UNIVERSIDAD DE VALLADOLID

ESCUELA DE INGENIERIAS INDUSTRIALES

Grado en Ingeniería en Organización Industrial

Principles of Revenue Management and their applications

Autor:

Oliveri Martínez-Pardo, Gonzalo

Responsable de Intercambio en la UVa

Gento Municio, Ángel Manuel

Universidad de destino

Hochschule Albstadt-Sigmaringen

Valladolid, 08/2017.

TFG REALIZADO EN PROGRAMA DE INTERCAMBIO

TÍTULO:	Principles of Revenue-Management and their applications
ALUMNO:	Gonzalo Oliveri Martínez-Pardo
FECHA:	24/07/2017
CENTRO:	Hochschule Albstadt-Sigmaringen
TUTOR:	Dr. Bernd Strauss

BACHELOR THESIS

“Principles of Revenue-Management and their Applications”

AUTHOR:

GONZALO OLIVERI MARTÍNEZ-PARDO

Albstadt, 24-07-2017

Hochschule
Albstadt-Sigmaringen

Albstadt-Sigmaringen University

Eidesstattliche Erklärung / Affidavit

Ich versichere, die Arbeit selbständig verfasst zu haben und keine anderen Quellen und Hilfsmittel benutzt zu haben. Alle Stellen, die wörtlich oder sinngemäß aus Veröffentlichungen entnommen sind, habe ich als solche kenntlich gemacht.

I hereby declare that this project work has been written only by the undersigned and without any assistance from third parties.

Furthermore, I confirm that no sources have been used in the preparation of this work other than those indicated in the thesis itself.

Albstadt, (Date) _______________ *(Signature):*_____________________________

Index

1. INTRODUCTION 11
   1.1. CONCEPTUAL FRAMEWORK OF REVENUE MANAGEMENT 11
      1.1.1. Management decisions 11
      1.1.2. Properties that encourage the application of Revenue Management 12
      1.1.3. The Revenue Management process 14
   1.2. HISTORY 15
   1.3. REVENUE MANAGEMENT IN 21th CENTURY 19
      1.3.1. Revenue Management in the Industry 19
      1.3.2. How to conduct Revenue Management in a company 20
      1.3.3. Important factors in Revenue Management 20
2. KEY IDEAS 22
   2.1. QUANTITY-BASED METHOD 22
      2.1.1. Types of controls 23
      2.1.2. Overbooking 26
   2.2. PRICE-BASED METHOD 28
      2.2.1. Factors must be considered in dynamic-pricing models 28
      2.2.2. Customers segmentation in RM pricing 29
      2.2.3. How to set prices 31
   2.3. PRICE-BASED VS QUANTITY-BASED 32
3. MODELS 34
   3.1. CAPACITY CONTROL MODELS 34
      3.1.1. n-Class model 34
      3.1.2. Heuristic models 39
   3.2. STATIC OVERBOOKING MODEL: THE BINOMIAL MODEL 45
      3.2.1. Overbooking based on Service-Level Criteria 47
      3.2.2. Overbooking based on Economic criteria 48
   3.3. DYNAMIC PRICING MODEL 50
      3.3.1. Model´s enunciate 50
      3.3.2. Solving the problem 51
      3.3.3. Example 52
4. USES IN REAL WORLD 57
   4.1. Hotels 57
   4.2. Rental car 58
   4.3. Manufacturing 59
   4.4. Sport centres 59
   4.5. Electricity generation and transmission 60

4.6. Online travel portals 61
4.7. Restaurants 62
4.8. Passenger railways 63
4.9. Theatres and Sport events 63
4.10. Freight 64
4.11. Tour operators 64
4.12. Programs used in Hotel industry 65
4.12.1. STR 65
4.12.2. IDeaS 66
4.12.3. HotelsDot 66
4.12.4. Modern Revenue 66
5. CONCLUSIONS 67
6. APPENDIX: 68
6.1. Appendix 1: Table of normal distribution 68
6.2. Appendix 2: Graphic of c.d.f. distribution 69
7. BIBLIOGRAPHY 70

List of tables

Table 1: Management decisions (contents from [2]) 11

Table 2: Properties that encourage the application of RM (contents from [3] p.4-7) 13

Table 3: Distribution of the full rate demand [35] 17

Table 4: Important factors in dynamic pricing models (contents from [2] p.182-187) ... 29

Table 5: Matrix of segmentation [34] 30

Table 6: Different kind of rate fences (Information from [3] p.15-16) 31

Table 7: Prices and demand data for each class 37

Table 8: Results of the example 3.1.1.2. 39

Table 9: Demand data for each class [2] 41

Table 10: Results of heuristic models and comparison with the optimal policy [2] 45

Table 11: Expected revenues obtained for different capacities and comparison with the optimal [2] 45

Table 12: Binomial approximation overbooking probabilities [2] 48

Table 13: Results of the first numerical example 3.3.3. 55

Table 14: Results of the second numerical example 3.3.3. 56

Table 15: How hotels satisfy Revenue Management favourable conditions 57

Table 16: How Rental car companies satisfy R M favourable conditions 58

Table 17: How sport centres satisfy R M favourable conditions 60

Table 18: How electricity companies satisfy R M favourable conditions 61

Table 19: How online travel portals satisfy R M favourable conditions 62

Table 20: How restaurants satisfy R M favourable conditions 62

Table 21: How satisfy Theatres and sport events RM conditions 63

Table 22: How travel operators satisfy R M favourable conditions 65

List of figures

Figure 1: Revenue Management process (By my own) 14

Figure 2: Important factors in the future (Graphic from [6] p.23) 20

Figure 3: Booking limits (b_i) and protection levels example (y_i) [2] 23

Figure 4: Booking limits (b), protection levels (y) and bid prices ($\pi(x)$) relation [2]...... 25

Figure 5: Optimal protection level y_i^* in the model [2] 36

Figure 6: Overbooking limits and reservations over the time [18] 46

Figure 7: Table of normal distribution [24] 68

Figure 8: Graphic of c.d.f. distribution [25] 69

1. INTRODUCTION

The intention of this thesis is to show some ideas about the concept of Revenue Management. This thesis begins with an introduction that contents general ideas about Revenue Management. Later, it is explained in more detail some key ideas of RM and some basic models in RM. Afterward some applications in the real world and software used in RM are presented and finally there are some conclusions.

1.1. CONCEPTUAL FRAMEWORK OF REVENUE MANAGEMENT

Revenue management is defined as the collection of strategies and tactics firms use to scientifically manage demand for their perishable products and services [1].

Every seller faces several fundamental decisions. You want to sell when market conditions are most favourable; you want the right price (not so high that you put off potential customers and not so low that you lose out on profits) and you want to know how customers value your product. Businesses face these and other complex decision with the aim of maximize benefits [1].

1.1.1. Management decisions

Revenue management (RM) is concerned with the methodology and systems required to make demand **management decisions**. Next table (Table 1) shows the three categories of management decisions showed in *Theory and Practice of Revenue Management* [2].

MANAGEMENT DECISION	DECISIONS ABOUT
Structural decisions	- Which sales channel to use for selling the products or service. - Which segmentation mechanism to use. - Which terms of trade to use. - Which set of products to offer.
Prices decisions	- How to set prices and individual offers. - How to price over time. - How to establish different category prices. - How to set discounts over the product lifetime.
Quantity decisions	- Whether accept or reject an offer. - How to set the capacity to different products, segments or channels. - When refuse a sale and sale this product later.

Table 1: Management decisions (contents from [2])

To decide which of these decisions is the most important depends on the context. Structural decisions about channels for selling and the tools for grouping products are strategic decisions that are not taken frequently. Firms also should take decisions about prices or quantity which change their ability to adjust price or quantities on a tactical level. The ability to adjust quantities may be a function of the flexibility of the supply process and the costs of real locating capacity and inventory. For example, Airlines use capacity controls as a tactic because of the fact that the different products that they sell (different ticket types sold at different times and under different terms) are all provided using the same, homogeneous capacity [1].

Revenue Management can be qualified as being either **quantity-based RM** or **price-based RM** if it uses capacity-allocation decisions or price decisions as the first tactical tool for managing demand. These two terms are explained more extensively in second chapter.

1.1.2. Properties that encourage the application of Revenue Management

The application of Revenue Management has been most effective when it is applied to operations that have the properties [3] (p.4-7) shown in the table 2:

PROPERTY	EXPLANATION	EXAMPLE
Relatively fixed capacity	The production is constant, it is not able to increment the production in a brief time period	In each flight, there are a limited number of seats and it is not possible to increment the capacity.
Perishable inventory	Attributes cannot be stored. The attributes that have not been consumed cannot be stored to do it later.	If there are available seats for a flight today, these seats cannot be stored for another flight that is full.
Inventoried demand	Most industries that employ RM use reservations to anticipate and to control the future demand by the selling of products before the consume.	Hotels and airlines use reservations to sell and control their inventory before it is consumed.
Stochastic demand	Most of demand varies over time.	Consider the demand for air travel. It varies due to season, time of day, day of week, holidays, …
High fixed costs	The combination of high fixed costs and low variable costs gives more incentive to fill their unused capacity	In a flight, there are high fixed costs (plane, staff, …) and insignificant variable costs for selling an unused seat. Airlines are interested in selling all the capacity even a low price.
Low variable costs		
Customer heterogeneity	Customers value the product differently. Some customers prefer less service and lower-price but others are willing to pay more for a better service. Then it is necessary to make a segmentation of customers.	A student or family in holidays are willing to travel at uncommon hours or days for a cheaper ticket while a business man prefers a more convenient schedule although he has to pay more money.

Table 2: Properties that encourage the application of RM (contents from [3] p.4-7)

The combination of these characteristics makes the application of RM more effective because they allow the possibility of offering different rates for different customers over time.

1.1.3. *The Revenue Management process*

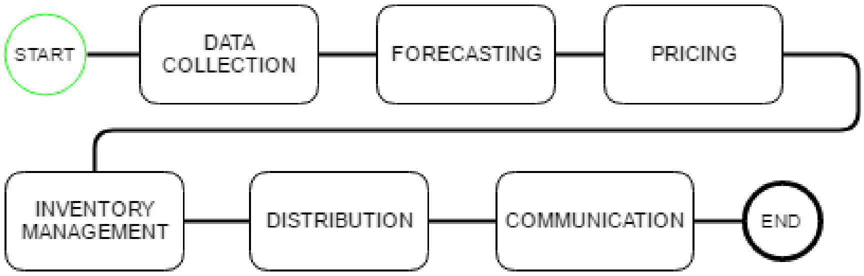


Figure 1: Revenue Management process (By my own)

Stuart-Hill in an article [4] shows the process that most practitioners follow today involves a continuous cycle of six basic steps:

- Step 1: Data collection
  The best decisions are often a mix of intuition and science. In order to understand the impact of these decisions, it is recommendable to store data and to use those to help make better decisions in the future.
  Automation and decision support tools have evolved as the revenue management discipline has grown and provide business insights that humans are not able to calculate on their own. The ideal solution is when automation provides insights for the revenue management professional and the revenue management professional provides appropriate feedback within the system.

- Step 2: Forecasting
  Predicting future demand based on past results is a good starting point to develop a strategy. Of course, changes in market conditions, competitive supply, capacity constraints, special events, floating holidays, weather conditions, economic climate and other external factors need some additional thought around true demand potential; but the past results provide help to make good predictions and to be prepared against the variations produced by these factors.

- Step 3: Pricing
  People who unknow the scope of the discipline often think that revenue management simply consists in changing prices, or making discounts. The fact is that pricing is far more complex than this.
  Basic economic theory would dictate that equilibrium pricing, where the demand curves meets the supply curve, would be optimal. Unfortunately, inventory is perishable so this approach would not utilize all the resource's potential.

- Step 4: Inventory Management
  Once future demand has been forecasted and pricing strategies determined, resource allocation or inventory management comes into play. Inventory management simply means opening or restricting inventory for sale based on certain conditions as the available capacity.

- Step 5: Distribution
  Distribution refers to how guests find and book a property. Telephone, Online Travel Agencies, Global Distribution Systems and Internet Booking Engines are all examples of distribution channels. Each one may offer a different value proposition, but each one requires that a revenue management professional think through how to best leverage the potential that each one represents.

- Step 6: Communication
  Communication is crucial because Revenue Management requires that a firm must continually re-evaluate their prices, products, and processes in order to maximize revenue. In a dynamic market, an effective Revenue Management System constantly re-evaluates the variables involved in order to move dynamically with the market. Then communication is causal because the information is constantly varying.

Of course, once a strategy has been implemented the process begins again with Step 1 - tracking. This process is one of continuous improvement; however, some elements of experimentation will depend of the probability. Then, some actions will have a positive impact and others won't. The secret is when there is a fail, find it quickly and ensure learning is documented.

In few words, Revenue Management's basis is "*to sell the correct product to the right client at the right price in the right moment*". But to understand in a better way what is Revenue Management it should be recommendable to take a look at its origins.

1.2. HISTORY

The interest in revenue management practices started during the 70s with the researches of Littlewood (1972) on overbooking in airlines. These researches have as a result, the rule known as Littlewood's rule.

Littlewood proposed that when a product has two different prices, the product must be sold at the lower price until the probability of selling at the higher-price is higher than the relation between the two prices.

- LITTLEWOOD RULE:

  Littlewood considered a flight with two different rates, $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ (with $\boldsymbol{p_1} > \boldsymbol{p_2}$), and demand of $\boldsymbol{p_1}$ is sold after the demand of $\boldsymbol{p_2}$. Littlewood proposed that the product is sold at the lower price until the probability of selling at the higher-price is higher than the relation between the two prices.

$$\boldsymbol{P(p_1) > \frac{p_2}{p_1}} \quad \begin{cases} P(p_1) : Probability\ of\ selling\ product\ at\ price\ 1 \\ p_2: lower - price \\ p_1: higher - price \end{cases}$$

The solution can be obtained from the relationship marginal revenue/loss. To get the solution, it is supposed there are $\boldsymbol{n}$ vacant units and one class 2 client wants to buy a seat. In case of the airline regret the purchase, it will be sold $\boldsymbol{n}$ units at $\boldsymbol{p_1}$ only in the case if demand of class 1 is higher or equal than $\boldsymbol{n(D_1 \geq n)}$. As a result, marginal revenue of the booking of this seat for class 1 will be $\boldsymbol{p_1 * P(D_1 \geq n)}$. Finally, it is logic to accept class 2 clients always that $\boldsymbol{p_2}$ would be equal or higher than this marginal revenue $\boldsymbol{p_2 \geq\ p_1 * P(D_1 \geq n)}$.

