PREFACE

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Generally speaking, the term nonautonomous dynamics refers to the systematic use of dynamical tools to study the solutions of differential or difference equations with time-varying coefficients. The nature of the time variance may range from periodicity at one extreme, through Bohr almost periodicity, Birkhoff recurrence, Poisson recurrence etc. to stochasticity at the other extreme. The "dynamical tools" include almost everywhere Lyapunov exponents, exponential splittings, rotation numbers, and the theory of cocycles, but are by no means limited to these. Of course in practise one uses whatever "works" in the context of a given problem, so one usually finds dynamical methods used in conjunction with those of numerical analysis, spectral theory, the calculus of variations, and many other fields. The reader will find illustrations of this fact in all the papers of the present volume.

Each of the papers presented here contains a detailed introduction, which describes its contents and provides sufficient background material to orient the reader with respect to the related literature and, more generally, to the scientific context in which it was conceived. Nevertheless, we present here a brief description of each contribution and some comments concerning the relations between them.

It is often convenient to study a nonautonomous differential/difference system by introducing a so-called driving flow; one obtains a family of systems whose solutions define a cocycle over that flow. This situation can be achieved via a Bebutov-type construction if the coefficients of the system are bounded; that is to say, one closes the set of translates of the coefficient function(s) in some topology. The result of this construction is sometimes called the Bebutov hull of the system. It should also be said that a good number of problems are posed with a driving flow already present; e.g. this is often the case in what is known as stochastic dynamics. In any case, the cocycle gives rise to a skew-product flow, which lives on the product of the phase space of the driving flow with the phase space of the differential/difference system.

Several of the papers in this volume treat problems formulated in the driving flow/cocycle framework, as we now indicate.

Campos, Obaya and Tarallo study some consequences of their Fredholm alternative theory for linear nonhomogeneous ODEs with Birkhoff recurrent coefficients, and generalize a well-known result of Cieutat and Haraux.

Cong and Son consider bounded linear random ODEs, that is, the coefficients are bounded and are driven by an ergodic flow on a probability space. In this context they prove the openness and density of the integral separation property in the space of coefficients, in the topology of the L^∞ norm.

Damanik, Fillmore, Lukic and Jessen use dynamical methods to study the spectral theory of CMV matrices; these are intimately related to the (Szego) cocycles which arise in the study of orthogonal polynomials on the unit circle. As a preliminary to this they give a unified treatment of the theory of uniform hyperbolicity of two-dimensional cocycles.

Fabbri, Novo, Núñez and Obaya study time-dependent linear control processes when the condition of uniform null controllability does not hold. They discuss the reachable set as a function of points in the driving flow; they make use of certain constructions arising in the linear-quadratic control theory.

Figueras and Haro consider a one-parameter family of forced n-dimensional maps where the driving flow is an irrational rotation of the circle. They assume that the family admits a corresponding family of invariant curves, and study the fractalization process (breakdown of the invariant curve) which can occur in this situation.

Franca and Johnson work out a theory of the "perturbed Andronov-Hopf bifurcation pattern". They develop an appropriate integral manifold theory, then discuss the flow on the integral manifold, where for example the so-called Furstenberg flows may arise.

Jorba, Rabassa and Tatjer study a forced one-dimensional map, where the driving flow is an irrational rotation of the circle. They introduce a generalization of a renormalization operator which is a standard tool in the context of one-dimensional maps, and discuss its properties.

Mierczynski and Shen consider linear parabolic PDEs whose coefficients are driven by an ergodic flow on a probability space. They derive some estimates of the principal Lyapunov exponent of such a family of PDEs; this quantity is important in questions of stability.

Nerurkar considers forced linear oscillators, which are modelled by linear nonhomogeneous ODEs with time-varying coefficients. The forced quantum oscillator is an important example. He shows that, if the driving flow is ergodic, then so is the "generic" oscillator.

Sometimes one is interested in the Bebutov hull of a system in and of itself. This is the case in the paper of Johnson and Zampogni, who consider certain compact translation-invariant subsets of Lundina's generalized reflectionless Schroedinger potentials. They discuss subsets of this type which do not satisfy the "property P" and others with a discontinuous Lyapunov exponent and with a non-injective divisor map.

Some authors approach the study of the solutions of a nonautonomous differential resp. discrete system by taking the driving flow to be that of translation on the real line resp. the integers (i.e., one does not close the set of translates of the system but takes that set "as it is"). One thus arrives at the concept of a two-parameter process, in which one parameter is the time variable and the other is the translation variable. Two papers in this volume illustrate how this approach can be used; we discuss their contents.

Caraballo, Langa and Valero study the pullback attractor of a class of one-dimensional inclusions. They develop and use the language of multivalued processes. It turns out that for the class they study, the pullback attractor consists of all complete bounded trajectories.

Poetzsche and Russ state and prove a version of the Hartman-Grobman theorem in an infinite-dimensional context, namely that of abstract evolution equations. The method of proof is based on invariant manifolds and foliations. They then give an application to reaction-diffusion equations which generalizes results of previous authors.

Often the driving flow formalism is not needed in the study of a nonautonomous differential or difference system; for example, exponential dichotomy theory can certainly be applied to a single system. It also happens that a problem regarding an autonomous system can be studied by the introduction of some related nonautonomous equation; think for example of the linearization of an autonomous equation along a nonconstant solution. We now indicate several papers in this volume which illustrate these observations.

Battelli and Feckan study certain implicit ordinary differential equations which model RLC electrical circuits. Using the theory of exponential dichotomies, they study the bifurcation of bounded solutions connecting singularities in finite time, and their approximation by shadowed periodic solutions.

Budisic, Siegmund, Son and Mezic study the finite-time behavior of the velocity field of a fluid, the goal being to detect organizing geometrical structures in real time from data regarding that velocity field. They work out a mesochronic (time-averaged) method for determining hyperbolic resp. elliptic regions, and apply it to several examples.

Dambrosio and Papini prove the existence of multiple homoclinic orbits for a nonlinear Schroedinger equation with steplike potential. They construct stable and unstable continua using a Wazewski-type method, and join them using a technique of stretched rectangles.

Dieci and Elia consider a piecewise smooth differential system in the neighborhood of a discontinuity manifold of codimension 2. Though the codimension 1 case is fairly well understood (think sliding motion), the case they consider is not, and they evaluate the dynamics on and near the manifold via "Euler iterates with random uniformly distributed steps".

Fortunati and Wiggins work out a normal form theory in the neighorhood of a hyperbolic equilibrium for Hamiltonian systems which are analytic and aperiodic in time. They use time as a parameter and define an infinite sequence of coordinate transforms which takes the Hamiltonian function to a time-dependent normal form of Moser type.

As the reader will see, the papers in this volume present an up-to-date picture of an ample range of activity in the field of nonautonomous dynamical systems. New results and material for study are present in abundance. We wish to thank the colleagues who contributed to this special volume of the DCDS-S and thereby made it possible. Thanks are also due to Prof. Alain Miranville for his invitation to prepare it, and to the AIMS staff for their expert work in its publication.

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