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## A new procedure based on time domain indicators for optimal TMD tuning on footbridges

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### Abstract

Structures subjected to excitations like human induced vibrations may produce large accelerations and serviceability limit state problems. Passive, semi-active and active vibration controls have been proposed as possible solutions to reduce the vibration level at civil structures such as bridges, multi-storey buildings or slender floor structures, among others. It is known that Tuned Mass Dampers (TMD) mitigates the vibration response of a structure by increasing its damping through the application of inertial forces generated in response to the movement of the structure. Recently, different TMD implementations have been proposed in order to improve their tuning. In the case of structures with spatially distributed and closely spaced natural frequencies the TMD design may not be obvious because Den Hartog's theory may not be applied due coupling effects between the modes of the structures and the used TMD's.

In this work, alternative design techniques are applied for the case under study consisting on a simplified model of a footbridge with a main span 40m long. The first two modes are at 2.104Hz and 2.505Hz, both in the range prone to be excited by walking or jogging (90 to 180 bpm). Also the third one (at 3.18Hz) could be excited by runners. Once the finite element model is calibrated, mitigation devices (one or two TMDs) are proposed. The problem of finding their optimal location and tuning is not a simple one. Note that any strategy based on FRFs (frequency response functions) is not only tedious (several FRFs should be considered along the footbridge) but also limited, as the pedestrian crossing is a transient effect and FRFs are for steady states. Some standard analysis are carried out for a range of crossings at different paces and several simulations are carried out considering multiple scenarios in which the TMDs are attached to different locations. After studying several proposals, the best solutions are compared also in the frequency domain in order to extract some interesting and nor obvious conclusions.

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## 1. Introduction

Although in the past, civil engineering sector made extensive use of approximate models to estimate the dynamic response of bridge type structures, nowadays is usual to model the structure using current CAE abilities. Simple discrete models have proved insufficient for the accurate modelling of slender footbridge structures as they cannot represent some effects as the closely spaced modes of vibration which frequently occur in practice. Additionally, modern footbridges become increasingly slender and prone to oscillate under pedestrian loading, so there is a much greater need for vibrations to be considered at the design stage. Having the FE methods the capability for the accurate modelling of the dynamic behaviour, and becoming CAE software more affordable, civil engineering practitioners do not hesitate in their use. However, with regard to lively structural design, there is a lack of expertise in FE modelling, particularly with regard to their vibration serviceability performance, being not rare that the model does not match with the real structure. The way forward for developing such expertise is by linking modal testing and FE analysis by the updating of the models of representative structures and extract general design guidelines. This type of approach is the usual in, for example, the aerospace engineering sectors [5,6], but it is only recently that the civil engineering community has begun to adopt this advanced technology [7,9].

The aim of the paper is to describe a procedure for the use of updated FE models with TMDs attached to estimate the response in terms of accelerations and evaluate the serviceability of the assembly.

### 1.1. Structure description

The footbridge under study is an urban link with several minor access spans and one main 40m long arched central lively span. Most of the structural members are constructed using tubular steel profiles. An aerial photograph of the footbridge and 3D isometric view of its FE model is depicted in Figure 1. More information about the structure can be found in [11]. Updated mode-shapes 1 and 2 are shown in Figure 2.a. The structural damping (Rayleigh type) was set to 0.32%.



Fig. 1: Footbridge under study: photograph and numerical FE model

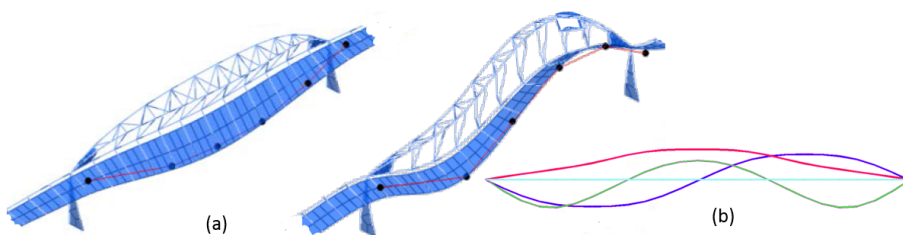


Fig. 2: Footbridge under study: photograph and numerical FE model

Although it would be possible to apply the methodology using the 3D F.E. model, in order to make more efficient simulations, a less time consuming F.E. model has been created. In this case, just a 2D equivalent structure meshed in 12 elements and modeled with Euler straight beam elements has been used. Both models exhibit similar vertical modes (see Figure 2.b), which are the interesting ones for the problem under study (pedestrian loading vertical response). Being  $L$  the length of the footbridge, note that the second mode-shape has a node at  $6L/12$  and the nodes of the third

mode-shapes are located at  $4L/12$  and  $8L/12$ . In the next section, the dynamic problem is established using the state space approach.

