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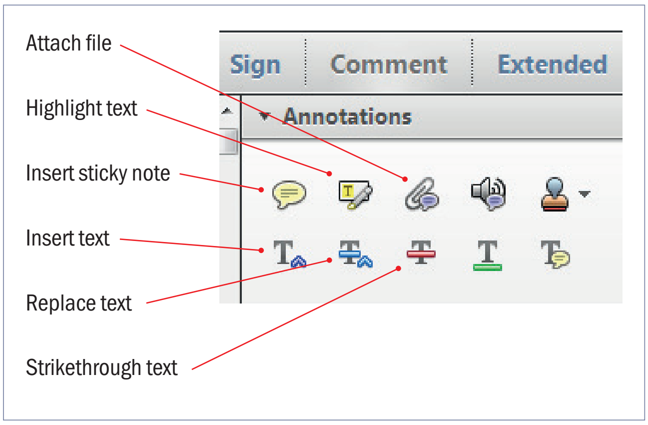

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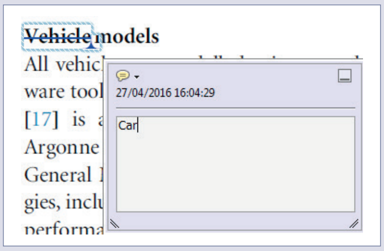

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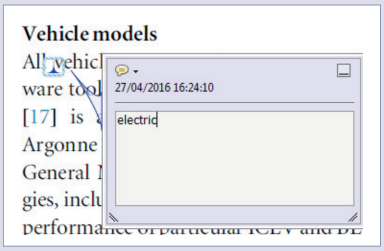

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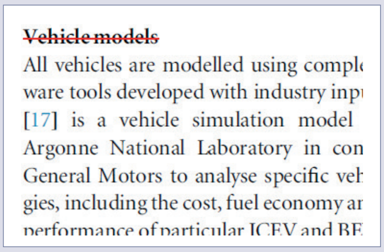

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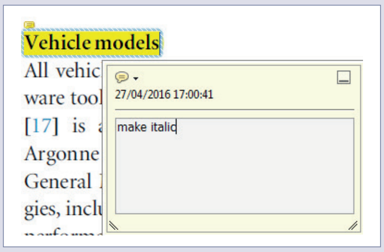

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Measuring the coefficient of restitution and more: a simple experiment to promote students' critical thinking and autonomous work

Manuel Á González[1], **Miguel Á González**[2], **Jesús Vegas**[3] **and César Llamas**[3]

[1] Departamento de Física Aplicada, E.I. Informática, Universidad de Valladolid, 47011 Valladolid, Spain
[2] Departamento de Física de la Materia Condensada, E.I. Industriales, Universidad de Valladolid, 47011 Valladolid, Spain
[3] Departamento de Informática, E.I. Informática, Universidad de Valladolid, 47011 Valladolid, Spain AQ1

E-mail: manuelgd@termo.uva.es

Abstract
A simple experiment on the determination of the coefficient of restitution of different materials is taken as the basis of an extendable work that can be done by the students in an autonomous way. On the whole, the work described in this paper would involve concepts of kinematics, materials science, air drag and buoyancy, and would help students to think of physics as a whole subject instead of a set of, more or less, isolated parts. The experiment can be done either in teaching laboratories or as an autonomous work by students at home. Students' smartphones and cheap balls of different materials are the only experimental materials required to do the experiment. The proposed work also permits the students to analyse the limitations of a physical model used in the experiment by analysing the approximations considered in it, and then enhancing their critical thinking.

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1. Introduction

Students' smartphones can play a very useful role increasing the motivation, interest and learning outcomes of students of physics according to recent works [1, 2]. The authors of these works call this effect 'material-aided situated learning'.

Simple experiments in which smartphones are used to measure directly kinematic, acoustic, optical or magnetic magnitudes are, in general, easily implemented, providing students with an easy way of testing the theoretical concepts learned in the classroom. Also, the available sensors in smartphones can be used in experiments where different branches of physics are combined together, showing students the unity of physics beyond the partitioned idea that, sometimes,

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they receive due to the organization of textbooks or subjects. For example, the authors of a recent work [3] used only a smartphone to analyse the vibrations of metallic rods, showing how sound measurements using a smartphone allow students to easily study properties of the materials as their Young's modulus and density.

The work proposed in this paper starts with a simple method to study the dependence of the coefficient of restitution, e, of different materials by listening to the noise made by bouncing balls using only a ball, a metre stick and a smartphone, in what, thus, can be an easy to do 'at home' experiment. Measuring e for different materials using balls is an usual teaching activity that has been discussed in different works [4–12]. The method described here has less experimental difficulty than required in other methods [4, 6, 8, 13, 14], which reduces possible sources of experimental error and also allows students to measure smaller values of e. In addition, the method described here requires less numerical analysis than other previous methods [8], what makes it suitable for students with lower mathematical skills. The main difference between the above mentioned works and the work described in this paper is that here we propose and evaluate simple activities to go beyond the determination of the coefficient of restitution and help students analyse the physical model and the approximations considered in the experiment. In this way we foster students' critical thinking and teach them to behave like scientist while, at the same time, learning different physics concepts or experimental techniques. These additional activities include analysing the effect of temperature and speed on the coefficient of restitution, and then on the behavior of materials, as well as the effect of air drag in the experimental results.

