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# Electoral Mathematics and Asymmetrical Treatment to Political Parties: The Mexican Case

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***Abstract*—**The Mexican Chamber of Deputies is composed of 500 representatives: 300 of them elected by relative majority and another 200 ones elected through proportional representation in five electoral clusters (constituencies) with 40 representatives each. In this mixed-member electoral system, the seats distribution of proportional representation is not independent of the election by relative majority, as it attempts to correct representation imbalances produced in single-member districts. This two-fold structure has been maintained in the successive electoral reforms carried out along the last three decades (eight from 1986 to 2014). In all of them, the election process of 200 seats becomes complex: Formulas in the Law are difficult to understand and to be interpreted. This paper analyzes the Mexican electoral system after the electoral reform of 2014, which was applied for the first time in 2015. The research focuses on contradictions and issues of applicability, in particular situations where seats allocation is affected by ambiguity in the law and where asymmetrical treatment of political parties arises. Due to these facts, a proposal of electoral reform will be presented. It is intended to be simpler, clearer, and more enduring than the current system. Furthermore, this model is more suitable for producing electoral outcomes free of contradictions and paradoxes. This approach would allow a fair treatment of political parties and as a result an improved opportunity to exercise democracy.

***Keywords*—**Apportionment paradoxes, biproportional representation, electoral mathematics, electoral reform, Mexican electoral system, proportional representation, political asymmetry.

## I. Introduction

THE constitutionalization of political parties in Mexico took place with the 1977 electoral reform. Since then, the Mexican electoral system used in the election of the Chamber of Deputies (also called the Lower House of the Congress) has been mixed. Initially, the Chamber was composed for 300 deputies elected by relative majority (RM) and 100 by proportional representation (PR). The Deputies of PR were only assigned to parties with less than 60 RM seats reaching at least 1.5% of the national vote [1].

The allocation of PR seats was modified in the electoral reform of 1986-1987. The number of deputies of PR increased to 200. The reform also stated that no political party could have more than 350 deputies (70% of the Chamber) even if they had had a higher percentage of votes. Even more, a governability clause was incorporated to guarantee an absolute majority of the Chamber of Deputies for the party with the most seats [2].

The 1990 electoral reform modified the previous governability clause. In order to reach an absolute majority, a sufficient number of seats was assigned to the party that having obtained the most seats by RM also achieved at least 35% of them. On the other hand, all the parties with at least 35% of the national vote will be given two more seats for each 1% of the votes obtained above 35% and up to 75% [3].

In the next electoral reform (1993-1994), the maximum representation limit for each political party in the Chamber of Deputies was reduced from 350 to 315 seats [4]. It was the 1996 reform which established that no political party could obtain more than 300 deputies in total and that the percentage of deputies for any political party could not exceed 8% of its percentage of the national vote, except for parties that reached this difference by winning districts by RM. Furthermore, the legal threshold to have access to PR seats was increased to 2% [5].

Electoral reforms in 2007-2008 did not consider changes in the method of seat allocations for political parties [6]. The last electoral political reform in 2014, named *Ley General de Institutions y Procedimientos Electorales (LGIPE),* increased from 2% to 3% the minimum percentage required to have right to the allocation of PR seats [7]. It is important to emphasize that, along these processes, the highest remainders method has been used in the distribution of seats. However, the implementation description of the electoral system has always been tedious and ambiguous.

Regarding the electoral system in Mexico, some studies have exposed failures in its electoral law and shown non-applicability in some situations. Furthermore, they have proven how the party in power, the PRI, was unfairly favored [8]-[10]. The hegemonic role of PRI party, which had normally achieved the control in the Chamber, is also analyzed in [11] in terms of number of seats that the parties could reach and those that actually they have reached.

There also exist qualitative studies on the effects of electoral reforms in Mexico, highlighting the political, economic and social circumstances that gave rise to the different electoral reforms. In this way, [12] explains the results of these electoral reforms in terms of the development of democracy in Mexico and illustrates the difference in treatment among political parties.

The paper is structured as follows: Section II introduces the

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basic concepts. Section III presents the implementation of the Mexican Electoral Law for the 2015 election, showing how the obtained results yield inequities concerning the treatment to political parties. Section IV deals with inconsistencies and paradoxes of the Mexican electoral system, considering specific examples that clearly make evident such failures. Section V shows how the implementation of the Electoral Law causes asymmetrical treatment of political parties: Specifically, big parties take advantage of the small ones. In Section VI a proposal for electoral reform in Mexico with a fairer treatment to political parties is suggested. Finally, in Section VII, some conclusions are presented.

## II. Basic Concepts

### A. Notation

Let $V$ be the number of voters, $n$ the number of parties and $S$ the number of seats to be distributed; $(V_1, V_2, \dots, V_n)$ is the vector of votes obtained by each party, so that $V = \sum_{i=1}^{n} V_i$, and $(S_1, S_2, \dots, S_n)$ is the vector of seats assigned to each party, where $S = \sum_{i=1}^{n} S_i$; finally, $v_i$ and $s_i$ are the proportion of votes and seats that party $i$ receives, respectively. Thus, $v_i = V_i/V$ and $s_i = S_i/S$ .

The quota is the number of seats that the party $i$ should receive in exact proportionality after obtaining $V_i$ votes. That is, the quota for party $i$ is $q_i = \frac{V_i}{V} S$, $i = 1,2, \dots, n$.

