New approach to measure preference stability

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***Abstract*—A non-traditional approach about the measurement of agents' preference stability is introduced. This contribution focus on measuring preference consensus at different moments under the assumption of considering the following evaluations: approved, undecided and disapproved. To this aim, the concept of *preference stability measure* is defined as well as a particular one, the *sequential preference stability measure*, taking into account any two successive time moments. Finally and in order to highlight the good behaviour of novel measures, some properties are also provided.**

I. Introduction

Several research fields such as Economics, Social Choice, Marketing, Decision Analysis have been paying attention to intertemporal decision making problems.

In the traditional theory literature, preferences have mainly been considered constant along time [1], but some of current studies are focused on checking if preferences are constant over time [2]. From an empirical point of view, preference stability has been studied using small samples in short time periods considering the following type of preferences, the *risk preferences* [3], [4]. In recent years, there has been an increasing interest in works about time preference [5], [6], [7], but only a few contributions study the stability of social preferences [8].

From another point of view, there has been an increase in the number of studies that considers changes in preferences as consequence of shocks such as illness, civil wars, natural disasters, etc. [9], [10], [11], [12]. In addition, other research areas like Game Theory have been dealing with the aforementioned problem [13], [14], [15].

Taking into account the previous literature on measurement of preference stability, this contribution addresses an intertemporal decision making problem where agents or experts express their opinions on an alternative/candidate/option over different time moments. Particularly, agents express their opinions on the alternative under study at different times showing their approving, indecision or disapproving on it.

Under the assumption of this framework, the objective of this contribution is to determine how much stability agents' opinions conveys to the group on the alternative along time. For this propose, a new approach to measure preference stability from a non-traditional perspective is defined, the *preference stability measure*. This measurement takes values in the unit interval considering value 1 full stability and value 0 total lack of stability. Moreover, an specific formulation of the preference stability measure is introduced, the *sequential preference stability measure* as well as a study of its analytic properties. Under this approach, the stability of preferences is understood like the probability that for a randomly chosen moment of time, two randomly chosen agents have the same opinion at such a time and its consecutive.

The paper is structured as follows. Section 2 establishes the notation necessary to be the contribution self-contained. Section 3 introduces our proposal to measure preference stability: the *preference stability measure*. Moreover, an specific type of this measure, the *sequential preference stability measure*, is presented as well as its properties. Finally, some concluding remarks are provided.

II. Notation

Let $\mathbf{N} = \{1, 2, ..., N\}$ a set of agents or experts. Agents express their opinions on an alternative, x, at different time moments $\mathbf{T} = \{t_1, \ldots, t_T\}$.

From now on, the notation used to formalize theses assessments is the following:

Definition 1: A *temporal preference profile* of a set of agents $\mathbf{N}$ on an alternative x at T different time moments is an $N \times T$ matrix

$$\mathbf{P} = \begin{pmatrix} P_{1t_1} & \ldots & P_{1t_T} \\ \vdots & \ddots & \vdots \\ P_{Nt_1} & \ldots & P_{Nt_T} \end{pmatrix}_{N \times T}$$

where P_{it_j} is the opinion of the agent i over alternative x at t_j moment, in the sense

$$P_{it_j} = \begin{cases} 1 & \text{if agent } i \text{ approves } x \text{ at the } t_j \text{ time}, \\ 0.5 & \text{if agent } i \text{ is undecided on } x \text{ at the } t_j \text{ time}, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathbb{P}_{N \times T}$ denote the set of all such $N \times T$ matrices. For simplicity of notation, $(1)_{N \times T}$ is the $N \times T$ matrix whose cells are universally equal to 1, $(0.5)_{N \times T}$ is the $N \times T$

matrix whose cells are universally equal to 0.5 and $(0)_{N\times T}$ is the $N\times T$ matrix whose cells are universally equal to 0.

A temporal preference profile $\mathbf{P}$ is *unanimous* if alternative x is approved (resp. undecided or disapproved) over $\mathbf{T}$ by all agents. In matrix terms, if the time preference profile $\mathbf{P}\in\mathbb{P}_{N\times T}$ is constant, $\mathbf{P}=(1)_{N\times T}$ (resp. $\mathbf{P}=(0.5)_{N\times T}$ or $\mathbf{P}=(0)_{N\times T}$).