For lower level or less motivated students, their interest in the experiment can be reinforced by proposing they compare their experimental results for different sport balls with the official ball specifications [13]. On the other hand, for physics or engineering students, this experiment and the techniques used in it can be specially interesting as similar techniques are used in research in materials science [15–19], to study materials behaviour and hysteresis [20], or even granular materials [21].

2. Theory

The coefficient of restitution (COR) of a collision between two bodies of equal mass is defined as the ratio between the relative normal components of the velocities after and before a collision:

$$e = \frac{|\,(\vec{v}_{2a} - \vec{v}_{1a}) \cdot \vec{u}_{12}\,|}{|\,(\vec{v}_{2b} - \vec{v}_{1b}) \cdot \vec{u}_{12}\,|} \tag{1}$$

where $\vec{u}_{12}$ represents the unit vector pointing in the particles' inter-centers direction. This coefficient is usually employed as a measure of the elasticity of a collision, as for elastic collisions the value of the coefficient of restitution is 1, while it decreases towards 0 as the collision becomes more inelastic. However, this is strictly true whenever there is no transfer of energy between kinetic and rotational energies. For these reasons the COR is usually measured in normal collisions between spheres or between a sphere and a large and much heavier flat surface which is at rest before the collision and that also remains at rest after it.

If we consider the collision of a vertically falling spheric ball on a static hard floor, then (1) reduces to

$$e = \frac{v_a}{v_b}, \tag{2}$$

taking into account only the speed of the ball before and after the collision.

One can find experiments similar to the one described here where the measurements of collision times and later calculations to obtain the COR are performed considering the approximations:

(A) the COR does not depend on the speed of the falling object,
(B) the movement of the bouncing ball is vertical and the ball does not spin,
(C) the time of contact with the floor during the impact is negligible,
(D) the friction with the air is negligible.

In this work we propose students assume these approximations initially valid and then analyse the validity of the model. In this way, we want them to gain an insight into the influence of different paramaters influencing such a simple experiment, and learn to behave like physicists.

Under the above mentioned approximations, the speeds of a bouncing ball before and after bounce ith are calculated using

$$v_{bi} = \frac{1}{2} g \left(t_i - t_{i-1}\right), \qquad v_{ai} = \frac{1}{2} g \left(t_{i+1} - t_i\right) \tag{3}$$

where t_i is the time of the ith bounce. Then, the COR (2) results:

$$e = \frac{t_{i+1} - t_i}{t_i - t_{i-1}} = \frac{\Delta t_{i+1}}{\Delta t_i}. \tag{4}$$

See figure 1 for a graphical definition of the variables used in these expressions. In the experiment described in this paper, assuming valid approximations (A) to (D), the students will drop several balls from different initial heights and obtain the value of e for each ball by fitting the recorded values of the differences $\left(t_{i+1} - t_i\right)$ versus $\left(t_i - t_{i-1}\right)$.

On the other hand, if we use the initial height, h_0, from which the ball is dropped, the COR can also be obtained using:

$$e = \frac{\frac{1}{2} g \Delta t_1}{\sqrt{2 g h_0}}. \tag{5}$$

Alternatively, equations (4) and (5) can be used to obtain the value of g, as it is done in [11].

As it is well known, the COR actually depends on the velocity [22], and decreases with increasing speed as a result of the larger deformation of the ball. However in this simple experiment we are going to assume initially that it is independent of the velocity, as it is also considered in similar experiments. Students can discuss later this approximation by analysing the results obtained by dropping the ball from different initial heights and using equations (4) and (5) to obtain e assuming known the value of g.

3. Experiment

In the work described in this paper, the sound recording was performed using a smartphone running the app Audia [23] developed by our group. Many other apps are available in the app store and students can use the one they are more comfortable with. We have used a smartphone to do this experiment in order to check the possibility of proposing the experiment as an autonomous work that the students can do by themselves at home. For the experiment students would only need a ball, a measuring tape and their smartphone, as can be seen in figure 2, without requiring any additional experimental equipment.

As stated above, we propose here an experiment that can be performed with different intensity and adapted to the knowledge and capabilities of students of different levels. Then, the experiment will be described in this paper divided into different subsections corresponding to different possible activities.

The COR for a collision really depends on the nature of the two bodies that collide, the ball and the horizontal surface of the floor in this experiment. Then, in order to avoid the influence of the floor on the measured COR, students are advised to choose a hard floor. The interested students can also extend the work described here and study the influence of different floors hardnesses on the experimental results for the same balls as an additional experiment.

3.1. Level one: determining the COR

The simplest version of the experiment, assuming valid approximations (A) to (D) in section 2, consists of dropping a ball and listening to the noise produced in different consecutive bounces of the ball.

Figure 3 shows a screenshot of the smartphone that has recorded four different bounces of the dropped ball. Time measurements can be done directly on the smartphone screen or can be exported to a computer for a more precise measurement.

For the work described in this paper we have used three balls of different materials: a table tennis ball made of celluloid plastic, a foam ball (stress ball) made of closed-cell polyurethane foam rubber, and a golf ball that has a more complex multilayered structure. The characteristics of these balls are given in table 1. These are examples of cheap balls of different materials that the students can get without difficulty, so that this experiment can be easily done either in the classroom or at the student's home, in order to improve their personal autonomous work, curiosity and interest in physics. The use of the students' own smartphones also aims to facilitate the students' autonomous work while also increasing their learning by using their own devices [1].