The lower quota is the closest integer number that does not exceed $q_i$; it will be denoted by $\lfloor q_i \rfloor$. Likewise, the upper quota is the smallest integer number bigger than or equal to $q_i$; it will be denoted by $\lceil q_i \rceil$. Those apportionment methods that will be used in this paper (Highest Remainder, Sainte-Laguë and Biproportional apportionment method) will be introduced in what follows (see [13] for more details).

Under the Highest Reminder method (HR), also called Hamilton Rule, each party first receives as many seats as the integer part of its quota, $\lfloor q_i \rfloor$. Then, the remainders $(q_i - \lfloor q_i \rfloor)$ are ordered from the largest to the smallest one. Finally, the remaining seats are assigned to parties with highest reminders up to $S$ seats are completed.

Saint-Laguë method (also called Webster rule) is one of the apportionment divisor methods which considers for each party successive quotients of its number of obtained votes $V_i$, calculated as

$$\frac{V_i}{s+1/2}$$

where $s = 0,1,2 \dots S-1$.

The $S$ highest quotients determine both the number of seats for each party and the order in which they are allocated. For practical purposes the previous quotients are equivalent to those obtained dividing $V_i$ by $s = 1,3,5 \dots$

Biproportional apportionment methods has been described by Balinski and Pukelsheim [14] as:

> […] a novel approach of translating electoral votes into parliamentary seats. A two-way proportionality is achieved, to districts relative to their populations, and to parties relative to their total votes. The methods apply when the electoral region is subdivided into several electoral districts, each with a prespecified "district magnitude", that is, the number of seats per district. The input data thus consists of a matrix with rows and columns corresponding to districts and parties, and entries to party votes in districts. A biproportional apportionment method converts the party votes into an apportionment matrix of corresponding seat-numbers such that, within a district, the sum of the seat-numbers matches the prespecified district magnitude, while within a party, the seat-numbers sum to the overall party seats that are proportional to the vote totals across the whole electoral region.

Due to the calculus difficulty (pointed out in [15]), computer programming is necessary to perform biproportional method. Currently there exists free software, called BAZI, which is simple to use [16].

### B. General Description of LGIPE

The current Mexican Chamber of Deputies has 500 members. 300 of them are elected by RM in 300 uninominal districts. The remaining 200 seats are allocated under PR through the system of regional lists considering 5 constituencies made up of 40 seats each.

In order to obtain its regional lists registration, a political party must present RM candidates in at least 200 districts. Once this requirement satisfied, any political party that reaches at least 3% of the Valid Cast Votes (sum of the votes casted at the polls minus null votes and votes for unregistered candidates) will be allowed to participate in the distribution of PR deputies.

Concerning the number of seats that a party can reach in total (RM and PR modalities), there are two restrictions. On one hand, no political party can obtain more than 300 seats in total. And, on the other hand, the percentage of deputies obtained by a political party cannot exceed 8% of Effective National Votes (VCV minus the number of votes obtained by independent candidates and by political parties that did not reach 3% of VCV). The second limit does not apply to political parties obtaining under RM a percentage of total Chamber seats higher than the sum of its percentage of ENV plus 8%. For example, if a political party obtains 35% of ENV, it cannot receive more than 43% of all seats in the Chamber; that is, no more than 215 of the 500 deputies.

The allocation of the 200 PR seats is codified in articles 14-21 of LGIPE. The implementation of these articles is developed in the next section for 2015 elections results.

## III. Implementation of LGIPE on 2015 Elections

In this section, the 2015 PR seats allocation results will be derived step-by-step. Data of votes in this section are obtained in [17].

First of all, Table I shows the basic data for applying the allocation method: VCV and ENV.

TABLE I
VCV AND ENV FOR 2015 ELECTIONS [17]

| | |
|---|---|
| Total vote emitted | 39,864,082 |
| Nulls votes | 1,900,449 |
| Not registered | 52,371 |
| VCV | 37,911,262 |
| Independent candidates | 225,029 |
| Parties under 3% VCV | 1,990,817 |
| ENV | 35,695,416 |

The votes and VCV percentages for each political party, the null votes and the votes cast for Independent (IND) and Non-Registered (NR) candidates in 2015 elections are shown in Table II. Likewise, results for RM seats appear in last column of Table II. It can be observed that PT and Humanista parties did not get 3% of VCV, so they had no right to participate in the distribution of PR seats. The remaining parties were allowed to participate in the allocation of PR seats.

TABLE II
DEPUTY ELECTIONS 2015 [18]

| Party | Votes | VCV (%) | RM seats |
|---|---|---|---|
| PRI | 11,636,957 | 30.70 | 155 |
| PAN | 8,377,535 | 22.10 | 56 |
| PRD | 4,335,321 | 11.44 | 28 |
| MORENA | 3,345,712 | 8.83 | 14 |
| PVEM | 2,757,170 | 7.27 | 29 |
| MC | 2,431,063 | 6.41 | 10 |
| NA | 1,486,626 | 3.92 | 1 |
| PES | 1,325,032 | 3.50 | 0 |
| PT | 1,134,101 | 2.99 | 6 |
| Humanista | 856,716 | 2.26 | 0 |
| Nulls | 1,900,449 | - | - |
| IND | 225,029 | 0.59 | 1 |
| NR | 52,371 | - | - |
| Total | 39,864,082 | 100 | 300 |