Any permutation σ of the agents $\{1,2,...,N\}$ determines a temporal preference profile $\mathbf{P}^\sigma$ by permutation of the rows of $\mathbf{P}$, that is, row i of the profile $\mathbf{P}^\sigma$ is row $\sigma(i)$ of the profile $\mathbf{P}$.

For each temporal preference profile $\mathbf{P}$, $\mathbf{P}_S$ is the restriction to a subset of agents, an *agent-subprofile* on the agents in $S\subseteq\mathbf{N}$, and it emerges from selecting the rows of $\mathbf{P}$ that are associated with the respective agents in S.

For each temporal preference profile $\mathbf{P}$, $\mathbf{P}^I$ is the restriction to a subset of consecutive moments of time, *temporal-subprofile* on the moments of time in $I\subseteq\mathbf{T}$, and it emerges from selecting consecutive columns of $\mathbf{P}$ that are associated with the respective moments of time in I. Any partition $\{I_1,\dots,I_p\}$ of $\mathbf{P}$ generates a decomposition of $\mathbf{P}$ into temporal-subprofiles $\mathbf{P}^{I_1},\dots,\mathbf{P}^{I_p}$ where $\mathbf{P}^{I_1}\cup\ldots\cup\mathbf{P}^{I_p}=\mathbf{P}$.

An *extension* of a temporal preference profile $\mathbf{P}$ of a group of agents $\mathbf{N}$ at $\mathbf{T}=\{t_1,\dots,t_T\}$ is a temporal preference profile $\overline{\mathbf{P}}$ at $\overline{\mathbf{T}}=\{t_1,\dots,t_T,t_{T+1},\dots,t_{T+q}\}$ such that the restriction of $\overline{\mathbf{P}}$ to the first T moments of time of $\overline{\mathbf{T}}$ coincides with $\mathbf{P}$.

A *replication* of a temporal preference profile $\mathbf{P}$ of a group of agents $\mathbf{N}$ on alternative x is the temporal preference profile $\mathbf{P}\uplus\mathbf{P}\in\mathbb{P}_{2N\times T}$ obtained by duplicating each row of $\mathbf{P}$, in the sense that rows r and $N+r$ of $\mathbf{P}\uplus\mathbf{P}$ are row r of $\mathbf{P}$, for each $r=1,...,N$.

For each temporal preference profile $\mathbf{P}$ on alternative x, $n_0^{t_j}$ denotes the number of agents that disapprove x at the t_j moment of time, $n_{0.5}^{t_j}$ denotes the number of agents that are undecided on x at t_j, and $n_1^{t_j}$ denotes the number of agents that approve alternative x at the t_j moment of time. Therefore, $N=n_0^{t_j}+n_{0.5}^{t_j}+n_1^{t_j}$ for each $t_j\in\mathbf{T}$. See Table I for enhancing the understanding.

In addition, $n_{0,0}^{t_j,t_{j+1}}$ denotes the number of agents that disapprove alternative x at t_j and keep their opinion at the following point of time t_{j+1}. Analogously, $n_{0.5,0.5}^{t_j,t_{j+1}}$ denotes the number of agents that are undecided on alternative x at t_j and t_{j+1}. In the same vein, $n_{1,1}^{t_j,t_{j+1}}$ denotes the number of agents that approve alternative x at t_j and keep their opinion at the following point of time t_{j+1}.

In this regard, $n_{0,1}^{t_j,t_{j+1}}$ is the number of agents that disapprove alternative x at t_j but change their opinion at t_{j+1}, and $n_{1,0}^{t_j,t_{j+1}}$ is the number of agents that approve alternative k at t_j but change their opinion at t_{j+1}. $n_{0.5,1}^{t_j,t_{j+1}}$ and $n_{0.5,0}^{t_j,t_{j+1}}$ denote the number of agents that are undecided at t_j but change their opinion at t_{j+1} for approving or disapproving x, respectively. Similarly, $n_{0,0.5}^{t_j,t_{j+1}}$ and $n_{1,0.5}^{t_j,t_{j+1}}$ denote the number of agents that disapprove and approve x at t_j, respectively, but change their opinion at t_{j+1} for undecided. For each $t_j\in\mathbf{T}$, $n_0^{t_j}=n_{0,0}^{t_j,t_{j+1}}+n_{0,0.5}^{t_j,t_{j+1}}+n_{0,1}^{t_j,t_{j+1}}$, $n_{0.5}^{t_j}=n_{0.5,0}^{t_j,t_{j+1}}+n_{0.5,0.5}^{t_j,t_{j+1}}+n_{0.5,1}^{t_j,t_{j+1}}$ and likewise $n_1^{t_j}=n_{1,1}^{t_j,t_{j+1}}+n_{1,0.5}^{t_j,t_{j+1}}+n_{1,0}^{t_j,t_{j+1}}$.