This simple experiment, with small variations, has been previously described in different works [11, 24]. A little improvement, assuming valid approximation (A) above, would consist

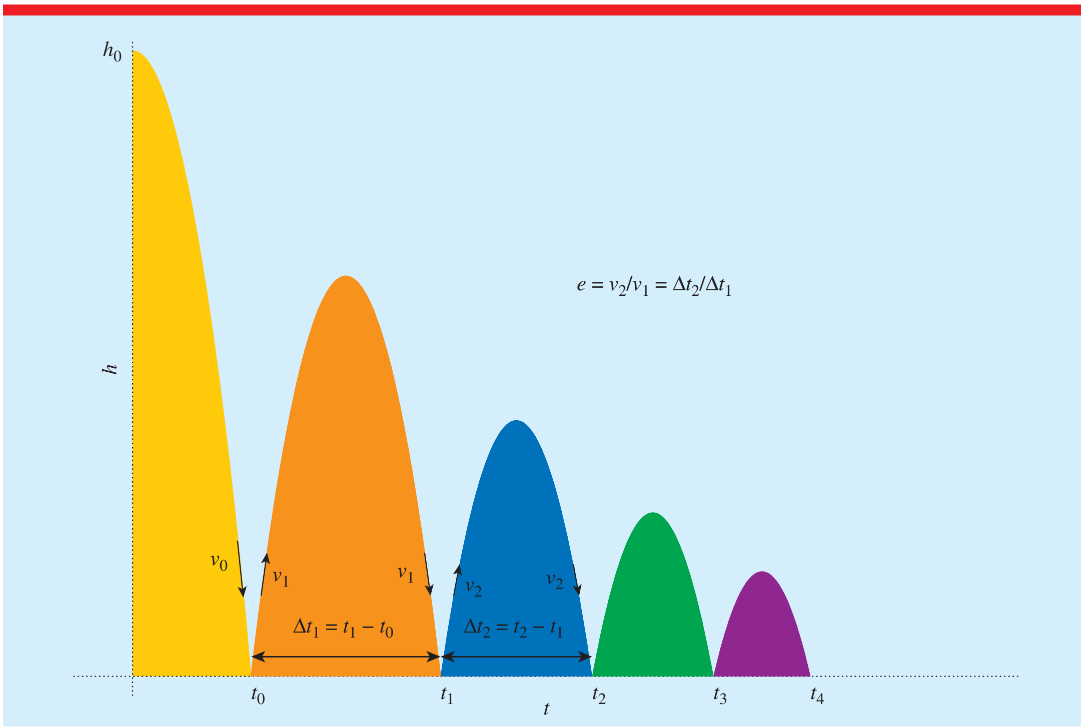

Figure 1. Definition of the COR as the ratio of consecutive bouncing times.

of proposing students drop the ball from different initial heights and then obtain the COR as the slope calculated fitting Δt_2 versus Δt_1 for different initial dropping heights. In this way they can reduce the random experimental noise of a single measurement and obtain an estimation of the accuracy of the COR calculated by using the slope error. This is the procedure we have followed here. Figure 4 shows results obtained with the balls at $T = 298$ K. As can be seen there, the points corresponding to the different times of flight recorded dropping each ball from different initial heights follow well the linear trend. Fittings in figure 4 were calculated following equation (4), and thus forcing the intercept to be 0. As can be seen in figure 4, for the worst case (table tennis ball) the inaccuracy in the value of the calculated COR is a little higher than the 1%. One can also see that the fitting for the table tennis ball is the worst of the three examples shown in the figure. In fact, if we would allow a free calculation of both the intercept and the slope in the table tennis ball fitting we would obtain for the value of the slope, a COR, of 0.68 (nearly a 24% lower than the slope calculated forcing the intercept to be null), also in disagreement with the values obtained from measurements of the table tennis ball at temperatures close to $T = 298$ K. On the other hand, for the golf and the foam balls the differences between the slopes obtained from both types of fittings are much smaller, changing from 0.88 to 0.90 for the golf ball and from 0.67 to 0.61 for the foam ball. As will be shown below, the worst fitting for the table tennis data may be due to the larger dependence of its COR with speed, and then with the initial dropping height.

3.2. Level two: dependence of the COR with temperature

The work described in section 3.1 can be extended without much difficulty proposing students study the influence of temperature on the behaviour of different materials. For this a refrigerator and an oven will be enough. Of course, special care must be taken to avoid burns. Students must be advised to use protective gloves to manipulate the balls when low or high temperatures are considered.

Figure 2. Experimental equipment for the simplest version of the experiment. Students only need a metre, a ball and their smartphone.

Also some care must be taken to avoid balls melting in the oven depending on their material and the oven temperature.

