(S_1, \delta_1, Z_1), \ldots, (S_N, \delta_N, Z_N)$, where (S_i, δ_i) are the explanatory variables and Z_i is the response variable. We assume a model of the kind $Z_i = g(S_i, \delta_i) + \epsilon_i$, where $g(S, \delta)$ is an unknown function and ϵ_i is an error term, representing random errors in the observations or variability from sources not included in the (S_i, δ_i) observations. The errors ϵ_i are assumed to be independent and identically distributed with mean 0 and finite variance. The estimate has the closed-form

$$\hat{g}(S, \delta) = \frac{\sum_{i=1}^{N} w_i(S, \delta) Z_i}{\sum_{i=1}^{N} w_i(S, \delta)},$$

where $w_i(S, \delta) = K\left(\frac{S - S_i}{h_S}\right) K\left(\frac{\delta - \delta_i}{h_\delta}\right)$ are weight functions (we use the multivariate Gaussian kernel which is also widely used in the literature) and h_S, h_δ the bandwidths, see [19].

4. Empirical application

In this section, we show how practitioners can implement the approach in Section 3 for pricing commodity futures in the markets. Moreover, we analyse the effect of adding jumps to the commodity spot price over the futures prices.

In this empirical application, we use the commodity spot price and the convenience yield as state variables, which are frequently used in the literature, see for example [3,4]. We assume that the spot price follows a jump–diffusion process, because commodity prices usually suffer from abrupt changes in the markets, see [6]. However, we assume that the convenience yield is a diffusion process because its behaviour is not affected by extreme changes, see for example [9].

Table 1
Summary of the statistics on the natural gas spot price and its first differences, January 2004–December 2014.

Variable	N	Mean	Std. dev.	Max	Min
S_t	2735	5.4732	2.3070	15.3900	1.8200
$S_{t+1} - S_t$	2734	−0.0011	0.2665	2.5000	−1.9100

For simplicity and tractability and as usual in the literature, we also assume that the distribution of the jump size under $\mathcal{Q}$ measure is known and equal to the distribution under $\mathcal{P}$ measure. This means that all risk premium related to the jump are artificially absorbed by the change in the intensity of the jump from λ under the physical measure to $\lambda^{\mathcal{Q}}$ under the risk-neutral measure, see [20]. Moreover, we assume in (4) that $J(S, \delta, Y) = Y$, where Y is a random variable which follows a normal distribution $N(0, \sigma_Y)$, see [9,21,22] among others. Therefore, $E_Y^{\mathcal{Q}}[Y] = E_Y[Y] = 0$.

In order to show how the approach in Section 3 can be implemented, we will price natural gas futures with daily data from the NYMEX. Natural gas spot and futures prices were obtained from the Energy Information Administration of the U.S. Department of Energy (E.I.A. database) and Quandl platform. The sample period covers from January 2004 to April 2015. Fig. 1 plots the natural gas spot price data and its first differences. We also consider futures prices with maturities equal to 1, 2, 3, 4, 6, 9, 12, 18, 24, 36 and 44 months. We use data from January 2004 to December 2014 for estimating the risk-neutral functions. Table 1 summarizes the in-sample data. We keep data from January to April 2015 as out-of-sample data in order to evaluate the results of our approach.

As it is well known in the literature, the convenience yield is not observed in the markets. Then, following [3], we approximate it by the following result

$$\delta_{T-1,T} = r_{T-1,T} - 12 \ln \left[\frac{F(t, S, \delta; T)}{F(t, S, \delta; T-1)} \right],$$

where $r_{T-1,T}$ denotes the $T-1$ period ahead annualized one month riskless forward interest rate. We obtain this forward interest rate with two daily T-Bill rates with maturities as close as possible to the futures contracts' ones in order to compute $\delta_{1,2}$, the one-month ahead annualized convenience yield. The latter is identified with the instantaneous convenience yield $\delta_{0,1}$ in this study, see [3] for more details. T-Bill rates are obtained from the Federal Reserve h.15 database.

First, we obtain the compensated risk-neutral drift of the spot price. We approximate the partial derivative in (6), using numerical differentiation (9), with futures prices with maturities equal to 1, 2, 3 and 4 months. Then, we estimate it by means of the Nadaraya–Watson estimator. Secondly, in order to obtain the risk-neutral jump intensity, we approximate the partial derivatives $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial (SF)}{\partial T}|_{T=t}$ in (7), using numerical differentiation (9) with spot prices and futures prices with maturities equal to 1, 2, 3 and 4 months.

In order to estimate the functions of the stochastic processes of S and δ under the physical measure, we use the following result.

Theorem 2. *If S and δ solve (1)–(2), then*

$$M_S^1(S, \delta) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[S(t + \Delta t) - S(t) | S(t) = S, \delta(t) = \delta] = \mu_S(S, \delta) + \lambda(S, \delta) E_Y[Y], \tag{10}$$

$$M_S^2(S, \delta) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[(S(t + \Delta t) - S(t))^2 | S(t) = S, \delta(t) = \delta] = \sigma_S^2(S, \delta) + \lambda(S, \delta) E_Y[Y^2], \tag{11}$$

$$M_S^k(S, \delta) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[(S(t + \Delta t) - S(t))^k | S(t) = S, \delta(t) = \delta] = \lambda(S, \delta) E_Y[Y^k], \quad k \geq 3, \tag{12}$$

$$M_\delta^2(S, \delta) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[(\delta(t + \Delta t) - \delta(t))^2 | S(t) = S, \delta(t) = \delta] = \sigma_\delta^2(S, \delta), \tag{13}$$

$$Cov(S, \delta) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[(S(t + \Delta t) - S(t))(\delta(t + \Delta t) - \delta(t))^2 | S(t) = S, \delta(t) = \delta] = \rho(S, \delta) \sigma_S(S, \delta) \sigma_\delta(S, \delta). \tag{14}$$

This theorem can be proved with the infinitesimal operator (see [12]), as in [23,24], for diffusion processes, and in [14,15], for jump–diffusion processes.

As we have previously assumed, the jump size distribution under $\mathcal{Q}$ measure is known and equal to the distribution under $\mathcal{P}$ measure and $Y \rightsquigarrow N(0, \sigma_Y^2)$, then $E_Y^{\mathcal{Q}}[Y] = E_Y[Y] = 0$ and $\sigma_Y^2 = E_Y^{\mathcal{Q}}[Y^2] = E_Y[Y^2]$. Therefore, we estimate σ_Y^2 and the volatility of the spot price σ_S, by means of the moment equations of a jump–diffusion process in Theorem 2, when the jump amplitude follows a normal distribution. More precisely, as $Y \rightsquigarrow N(0, \sigma_Y^2)$, then:

$$E_Y[Y^{2k}] = \sigma_Y^{2k} \prod_{n=1}^{k} (2k - 1),$$

$$E_Y[Y^{2k-1}] = 0, \quad k = 1, 2, 3, \ldots.$$

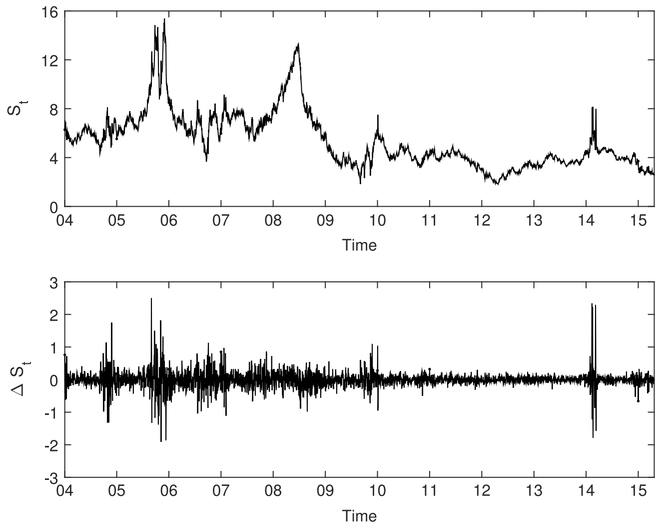

Fig. 1. Daily natural gas spot price and its first differences from January 2004 to April 2015.

In order to estimate σ_Y^2 and σ_S^2, We use moments (11) and (12) with $k = 4$ and 6 and the Nadaraya–Watson estimator. Then, we replace these values and the approximations of the partial derivatives $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial (SF)}{\partial T}|_{T=t}$ in (7) and we estimate the risk-neutral jump intensity of the spot price with the Nadaraya–Watson estimator. As the convenience yield is a diffusion process, we can estimate its risk-neutral drift by means of (8) where $\rho\sigma_S\sigma_X = \mathrm{Cov}(S, \delta)$. In order to estimate this covariance, we use the moment condition (14) and the Nadaraya–Watson estimator, see [24] for more details. Then, we replace the estimated covariance and the approximations of $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial (\delta F)}{\partial T}|_{T=t}$ in (8) and we get the risk-neutral drift of the convenience yield by means of the Nadaraya–Watson estimator. Finally, the volatility of the convenience yield under $\mathcal{P}$ measure is equal to the volatility under $\mathcal{Q}$ measure, we estimate σ_δ by means of the second order moment (13) and convenience yield data.

In order to price natural gas futures, we use the Monte Carlo simulation approach because it is widely used by practitioners in the markets, especially for multifactorial models because of its simplicity and efficiency.

The approach we propose in this paper is a jump–diffusion extension of the one proposed by Gómez-Valle and Martínez-Rodríguez [16] for a diffusion commodity model. Therefore, we can use both approaches for examining the effect of adding jumps to the commodity spot price over the commodity futures prices.

Gómez-Valle and Martínez-Rodríguez [16] assume that the futures price depends on the same two factors: the commodity spot price and the convenience yield. More precisely, they assume that these factors follow this joint diffusion stochastic process under $\mathcal{Q}$ measure:

$$dS = \left(\mu_S - \sigma_S\theta^{W_S}\right) dt + \sigma_S dW_S^{\mathcal{Q}}, \tag{15}$$

$$d\delta = \left(\mu_\delta - \sigma_\delta\theta^{W_\delta}\right) dt + \sigma_\delta dW_\delta^{\mathcal{Q}}, \tag{16}$$

with $Cov(W_S^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}) = \rho t$.

We also estimate model (15)–(16) directly from data in the NYMEX, using the approach by Gómez-Valle and Martínez-Rodríguez [16], and obtain the natural gas futures prices with the method of Monte Carlo.

In order to analyse the effect of adding jumps to the commodity price over the futures prices, we make some comparisons between the futures prices obtained with the jump–diffusion model (JDM) and with the diffusion model (DM). We use the root mean square error (RMSE), the percentage root mean square error (PRMSE) and the mean absolute error (MAE) for the out-of-sample period of time as measures of error:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(F_t - \hat{F}_t\right)^2},$$

$$PRMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\frac{F_t - \hat{F}_t}{F_t}\right)^2},$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|F_t - \hat{F}_t|,$$

where n is the number of observations, F_t is the market futures price and $\hat{F}_t$ is the predicted futures price of the different models.

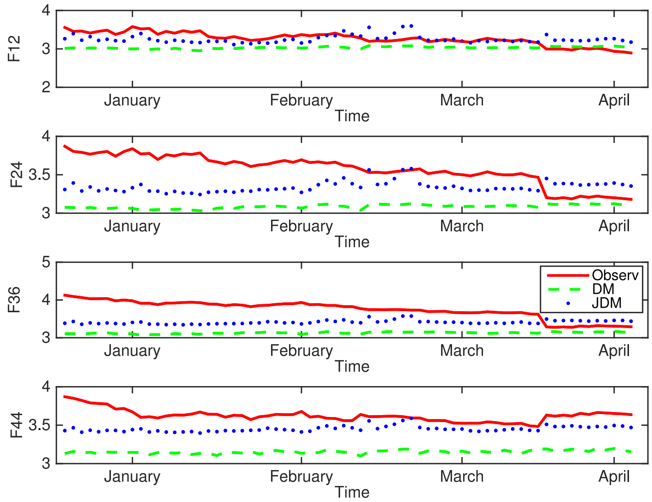

Fig. 2. Natural gas futures prices (January 2015–April 2015) with maturities: 12, 24, 36 and 44 months. The observed futures prices on the NYMEX are the red solid line, the DM futures prices are the green dash line and the JDM futures prices are the blue dotted line.

For the out of sample period of time, in Table 2 we show the values of these different measures of error in the futures prices with different maturities. For short maturities (6 months and 9 months) the DM provides slightly lower errors than the JDM, except for the shortest maturity (1 month). However, for the longest maturities the JDM provides smaller errors than the DM. Moreover, the higher the maturity the higher the differences between the two models.

In Fig. 2, we plot the observed futures prices and those estimated with the DM and the JDM along the out-of-sample period of time for different maturities. As seen from the figure, for the different maturities, the observed futures prices are, in general, over the prices obtained with DM and JDM. More precisely, the prices with the DM are nearly always smaller than those with the JDM. Furthermore, the higher the maturity, the higher the differences.

In sum, JDM and DM underprice the futures prices and the prices obtained with the JDM are, in general, closer to the observed prices than those obtained with the DM. Hence this fact supports the use of jump processes when modelling the commodity price dynamics in order to price commodity futures, especially for high maturities.

5. Conclusions

In order to price commodity derivatives with jump–diffusion processes, we provide a novel procedure based on the estimation of the drifts and jump intensities of the risk-neutral processes. This technique is notable because neither the physical drift, nor the market prices of risk have to be estimated. As a consequence, it is not necessary to make arbitrary assumptions about the market prices of risk as usual in the literature, when a closed-form solution is not known. Furthermore, as we estimate the risk-neutral drifts directly from data in the market, we reduce the misspecification error because we do not have to estimate the physical drifts. Finally, this approach is adaptable: both parametric and nonparametric methods could be used to estimate the required functions.

We show the practical interest of this new approach in an empirical experiment with NYMEX data and we analyse the effect of adding jumps to the commodity spot price over the futures prices.

Acknowledgements

The authors are grateful to the anonymous referees for their constructive suggestions and helpful comments. L. Gómez-Valle and J. Martínez-Rodríguez were supported in part by the GIR Optimización Dinámica, Finanzas Matemáticas y Utilidad Recursiva of the University of Valladolid and the projects: VA191U13 of the Consejería de Educación, JCyL and MTM2014-56022-C2-2-P of the Spanish Ministerio de Economía y Competitividad and European FEDER Funds.

Appendix

In this appendix, we prove Theorem 1 in Section 3.

Proof of Theorem 1. We consider the integral form of (4)

$$S(T+h) - S(T) = \int_T^{T+h} \left(\mu_S - \sigma_S \theta^{W_S} + \lambda^{\mathbb{Q}} E_Y^{\mathbb{Q}}[J]\right)(z)dz + \int_T^{T+h} \sigma_S dW_S^{\mathbb{Q}}(z) + \int_T^{T+h} J d\tilde{N}(z).$$

Table 2
Measures of error: MAE, RMSE and PRMSE, for the out of sample period of time, January 2015–April 2015, with the diffusion and jump–diffusion model.

	DM			JDM		
	RMSE	PRMSE	MAE	RMSE	PRMSE	MAE
F1	2.019×10^{-1}	7.2%	1.565×10^{-1}	1.929×10^{-1}	6.9%	1.448×10^{-1}
F6	1.424×10^{-1}	4.9%	1.209×10^{-1}	2.567×10^{-1}	8.8%	2.271×10^{-1}
F9	1.151×10^{-1}	3.7%	9.852×10^{-2}	1.666×10^{-1}	5.4%	1.290×10^{-1}
F12	2.993×10^{-1}	9.1%	2.637×10^{-1}	1.858×10^{-1}	5.7%	1.509×10^{-1}
F18	1.964×10^{-1}	6.1%	1.585×10^{-1}	1.711×10^{-1}	5.3%	1.399×10^{-1}
F24	5.280×10^{-1}	14.8%	4.839×10^{-1}	3.218×10^{-1}	9.0%	2.892×10^{-1}
F36	6.677×10^{-1}	17.8%	6.206×10^{-1}	4.229×10^{-1}	11.3%	3.865×10^{-1}
F44	4.780×10^{-1}	13.2%	4.706×10^{-1}	1.933×10^{-1}	5.3%	1.711×10^{-1}

Taking into account (3) and that the expected value of stochastic integral is zero, we calculate the expectation with respect to $\mathcal{Q}$ measure, and we obtain

$$F(t, S, \delta; T + h) - F(t, S, \delta; T) = \int_T^{T+h} E^{\mathcal{Q}}[(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J_S])(z)|S(t) = S, \delta(t) = \delta]dz. \tag{17}$$

If we divide by h and take limits in (17), we get (6).