t_j \ t_{j+1}	No	Undecided	Yes	
No	$n_{0,0}^{t_j,t_{j+1}}$	$n_{0,0.5}^{t_j,t_{j+1}}$	$n_{0,1}^{t_j,t_{j+1}}$	$n_0^{t_j}$
Undecided	$n_{0.5,0}^{t_j,t_{j+1}}$	$n_{0.5,0.5}^{t_j,t_{j+1}}$	$n_{0.5,1}^{t_j,t_{j+1}}$	$n_{0.5}^{t_j}$
Yes	$n_{1,0}^{t_j,t_{j+1}}$	$n_{1,0.5}^{t_j,t_{j+1}}$	$n_{1,1}^{t_j,t_{j+1}}$	$n_1^{t_j}$
	$n_0^{t_{j+1}}$	$n_{0.5}^{t_{j+1}}$	$n_1^{t_{j+1}}$	N

Table I: Condensed table of notation

For the purpose of clarifying the use of the previous notation, the following illustrative example is introduced.

Example 1: Let $\mathbf{N}=\{1,2,\dots,12\}$ be a set of twelve agents that express their opinions on alternative x along four consecutive moments of time $\mathbf{T}=\{t_1,t_2,t_3,t_4\}$. Their temporal preference profile is:

$$\mathbf{P}=\begin{pmatrix} P_{1t_1} & \dots & P_{1t_4} \\ \vdots & \ddots & \vdots \\ P_{12t_1} & \dots & P_{12t_4}\end{pmatrix}=\begin{pmatrix} 0 & 0 & 0.5 & 0.0 \\ 0.5 & 0 & 1.0 & 0.5 \\ 1 & 0.0 & 1 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0.5 & 1 & 0 & 1 \\ 0.5 & 0.5 & 1 & 0.5 \\ 1 & 1 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \\ 0.5 & 0 & 0 & 0 \end{pmatrix}_{12\times 4}$$

This temporal preference profile can be summarized in a table containing the number of agents who approve, are undecided or disapprove alternative x at each moment of time t_j as well as the number of agents that keep or change their opinion during consecutive time moments (see Table II).

III. The preference stability measure: Definition and properties

In this section, our proposal of preference stability measure as well as its properties are introduced. Concretely, the notion of preference stability is considered in the same vein that the notion of Bosch's consensus [16]. This seems natural because the measurement of preference stability resembles the notion

t_1 \ t_2	No	Undecided	Yes	
No	$n_{0,0}^{t_1,t_2}=2$	$n_{0,0.5}^{t_1,t_2}=0$	$n_{0,1}^{t_1,t_2}=1$	$n_0^{t_1}=3$
Undecided	$n_{0.5,0}^{t_1,t_2}=2$	$n_{0.5,0.5}^{t_1,t_2}=2$	$n_{0.5,1}^{t_1,t_2}=2$	$n_{0.5}^{t_1}=6$
Yes	$n_{1,0}^{t_1,t_2}=2$	$n_{1,0.5}^{t_1,t_2}=0$	$n_{1,1}^{t_1,t_2}=1$	$n_1^{t_1}=3$
	$n_0^{t_2}=6$	$n_{0.5}^{t_2}=2$	$n_1^{t_2}=4$	$N=12$

