Determination of the optimal set of b-values for ADC mapping under a Rician noise assumption

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Synopsis(100/100 words): Mapping of the apparent diffusion coefficient (ADC), estimated from a set of diffusion-weighted (DW) images acquired with different b-values, often suffers from low SNR, which can introduce large variance in ADC maps. Unfortunately, there is no consensus on the optimal b-values to maximize the noise performance of ADC map. In this work, we determine the optimal b-values to maximize the noise performance of ADC mapping by using a Cramér-Rao Lower Bound (CRLB) approach under realistic noise assumptions. The strong agreement between the CRLB-based analysis, Monte-Carlo simulations, and ADC phantom experiment, suggests the utility of this approach to optimize DW-MRI acquisitions.

Purpose (731/750 words): Mapping of the apparent diffusion coefficient (ADC) from a set of diffusion-weighted (DW) images acquired with different b-values, often suffers from low SNR, which can introduce large variance in ADC maps. Unfortunately, there is currently no consensus on the optimal set of b-values to maximize the noise performance (ie: minimize the variance) of ADC mapping. The purpose of this work is to optimize the set of b-values for ADC mapping, using Cramér-Rao Lower Bound[1] (CRLB) analysis under the assumption of realistic Rician distributed data, noise distribution commonly present in MRI[2].

Methods:

__Determination of optimal b-values:__ The ADC mapping signal model is described by $S(b)=S_0 e^{-bADC}$. In order to maximize the noise performance of ADC estimation we optimize the set of b-values via the CRLB approach. Such an approach was previously employed, in related contexts, in Refs[3,4] under Gaussian noise assumptions, and by Ref[5] for the Kurtosis model under a Rician assumption. Given a set of independent Rician distributed observations of the ADC signal model, S_k where $k \in [1,...,K]$, with the same noise level, the CRLB of the ADC ($CRLB_{ADC}$) is given by the element (2,2) of the inverse of the Fisher Information Matrix[1] (Eq. 2), where I_n is described in Ref[5]. Therefore, for a given target ADC, noise level, and number of b-values (K), the determination of the set of K b-values that maximizes the noise performance of the ADC estimation is performed by an iterative brute force algorithm. The iterative algorithm starts from a set of b-values composed only of b-value=0 s/mm² and iteratively adds to the set the b-value that achieves minimum $CRLB_{ADC}$ among a large set of b-value candidates. The proposed algorithm iterates until a set of K b-values is completed. Table 1 shows the pool of candidate b-values and other parameters employed.

Eq. 2:
$$FIM=\begin{bmatrix} \sum^{K} e^{-2b_k ADC} I_n(S_k,\sigma) & -\sum^{K} S_0 b_k e^{-2b_k ADC} I_n(S_k,\sigma) \\ -\sum^{K} S_0 b_k e^{-2b_k ADC} I_n(S_k,\sigma) & \sum^{K} S_0^2 b_k^2 e^{-2b_k ADC} I_n(S_k,\sigma) \end{bmatrix}$$

__Validation of optimal b-values__: The sets of b-values obtained theoretically with the proposed CRLB-based approach are compared to those obtained experimentally from:
1) Monte-Carlo simulations, which included 13 different true ADCs (4000 simulated pixels each) with a Rician distribution and parameters from Table 1.
2) An ADC phantom[6] experiment consisting of 13 vials with different ADCs at room temperature. The DW-MRI acquisition was performed at 1.5T (GE Healthcare, Waukesha, WI) using a standard single shot EPI sequence with the following parameters: slice thickness of 5 mm, FOV=24cm x 24cm, matrix size of 144x144, TE=111 ms, and 41 b-values uniformly distributed between 0-2000 s/mm². Further, this acquisition was repeated 16 consecutive times (discarding the first three repetitions to avoid steady-state effects) to enable voxel-wise determination of ADC estimation statistics.