## 2. Space state modeling

The set of dynamic equations to be solved is  $\mathbb{M}\ddot{q} + \mathbb{C}\dot{q} + \mathbb{K}q = F$ , where  $\mathbb{M}$  is the mass matrix and  $\mathbb{K}$  is the stiffness matrix of the structure.  $\mathbb{C} = (V\tilde{\mathbb{C}}V^T)^{-1}$  is the damping matrix, evaluated through modal damping values  $\xi_i$ , where  $V$  is the modes matrix  $V = (\phi_1, \phi_2, \dots)$  and  $\tilde{\mathbb{C}}$  is the modal damping matrix, a diagonal matrix defined as  $\tilde{\mathbb{C}} = \text{diag}(2\xi_i\omega_i)$ .

Only bending degrees of freedom are considered (vertical deflection and planar rotation). The vector of degrees of freedom is shown in Equation 1.

$$q = [v_1 \ \theta_1 \ v_2 \ \theta_2 \ \cdots \ v_n \ \theta_n]^T \quad (1)$$

Standard procedures for meshing and assembling must be applied to all the matrices. Also boundary conditions of simple supported beam must be included in the former formulation.  $F$  in Equation ?? is the input forces vector, affecting to the whole model. It can be expressed in terms of an harmonic function  $u(t)$  and a vector of amplitudes  $F_0$ , as  $F = F_0 u(t)$ .

Any TMD means an additional degree of freedom to be added in the matrix formulation. For that, when  $m$  TMDs are considered to be included in the model the vector of degrees of freedom  $q$  changes as shown in Equation 2 to take into account the displacements of those TMDs ( $w_i$ ).

$$q' = [q^T \ w_1 \ w_2 \ \cdots \ w_n]^T \quad (2)$$

And for any TMD the corresponding matrices to be assembled are shown in Equation 3, where  $m_t$ ,  $c_t$ ,  $k_t$  are the physical properties of a single TMD.

$$M_t = \begin{bmatrix} 0 & 0 \\ 0 & m_t \end{bmatrix} \quad C_t = \begin{bmatrix} c_t & -c_t \\ -c_t & c_t \end{bmatrix} \quad K_t = \begin{bmatrix} k_t & -k_t \\ -k_t & k_t \end{bmatrix} \quad (3)$$

Considering as input the former hamonic function  $u(t)$  and as output just the acceleration in a set of selected degrees of freedom, the dynamic equations of motion can be rewritten in terms of a state vector  $x$  (Equation 4) and the typical state-space matrices  $A$ ,  $B$ ,  $C$  and  $D$ . A definition of matrices  $A$  and  $B$  is also provided in Equation 4, where  $\emptyset_{n \times n}$  is an square zero matrix and  $I_{x \times n}$  is the identity one. Matrices  $C$  and  $D$  are a set of rows of  $A$  and  $B$  corresponding to those degrees of freedom whose acceleration is wanted to be the output of the model.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad A = \begin{bmatrix} \emptyset_{n \times n} & I_{x \times n} \\ -\mathbb{M}^{-1}\mathbb{K} & -\mathbb{M}^{-1}\mathbb{C} \end{bmatrix} \quad B = \begin{bmatrix} \emptyset_{n \times n} \\ \mathbb{M}^{-1}F_0 \end{bmatrix} \quad (4)$$

Once the system is established, the transfer function  $G(s)$  between the input  $u(t)$  and an output acceleration can be obtained after some manipulations and it is shown in Equation 5, where  $\mathcal{L}$  is the Laplace operator.

$$G(s) = \frac{\ddot{Q}(s)}{U(s)} = \frac{\mathcal{L}\{\ddot{q}(t)\}}{\mathcal{L}\{u(t)\}} = C(sI_{2n \times 2n} - A)^{-1}B + D \quad (5)$$

The frequency response function (FRF) to be used in the following sections is just the magnitude of the Bode diagram of the former transfer function. The whole calculation is carried out in the Matlab environment using the standard functions: *ss()* for creating the state-space model, *tf()* for obtaining the transfer function of the model and *bode()* for calculating its Bode diagram. Also the function *lsim()* would be used to obtain the time response of the model.