Using the risk-neutral process (4) and the Ito's product rule, see [11] we have

$$d(S^2) = \left(2S(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J]) + \sigma_S^2 + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J^2]\right) dt + 2S\sigma_S dW_S^{\mathcal{Q}} + (2SJ + J^2)d\tilde{N}^{\mathcal{Q}}. \tag{18}$$

If we consider the integral form of (18), with a similar reasoning to one used for the equality (6), we obtain (7).

Finally, we use the risk-neutral processes (4) and (5) and with the Ito's product rule, we have

$$d(S\delta) = \left(\delta(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J]) + S(\mu_\delta - \sigma_\delta\theta^{W_\delta}) + \rho\sigma_S\sigma_\delta\right) dt + \delta\sigma_S dW_S^{\mathcal{Q}} + S\sigma_\delta dW_\delta^{\mathcal{Q}} + \delta J d\tilde{N}^{\mathcal{Q}}. \tag{19}$$

Once more, if we consider the integral form of (19), with a similar reasoning to our analysis above we get (8).

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Chapter 4

The jump size distribution of the commodity spot price and its effect on futures and option prices

[This chapter has been previously published (jointly with Lourdes Gómez-Valle and Julia Martínez-Rodríguez) in *Abstract and Applied Analysis*, vol. 2017, 10 Pages, Article ID 3286549, 2017.]

Hindawi
Abstract and Applied Analysis
Volume 2017, Article ID 3286549, 10 pages
https://doi.org/10.1155/2017/3286549

Research Article

The Jump Size Distribution of the Commodity Spot Price and Its Effect on Futures and Option Prices

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Received 12 July 2017; Accepted 30 August 2017; Published 18 October 2017

Academic Editor: Lucas Jodar

In this paper, we analyze the role of the jump size distribution in the US natural gas prices when valuing natural gas futures traded at New York Mercantile Exchange (NYMEX) and we observe that a jump-diffusion model always provides lower errors than a diffusion model. Moreover, we also show that although the Normal distribution offers lower errors for short maturities, the Exponential distribution is quite accurate for long maturities. We also price natural gas options and we see that, in general, the model with the Normal jump size distribution underprices these options with respect to the Exponential distribution. Finally, we obtain the futures risk premia in both cases and we observe that for long maturities the term structure of the risk premia is negative. Moreover, the Exponential distribution provides the highest premia in absolute value.

1. Introduction

In the literature, the commodity price usually follows a diffusion process with continuous paths when pricing commodity derivatives. Although this assumption is very attractive because of its computational, convenience, theoretical derivation and statistical properties, [1–4] others found significant evidence of the presence of jumps in commodity prices.

In traditional jump-diffusion commodity models, the functions of the stochastic processes and the market prices of risk are usually specified as simple parametric functions, for pure tractability and simplicity. Furthermore, the functions of the models are usually chosen to provide an affine model which has a known closed-form solution. For example, [5] considers a three-factor model where the spot price follows a jump-diffusion stochastic process. In [6] existing commodity valuation models were extended to allow for stochastic volatility and simultaneous jumps in the spot price and volatility. The standard geometric Brownian motion augmented by jumps was used by [7] to describe the underlying spot and the mean reverting diffusion processes for the interest rate and convenience yield in gold and copper price models. In [8] a seasonal mean reverting model with jumps and Heston-type stochastic volatility is analyzed.

We consider, in this paper, a two-factor jump-diffusion commodity model, where one of the factors is the commodity spot price and the other is the convenience yield. These factors are often used in the commodity literature. For example, [9, 10] propose affine models with these two factors, though they do not consider jumps. Then, all the functions can be easily estimated and the commodity derivatives priced. However, there is not any empirical evidence or consensus that affine models are the best models to price commodity futures. Furthermore, the market prices of risk are not observed in the markets. If we considered other more realistic functions for the state variables or the market prices of risk or even a nonparametric approach, then, the model would not be affine anymore, a closed-form solution could not be obtained, and, therefore, the estimation of the market prices of risk would not be possible. However, [11] shows a new approach to estimate the whole functions of the model although a closed-form solution is not known. They even apply it to a jump-diffusion model where the jump follows a Normal distribution. Finally, they estimate the whole functions with a nonparametric technique in order to avoid imposing arbitrary functions on the model.

Other authors have found seasonal patterns in commodity markets and this fact has been taken into account in their models; see [12–15].

In this paper, we price natural gas futures assuming that the spot price follows a diffusion process and, then, we also consider a jump-diffusion process with a Normal jump size distribution as in [11] but for a higher prediction period of time. Moreover, we also assume that the jump size follows an Exponential distribution in order to make some comparisons and analyze the role of the jump size distribution. We find that for short maturities the Normal distribution provides more accurate futures prices. However, the Exponential distribution shows the lowest error for long maturities. Furthermore, for long maturities, the models with both distributions underprice the futures in the market, but the futures prices with the Exponential distribution are higher than with the Normal distribution. Moreover, they are closer to the observed ones. Then, in order to complement [11], we also price futures options when the jump is not taken into account and when Normal as well as an Exponential jump size distributions are considered. In this case, we see that the differences between the prices are higher (in particular for out of money options).

Futures prices are potentially a valuable source of information on market expectations of asset prices. In fact, financial investors use futures contracts to hedge against commodity price risk. However, exploiting this information is difficult in practice, because of the presence of a risk premium between the current futures price and the expected spot price of the underlying asset. Moreover, understanding this premium is very important; see [16]. Therefore, in this paper, we also show an out-of-sample analysis of the natural gas futures risk premia. We find that the risk premium with the Exponential distribution is negative more times than with the Normal distribution. In all the cases, we use natural gas data traded at NYMEX and a nonparametric approach to estimate the whole functions of the two-factor model.

The rest of the paper is organized as follows. Section 2 shows a two-factor jump-diffusion model to price commodity derivatives. Section 3 prices futures with a diffusion model and a jump-diffusion model, when the jump size follows a Normal as well as an Exponential distribution. Then a comparison is made. Section 4 compares futures option prices when the jump follows a Normal or an Exponential distribution. Section 5 analyzes the futures risk premium and, finally, Section 6 concludes. All the implementation has been done using MATLAB software.

2. The Valuation Model

In this section, we introduce a commodity model with two state variables: the spot price and the convenience yield, for pricing commodity derivatives; see also [11, 17]. We assume that the spot price follows a jump-diffusion process, because commodity prices usually suffer from abrupt changes in the markets; see [1]. However, we assume that the convenience yield is a diffusion process because its behaviour is not affected by extreme changes; see, for example, [6].

Define $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ as a complete filtered probability space which satisfies the usual conditions where $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration; see [18–20]. Let S be the spot price and δ the instantaneous convenience yield. We assume that these factors follow this joint jump-diffusion stochastic process:

$$\begin{aligned} dS(t) &= \mu_S(S(t), \delta(t))\,dt + \sigma_S(S(t), \delta(t))\,dW_S(t) \\ &\quad + dJ(t), \\ d\delta(t) &= \mu_\delta(S(t), \delta(t))\,dt + \sigma_\delta(S(t), \delta(t))\,dW_\delta(t), \end{aligned} \tag{1}$$

where μ_S and μ_δ are the drifts and σ_S and σ_δ the volatilities. Moreover, W_S and W_δ are Wiener processes and the impact of the jump is given by the compound Poisson process, $J(t) = \sum_{i=1}^{N(t)} Y_i$, with jump times $(\tau_i)_{i\geq 1}$, where $N(t)$ represents a Poisson process with intensity $\lambda(S, \delta)$ and $Y_1, Y_2, \ldots$ is a sequence of identically distributed random variables with a probability distribution Π. We assume that W_S and W_δ are independent of N, but the standard Brownian motions are correlated with

$$[W_S, W_\delta](t) = \rho t. \tag{2}$$

We also suppose that the jump magnitudes and the jump arrivals time are uncorrelated with the diffusion parts of the processes. We assume that the functions $\mu_S, \mu_\delta, \sigma_S, \sigma_\delta, \lambda$ and Π satisfy suitable regularity conditions: see [20, 21]. Under the above assumptions, a commodity futures price at time t with maturity at time T, $t \leq T$, can be expressed as $F(t, S, \delta; T)$ and at maturity it verifies that $F(T, S, \delta; T) = S$.

We assume that the market is arbitrage-free. Then, there exists an equivalent martingale measure, $\mathcal{Q}$-measure, which is known as the risk-neutral measure; see extended Girsanov-type measure transformation in [22]. The state variables of the model (1) under the risk-neutral measure are as follows:

$$\begin{aligned} dS &= \left(\mu_S - \sigma_S \theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[Y_1]\right) dt + \sigma_S dW_S^{\mathcal{Q}} \\ &\quad + d\widetilde{J}^{\mathcal{Q}}(t), \\ d\delta &= \left(\mu_\delta - \sigma_\delta \theta^{W_\delta}\right) dt + \sigma_\delta dW_\delta^{\mathcal{Q}}, \end{aligned} \tag{3}$$

where $W_S^{\mathcal{Q}}$ and $W_\delta^{\mathcal{Q}}$ are the Wiener processes under the risk-neutral measure and $[W_S^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}](t) = \rho t$. The market prices of risk associated with W_S and W_δ Wiener processes are $\theta^{W_S}(S, \delta)$ and $\theta^{W_\delta}(S, \delta)$, respectively. Finally, $\widetilde{J}^{\mathcal{Q}}(t) = \sum_{i=1}^{N^{\mathcal{Q}}(t)} Y_i - \lambda^{\mathcal{Q}} t E_Y^{\mathcal{Q}}[Y_1]$ is the compensated compound Poisson process under $\mathcal{Q}$-measure, the intensity of the Poisson process $N^{\mathcal{Q}}(t)$ is $\lambda^{\mathcal{Q}}(S, \delta)$, and $E^{\mathcal{Q}}$ denotes the expectation under the $\mathcal{Q}$-measure. Then, the futures price can be expressed as

$$F(t, S, \delta; T) = E^{\mathcal{Q}}[S(T) \mid S(t) = S, \delta(t) = \delta]. \tag{4}$$

Let $V(t, S, \delta, T_2; T_1)$ be the price of a European call option that matures on T_1 on a futures contract that expires at T_2, $T_1 \leq T_2$, and K is the strike price. Then, analogously to (4), an

European commodity futures option is priced as the expected discounted payoff under the $\mathbb{Q}$-measure; see [6, 22]:

$$\begin{aligned} V\left(t,S,\delta,T_2;T_1\right) = E^{\mathbb{Q}}\Big[e^{-\int_t^{T_1} r(u)du} \\ \cdot \max\left(F\left(T_1,S\left(T_1\right),\delta\left(T_1\right);T_2\right)-K,0\right) \mid S(t) \\ = S,\delta(t)=\delta\Big], \end{aligned} \tag{5}$$

where r denotes the instantaneous risk-free interest rate, which is assumed to be constant. Moreover, $\tau_1 = T_1 - t$ and $\tau_2 = T_2 - T_1$ are the maturity of the option contract and futures contract, respectively.

3. Valuation of Commodity Futures with NYMEX Data

In this section, by means of an empirical application with natural gas NYMEX data, we illustrate the advantages and disadvantages of modelling the spot price with a jump-diffusion process with an Exponential distribution and a Normal distribution. In all the cases, we use the approach, the nonparametric techniques and the in-sample data (January 2004–December 2014) as in [11], to estimate the risk-neutral functions. However, we increase the out-of-sample period where we price the natural gas derivatives from January till July 2015.

In this empirical application, we use the model stated in Section 2, where the factors are the commodity spot price and the convenience yield. For simplicity and tractability and as usual in the literature, we also assume that the distribution of the jump size under $\mathbb{Q}$-measure is known and equal to the distribution under $\mathscr{P}$-measure. This means that all risk premium related to the jump is artificially absorbed by the change in the intensity of the jump from λ under the physical measure to $\lambda^{\mathbb{Q}}$ under the risk-neutral measure; see [8, 11, 23]. Moreover, we assume the jump size follows a Normal distribution $N(0,\sigma_Y)$ (see [11]) or an Exponential distribution $\text{Exp}\,(\sigma_Y)$ (see [6, 24, 25]) among others.

In order to price natural gas futures, we use daily natural gas data from the NYMEX in Quandl platform. Natural gas spot prices were obtained from the U.S. Energy Information Administration (EIA). The sample period covers from January 2004 to July 2015. More precisely, we use data from January 2004 to December 2014 to estimate the risk-neutral functions as in [11] and, then, we keep data from January to July 2015 to make our out-of-sample analysis of the futures prices.

As it is well known in the literature, the convenience yield is not observed in the markets. Then, following [9], we approximate it by the following result

$$\delta_{T-1,T} = r_{T-1,T} - 12\ln\left[\frac{F(t,S,\delta;T)}{F(t,S,\delta;T-1)}\right], \tag{6}$$

where $r_{T-1,T}$ denotes the forward interest rate between $T-1$ and T. We obtain this forward interest rate with two daily T-Bill rates with maturities as close as possible to the futures contracts' ones in order to compute $\delta_{1,2}$, the one-month ahead annualized convenience yield. The latter is identified with the instantaneous convenience yield $\delta_{0,1}$; see [9, 11] for more details.

In order to estimate the risk-neutral functions of the jump-diffusion models, we follow the same approach as [11]. Note that similar techniques have been proposed for interest rate derivatives; see [26, 27].

Firstly, we obtain the compensated risk-neutral drift of the spot price by means of the following equality which relates the futures slope in the origin with the drift of the spot in the stochastic process under $\mathbb{Q}$-measure; see [11] for more detail:

$$\frac{\partial F}{\partial T}(t,S,\delta;t) = \left(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathbb{Q}} E_Y\left[Y_1\right]\right)(t). \tag{7}$$

We approximate the partial derivative by means of numerical differentiation

$$\left.\frac{\partial g}{\partial T}\right|_{T=t} = \frac{-25g(t) + 48g(t+\Delta) - 36g(t+2\Delta) + 16g(t+3\Delta) - 3g(t+4\Delta)}{12\Delta} + O\left(\Delta^4\right), \tag{8}$$

with futures prices with maturities equal to 1, 2, 3, and 4 months. Then, we estimate it by means of the Nadaraya-Watson estimator; see [28] for more details on this estimation technique.

Secondly, for the risk-neutral jump intensity, we use a result proposed in [11] which relates the futures slope in the origin with the spot price, spot price volatility, and parameters of jump size distribution under $\mathbb{Q}$-measure:

$$\frac{\partial(SF)}{\partial T}(t,S,\delta;t) = \left(2S\frac{\partial F}{\partial T} + \sigma_S^2 + \lambda^{\mathbb{Q}} E_Y\left[Y_1^2\right]\right)(t). \tag{9}$$

Initially, [11] assumed that the jump size followed a Normal distribution as $Y_1 \rightsquigarrow N(0,\sigma_Y^2)$, then, $E_Y[Y_1] = 0$, and $\sigma_Y^2 = E_Y[Y_1^2]$. Furthermore, it is well known that

$$\begin{aligned} E_Y\left[Y_1^{2k}\right] &= \sigma_Y^{2k}\prod_{n=1}^{k}(2k-1), \\ E_Y\left[Y_1^{2k-1}\right] &= 0, \quad k = 1,2,3,\ldots. \end{aligned} \tag{10}$$

In this paper we also assume that the jump size follows an Exponential distribution as $Y_1 \rightsquigarrow \text{Exp}\,(\sigma_Y)$; then:

$$E_Y\left[Y_1^k\right] = k!\sigma_Y^k, \quad k = 1,2,3,\ldots. \tag{11}$$

This jump size distribution has also been considered by [29] for the volatility and [30] for interest rates. This assumption could be useful for pricing during periods in which positive jumps are expected to dominate negative jumps, for example, coming out of an economic crisis (see [30]) or in certain economic regimes (see [31]).

With both distributions, the parameters of the jump size distribution and the spot price volatility, σ_S, are estimated by means of a system of moment equations of a jump-diffusion process (see [11, 32, 33]):

$$\begin{aligned} &M_S^2(S,\delta) \\ &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}E\left[(S(t+\Delta t)-S(t))^2 \mid S(t)=S,\delta(t)=\delta\right] \\ &= \sigma_S^2(S,\delta)+\lambda(S,\delta)E_Y\left[Y_1^2\right], \\ &M_S^k(S,\delta) \\ &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}E\left[(S(t+\Delta t)-S(t))^k \mid S(t)=S,\delta(t)=\delta\right] \\ &= \lambda(S,\delta)E_Y\left[Y_1^k\right], \quad k\geq 3. \end{aligned} \tag{12}$$

More precisely, we use moments M_S^2, M_S^4, and M_S^6 for the Normal distribution and moments M_S^2, M_S^3, and M_S^4 for the Exponential distribution; see, for example, [34, 35], respectively. Then, Nadaraya-Watson estimator is applied. Once we estimate the parameters of the jump size distribution and the spot volatility and approximate the previous partial derivatives $(\partial F/\partial T)|_{T=t}$ and $(\partial(SF)/\partial T)|_{T=t}$, we replace them in (9). Then, we estimate the risk-neutral jump intensity of the spot price with the Nadaraya-Watson estimator.