t_2 \ t_3	No	Undecided	Yes	
No	$n_{0,0}^{t_2,t_3}=3$	$n_{0,0.5}^{t_2,t_3}=1$	$n_{0,1}^{t_2,t_3}=2$	$n_0^{t_2}=6$
Undecided	$n_{0.5,0}^{t_2,t_3}=0$	$n_{0.5,0.5}^{t_2,t_3}=1$	$n_{0.5,1}^{t_2,t_3}=1$	$n_{0.5}^{t_2}=2$
Yes	$n_{1,0}^{t_2,t_3}=2$	$n_{1,0.5}^{t_2,t_3}=1$	$n_{1,1}^{t_2,t_3}=1$	$n_1^{t_2}=4$
	$n_0^{t_3}=5$	$n_{0.5}^{t_3}=3$	$n_1^{t_3}=4$	$N=12$

t_3 \ t_4	No	Undecided	Yes	
No	$n_{0,0}^{t_3,t_4}=3$	$n_{0,0.5}^{t_3,t_4}=0$	$n_{0,1}^{t_3,t_4}=2$	$n_0^{t_3}=5$
Undecided	$n_{0.5,0}^{t_3,t_4}=2$	$n_{0.5,0.5}^{t_3,t_4}=0$	$n_{0.5,1}^{t_3,t_4}=1$	$n_{0.5}^{t_3}=3$
Yes	$n_{1,0}^{t_3,t_4}=1$	$n_{1,0.5}^{t_3,t_4}=3$	$n_{1,1}^{t_3,t_4}=0$	$n_1^{t_3}=4$
	$n_0^{t_4}=6$	$n_{0.5}^{t_4}=3$	$n_1^{t_4}=3$	$N=12$

Table II: Condensed table of notation for Example 1

of measurement of consensus over time, in the sense that the maximum value captures the notion of full stability, that is unanimity along time, while the minimum value captures the notion of total lack of stability, that is, total disagreement along time.

From the Social Choice literature, it is possible to point out the consensus measurement proposed by Alcalde-Unzu and Vorsatz [17], Alcantud et al. [18], Alcantud, de Andrés Calle and Cascón [19], García-Lapresta and Pérez-Román [20] and González-Arteaga et al. [21]. Additionally, there are several studies related to consensus problem from the Decision Making Theory like the approaches proposed by González-Arteaga et al. [22], González-Pachón and Romero [23], González-Pachón et al. [24], Herrera-Viedma et al. [25], and so on.

Taking into account the aforementioned arguments, our novel approach to measure preference stability is now presented.

Definition 2: A *preference stability measure* for a group of agents $\mathbf{N} = \{1, ..., N\}$ on an alternative x is a mapping

$$\psi : \mathbb{P}_{N\times T} \rightarrow [0,1]$$

that assigns a number $\psi(\mathbf{P}) \in [0,1]$ to each temporal preference profile $\mathbf{P}$, with the properties:

i) $\psi(\mathbf{P}) = 1$ if and only if $\mathbf{P}$ is unanimous (full stability).
ii) $\psi(\mathbf{P}^\sigma) = \psi(\mathbf{P})$ for each permutation σ of the agents and $\mathbf{P} \in \mathbb{P}_{N\times T}$ (anonymity).

A preference stability measure is a collection of preference stability measures for each group of agents **N**.

Our proposal unlike Bosch's contribution does not require *neutrality* property, time moments can be exchanged, due to the fact that time order is an essential aspect to measure the stability of preferences.

Now a particular preference stability measure is introduced. Formally:

Definition 3: The *sequential preference stability measure* for a group of agents $\mathbf{N} = \{1, ..., N\}$ on an alternative x is the mapping $\psi_S : \mathbb{P}_{N\times T} \rightarrow [0,1]$ given by

$$\begin{aligned}
\psi_S(\mathbf{P}) &= \\
&= \frac{1}{T-1}\cdot\frac{\sum_{j=1}^{j=T-1} n_{0,0}^{t_j,t_{j+1}}\cdot(n_{0,0}^{t_j,t_{j+1}}-1)}{N(N-1)} \\
&+ \frac{1}{T-1}\cdot\frac{\sum_{j=1}^{j=T-1} n_{0.5,0.5}^{t_j,t_{j+1}}\cdot(n_{0.5,0.5}^{t_j,t_{j+1}}-1)}{N(N-1)} \\
&+ \frac{1}{T-1}\cdot\frac{\sum_{j=1}^{j=T-1} n_{1,1}^{t_j,t_{j+1}}\cdot(n_{1,1}^{t_j,t_{j+1}}-1)}{N(N-1)}
\end{aligned}$$

Intuitively, it measures the probability that for a randomly chosen moment of time, two randomly chosen agents of a group have the same opinion upon an alternative at the moment of time selected and its consecutive.