## 3. Tuning for minimizing FRFs values

For all the studied cases, a tonne inertial moving mass is added (being the 2% of the total mass of the simplified model of the footbridge) in one or two TMDs (500kg each). The TMD may be tuned to mode 1 ( $w_1$ ) or mode 2 ( $w_2$ )

and located in some point ( $p$ ) along the beam. To simplified the continuous problem, only 11 discrete locations are considered (as the structure is discretized in 12 beam elements) so point  $p$  is at  $jL/12$  from the left end of the structure. The notation will be  $TMDi(m; \omega; \xi; jL/12)$  which means that a TMD number  $i$  with a moving mass of  $m$  kg is tuned at  $\omega$  Hz with a damping of  $\xi$  and located in  $jL/12$ .

In this section, the numerical values for the estimation of the response are obtained from the FRFs. In all the cases, the excitation point (force) and the response (acceleration) will be the same. The standard approach to mitigate the response around mode 1 is to install a TMD in the middle of the span. The properties of this *optimum* TMD have been calculated in order to minimise the peak response of the FRF in the band of the 1<sup>st</sup> mode, resulting in TMD1(1000; 2.0446; 0.131; 6L/12). Figure 3a shows the FRF at point 6L/12 for the original footbridge (in blue) and for the bridge with that TMD attached (in red).

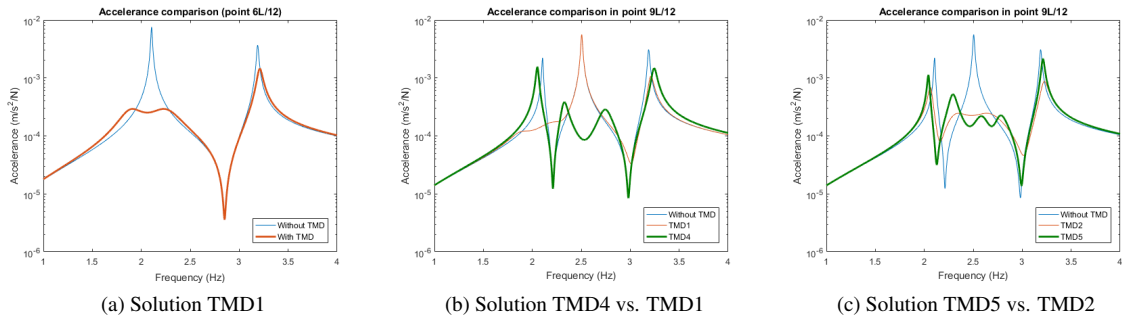


Fig. 3: FRF for different TMDs

Even if this is the most common approach, the solution is only optimum when the excitation of the structure is harmonic at frequencies around  $\omega_1$  and lasts enough time for the structure to reach the stationary response. Otherwise, the solution presented is only a *good enough* approximation and further analysis (see Section 4) would be necessary to be carried out.

#### 4. Performance indicators based on the time response under a moving force

Assuming the usual force to be as Equation 6 for a pedestrian of 70 kg walking at 1.2 m/s and a  $\omega_p$  pace, the acceleration time response can be computed with the Matlab function *lsim()* in any node  $jL/12$ . For any of those responses, the Root Mean Square Trend (RMS trend) from ISO 2631 can be computed with Equation 7.

$$F_p(t) = G + \sum_{i=1}^n G\alpha_i \sin(i\omega_p t - \phi_i) \quad (6)$$

$$RMS_{jL/12}(t_0) = \sqrt{\frac{1}{\tau} \int_{t_0-\tau}^{t_0} a(t)^2 dt} \quad (7)$$

The maximum value of the RMS trend can be also evaluated, obtaining the Maximum Transient Vibration Value ( $MTVV_{jL/12} = \max(RMS_{jL/12}(t_0))$ ). Figure 4a shows the acceleration of node 7L/12 during the crossing at pace of  $\omega_p = \omega_1$  (resonant at mode 1), that turns out to be the node with the highest MTVV value at that pace. Also Figure 4a shows the same values for the case TMD1 is installed.

Figure 4b shows all the RMS trend curves for the 11 selected nodes. The average of the 11 MTVV values (0.04448) might look a good serviceability indicator, as according to ISO 10137, MTVV values are supposed to be correlated with the human motion perception. We denote this new time indicator by  $mtvv(\omega_p)$ , defined in Equation 8, where  $\omega_p$  is the pace of the pedestrian.