As $\boldsymbol{n}$ is decreasing, after the moment that $\boldsymbol{p_1 * P(D_1 \geq n)}$ is higher than $\boldsymbol{p_2}$, it will be always higher, then exists an optimal protection level $\boldsymbol{y_1^*}$ such that we accept class 2 until the moment that available capacity is equal or less than $\boldsymbol{y_1^*}$. Then, $\boldsymbol{y_1^*}$ satisfies:

$$\boldsymbol{p_2 <\ p_1 * P(D_1 \geq y_1^*)}\ and\ \boldsymbol{p_2 \geq\ p_1 * P(D_1 \geq y_1^* + 1)}$$

If the demand is modelled by the continuous distribution $\boldsymbol{F_1(x)}$, Then, we have:

$\boldsymbol{p_2 =\ p_1 * P(D_1 \geq y_1^*)}$ equivalently, $\boldsymbol{y_1^* = F_1^{-1}(1 - \frac{p_2}{p_1})}$

that is known as *Littlewood's rule*.

Now, it is shown a little example [35] to understand better how it works:

An airline offers two different rates for the same flight: full rate ($\boldsymbol{p_1 = 418}$€) and Economic rate ($\boldsymbol{p_2 = 218}$€). The capacity is $\boldsymbol{C = 230}$ and the demand for the full rate is distributed as it is shown in the next table:

Demand (Q)	P(d=Q)	P(d<=Q)
40	0,02	0,25
41	0,06	0,31
42	0,04	0,35
43	0,01	0,36
44	0,06	0,42
45	0,07	0,49
46	0,02	0,51
47	0,03	0,54
48	0,03	0,57
49	0,05	0,62
50	0,03	0,65
51	0,05	0,70
52	0,04	0,74
53	0,06	0,80
54	0,09	0,89
55	0,11	1,00

Table 3: Distribution of the full rate demand [35]

Now, it is determined the number of Full seats and the limit of Economic seats by the Littlewood´s rule. To get it, it is determined the highest value of Q that satisfy:

$$\boldsymbol{p_1 * P(D_1 \geq Q) + 0 * P(D_1 < Q) \geq p_2}$$

$$\boldsymbol{p_1 * P(D_1 \geq Q) \geq p_2}$$

$$\boldsymbol{P(D_1 \geq Q) \geq \frac{p_2}{p_1}}$$

$$\boldsymbol{1 - P(D_1 < Q) \geq \frac{p_2}{p_1}}$$

$$\boldsymbol{P(D_1 < Q) \leq 1 - \frac{p_2}{p_1} = 1 - \frac{218}{440} = 0,504}$$

Now, it is looked for the value that satisfy it:

For $\boldsymbol{Q = 45}$, $\boldsymbol{P(D_1 \leq Q) = 0,49}$ and $\boldsymbol{P(D_1 = Q) = 0,07}$

Then, $\boldsymbol{P(D_1 < Q) = P(D_1 \leq Q) - P(D_1 = Q) = 0,49 - 0,07 = 0,42 < 0,504}$

For $\boldsymbol{Q = 46}$, $\boldsymbol{P(D_1 \leq Q) = 0,51}$ and $\boldsymbol{P(D_1 = Q) = 0,02}$
Then, $\boldsymbol{P(D_1 < Q) = P(D_1 \leq Q) - P(D_1 = Q) = 0,51 - 0,02 = 0,49 < 0,504}$

For $\boldsymbol{Q} = \mathbf{47}$, $\boldsymbol{P}(\boldsymbol{D_1} \leq \boldsymbol{Q}) = \mathbf{0,54}$ and $\boldsymbol{P}(\boldsymbol{D_1} = \boldsymbol{Q}) = \mathbf{0,03}$

Then, $\boldsymbol{P}(\boldsymbol{D_1} < \boldsymbol{Q}) = \boldsymbol{P}(\boldsymbol{D_1} \leq \boldsymbol{Q}) - \boldsymbol{P}(\boldsymbol{D_1} = \boldsymbol{Q}) = \mathbf{0,54} - \mathbf{0,03} = \mathbf{0,51} > \mathbf{0,504}$

The highest value of **Q** that satisfy the Littlewood's rule is **Q=46**; that is the number of Full seats. As a consequence, **230-46=184**; are the maximum number of seats accepted for the Economic class.

During the first years of Revenue Management was the basis of the algorithms used by the automatized systems of Revenue Management.

October of 1978 was a crucial moment during the beginnings of Revenue Management because it was the year when the airline deregulation was signed in USA. This supposed the end of the monopoly protection of the biggest airlines in the country and the entrance of new competitors and the free airfares.

The entrance of new companies supposed many problems for the big six that are the companies which had been protected by the government from 1937 until 1977 (American Airlines, Delta Air Lines, USAir (now belongs to AA), Continental Airlines (acquired by United Airlines), United Airlines and Eastern Airlines (dissolved in 1991)), in fact some of them disappeared and the others offered discounted rates. One of these new companies was People Express, this low-cost company appeared in 1981 and gain much importance and it was a big threat for the big six.

The answer of American Airlines came in December of 1985 with discounted rates (up to 80% lower than the usual rate) called 'Ultimate Super-Saver fares'(uss). The difference with other discounted rates was DINAMO (Dynamic Inventory Optimization and Maintenance Optimizer) that represents the first large-scale RM system development in the industry.

DINAMO is the automatized system developed by American Airlines to include Littlewood´s rule in their decisions and reduced the proportion of flights requiring analyst review from 100 percent to approximately 5 percent [5]. DINAMO calculated dynamically the probability of sale the last seat in the highest rate using the Littlewood´s rule. It optimized the quantity of seats that were offered with 'ultimate supersaver' rate. Due to the success of DINAMO People Express broke down and was sold to Continental Airlines a year later and the revenues of American Airlines grew up $1.4 billion during the next three years.

Donald Burr, CEO of People Express, summarized the reasons behind the company's failure: *"We were a vibrant, profitable company from 1981 to 1985, and then we tipped right over into losing $50 million a month. We were still the same company. What* *changed was American's ability to do widespread Yield Management in every one of our markets. ...We did a lot of things right. But we didn't get our hands around Yield Management and automation issues. ... [If I were to do it again,] the number one priority on my list every day would be to see that my people got the best information technology tools. In my view, that's what drives airline revenues today more than any other factor—more than service, more than planes, more than routes."*

These is the moment when the airline industry provided a concrete example of the great impact that the revenue management can have on the operations of a company. At this stage, however, much of the work was about capacity management and overbooking and only a little about dynamic pricing policies. In these original models, prices were assumed to be fixed and managers only decide about how many seats sell in each class.

During the 1990s the increasing interest in revenue management made evident the different applications that can be considered. Models became industry specific with a higher complexity. Moreover, is during this decade when prices policies became an important component of the revenue management activities.

As we said, the airline industry leaded the use of revenue management techniques in terms of capacity/seat control and dynamic pricing.

1.3. REVENUE MANAGEMENT IN 21th CENTURY

What is special about revenue management is not which decisions are made but rather how are decisions made to get the aim of increasing benefits in every budget that you can use; then, what develop is the tools that Revenue Management to get the goal [17]. The new about RM is the method of decision making, that is driven by two complementary points. First, the scientific advances allow computing optimal real-time solutions to complex decision problems. The other point resides in that the advances in information technology provide the capability to automate transactions, store and implement the data and manage detailed demand-management decisions [17].

1.3.1. Revenue Management in the Industry

There are other examples besides airlines of the success of Revenue management, as the case of hotels. First in hotels, RM only was applied to rent bedrooms. But nowadays it is used in every part in the hotel (restaurant, parking, golf, spa, …) in response to the increments caused by the application of RM in bedrooms management. As Kimes said "each squared meter in the hotel is susceptible of generate revenues" [6]. This high grow has taken place in less than 20 years.

Revenue Management allows increasing the benefits and it can be used in too many different industries. The RM's popularity has grown because, nowadays, methods of decision making in RM are growing by big steps as consequence of all the developments around Internet, software programs and big-data; that permit to work easier and more efficiently than some years ago. In fact, until 1990s Revenue Management only was useful as philosophy to manage in airline industry but now it has reached other sectors [17].

Some of the industries that use Revenue Management are explained in more detail in chapter 4.

1.3.2. How to conduct Revenue Management in a company

The future of how to manage the RM in a company is already uncertain. In the case of hotel chains, Kimes´ investigation [6] (p.5) shows two tendencies of how to solve the management.

First tendency is thought that in the future, Revenue Management department will be an independent department which combines Marketing and Business because the aim is to sell in the most efficient way as possible but without damaging the image of the product.

The second tendency talks about a future where the tendency in management in hotel chains will be the centralization or a hybrid model that consists in that centrals transmits a general view and hotels offer a concrete view of each case. For example, the group NH hotels has the management centralized for all hotels but there are some concrete management decisions those are taken by each hotel.

1.3.3. Important factors in Revenue Management

In the same investigation [6], Kimes conducted a survey among professionals in the field. Results shown that technology is the most important RM influencing factor, as it is showed in figure 2:

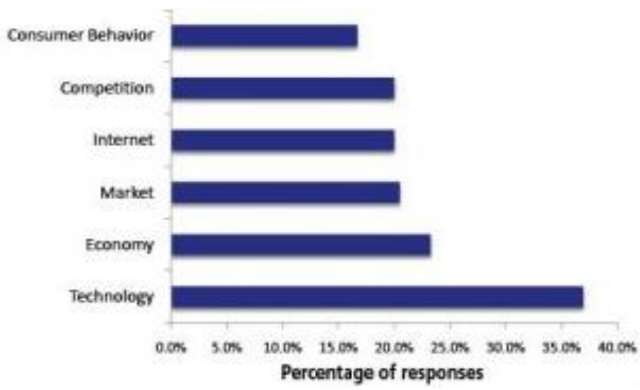


Figure 2: Important factors in the future (Graphic from [6] p.23)

The graphic shows the future influence of the different RM influencing factors among the respondents. Consumer behaviour, Competition, Internet, Market segmentation and Economy are factors that influence in products' prices. But Technology is the most important one because is the tool to take advantage respect the influence of the other factors.

Although these results come from a survey among professionals in Revenue Management, they could be the same for other general consumer choice situations.

As we all know now we are living a technologic-boom caused by the constant development. Technology is growing exponentially and Revenue Management take advantage of that because tools used to take decisions are being developed at the same time.

The reasons of why other industries bet for Revenue Management are that it is a new way to work that is already in development phase and that the industries that are using RM are obtaining great results.

2. KEY IDEAS

As it is shown in the Introduction, in Revenue Management we can distinguish two methods to work. The first, and older, is the Quantity-based method. This method has as main idea the control of the quantity of products are sold with each price. In the field of airlines, it would be the method that help to make decisions on how seats must be sold in each flight at each rate to maximize profits.

The second method is the Price-based Revenue Management. This method works to control and to adjust the prices to maximize their benefits.

2.1. QUANTITY-BASED METHOD

Quantity-based method has as main idea the control of the quantity of products we sold with each price. In the field of airlines, it would be the method that help to make decisions on how seats we must sell in each flight at each rate to maximize profits [2].

To explain it better, it will be used an example. It is the case of a single leg flight or a night in a hotel. For trying to sell every seats or rooms, companies offer two different rates the business-rate (higher-price) and the tourist-rate (more economic price). The question is now how many products must be sold at each price for the optimization of benefits. The problem is the customer heterogeneity that consists in that different people rate the same product at a different value. Then, if you offer too many business-rate products, not all the products will be sold. In the other case if you don´t offer enough business-rate products, you will sell some tourist-rate products that could have been sold more expensively.

Quantity-Based RM use some tools to give an answer of the question and to offer some optimized solutions in real-time that helps in the decisions about how many products offer in each fare.

The central problem is how to assign optimally the capacity of resources to the different customers. This assignation must be done dynamically as demand materializes and with considerable uncertainty about the quantity or composition of future demand. To make this assignation widely it is crucial to make a good segmentation of the demand in function of their willingness to pay different rates.

There are some different ways to make a segmentation but it is necessarily to consider two aspects: first, the distribution channel used by customers to make their bookings; and second, the price, that is the tool to segment the customers offering different rates in function of extra-services, purchase channel, … There are some requirements to know that the segmentation is correct [33]:

- Homogeneity in the segment.
- Heterogeneity between segments.
- Stability of segments.
- Segments must be identifiable and measurable.
- Segments must be accessible and manageable.
- Segments must be large enough to be profitable.

Now the different controls used to control the future demand are shown.

2.1.1. *Types of controls*

To work efficiently, companies must realise some controls. In the travel Industry, the availability of bookings is controlled every time because the data saved and implemented every moment and the system works dynamically. These controls [17] are:

2.1.1.1. **Booking limits:**

These limits are responsible of restricting the amount of capacity that can be sold to each class at a given point in time. For example, if the booking limit is 24 on class 1, it indicates that the most amount of capacity that can be sold to customers in class 1 is 24.

Booking classes can be partitioned or nested. The partitioned booking limit divides the capacity in separate blocks for each class. The sum of booking limits of all the classes must be equal than the whole capacity.

In the case of a flight, with three different classes and a whole capacity of 30 passengers we have these booking limits: 12(class 1), 10 (class 2), 8 (class 3). The limits can be changed but the sum of all must be 30.

Then when, we have sold 12 of class 1, class 1 would be closed. This could be undesirable when class 1 has higher revenues than the other classes and the amount allocated to class 1 are sold out.

Nested booking limit works in a different way to solve the problem with the partitioned booking limits. In this case, also there are different booking limits for each class, but the difference resides in that higher classes have access to all the capacity of the lower classes. For the case we saw before, the nested booking limits would be b1=30 for the set 1 (because class 1 is the highest then it has access to the units of the other two classes), b2=18 for set 2 (includes the 10 units for class 2 and the 8 for class 3) and b3=8 for set 3 (because it is the third lower class, it only has access its own units). It is shown in the Figure 3.

Nested booking limits are used by almost every reservations system because they allow to solve the problem of capacity being simultaneously unavailable for a high class yet available for lower classes.

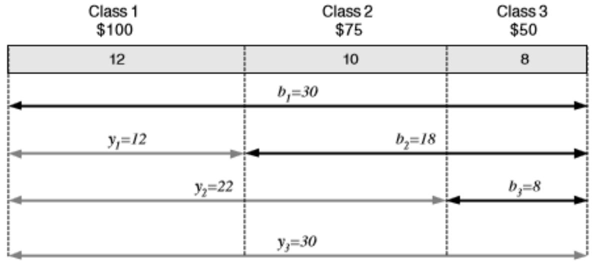


Figure 3: Booking limits (b_i) and protection levels example (y_i) [2]

2.1.1.2. Protection levels

Protection levels are used to reserve an amount of capacity for a class or set of classes. As in booking limits, protection levels can also be nested or partitioned.