It is easy to check that Definition 3 provides a preference stability measure.

Example 2: For the temporal preference profile in Example 1, the computations obtained are the following:

$$\begin{aligned}
\psi_S(\mathbf{P}) &= \\
&= \frac{1}{3}\cdot\frac{2(2-1)+3(3-1)+3(3-1)}{12(11)}+ \\
&+ \frac{1}{3}\cdot\frac{2(2-1)+1(1-1)+0(0-1)}{12(11)}+ \\
&+ \frac{1}{3}\cdot\frac{1(1-1)+1(1-1)+0(0-1)}{12(11)} = \\
&= 0.04
\end{aligned}$$

In this case, the sequential stability measure takes a value near zero because the opinions of two agents hardly ever coincidence in two successive time moments.

Some desirable properties of the sequential preference stability measure are defined bellow.

Properties[1]*:*

1) **Reversal invariance** among decided agents: This property shows that the main aspect of the sequential preference stability measure is the stability of agents' opinions more than an specific value. If the 0's are changed for 1's and vice verse (undecided agents do not change), then the sequential preference stability measure reminds equal. Formally:

   Let $\mathbf{P}^c$ be the complementary temporal preference profile of $\mathbf{P}$ defined by $\mathbf{P}^c = (1)_{N\times T} - \mathbf{P}$. If ψ_S verifies reversal invariance among decided agents then $\psi_S(\mathbf{P}^c) = \psi_S(\mathbf{P})$.

2) **Temporal reducibility**: It means that the stability of a temporal preference profile is the average of the sequential preference stability measures of all its consecutive temporal-subprofiles of two consecutive moments of time. Formally:

   Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a temporal preference profile. We say that ψ_S verifies time-reducibility if

$$\psi_S(\mathbf{P}) = \frac{1}{T-1}\sum_{j=1}^{T-1}\psi_S(\mathbf{P}^{I_{j,j+1}})$$

   where $\mathbf{P}^{I_{j,j+1}} \in \mathbb{P}_{N\times 2}$ is the temporal-subprofile of $\mathbf{P}$ containing the columns corresponding to times t_j and t_{j+1}.

3) **Convexity**: It means the sequential preference stability measure of a temporal preference profile is a weighted average of the measures of any decomposition of $\mathbf{P}$ into consecutive temporal-subprofiles. Formally:

   For each temporal preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$, and each decomposition of $\mathbf{P}$ into two consecutive temporal-subprofiles, $\mathbf{P}^{I_1} \in \mathbb{P}_{N\times(k_1+1)}$ and $\mathbf{P}^{I_2} \in \mathbb{P}_{N\times(T-k_1)}$ with $I_1 = \{t_1,\dots,t_{k_1+1}\}$ and $I_2 = \{t_{k_1+1},\dots,t_T\}$, and $(\mid I_1 \mid -1) + (\mid I_2 \mid -1) = T-1$

$$\psi_S(\mathbf{P}) = \frac{(\mid I_1 \mid -1)\cdot\, \psi_S(\mathbf{P}^{I_1}) + (\mid I_2 \mid -1)\cdot\, \psi_S(\mathbf{P}^{I_2})}{T-1}$$

4) **Replication monotonicity**: When a non-unanimous temporal preference profile is replicated, its sequential preference stability measure increases. Formally:

   Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a non unanimous temporal preference profile then

$$\psi_S(\mathbf{P} \uplus \mathbf{P}) > \psi_S(\mathbf{P})$$

   In addition, for an unanimous time preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$, by Definition 3, ψ_S verifies

$$\psi_S(\mathbf{P} \uplus \mathbf{P}) = \psi_S(\mathbf{P}) = 1$$

5) **Minimum stability**: If all agents express their opinions at t_j and change their opinions at t_{j+1}, then the sequential preference stability measure takes a zero value. It also happens when there are at most two agents keeping their opinion at t_j and t_{j+1}, but their opinions do not coincide each other. Formally:

   Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a temporal preference profile such that there is at most one agent who has the same opinion at t_j and t_{j+1} for $j \in \{1,\dots T\}$, that is, $n_{0,0}^{t_j,t_{j+1}} \leq 1$, $n_{0.5,0.5}^{t_j,t_{j+1}} \leq 1$ and $n_{1,1}^{t_j,t_{j+1}} \leq 1$ for all $j \in \mathbf{T}$. Then, $\psi_S(\mathbf{P}) = 0$.