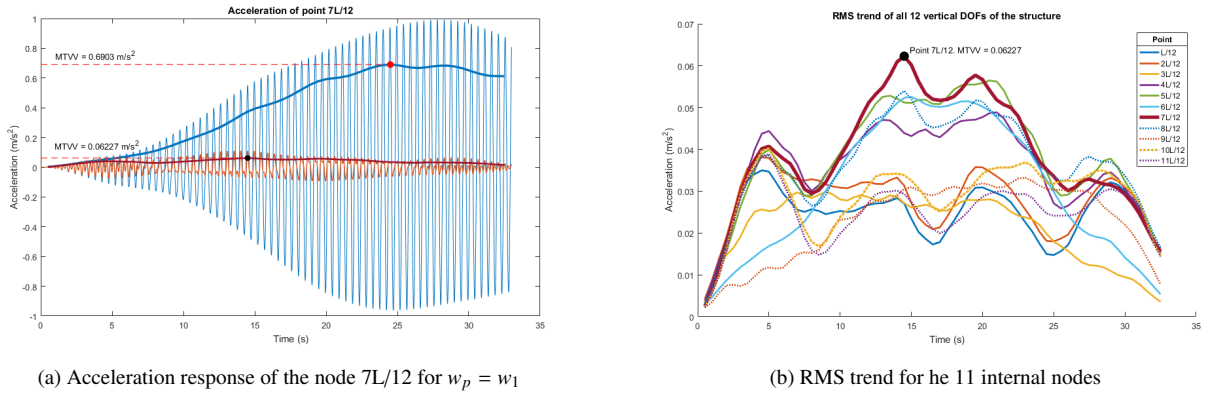


Fig. 4: Time responses

$$\overline{mtvv}(\omega_p) = \frac{1}{11} \sum_{j=1}^{11} MTVV_{jL/12} \quad (8)$$

Table 1 shows the  $\overline{mtvv}$  values for crossing at  $\omega_1$  pace and  $\omega_2$  pace along the original footbridge (0.43073 and 0.33018) and also when the TMD1(1000; 2.0446; 0.131; 6L/12) is installed (0.04448 and 0.3305). Note that in this last case  $\overline{mtvv}$  value is almost the same, as the TMD is tuned to mode 1.  $\omega_1$  and  $\omega_2$  paces seem to be the more risky ones as resonance can appear but with the TMD1 installed other paces around  $\omega_1$  (lets say, according to Figure 3a, from 1.9 to 2.3 Hz) are equally risky.

For crossings at any other pace, the values of  $\overline{mtvv}$  are different. In order to propose a time indicator valid for a significant pace range (from  $\omega_a$  to  $\omega_b$ ), the following one can be defined.

$$\overline{\overline{mtvv}} = \frac{1}{n} \sum_{j=0}^n \overline{mtvv} \left( \omega_a + \frac{\omega_b - \omega_a}{n} j \right) \quad (9)$$

In the range of walking, jogging or running paces,  $\omega_a$  and  $\omega_b$  are the range where the accelerances (FRF) are high. The definition of  $n$  is an attempt to discretise the continuous problem. In the case under study, 9 crossing paces between 1.9 and 2.7 Hz (separated 0.1 Hz) are selected.  $\overline{\overline{mtvv}}$  values are also shown in Table 1. together with the relative changes regarding values for the footbridge without TMD.

Table 1:  $\overline{\overline{mtvv}}$  values for each node

	$\omega_1$	Performance	$\omega_2$	Performance	(1.9, 2.7)	Performance
(no TMD)	0.43073	-	0.33018	-	0.07506	-
TMD1	0.04448	89.67%	0.33055	-0.11%	0.05064	32.54%
TMD2	0.14174	67.09%	0.04501	86.37%	0.04059	45.93%
TMD3	0.05542	87.13%	0.06528	80.23%	0.04315	42.52%

Table 1 also shows two new scenarios. The former scenario (row TMD1) attended for installing a 1000 kg TMD to mitigate the first mode. Now, in the scenario TMD2 two 500 kg TMDs are going to be installed at locations 3L/12 and 9L/12 to mitigate the second mode. The optimum solution, attending FRF response (Figure 5a) is TMD2([500, 500]; [2.4618, 2.4618]; [0.116, 0.116]; [3L/12, 9L/12]). In the scenario TMD3 a TMD of 500 kg is installed at 6L/12 to mitigate the first mode and another one at 3L/12 to mitigate the second one, resulting (see Figure 5b) in TMD3([500, 500]; [2.073, 2.496]; [0.1, 0.08]; [6L/12, 3L/12]). Note in Table 1 the performance in all cases.