As the convenience yield follows a diffusion process, we estimate its risk-neutral drift by means of

$$\begin{aligned} &\frac{\partial(\delta F)}{\partial T}(t,S,\delta;t) \\ &\quad= \left(\delta\frac{\partial F}{\partial T}+S\left(\mu_\delta-\sigma_\delta\theta^{W_\delta}\right)+\rho\sigma_S\sigma_\delta\right)(t); \end{aligned} \tag{13}$$

see [11]. In order to estimate the correlation, we use the moment

$$\begin{aligned} M_{S,\delta}^1(S,\delta) &= \lim_{\Delta t\to 0}\frac{1}{\Delta t} \\ &\cdot E\left[(S(t+\Delta t)-S(t))(\delta(t+\Delta t)-\delta(t)) \mid S(t)\right. \\ &\left.= S,\delta(t)=\delta\right] = \rho(S,\delta)\sigma_S(S,\delta)\sigma_\delta(S,\delta), \end{aligned} \tag{14}$$

and the Nadaraya-Watson estimator; see [36] for more details. Later, we replace the estimated covariance and the approximations of $(\partial F/\partial T)|_{T=t}$ and $(\partial(\delta F)/\partial T)|_{T=t}$ in (13) and we estimate the risk-neutral drift of the convenience yield by means of the Nadaraya-Watson estimator.

Finally, the volatility of the convenience yield under $\mathscr{P}$-measure is equal to the volatility under $\mathscr{Q}$-measure. Hence, we estimate σ_δ by means of the second order moment of a diffusion process:

$$\begin{aligned} M_\delta^2(S,\delta) &= \lim_{\Delta t\to 0}\frac{1}{\Delta t} \\ &\cdot E\left[(\delta(t+\Delta t)-\delta(t))^2 \mid S(t)=S,\delta(t)=\delta\right] \\ &= \sigma_\delta^2(S,\delta), \end{aligned} \tag{15}$$

and Nadaraya-Watson estimator, with spot and convenience yield data.

Up to this point, we have focused on the estimation of the risk-neutral functions of jump-diffusion processes. If we assume that the spot price follows a diffusion stochastic process, the factors of the model will follow this joint diffusion stochastic process under $\mathscr{Q}$-measure:

$$\begin{aligned} dS &= \left(\mu_S-\sigma_S\theta^{W_S}\right)dt+\sigma_S dW_S^{\mathscr{Q}}, \\ d\delta &= \left(\mu_\delta-\sigma_\delta\theta^{W_\delta}\right)dt+\sigma_\delta dW_\delta^{\mathscr{Q}}, \end{aligned} \tag{16}$$

with $[W_S^{\mathscr{Q}}, W_\delta^{\mathscr{Q}}](t)=\rho t$.

The estimation of these functions is made by means of the approach in [37] and the Nadaraya-Watson estimator, with the same natural gas data and numerical differentiation approximation as the jump-diffusion model.

For analyzing the effect of the jumps on the natural gas futures prices, we price natural gas futures with a diffusion model (DM) as well as a jump-diffusion model with a Normal jump size distribution (JDMN) and an Exponential distribution (JMDExp). In order to price natural gas futures it is necessary to solve a partial integrodifferential equation or, equivalently, by means of Feynman-Kac Theorem the expectation in (4). As we use nonparametric methods a closed-form solution cannot be found. Recently, several numerical methods have been developed to solve this kind of problems; see [38, 39].

In this paper, we use the Monte Carlo simulation approach because it is widely used by practitioners in the markets, especially for multiple factor models because of its simplicity and efficiency, [40]. More precisely, we consider 5000 simulations and a daily time step, $\Delta t = 1/250$. We price natural gas futures with maturities from 1 to 44 months and we compare them with those traded at NYMEX along the out-of-sample (January–July 2015). As measures of error, we use the root mean square error (RMSE) and the percentage root mean square error (PRMSE) for the out-of-sample:

$$\begin{aligned} \text{RMSE} &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(F_t-\widehat{F}_t\right)^2}, \\ \text{PRMSE} &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\frac{F_t-\widehat{F}_t}{F_t}\right)^2}, \end{aligned} \tag{17}$$

where n is the number of observations, F_t is the futures price traded at NYMEX, and $\widehat{F}_t$ is the predicted futures price with the different models.

Table 1: RMSE and PRMSE for the out-of-sample (January–July 2015) for DM, JDMN, and JDMExp models.

	RMSE			PRMSE		
	DM	JDMN	JDMExp	DM	JDMN	JDMExp
F1	0.1582	0.1362	0.1398	5.7376	4.9326	5.0514
F6	0.1636	0.1744	0.1893	5.4787	5.9201	6.4430
F9	0.1472	0.1162	0.1531	4.5256	3.6732	4.9852
F12	0.2113	0.1529	0.1590	6.2670	4.6156	5.1349
F18	0.2651	0.2012	0.1466	7.7642	5.8706	4.5386
F24	0.3757	0.3172	0.2176	10.3069	8.6277	6.0603
F30	0.3826	0.3141	0.1630	10.7959	8.8260	4.5345
F36	0.4753	0.4199	0.3126	12.4319	10.8809	8.0727
F42	0.4295	0.3661	0.2254	11.8569	10.0716	6.1337
F44	0.5065	0.4413	0.2953	13.7741	11.9909	7.9929

Table 1 shows a summary of the RMSE and PRMSE of the different models for the out-of-sample and for several maturities. F1 is the futures price with a maturity of 1 month, F6 with six months, and so on. In this table, we show that for short maturities the RMSE are usually lower than for long maturities. Besides, for very short maturities sometimes the diffusion model prices natural gas futures quite accurately, as for F6. However, for F1 and for maturities higher or equal to 9 months, jump-diffusion models provide lower errors than the diffusion model as in [11]. Moreover, for maturities lower than 18 months the JDMN is more accurate than the JDMExp, but for long maturities (higher or equal to 18 months) the results change and the JDMExp displays lower errors than the JDMN. Therefore, depending on the maturity of the futures to price, some models are more accurate than others. As far as the PRMSE is concerned, we reach the same conclusion but, for maturities longer or equal than 36, the differences between the relative error of the JDMN and JDMExp are higher.

We now turn our attention to the absolute errors along the out-of-sample for some maturities. Figure 1 plots the absolute errors of the considered models for some maturities such as 6, 18, and 44 months. We show only these maturities because the behaviour of the rest is analogous. For example, for a maturity of 6 months, we observe that the errors of the DM are the lowest along the first months of the out-of-sample, although it changes for the last months. For longer maturities, for example 18 months, the JDExp model provides the lowest errors for a great number of months, followed by the JDN. Finally, when we consider the longest available maturity, the JDExp model is clearly the most accurate.

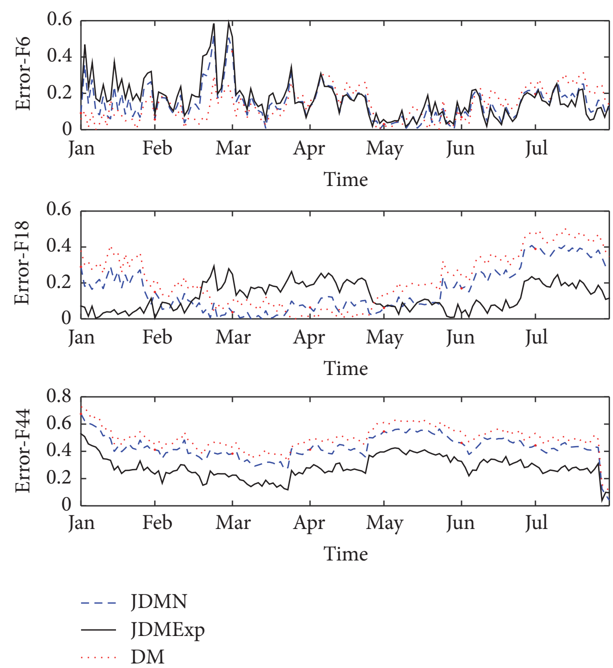

Figure 1: Absolute error of the futures prices for the out-of-sample (January–July 2015) with maturities: 6, 18, and 44 months. The absolute error for the DM is the red dotted line, the JDMN is the blue dash line, and the JDMExp is the black solid line.

If we analyze the price behaviour along the out-of-sample, we observe high changes for short maturities, but they decrease when we increase the maturity. That is, the longer the maturity, the lower the price variations along the time. In order to illustrate this result, in Figure 2, we plot the futures prices traded at NYMEX and those priced with the different models considered in this paper (DM, JDMN, and JDMExp). As we can see in this figure, the highest variations are for F6 and the lowest are for F44. Focusing on the estimated prices, we observe that, in general, the DM provides the lowest prices and the JDMExp the highest prices for each maturity along the time for some maturities. We observe that the NYMEX and estimated futures prices usually rise when the maturity increases, but the rate of rising of the market prices is higher than the rate of the estimated prices with the different models. We also see that the estimated models overprice the NYMEX F6 futures in several months. However, in most of the cases, the JDMN and the DM underprice the NYMEX futures for a maturity of 18 months. Finally, for a maturity of 44 months, the whole estimated models underprice the NYMEX futures. Then, the higher the maturity the higher the possibility for natural gas futures to be underpriced by the different models, especially by the DM.

In conclusion, as in [11], the jump-diffusion models provide lower errors than the diffusion model apart from some

Table 2: Ratios between the JDMN and JDMExp option prices.

Strike	95%				100%				105%			
$\tau_1 \backslash \tau_2$	3 m	6 m	9 m	12 m	3 m	6 m	9 m	12 m	3 m	6 m	9 m	12 m
3 m	0.91	0.76	0.65	0.59	0.91	0.70	0.52	0.42	0.94	0.63	0.36	0.19
6 m	0.88	0.73	0.63	0.58	0.88	0.68	0.52	0.43	0.90	0.62	0.40	0.24
9 m	0.80	0.68	0.60	0.58	0.79	0.63	0.50	0.43	0.79	0.56	0.37	0.26
12 m	0.80	0.69	0.62	0.60	0.78	0.63	0.51	0.46	0.78	0.57	0.40	0.29

short maturities. Hence, this fact supports the use of jump processes when modelling the commodity price dynamics for pricing natural gas futures. As far as the jump size distribution is concerned, the JDMN prices are, in general, lower than the JDMExp prices. This is consistent with the assumptions made for the jump size distribution in Section 3. Under the Normal distribution the average jump size is zero, whereas under the Exponential distribution the average jump size is positive. Therefore, average impact of the jumps on the spot prices under the Normal distribution should be lower than under the Exponential distribution. Moreover, the Normal distribution provides the lowest error for maturities shorter than or equal 12 months, but the Exponential distribution is more accurate for longer maturities. This fact could be due to the very low natural gas spot price during the prediction period of time. Furthermore, investors in the market should take care of the possible overpricing or underpricing of these models depending on the maturity of the futures.

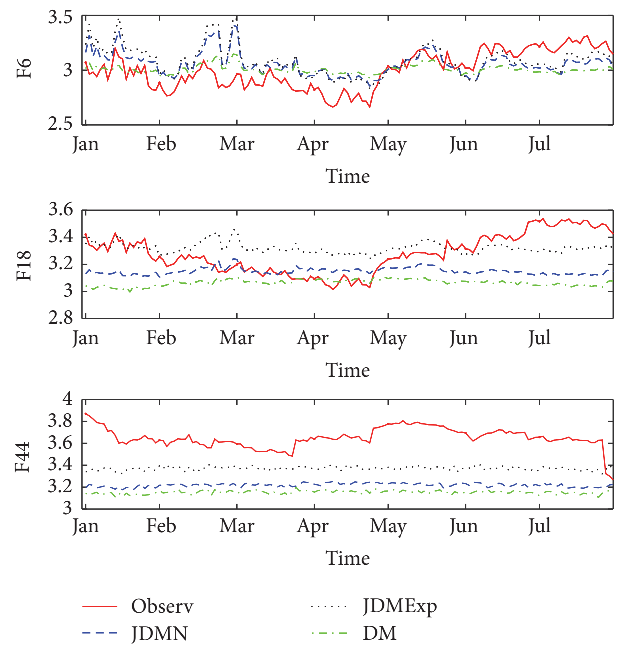

Figure 2: Natural gas futures prices (January–July 2015) with maturities: 6, 18, and 44 months. The NYMEX futures prices are the red solid line, the DM is the green dashed dotted line, the JDMN is the blue dash line, and the JDMExp is the black dotted line.

4. Valuation of Futures Options

In the previous section we have already seen the superiority of the jump-diffusion models over the diffusion models for pricing natural gas futures and that, for long maturities, the model with an Exponential distribution is more accurate than the other models. Hence, in this section we present the effect of the different jump size distributions on a different natural gas derivative: a futures option. In order to price this European call option we use the same NYMEX data and estimation methodology than in the previous section, but, now, the Monte Carlo method approximates (5) with 5000 simulations and a daily time step ($\Delta t = 1/250$).

We assume different option maturities, τ_1, such as 3, 6, 9, and 12 months, and different futures maturities, τ_2, equal to 3, 6, 9, and 12 months. We also assume that the strike price, which is a percentage of the natural gas spot price at the moment of its pricing, is equal to 95%, 100%, and 105%. Therefore, the options are priced in the money, at the money, and out of the money, respectively.

As the instantaneous interest rate is not observable, we use the three-month Treasury Bill rates of the US Federal Reserve at the valuation moment as a proxy. In the term structure literature, this Treasury Bill rate is also usually considered as a proxy of the instantaneous interest rate; see, for example, [33].

In this paper, we price the futures options the first day of the out-of-sample data, that is, on January 3, 2015, and we observe that the higher the strike price, the lower the option price. However, conclusions do not change if we consider other different days of the out-of-sample for valuation.

As we do not have observations of European natural gas option prices for different maturities, we compare the prices when the Normal and Exponential jump sizes are considered. In Table 2, we show some ratios between the JDMN and JDMExp for different strike prices and maturities on January 3, 2015. As we can see, for options and underlying futures with short maturities (3 months) the ratios are higher than 90%. The main reason is that the futures prices with short maturities are quite similar for both distributions, although the futures prices with the Normal distribution are slightly lower. However, as we increase the maturities, especially of the futures, the ratios decrease considerably till 19%. This fact is consistent with the high differences between the futures prices with both distributions when the maturity increases. Moreover, these differences are even higher because the futures price is the underlying of the option. Therefore, we conjecture that, in order to price futures options accurately, other stochastic variables should be considered in the model, such as the volatility or interest rates.

This result can be very interesting for practitioners, because they should take into account the fact that the Exponential jump size distribution overprices option prices with respect to the Normal distribution, which is consistent with the results obtained in the previous section for jump-diffusion futures prices. Finally, we see that the higher the strike price the lower the ratio. Therefore, the highest price differences can be found for the out of the money options.

5. Futures Risk Premium

The futures risk premium provides a link between natural gas futures and expected spot prices and it is a key measure in risk management. In particular, the term structure of commodity risk premia supplies additional information about the role of the net hedging pressure. Then, it is an important factor in understanding the markets and it deserves great attention.

In the literature, the risk premium is defined as the difference between the expected future spot price and the futures price; see [25, 41] among others:

$$RP = E\left[S\left(T\right) \mid S\left(t\right) = S, \delta\left(t\right) = \delta\right] - F\left(t, S, \delta; T\right). \tag{18}$$

Therefore, the risk premium is the reward for holding a risk rather than a risk-free investment; see [41]. In energy markets, the sign of the risk premium usually changes along the time, with the maturity of the futures and even with the market and the commodity; see for example [42].

On the one hand, commodity consumers may enter into a long position in futures contracts, because they want to insure against future increases in the spot price, so they accept prices over the expected spot price. On the other hand, commodity producers may enter into a short position in futures contracts because they wish to hedge their revenue risk. Since this decision is taken in advance, they accept prices below the expected spot price. Then, if the activity of consumers is greater than that of producers, there will be an excess of commercial participants looking to enter a long position. In this case, the net hedging pressure theory establishes that the futures price will be higher than the expected future spot price to induce speculators to balance the market by taking a short position. In contrast, if the hedging activity of producers is greater than that of consumers, there will be an excess of commercial participants looking to enter a short position. Then, the expected future spot price will be higher than the futures price to induce speculators to balance the market by taking a long position. Therefore, the commodity futures risk premia (in absolute value) can be seen as the return that speculators expect to receive to compensate the market; see [42].