6) **Breaking minimum stability**: In order to break the minimum stability it is needed that at least the opinions of two agents coincide at the same moment of time and the next one. Formally:

   Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a temporal preference profile such that there exists at least a k, $k \in \mathbf{T}$, such that $n_{0,0}^{t_k,t_{k+1}} > 1$ or $n_{0.5,0.5}^{t_k,t_{k+1}} > 1$ or $n_{1,1}^{t_k,t_{k+1}} > 1$, then $\psi_S(\mathbf{P}) > 0$.

7) **Temporal monotonicity**: Consider two temporal preference profiles, $\mathbf{P}$ and $\mathbf{P'}$, that coincide in all their elements excepting the opinion of an agent $m \in \mathbf{N}$, at t_k and t_{k+1}. Concretely, this agent has different opinion at t_k and t_{k+1} in $\mathbf{P}$: $P_{mt_j} \neq P_{mt_{j+1}}$, and the agent's opinion is the same at t_k and t_{k+1} in $\mathbf{P'}$: $P'_{mt_j} = P'_{mt_{j+1}}$. In this case, the sequential preference stability measure verifies $\psi_S(\mathbf{P'}) \geq \psi_S(\mathbf{P})$. Formally:

   Let $\mathbf{P}, \mathbf{P'} \in \mathbb{P}_{N\times T}$ be temporal preference profiles such that:

   a) $P_{it_j} = P'_{it_j}$, $i \in \{\mathbf{N}\setminus\{m\}\}$, $t_j \in \{\mathbf{T}\setminus\{t_k,t_{k+1}\}\}$,
   b) $P_{mt_k} \neq P_{mt_{k+1}}$, $m \in \mathbf{N}$, $t_k, t_{k+1} \in \mathbf{T}$,
   c) $P'_{mt_k} = P'_{mt_{k+1}}$, $m \in \mathbf{N}$, $t_k, t_{k+1} \in \mathbf{T}$.

   Then, $\psi_S(\mathbf{P'}) \geq \psi_S(\mathbf{P})$.

8) **Convergence to full stability**: If new moments of time are repeatedly introduced into the problem and all agents have the same opinion at them, then the sequential preference stability measure approaches 1. Formally:

   Suppose that q moments of time $t_{T+1},\dots t_{T+q}$ are added to $\mathbf{T}$, and at these new moments of time the alternative x is unanimously approved (resp. unanimously

[1]The proofs of the properties are not included in this contribution because the limited space, but they can be provided if they were required.

undecided or unanimously disapproved) by all agents. If the introduction of new moments of time does not affect agents' opinions in past times, then the sequential time cohesiveness measure of the extended temporal preference profile $\overline{\mathbf{P}}^{(q)} \in \mathbb{P}_{N\times(T+q)}$ approaches 1 when q tends to infinity.

$$\lim_{q\to\infty} \psi_S(\overline{\mathbf{P}}^{(\mathbf{q})}) = 1$$

IV. Concluding remarks

Research in the subject of preference stability has made progress mostly in Economics. The aim of this paper is to manage the problem of measuring preference stability from a non-traditional perspective. In order to set forth the context of our research a framework is establihsed where agents express their opinions on an alternative at different moments considering the following evaluations: approved, undecided and disapproved. The general notion of *preference stability measure* is introduced. Then, a specific formulation is developed with particular regard to any two successive time moments. In this way, the *sequential preference stability measure* is proposed. Moreover, some meaningful properties which make our proposal compelling are also provided.

Overall, the proposals of this contribution have a range of implications for future research. Many problems on preference stability from a diversity of fields can be faced by our approach such as the consumers' preferences, risk preference, and so on.

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