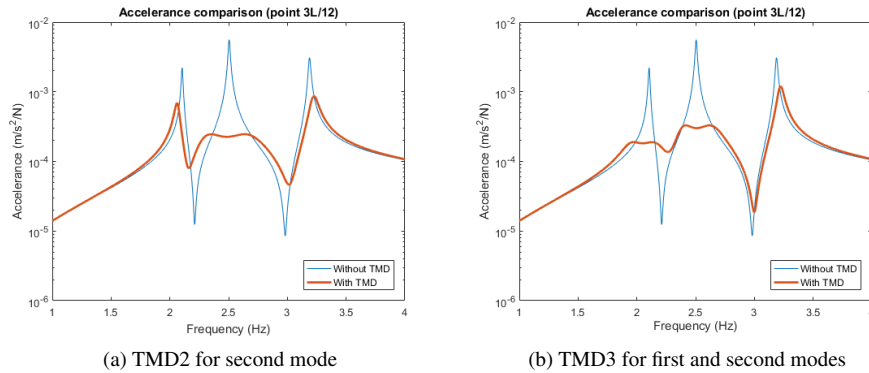


Fig. 5: Accelerances of cases TMD2 and TMD3 at 3L/12

Once the  $\overline{mtvv}$  indicator is defined, it would be an engineering target to choose the location and tuning of the TMDs that minimise it, meaning that the response of the footbridge under a range of transient excitations is optimum. If only one TMD is going to be installed and the several crossing from 1.9 to 2.7 Hz (separated 0.1 Hz) are to be considered, the solution that minimise the indicator is TMD4(1000; 2.537; 0.044; 9L/12). Figure 3b shows the corresponding FRF function at point 9L/12 (compared with the original case and with TMD1). In the case with 500 kg TMDs, the optimum solution (see Figure 3c) is TMD5([500, 500]; [2.4684, 2.621]; [0.0414, 0.0314]; [9L/12, 4L/12]). Table 2 summarises the response not only for the optimised range (1.9 to 2.7 Hz) but also for crossings at  $\omega_1$  and  $\omega_2$ .

Table 2:  $\overline{mtvv}$  values for each node

	$\omega_1$	Performance	$\omega_2$	Performance	(1.9, 2.7)	Performance
(no TMD)	0.43073	-	0.33018	-	0.07506	-
TMD4	0.11772	72.67%	0.04263	87.09%	0.04096	45.43%
TMD5	0.10286	76.12%	0.03889	88.22%	0.03361	55.23%

Optimum design parameters in the cases TMD4 and TMD5 were found using the genetic algorithm of Matlab (function *ga()*). Any other solution obtained trying to minimise accelerances results in TMD designs not as good as TMD4 (only one TMD) or TMD5 (two TMDs). Table 3 shows  $\overline{mtvv}$  and  $\overline{mtvv}$  values for the optimum designs TMD6, TMD7 and TMD8 where the optimization criterium is not to lower the accelerance peaks around  $\omega_1$  and/or  $\omega_2$  but the area under the accelerance curve in the  $\pm 20\%$  range around them. When just one TMD tuned at first mode is sought, the best solution is TMD6(1000; 2.0997; 0.0971; 4L/12). When second mode is reduced with two TMDs, TMD7([500, 500]; [2.5514, 2.584]; [0.1012, 0.1057]; [4L/12, 8L/12]) is the best alternative. And if both modes are taken into consideration, TMD8([500, 500]; [2.5331, 2.0913]; [0.0794, 0.0976]; [3L/12, 7L/12]) is the best solution. Note that both indicators values for all the three *best* solutions are worse than those for TMD4 and TMD5.

Table 3:  $\overline{mtvv}$  values for each node

	$\omega_1$	Performance	$\omega_2$	Performance	(1.9, 2.7)	Performance
(no TMD)	0.43073	-	0.33018	-	0.07506	-
TMD6	0.06387	85.17%	0.08540	74.14%	0.04539	39.53%
TMD7	0.08029	81.36%	0.04664	85.88%	0.03885	48.24%
TMD8	0.05851	86.42%	0.06507	80.29%	0.04012	46.55%

## 5. Conclusions

The engineering problem to mitigate the vibrations of slender structures under transient loadings is not a simple one. The problem of finding the best number of TMDs, their location and tuning can be addressed via frequency domain or via time domain. Traditional tuning strategies are based on minimising acceleration peaks around problematic vibration modes. With this strategy, proposal TMD1 only gets an overall 32.54% of performance versus 45.43% of TMD4 (tuned using the new proposed time strategy), both solutions using only one TMD. When two TMDs can be installed, performance improves from 44.93% (TMD2, based on frequency domain) to 55.23% (TMD5, designed under the new approach).

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