It is important to explain carefully the importance of thermal equilibrium so that the students keep the experimental balls within the refrigerator, the freezer or the oven for enough time to have them thermalized. This is another added value of the experiment, as the understanding of this thermodynamic concept, and the necessary experimental care, is important to obtain good results. Not having well thermalized balls before the experiment can increase, notably, the dispersion of the experimental results. In our case, to avoid heating or cooling of the balls during the experiment, we kept several balls of each type within the refrigerator/freezer or the oven. Then, we did the measurement corresponding to one drop from a given height with one of them, and then placed it immediately again in the refrigerator/freezer or oven and took another different ball for the next height measurement. Using the freezer, the refrigerator and the oven allows us to span our measurements safely to a range of temperatures between, approximately, 260 K and 360 K. In this work we also have extended the above mentioned range of temperatures using liquid nitrogen (77 K) and dry ice (195 K), and an oven that allowed us to reach temperatures up to 383 K. The temperatures considered in the

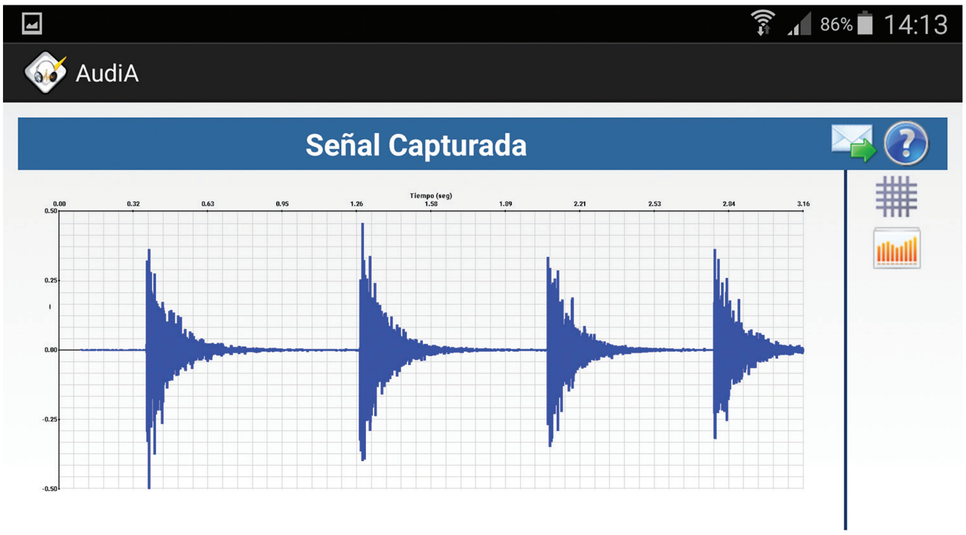

Figure 3. Example of the noise recording using the smartphone with the app AudiA. Students can use their fingers to move along the time scale in the screen and do the time measurements easily.

experiment and how we got them are shown in table 2. The two lowest temperatures cannot be obtained easily outside the laboratory but measurements at the other temperatures can be easily done by the students at home. It must also be taken into account that we couldn't do measurements with the golf ball because below $T = 195$ K, due to its multilayered structure and the different thermal behavior of its compound materials, its surface layer tears apart at low temperatures.

Once the calculations of the COR of each ball are obtained, students can compare the different behaviors of the materials that compose the used balls. Figure 5 shows the comparison of the results obtained in our experiment for all the temperatures. COR data in that figure show error bars calculated from the fitting errors. Lines connecting experimental points in figure 5 are smooth Bézier curves [25], but the students can also be proposed to perform polynomial fittings to characterise, in a simple way, the dependence of the COR of each material with temperature. Then, this experiment, performed with materials that the students can have at home allow them to study easily the behavior of different materials by using their smartphones as measurement devices. As can be seen in figure 5, foam ball behaviour shows a trend change around 200 K. We also had more experimental noise in our measurements of the foam ball COR when using frozen carbon dioxide (195 K). In order to understand the change in the mechanical properties of the foam ball at high and low temperatures, a differential scanning calorimetry (DSC) measurement was carried out. The results showed that the foam ball undergoes a glass transition temperature (Tg) at around 213 K and a melting transition at around $T = 328$ K, where we can see another change in trend. The temperatures of both transitions are well correlated with the observed changes in its mechanical properties. As a result, it can be concluded that the foam ball is a polymer, most likely PDM rubber (ethylene propylene diene monomer (M-class) rubber), that consists of a mixture of crystalline and amorphous phases. Of course a DSC is far from the reach of many laboratories, but this result shows us how students can learn about the importance of these changes in materials behaviour by analyzing their COR results, and that a simple experiment as the one described in this work can also be proposed to students

Table 1. Characteristics of the balls used in the experiment described in this work.

Ball	Diameter (m) $\Phi = 2R$	Mass (kg) m
Golf	$4.24 \cdot 10^{-2}$	$45.49 \cdot 10^{-3}$
Table tennis	$3.95 \cdot 10^{-2}$	$2.70 \cdot 10^{-3}$
Foam	$4.485 \cdot 10^{-2}$	$6.99 \cdot 10^{-3}$