TABLE III
MAXIMUM LIMIT OF SEATS BY PARTY

| Party | Votes | ENV (%) | ENV(%)+8 | Limit of seats by party |
|---|---|---|---|---|
| PRI | 11,636,957 | 32.60 | 40.60 | 203 |
| PAN | 8,377,535 | 23.47 | 31.47 | 157 |
| PRD | 4,335,321 | 12.15 | 20.15 | 100 |
| MORENA | 3,345,712 | 9.37 | 17.37 | 86 |
| PVEM | 2,757,170 | 7.72 | 15.72 | 78 |
| MC | 2,431,063 | 6.81 | 14.81 | 74 |
| NA | 1,486,626 | 4.17 | 12.16 | 60 |
| PES | 1,325,032 | 3.71 | 11.71 | 58 |
| Total | 35,695,416 | 100 | | |

Once calculated the VCV and known what parties do not reach the 3% threshold, the next step consists in determining the ENV percentage for each party and checking if there is some party out of the established limits: No party may seat more than 300 deputies and no party's seat share may exceed more than eight percent its ENV share, as mentioned. These requirements can be checked in Table III. Now, articles 16, 17.1 and 17.3 of the LGIPE [7] establish how to assign the PR seats (English version of LGIPE deals with "districts" instead of):

*Article 16.*

1. For the allocation of representatives by proportional representation according to the provisions of section III of article 54 of the Constitution, a pure proportionality formula integrated by the following elements will be applied:

a) Natural Quotient, and
b) Largest Remainder.

2. Natural Quotient: is the result of dividing the national cast votes by the 200 representatives by proportional representation.
3. Largest Remainder: is the highest remainder between the remaining votes cast for each political party, once the allocation of seats is done by the natural quotient. The largest remainder is used when there are still seats left to distribute.

*Article 17.*

1. Once the formula mentioned in the previous article has been developed, the following procedure will be observed:

a) It will be decided which representatives will be allocated to each political party, according to the number of times that its votes contain the natural quotient, and
b) The representatives that will be distributed by largest remainder, if there are remaining seats to assign after applying the natural quotient, follow the descending order of the votes not used for each of the political parties in the distribution of seats.

*[…]*

3. Once the excess number of representatives by proportional representation has been determined, the political party affected by the terms of the previous paragraph will be allocated its corresponding seats for each district, according to the following terms:

a) The distribution ratio will be obtained by dividing the total number of votes of the political party which finds itself in this situation by the seats to be allocated to the same political party;
b) The votes obtained by the political party in each electoral district will be divided by the distribution ratio, allocating in whole numbers the seats for each of the districts, and
c) If there are still representatives left to be allocated, the largest remainder method will be used, as stated in the previous article.

Table IV shows the implementation of the procedure described for 2015 elections. To this aim, the Natural Quotient (NQ) appearing in the Mexican Law is:

$$\text{NQ} = \frac{35{,}695{,}416}{200} = 178{,}477.08$$

Notice that the method appearing in Articles 16 and 17.1 exactly corresponds with the Highest Remainder method described in Section II, where for each party, Votes/NQ coincides with its quota.

TABLE IV
ESTIMATION OF PR SEATS BY PARTY

| Party | Votes/NQ | Assigned seats | Unused votes | Adjustment seats | HR seats |
|---|---|---|---|---|---|
| PRI | 65.2 | 65 | 35,947 | | 65 |
| PAN | 46.94 | 46 | 167,589 | 1 | 47 |
| PRD | 24.29 | 24 | 51,871 | | 24 |
| MORENA | 18.75 | 18 | 133,125 | 1 | 19 |
| PVEM | 15.45 | 15 | 80,014 | 1 | 16 |
| MC | 13.62 | 13 | 110,861 | 1 | 14 |
| NA | 8.33 | 8 | 58,809 | | 8 |
| PES | 7.42 | 7 | 75,692 | | 7 |
| Total | 200 | 196 | - | 4 | 200 |

Table V summarizes the number of RM and PR seats obtained by each party at this moment.

TABLE V
RM AND PR ESTIMATED SEATS BY PARTY

| Party | RM seats | HR seats | Total assignment | Limit of seats by party |
|---|---|---|---|---|
| PRI | 155 | 65 | 220 | 203 |
| PAN | 56 | 47 | 103 | 157 |
| PRD | 28 | 24 | 52 | 100 |
| MORENA | 14 | 19 | 33 | 86 |
| PVEM | 29 | 16 | 45 | 78 |
| MC | 10 | 14 | 24 | 74 |
| NA | 1 | 8 | 9 | 60 |
| PES | 0 | 7 | 7 | 58 |
| PT | 6 | 0 | 0 | |
| Total | 300 | 200 | - | - |

There is not party over 300 seats. On the other hand, it can be observed that PRI party is the only one that exceeds the limit given by its percentage of ENV plus eight points (with 17 seats). For this reason, PRI gets just 48 HR seats instead of those 65 initially assigned.

In order to allocate such 48 seats per constituency according to the obtained votes in each one, the distribution ratio (DR) considered in article 17.3 is:

$$\text{DR} = \frac{11{,}636{,}957}{48} = 242{,}436.60$$

Dividing the PRI votes in each constituency by this coefficient the corresponding number of seats for this party is obtained (Table VI).