The optimal sets of b-values for the Monte-Carlo simulations and the ADC phantom experiment were obtained iteratively. At each iteration, the b-value that minimizes the variance of the ADC estimation among all candidate b-values is added to the selected set. The procedure is analogous to the one employed in the CRLB-based optimization, but using the experimental variance instead of the $CRLB_{ADC}$ (note that 13 repetitions are available for both the Monte-Carlo simulations and the ADC phantom experiment). Each ADC estimation was performed pixel-wise via a Maximum Likelihood[7] estimator (ML).

Results: Table 2 shows the optimal sets of b-values obtained from the CRLB-based theoretical analysis, Monte-Carlo simulations, and ADC phantom experiment for one of the vials ($ADC=2.1\cdot10^{-3}mm^2/s$). Further, Figure 1 shows, for the same vial, the evolution of the $CRLB_{ADC}$ and experimental variances with respect to the number of b-values employed. Figure 2 shows a color-coded comparison of the optimal sets of b-values for all the vials in Table 1.

Discussion: Results indicate a strong agreement between optimal the sets of b-values obtained using the proposed CRLB-based theoretical analysis and those found experimentally both by the Monte-Carlo simulations and by the ADC phantom experiment. This illustrates the potential of the proposed CRLB-based approach to maximize the noise performance (minimize the variance) of ADC mapping by optimizing the choice of acquired b-values. In order to achieve a procedure that is suitable for the selection of the set of b-values in clinical settings, further validation is still required, considering realistic distributions of target ADCs, the effect of T_2 relaxation, and *in-vivo* results.

Conclusion: The optimization of the set of b-values is critical to maximize the noise performance (ie: minimize variance) of the ADC estimation in DW-MRI. The proposed approach may help optimize and standardize DW-MRI acquisitions by computing the optimal set of b-values for a target ADC.

References:

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Acknowledgements: The authors acknowledge grant TEC2013-44194-P from the Ministerio de Economía y Competitividad of Spain.

Table 1: Parameters of interest obtained from the ADC phantom experiment. These values were also applied for CRLB-based optimization as well as in the Monte-Carlo simulations.
*The CRLB-based optimization is under the assumption of NEX=13.

	VIAL 1	VIAL 2	VIAL 3	VIAL 4	VIAL 5	VIAL 6	VIAL 7	VIAL 8	VIAL 9	VIAL 10	VIAL 11	VIAL 12	VIAL 13
ADC ($\cdot 10^{-3}$ mm^2/s)	0.3	0.32	0.54	0.54	0.[illegible]7	0.[illegible]7	1.2	1.3	1.6	1.7	2.1	2.1	2.1
SNR of b_0 image	42	53	35	49	29	23	40	25	51	53	58	47	40
NEX*	13												
b-values (s/mm^2)	0, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 950, 1000, 1050, 1100, 1150, 1200, 1250, 1300, 1350, 1400, 1450, 1500, 1550, 1600, 1650, 1700, 1750, 1800, 1850, 1900, 1950,2000												

Table 2. Results of the b-value optimization for vial 1[illegible] (ADC = $2.1\cdot10^{-3}$ mm^2/s). b-values appear in the same order they were included in the optimal sets.

Number of b-values (K)	Optimal sets of b-values (s/mm^2) (CRLB-based analysis, Monte-Carlo simulations, ADC phantom experiment)
K=2	[illegible] 0 [illegible] 0 [illegible]
K=3	0 550 550 0 500 500 0 450 [illegible]
K=4	0 550 [illegible] 600 [illegible] 0 450 600 650
K=5	0 550 [illegible] [illegible] 600 0 500 [illegible] 550 [illegible] 0 450 [illegible] [illegible] 650
K=6	0 550 550 600 600 650 0 500 500 550 750 700 0 450 600 650 650 650
K=7	0 550 [illegible] [illegible] [illegible] 650 0 0 500 500 550 750 700 0 0 450 [illegible] [illegible] [illegible] 650 0
K=8	0 550 550 600 600 650 0 600 0 500 500 550 750 700 0 500 0 450 600 [illegible] [illegible] [illegible] 0 650
K=9	0 550 550 [illegible] [illegible] [illegible] [illegible] 600 600 0 500 500 550 750 700 [illegible] 500 500 0 450 600 650 650 650 [illegible] 650 650