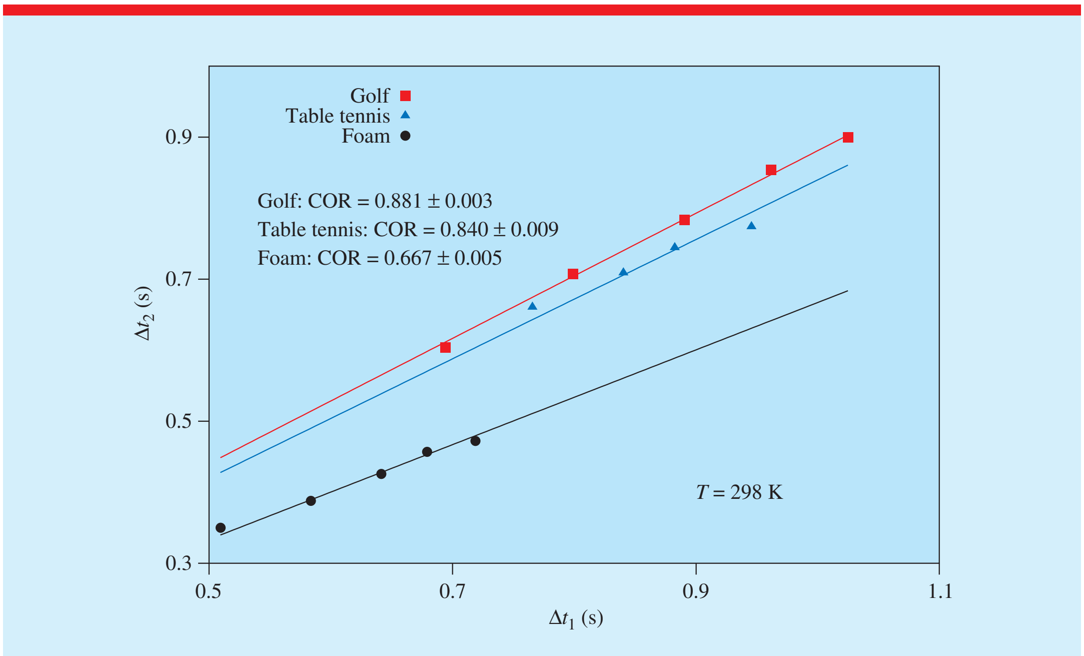

Figure 4. Experimental determination of the COR fitting the time interval of the second bounce versus that of the first bounce when a ball is dropped from different, but similar, initial heights.

of materials physics. On the other hand, different polymers can be chosen with glass transition temperatures in a range of temperatures easily available for students' experiments [26].

3.3. Level three: analysis of the model approximations

3.3.1. Dependence with the speed. In order to understand the limitations of the experiment, students can also analyse the applicability of approximations (A) to (D). This additional work would allow students to better understand the character of the physics explained in the classroom, as well as of the experiments they do in the laboratory. In fact, many students have a poor knowledge of physics as a collection of mathematical formulas without understanding the physical model that supports them [27, 28], and analysis by themselves on the validity of the different approximations in the experiment can help them to think like physicists. Thus, the analysis of the approximations considered in the experiment aims to encourage students to think of the experiment as a physical process instead of following a *cooking recipe* to measure an unknown quantity. With this we reinforce the students' scientific thinking [27].

Table 2. Set of temperatures considered in the experiment.

Temperature (K)	Method
77	Bath in liquid nitrogen
195	Bath in frozen carbon dioxide
253	Freezer
278	Refrigerator
298	Ambient temperature
313, 323, 338, 358, 368, 383	Oven

Students can easily analyse how good approximation (A), that considers that the COR does not depend on the velocity, is. As the experimental balls have been dropped from different known initial heights h_0, the COR can also be calculated from the ratio between the velocity after and before the first bounce using equation (5). As initial heights range between 0.75 m and 1.75 m, then the balls velocities before the first impact on the floor range, approximately, between 3.83 m s^{-1} and 5.86 m s^{-1} (assuming friction with air negligible), what permits one to carry forward a simple study of the dependence of the COR with the impact speed. From that analysis the students can see that the COR decreases with

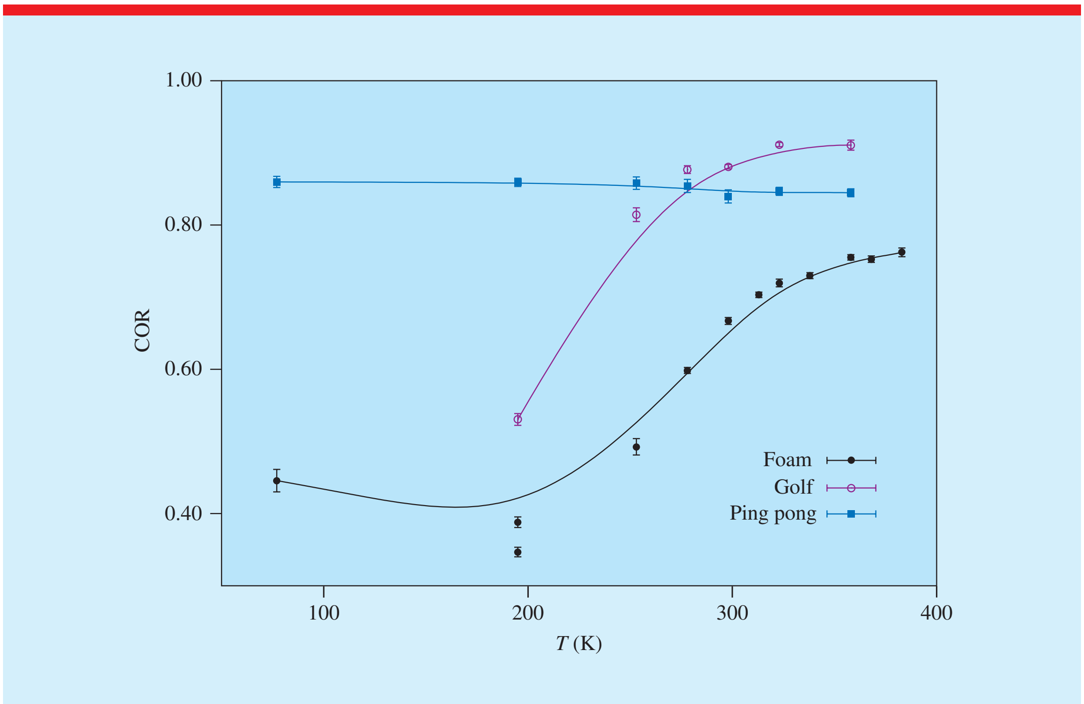

Figure 5. Dependence of the COR of three different balls, a foam ball, a golf ball and a table tennis ball, on temperature. Measurements were done using only a smartphone as experimental recording device.