TABLE VI
PRI SEATS BY CONSTITUENCY

| Cons | Votes PRI | Votes PRI /DR | Deputies number | Unused votes | Reman. seats | Total seats |
|---|---|---|---|---|---|---|
| 1st | 2,336,569 | 9.6378 | 9 | 154,640 | 1 | 10 |
| 2nd | 2,689,712 | 11.0945 | 11 | 22,909 | | 11 |
| 3rd | 2,334,043 | 9.6274 | 9 | 152,114 | 1 | 10 |
| 4th | 1,585,747 | 6.54087 | 6 | 131,127 | | 6 |
| 5th | 2,690,886 | 11.0993 | 11 | 24,083 | | 11 |
| Total | 11,636,957 | | 46 | | | 48 |

Once the distribution of seats per constituency of the party that exceeds the limit has been done, the remaining seats are object of a new distribution among the other parties, taking into account ENV, is presented in Table VI.

*Article 18.1a*

*[…]*

II. The effective national votes will be divided by the number of seats of parliament to be allocated, to obtain a new natural quotient;
III. The effective national votes obtained by each political party will be divided by the new natural quotient. The result in whole numbers will be the total number of representatives to be allocated to each political party, and
IV. If there would still be seats left to be distributed, these would be allocated according to the largest remainders of the political parties.

The new natural quotient for the 2015 elections is:

$$\text{New NQ} = \frac{35{,}695{,}416 - 11{,}636{,}957}{200 - 48} = 152{,}303.59$$

The total number of seats for the remaining parties is obtained with this new natural quotient, as appears in Table VII.

TABLE VII
SEATS FOR THE REMAINING PARTIES IN 2015

| Party | Votes | Seats | Not used Votes | Seats adjustment | Total seats / party |
|---|---|---|---|---|---|
| PAN | 8,377,535 | 52 | 147,010 | 1 | 53 |
| PRD | 4,335,321 | 27 | 61,779 | | 27 |
| MORENA | 3,345,712 | 21 | 21,846 | | 21 |
| PVEM | 2,757,170 | 17 | 66,421 | 1 | 18 |
| MC | 2,431,063 | 15 | 56,873 | | 15 |
| NA | 1,486,626 | 9 | 62,112 | 1 | 10 |
| PES | 1,325,032 | 8 | 58,797 | | 8 |

Now, the procedure to distribute the seats of each party among the five constituencies is described in Article 18.2 of the LGIPE [7], it says:

*Article 18.2*

*[…]*

b) The effective vote by electoral districts will be divided by the number of seats to be allocated in each multi-member regional electoral district in order to obtain the distribution ratio in each district;
c) The effective vote for each political party in each of the multi-member regional electoral district will be divided by the distribution ratio, and the result in whole numbers is the total number of representatives to be allocated in each multi-member regional electoral district, and
d) If some seats are left to be distributed to the political parties after applying the distribution ratio, the largest remainder of votes that each political party received in the electoral districts will be used, in descending order, until they are exhausted, so that each multi-member regional electoral district has forty representatives.

According to the described process, the quotients of distribution in each district appear in Table VIII. With these results and the ENV for each party, seats for the remaining

parties per constituency are obtained (Table IX).

TABLE VIII
DISTRIBUTION COEFFICIENT FOR EACH CONSTITUENCY

| Cons | Votes | Votes PRI | Effective votes | PRI seats | Remaining seats | DC |
|---|---|---|---|---|---|---|
| 1st | 6,629,435 | 2,336,569 | 4,292,866 | 10 | 30 | 143,095.53 |
| 2nd | 7,876,851 | 2,689,712 | 5,187,139 | 11 | 29 | 178,866.86 |
| 3rd | 7,086,446 | 2,334,043 | 4,752,403 | 10 | 30 | 158,413.43 |
| 4th | 6,467,060 | 1,585,747 | 4,881,313 | 6 | 34 | 143,568.03 |
| 5th | 7,635,624 | 2,690,886 | 4,944,738 | 11 | 29 | 170,508.21 |