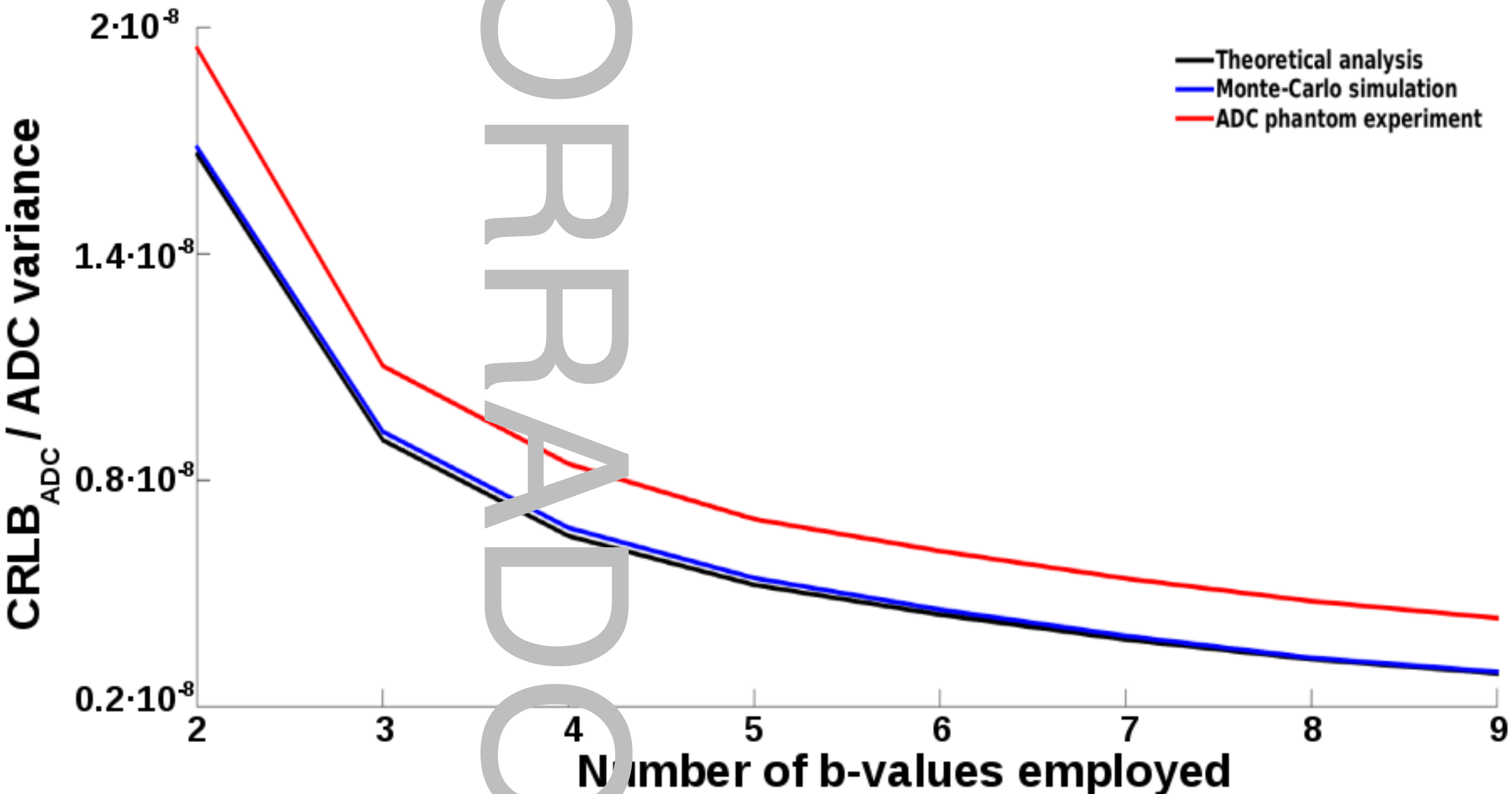

Figure 1: Evolution of the minimum $CRLB_{ADC}$ of the CRLB-based analysis and minimum variance of the Monte-Carlo simulations and the ADC phantom experiment for vial 12 as the number of employed b-values increases (ADC = $2.1 \cdot 10^{-3}$ mm^2/s, SNR=47).