increasing speed for the three studied balls and all the considered temperatures. Beyond the simple fact of the decrease of the COR with speed, this result allows the students to discuss the validity of approximation (A) for the different materials and temperatures. They can also analyse quantitatively here the dependence of the COR with the velocity. For that we fitted our experimental data to expressions of the type [15, 29]:

$$e = cv^{-a}. \qquad (6)$$

From the results in these fittings the students can discuss the dependence of the COR with the impact speed and the temperature for each material more accurately. A detailed description of some results of the analysis performed by us is given in the supplementary material (stacks.iop.org/PhysEd/00/000000/mmedia) accompanying this paper.

For interested students, a nice qualitative description, based on Hertz model [30, 31], of the causes of the decrease of the COR with increasing speed can be found in [22], which may be an easy read for many undergraduate students. These results can help students understand that the COR is not a constant property of the materials, as it is many times presented in textbooks, but it also depends on the dynamics of the collision and helps characterize the way in which energy dissipation occurs [21].

Similar to the simple study done above on the dependence of the COR with the impact speed, another work that can be proposed to students is to study the constancy of the COR between different bounces. For this, a large number of bounces can be recorded, and the change in the COR obtained from different bounces will be analysed. But in this work we have used a different, simpler method, that can also be followed by the students in their autonomous work. Here we compare only the COR obtained in the first and in the second bounces. If we depict the COR of the second bounce versus the COR of the first one for different temperatures we should obtain a straight line with slope close to 1 if the COR remains (nearly) constant in consecutive bounces. Differences between the experimental slope and the value of 1 give the students an indication of the accuracy of this approximation. Results of this analysis from our measurements, and a short discussion are shown in the supplementary material of this paper.

If students have analysed the dependence of the COR with the speed (or with the initial height) now they could also re-analyse the dependence of the COR with the temperature for different heights independently, instead of the average calculation shown in figure 5. So, they can obtain the joint dependence of the COR with temperature and velocity simultaneously, as shown in figure 6 for the golf and table tennis balls. In that figure the COR values calculated using equations (4) and (5) are represented with 3D surfaces that show the COR dependence on v and T as an example of the results that can be obtained by the students. These surfaces can be calculated easily, for example, by fitting the experimental values of the COR versus v using equation (6) for constant values of T, and then fitting the parameters c and a obtained in those fittings, using simple polynomials to approximate their dependence on the temperature.

3.3.2. Analysis of the verticality and spinning of the movement. The approximations (A) to (D) are also easy to check and can give the students an interesting insight into the physics of a bouncing ball and on the air drag and buoyancy effects beyond the initially expected learning results of the experiment.

For this work the students can record the bounces with their smartphone cameras and later use the free video analysis software Tracker [32] in a computer for the analysis. Other works have used Tracker in order to determine the heights reached by the ball for different bounces [10] (this task can be also proposed to the students) but we propose to use it in a different way in this work. On the other hand, students using iOS mobile devices can also use them to do the same video analysis using the app video-physics [33], without requiring any computer.

From the analysis of the trajectories of balls falling from different heights, students can see that the lateral displacement can be negligible in comparison with the total distance traveled be the bouncing ball, as can be seen in the discussion detailed in the supplementary material to this paper. In our recorded experiments the maximum measured horizontal displacement was 0.18 m after the (table tennis) ball had bounced 7 times.

In the same way, the analysis of the recorded videos is useful to determine the ball spinning. From our recorded videos, we observed that even in the case of maximum spinning, the rotational kinetic energy was nearly two orders of magnitude smaller than the kinetic translational energy. More details of these analysis are shown in the supplementary material to this paper.

From similar data, the students can analyse by themselves the quality of these approximations as well as the importance of the experimental care in order to obtain a more vertical movement of the balls without spinning. In our experiments we have seen that for both conditions to be fulfilled, the care in the initial release is an important factor in obtaining the desired conditions. Other factors that can influence the nearly nonspinning vertical movement are the smoothness of the impacting surface and the irregularities in the ball surface. These aspects were also discussed as sources of error in similar experiments, though performed with much more expensive experimental equipment [34, 35].

Also approximation (C) can be easily checked with this simple method. Using video recordings with 240 fps, we can establish an upper value for the impact duration that, in the worst case, is 100 time lower than a bounce flight time. More details of this analysis are described in the supplementary material of this paper.

3.3.3. Analysis of the air drag and buoyancy. The students can also use the recorded videos to study the influence of air drag and buoyancy. This additional work, that includes friction and fluids concepts, may require the students to have more theoretical and experimental skills and the results, obtained from the videos recorded with the students' smartphones, may not be conclusive due to the possible experimental inaccuracies, as the differences between times or speeds calculated with or without considering air drag and buoyancy may be under the precision of the recorded video. The first step in this last part of the extended experiment would consist on determining the Reynolds number (Re) of the balls used in the measurements.