In this section, we obtain the natural gas futures risk premia for the out-of-sample (January–July 2015). We use the natural gas futures prices traded at NYMEX for maturities between 1 and 24 months, but we also need to calculate $E[S(T) \mid S(t) = S, \delta(t) = \delta]$. In this case, the functions of the stochastic processes (1) are estimated directly from the moment conditions for the different jump distributions; see, for example, [34] for the Normal distribution and [35] for the Exponential distribution. The market prices of risk are not taken into account because there is not a change from the physical to the risk-neutral measure. Then, these estimated functions are used to obtain $E[S(T) \mid S(t) = S, \delta(t) = \delta]$ by means of Monte Carlo simulation approach, with 5000 simulations and a daily time step ($\Delta t = 1/250$).

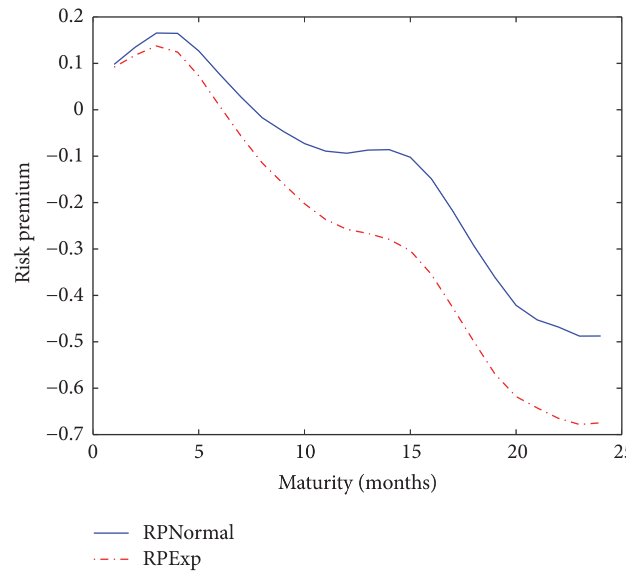

Figure 3: The risk premium as a function of time to maturity for the JDMN and JDMExp models.

Figure 3 shows the term structure of natural gas risk premia with the Normal and the Exponential jump size distributions, hereafter RPNormal and RPExp, respectively. We calculate these values like the mean of the risk premia, for each maturity, in the out-of-sample. In this figure, both RPNormal and RPExp have, in general, the same behaviour although the risk premium under the Normal jump size distribution is always higher than the risk premium under the Exponential distribution. This fact is consistent with the mean of the distributions considered in each case. Furthermore, as it can be seen in Figure 3, the risk premium is positive for short maturities (approximately, up to 7 or 8 months for the Exponential and the Normal distribution, resp.). Following the net hedging pressure theory, for these short maturities the activity of the producers is higher than the consumers activity and the risk premium is the average return that speculators would receive by entering a long position in the natural gas futures markets and holding the futures to expiration. This means that the futures prices are below the expected spot prices and the futures curve is said to be normally backwardated; see [43]. However, for maturities higher than 7 or 8 months the risk premium starts to be negative. In this case, the futures prices are above the expected spot prices and, then, the curve is said to be in Normal contango; see [43]. Following the net hedging pressure theory, consumers have to offer an incentive to induce speculators to enter a short position, and the absolute value of the risk premia is the return that speculators expect to receive for balancing the market. More precisely, in general, the higher the maturity the more negative the risk premium and, then, speculators expect to receive a higher compensation to balance the market.

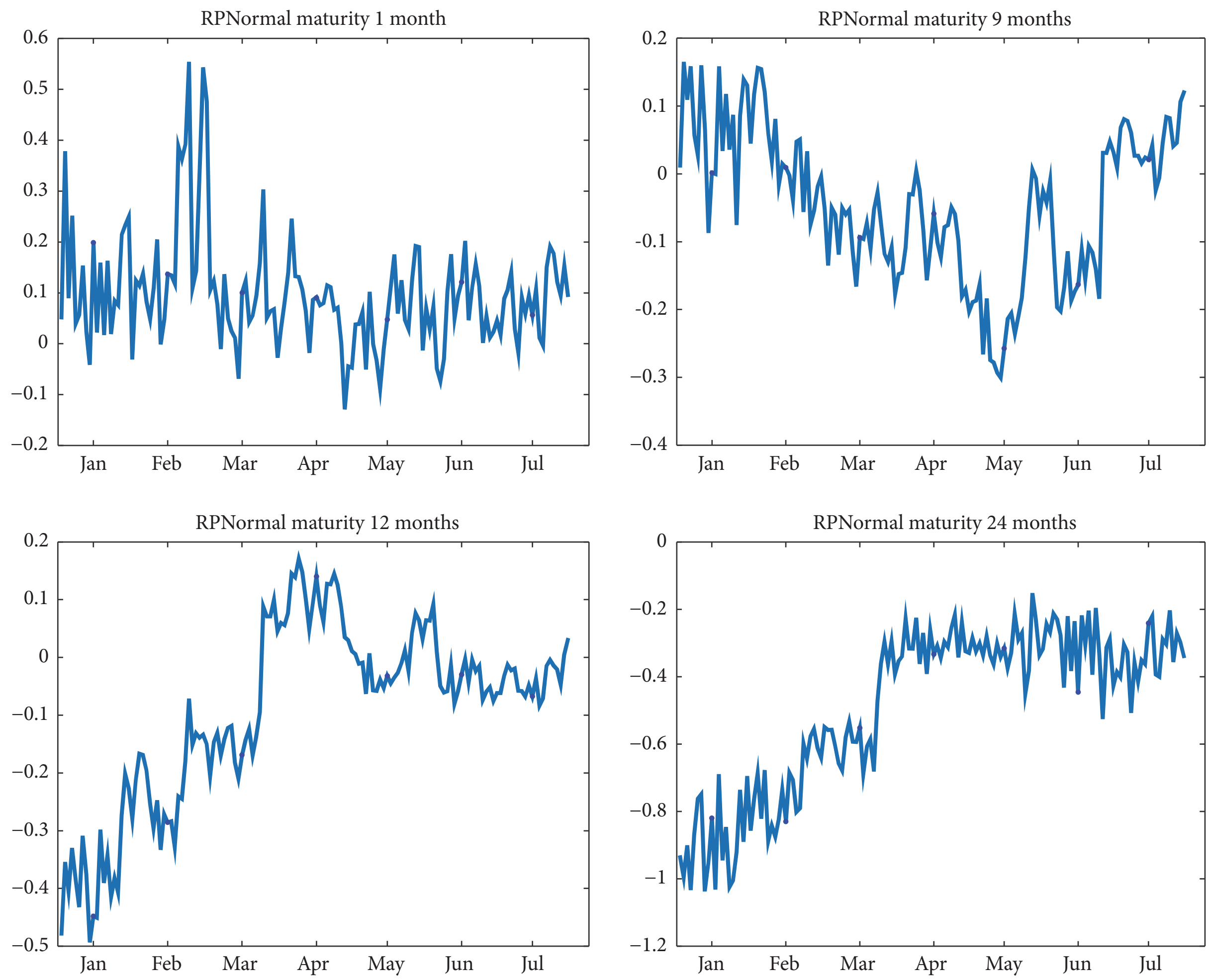

Figure 4: The risk premium for the JDMN model along the out-of-sample, for maturities 1, 9, 12, and 24 months.

In Figures 4 and 5, we plot the estimated risk premium as a function of time when the jump size follows a Normal and an Exponential distribution, respectively. These figures show that there is mixed evidence of the sign of the risk premium and, besides, the risk premia are strongly time-varying. Hence, the activity of speculators is also time-varying. In Figure 3 we saw that the risk premium for very short maturities was positive; however, in Figure 4 we see that it is not always positive but it is on average. Therefore, in general, the futures price is a downward biased predictor of the expected spot price for short maturities. However, for longer maturities, we see that the risk premium is usually negative, apart from maturities longer than 12 months for the Exponential distribution and longer than 24 months for the Normal distribution, where it is always negative. Then, for maturities longer than 6 months, the futures price is an upward biased predictor of the expected spot price as a whole.

6. Conclusions

In this paper, we make mainly two contributions. Firstly, we apply the approach in [11] for pricing natural gas futures, but we assume that the jump size follows an Exponential distribution. We use the data and nonparametric techniques to estimate all the risk-neutral functions of the model as in [11]. Then, considering a higher out-of-sample period, we show that considering a jump-diffusion model provides lower errors than a diffusion model when pricing futures. Furthermore, we also show that the Normal distribution is the best assumption to price short maturity futures. However, the Exponential distribution provides lower errors when pricing long maturity futures.

The second contribution comes through the use of [11] approach and data to price natural gas options and risk premia. We find that, in general, the model with the Exponential distribution overprices option prices with respect to the Normal distribution. We think that, in order to price options more accurately, other state variables should be taken into account.

As far as the risk premia is concerned, we find that this premium is negative more times with the Exponential distribution than with the Normal distribution. These facts should be taken into account when a jump-diffusion is applied to price commodity futures or options.

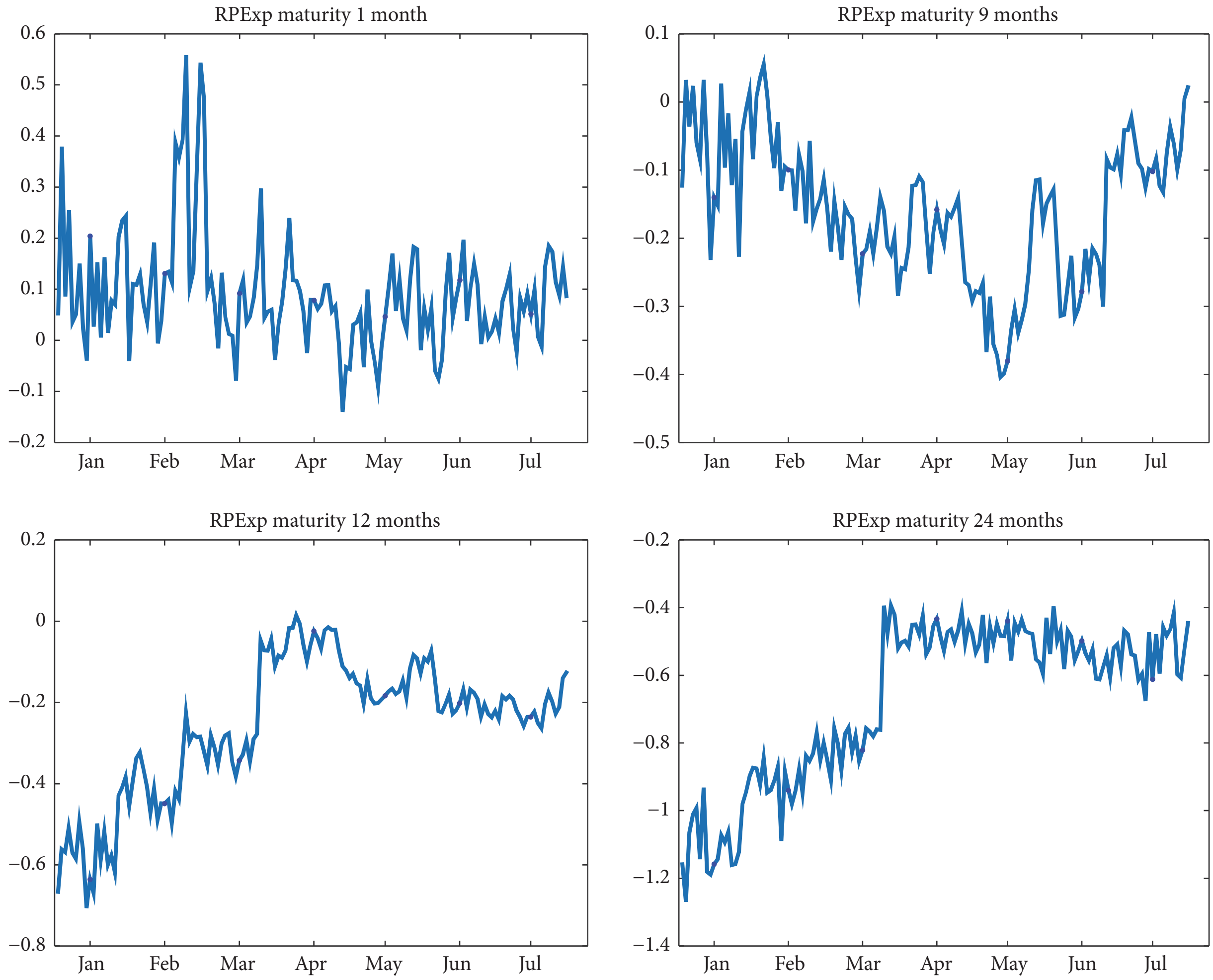

Figure 5: The risk premium for the JDMExp model along the out-of-sample, for maturities 1, 9, 12, and 24 months.

Both of these contributions open opportunities for further work. On the one hand, we could consider that the distribution of the jump size under $\mathcal{Q}$-measure is not equal to the distribution under $\mathcal{P}$-measure. In this case, we would have to obtain an additional relation to estimate the parameter of the jump size distribution under the risk-neutral measure. On the other hand, it is straightforward to see that a more realistic model should include the effect of seasonality, especially in natural gas markets.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

L. Gómez-Valle and J. Martínez-Rodríguez were supported in part by the GIR Optimización Dinámica, Finanzas Matemáticas y Utilidad Recursiva of the University of Valladolid, and the Projects VA191U13 of the Consejería de Educación, JCyL and MTM2014-56022-C2-2-P of the Spanish Ministerio de Economía y Competitividad and European FEDER Funds.

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Chapter 5

A multiplicative seasonal component in commodity derivative pricing

[This chapter has been previously published (jointly with Lourdes Gómez-Valle and Julia Martínez-Rodríguez) in *Journal of Computational and Applied Mathematics*, vol 330, pp. 835-847, 2018.]

Journal of Computational and Applied Mathematics 330 (2018) 835–847

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A multiplicative seasonal component in commodity derivative pricing

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ARTICLE INFO

Article history:
Received 24 November 2016
Received in revised form 5 May 2017

JEL classification:
G13
G17

Keywords:
Jump–diffusion stochastic processes
Seasonality
Risk-neutral measure
Numerical differentiation
Nonparametric estimation
Risk premium

ABSTRACT

In this paper, we focus on a seasonal jump–diffusion model to price commodity derivatives. We propose a novel approach to estimate the functions of the risk-neutral processes directly from data in the market, even when a closed-form solution for the model is not known. Then, this new approach is applied to price some natural gas derivative contracts traded at New York Mercantile Exchange (NYMEX). Moreover, we use nonparametric estimation techniques in order to avoid arbitrary restrictions on the model. After applying this approach, we find that a jump–diffusion model allowing for seasonality outperforms a standard jump–diffusion model to price natural gas futures. Furthermore, we also show that there are considerable differences in the option prices and the risk premium when we consider seasonality or not. These results have important implications for practitioners in the market.

1. Introduction

In the 1990s, and especially the 2000s, commodity derivatives have become an important component of many investors portfolios. In particular, pension funds and other portfolio managers have considered commodities as an independent asset class that, when combined with traditional stock and bond portfolios, can improve the risk-return performance. Most practitioners use simple models, such as the models which are the basis for the Black–Scholes option pricing formula, to analyze commodity prices and price commodity derivatives. Nevertheless, these models are extremely limited and do not answer questions related to the effects of speculation, see [1].

When pricing commodity derivatives, the special features of these markets should be considered. In the literature, the commodity price is usually assumed to follow a mean-reverting diffusion process, because of the dynamics of the supply and demand, see [2,3]. However, nowadays, the commodity prices suffer from abrupt changes and empirical studies find significant evidence about the presence of jumps in commodity processes, see [4,5]. Then, this fact has also been considered in the commodity pricing literature, see, among others, [6–9]. Lastly, there are numerous studies, e.g. [10,11], which have documented that commodity prices show a seasonal behavior. Therefore, this property has been taken into account in several models in the literature. Lucia and Schwartz [12], Cartea and Figueroa [13], and Li et al. [14] considered the seasonality in electricity markets, García et al. [15] in the natural gas markets, Kyriakou et al. [9] in petroleum commodities and Back et al. [11] in the soybean, corn, heating oil and natural gas markets. Recently, Arismendi et al. [16] analyze the importance of the seasonal behavior in the volatility to price commodity options.