A partitioned protection level is essentiality the same than a partitioned booking limit. A booking limit of 10 on class 2 sales is equivalent to a protection level of 10 units of capacity for class 2.

Nested protection levels are again defined for sets of classes but in reverse order. For nested booking limits, each set of classes includes a class and the lower class (in the example b1 includes all classes, b2 includes class 2 and 3 and b3 only includes class 3).

Instead for nested protection levels each set of classes includes its class and higher classes. Then in the example we would have these protection levels: y1=12 for set 1(only includes class 1 because is the highest), y2=22 for set 2 (includes class 2 and higher, class 1) and y3=30 (includes all classes because third class is the lowest). Thus, we have 12 units reserved for class 1, 22 for classes 1 and 2 and 30 for every class.

Then, if 12 units of class 1 are sold you can continue selling because there are reserved 30 seats for the three classes then class 1 has already access. In the other hand, if 8 units of class 3 are sold, you can´t sell more class 3 because all the other units are reserved for classes 1 and 2.

In the figure 3 (page before), we can see that the booking limit for class (set) j is simply the capacity (C) minus the protection level for set j-1.

$$\boldsymbol{b_j = C - y_{j-1}, j = 2, \dots, n}$$

Besides $b_1 = y_n = C$ because the booking limit for highest class is all the capacity and the protection level for all classes combined (the lowest, that includes every class higher) is al capacity too.

2.1.1.3. Standard/ Theft Nesting:

There are two different process for using nested booking limits or nested protection levels. The standard process starts with C units of capacity. It is accepted a booking for class j when there is capacity available and the total request accepted for class j and lower (j, j+1, …, n) is less than the booking limit b_j. (also, the current capacity available is more than the protection level y_{j-1} for classes higher than j).

In theft nesting process, a booking in class j don´t reduce the units for class j, and takes the allocation of the lower classes. This is the same than keeping y_j units of capacity protected for future demand from class j and higher. In other words, when we accepted a request for class j, we continue to reserve y_j units for class j and higher, and to reduce the allocation for classes j+1, j+2, ..., n. In contrast, under standard nesting, when we accept a request from class j we only reduce by one the capacity we protect for future demand from class j and higher. In the example of the figure 3, if we would accept a request for class 1, y_1 would continue valuing 12; but y_2 and y_3 would be reduced: y_2=21 and y_3=29.

2.1.1.4. Bid prices:

What makes bid-price controls special from booking limits and protection levels is that bide-price controls are based in the revenues from each sale, rather than class-based controls. A bid-price control establishes a threshold price (which may depend on variables such as the remaining capacity or time) and request for booking is accepted if its revenue exceeds the threshold price and rejected if it is less than the threshold price.

Firstly bid-price controls look simpler than protection-level and booking-limit controls because they only need set only a single threshold value at any point in time. However, to work effectively, bid prices must be actualized after each sale and this usually needs to store a table of bid-prices values directed by the current remaining capacity, current time, or both.

The figure 4 shows how can be used bid prices to implement the same example as the other two types of control where the bid price π(x) is planned as a function of the remaining capacity x. When there are between 22 and 30 available units bid-price is lower than 50$ (all the three classes are accepted). With 13 to 22 units, bid-price is between 50$ and 75$ (only classes 1 and 2 are accepted). And when there are 12 or less units available, bid-price is over than 75$ but less than 100$ (only class 1 demand is accepted).

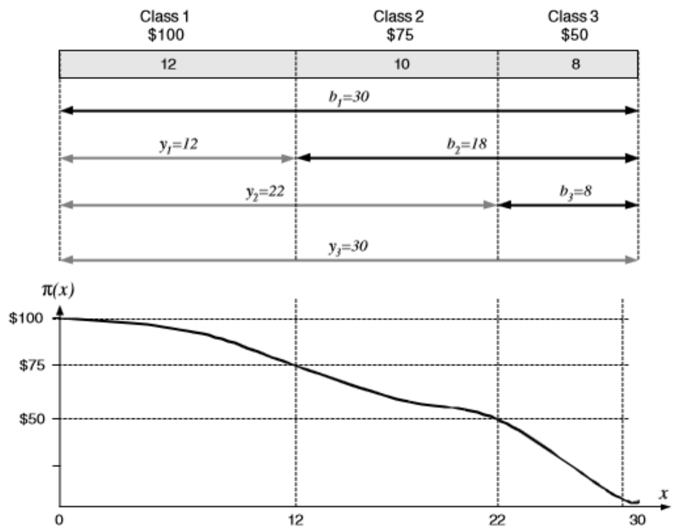


Figure 4: Relationship between booking limits (b), protection levels (y) and bid prices (π(x)) [2]

Some experts criticize Bid-price because consider it unsafe because use only a threshold price as the only control. That means that RM system sell an unlimited number of units to any class whose revenues exceed the bid-price threshold. However, this is truth only when the bid price is not actualized. As it is shown in the figure 4, if the bid-price is function of remaining capacity (bid-price is actualized after each purchase), then it works like the other two methods, closing off capacity to successively higher classes as capacity is consumed. This function is necessary because a simple static threshold is indeed a somewhat dangerous form of control.

One advantage of bid-prices controls is the ability to discriminate because it is based on revenue rather than on class. Often some customers with different willingness to pay are in a single class using an average price as the price for this class. However, if actualized revenue information can be used for each request, then a bid-price control can selectively accept only the higher revenue requests in a class, while the other controls can only accept or reject all requests of a class. But if it is not possible to observe the exact revenue at the time of reservation, then this advantage is lost.

Bid-prices don´t segment the demand in different closed class with the average price. In the example that we are using during the explanation, it is supposed that there are only seats available for class 1. If we use the other two controls, you only accept people that are willing to pay 100$ and reject the others and maybe it is not possible sell all of them. But using bid prices some customers who are willing to pay between 75$ and 100$ are accepted when this price is higher than bid-price. This fact provides the airline the opportunity of selling more seats.

It could be thought that bid-prices are controls for dynamic pricing but it is not truth because what this control vary is the minimum price that must be accepted for each available capacity to increase the revenues.

Figure 3 shown the examples that we use in the three controls. At the top, it is shown the relationship between booking limits and protection levels. At the bottom, we can see how would vary bid-prices in function of the available capacity.

2.1.2. <u>Overbooking</u>

Some experts [2] consider overbooking as essential activity in Revenue Management. Overbooking is distinct from the two models; indeed, overbooking is the oldest of RM practices. It is an important activity although it is not the principal strategy. The objective of overbooking is to maximize the capacity utilization of the system in the presence of cancelations and, consequently, the benefits.

As most people know, overbooking essentially consists in accepting more reservations than units you have available [11]. It is applicable when the follow characteristics [12] are fulfilled:

- Capacity is constrained and perishable and bookings are accepted for future use.
- Customers can cancel or no-show.
- The cost of denying service to a costumer with a booking is relatively low.

Despite most of customers have bad perception of Overbooking, companies continue using it; but, why Overbooking? The reason is simple, companies that accept advance bookings with refundable sales run the risk of cancellations. Overbooking only is a strategy that firms for protecting themselves from this risk and thus it increases the utilization of capacity and maximize revenue for the firm. But it must be well implemented because it is useful only if it works well. If the Overbooking is not well implemented it only increment the costs and reduce the revenues. Thus, it is necessary to face some problems. One problem is facing the legal and regulatory implications of failing the booking contract. Companies must have operational policies and procedures used when a service must be denied.

When there are oversales, managing the compensation and the selection of customers can have a significant impact on denied-service costs and customers perception of overbooking. We next look at the main issues involved in managing oversales [2]:

- <u>Compensation of Denied Service:</u> Legally mandated compensations often specify a payment of monetary damage; but it is often inadequate for the customers' perception. It is usually more effective to offer a substitute service (more quality service), because sometimes customers are not interested in an economic compensation, they only want to enjoy the service then they are usually more satisfied when they receive a more quality service than an economical compensation.

- <u>Selection of customers:</u> The selection of customers who will be denied also can have a significant impact in firm´s costs and in customers´ perception. From a legal standpoint, selection must not be discriminatory.
  The most intuitive option for allocating a service to customers it is to do it on a first-come, first serve (FCFV). It means that customers are served when they arrive until the service is full. After the service is full, the rest of consumers are denied. This option has the point that it is fair and encourage customers to arrive on time; but it is not desirable in some industries. Some firms try to select customers who pay lower-rates because the compensation cost often less than the compensation for the other customers.

- <u>Oversale auctions:</u> an alternative method is conducting an auction to attract volunteers to give up their reservations in exchange for an economic compensation. This practice is very typical in airlines, but it does not always work. An example of it was some months ago, United Airlines had an Overbooking problem in a flight and they offer some money and a pleasant hotel in exchange for taking a flight the following day. Nobody wanted to accept the offer, then firm chose five randomly people. The problem came when one of the travellers selected was a Vietnamese doctor who did not want to accept it. He was forcibly thrown out while other travellers filmed the scene. In few hours, the video was in all the social networks in many countries. This fact affected the image of the company and they had to pay a big compensation for the customer.

2.2. PRICE-BASED METHOD

Price-based method (or dynamic pricing) use prices rather than quantity controls as primary variables used to manage demand. Little of the research published mentioned price as a variable before 1995; price was considered as an exogenous variable that does not vary in the model. But, given that any RM decision is a function of price and duration, it is essential that RM models include information on the relationship between price and demand.

Price can be used in two ways: to determine the optimal prices and to determine who should pay which price. What is special in RM pricing is the heterogeneous demand. When it exists, firms can select the customers willing to pay the most. Companies that use RM successfully generally shows a strong positive correlation between their capacity utilization percentage and their average rate per person [7].

2.2.1. Factors must be considered in dynamic-pricing models

Segmentation of customers is also essential in dynamic-pricing problems. To made this segmentation in a good way it is necessary to consider some factors [2]; how customers behave over the time and the state of market conditions.

2.2.1.1. Sophistication level of customers:

In dynamic-pricing models we distinct between two kinds of customers. Some of the models assume the customers as **myopic customers,** those who buy as soon as the offered price is less than their willingness to pay. Other models consider **strategic-customers**, those who optimize their purchase in response to the pricing strategies of the firms [2] (p.182-184). Models that consider strategic-customer are more realistic than those who consider myopic-customers but, considering strategic-customer complicates the estimation and the analysis of optimal pricing strategies.

2.2.1.2. Population size:

Models works in two different ways: **Infinite-population Models** and **Finite-population Models**. The difference is that in infinite-population models the distribution of number of customers and their inclination to pay are not affected by the history of demand. In economics that is known as nondurable-goods assumption, that is assumed because make the problem simplest.

In contrast, in finite population models both distributions are affected by the history of demand. It means that when one customer of the population purchases, the population of potential customers is reduced. That is termed durable-goods assumption because when one customer buys a product, he will not another product in a short period of time.

2.2.1.3. Market conditions:

The other factor that must be considered is the level of competition in the market. Many models are **monopoly models**, where the demand depends only its own price and not on the price of competitors, but this assumption is usually not realistic.

Oligopoly models are those in which the price-decisions of competitors are modelled and computed. But these models are not popular in practice because of the difficulty in collecting competitors' data and some predictions of the price response are poor.

The other models are **perfectly competitive models**, in those it is assumed that are many firms that offer the same conditions and the output of each firm is small related with the market size. Then, one firm cannot influence market prices. Essentially, each firm can sell as much at it wants under the market price but nothing at higher prices [2] (p.185-187).

FACTOR	MODELS
Sophistication level of customers	Myopic customers
	Strategic customers
Population size	Infinite-population model
	Finite-population model
Market conditions	Monopoly model
	Oligopoly model
	Perfectly competitive model

Table 4: Important factors in dynamic pricing models (contents from [2] p.182-187)

2.2.2. Customers segmentation in RM pricing

There are some different methods to do the segmentation. But, generally the process works step by step. First of all, the customers are distributed in broad segments as for example French families, American businessmen, … After this first segmentation, it is followed by segmentation niches; for example, in hotels, kind of accommodation, length of stay, usual customers, … The way to do the segmentation depends on what wants the company to know about the customers [34].

When the segmentation is completed by the company, it is possible to do a matrix including the characteristics of each segment. The following table shows an example:

CARACTERISTICAS	**SEGMENTO ANXIUS**	**SEGMENTO PACIENT**	**SEGMENTO FREESTYLE**
	SOCIO DEMOGRAFICO		
Revenues	Between 21,000 and 35,000 per year	Between 35,000 and 65,000 per year	Between 16,000 and 21,000 per year
Occupation	Technologic	Executives	Students
Social status	Couples	Empty nest	Single youths
Locality	France	United States	Germany
	PURPOSE OF THE TRIP		
Recreational	25%	9%	41%
Educational	16%	2%	63%
Business	2%	42%	2%
	PROFITS		
Quantitative			
Food and drinks importance	3 of 5	4 of 5	2 of 5
Price importance	4 of 5	3 of 5	5 of 5
Cualitativo			
Service importance	4 of 5	5 of 5	3 of 5
Accommodation importance	3 of 5	4 of 5	4 of 5

Table 5: Matrix of segmentation [34]

When the matrix is already done, it allows the possibility to distribute the heterogeneous demand in more homogeneous niches.

Simon and Dolan [8] explain illustratively how firms charge different prices to different customer segments for essentially the same service as follows:

The basic idea of price customization is simple: Have people pay prices based on the value they put on the product. Obviously, you can’t just hang out a sign saying: “Pay me what it’s worth to you,” or “It’s $80 if you value it that much but only $40 if you don’t.” You have to find a way to segment customers by their valuations. In a sense, you have to “build a fence” *between high-value customers and low value customers so the "high" buyers can't take advantage of the low price.*

In other words, if a company charge different rates for essentially the same product, it must differentiate the rates with any special characteristic between the different rates to avoid that customers willing to pay a high rate, don´t take advantage of the low rate. For example, in a hotel, consider two tariffs 100€ and 75 €. Customers paying 100€ have additional services as free breakfast or a more desirable room. In this way, high-value customers pay the high rate because they prefer the additional services.

Essentially, rate fences allow companies to restrict lower prices to customer segments that are willing to accept certain restrictions on their purchase and consumption experiences [3]. We can distinguish two kind of rate fences, shown in the table 4.