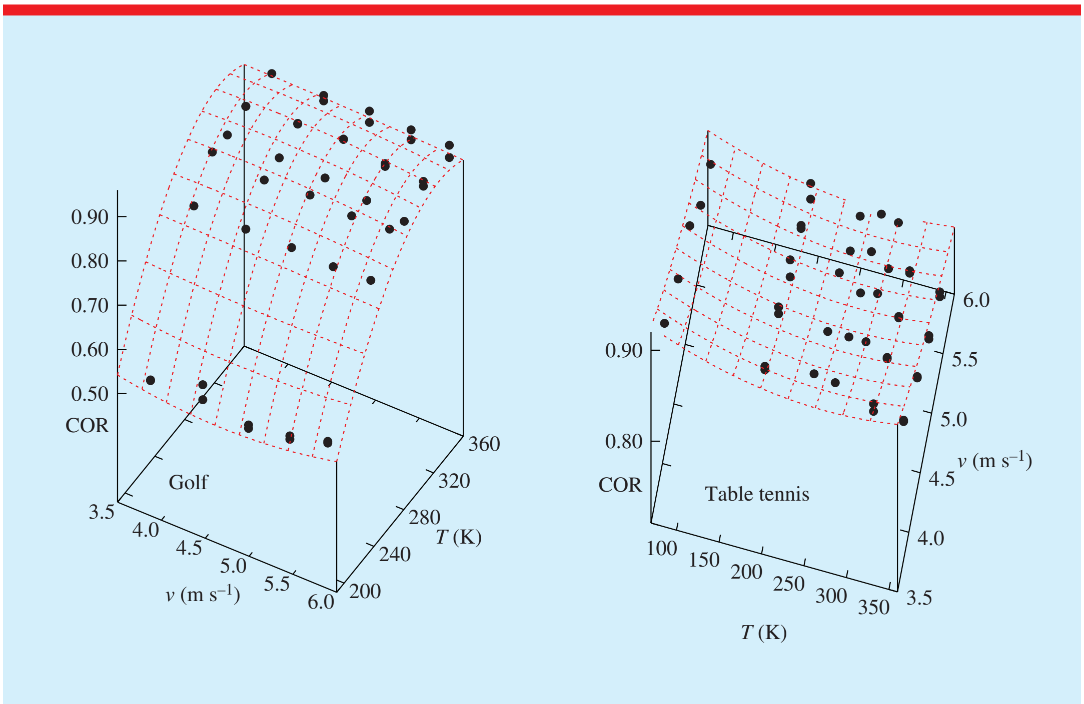

Figure 6. Dependence of the COR with the temperature for the golf ball (left) and the table tennis ball (right). Notice that both figures have different orientations in order to improve their visibility according to the different dependences of the COR on temperature and velocity for both balls.

$$\mathrm{Re} = \rho \frac{lv}{\eta}. \tag{7}$$

For the calculation of Re for the balls dropped in air, ρ is the density of air, l the characteristic length scale of the object, that for the balls corresponds to their diameter, η the dynamic viscosity of the air, and v is the balls' velocity. If we consider speeds of the order of 2 to 6 m s^{-1} (corresponding to final speeds of a falling ball from heights between approximately 0.20 and 1.75 m) and air properties for temperatures between 273.15 and 373.15 K, the values of *Re* for the three balls range between approximately $8 \cdot 10^3$ and $1.6 \cdot 10^4$, so that we are under the conditions of a resistive force proportional to the square of the speed $F(v) = 0.2\rho\pi R^2 v^2$, and the problem of the ball thrown upwards vertically can be solved analytically, neglecting the short times when the speed of the ball is low enough and then when the quadratic drag is no longer valid [36]. In fact, the influence of those short times on the air drag calculation was studied for a table tennis ball and it was determined to be negligible [37].

Then, using expressions (30), (33) and (43)–(45) in [36] the students can obtain the time of maximum height, the value of maximum height, the time of descent, the total time of flight and the speed of impact on the floor, respectively. As all the mentioned parameters depend on the initial speed of the ball, v_0, it could be possible to calculate them if students measure the values of v_0 for the different bounces with enough accuracy. However, we have seen two problems in this procedure that can lead to misestimations of v_0. The first problem is that it can be difficult to establish accurately the position of the ball in a video frame when the ball speed is higher, what induces an error in the determination of the ball speed. In order to reduce this problem we tried averaging the speeds calculated from three consecutive video frames (though evidently the speed isn't constant). The second problem can arise from a parallax error if the plane of the movement of the ball and the plane of the considered calibration rule used in the video aren't the same. Students can fall easily into this mistake if they use a wall to hold the calibration rule while dropping the ball