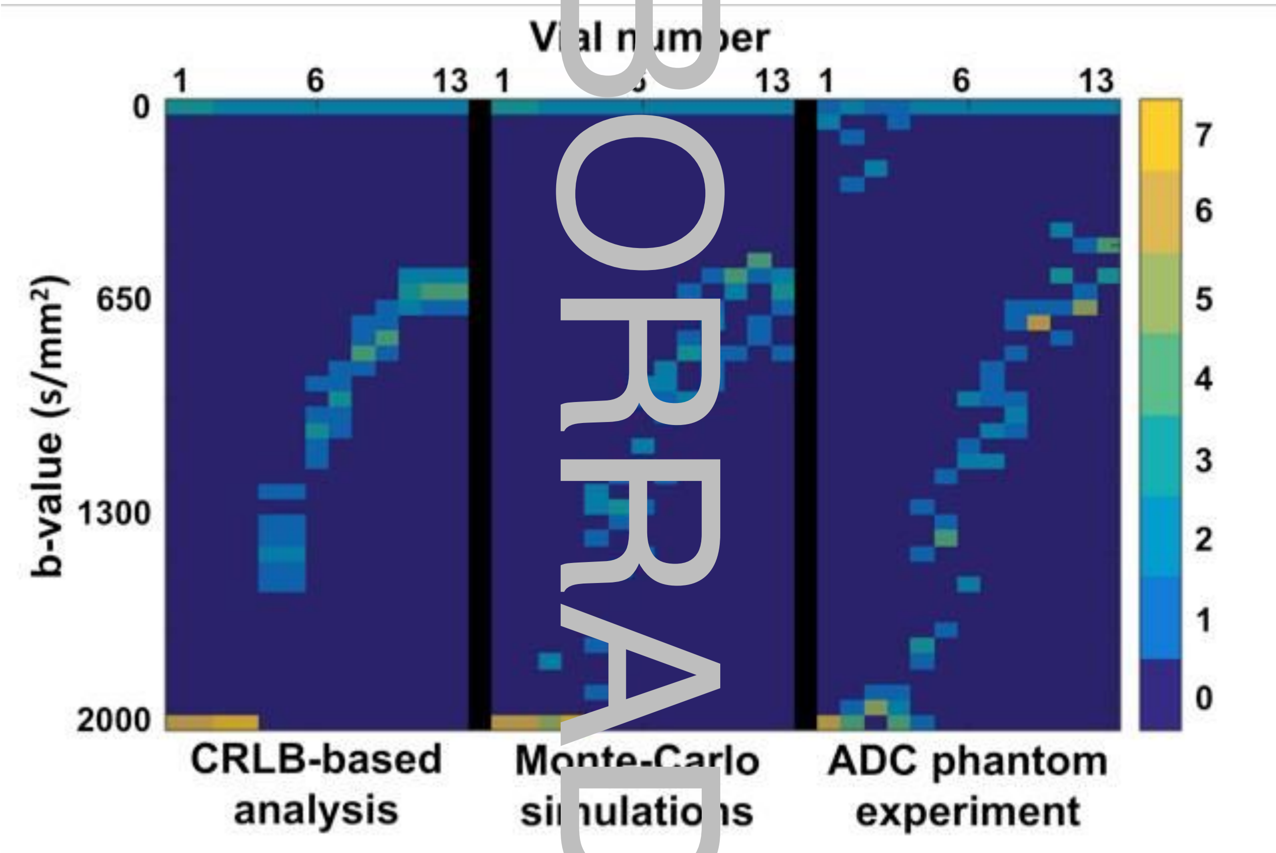

Figure 2: Optimized set of K=9 b-values for the 13 vials in the theoretical, Monte-Carlo simulations, and ADC phantom experiments. Each column shows the optimal set of b-values found for a vial, where the color code indicates how many times a certain b-value was included in the optimal set.