FENCES	Explanation	Example
Physical fences	Refer to product differences as the seat location or extra services	A most desirable room or free breakfast are physical fences because the service offered is different
Unphysical fences	There are not differences in the product; there are differences in the consumption or the transaction for the same basic services	Discounts for booking in the web-site or for buying in advance are unphysical fences because despite the same service, rates are different

Table 6: Different kind of rate fences (Information from [3] p.15-16)

2.2.3. How to set prices

Most of pricing practices are still non-mathematically based; most Revenue Management prices are set with **competitive pricing** or through **negotiation**. It results in a large number of prices that must be placed into rate categories than can be controlled by the Revenue Management system [3] (p.16-21).

2.2.3.1. Competitive pricing:

The growth of the online market motivated the increase of the importance of competitive pricing because consumers can easily compare prices among competitors by going to some of the Internet travel sites. They can also compare the price for a particular company across the different distribution channels.

These Internet travel sites provoke two different feelings for travel firms. In the first hand, firms like these sites because of the increased visibility and sales of their products. In the other hand, firms don´t like these intermediates because often char sales commissions of 20-30%. In addition, it is important when a firm use different distribution channels that they conserve the same price in each channel because of the potential impact on customer satisfaction.

Firms usually has four sources for obtain the competitive information:

- Shopping: Phone calls to competitors for inquiring about their rates and availability.

- Global Distribution Systems (GDS): Many pricing analysts use GDSs to determine what the competition is charging to different products and use this information to make adjustments in their own prices.

- Third Party Data Providers: Third party systems search competitive websites on at least a daily basis and provide information on what their competition is charging in various markets.

- Electronic Distribution Systems: Many of the online distribution systems provide their clients with competitive pricing information.

These four sources of data can be used to evaluate current pricing policies.

2.2.3.2. Negotiation:

A considerable portion of prices are set through negotiation. Prices are generally negotiated as are the rates offered to large corporate accounts. The prices are based on demand, the forecasted number of inventory units that will be used, when usage is probably to occur, the auxiliary revenue associated with the business, and the long-term value of the business to the firm [3] (p.21). Negotiation is usually used by tour operators to get good prices from airlines, hotels and rental car companies.

2.3. PRICE-BASED VS QUANTITY-BASED

Industries that use Revenue management has to decide between to use quantity-based RM either to use price-based RM. Even in the same industry, some firms use quantity-based RM while other industries use price-based RM. This the case of airlines, where low-cost firms usually use dynamic pricing because they don´t offer many extra services; while traditional airlines usually use quantity-based RM because they can differentiate the different classes better due to the higher quantity of services to offer. But how to make the decision of which to use? There is not a concrete answer, but in essence it boils down to ask how able is a firm to vary quantity or price in repose to changes in market conditions. This ability is determined by the engagements a firm make, its level of flexibility in supplying products or services, and the costs of making quantity or price changes [2].

Consider airlines, some companies establish prices for their various fare products in advance of taking bookings due to advertising constraints, distribution constraints and to simplify the prices´ management. This limits their ability to use price to manage the demand, that varies considerable and is uncertain at the time of the price posting. But these limitations can be supplied by the flexibility to decide how many seats reserve for each class because all fare products sold share a homogenous seat capacity. This combination of price constrains and flexibility on the supply side makes quantity-based RM a useful tactic. In the case of apparel retailing firms must order quantities in advance of a sales season and may have certain stocking in each store. Often, it is difficult or costly reorder stock but, at the same time, it is easier change prices that requires only changing labels and data entries into a point-of-sale system. In these and other situations, when it is easier varying prices than units, it is usually more useful to use dynamic pricing. This does not mean that in these industries firms must use always the same options. Despite it is usually more recommendable, sometimes in certain cases it is better use the other tactic.

However, when it exists the choice between both tactics, there is an argument of that dynamic pricing is better option. The reason is that quantity-based RM rations the quantity sold to different products or segments of customers. However, these rationings involve reducing sales by limiting supply. If there is price-flexibility, however, we can reduce sales by increasing price. This achieves the same quantity-reducing function as rationing, but does it more profitably because by increasing price we both reduce sales and increase revenue at the same time. In practice, of course, firms rarely have the luxury of choosing price and quantity flexibility. Therefore, practical business constraints dictate which tactical response—price- or quantity based RM (or a mixture of both)—is most appropriate in any given business context [10].

3. MODELS

In this section, some basic models are shown used in Revenue Management.These models are divided between capacity control models (Quantity-based), dynamic pricing models (Price-based) and overbooking models.

3.1. CAPACITY CONTROL MODELS

The capacity control models are the models for quantity based Revenue Management. Although there are more models, it is only explained static models because they are better to understand how RM works.

Static models are models that works only with a single resource and makes the following assumptions:

- Demand for different classes arrives in different intervals.
- Demands for different classes are different random variables.
- Demand for a class does not depend of the capacity controls or the availability of the other classes.
- Units cannot be sold in group, they must be purchased individually or, if they are sold in group, firm can accept only some units and regret the others.
- It is assumed risk-neutrality because the aim is maximizing average revenue and firms usually make decisions for many products sold repeatedly.

Now we start with some of the models. The first and most basic model is the Littlewood´s Two class model that was explained in the introduction (chapter 1.2. History); then it will begin with the n-Class Model.

3.1.1. n-Class model

3.1.1.1. Explanation of the model

These models consider a more general case than Littlewood´s Two class model because there are considered n>2 classes that is more realistic. As in Littlewood´s Two class model we also assume that demand arrives in n stages in increasing order of their revenue values, i.e. from lowest price to highest price. Then we have $p_1>p_2>\ldots>p_n$ and firstly we sell at p_n price to the demand of class n, after at p_{n-1} for class $n-1$, …, until p_1 for class 1. We will index the class (or stage) by i.

This model formulates this problem as a dynamic program in the classes with remaining capacity x and discrete demand. In each class i, occurs the following sequence [13]:

1.- Realization of the demand D_i.

2.- Decide on a quantity of customers $u_i <= x_i$ to accept in this class (beginning in class n). u_i is function of i, x_i and D_i.

3.- Revenue $p_i u_i$ is collected and we start the process in the stage $i-1$ with $x_i - u_i$ available capacity.

This sequence does not represent the reality because the decision about u_i has to be made before observing all the demand D_i. but it is a good approximation because this assumption is not restrictive.

It is denoted the value function at the start of each stage i as $V_i(x_i)$. When D_i is met, the value u_i is chosen to maximize the revenue of the stage i plus the estimate revenue of next stages.

$$p_i u_i + V_{i-1}(x-u) \qquad \text{with } 0 <= u <= \min \{D_i, x_i\}$$

The value function $V_i(x_i)$ is the expected value of this optimization known as Bellman equation [2]:

$$\boldsymbol{V_i}(\boldsymbol{x}) = \boldsymbol{E}\left[\max_{0 \le u \le min\{D_i, x\}} \{\boldsymbol{p_i u} + \boldsymbol{V_{i-1}}(\boldsymbol{x}-\boldsymbol{u})\}\right] \qquad \text{with } V_0(x)=0,\ x=0, 1, \dots, C$$

The value u^* that maximize this function for each stage i is an optimal control policy for this model. And the expected marginal value of capacity (that represents the incremental value of the xth unit of capacity) at stage i would be:

$$\Delta \boldsymbol{V_i}(\boldsymbol{x}) \equiv \boldsymbol{V_i}(\boldsymbol{x}) - \boldsymbol{V_i}(\boldsymbol{x}-\mathbf{1})$$

The resulting optimal control can be expressed in terms of optimal protection levels (that reserve an amount of capacity classes from i to 1; see 2.1.1.1.):

$$\boldsymbol{y_i^*} \equiv \mathbf{max}\{\boldsymbol{x} : \boldsymbol{p_{i+1}} < \Delta \boldsymbol{V_i}(\boldsymbol{x})\} \qquad \text{with } \mathbf{i= 1, 2, \dots, n-1}$$

Then, the optimal control at stage **i+1** is:

$$\boldsymbol{u}^*(\boldsymbol{i}+\mathbf{1}, \boldsymbol{x}, \boldsymbol{D_{i+1}}) = \mathbf{min}\{(\boldsymbol{x} - \boldsymbol{y_i^*})^+, \boldsymbol{D_{i+1}}\}$$

Where $(\boldsymbol{x} - \boldsymbol{y_i^*})^+$ represents the remaining capacity in excess of the protection level, that is the maximum capacity we desire to sell to class **i+1**.

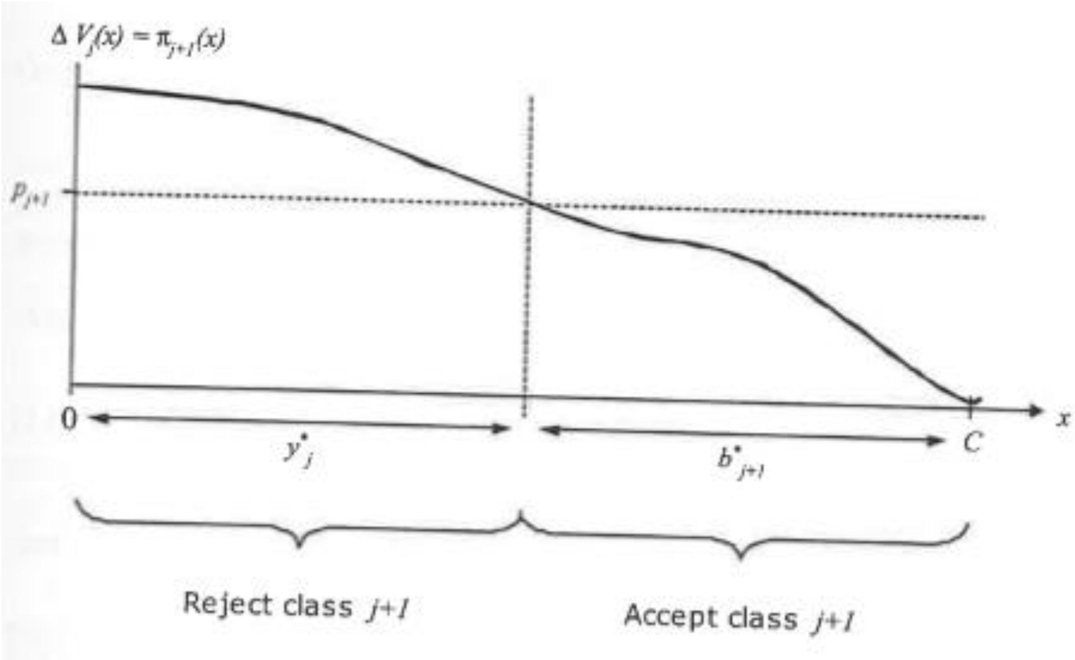


Figure 5: Optimal protection level y_i^* in the model [2]

As it is shown in the figure 4, class i+1 is accepted when $\Delta \boldsymbol{V_i}(\boldsymbol{x})$ is lower than p_{i+1}. When it is higher, it means that we must reject this offer because if we reserve it for the class i, revenue it will be higher.

In the picture, it is also shown the relation with the other two kinds of controls. Optimal nested booking limits are defined as:

$$\boldsymbol{b_i^*} \equiv \boldsymbol{C} - \boldsymbol{y_{i-1}^*} \qquad \text{with i = 2, …, n}$$

$b_1^* \equiv C$ and the booking limit control in stage i+1 would be:

$$\boldsymbol{u}^*(\boldsymbol{i}+\mathbf{1}, \boldsymbol{x}, \boldsymbol{D_{i+1}}) = \mathbf{min}\{\boldsymbol{b_{i+1}} - (\boldsymbol{c}-\boldsymbol{x})^+, \boldsymbol{D_{i+1}}\}$$

And we can define the bid price by:

$$\boldsymbol{\pi_{i+1}}(\boldsymbol{x}) \equiv \Delta \boldsymbol{V_i}(\boldsymbol{x})$$

Then the optimal control will be:

$$\boldsymbol{u}^*(\boldsymbol{i}+\mathbf{1}, \boldsymbol{x}, \boldsymbol{D_{i+1}}) = \begin{cases} \mathbf{0} & \boldsymbol{if}\ \boldsymbol{p_{i+1}} < \boldsymbol{\pi_{i+1}}(\boldsymbol{x}) \\ \boldsymbol{max}\{\boldsymbol{z}: \boldsymbol{p_{i+1}} \geq \boldsymbol{\pi_{i+1}}(\boldsymbol{x})\} & \boldsymbol{otherwise} \end{cases}$$

In words, it is that we accept class i+1 only if bid price is lower than the price of this rate.

3.1.1.2. Example

Now it is shown an example to understand better this model. This example is not useful in real life because data of demand are known and results are very intuitive but it is useful to understand how this model works.

It is supposed that for a single-leg flight with capacity $\boldsymbol{C = 100}$ there are four different rates. Those prices are $\boldsymbol{p_1 = 100}$€ ; $\boldsymbol{p_2 = 75}$€; $\boldsymbol{p_3 = 50}$€; $\boldsymbol{p_4 = 25}$€ . Firstly, it is sold at price $\boldsymbol{p_4}$ to the demand $\boldsymbol{D_4 = 60}$; after at $\boldsymbol{p_3}$ to the demand $\boldsymbol{D_3 = 45}$; then $\boldsymbol{p_2}$ to the demand $\boldsymbol{D_2 = 25}$; and finally, $\boldsymbol{p_1}$ to the demand $\boldsymbol{D_1 = 10}$.

Class (i)	Price (i)	Demand (i)
1	**100**	**10**
2	**75**	**25**
3	**50**	**45**
4	**25**	**60**

Table 7: Prices and demand data for each class

The objective is to maximize for each class i:

$$\boldsymbol{V_i(x) = p_i u_i + V_{i-1}(x_i - u_i) \qquad with\ 0 \le u_i \le \min\{D_i, x\}}$$

Where $\boldsymbol{u}$ is the optimal quantity to accept in this class.

The optimal $\boldsymbol{u}^*$ that maximize the function can be expressed as a protection level and we have:

$$\boldsymbol{y_i^* \equiv \max\{x : p_{i+1} < \Delta V_i(x_i)\} \quad where \ \ \Delta V_i(x_i) \equiv \ V_i(x_i) - V_i(x_i - 1)}$$

In cases where the demand is known, the optimal control at class $\boldsymbol{i + 1}$ would be:

$$\boldsymbol{u_i{}^*(i + 1, C, D_{i+1}) = \min\{(C - y_i^*)^+, D_{i+1}\}}$$

In spite of it is sold first at p_4 than the others, as the demand it is known, firstly, it is necessary calculate the protection levels beginning from $\boldsymbol{y_1^*}$**.**

It is logical that the protection level will be the same than the demand because is the class with higher revenues and there is enough capacity for all the demand.

$$y_1^* \equiv \max\{x : 75 < \Delta V_1(x)\} \quad where \;\; \Delta V_1(x) \equiv \; V_1(x) - V_1(x-1)$$

$$\Delta V_1(10) \equiv \; V_1(10) - V_1(9) = 10p_1 - 9p_1 = p_1 = 100$$

$$\Delta V_1(11) \equiv \; V_1(11) - V_1(10) = 10p_1 - 10p_1 = 0$$

Then, $y_1^* = u_1^* = 10$.