TABLE IX
PARTY SEATS PER CONSTITUENCY

| Party | Cons | Effective votes | DR /Cons. | Quota | Seats | Remaining Votes | Add Seats |
|---|---|---|---|---|---|---|---|
| PAN | 1st | 1,790,937 | 143,095.53 | 12.52 | 12 | 73,791 | 1 |
| PAN | 2nd | 2,707,710 | 178,866.86 | 15.14 | 15 | 24,707 | |
| PAN | 3rd | 1,280,757 | 158,413.43 | 8.08 | 8 | 13,450 | |
| PAN | 4th | 1,147,713 | 143,568.03 | 7.99 | 7 | 142,737 | 1 |
| PAN | 5th | 1,450,418 | 170,508.21 | 8.51 | 8 | 86,352 | 1 |
| PRD | 1st | 316,598 | 143,095.53 | 2.21 | 2 | 30,407 | |
| PRD | 2nd | 479,996 | 178,866.86 | 2.68 | 2 | 122,262 | 1 |
| PRD | 3rd | 922,941 | 158,413.43 | 5.83 | 5 | 130,874 | 1 |
| PRD | 4th | 1,259,498 | 143,568.03 | 8.77 | 8 | 110,954 | |
| PRD | 5th | 1,356,288 | 170,508.21 | 7.95 | 7 | 162,731 | 1 |
| MORENA | 1st | 365,306 | 143,095.53 | 2.55 | 2 | 79,115 | |
| MORENA | 2nd | 342,972 | 178,866.86 | 1.92 | 1 | 164,105 | 1 |
| MORENA | 3rd | 806,798 | 158,413.43 | 5.09 | 5 | 14,731 | |
| MORENA | 4th | 1,096,758 | 143,568.03 | 7.64 | 7 | 91,782 | 1 |
| MORENA | 5th | 733,878 | 170,508.21 | 4.30 | 4 | 51,845 | |
| PVEM | 1st | 299,898 | 143,095.53 | 2.10 | 2 | 13,707 | |
| PVEM | 2nd | 569,775 | 178,866.86 | 3.19 | 3 | 33,174 | 1 |
| PVEM | 3rd | 1,141,491 | 158,413.43 | 7.21 | 7 | 32,597 | |
| PVEM | 4th | 401,659 | 143,568.03 | 2.80 | 2 | 114,523 | 1 |
| PVEM | 5th | 344,347 | 170,508.21 | 2.02 | 2 | 3,331 | |
| MC | 1st | 1,026,591 | 143,095.53 | 7.17 | 7 | 24,922 | |
| MC | 2nd | 465,741 | 178,866.86 | 2.60 | 2 | 108,007 | |
| MC | 3rd | 225,516 | 158,413.43 | 1.42 | 1 | 67,103 | |
| MC | 4th | 366,648 | 143,568.03 | 2.55 | 2 | 79,512 | 1 |
| MC | 5th | 346,567 | 170,508.21 | 2.03 | 2 | 5,551 | |
| NA | 1st | 286,959 | 143,095.53 | 2.01 | 2 | 768 | |
| NA | 2nd | 364,309 | 178,866.86 | 2.04 | 2 | 6,575 | |
| NA | 3rd | 208,688 | 158,413.43 | 1.32 | 1 | 50,275 | 1 |
| NA | 4th | 299,482 | 143,568.03 | 2.09 | 2 | 12,346 | |
| NA | 5th | 327,188 | 170,508.21 | 1.92 | 1 | 156,680 | 1 |
| PES | 1st | 206,577 | 143,095.53 | 1.44 | 1 | 63,481 | 1 |
| PES | 2nd | 256,636 | 178,866.86 | 1.43 | 1 | 77,769 | |
| PES | 3rd | 166,212 | 158,413.43 | 1.05 | 1 | 7,799 | |
| PES | 4th | 309,555 | 143,568.03 | 2.16 | 2 | 22,419 | |
| PES | 5th | 386,052 | 170,508.21 | 2.26 | 2 | 45,036 | |

Summing up, the application of the law produces the PR seats allocation appearing in Table X.

Notice that the mere implementation of the law does not always guaranty an exact allocation of 40 seats per constituency. In order to avoid this problem, one seat that should correspond to MC party in district 2 was transferred to the same party in district 4. The reason of this adjustment was that 108,007 votes of MC did not obtain representation in district 4, while with 79,512 votes in district 2 one seat were assigned. A similar fact occurred with PES (one seat from this party was transferred from district 2 to 1). With these seats movements, exactly 40 seats per constituency were achieved. Consequently, the law favors the biggest parties to assure their seats in the corresponding constituencies, while the smallest ones sometimes have to transfer seats among constituencies. Therefore, an asymmetrical treatment to some parties with respect to others may arise.

TABLE X
ALLOCATION OF TOTAL SEATS PER POLITICAL PARTY

| | Constituency | | | | | |
|---|---|---|---|---|---|---|
| Party | 1st | 2nd | 3rd | 4th | 5th | Total |
| PRI | 10 | 11 | 10 | 6 | 11 | 48 |
| PAN | 13 | 15 | 8 | 8 | 9 | 53 |
| PRD | 2 | 3 | 6 | 8 | 8 | 27 |
| MORENA | 2 | 2 | 5 | 8 | 4 | 21 |
| PVEM | 2 | 4 | 7 | 3 | 2 | 18 |
| MC | 7 | 3 | 1 | 2 | 2 | 15 |
| NA | 2 | 2 | 2 | 2 | 2 | 10 |
| PES | 1 | 2 | 1 | 2 | 2 | 8 |
| Independent | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 39 | 42 | 40 | 39 | 40 | 200 |

## IV. INCONSISTENCIES AND PARADOXES AFFLICTING THE MEXICAN ELECTORAL SYSTEM

The previous section showed an asymmetrical treatment among parties, where two small ones had to transfer a seat among constituencies. Nonetheless, they maintained their amount of seats in a global way.

Now a hypothetical example will demonstrate that this kind of agreements is not always possible and some party might be forced to lose seats. As a consequence, the formulas described in the Mexican electoral law may yield inconsistencies that directly demonstrate its inapplicability.

First of all, suppose that there exists eight parties whose RM seats appear in Table XI and also assume that the votes' distribution per constituency is reflected in Table XII. Notice that, although this is an unreal example, the number of votes for each party, if multiplied by 1000, would be similar to those obtained in 2015 by the participant parties.

First, all 200 PR seats are distributed using the highest remainder method (Table XIII). Notice that P1 exceeds 17 seats the limit of those it can obtain. Thus, P1 only receives 48 PR seats.