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http://dx.doi.org/10.1016/j.cam.2017.05.014
0377-0427/

In the commodity pricing literature, the functions of the stochastic processes and the market prices of risk are usually assumed as simple parametric functions, for pure tractability and simplicity. Moreover, the whole functions are usually chosen to provide an affine model which has a known closed-form solution. However, there is not any empirical evidence either consensus about affine models are the best models to price commodity futures. Furthermore, the market prices of risk are not observed in the markets. Hence, if we consider other more realistic functions for the stochastic variables or the market prices of risk or even a nonparametric approach then, the model would not be affine anymore, a closed-form solution could not be obtained and the estimation of the market prices of risk would not be possible. This problem was solved by [17] but for a jump–diffusion model without seasonality. Then, we contribute to the literature by filling this gap for a seasonal jump–diffusion model.

This paper is mainly concerned with the importance of the accurate estimation of the whole functions of a jump–diffusion pricing commodity derivative model with a seasonal component, although a closed-form solution was not known. We obtain some results to estimate the whole functions of a two-factor jump–diffusion model with seasonality under the risk-neutral measure, directly from market data. In our model, we assume that one of the factors is the commodity spot price and the other is the convenience yield. These factors are often used in the commodity literature. For example, Gibson and Schwartz [2] and Schwartz [3] also consider these two factors, though they consider neither jumps nor seasonality. Gómez-Valle et al. [17] use this two-factor model with a diffusion and a jump–diffusion process, but they do not take into account the seasonality. Furthermore, we use a nonparametric approach to estimate the functions of the risk-neutral stochastic processes so as to avoid imposing arbitrary restrictions on the model. To our knowledge, this is the first research that explicitly estimates the functions of the risk-neutral stochastic processes with a nonparametric technique in a commodity two-factor seasonal jump–diffusion model to price commodity derivatives. Then, we analyze the role of the seasonality for some natural gas derivatives. As a result of the great attention that partitioners have devoted to the risk premium, we also analyze the influence of the seasonality on this premium.

The rest of the paper is organized as follows. Section 2 develops a two-factor jump–diffusion model with seasonality to price commodity derivatives. Section 3 proposes some novel results to estimate the whole functions of a derivative pricing model with jumps and seasonality, directly from market data, even when a closed-form solution is not known. Section 4 shows how to deal with the seasonality in the natural gas market and how to implement the approach in Section 3 with a nonparametric technique. Section 5 prices futures and options with the approach in Section 3 and data in Section 4 and shows the effect of taking into account the seasonality. Section 6 analyzes the futures risk premium and finally, Section 7 concludes. All the implementations have been done using MATLAB software.

2. The valuation model

In this section, we discuss the two-factor jump–diffusion model with seasonality that we use to price commodity derivatives. This research assumes that the two factors are the dynamics of the spot price, S, and the instantaneous convenience yield, δ. However, the proposed model works with the natural logarithm of the spot price instead of the spot price itself, see, among others, [12]. In a similar way, we could obtain the same theoretical results if we considered another state variable instead of the convenience yield.

Define $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ as a complete filtered probability space which satisfies the usual conditions where $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration, see [18–20]. We describe the behavior of the log-price process in terms of two types of components. The first one is a predictable deterministic component which takes into account regularities in the evolution of the spot price, that is, a deterministic trend and any periodic behavior. This component is represented by a known deterministic function of time. The second component X is stochastic and both of them verify that

$$\ln S(t) = f(t)X(t), \quad t \in [0, \infty). \tag{1}$$

In particular, we assume that X follows a jump–diffusion stochastic process.

As far as the second factor of the model is concerned, the convenience yield is assumed to follow a diffusion process, see for example [7]. However, we assume that X follows a jump–diffusion process because commodity prices usually suffer from abrupt changes in the markets, see [4]. Therefore, the factors of the model follow this joint stochastic process:[1]

$$X(t) = X(0) + \int_0^t \mu_X(X(z), \delta(z))dz + \int_0^t \sigma_X(X(z), \delta(z))dW_X(z) + \int_0^t c(X(z-), \delta(z))dJ(z), \tag{2}$$

$$\delta(t) = \delta(0) + \int_0^t \mu_\delta(X(z), \delta(z))dz + \int_0^t \sigma_\delta(X(z), \delta(z))dW_\delta(z), \tag{3}$$

where μ_X and μ_δ are the drifts and σ_X and σ_δ the volatilities. Moreover, W_X and W_δ are Wiener processes and the impact of the jump is given by the function c and the compound Poisson process, $J(t) = \sum_{i=1}^{N(t)} Y_i$, with jump times $(\tau_i)_{i\geq 1}$, where $N(t)$ represents a Poisson process with intensity $\lambda(X, \delta)$ and $Y_1, Y_2, \ldots$ is a sequence of identically distributed random variables

[1] S is right-continuous (cadlag, see [20]) and we denote the left limit $X(t-) = \lim_{z\uparrow t} X(z)$. However, for notational clarity the pre-jump values $X(t-)$ will be added only when necessary to avoid confusion and otherwise, they will be assumed implied.

with a Gaussian probability distribution $\Pi, \mathcal{N}(0, \sigma_Y^2)$. We assume that W_X, W_δ and the jump size distribution are independent of N, but the standard Brownian motions are correlated with:

$$[W_X, W_\delta](t) = \rho t.$$

We also assume that the jump magnitude and jump arrival times are uncorrelated with the diffusion parts of the processes. Lastly, we suppose that the functions $\mu_X, \mu_\delta, \sigma_X, \sigma_\delta, \lambda$ and Π satisfy suitable regularity conditions provided in Appendix.

We assume that the market is arbitrage-free. Then, there exists an equivalent martingale measure, $\mathcal{Q}$-measure, which is known as the risk-neutral measure, see extended Girsanov-type measure transformation in [21,22]. The state variables of the model (2)–(3) under the risk-neutral measure, are as follows:

$$X(t) = X(0) + \int_0^t \left(\mu_X - \sigma_X \theta^{W_X}\right) dz + \int_0^t \sigma_X dW_X^{\mathcal{Q}}(z) + \int_0^t c(X(z-), \delta(z)) \tilde{J}^{\mathcal{Q}}(z), \tag{4}$$

$$\delta(t) = \delta(0) + \int_0^t \left(\mu_\delta - \sigma_\delta \theta^{W_\delta}\right) dz + \int_0^t \sigma_\delta dW_\delta^{\mathcal{Q}}(z), \tag{5}$$

where $W_X^{\mathcal{Q}}$ and $W_\delta^{\mathcal{Q}}$ are the Wiener processes under $\mathcal{Q}$-measure and $[W_X^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}](t) = \rho t$. The market prices of risk associated to W_X and W_δ Wiener processes are $\theta^{W_X}(X, \delta)$ and $\theta^{W_\delta}(X, \delta)$, respectively. Finally, $\tilde{J}^{\mathcal{Q}}(t) = \sum_{i=1}^{N^{\mathcal{Q}}(t)} Y_i - \lambda^{\mathcal{Q}} t E_Y^{\mathcal{Q}}[Y_1]$ is the compensated compound Poisson process under $\mathcal{Q}$-measure and the intensity of the Poisson process $N^{\mathcal{Q}}(t)$ is $\lambda^{\mathcal{Q}}(X, \delta)$.

Moreover, we will consider the function $c(X, \delta) = 1$ in (2) and (4) and, as usual in the literature, for simplicity and tractability, we assume that the jump size distribution under $\mathcal{Q}$-measure of the jump–diffusion process is known and equal to the distribution under $\mathcal{P}$-measure. This means that all risk premia related to the jump are artificially absorbed by the change in the intensity of the jump from λ under the physical measure to $\lambda^{\mathcal{Q}}$ under the risk-neutral measure, see [9,23]. Then, $E_Y^{\mathcal{Q}}[Y_1] = E_Y[Y_1]$.

As $S(t) = h(t, X(t)) = e^{f(t)X(t)}$ and provided that the function f satisfies the appropriate regularity conditions, we can apply the Itô formula for jump–diffusion processes, see [20,24]. Then, the spot price under the risk-neutral measure is the solution to the following stochastic differential equation (we use the differential notation to simplify, see [20]):

$$dS = S\left(\frac{f' \ln S}{f} + f(\mu_X - \sigma_X \theta^{W_X}) + \frac{1}{2}\sigma_X^2 f^2\right) dt + \sigma_X f^2 S dW_X^Q + d\left(\sum_{i=1}^{N^{\mathcal{Q}}(t)} S(\tau_i-)(e^{fY_i} - 1)\right). \tag{6}$$

As $Y_1 \rightsquigarrow N(0, \sigma_Y^2)$, then e^{fY_1} follows a log-normal distribution, $e^{fY_1} \rightsquigarrow \mathcal{L}N(0, f^2\sigma_Y^2)$, and we obtain:

$$E_{Y_1}[e^{fY_1} - 1] = e^{\frac{f^2\sigma_Y^2}{2}} - 1. \tag{7}$$

Moreover, as the compensated Poisson process is a martingale, then we show the compensated process of (6):

$$\begin{aligned} dS &= S\left(\frac{f' \ln S}{f} + f(\mu_X - \sigma_X \theta^{W_X}) + \frac{1}{2}\sigma_X^2 f^2 + \lambda^Q \left(e^{\frac{f\sigma_Y^2}{2}} - 1\right)\right) dt + \sigma_X f^2 dW_X^Q \\ &\quad + \left[d\left(\sum_{i=1}^{N^{\mathcal{Q}}(t)} S(\tau_i-)(e^{fY_i} - 1)\right) - S(t-)\lambda^Q \left(e^{\frac{f\sigma_Y^2}{2}} - 1\right) dt\right]. \end{aligned} \tag{8}$$

If seasonality is not considered in the model: $\ln S = X$ then, $f \equiv 1$ and $f' \equiv 0$. Therefore, the spot price under the risk-neutral measure follows this stochastic differential equation:

$$\begin{aligned} dS &= \left((\mu_X - \sigma_X \theta^{W_X}) + \frac{1}{2}\sigma_X^2 + \lambda^Q \left(e^{\frac{\sigma_Y^2}{2}} - 1\right)\right) dt + \sigma_X dW_X^Q \\ &\quad + \left[d\left(\sum_{i=1}^{N^{\mathcal{Q}}(t)} S(\tau_i-)(e^{Y_i} - 1)\right) - S(t-)\lambda^Q \left(e^{\frac{\sigma_Y^2}{2}} - 1\right) dt\right]. \end{aligned} \tag{9}$$

A commodity futures price at time t with maturity at time T, $t \leq T$, under the above assumptions, can be expressed as $F(t, S, \delta; T)$ and at maturity it verifies that $F(T, S, \delta; T) = S(T)$, and the futures price can be expressed by

$$F(t, S, \delta; T) = E^{\mathcal{Q}}[S(T) | S(t) = S, \delta(t) = \delta]. \tag{10}$$

Let $V(t, S, \delta, T_2; T_1)$ be the price of a European call option that matures on T_1 on a futures contract that expires at T_2, $T_1 \leq T_2$, and K is the strike price. Then, analogously to (10), an European commodity futures option is priced as the expected discounted payoff under the $\mathcal{Q}$-measure, see [7,22],

$$V(t, S, \delta, T_2; T_1) = E^{\mathcal{Q}}\left[e^{-\int_t^{T_1} r(u)\,du} \max(F(T_1, S(T_1), \delta(T_1); T_2) - K, 0) \,|\, S(t) = S, \delta(t) = \delta\right], \tag{11}$$

where r denotes the instantaneous risk-free interest rate, which is assumed to be constant. Moreover, $\tau_1 = T_1 - t$ and $\tau_2 = T_2 - T_1$ are the maturity of the option contract and futures contract, respectively.

3. Exact results and approximations

Researchers have devoted the greatest attention to affine models such as [3,6–8]. One of the main reasons is that a closed-form solution for the commodity futures price is known. Moreover, this fact allows the application of different estimation techniques, like the Kalman Filter or Maximum Likelihood, and the market price of risk estimation is simple, although sometimes statistically insignificant see, among others, [11,25]. However, there is neither evidence nor consensus that affine models are the most suitable for pricing futures contracts.

In the literature, to the best of our knowledge, there is no approach to estimate the market prices of risk in jump–diffusion models with seasonality, unless a closed-form solution is known. Bandi and Nguyen [26] and Johannes [27] show how to estimate nonparametrically the functions of a jump–diffusion process by means of their moment equations for interest rate models. However, this approach does not show how to estimate the market prices of risk. Gómez-Valle et al.[17] propose some results to estimate the risk-neutral functions for jump–diffusion processes directly from market data in a commodity derivative model, but they do not consider seasonality. Therefore, in this section, we contribute to the literature by proposing a new approach to estimate the functions of the risk-neutral jump–diffusion stochastic factors of a seasonal commodity model directly from market data. Then, we can price commodity derivatives without estimating the unobservable market prices of risk, although a closed-form solution of the futures price was not known.

In the following result we prove several equalities which relate the slope of the futures curve with the risk-neutral drift, the volatility and the covariance of the stochastic variables.

Theorem 1. *Let $F(t,S,\delta;T)$ be the price of a commodity future, with $\ln S = fX$, and X and δ follow the joint stochastic processes given by (4)– (5), then:*

$$\frac{\partial F}{\partial T}(t,S,\delta;T) = F(t,S,\delta;T)\left(\frac{f'\ln S}{f} + f(\mu_X - \sigma_X\theta^{W_X}) + \frac{1}{2}\sigma_X^2 f^2 + \lambda^{\mathbb{Q}} E_Y[e^{fY_1} - 1]\right)(T), \tag{12}$$

$$\frac{\partial (SF)}{\partial T}(t,S,\delta;T) = F(t,S,\delta;T)\left(2\frac{\partial F}{\partial T} + S(f^4\sigma_X^2 + \lambda^{\mathbb{Q}} E_Y[(e^{fY_1} - 1)^2])\right)(T), \tag{13}$$

$$\frac{\partial (\delta F)}{\partial T}(t,S,\delta;T) = \left(\delta\frac{\partial F}{\partial T} + S(\mu_\delta - \sigma_\delta\theta^{W_\delta}) + Sf^2\rho\,\sigma_X\sigma_\delta\right)(T). \tag{14}$$

The derivatives above should be assumed as right derivatives when $T \in (\tau_i)_{i\geq 1}$, that is, when T is a jump time.

We prove these results by means of (10). The detailed proof of this theorem can be found in Appendix. Analogous results, but for diffusion and jump–diffusion processes without seasonality, are also shown in [17,28].

If we consider that $X = \ln S$ that is, the seasonality is not taken into account, then the expressions (12)–(14) are reduced to

$$\frac{\partial F}{\partial T}(t,S,\delta;T) = F(t,S,\delta;T)\left(\mu_X - \sigma_X\theta^{W_X} + \frac{1}{2}\sigma_X^2 + \lambda^{\mathbb{Q}} E_Y[\mathrm{e}^{Y_1} - 1]\right)(T), \tag{15}$$

$$\frac{\partial (SF)}{\partial T}(t,S,\delta;T) = F(t,S,\delta;T)\left(2\frac{\partial F}{\partial T} + S(\sigma_X^2 + \lambda^{\mathbb{Q}} E_Y[(\mathrm{e}^{Y_1} - 1)^2])\right)(T), \tag{16}$$

$$\frac{\partial (\delta F)}{\partial T}(t,S,\delta;T) = \left(\delta\frac{\partial F}{\partial T} + S(\mu_\delta - \sigma_\delta\theta^{W_\delta}) + S\,\rho\,\sigma_X\sigma_\delta\right)(T). \tag{17}$$

4. Implementation and estimation

In this section, we illustrate how practitioners can apply the novel approach in Section 3 to estimate the functions of the risk-neutral processes of the state variables. Particularly, we apply it to the Henry Hub natural gas data because, as mentioned in the literature, there is a high evidence of seasonality in this commodity market, see, among others, [11,15]. Then, we use a nonparametric estimation technique in order to avoid imposing arbitrary restrictions on the model functions.

Henry Hub natural gas spot price data traded at NYMEX were obtained from the US Energy Information Administration (EIA). Fig. 1 depicts the daily Henry Hub natural gas spot price and its first differences from January 2004 to July 2015. This figure shows the behavior of the in-sample data (January 2004–December 2014) and the out-of-sample data (January–July 2015) that we use in the paper.

As in our model we consider (1), we use the natural logarithm of the spot price instead of the spot price. Therefore, in Table 1, we summarize the main statistics of the log-spot price. We see that both skewness and kurtosis values indicate that the log-spot price series does not follow a Normal distribution, which is further confirmed by results of the Jarque–Bera test (JB) for normality in the last column (p-values on brackets). In fact, this series exhibits significant excess kurtosis meaning that as compared to a Normal distribution, it has higher and sharper central peak and longer fatter tails. Hence, a jump–diffusion process is more suitable for our model than a diffusion process.