When the demand it is known, it is not necessary calculate the marginal revenues because if we try to sell the eleventh unit in class 1, the revenues will be 0 because it is not possible to sell more than demand. In cases where demand is unknown, it would depend of the probability of selling the eleventh unit. If $75 > p_1 P(D_1 \geq 11)$, then the protection level y_1^* would be also 10, but if $75 \leq p_1 P(D_1 \geq 11)$, then it is possible continue selling at p_1 until $75 > p_1 P(D_1 \geq y_1^* + 1)$.

Now it continues calculating the other booking limits:

The optimal control at class 2 would be:

$$u^*(2, C, D_2) = \min\{(C - y_1^*)^+, D_2\}$$

As $x - y_1^* = 90$ and $D_2 = 25$; then $u_2^* = 25$ and $y_2^* = y_1^* + u_2^* = 35$

The optimal control at class 3 would be:

$$u^*(3, C, D_3) = \min\{(C - y_2^*)^+, D_3\}$$

As $C - y_2^* = 65$ and $D_3 = 45$; then $u_3^* = 45$ and $y_3^* = y_2^* + u_3^* = 80$

The optimal control at class 4 would be:

$$u^*(4, C, D_4) = \min\{(C - y_3^*)^+, D_4\}$$

As $C - y_3^* = 20$ and $D_4 = 60$; then $u_4^* = 20$ and $y_4^* = y_3^* + u_4^* = 100$

Results:

CLASS (i)	u_i^*	y_i^*
1	10	10
2	25	35
3	45	80
4	20	100

Table 8: Results of the example 3.1.1.2.

With these results, we obtain:

$$\boldsymbol{V_1(10) = 100 * 10 + 0 = 1000}€.$$

$$\boldsymbol{V_2(35) = 75 * 25 + 1000 = 2875}€.$$

$$\boldsymbol{V_3(80) = 50 * 45 + 2875 = 5125}€.$$

$$\boldsymbol{V_4(100) = 25 * 20 + 5125 = 5625}€.$$

With the data that we have, the maxim revenue that we can obtain is 5625€. To get it, we must:

1- To sell 20 units of class 4 and to close class 4 when 20th unit is sold.
2- To open class 3 and sell 45 units. When 65th unit is sold, class 3 must be closed
3- To open class 2 and sell 25 units. When 90th unit is sold, class 2 must be closed
4- To open class 1 and sell the last 10 units.

3.1.2. Heuristic models

Most airline Revenue Management systems use heuristics to compute booking limits and protection levels in single-resource problems [2].

There are other routines for finding optimal controls exist but Heuristic models are very useful because they are easier to implement, quicker to run and generate satisfactory results [13].

Now we will explain in more detail the two most popular heuristics; the two versions of Expected Marginal Seat Revenue (EMSR-a and EMSR-b) that produce results very close to the optimal.

3.1.2.1. EMSR-a

EMSR-a is based on the idea of applying Littlewood's rule to successive pairs of classes and to add the protection levels.

Consider class j+1 with demand D_{j+1} and price p_{j+1}. We are interested in computing how much capacity reserve for the remaining classes (j, j-1, …,1). To do it we consider all the remaining classes as a single class k. Now we consider only the classes j+1 and k; and we apply Littlewood´s rule [14]:

$$\boldsymbol{P\left(D_k \geq y_k^{j+1}\right) = \frac{p_{j+1}}{p_k}}$$

We repeat the process from k=j to k=1 to obtain all the protection levels $\boldsymbol{y_k^{j+1}}$. After finding the protection levels for all future classes k, we use these individual protection levels to compute the protection level y_j in this way:

$$y_j = \sum_{k=1}^{j} y_k^{j+1}$$

After obtaining the first protection level, we repeat this process for each class j.

We can see that EMSR-a is very simple. That is a good point because it is easy to implement, but the problem with it is that is excessively conservative and, in some cases, produce protection levels that are not very close to the optimal.

The problem is because EMSR-a ignores the statistical averaging effect of aggregating demand across classes when only considering pairs of classes k, j+1 [13]. This difference increases when there are a large number of classes whose revenues are close together.

3.1.2.2. EMSR-b

EMSR-b is an alternative that avoids the problem of EMSR-a. It also reduces the problem to two classes at each class j. The difference is that instead of aggregating protection levels as in EMSR-a, demands are aggregated and all future classes are treated as one. EMSR-b find each protection level y_j with the following process [13]:

Consider class j+1 and define the aggregated future demand for classes j, j-1, …,1 by:

$$S_j = \sum_{k=1}^{j} D_k$$

And calculate the weighted-average revenue from classes 1, …, j defined by:

$$\bar{p}_j = \frac{\sum_{k=1}^{j} p_k E[D_k]}{\sum_{k=1}^{j} E[D_k]}$$

Now, the protection level y_j is determined by Littlewood's rule:

$$P(S_j > y_j) = \frac{p_{j+1}}{\bar{p}_j}$$

It is common to assume demand for each class j independent and normally distributed with mean μ and variance σ^2. Then [2]:

$$y_j = \mu + z_\alpha \sigma$$

Where $\mu = \sum_{k=1}^{j} \mu_k$; $\sigma^2 = \sum_{k=1}^{j} \sigma_k^2$; $z_\alpha = \Phi^{-1}(1 - p_{j+1}/\bar{p}_j)$

*$\Phi^{-1}(x)$ is the inverse of standard normal c. d. f.

Now, this process is repeated for each class j to obtain all the protection levels.

3.1.2.3. Example

In this example [2] there are four classes and demand is as normally distributed. In the table 6 it is shown the data of the demand and the protection levels obtained by EMSR-a, EMSR-b and the optimal policy for each class.

Class (j)	**P(j)**	$\boldsymbol{\mu(j)}$	$\boldsymbol{\sigma(j)}$
1	**1050**	**17,3**	**5,8**
2	**950**	**45,1**	**15,0**
3	**699**	**39,6**	**13,2**
4	**520**	**34,0**	**11,3**

Table 9: Demand data for each class [2]

Now it is only shown how are obtained the protection levels for class 3 (because the other protection levels follow the same process):

- EMSR-a:

Protection level class 3: $y_3 = \sum_{k=1}^{3} y_k^4 = y_1^4 + y_2^4 + y_3^4$

$$P(D_3 > y_3^4) = \frac{p_4}{p_3} = \frac{520}{699} = 0{,}744$$

$$P(D_3 > y_3^4) = 1 - P(D_k \leq y_3^4) = 0{,}744$$

$$P(D_3 \leq y_3^4) = 0{,}256$$

In the table of normal distribution, we can see that for this probability,

$$P(Z \leq z_0) = 0{,}744;\ z_0 = 0{,}656$$

Then, $P(Z \leq z_0) = 0{,}256;\ z_0 = -\,0{,}656$

$$z_0 = \frac{X - \mu_3}{\sigma_3} = \frac{X - 39{,}6}{13{,}2} = -0{,}656$$

$$X = -0{,}656 * 13{,}2 + 39{,}6 = 30{,}94$$

Then we have $X = y_3^4 = 30{,}94$

$$P(D_2 > y_2^4) = \frac{p_4}{p_2} = \frac{520}{950} = 0{,}547$$

$$P(D_2 > y_2^4) = 1 - P(D_2 \leq y_2^4) = 0{,}547$$

$$P(D_2 \leq y_2^4) = 0{,}453$$

In the table of normal distribution (Apendix), we can see that for this probability,

$$P(Z \leq z_0) = 0{,}547;\ z_0 = 0{,}12$$

Then, $P(Z \leq z_0) = 0{,}453;\ z_0 = -\,0{,}12$

$$z_0 = \frac{X - \mu_2}{\sigma_2} = \frac{X - 45{,}1}{15} = -0{,}12$$

$$X = -0{,}12 * 15 + 45{,}1 = 43{,}3$$

Then we have $X = y_2^4 = 43{,}3$

$$P(D_1 > y_1^4) = \frac{p_4}{p_1} = \frac{520}{1050} = 0{,}495$$

$$P(D_1 > y_1^4) = 1 - P(D_1 \leq y_1^4) = 0{,}495$$

$$P(D_1 \leq y_1^4) = 0{,}505$$

In the table of normal distribution, we can see that for this probability,

$$P(Z \leq z_0) = 0{,}505;\ z_0 = 0{,}013$$

Then

$$z_0 = \frac{X - \mu_1}{\sigma_1} = \frac{X - 17{,}3}{5{,}8} = 0{,}013$$

$$X = 0{,}013 * 5{,}8 + 17{,}3 = 17{,}37$$

Then we have $X = y_1^4 = 17{,}4$

Now we can obtain the protection level by EMSR-a for class 3:

$$y_3 = \sum_{k=1}^{3} y_k^4 = y_1^4 + y_2^4 + y_3^4 = 30{,}94 + 43{,}3 + 17{,}37 = 91{,}61$$

- EMSR-b:

The protection level for class 3 when demand is normally distributed, is determined by:

$$y_3 = \mu + z_\alpha \sigma$$

Where

$\mu = \sum_{k=1}^{3} \mu_k = \mu_1 + \mu_2 + \mu_3 = 17{,}3 + 45{,}1 + 39{,}6 = 102$

$\sigma^2 = \sum_{k=1}^{3} \sigma_k^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 5{,}8^2 + 15^2 + 13{,}2^2 = 432{,}88; \sigma = 20{,}8$

$z_\alpha = \Phi^{-1}(1 - p_{j+1}/\bar{p}_j)$; ($\Phi^{-1}(x)$ is the inverse of standard normal c. d. f.)

To obtain z_α we need know $\bar{p}_j$ that is determined by:

$$\bar{p}_3 = \frac{\sum_{k=1}^{3} p_k E[D_k]}{\sum_{k=1}^{j} E[D_k]} = \frac{p_1 E[D_1] + p_2 E[D_2] + p_3 E[D_3]}{E[D_1] + E[D_2] + E[D_3]}$$
$$= \frac{1050 * 17{,}3 + 950 * 45{,}1 + 699 * 39{,}6}{17{,}3 + 45{,}1 + 39{,}6} = \frac{18165 + 42845 + 27680}{102}$$
$$= \frac{88690}{102} = 869{,}5\$$$

Then, we have:

$$z_\alpha = \Phi^{-1}(1 - p_4/\bar{p}_3) = \Phi^{-1}(1 - 520/869{,}5) = \Phi^{-1}(0{,}402)$$

In the figure of Appendix 2 it is shown the cdf distribution. It is possible to take the answer from the graphic but it is more exact if it is used the table of normal distribution (Appendix 1).

To get the answer, it is necessary to search the data $1 - 0{,}402 = 0{,}598$ in the table (that is the opposite) or directly 0,402 in the graphic.

Then, it is obtained z_α:

$$z_\alpha = \Phi^{-1}(0{,}402) = -0{,}25$$

Now it is possible to obtain the protection level:

$$y_3 = \mu + z_\alpha \sigma = 102 - 0{,}25 * 20{,}8 = 96{,}8$$

To get the other protection levels it is used the same process.

After doing the same process for all classes we obtain the following protection levels:

Class(j)	Optimal value	EMSR-a	EMSR-b
1	9,7	9,8	9,8
2	54,0	50,4	53,2
3	98,2	91,6	96,8
4	C	C	C

Table 10: Results of heuristic models and comparison with the optimal policy [2]

When all the protection levels are obtained, it is possible to obtain the expected revenues for different capacities. Results from a simulation study [2] are shown in the table 8:

	OPT	EMSR-a		EMSR-b	
C	Rev.	Rev.	%Sub opt	Rev.	%Sub opt
80	67.512	67.462	0,07%	67.516	-0,01%
90	74.003	73.950	0,07%	74.000	0,00%
100	79.429	79.164	0,33%	79.426	0,00%
110	84.884	84.554	0,39%	84.862	0,03%
120	89.879	89.668	0,23%	89.875	0,00%
130	95.054	94.899	0,16%	95.045	0,01%
140	99.072	99.004	0,07%	99.068	0,00%
150	102.346	102.339	0,01%	102.346	0,00%

Table 11: Expected revenues obtained for different capacities and comparison with the optimal [2]

It is possible to observe that the difference between the heuristics (specially the EMSR-b) and the optimal policy is very little. That is the reason why they are very popular [2].

3.2. STATIC OVERBOOKING MODEL: THE BINOMIAL MODEL

Static overbooking models are used to make overbooking decisions. In static models, the dynamics of costumer cancellations and new reservation requests over time are ignored.

Static models use estimations of cancellation rates from the current time until the day of service to determine overbooking limit (maximum number of reservations to accept at the current time). This overbooking limit is recalculated periodically to reflect changing state and cancellation probabilities over time, resulting in overbooking limits that also vary over time.

The figure 6 shows how current overbooking limit gives the maximum number of reservations to accept at any time.

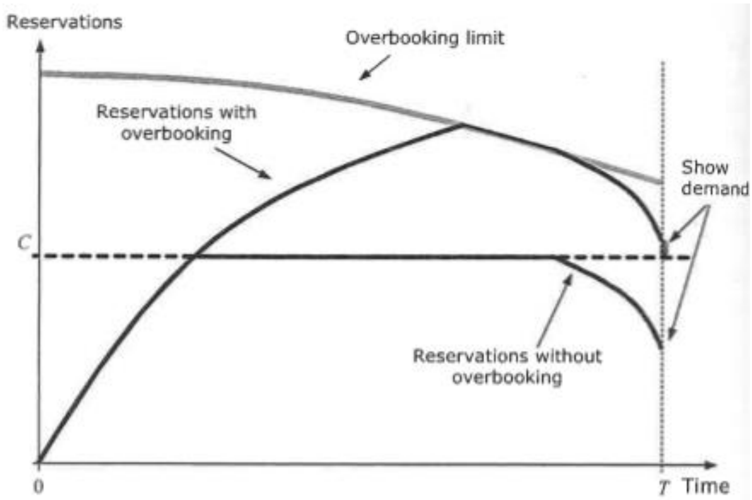


Figure 6: Overbooking limits and reservations over the time [18]

The overbooking decisions are affected by two events: **no-shows**, that is when a costumer does not show up at the time of service; and **cancellations**, that is when a costumer annuls his reservation before the time of service.