TABLE XI
NUMBER OF SEATS PER PARTY

| Party | RM seats |
|---|---|
| P1 | 155 |
| P2 | 56 |
| P3 | 29 |
| P4 | 28 |
| P5 | 14 |
| P6 | 10 |
| P7 | 1 |
| P8 | 0 |
| Independent | 7 |

TABLE XII
NUMBER OF VOTES PER PARTY PER CONSTITUENCY

| | Constituency | | | | | |
|---|---|---|---|---|---|---|
| Party | 1st | 2nd | 3rd | 4th | 5th | Total |
| P1 | 2440 | 2700 | 2400 | 1600 | 2670 | 11810 |
| P2 | 3600 | 3600 | 360 | 320 | 600 | 8480 |
| P3 | 800 | 800 | 720 | 1280 | 720 | 4320 |
| P4 | 400 | 200 | 480 | 1920 | 360 | 3360 |
| P5 | 400 | 200 | 600 | 960 | 720 | 2880 |
| P6 | 400 | 400 | 600 | 640 | 360 | 2400 |
| P7 | 200 | 400 | 480 | 160 | 360 | 1600 |
| P8 | 200 | 200 | 360 | 160 | 360 | 1280 |
| Total | 8440 | 8500 | 6000 | 7040 | 6150 | 36130 |

TABLE XIII
MAXIMUM NUMBER OF SEATS AND REMAINDERS PER PARTY PER CONSTITUENCY

| Party | Votes | %Votes | PR seats | %Max | Seats. Max. | Exceeding |
|---|---|---|---|---|---|---|
| P1 | 11810 | 32.69 | 65 | 40.69 | 203 | 17 |
| P2 | 8480 | 23.47 | 47 | 31.47 | 157 | 0 |
| P3 | 4320 | 11.96 | 24 | 19.96 | 99 | 0 |
| P4 | 3360 | 9.30 | 19 | 17.30 | 86 | 0 |
| P5 | ***2880*** | 7.97 | 16 | 15.97 | 79 | 0 |
| P6 | 2400 | 6.64 | 13 | 14.64 | 73 | 0 |
| P7 | 1600 | 4.43 | 9 | 12.43 | 62 | 0 |
| P8 | 1280 | 3.54 | 7 | 11.54 | 57 | 0 |
| Total | | 100 | 200 | | | |

The distribution of seats per constituency of P1 according to Article 17 of the LGIPE is presented in Table XIV. In this case, the corresponding DR is

$$\text{DR of P1} = \frac{11{,}810}{48} = 246.04$$

Then, the remaining 152 seats are distributed among the other parties. The new NQ is 24320/152 = 160. The result is shown in Table XV.

TABLE XIV
TOTAL NUMBER OF SEATS OBTAINED BY P1 PER CONSTITUENCY

| Const | Votes | Seats number | Remaining voting | Adjustment seats | Total seats |
|---|---|---|---|---|---|
| 1st | 2,440 | 9 | 226 | 1 | 10 |
| 2nd | 2,700 | 10 | 240 | 1 | 11 |
| 3rd | 2,400 | 9 | 186 | 1 | 10 |
| 4th | 1,600 | 6 | 124 | | 6 |
| 5th | 2,670 | 10 | 210 | 1 | 11 |
| Total | 11,810 | 44 | | | 48 |

TABLE XV
TOTAL NUMBER OF SEATS OBTAINED PER REMAINING PARTIES

| Party | Votes | Seats | Total PR seats |
|---|---|---|---|
| P2 | 8480 | 53 | 53 |
| P3 | 4320 | 27 | 27 |
| P4 | 3360 | 21 | 21 |
| P5 | 2880 | 18 | 18 |
| P6 | 2400 | 15 | 15 |
| P7 | 1600 | 10 | 10 |
| P8 | 1280 | 8 | 8 |
| Total | 24320 | 152 | 152 |

DRs for multi-member constituencies are displayed in Table XVI. Using these DRs, the allocation of PR seats per constituency is displayed in Table XVII.

TABLE XVI
DISTRIBUTION RATIO FOR CONSTITUENCY

| Constituency | Votes | Available seats | DR |
|---|---|---|---|
| 1ª | 6,000 | 30 | 200 |
| 2ª | 5,800 | 29 | 200 |
| 3ª | 3,600 | 30 | 120 |
| 4ª | 5,440 | 34 | 160 |
| 5ª | 3,480 | 29 | 120 |
| Total | 24,320 | 152 | |

TABLE XVII
FINAL ASSIGNMENT OF SEATS PER PARTY PER CONSTITUENCY

| Party/Const. | 1st | 2nd | 3rd | 4th | 5th | Seats |
|---|---|---|---|---|---|---|
| P1 | 10 | 11 | 10 | 6 | 11 | 48 |
| P2 | 18 | 18 | 3 | 2 | 5 | 46 |
| P3 | 4 | 4 | 6 | 8 | 6 | 28 |
| P4 | 2 | 1 | 4 | 12 | 3 | 22 |
| P5 | 2 | 1 | 5 | 6 | 6 | 20 |
| P6 | 2 | 2 | 5 | 4 | 3 | 16 |
| P7 | 1 | 2 | 4 | 1 | 3 | 11 |
| P8 | 1 | 1 | 3 | 1 | 3 | 9 |
| Total | 40 | 40 | 40 | 40 | 40 | 200 |

It can be observed that, for parties P2 to P8, the numbers of obtained seats in Table XVII differ from those appearing in Table XV. This is an example where the Mexican electoral system is inconsistent.