In this paper, we consider the seasonality of the natural gas. However, the seasonality can be taken in different ways. Therefore, we make a seasonal decomposition of the log-spot price time series (with monthly data) and calculate the seasonal

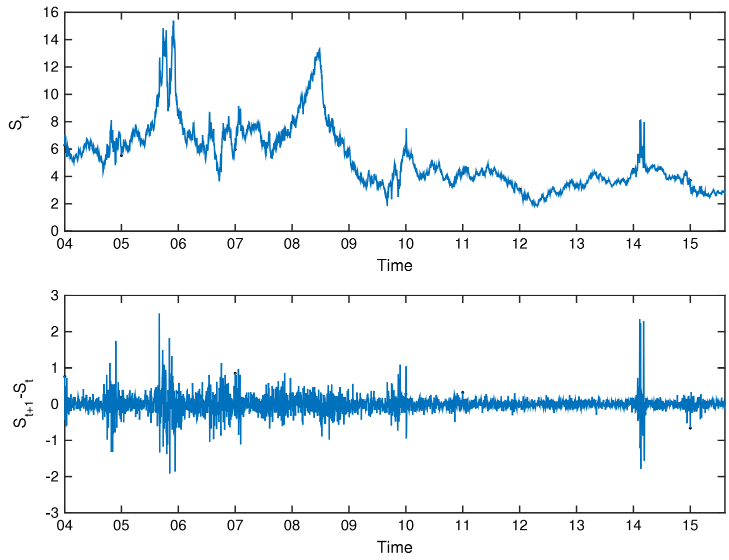

Fig. 1. Daily Henry Hub natural gas spot price and its first differences, January 2004–July 2015.

Table 1
Summary of the statistics on the Henry Hub log-natural gas spot price and its first differences, January 2004–December 2014.

Variable	N	Mean	Std. dev.	Max	Min	Skewness	Kurtosis	JB
$\ln S_t$	2735	1.6199	0.3957	2.7337	0.5988	0.1973	2.7310	25.9932 (0.0000)
$\ln S_{t+1} - \ln S_t$	2734	−0.0003	0.0430	0.3901	−0.2784	0.6765	14.6122	15570 (0.0000)

Table 2
Estimated parameters of the approximated seasonal function.

a_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
1	0.00484	0.0155	0.02147	0.00571	−0.00785	0.00165	−0.00310	0.00197

variations with an additive and multiplicative model. We obtain the coefficients of variations in both cases and we find that the coefficient is much lower in the multiplicative model than in the additive model. This is the reason why we use the multiplicative decomposition as in (1).

We choose as the deterministic seasonal function, f, a fit of the monthly average of the log-spot price with a fourth-order Fourier approximation as follows

$$f(t) = a_0 + \sum_{k=1}^{4}(a_k \cos(2k\pi t) + b_k \sin(2k\pi t)) \quad a_0, a_k, b_k \in \mathbb{R},\ k = 1, \ldots, 4.$$

The parameters are estimated by means of *fit*, a MATLAB-function which uses Least Squares. Table 2 shows the parameter values with R-square 0.9319 and standard error 0.0135.

The model we propose in Section 2 to price commodity derivatives has two factors: the spot price and the convenience yield. As it is well known in the literature, the convenience yield is not observed in the markets. Then, as in [2], we approximate it by the following formula

$$\delta_{T-1,T} = r_{T-1,T} - 12 \ln \left[\frac{F(t, S, \delta; T)}{F(t, S, \delta; T-1)} \right], \tag{18}$$

where $r_{T-1,T}$ denotes the $T-1$ period ahead annualized one month riskless forward interest rate. We obtain this forward interest rate with two daily T-Bill rates with maturities as close as possible to the futures contracts' ones in order to compute $\delta_{1,2}$, the one-month ahead annualized convenience yield. The latter is identified with the instantaneous convenience yield $\delta_{0,1}$, see [2] for more details. T-Bill rates are obtained from the Federal Reserve h.15 database. We also use the natural gas futures contracts with the shortest maturities (1 and 2 months) traded at NYMEX. This data is taken from the EIA database.

So as to price commodity derivatives we need to estimate the functions of the risk-neutral spot price and convenience yield processes. Then, we have to estimate the different terms in (5) and (8), but the risk-neutral variables are not observed.

For this purpose, the risk-neutral functions of the jump–diffusion component, X, and the convenience yield δ are estimated by means of the jump–diffusion moment equations and Theorem 1 (we consider $T = t$ and then, $F(t, S, \delta; t) = S(t)$).

Following [17], the estimation of the volatility of X (σ_X) and the distribution parameters of Y_1 is done with the jump–diffusion process moment equations (see [17,26,27]),

$$M_X(X,\delta) = \lim_{\Delta t\downarrow 0} \frac{1}{\Delta t} E[X(t+\Delta t) - X(t)|X(t) = X, \delta(t) = \delta] = \mu_X + \lambda(X,\delta)E_Y[Y_1], \quad (19)$$

$$M_X^2(X,\delta) = \lim_{\Delta t\downarrow 0} \frac{1}{\Delta t} E[(X(t+\Delta t) - X(t))^2|X(t) = X, \delta(t) = \delta] = \sigma_X^2(X,\delta) + \lambda(X,\delta)E_Y[Y_1^2], \quad (20)$$

$$M_X^k(X,\delta) = \lim_{\Delta t\downarrow 0} \frac{1}{\Delta t} E[(X(t+\Delta t) - X(t))^k|X(t) = X, \delta(t) = \delta] = \lambda(X,\delta)E_Y[Y_1^k], \quad k \geq 3. \quad (21)$$

As we assume that the jump size follows a Normal distribution $Y_1 \rightsquigarrow N(0, \sigma_Y^2)$. Then, $\mu_Y = E_Y^Q[Y_1] = E_Y[Y_1] = 0$ and $\sigma_Y^2 = E_Y^Q[Y_1^2] = E_Y[Y_1^2]$. Furthermore, under this assumption of normality it is well known that

$$E_Y[Y_1^{2k}] = \sigma_Y^{2k} \prod_{n=1}^{k}(2k-1),$$
$$E_Y[Y_1^{2k-1}] = 0, \quad k = 1, 2, 3, \ldots.$$

In this paper, we use a nonparametric approach to estimate the whole functions and in consequence, we avoid to impose arbitrary restrictions on the functions of the stochastic variables of the model. To our knowledge, in the literature, nonparametric methods are not usually used for commodity derivative pricing, apart from [17,28] but they do not consider the seasonality.

Suppose a data set consists of N observations, $(X_1, \delta_1, Z_1), \ldots, (X_N, \delta_N, Z_N)$, where (X_i, δ_i) are the explanatory variables and Z_i is the response variable. We assume a model of the kind $Z_i = g(X_i, \delta_i) + \epsilon_i$, where $g(X, \delta)$ is an unknown function and ϵ_i is an error term, representing random errors in the observations or variability from sources not included in the (X_i, δ_i) observations. The errors ϵ_i are assumed to be independent and identically distributed with mean zero and finite variance. The estimate has the closed-form

$$\hat{g}(X,\delta) = \sum_{i=1}^{N} W_i(X,\delta)Z_i,$$

where $W_i(X, \delta)$ is the Nadaraya–Watson product weight function:

$$W_i(X,\delta) = \frac{K_{h_X}(X - X_i)K_{h_\delta}(\delta - \delta_i)}{\sum_{j=1}^{N} K_{h_X}(X - X_j)K_{h_\delta}(\delta - \delta_j)},$$

K is the Gaussian Kernel and h_X and h_δ the bandwidths or smoothing parameters, see [29]. Theoretical results for kernel regression estimators show that the optimal bandwidths will be proportional to $N^{-1/6}$. Then, we consider that the bandwidths are as follows $h_X = \Phi_X \hat{\sigma}_1 N^{-1/6}$ and $h_\delta = \Phi_\delta \hat{\sigma}_2 N^{-1/6}$, where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the standard deviation estimates of X and δ, respectively, and Φ_X and Φ_δ are the scaling factors, see [30,31], for further details.

Then, for estimating the variance of X, σ_X^2, and the distribution parameter of the random variable Y_1, σ_Y^2, the Nadaraya–Watson estimator is applied to the moment conditions (20)–(21) with data in the market. That is, we obtain the spot price $S(t)$ from the market and we calculate $X(t)$ with the relation (1). The convenience yield observations $\delta(t)$ are obtained by means of (18). Then, we have the observations (X_i, δ_i) and the response variable Z_i is $(X_{i+1} - X_i)^2$ and $(X_{i+1} - X_i)^k$ in (20) and (21), respectively. In order to estimate σ_Y^2, we use moment (21) with $k = 4$ and 6, then, we use moment (20) for estimating σ_X^2, as in [27].

As far as the rest of functions is concerned, we use Theorem 1 because we have not got observations of the risk-neutral variables. As $e^{fY_1} \rightsquigarrow \mathcal{L}N(0, f^2\sigma_Y^2)$, then the expectations in (12) and (13) are replaced by (7) and

$$E_Y[(e^{fY_1} - 1)^2] = e^{f^2\sigma_Y^2} + 1 - 2e^{\frac{f^2\sigma_Y^2}{2}}.$$

Firstly, we estimate the risk-neutral jump intensity λ^Q using (13). We approximate the partial derivatives $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial (SF)}{\partial T}|_{T=t}$ by means of numerical differentiation. More precisely, we obtain a fourth order approximation to the slopes by the well-known forward difference formula:

$$\left.\frac{\partial g}{\partial T}\right|_{T=t} = \frac{-25g(t) + 48g(t+\Delta) - 36g(t+2\Delta) + 16g(t+3\Delta) - 3g(t+4\Delta)}{12\Delta} + O(\Delta^4). \quad (22)$$

This approximation allows us to consider a high spectrum of futures maturities for a better approximation of the slopes. More precisely, we use natural gas futures prices with maturities equal to 1, 2, 3 and 4 months which are traded at NYMEX and obtained from the Quandl platform. We replace in (13) the estimated volatility $\hat{\sigma}_X$, the distribution parameter of the

Table 3
RMSE of the futures prices for the out-of-sample, January–July 2015, for NSM and SM.

Futures	F1	F6	F9	F12	F18	F24	F30	F36	F42	F44
NSM	0.1607	0.2445	0.1446	0.1797	0.1435	0.2490	0.2346	0.3624	0.3088	0.3850
SM	0.1232	0.1473	0.0920	0.1577	0.0949	0.1847	0.1099	0.2453	0.1674	0.2595

random variable Y_1, the approximated seasonal function and the approximations of $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial(SF)}{\partial T}|_{T=t}$. Next, we apply the Nadaraya–Watson estimator and we approximate the jump intensity of the risk-neutral spot price.

Secondly, we replace the estimators of $\sigma_X, \sigma_Y, \lambda^{\mathcal{Q}}$, the approximated deterministic seasonal function f and the numerical approximation of $\frac{\partial F}{\partial T}|_{T=t}$ in (12) and we apply the Nadaraya–Watson estimator to estimate the risk-neutral drift of X.

Thirdly, the risk-neutral convenience yield functions are also estimated. For as much as this variable follows a diffusion process, its estimation is easier. To this end, previously, we have to approximate numerically $\frac{\partial(\delta F)}{\partial T}|_{T=t}$ by means of (22) and estimate $\rho\sigma_S\sigma_X$ with the moment:

$$M_{X,\delta}(X,\delta) = \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} E[(X(t+\Delta t) - X(t))(\delta(t+\Delta t) - \delta(t))|X(t) = X, \delta(t) = \delta] = \rho(X,\delta)\sigma_X(X,\delta)\sigma_\delta(X,\delta),$$

and the Nadaraya–Watson estimator, see [31] and [17] for more details. Then, we replace this estimated covariance and the numerical approximations of $\frac{\partial F}{\partial T}|_{T=t}$ and $\frac{\partial(\delta F)}{\partial T}|_{T=t}$ in (14) for getting the risk-neutral drift estimator of the convenience yield by means of the Nadaraya–Watson estimator.

Finally, the volatility of the convenience yield under $\mathcal{P}$-measure is equal to the volatility under $\mathcal{Q}$-measure. Hence, we estimate σ_δ by means of the second order moment of a diffusion process, see [31],

$$M_\delta^2(X,\delta) = \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} E[(\delta(t+\Delta t) - \delta(t))^2|X(t) = X, \delta(t) = \delta] = \sigma_\delta^2(X,\delta),$$

and the Nadaraya Watson estimator.

5. Commodity derivatives pricing

In this section, we price different natural gas derivatives, such as futures and futures options, by means of the approach in Section 3. Finally, we study the role of the seasonality when pricing these natural gas derivatives. In order to analyze the efficiency of this approach, we price natural gas futures traded at NYMEX with maturities from 1 to 44 months. As in the previous section, we use data from January 2004 to December 2014 for the estimation. Then, data from January to July 2015 have been selected as out-of-sample data for analyzing the accuracy of this approach with data from the EIA and the Quandl platform.

As we have previously stated in Section 4, we consider a nonparametric approach to estimate the whole functions of the risk-neutral stochastic processes.[2] Therefore a closed-form solution for this model cannot be obtained. Hence, a numerical method is necessary to get a solution for the problems (10) and (11). We use the Monte Carlo simulation approach because it is widely used by practitioners in the markets and in the literature, especially for multiple-factor models because of its simplicity and efficiency, see [32].

For analyzing the role of the seasonality in the natural gas derivatives, it is interesting to also price the derivatives with a jump–diffusion model without seasonality. Then, we consider that $X = \ln S$ and as a consequence, the stochastic process followed by the spot price is (9). In order to estimate the risk-neutral functions of the model, we use (15)–(17) in Section 3 and the same approach mentioned in Section 4.

The natural gas futures are priced by means of (10) and Monte Carlo simulation approach, with 5000 simulations and a daily time step (1/250), for the out-of sample (January–July 2015).

So as to make comparisons, we use the root mean square error (RMSE) for the out-of-sample:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(F_t - \hat{F}_t\right)^2},$$

where n is the number of observations, F_t is the NYMEX futures price and $\hat{F}_t$ is the predicted futures price with the different models.

Table 3 shows the RMSE of the NYMEX futures prices with the different models and for several maturities. F1 means the futures with a maturity of 1 month, F6 with six months and so on. In this table, we show that, for the whole maturities, the RMSE are always lower with the SM than with the NSM. Hence, the superiority of the seasonal model is clear.

[2] All the scaling factors in the bandwidths (h_X, h_δ) take values $\Phi_X \in [1, 8]$ and $\Phi_\delta \in [3, 10]$.