This explanation is based on [2]. The binomial model considers these two events (no-shows and cancellations) together and make the following assumptions:

- Customers cancelations are independent.
- All customers have the same probability of cancelling.
- Cancellation probability only depends on the time to the service and it is independent of the time when the reservation was made.

Under these assumptions, the number of customers who will show up when there are ***y*** reservations, is denoted ***Z(y)*** and is binomially distributed a probability mass function:

$$P_y(z) = P(Z(y) = z) = \binom{y}{z} q^z (1-q)^{y-z}, \qquad with\ z = 0, 1, \dots, y.$$

And with cumulate distribution function:

$$F_y(z) = P(Z(y) \leq z) = \sum_{k=0}^{z} \binom{y}{k} q^k (1-q)^{y-k}$$

Where ***q*** is the probability that a reservation currently on hand shows up at the time of service (1-q represents the probability of customers cancel the service before the time of service).

It is usual to work with the complement of the distribution F_y, defined by:

$$\bar{F}_y(z) = P(Z(y) > z) = 1 - F_y(z)$$

3.2.1. Overbooking based on Service-Level Criteria

We have two different level of service:

- ***Type 1 service level*** that is the probability of oversale at the time of the service.
- ***Type 2 service level*** that is the fraction of customers who must be denied service.

The overbooking limit is denoted ***x*** and firm accept reservations until ***y=x***.

Type 1 service level is given by:

$$s_1(x) = P(Z(x) > C) = \bar{F}_x(C) = 1 - F_x(C)$$

If the number of reservations ***y*** is equal than the overbooking limit ***x***, $s_1(x)$ represents the probability of having to deny the service to one or more customers.

Type 2 service level is defined by:

$$s_2(x) = \frac{E[(Z(x) - C)^+]}{E[Z(x)]} = \frac{\sum_{k=C+1}^{x}(k - C)P_x(k)}{x(1 - q)}$$

It can be simplified resulting:

$$s_2(x) = \bar{F}_{x-1}(C - 1) - \frac{C}{qx}\bar{F}_x(C) = 1 - F_{x-1}(C - 1) - \frac{C}{qx}(1 - F_x(C))$$

That is more useful for computations.

In practice, firstly it is specified a service level and then it is searched the largest booking level that satisfy this service.

Table 12 shows an example [2] with C=150 and q=0,85

X	$\overline{F}_{x-1}(C-1)$	$s_1(x)$	$s_2(x)$
165	**0,00979**	**0,00904**	**0,00012**
166	**0,01726**	**0,02603**	**0,00022**
167	**0,2883**	**0,02692**	**0,00039**
168	**0,4577**	**0,04294**	**0,00066**
169	**0,6935**	**0,06539**	**0,00107**
170	**0,10062**	**0,09533**	**0,00166**
171	**0,14025**	**0,13351**	**0,00247**
172	**0,18838**	**0,18015**	**0,00355**
173	**0,24449**	**0,23484**	**0,00494**
174	**0,30743**	**0,29654**	**0,00668**
175	**0,37549**	**0,36365**	**0,00879**

Table 12: Binomial approximation overbooking probabilities [2]

For this example, if we specify a service level of **0,1%** of customers to be denied service, the maximum of reservations we should accept is **x=168** because **s_2(168)=0,00066=0,066%** that is lower than the service level specified and **s_2(169)=0,00107=0,107%** that is higher than the service level specified.

3.2.2. Overbooking based on Economic criteria

There is another alternative to set overbooking limits that is based on using an economic criterion. It requires an estimation of the cost from not accepting additional reservations and other of the cost of denied service.

In this model, we define by ***z*** as the number of customers who show up on the day of the service and ***c(z)*** is the denied-service cost (it is assumed that ***c(z)*** is an increasing convex function). Then we have:

$$c(z) = h(z - C)^{+}$$

Where ***h*** is the constant marginal cost for each denied-service.

The model is more realistic under the assumption of the marginal cost increase when an additional customer is denied service because it is assumed the need to offer higher compensations. To get it, we use ***p*** that represents the marginal revenue generated by an additional booking; then the total revenue from ***y*** reservations is:

$$V(y) = py - E[c(Z(y))]$$

Where $Z(y)$ is the number of customers who show up on the day of service out of ***y*** reservations; and $E[c(Z(y))]$ the estimated denied-service costs.

As the function ***c(·)*** ins convex, then ***V(·)*** is concave. As a consequence, it is optimal to accept the yth reservation only if $\Delta V(y) = V(y) - V(y-1)$ is positive and regret it when it is negative. Then, the optimal booking limit x* is the highest x that satisfy:

$$\Delta V(x) = px - E\big[c\big(Z(x)\big)\big] - p(x-1) + E\big[c\big(Z(x-1)\big)\big] \geq 0$$

If we continue operating:

$$E\big[c\big(Z(x)\big)\big] - E\big[c\big(Z(x-1)\big)\big] \leq p$$

For the binomial model whit constant marginal costs, this is reduced to

$$hqP(Z(x-1) \geq C) \leq p$$

It means that the offer is accepted when the marginal denied-boarding penalty multiplied by the probability of having to denied the service is less than the price.

It also can be expressed as:

$$\bar{F}_{x-1}(C-1) \leq \frac{p}{hq}$$

or

$$1 - F_{x-1}(C-1) \leq \frac{p}{hq}$$

In the same example used for the service-level criteria where C=150; q=0,85; h=500$; p=100$ (the economic data are only in this example because they were not necessary in the for the service-level criteria). For this data, ***p/hq = 0,235***. In table 12 is shown that the optimal booking limit would be x* = 172 because is the largest value for x that $\boldsymbol{\bar{F}_{x-1}(C-1) < 0,235}$.

For the Overbooking based in economic criteria it is important made good estimations of the marginal cost h. It is possible to obtain an imputed cost h from a booking level x* according to Type 2 service levels.

It is used the equal case of the formula obtained before and it results:

$$h = \frac{p}{q\bar{F}_{x-1}(C-1)}$$

In the same example used during the explanation for the service-level criteria, it was obtained that for no more than ***0,1% ($s_2(y) \leq 0,001$)*** of customers denied service, optimal overbooking limit was ***x*=168***. Then, $\bar{F}_{x-1}(168-1) = 0{,}02883$ (Figure 7) and, as p=100$ and q=0,85; it is possible to obtain the imputed cost of denied service:

$$h = \frac{100}{0{,}85 * 0{,}2883} = 408\$$$

3.3. DYNAMIC PRICING MODEL

The model that we will explain in this section works under the following factors that we commented in chapter 2.2.1.:

- Sophistication level of customers: Myopic customers.
- Population size: Infinite population.
- Market conditions: Monopoly.

Working under these factors is not useful for a real-life problem but it is easier to understand dynamic-pricing working under these assumptions than in a real-life problem.

3.3.1. *Model´s enunciate*

This model works with the initial inventory s_0 that is the maximal quantity that can be sold by time T. It is assumed that demand appears at times 1, 2, …, T and x_t is de demand at time t. The price of the item at time t is p_t that is function of the demand and the time:

$$p_t = p(x_t, t)$$

We assume that there is one-to-one relationship between demand and price at any time. Then, when the price is fixed, the demand follows this function:

$$x_t = x(p_t, t)$$

We also assume that demand and price are inversely proportional; when the price is lower, the demand is higher and when the price tend to the maximal, demand tends to zero.

Under these premises, we can express the problem:

- Objective Function:

$$\text{Maximize } \sum_{t=1}^{T} p_t x_t$$

- Subject to the restrictions:

$$\sum_{t=1}^{T} x_t \le s_0$$

$$x_t \ge 0 \ \ for \ \ t \in \{1, 2, \dots, T\}$$

$$x_t \le x(p_t^{min}, t) \ \ for \ \ t \in \{1, 2, \dots, T\}$$

The objective function is to maximize the total revenue. The first restriction is obvious because the total demand at T cannot be higher than the initial inventory. The second restriction is only to make sure that demand is never negative. And the third restriction is to limit the demand with its highest value at any time.

3.3.2. Solving the problem

The resolution of this problem is based on [15]:

To solve this problem, we use the Kuhn and Tucker approach based on Lagrange multipliers. As we said before, ***p_t*** is function of ***x_t***, then we can consider ***$p_t x_t$*** as a function of ***x_t***. Knowing it, the Lagrangian will be:

$$L(x_1, \dots, x_T, \lambda, \mu_1, \dots, \mu_T, l_1, \dots, l_T) = \\ = \sum_{t=1}^{T} p(x_t, t) x_t - \lambda(\sum_{t=1}^{T} x_t - s_0) + \sum_{t=1}^{T} \mu_t x_t \ - \sum_{t=1}^{T} l_t (x_t - x\left(p_t^{min}, t\right))$$

The aim is to solve the T equations:

$\frac{\partial L}{\partial x_t} = 0 \ \ for \ \ t \in \{1, 2, \dots, T\}$9

Under the following conditions:

$$\lambda(\sum_{t=1}^{T} x_t - s_0) = 0$$

$$for\ t \in \{1,2,\dots,T\} \begin{cases} \mu_t x_t = 0 \\ l_t\left(x_t - x\left(p_t^{min}, t\right)\right) = 0 \end{cases}$$

Now we have 3T+1 equations and 3T+1 unknowns $(x_1,\dots,x_T,\lambda,\mu_1,\dots,\mu_T,l_1,\dots,l_T)$

Only is possible to get a solution of the system of equations if $\lambda \geq 0$; $\mu_T \geq 0$; $l_T \geq 0$ and under the restrictions that we commented when we express the problem before.

In the last 2 conditions, we can appreciate:

$\lambda = 0$ or/and $\sum_{t=1}^{T} x_t = s_0$

$\mu_t = 0$ or/and $x_t = 0$ for $t \in \{1,2,\dots,T\}$

$l_t = 0$ or/and $x_t = x\left(p_t^{min}, t\right)$ for $t \in \{1,2,\dots,T\}$

3.3.3. <u>Example</u>

Now it is shown a resolution when we have the price function.

We consider the price function:

$$p(x_t, t) = (A - Bx_t)\frac{D}{D-t}$$

Where A, B and D are positive constants.

Then, it is truth that:

$$x(p_t, t) = \frac{1}{B}(A - p_t\frac{D-t}{D})$$

We can make two affirmations:

- Price is a increasing function of ***t*** (when ***t*** is higher, price is lower).
- Demand will be always lower than $\frac{A}{B}$ because if it was higher, it means that cost would be negative.

Now we must solve the following problem:

Objective function:

$$Maximize \sum_{t=1}^{T} x_t(A - Bx_t)\frac{D}{D-t}$$

Subject to the restrictions:

$$\sum_{t=1}^{T} x_t \leq s_0$$

$$x_t \geq 0 \ \ for \ \ t \in \{1, 2, \ldots, T\}$$

$$x_t \leq \frac{A}{B} \ \ for \ \ t \in \{1, 2, \ldots, T\}$$

$$D > T$$

Now that we have the objective function and the restrictions, we can determine the Langrangian:

$$L(x_1, \ldots, x_T, \lambda, \mu_1, \ldots, \mu_T, l_1, \ldots, l_T) =$$
$$= \sum_{t=1}^{T}(Ax_t - Bx_t^2)\frac{D}{D-t} - \lambda(\sum_{t=1}^{T} x_t - s_0) + \sum_{t=1}^{T} \mu_t x_t \ - \sum_{t=1}^{T} l_t(x_t - \frac{A}{B})$$

Now, the aim will be to solve these T equations:

$$(A - 2Bx_t)\frac{D}{D-t} - \lambda + \mu_t - l_t = 0 \quad for\, t \in \{1, 2, \ldots, T\}$$

Under the following conditions:

$$\lambda(\sum_{t=1}^{T} x_t - s_0) = 0$$

$$for\, t \in \{1, 2, \ldots, T\} \begin{cases} \mu_t x_t = 0 \\ l_t\left(x_t - \dfrac{A}{B}\right) = 0 \end{cases}$$

Now we have three cases $for\, each\, t \in \{1, 2, \ldots, T\}$:

1. $x_t = 0$; then

   $l_t = 0$ then $\lambda = \frac{AD}{D+t} + \mu_t > 0$ and $\sum_{t=1}^{T} x_t = s_0$

2. $x_t = \frac{A}{B}$; then

   $\mu_t = 0$ and $-\frac{AD}{D-t} = \lambda + l_t$. That is not possible because the left-hand side is always negative and the other side is at least zero.

3. $x_t \in (0, \frac{A}{B})$; then

   $\mu_t = l_t = 0$ and $(A - 2Bx_t)\frac{D}{D-t} = \lambda$

Finally, we assume that $x_t \in (0, \frac{A}{B})$ Where $\mu_t = l_t = 0$ and $(A - 2Bx_t)\frac{D}{D-t} = \lambda$

Consequently, $x_t \in (0, \frac{A}{2B}]$ and $x_t = \frac{1}{2B}(A - \lambda\frac{D-t}{D})$

Now we have to consider two cases:

1. $x_t = \frac{A}{2B}$; then

   $(A - 2Bx_t)\frac{D}{D-t} = \lambda = 0$ and $\sum_{t=1}^{T} x_t \leq s_0$

2. $x_t \in (0, \frac{A}{2B})$; then

   $\lambda > 0$ and $\sum_{t=1}^{T} x_t = s_0$

We consider $Y = \{t | t \in \{1, 2, ..., T\}, x_t > 0\}$ where N_y is the number of elements of Y.

The first case is only possible if $T\frac{A}{2B} \leq s_0$

For the second case, if $N_y\frac{A}{2B} \geq s_0$ then $\sum_{t=1}^{T} x_t = s_0$

As $x_t = \frac{1}{2B}(A - \lambda\frac{D-t}{D})$ and $\sum_{t=1}^{T} x_t = s_0$ becomes:

$\sum_{t \in Y} \frac{1}{2B}\left(A - \lambda\frac{D-t}{D}\right) = s_0$ and $\lambda = \frac{N_y AD - 2BDs_0}{N_y D - \sum_{t \in Y} t}$

Finally, we substitute λ in $x_t = \frac{1}{2B}(A - \lambda\frac{D-t}{D})$ and we obtain:

$$x_t = \frac{1}{2B}(A - \frac{N_y AD - 2BDs_0}{N_y D - \sum_{t \in Y} t}\frac{D-t}{D})$$

- Numerical example:

We suppose that for the same function, A=200; B=10; D=20; the initial inventory level is 150 and there are T=10 different classes.