Now it will be shown how the so-called No-show paradox (The No-show paradox occurs when part of the electorate may be better off by not voting than voting (see [19] and [20]) may arise in the Mexican electoral system. Consider data of the 2015 elections in Mexico, and suppose that 900 abstaining electors decided to vote for NA party. Also assume that this do not affect the RM results (Table XVIII). In such situation, the maximum number of seats for PRI party decreases one unit (from 203 to 202) as shown in Table XIX.

TABLE XVIII
FINAL ASSIGNMENT OF RM SEATS PER PARTY

| Party | Votes | RM seats |
|---|---|---|
| PRI | 11,636,957 | 155 |
| PAN | 8,377,535 | 56 |
| PRD | 4,335,321 | 28 |
| MORENA | 3,345,712 | 14 |
| PVEM | 2,757,170 | 29 |
| MC | 2,431,063 | 10 |
| NA | 1,486,626+900 | 1 |
| PES | 1,325,032 | 0 |
| PT | 1,134,101 | 6 |
| IND | 225,029 | 1 |

Now, PRI party receives 65-18=47 seats and the remaining 153 seats must be distributed among the other parties. The result appears in Table XX.

TABLE XIX
MAXIMUM AND EXCEEDING SEATS PER PARTY

| Party | Votes | Votes % | PR seats | % Max | Seats. Max. | Seats excess |
|---|---|---|---|---|---|---|
| PRI | 11,636,957 | 32.59 | 65 | 30.59 | 202 | 18 |
| PAN | 8,377,535 | 23.47 | 47 | 31.47 | 157 | 0 |
| PRD | 4,335,321 | 12.15 | 24 | 20.15 | 100 | 0 |
| MORENA | 3,345,712 | 9.37 | 19 | 17.37 | 86 | 0 |
| PVEM | 2,757,170 | 7.72 | 16 | 15.72 | 78 | 0 |
| MC | 2,431,063 | 6.81 | 14 | 14.81 | 74 | 0 |
| NA | 1,486,626+900 | 4.17 | 8 | 12.17 | 60 | 0 |
| PES | 1,325,032 | 3.72 | 7 | 11.72 | 58 | 0 |

TABLE XX
PR SEATS PER PARTY

| Party | Votes | Quota | PR Seats |
|---|---|---|---|
| PAN | 8,377,535 | 53.27 | 53 |
| PRD | 4,335,321 | 27.58 | 28 |
| MORENA | 3,345,712 | 21.28 | 21 |
| PVEM | 2,757,170 | 17.53 | 18 |
| MC | 2,431,063 | 15.46 | 16 |
| NA | 1,486,626 | 9.45 | 9 |
| PES | 1,325,932 | 8.43 | 8 |
| Total | 24,059,359 | 153 | 153 |

Comparing with the 2015 results (see Table X), it can be observed that NA, with 900 more votes, would obtain one seat less. In other words, these voters participation would have been against their interests. Moreover, notice that two well-known apportionment paradoxes of PR (see [12]) are involved in this fact. On one hand, when allocating all 200 PR seats, the Population paradox arises: NA loses one seat in spite of obtaining 900 more votes. On the other hand, when allotting the 153 remaining seats (once the PRI excluded), a stronger version of the Alabama paradox also appears: With one more seat in the PR apportionment (153 instead of 152), and 900 more votes, NA would lose one seat.

## V. EFFECTS OF THE THRESHOLD CHANGE ON THE TREATMENT TO POLITICAL PARTIES

The 2014 reform established a more restrictive requirement than previous electoral laws for parties to participate in PR allocation, increasing from 2% to 3% the exclusion threshold.

In this way, when applied the new electoral law in 2015, such change excluded two parties (concretely, the PT and the Humanist parties) that would participate with the previous codification. Even more, it should be noted that PT party obtained a little more than 2.99% of the VCV, so that with 0.001% more votes (3,337), this party would have received at least 6 seats. Thus, the threshold outlined in the LGIPE means that 1,134,101 citizens did not obtain representation in the Chamber. On the other hand, with 190,931 votes more than those obtained by PT, the PES obtained 8 PR seats, being this amount of votes just a little more than that required to obtain a seat (see last column in Table X).

The next hypothetical situation compares the real voting data from 2015 elections with those obtained if PT party had got 3,337 additional votes (needed for getting 3% of the VCV).