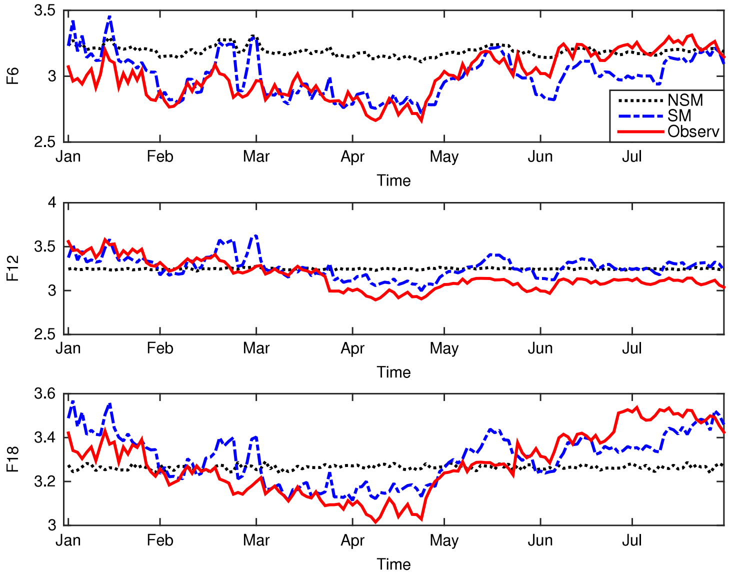

Fig. 2. Natural gas futures prices (January–July 2015) with maturities: 6, 12 and 18 months. The observed NYMEX futures prices are the red-solid line, the SM is the blue dashed-line and the NSM the black-dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

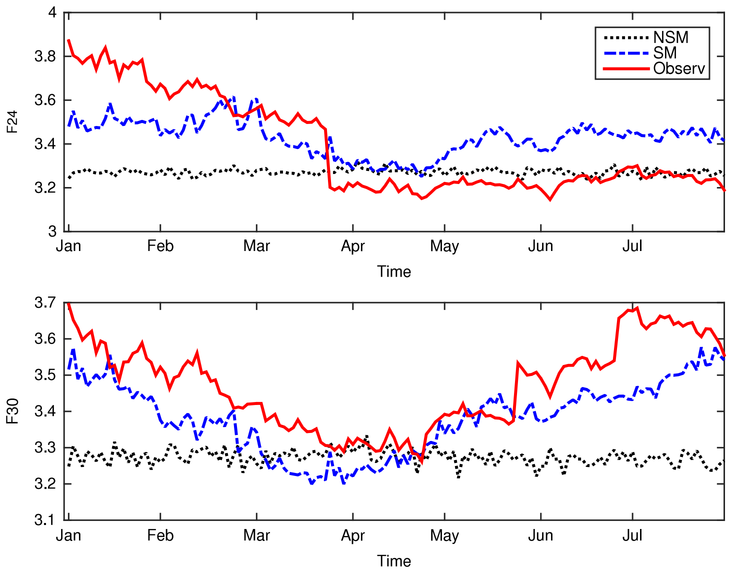

Fig. 3. Natural gas futures prices (January–July 2015) with maturities: 24 and 30 months. The observed NYMEX futures prices are the red solid line, the SM is the blue dashed line and the NSM the black dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In order to gain some insight into the causes of the discrepancies between real futures prices and theoretical prices, Figs. 2, 3 and 4 allow for graphical comparison of the prices provided by SM and NSM with the observed futures prices in the markets. More precisely, Fig. 2 shows the futures prices for maturities of 6, 12 and 18 months, Fig. 3 for maturities of 24 and 30 months and, lastly, Fig. 4 for maturities of 36 and 44 months. As it can be seen from these figures, in general, the SM prices are closer to the futures prices traded at NYMEX than the NSM prices for the whole out-of-sample and for short and long maturities. These figures show that SM reflects the changes of the prices along the time more accurately, while NSM models do not nearly change. Furthermore, for short maturities, the SM and the NSM, in general, overprice the natural gas futures traded at NYMEX, see Fig. 2. However, this fact changes with the maturities, whenever the maturity increases the SM and the NSM start to underprice the futures traded at NYMEX.

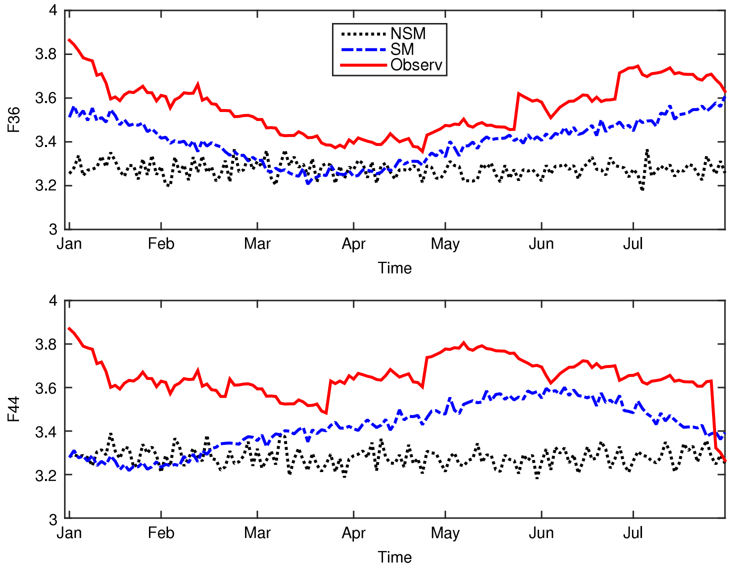

Fig. 4. Natural gas futures prices (January–July 2015) with maturities: 36 and 44 months. The observed NYMEX futures prices are the red solid line, the SM is the blue dashed line and the NSM the black dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4
Ratios between the NSM and SM option prices.

Strike	Maturity			
	3 months	6 months	9 months	12 months
90%	1.20	0.75	0.50	0.37
100%	1.39	0.78	0.49	0.36
110%	1.70	0.81	0.48	0.34

In summary, these results show that when the seasonality is taking into account in the model we obtain more accurate prices and therefore, lower errors.

It is usually shown in the literature that the errors in pricing commodity futures are magnified when pricing futures options. Then, in this section, we also analyze the differences between the option prices with the SM and NSM. Commodity options are generally not written on the commodity itself, but on futures contracts. This fact ensures high liquidity of the underlying, as the trading is more common on the futures than on the spot market [11].

In order to price natural gas options, we use the same NYMEX data and estimation methodology than in Section 4, but now, the Monte Carlo method approximates (11). Here, we also run 5000 simulations and we consider a daily time step (1/250). We assume that the maturity of the option is equal to the underlying futures contract. In Table 4, we show the ratio between the option prices obtained with the NSM and the SM (NSM/SM), for several strike prices and maturities. We observe that for a maturity of 3 months, the SM underprices with respect to the NSM. However, for longer maturities, that is higher or equal to 6 months, the SM overprices the natural gas options. This fact should be taking into account by practitioners in the markets when pricing options with any of these models.

6. The risk premium

The analysis of the futures risk premia deserves great attention in the literature because it is a key measure in risk management. They affect the costs and benefits of hedging a spot contract and the diversification benefits that result from including futures in investment portfolios. Moreover, they also play a prevailing role for economic agents who decide on their production, storage and consumption by considering the futures prices as indicators of future spot prices, see [33].

It is very well-known that futures prices deviate from expected future spot prices because of the risk premia that traders expect to earn or pay when trading in futures markets. Therefore, following [34,35], we consider the risk premium as the difference between the expected future spot price and the futures price, that is,

$$RP = E^{\mathcal{P}}[S(T)|S(t) = S, \delta(t) = \delta] - F(t, S, \delta; T). \tag{23}$$

Hereby, the futures price is the sum of the expected future spot price under the physical measure and a risk premium.

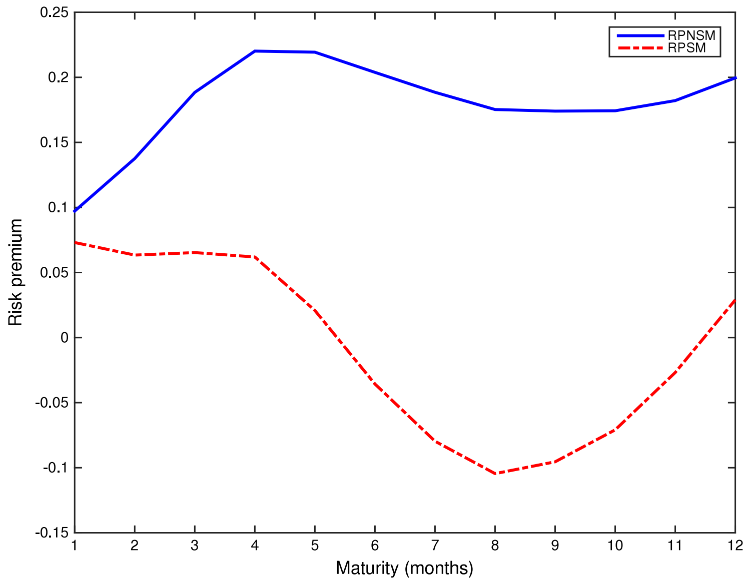

Fig. 5. The term structure of risk premium for the SM and NSM: RPSM and RPNSM, respectively.

In this section, we obtain the natural gas futures risk premium for the out of sample (January–July 2015) with (23) and data in Section 4. We use the natural gas futures prices traded at NYMEX with maturities till 1 year. In order to calculate the $E^{\mathcal{P}}[S(T)|S(t) = S, \delta(t) = \delta]$, the functions of the stochastic process (2)–(3) are estimated directly from moment conditions (19)–(21) and data in the market, because they are under $\mathcal{P}$-measure. Then, Monte Carlo method is applied to the expectation under $\mathcal{P}$-measure in (23), with 5000 simulations and a daily time step (1/250).

To investigate natural gas futures risk premia, first, we obtain the term structure as the mean of the risk premia for each maturity. We plot this term structure in Fig. 5 with two models: seasonal model and nonseasonal model. Consistent with most previous studies [36], neither the risk premium with seasonality (RPSM) nor the risk premium without seasonality (RPNSM) are constant along the term structure. In fact, it varies across the different maturities. Moreover, the differences in size and sign between the RPSM and RPNSM are very important. More precisely, the RPNSM is always positive but the RPSM is positive till 4 months and then, it starts to be negative. As [36] states, if speculators require a term premium to compensate for price uncertainty over a long period of time, the commodity risk premium should be larger (in absolute terms) for longer maturity futures. As it can be seen from Fig. 5, this is in general true for the seasonal model, although not for all maturities. The RPSM is very low for short maturities which is usually associated to the effect of financialisation of the commodity markets, see [36]. Moreover, the RPSM is lower (in absolute terms) than the RPNSM. Then, the producers or consumers would pay a lower premium if the SM model was used for pricing the natural gas futures.

Finally, Fig. 6 plots the RPSM and the RPNSM as a function of time along the out-of sample for some short maturities. In general, these graphs show that the futures risk premia are strongly time-varying and take negative and positive values, specially the RPSM. Therefore, the activity of speculators is also time varying. For a maturity of one month, both risk-premia are quite similar and they are positive and negative. However, for a maturity of nine months both risk premia are very different. Meanwhile the RPSM is always negative, the RPNSM is positive on average.

7. Conclusions

In the commodity literature, different assumptions have been considered for modeling the commodity spot price. On the one hand, jump-diffusion processes are considered for describing the abrupt changes of the commodity prices in the markets. On the other hand, the seasonal behavior of some commodity prices is taken into account. They all assume parametric functions in the stochastic processes in order to find a closed-form solution and, then, they can estimate the market prices of risk.

In this paper, we assume that the commodity price follows a jump–diffusion process and we also add a seasonal component. Then, we prove several equalities which relate the slope of the futures curve with the functions of the risk-neutral processes. The main point of this result is that it allows estimating the whole risk-neutral functions directly from data in the markets, even when a closed form-solution for the pricing model is not known.

So as to analyze the empirical performance of this approach, we apply it to price some natural gas derivatives. We choose the natural gas because there is high empirical evidence of the abrupt changes and seasonality of the natural gas spot price, but other commodities could also be used. In order to show a more realistic behavior of the model, we use a

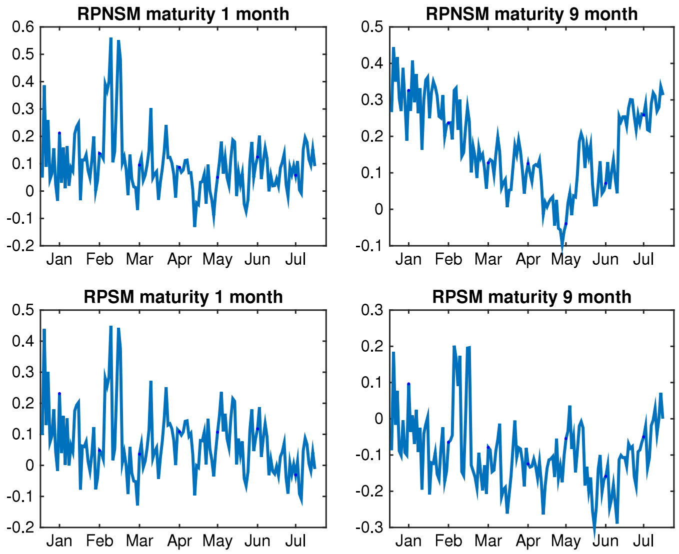

Fig. 6. The RPSM and the RPNSM along the out-of-sample (January–July 2015) for some different maturities.

nonparametric approach instead of imposing arbitrary restrictions to the functions. As a consequence, this pricing model would not have a closed-form solution. However, this is not a problem for this novel approach because all the functions can be estimated directly from data in the market. Then, we compare these futures prices with those obtained with a jump–diffusion model without seasonality. In general, we see that both models overprice the market futures prices for short maturities and underprice for long maturities. However, in all cases the futures prices obtained, when the seasonality is taking into account, are closer to the market prices. Moreover, its behavior is more realistic (similar to the market prices). In fact, taking into account this dynamic could have a great impact on the pricing of options.

This can be seen when we calculate option prices with both models. In this case, we observe high differences between the seasonal and the nonseasonal model. Finally, we analyze the risk premium of both models and the differences between them are also considerable. Therefore, in this paper we show that the jump–diffusion model with seasonality outperforms the standard jump–diffusion models.

Acknowledgments

The authors are grateful to the anonymous referees for their constructive suggestions and helpful comments. L. Gómez-Valle and J. Martínez-Rodríguez were supported in part by the GIR Optimización Dinámica, Finanzas Matemáticas y Utilidad Recursiva of the University of Valladolid and the projects: VA191U13 of the Consejería de Educación, JCyL and MTM2014-56022-C2-2-P of the Spanish Ministerio de Economía y Competitividad and ERDF.

Appendix

This appendix states the regularity conditions that guarantee the existence and uniqueness of the stochastic differential equations considered in this paper. These conditions are necessary to prove Theorem 1.

In the following assumptions, we consider the notation form of the functions in (2)–(3): $\mu = (\mu_X, \mu_\delta)$ and $\sigma = (\sigma_X, \sigma_\delta)$.

- **Assumption 1** The functions μ, σ and λ are twice continuously differentiable and we consider the function $c(X, \delta) = 1$ in (2) along this paper. Moreover, they satisfy local Lipschitz and growth conditions. That is, for every compact subset $D \subset \mathbb{R}^2$, there exists a constant C_1^D such that, for all $x, z \in D$,

$$|\mu(x) - \mu(z)| + |\sigma(x) - \sigma(z)| \leq C_1^D |x - z|.$$

- **Assumption 2** There exists a constant C_2 such that for any $x \in \mathbb{R}^2$,

$$|\mu(x)| + |\sigma(x)| + \lambda(x) \int_{\mathbb{R}} |y| \Pi(dy) \leq C_2(1 + |x|).$$

- **Assumption 3** For any $\alpha > 2$, there exist a constant C_3 such that for any $x \in \mathbb{R}^2$

$$\lambda(x) \int_{\mathbb{R}} |y|^\alpha \Pi(dy) \leq C_3(1 + |x|^\alpha).$$

- **Assumption 4** $\lambda(x) \geq 0$ and $\sigma^2(x) > 0$ on $\mathbb{R}^2$.

The above conditions on the model guarantee the existence and uniqueness of a cadlag strong solution to (2)–(3), see [24,26].

Then, we prove Theorem 1 in Section 3.

Proof of Theorem 1. We consider the integral form of the spot price stochastic equation (8)

$$S(T+h)-S(T)=\int_T^{T+h} S\left(\frac{f'\ln S}{f}+f(\mu_X-\sigma_X\theta^{W_X})+\frac{1}{2}\sigma_X^2 f^2+\lambda^{\mathcal{Q}}E_Y[e^{fY_1}-1]\right)(z)\,dz$$
$$+\int_T^{T+h}\sigma_X f^2 S dW_X^{\mathcal{Q}}(z)+\left[\left(\sum_{i=N^{\mathcal{Q}}(T+1)}^{N^{\mathcal{Q}}(T+h)} S(\tau_i-)(e^{fY_i}-1)\right)-\int_T^{T+h} S(z-)\lambda^{\mathcal{Q}}E_Y[e^{fY_1}-1]dz\right].$$

Then, we calculate the conditional expectation under $\mathcal{Q}$-measure in the above equality. Taking into account (10) and the fact that the Itô integral and compensated process are martingales, the conditional expectation under $\mathcal{Q}$-measure of the last terms is zero (that is, $E^{\mathcal{Q}}[\int_T^{T+h}\sigma_X f^2 S\,dW_X^{\mathcal{Q}}(z)\,|\,S(t)=s,\delta(t)=\delta]=E^{\mathcal{Q}}[\int_0^{T+h}\sigma_X f^2 S\,dW_X^{\mathcal{Q}}(z)\,|\,S(t)=s,\delta(t)=\delta]-E^{\mathcal{Q}}[\int_0^{T}\sigma_X f^2 S\,dW_X^{\mathcal{Q}}(z)\,|\,S(t)=s,\delta(t)=\delta]=0$ and analogously for the jump term). Then, we obtain

$$F(t,S,\delta;T+h)-F(t,S,\delta;T)=\int_T^{T+h}F(t,S,\delta;z)\left(\frac{f'\ln S}{f}+f(\mu_X-\sigma_X\theta^{W_X})+\frac{1}{2}\sigma_X^2f^2+\lambda^{\mathcal{Q}}E_Y[e^{fY_1}-1]\right)(z)\,dz.$$

Now dividing by h and taking limits, when h tends to 0[3] , leads to (12).