The objective function would be:

$$Maximize \sum_{t=1}^{T} x_t(200-10x_t)\frac{20}{20-t}$$

After determining the Langranian, the aim will be to solve these 10 equations:

$$(200-20x_t)\frac{20}{20-t} - \lambda + \mu_t - l_t = 0 \quad for\ t \in \{1,2,\dots,10\}$$

Under the following conditions:

$$\lambda(\sum_{t=1}^{T} x_t - s_0) = 0$$

$$for\ t \in \{1,2,\dots,10\} \begin{cases} \mu_t x_t = 0 \\ l_t\left(x_t - \frac{200}{10}\right) = 0 \end{cases}$$

It is assumed that $x_t \in (0,20\,)$ Where $\mu_t = l_t = 0$ and $(200-20x_t)\frac{20}{20-t} = \lambda$

Consequently, $x_t \in (0,10\,]$ and $x_t = \frac{1}{20}(200 - \lambda\frac{20-t}{20})$

As $T\frac{A}{2B} \leq s_0$ because $10\frac{200}{20} \leq 150$ then, the demand for each class is $\sum_{t=1}^{T} x_t = \frac{A}{2B} = \frac{200}{20} = 10.$

And prices are determined by the following function:

$$p(x_t,t) = (A-Bx_t)\frac{D}{D+t} = (200-10x_t)\frac{20}{20-t}$$

Results obtained are shown in the table 13:

Class	1	2	3	4	5	6	7	8	9	10
Demand	10	10	10	10	10	10	10	10	10	10
Price	105,26	111,11	117,65	125	133,33	142,86	153,84	166,67	181,82	200

Table 13: Results of the first numerical example 3.3.3.

Total demand = 100
Revenue = 14.375,4

In other example, it is supposed that for the same function, A=700; B=5; D=20; the initial inventory level is 150 and there are T=10 different classes.

The problem is solved in the same way until the moment when we determine the demand because in this case, $T\frac{A}{2B} \leq s_0$ is not truth because $10\frac{700}{10} > 150$.

Then, demand is obtained as follows:

$$x_t = \frac{1}{2B}\left(A - \frac{N_y AD - 2BDs_0}{N_y D - \sum_{t\in Y} t}\frac{D-t}{D}\right) = \frac{1}{10}(700 - \frac{N_y 14.000 - 200s_0}{N_y 20 - \sum_{t\in Y} t}\frac{20-t}{20})$$

In this case, $N_y = 9$ because for $N_y = 10$; demand for class 1 would be negative

And prices are determined by:

$$p(x_t, t) = (A - Bx_t)\frac{D}{D+t} = (700 - 5x_t)\frac{20}{20-t}$$

Table 14 shows the results obtained:

Class	1	2	3	4	5	6	7	8	9	10
Dem	2,44	5,99	9,55	13,09	16,63	20,19	23,75	27,29	30,93	0
Price	724	744,25	767,35	793,19	822,2	855,79	894,23	939,25	991,55	0

Table 14: Results of the second numerical example 3.3.3.

Total demand = 149,87 ≈ 150
Revenue = 132.425,99

4. USES IN REAL WORLD

In this chapter are shown some of the industries that can use Revenue Management because in these industries are the required market conditions to implement RM. Moreover, some programs used by Revenue managers in the hotel industry are exposed.

Airlines are not mentioned in this chapter because they are explained during the other chapters.

4.1. Hotels

The hotel industry is one of the industries where Revenue Management is more established. Hotels can be categorized in function of their size, their location or the kind of clients that they usually have.

As it was commented in chapter 1.3. Revenue Management was very successful when it was applying for the rooms´ location. This success brought the consequence of the application of Revenue Management in other service of the hotel.

Next table shows why it is very convenient to apply Revenue Management in hotel industry:

Relatively fixed capacity	**In hotels, there are a limited number of rooms and it is not possible to increment this number in a short-period.**
Perishable inventory	**Rooms those are available for one day cannot be stored for consuming later when the hotel is full.**
Inventoried demand	**Customers can make bookings for a room in the future. These bookings allow the company to vary prices when predicted demand is low or high.**
Stochastic demand	**Demand is not constant. Demand is not the same during holidays, that customers are usually families or friends than a week day that usually are businessmen.**
High fixed costs	**Majority of the costs in a hotel are the cost of the infrastructures that not depend of the demand.**
Low variable costs	**The cost that suppose another customer are not very high because the personal that work to offer the services is already contracted.**
Customer heterogeneity	**Customers in hotel industry are very different. A businessman who travel to work, has other preferences than a family that are traveling. Different customers are willing to pay different prices for different services.**

Table 15: How hotels satisfy Revenue Management favourable conditions

Hotels offer different rates in function of the quantity of demand. For example, during summer, hotels in the coast, offer higher rates than during the winter because demand is higher. Moreover, the offer different rates in function of the services that the customer is willing to pay. Hotels usually offer 1,2 or 3 people rooms that the price for each person is lower when the room has more capacity; or they offer the possibility of having breakfast included that suppose an extra-cost.

Other practice, it is to offer discounts for reservations that are made well in advance, because it allows the hotel the capacity of knowing the future demand. Often, when hotel has not many bookings for a period, the offer discounted-rates for this period to attract customers because each room that is available means lower revenues.

4.2. Rental car

Companies of this industry offer cars to the costumers for a temporally use. Customers can rent a car only for a day or for a long time. In next table, it is shown how car rental companies satisfy RM conditions and then some special characteristics of this industry.

Relatively fixed capacity	**Each company has a limited number of available cars of each class and these number cannot be reduced or incremented in a short-time period.**
Perishable inventory	**If a company has 20 cars and one day it has 5 available cars, the company cannot store these cars for rent those a day when the 20 cars are rented.**
Inventoried demand	**Customers can make bookings for renting a car in future dates. These bookings allow the company to vary prices when predicted demand is low or high.**
Stochastic demand	**Demand is not constant. Demand is not the same during holidays, that customers are usually families or friends than a week day that usually are businessmen.**
High fixed costs	**Majority of the costs in a rental car company are the costs of buying the cars and infrastructures that not depend of the demand of cars.**
Low variable costs	**The cost that suppose renting another car are not very high. They usually are some manage costs and the maintenance of the car.**
Customer heterogeneity	**Customers of rental car company are very different. A businessman that needs it to work, has other preferences than a family that wants the car for a week trip.**

Table 16: How Rental car companies satisfy R M favourable conditions

Now are shown some particularities of car rental industry [32]:

- Capacity in a rental car company is usually more flexible than other of the industries commented. Big companies frequently have different locations where they operate. It allows companies to vary the number of cars that they have in each location then they have more flexibility to adapt he capacity with the demand.

- Sometimes customers rent a car at one location and drop off at another. Company must control it because the capacity at each location depends of where customers take and drop off the cars.

- The price of each product depends of different factors: car type (class of the car), insurance options (rental car companies offer different insurance for their customers), pick up and drop off location, length of the rental and age or experience of the customer.

- The competence is high because all rental car companies are usually sited together at airports or train stations. A last-minute customer often has many alternatives to choose. This competence is a difference between rental car and other industries as airlines where a last-minute customer has not many options and he is willing to pay a higher price.

- Some companies and travel agencies negotiate special rates with car rental companies because they take a big volume of products.

4.3. Manufacturing

Manufacturing consists in the production of goods for their distribution and consume. Revenue Management is not very popular in Manufacturing industry because it not satisfies all the favourable conditions for applying RM. Nonetheless, some companies are developing technology systems to offer pricing optimization.

Manufacturing is a very big sector with a diverse set of firms and products. We can make a division in two ways of producing [23]:

- Make-to-stock: It consists in producing standardized products and usually in large volumes based on the forecast of future demand. The aim is to balance the need to satisfy the demand that is often variable and uncertain against costs of production and inventory.
  Dynamic pricing is not typically used in this kind of industry. However, it is common to offer discount prices when a product is in its end-of-life-cycle.

- Make-to-order: Other manufacturer companies produces smaller lots based on the orders from the customers. This way of works is typically used when they do not produce a good for consume and their customers are other manufacturers. Price decisions are made based on tactical factors as the estimated costs, but also on strategic factors, such as the value of the relationship with the customer.

Both kinds of companies use some pricing tactics offering discounts for the volume or offering some special changes in the product. For example, automobilism companies offer the same car with different extras that increase the price when a customer is willing to pay more for more facilities.

Revenue Management in manufacturing is different than in service industries because [31]:

- The inventory is less perishable because usually items can be stored for future demand.
- Capacity is fixed but how you can store and have inventory for a future demand, it is possible to produce more when the demand is low for using this production when the demand is higher than the capacity.
- Although fixed costs are often high they are usually less (in percentage) than the fixed costs in the other industries, variable costs are higher because of the material costs.
- In some manufacturing industries, there are customer heterogeneity but usually customers identify each company with the willing to pay of the customers because the name of the company is symbol of quality. For example, in automobilism industry, a Ferrari is only focused in a small part of the market that is willing to pay high prices and they are not interested in selling products to other kinds of customers.

4.4. Sport centres

Sport centres and gyms offer a place to do different sports where there are the necessary infrastructures to practise the different sports. In sport centres, it is possible to apply Revenue Management in spite of the RM favourable conditions are not strictly satisfied.

Relatively fixed capacity	**The infrastructures have a limited capacity. If there are only two tennis fields, it is not possible play more than two match at the same time.**
Perishable inventory	**Each day that the capacity is not completely used, it means a waste of money because other customers could be using it.**
Inventoried demand	**Demand is not strictly inventoried because usually customers pay a month rate and they can enjoy the centre whenever they want. Then, you only know that they can go to the centre but not exactly when.**
Stochastic demand	**There are some hours of the day when the demand is higher and others that demand is very low.**
High fixed costs	**Almost all cost come from the infrastructures and the personal that not depend of the customer**
Low variable costs	**Costs for a new customer are practically insignificant.**
Customer heterogeneity	**Demand is completely heterogeneity. In a sport centres, there are every kind of people, from children to ancients.**

Table 17: How sport centres satisfy R M favourable conditions

Sport centres usually offer different rates that restrict more or less the use of the infrastructures to face the problems of stochastic and inventoried demand. [28] Lower rates restrict the use of some parts of the infrastructures that are more demanded or they only cannot use the centre all the time (for example, only in the mornings). These different rates allow sport centres to offer low prices when the demand is low and higher prices when the purchased demand is high.

Sometimes, sport centres allow customers who pay lower-rates to use some of the service that they cannot whit their rate, paying extra-money for each use. In this way, when demand is supposed higher but it is not high, the sport centre does not lose customers who are usually willing to pay less.

4.5. Electricity generation and transmission

This sector is probably one of the first users of Revenue Management because it is motivated by the different ways to generate electricity and the difficulties to store the electricity [26]. Some countries as France usually used nuclear energy because of its high fixed costs and low variable costs. When this energy was not enough they used hydroelectricity energy; next step would be use thermal energy but only when it is necessary because of its higher variable costs.

For avoiding the use of high-variable costs energies, it is common to offer lower rates for hours when demand is lower (for example at nights). In this way, energy distributors get that discretional demand use electricity at this hours and they maximize their revenues because they avoid using the more expensive electricity sources.

Next table shows how electricity distributors satisfies the RM favourable conditions.

Relatively fixed capacity	**Electricity production capacity is not flexible because the infrastructures have a limited capacity and use other infrastructures is usually expensive.**
Perishable inventory	**Electricity is difficult to store or very expensive. To store electricity is not useful for the electricity distributors.**
Inventoried demand	**Customers usually contract the power that they need. The time that they use this power is variable but companies have data about consume and they can estimate the future consume.**
Stochastic demand	**Demand is higher during the day when factories are working than during nights. And in winter is higher because there is less solar light.**
High fixed costs	**Fixed costs are high in the most used electricity production ways because of the costs of the production and the distribution infrastructures.**
Low variable costs	**Variable costs are low in the most common ways to produce electricity as hydroelectric and nuclear energy or removable energies.**
Customer heterogeneity	**Variety of costumers is very high because electricity is used from a big factory to a one-person home. Then, their use is very different.**

Table 18: How electricity companies satisfy R M favourable conditions

4.6. Online travel portals

During the 21st century these portals have been very successful because they offer a great variety of accommodations with different prices that varying according to the quality and the dates. Customers usually introduce the place and the dates those are convenient for them and the portal offer different accommodations in hotels or apartments. Some of these portals also offer transports and sometimes some activities for their customers.

"In an industry like online travel with such wild competition and volatile demand, the importance of Revenue Management is infinite. By developing, interpreting, and implementing complex pricing and inventory management actions we manage to compete effectively and maximize potential revenues.

Travel corporations are increasingly being forced to compete on price. Price intelligence reports is a tool which allows us to stay on top of our competition, ensure that our prices are being optimized on a daily basis to reflect market changes and variable demand, and accurately estimate hotel traffic for future dates."

Nick Tsimbidaros

In the table 19, it is shown how online travel portals satisfy the RM favourable conditions.

Relatively fixed capacity	**As hotels or airlines, capacity is fixed because they have a limited number of accommodations and it is fixed.**
Perishable inventory	**They can´t store seats or accommodations those are available one day to sell it when the capacity is full. They lose this available product.**
Inventoried demand	**As in other industries that use revenue management, customers make reservations in advance of the use of these products.**
Stochastic demand	**Demand is higher during weekends and some seasons as summer or holidays.**
High fixed costs	**All costs are practically fixed because they are the costs of the software, the website and some contracts. The variable costs usually are only a percentage of each sold that is a payment for the company that offer the service as a hotel for example.**
Low variable costs	
Customer heterogeneity	**Demand is completely heterogeneity. Travel portals are used by both by a businessman and by a student or a family.**

Table 19: How online travel portals satisfy R M favourable conditions

Pricing will be set according to the market trend, prices will increase as capacity decreases and vice versa, in such a dynamic way that Revenue obtained will be maximum [26]. Travel portals offer fewer rates when the demand is lower to attract customers who are not willing to pay much. Moreover, they offer discounts when a booking is made long time before the consume.

4.7. Restaurants

Many restaurants vary their price structure under certain circumstances. Those price changes generally involve specific promotions, set menus, … An example of this promotions is the happy hour that consists in make discounts usually during the hours when demand is usually fewer [29] or the offer of a daily menu for lunch usually offering a two courses menu (whit smaller portions) with a more economic rate than the regular menu.