TABLE XXI
SEATS PER PARTY CONSIDERING SUPPOSED DATA AND SEATS ASSIGNED IN 2015 ELECTION

| Party | Votes 2015 | PR seats 2015 | New votes | New PR Seats |
|---|---|---|---|---|
| PRI | 11,636,957 | 48 | 11,636,957 | 42 |
| PAN | 8,377,535 | 53 | 8,377,535 | 53 |
| PRD | 4,335,321 | 27 | 4,335,321 | 27 |
| MORENA | 3,345,712 | 21 | 3,345,712 | 21 |
| PVEM | 2,757,170 | 18 | 2,757,170 | 17 |
| MC | 2,431,063 | 15 | 2,431,063 | 15 |
| NA | 1,486,626 | 10 | 1,486,626 | 10 |
| PES | 1,325,032 | 8 | 1,325,032 | 8 |
| PT | 1,134,101 | 0 | 1,137,438 | 7 |

It can be observed how the PRI, jointly with its partner in 2015 elections, PVEM, are the only benefitted parties from increasing the threshold barrier from 2% to 3%. This example shows again an asymmetrical treatment to political parties: notice how without those 3,337 additional votes of PT (needed to reach the threshold), the PRI obtains six more seats. Moreover, big parties take advantage of the votes that are obtained by independent candidates, because their votes are valid but they do not take part on the PR allocation. Again, a differentiated treatment is given to big political parties *versus* independent candidates.

## VI. PROPOSAL OF APPORTIONMENT METHOD FOR MEXICAN ELECTORAL SYSTEM

As shown along this paper, the current Mexican electoral system presents inconsistencies, paradoxes and asymmetrical treatment to political parties. Due to its serious problems, a new apportionment method for the Mexican electoral system is proposed, intended be clearer, applicable, free of the inconsistencies and paradoxes, and fairer with all political parties.

The proposal is: First, it is proposed to define the VCV as all votes cast, minus the sum of null votes, and those of unregistered candidates and independent candidates. The reason for the exclusion of independent candidates' votes relies on they do not participate in the allocation of PR seats.

Second, the seats allocation to the parties is obtained according to its percentage of the ENV, with the current threshold and limits, but using the Sainte-Laguë apportionment method (Simulations with biproportional apportionment methods have confirmed Sainte-Lagüe as one of the best procedures in order to avoid seats biases [21]).

Finally, with the implementation of biproportional apportionment method, the allocation of party seats per constituency is obtained, again using the Sainte Laguë apportionment method.

Applying the first two steps to 2015 Mexican elections the results would be those appearing in Table XXII. Notice that, with this proposal, PT party has participation on PR seats, which does not happen with the current system.

Finally, the obtained seats by each party are distributed among the five constituencies, using the biproportional method (Table XXIII). Hence, with this proposal all 200 PR seats have been distributed in a comprehensive way avoiding

inconsistences and paradoxes that may arise with the current system.

TABLE XXII
PR ESTIMATED SEATS PER PARTY (SAINTE-LAGUË METHOD)

| Party | Votes | Percentage of VCV | RM seats | PR seats |
|---|---|---|---|---|
| PRI | 11,636,957 | 30.88 | 155 | 42 |
| PAN | 8,377,535 | 22.23 | 56 | 53 |
| PRD | 4,335,321 | 11.50 | 28 | 28 |
| MORENA | 3,345,712 | 8.88 | 14 | 21 |
| PVEM | 2,757,170 | 7.32 | 29 | 17 |
| MC | 2,431,063 | 6.45 | 10 | 15 |
| NA | 1,486,626 | 3.94 | 1 | 9 |
| PES | 1,325,032 | 3.52 | 0 | 8 |
| PT | 1,134,101 | 3.01 | 6 | 7 |
| Humanista | 856,716 | 2.27 | 0 | - |
| Nulls | 1,900,449 | - | - | - |
| IND | 225,029 | - | 1 | - |
| NR | 52,371 | - | - | - |
| Total | 39,872,757 | 100 | 300 | 200 |

TABLE XXIII
ALLOCATION OF TOTAL SEATS PER POLITICAL PARTY BY BIPROPORTIONAL METHOD

| | Constituency | | | | | |
|---|---|---|---|---|---|---|
| Party | 1st | 2nd | 3rd | 4th | 5th | Total |
| PRI | 9 | 9 | 9 | 6 | 9 | 42 |
| PAN | 13 | 15 | 8 | 8 | 9 | 53 |
| PRD | 2 | 3 | 6 | 9 | 8 | 28 |
| MORENA | 3 | 2 | 5 | 7 | 4 | 21 |
| PVEM | 2 | 3 | 7 | 3 | 2 | 17 |
| MC | 7 | 3 | 1 | 2 | 2 | 15 |
| NA | 2 | 2 | 2 | 1 | 2 | 9 |
| PES | 1 | 2 | 1 | 2 | 2 | 8 |
| PT | 1 | 1 | 1 | 2 | 2 | 7 |
| Total | 40 | 40 | 40 | 40 | 40 | 200 |

## VII. CONCLUSIONS

Along this paper the Mexican Electoral System has been analyzed. To this aim, the 2015 electoral results (where internal negotiations for some parties were needed in order to reach a final allocation of PR seats) and other hypothetical (but plausible) data have been used. As shown, the application of the current law may lead to undesirable effects: Inconsistencies, paradoxes and asymmetrical treatment to political parties. It is important to point out such failures of the LGIPE because this might help to improve the Mexican electoral system and achieve a greater democratic quality in this country. Although it is known after the Balinski and Young theorem [13] that is not possible a perfect apportionment method for allocating seats, it is necessary to have a clear, fair and free of paradoxes electoral system, as much as possible.

Contrary to the current system, with the biproportional method proposed in this paper a distribution of seats without inconsistencies in any electoral situation is possible. The allocations based on this method will be free of drawbacks as those presented in this paper; even more, the differences in treatment to political parties will become reduced and agreements for interchanging their seats among the constituencies will not be needed.

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