Using Itô's product rule, see [22] and [20], and (8) we have

$$dS^2=S^2\left(2\left(\frac{f'\ln S}{f}+f(\mu_X-\sigma_X\theta^{W_X})+\frac{1}{2}\sigma_X^2f^2+\lambda^{\mathcal{Q}}E_Y^{\mathcal{Q}}[e^{fY_1}-1]\right)+f^4\sigma_X^2+\lambda^{\mathcal{Q}}E_Y^{\mathcal{Q}}[(e^{fY_1}-1)^2]\right)dt$$
$$+2S^2f^2\sigma_X dW_X^{\mathcal{Q}}(t)+\left[d\left(\sum_{i=1}^{N^{\mathcal{Q}}(t)}S(\tau_i-)^2(e^{2fY_i}-1)\right)-S(t-)^2\lambda^{\mathcal{Q}}E_Y[e^{2fY_1}-1]dt\right]. \quad (24)$$

With the integral form of (24) and using the same steps as above, we get (13).

Now, as earlier, using Itô's product rule and (8) and (5) we have

$$d(S\delta)=S\left(\mu_\delta-\sigma_\delta\theta^{W_\delta}+\delta\left(\frac{f'\ln S}{f}+f(\mu_X-\sigma_X\theta^{W_X})+\frac{1}{2}\sigma_X^2f^2\right)+\rho\sigma_X\sigma_\delta f^2+\delta\lambda^{\mathcal{Q}}E_Y^{\mathcal{Q}}[e^{fY_1}-1]\right)dt$$
$$+S\delta f^2\sigma_X dW_X^{\mathcal{Q}}(t)+S\sigma_\delta dW_\delta^{\mathcal{Q}}(t)+\left[d\left(\sum_{i=1}^{N^{\mathcal{Q}}(t)}S(\tau_i-)\delta(e^{fY_i}-1)\right)-S(t-)\delta\lambda^{\mathcal{Q}}E_Y[e^{fY_1}-1]dt\right]. \quad (25)$$

Using the similar reasoning above, we obtain (14).

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Conclusions

The importance of commodity derivative markets has grown considerably over recent decades. As a consequence, these markets have become an exciting topic for researchers who have proposed and analyzed different commodity derivative pricing models. In the literature, parametric models are usually considered to obtain a closed-form solution. However, there is not any empirical evidence showing that these models are the best to price commodity derivatives. In fact, when more realistic functions are considered in the models, a closed-form solution is not known. As the risk-neutral stochastic processes and the market prices of risk are not observable in the market, when a closed-form solution of the derivative is not known, these functions cannot be estimated.

In this research, we consider a two-factor model to price some commodity derivatives. The first factor is the commodity spot price and the second one is the convenience yield. The first primary assumption is that the spot price and convenience yield follow a jump-diffusion and diffusion processes, respectively. The second assumption is that the distribution of the jump size under the risk-neutral measure is known and equal to the distribution under the physical measure. This means that all risk premia related to the jump are artificially absorbed by the change of measure in the intensity of the jump. Furthermore, we extend this model assuming that the jump size follows a normal or an exponential distribution, and, finally, add a seasonal factor in the spot price. In all the cases, we prove some results which allow us to estimate the whole functions of the risk-neutral processes directly from data in the market. This new technique is used for analyzing the role of the jump and seasonality when pricing natural gas futures, options, and futures risk premia.

Firstly, we analyze the effect of considering a jump term in the spot price process when pricing natural gas futures traded at NYMEX. We find that the diffusion model provides slightly lower errors that the jump-diffusion model but only for short maturities. In contrast, the jump-diffusion model provides lower errors than the diffusion model for longer maturities. We also show that, in general, the diffusion and jump-diffusion models underprice natural gas futures for all maturities. Moreover, the futures prices obtained with the jump-diffusion model are closer to the market prices. Therefore, this fact supports the use of jump-diffusion processes when modelling the natural gas price dynamics in order to price futures, especially for long maturities.

Secondly, we analyze the role of the jump size distribution in the US natural gas derivative prices. We price natural gas futures assuming that the spot price follows a jump-diffusion process with a normal and an exponential jump size distribution. For short maturities, equal or lower than

18-months, we find that the normal distribution provides more accurate futures prices. However, for long maturities, the exponential distribution shows the lowest errors. Moreover, for maturities higher than 18-months, the exponential distribution provides higher prices than the normal distribution, but both distributions underprice the natural gas futures prices observed in the markets. Furthermore, the errors with normal and exponential distribution are higher for maturities longer or equal to 36-months. Maybe, this fact could also be due to the propagation error because of the use of Monte Carlo simulation approach.

In this research, we also price futures options with both jump size distributions. Then, we calculate some ratios between the futures option prices with the normal and exponential distributions for different strike prices and maturities, and we observe the following facts. For options with short maturities, the ratios are higher than ninety percent. However, if we increase the maturities, these ratios decrease considerably and the higher the strike price, the lower the ratio is. Therefore, the highest price differences can be found in the out of the money options.

As far as the term structure of natural gas futures risk premia is concerned, we also show that both risk premia (with normal and exponential distribution) have the same qualitative behaviour, they have a decreasing trend. Moreover, we observe that the risk premium under the normal jump size distribution is always higher than the risk premium under the exponential distribution. As commonly found in the literature, we also find that both risk premia vary strongly over time. This means that the activity of production companies and speculators also varies with time. Moreover, we see that for both distributions the risk premium is, on average, positive for short maturities and negative for long maturities. That is, for short maturities the futures prices are a downward biased predictor of the expected spot price, but an upward biased prediction for long maturities.

As there is some evidence of seasonality in some commodity markets, we add a seasonal component to the previous two-factor jump-diffusion commodity derivative pricing model. We assume that this component is a predictable deterministic function of time. Then, we prove some new results which allow us to estimate the whole risk-neutral functions of the model. In particular, we price natural gas futures and futures options traded at NYMEX with this approach and analyze the effect of the seasonality. We find that the errors in pricing natural gas futures are always lower when the seasonality is taken into account in the model. Moreover, we also make some comparisons between the option prices with both models. The ratios between the option prices with or without seasonality for several strikes and maturities show that the seasonal model underprices with respect to the non-seasonal model for a maturity of three months. However, the seasonal model overprices the natural gas options for longer maturities, and the higher the maturity, the lower the ratios are.

Besides, we analyze the influence of the seasonality on the future risk premium. We obtain the term structure of the natural gas risk premium with a seasonal and a non-seasonal model. We observe that the model without seasonality always provides positive risk premia, but the seasonal model shows negative risk premia for maturities higher than 6-months. As far as its behaviour over time is concerned, in both cases the risk premia are strongly time-varying, taking positive and negative values. Therefore, we can conclude that considering a seasonal component improves the pricing model in the natural gas market.

Finally, the main goal of this research is to provide an alternative approach to estimate the risk-neutral functions directly from data in the markets. In fact, we design some estimation techniques for seasonal models as well as non-seasonal models. These techniques can be applied even when the dynamics of the factors do not allow to find a closed-form solution for the derivative prices.

In this research topic, there are investigation lines to be carried on.

On the one hand, in order to explain better the behaviour of commodity derivative prices, other state variables could be considered in the model, for example, interest rates or spot price volatility. Moreover, jump terms could be added to these variables to pick up any abrupt change.

As far as the jump size distribution is concerned, we could consider that it is also affected by the change of measure instead of being artificially absorbed by the change in the jump intensity, as is usual in the literature.

On the other hand, taking into account that there are diverse kinds of commodity markets whose behaviours are very different (such as agricultural or metal commodities), distinct state variables and dynamics should be considered in the models, depending on the commodity. In particular, some of them have a strong seasonal pattern, especially energy and agricultural commodities. In our research, we considered a deterministic seasonal factor. However, we could also assume that the seasonal factors are trigonometric components generated by stochastic processes. In this case, additional complex estimation techniques would be necessary to deal with this problem.

Finally, in this thesis, the Monte Carlo simulation approach has been applied to obtain numerical solutions for the different pricing models analyzed. This method is easy to implement, but its accuracy is low. Therefore, efficient numerical methods could be designed and applied for obtaining more accurate prices. In this regard, the errors could be reduced, especially for higher maturities where the propagation of errors could be affecting the long-term commodity prices.

Conclusiones

En las últimas décadas los mercados de derivados de materias primas han experimentado un gran auge. Como consecuencia, los investigadores han mostrado un gran interés por los temas relacionados con este tipo de mercados, y han propuesto y analizado diferentes modelos para valorar derivados de materias primas.

En la literatura se suelen considerar modelos paramétricos afines para poder obtener una forma cerrada de la solución. Sin embargo, no existe ninguna evidencia empírica de que este tipo de modelos sean mejores para valorar los derivados. De hecho, cuando se consideran funciones más realistas no se suele disponer de una forma cerrada de la solución. En este caso, como los procesos neutrales al riesgo y los precios de riesgo del mercado no son observables en el mercado, estas funciones no se pueden estimar salvo que se conozca la solución.

En este trabajo consideramos un modelo de dos factores para valorar derivados de materias primas. El primer factor es el precio al contado de la materia prima y el segundo es el rendimiento de conveniencia. En primer lugar, suponemos que estos factores siguen un proceso de difusión con saltos y un proceso de difusión, respectivamente. En segundo lugar, consideramos que se conoce la distribución del tamaño de salto bajo la medida neutral al riesgo y es igual a la distribución bajo la medida física, es decir, la prima de riesgo asociada al salto es absorbida por el cambio de medida en la intensidad del salto. Además, extendemos el modelo considerando que el tamaño de salto sigue una distribución normal o una exponencial y, finalmente, añadimos una componente estacional en el precio al contado de la materia prima. En todos los casos, probamos resultados que permiten estimar todas las funciones de los procesos neutrales al riesgo directamente de los datos del mercado, y utilizamos esta nueva técnica para analizar el papel que tienen los saltos y la estacionalidad a la hora de valorar futuros, opciones y las primas de riesgo de los futuros de materias primas.

En primer lugar analizamos el efecto del término de salto en el proceso del precio al contado cuando valoramos futuros del gas natural negociados en el NYMEX. En este caso observamos que el modelo de difusión sin saltos proporciona errores ligeramente más pequeños que el de difusión con saltos pero para vencimientos cortos. Por el contrario, el modelo de difusión con saltos da lugar a errores más pequeños que el de difusión para vencimientos largos. En cualquier caso, observamos que, en general, ambos modelos infravaloran los precios de los futuros del gas natural para todos los vencimientos. Además, los precios de los futuros obtenidos con el modelo de difusión con saltos son más cercanos a los del mercado. Por tanto, este hecho respalda el uso de los procesos con saltos para modelizar la dinámica del precio al contado del gas natural a la hora de valorar futuros, especialmente para vencimientos largos.

En segundo lugar, analizamos el papel del salto y de la distribución del tamaño de salto en la valoración de derivados del gas natural de Estados Unidos. Valoramos los futuros suponiendo que los saltos siguen una distribución normal y una exponencial y obtenemos que, para vencimientos cortos menores o iguales a 18 meses la distribución normal proporciona precios más precisos. Sin embargo, para vencimientos largos la distribución exponencial da lugar a errores más pequeños. En cualquier caso, para vencimientos superiores a 18 meses la distribución exponencial proporciona precios más altos que la distribución normal, pero ambas infravaloran los precios del mercado. Además, los errores con ambas distribuciones son mayores para vencimientos superiores o iguales a 36 meses. Probablemente, este hecho pueda ser debido a la propagación del error que puede producirse al utilizar el método de Monte Carlo.

En este trabajo también valoramos opciones sobre futuros con ambas distribuciones, y calculamos los ratios entre los precios de las opciones con la distribución normal y con la exponencial para diferentes precios de ejercicio y diferentes vencimientos. Así, observamos que, para vencimientos cortos, los ratios son mayores del noventa por ciento; sin embargo, si aumentamos el vencimiento, los ratios decrecen considerablemente, y cuanto mayor es el precio de ejercicio, menor es el ratio. Por tanto, las diferencias mas grandes entre los precios de las opciones se producen en el *out of the money*.

En lo que se refiere a la estructura temporal de las primas de riesgo de los futuros del gas natural, también mostramos que ambas primas de riesgo (con la distribución normal y la exponencial) tienen el mismo comportamiento cualitativo, tienen una tendencia decreciente. Además, observamos que la prima de riesgo con los saltos siguiendo una distribución normal es siempre mayor que cuando siguen una distribución exponencial. Como es habitual en la literatura, encontramos que la prima de riesgo con ambas distribuciones tiene una gran variabilidad a lo largo del tiempo. Esto quiere decir que la actividad de los productores y los especuladores también varía a lo largo del tiempo. Además, observamos que, con ambas distribuciones, la prima de riesgo es, en media, positiva para vencimientos cortos y negativa para vencimientos largos. Es decir, para vencimientos cortos, los precios de los futuros proporcionan una predicción sesgada por defecto del precio al contado, pero para vencimientos largos la predicción es sesgada por exceso.

Dado que existe evidencia de estacionalidad en los mercados de algunas materias primas, introducimos una componente estacional en los modelos de valoración considerados previamente. Suponemos que esta componente viene dada por una función del tiempo determinista. Entonces, probamos algunos resultados que permiten estimar todas las funciones de los procesos neutrales al riesgo del modelo. Con esta nueva forma de estimación, valoramos los futuros del gas natural y las opciones sobre futuros negociados en el NYMEX, y analizamos el efecto de la estacionalidad en los precios. Obtenemos que los errores en los precios de los futuros del gas natural son menores para todos los vencimientos cuando se tiene en cuenta la estacionalidad en el modelo. Además, realizamos comparaciones entre los precios de las opciones con ambos modelos. Calculamos los ratios entre los precios de las opciones con o sin estacionalidad para varios precios de ejercicio y vencimientos, y mostramos que el modelo estacional infravalora los precios de las opciones del gas natural con respecto al no estacional para vencimientos cortos (hasta tres meses); sin embargo, sobrevalora para el resto, y cuanto mas largo es el vencimiento, menor es el ratio.

En este trabajo también analizamos la influencia de la estacionalidad en la prima de riesgo de los futuros. Así, calculamos la estructura temporal de la prima de riesgo de los futuros con estacionalidad y sin estacionalidad en el modelo, y observamos que el modelo sin estacionalidad siempre proporciona primas positivas, pero el modelo con estacionalidad presenta primas negativas para vencimientos mayores que 6 meses. En lo que se refiere a su comportamiento a lo largo del tiempo, en ambos casos la prima de riesgo presenta una gran variabilidad a lo largo del tiempo. Todo esto nos lleva a concluir que considerar una componente estacional mejora el modelo de valoración para el mercado del gas natural.

El principal objetivo de este trabajo es proporcionar un enfoque alternativo para estimar las funciones de los procesos neutrales al riesgo directamente de los datos del mercado. De hecho, diseñamos técnicas de estimación para modelos con estacionalidad así como sin estacionalidad. Estas técnicas se pueden utilizar incluso cuando la dinámica de las variables de estado no permite encontrar una forma cerrada de la solución del precio del derivado.

En los temas tratados en esta tesis hay lineas abiertas con las que continuar.

Por una lado, para explicar mejor el comportamiento de los precios de los derivados de materias primas se pueden introducir otras variables de estado en el modelo; por ejemplo, el tipo de interés o la volatilidad del precio al contado. Además, se pueden añadir términos de salto en estas variables para recoger sus cambios bruscos en el mercado.

En lo que se refiere a la distribución del tamaño de salto, se puede suponer que se esta se ve afectada por el cambio de medida en lugar de ser absorbido artificialmente por el cambio en la intensidad del salto, como es habitual en la literatura.

Por otro lado, teniendo en cuenta que hay diferentes tipos de materias primas con comportamientos muy distintos (tales como las agrícolas o los metales), se deberían considerar diferentes dinámicas en las variables de estado del modelo, dependiendo de la materia prima. En particular, algunas de ellas tienen un patrón estacional, especialmente las agrícolas y las de la energía. En este trabajo hemos considerado un factor estacional determinista; sin embargo, sería adecuado suponer que el factor estacional viene dado por componentes trigonométricos generados por procesos estocásticos. En este caso, para este nuevo problema, es necesario proporcionar nuevas técnicas de estimación adecuadas.

En este trabajo hemos utilizado el método de Monte Carlo para obtener una aproximación de los precios de los derivados en los diferentes modelos. Este método es fácil de implementar, pero su precisión es baja; por tanto, sería interesante diseñar método numéricos eficientes para aproximar la solución de la ecuación en derivadas parciales de valoración y así obtener precios más precisos. De esta forma, podríamos reducir los errores, especialmente para vencimientos largos.

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