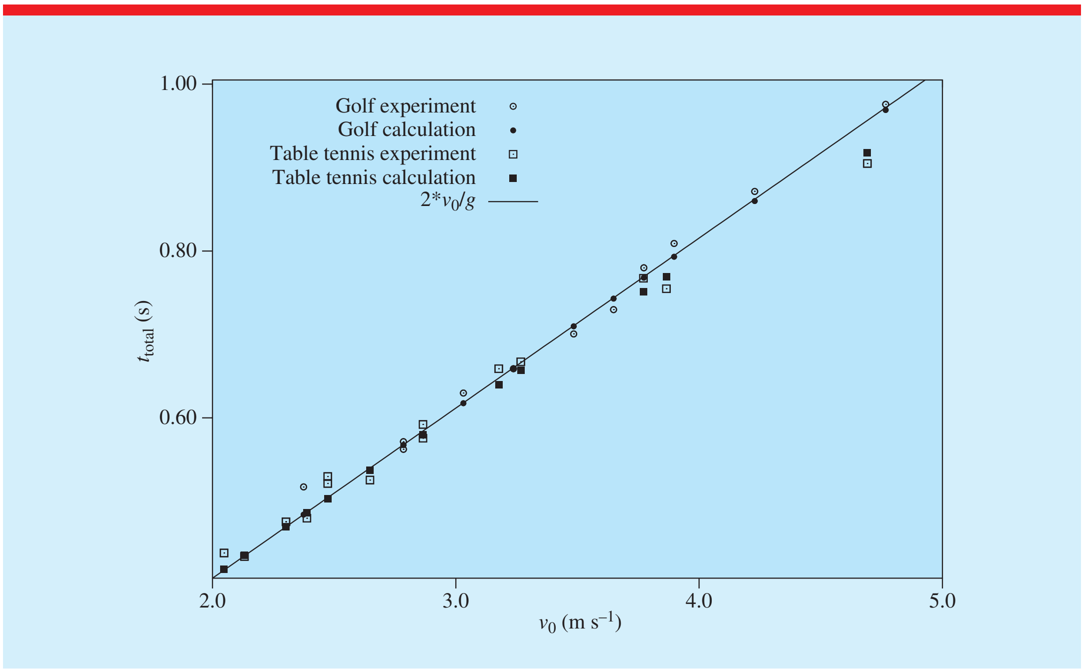

Figure 7. Comparison of the flight times measured versus the velocity after the bounce, v_0. Theoretical calculations obtained considering air drag and buoyancy are compared with the times measured in the videos and with the theoretical expression $t = 2 * v_0/g$ corresponding to the calculation without air drag nor buoyancy. Results for the golf and table tennis balls are shown in the figure.

a little apart from it to avoid bounces against the wall. In this case the speeds measured from the video will be overestimations of the actual speed.

An alternative method, in order to avoid the sources of error mentioned above, would consist on measuring the time employed by the ball in the flight from the floor after each bounce to the apex of the bounce trajectory. This time can be measured easily in the videos with an accuracy close to the frame length. This is the method we have followed here. Once this time is measured, other parameters of the trajectory can be obtained and compared with the values corresponding to an ideal movement without drag or buoyancy. Figure 7 shows some results obtained in this way for the golf and the table tennis balls. That figure compares the total flight times measured in the videos, the theoretical time of flight calculated considering air drag and buoyancy and the ideal time of flight $2v_0/g$. From that figure the students can see that for the golf ball, the heaviest of the three considered in the experiment, the effects of drag and buoyancy are negligible for all the speeds (and dropping heights) considered, which supports the approximation considered in the experiment. However for the table tennis ball, the lightest one, we can see how discrepancies between the values obtained without considering drag and the values of the experiment and calculated taking into account drag increase with increasing speed and are very noticeable for the highest values of v_0. Of course, these values correspond to the cases of largest flight trajectories, when the drag effects are more noticeable. This comparison can help students discuss the validity of approximation (D) and establish safe ranges or conditions to do the experiment.

An exact determination of the influence of drag for these balls, beyond the graphical comparison of figure 7 may be out of reach using students' smartphones. For example, for the golf ball the difference between the theoretical total flight times obtained with and without drag is around 10^{-3} s for a speed close to 4 m s^{-1}. That difference can only be accurately measured using cameras with frame speeds higher than the 240 fps used in our work. On the other hand, for the table tennis ball, that same difference is around

$2 \cdot 10^{-2}$ s that is more accessible to our measurement device, the smartphone.

4. Conclusions

A simple experiment that can be done in the teaching lab or at home independently by the students has been described. We have shown how this experiment can be developed with different intensity and can reach concepts on properties of materials, elasticity, kinematics, friction or buoyancy. It is also important to point out that the work described here allows students to work by themselves using materials that they already have, or that they can obtain easily, in order to improve their autonomous work and their interest in physics. The described technique not only focuses on different theoretical concepts, but also enhances the importance of analysing the theoretical framework of the used formulas and approximations. In this way the students can learn to work more like real physicists, realizing how many different concepts can appear in every simple experiment, and that while some approximations may be valid under certain conditions, outside them the approximations may not longer be valid. This work can also be extended to include, for example, the study of the influence of different impact surfaces, the dependence of the COR with the mass of the falling ball in order to study the influence of deformations or dents that can appear when elastic limited is overpassed or even analysing the dependence of the sound intensity with the speed of the ball. The combination in the same simple experiment described in this paper of different physical concepts and fields, together with various measurement methods requiring different analysis is another advantage of the proposed work. The use of students' smartphones allows them to perform the experiment nearly anywhere and anytime, opening up the possibility of learning beyond the walls of the traditional school and improving their autonomous work and interest in physics. It is also important to note that the use of a tool, the smartphone, that students use in their daily life helps connecting their learning with the students' life outside the classroom with positive effects in both their learning and motivation.

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