As the other industries that use Revenue Management, restaurants also satisfy RM favourable conditions:

Relatively fixed capacity	**The capacity in a restaurant is limited because of the number of seats those can be bussy at the same time.**
Perishable inventory	**Seats that has not been used during the lunch cannot be stored for the dinner when the restaurant is full.**
Inventoried demand	**Usually some restaurants offer the possibility of booking in advance although reservations are usually made a few days before or the same day.**
Stochastic demand	**Demand depends of the day of the week and the season. People usually go to restaurants more in weekend than week-days.**
High fixed costs	**Infrastructure and personal are a great part of the costs in a restaurant. A more customer basically only suppose the raw material that he will consume.**
Low variable costs	
Customer heterogeneity	**People who go to restaurants are very heterogenic. Customers can be families or groups of students, or can be groups of businessmen who are willing to pay more.**

Table 20: How restaurants satisfy R M favourable conditions

A difference between restaurants and the other industries is that is not common that customers pay in advance of the service. Sometimes customers who make a reservation don´t go to the restaurant and it can´t be a waste of money by the restaurant. For this reason, some restaurants that are usually full don´t accept bookings. The other significant difference with the others is that restaurants usually do not use a sophisticated method to collect data and make decisions; they often do it in an easily way because the volume of products that they offer is lower than in the other industries.

4.8. Passenger railways

The application of Revenue Management in this industry is impulse because of the preoccupation of using a high-costed infrastructure that has a low opportunity cost (rails from point A to point B cannot be changed if they are not useful) and the very few marginal costs for an additional passenger [26].

Some of the long-distance systems has a little occupation during some seasons during the year. That is the reason why is necessary to offer some special rates during the little occupation seasons to equilibrate the demand during the year. In this industry is not as important as in airlines the anticipation of some months to make reservations; it is only important when they sell a packet of seats for tour operators.

Characteristics between passenger railways and airlines are very similar then it is not necessary to show how passenger railways satisfy the RM favourable conditions.

4.9. Theatres and Sport events

To practice Revenue Management in theatres and sport events is very convenient because satisfy all the conditions to apply RM. This satisfaction of the conditions it is shown in the table

Relatively fixed capacity	**Theatres and sport stadiums have a limited number of sites that usually cannot be incremented or reduced.**
Perishable inventory	**Seats that are available for an actuation or a match cannot be sold in the future when all the locations are full.**
Inventoried demand	**Tickets are usually sold some time before the day of the event in the ticket office or in websites.**
Stochastic demand	**Demand is usually higher on weekends than during the week. Moreover, some actuations and matches are more attractive than others**
High fixed costs	**Almost every cost is fixed: the infrastructures, salaries of actors/sportmen and personal, …**
Low variable costs	**The increment in costs for another customer is insignificant**
Customer heterogeneity	**Customers are very different because people who like theatre and sports are not often willing to pay same prices.**

Table 21: How satisfy Theatres and sport events RM conditions

- We can see that satisfy all de conditions. In theatres and sport stadiums are offered different rates that can differ too much. This difference in prices usually depends of the location of the seat or the importance of the event. The locations that are more comfortable or have better vies are more expensive than the less-privilege seats. And, for example, customers are not willing to pay the same price for a ticket to watch the Champions League final match than a match of the second league.

- One special characteristic of this industry is that usually offers some season tickets that allows the customer to have a seat for all the events of the season [2]. Another modality is to sell some tickets that allows the customer to buy tickets at a lower price. The revenues of these kinds of tickets are lower than regular tickets but the benefits of these tickets give companies a fixed revenue that does not depend of the future demand.

4.10. Freight

Freight industry is the responsible of transporting objects or products from one place to another. It could be made by a variety modes of transports (airplanes, trucks, trains or ships). During the 90s Campbell and Morlok investigate the applicability of revenue management techniques to freight [30].

The difference between Freight and others of the industries is the variable capacity that allows reduce or increment the capacity. For example, in rail freight you can increase or reduce the number of wagons depending on the needs.

Freight companies usually offer different rates in function of the volume of products that a customer want to transport. Also, they offer better prices when customers notify in a good time than when they notify the same day of the travel. It is because that allows freight companies to store demand and to plan the transports to reduce the non-used places.

Demand is usually heterogenic because they transport every kind of products and materials with different sizes and conditions to travel. Moreover, demand is also stochastic because there are some seasons when factories reduce their work that demand is also reduced.

As the other industries that are able to Revenue Management, in freight industry fixed costs are high while the cost that suppose transport one more product from one point to another it is usually insignificant when it is necessary to do this trip because there are other products to transport.

4.11. Tour operators

Tour operators share the same Revenue Management problems than hotels and airlines because they sell packages of air/ground travel and lodgement. Usually they contract some amount of the capacity of the transport and the lodgement from third-party suppliers.

In the table 22, it is shown how satisfy the RM favourable conditions.

Relatively fixed capacity	**For each package, they have booked an amount of the capacity in transports and hotels that is fix and reduce or increment the amount means extra cost.**
Perishable inventory	**Each package has its own dates and it is not possible to sell more units of a package after the date of this service.**
Inventoried demand	**Customers reserve the packages before the date of the service**
Stochastic demand	**Demand is higher during holidays or when the weather is good for this activity than in other cases**
High fixed costs	**Usually tour operators pay for the amount of capacity that they contract independently of the customers that they have.**
Low variable costs	**Tour operators often pay each place independently of if they use all the capacity or not. Then, costs for a new customer is low.**
Customer heterogeneity	**The variety of customers is very high because works with groups as schools or families and also with particulars. Each customer is willing to pay different rates and to enjoy different services.**

Table 22: How travel operators satisfy R M favourable conditions

The fact of contract some amount of the capacity allows them the capacity to negotiate competitive prices. Then the challenge for tour operators is to manage flexible capacity and the different options offered with different costs [2].

Firstly, tour operators set the packages that they will sell that includes transport and lodgement; and in some cases, they also offer other service as activities or tourism visits.

After setting the packages they must establish the capacity for each package and set the prices for each package. As we commented before, the prices will depend of the quantity because marginal cost also depend of quantity.

Finally tour operators offer some discounts and promotions during the booking period to stimulate the demand.

4.12. Programs used in Hotel industry

Revenue managers use some programs to apply the Revenue Management in their companies. Some of the programs used in hotel industry are the followings:

4.12.1. STR

STR or STAR Program is used in the global hotel industry as a vital revenue management tool.

This program compare your occupation and rates data with the data of your competitors. It tracks and delivers monthly, weekly and daily data [36].

4.12.2. *IDeaS*

IDeaS Revenue Management System is another tool used by Revenue Managers in the Spanish hotel industry. It is used by different companies as Petit Palace to predict the occupation and stablish the rates depending on different data (arriving date, duration of the reservation process, duration of the stay, …) [37].

4.12.3. *HotelsDot*

HotelsDot is not a program used by companies. It is a company specialized in hotel Revenue Management that do it externally. When a hotel contracts their services, HotelsDot works as the Revenue Management department of the hotel. [38]

It is very useful to independent hotels or small hotel companies those have not a Revenue Management department.

4.12.4. *Modern Revenue*

Modern Revenue is a simply and integral software that optimize the management process in hotels; providing the optimal price of their rooms for each moment.

Modern Revenue works using some algorithms to analyse all the important data (occupation, historic, competence, demand, …) to provides a correct sales prevision. It provides the best rates to increase the marginal revenues per room. [38]

5. CONCLUSIONS

This thesis is born as a result of the investigation and study about the concept of Revenue Management that is a relatively new way to carry out the management in different industries.

The success of the application of Revenue Management could be observed in the airline industry that is where Revenue Management is born during the 70s. As a consequence of this success, other industries begin to apply RM during the 90s and the beginning of the 21st century.

The objective of this thesis is to provide an introduction to the Revenue Management because as it is commented, it has been very successful in the industries where it has been implemented.

Moreover, it is a very interesting concept because it is in developing process, especially in the industries where the concept is relatively new. I these industries there are much work to do behind this concept because although in other industries as airlines it is very developed, in each industry, Revenue Management work in a different way, with different models. In fact, different companies of the same industry, work in a different way depending on company´s priorities.

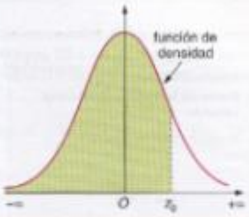
For these reasons, *“Principles of Revenue-Management and their applications*” is a good starting point to learn about this concept.

6. APPENDIX:

6.1. Appendix 1: Table of normal distribution

TABLA DE LA DISTRIBUCIÓN NORMAL $N(0, 1)$

$P(Z \le z_0) = \left\{ \begin{array}{c} \text{área del recinto} \\ \text{coloreado} \end{array} \right\}$



z_0	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9646	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Figure 7: Table of normal distribution [24]

6.2. Appendix 2: Graphic of c.d.f. distribution

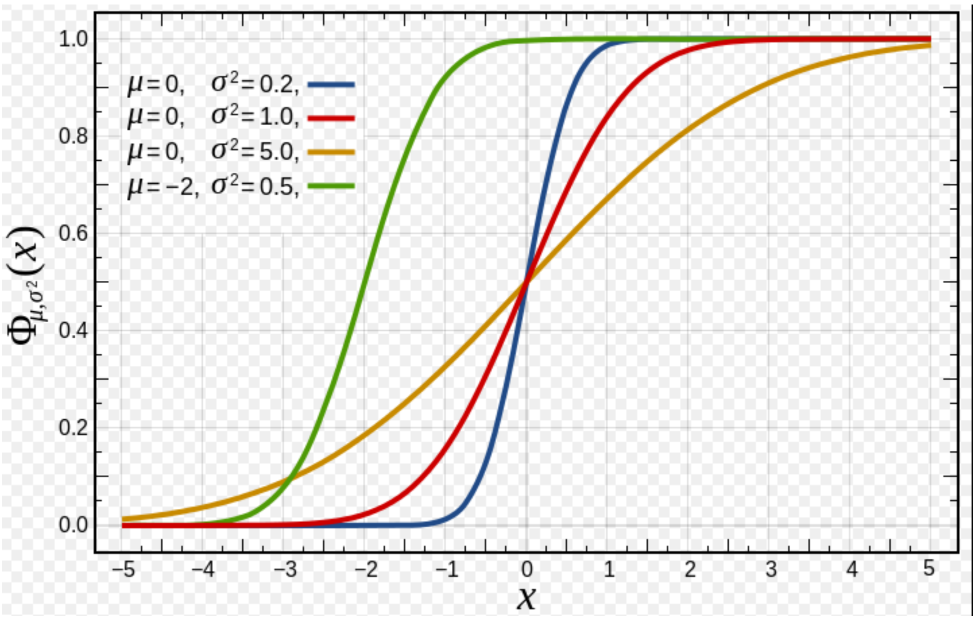


Figure 8: Graphic of c.d.f. distribution [25]

7. BIBLIOGRAPHY

[1] Kalyan T. Talluri, Itir Z. Karaesmen, Garrett J. van Ryzin and Gustavo J. Vulcano. 2008. Revenue Management: models and methods p.145-146.

[2] Kalyan T. Talluri and Garrett J. van Ryzin. 2004. The theory and practice of Revenue Management.

[3] Sheryl E. Kimes. 2013. Revenue Management: Advanced Strategies and Tools to Enhance Firm Profitability.

[4] Trevor Stuart-Hill. Aaaa. Revenue Management: An Overview on Past, Present and Future.

[5] Özalp Özer, Robert Phillips. 2012. The Oxford Handbook of Pricing Management (p.85).

[6] Sheryl E. Kimes. 2011. The Future of Hotel Revenue Management.

[7] L. Canina and C. A. Enz. 2006. Revenue management in US hotels.

[8] H. Simon and R. J. Dolan. 1998. Price customization (p.12).

[9] C. H. Lovelock and J. Wirtz. 2011. Services Marketing: People, Technology, Strategy.

[10] Kalyan T. Talluri and Garrett J. van Ryzin. 2005. An Introduction to Revenue Management (p.170-171).

[11] Alex Dietz, Natalie G. Osborn, and Tugrul Sanli. 2012. Factoring Upgrades into Overbooking Decisions for Hotels and Casinos.

[12] Phillips, Robert L. 2005. Pricing and Revenue Optimization.

[13] Brian Kallehauge. 2009. Background and introduction to Revenue Management (p.33-41).

[14] Peter P. Belobaba. 1987. Survey Paper—Airline Yield Management an Overview of Seat Inventory Control (p.67-71).

[15] A. Dolgui and J.M Proth. 2010. Supply Chain Engineering Useful Methods and Techniques (42-49).

[16] Garrett J. van Ryzin and Kalyan T. Talluri. 2005. An Introduction to Revenue Management.

[17] Linda Hatfield. 2010. El futuro del Revenue Management – Análisis completo del informe de Sheryl E-Kimes.

[18] Nur¸sen Aydın, S¸ Ilker Birbil, J. B. G. Frenk and Nilay Noyan. 2012. Single-Leg Airline Revenue Management With Overbooking.

[19] Daniel F. Otero, Raha Akhavan-Tabatabaei. 2015. A stochastic dynamic pricing model for the multiclass problems in the airline industry

[20] María Isabel Jurado Molina ;J. David Canca Ortíz. 2012. Una Introducción al Problema del Overbooking en el Transporte Aéreo

[21] Richard Klophaus and Stefan Pölt. Airline Overbooking Considering Passengers' Willingness to Pay.

[22] Gabriel Bitran and René Caldentey. 2003. An Overview of Pricing Models for Revenue Management.

[23] Tieming Liu. 2005. Revenue Management Models in the Manufacturing Industry

[24] http://ieszaframagon.com/matematicas/estadistica/var_aleatoria/tabla_normal.png

[25] https://es.wikipedia.org/wiki/Funci%C3%B3n_de_distribuci%C3%B3n#/media/File:Normal_Distribution_CDF.svg

[26] Alejandro Marchiona. 2005. Una visión estratégica sobre el Revenue Management (p.5-17).

[27] Luis Miguel Sanchez and Bhanu Chopra. 2005. Revenue Management for Travel Agents, online travel aggregators.

[28] José Navarro. 2013. ¿En qué sectores se aplica Revenue Management vs Estrategia de precios?

[29] Sheryl Kimes and Jochen Wirtz. 2016. Revenue Management in Restaurants: Unbundling Pricing for Reservations from the Core Service (p.3-6).

[30] Amstrong and Meissner. 2010. Railway Revenue Management: Overview and Models (p.3-6).

[31] Florian Defregger. 2009. Revenue Management for Manufacturing Companies (p.11-19).

[32] M. K. Geraghty and Ernest Johnson. 1997. Revenue Management saves national car rental (107-124).

[33] Juan Rodriguez. Revenue Management: La importancia de segmentar

[34] https://hospitalityrevenuemanagementstudentblog.com/2014/01/21/una-matriz-de-segmentacion/

[35] GEO Tutoriales. 2014. Técnica de Littlewood en Revenue Management para determinar niveles de protección

[36] https://www.str.com/products/star-program/en-us

[37] http://ideas.com/es/soluciones/productos/sistema-ideas-de-administracion-de-ingresos/

[38] Juan Daniel Nuñez. 2013. Las mejores soluciones de Revenue Management