

Trade Deals and/or On-Pack Coupons[☆]

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Abstract

The selection of either a pull or a push price promotion has mainly been investigated in contexts where manufacturers offer deals to consumers at the time of purchase or offer trade deals to retailers. This paper extends this framework to where manufacturers can offer either trade deals or rebate-like promotions to consumers such as on-pack coupons that stimulate the first and second purchases or a combination of the two promotion vehicles. It is demonstrated that the decision to implement either of the three promotion options critically depends, among other factors, on the percentage of first-time buyers who redeem their coupons at the second purchase. Particularly, a necessary condition to simultaneously offer both a trade deal and coupons is to have a positive coupon redemption rate. When possible, manufacturers prefer on-pack coupons over trade deals to take advantage of slippage and to further increase the overall demand via coupon-induced repeat purchase. Manufacturers are more likely to take the lion's share of channel profits.

Keywords: On-pack coupons, pull price promotions, push price promotions, trade deal

1. Introduction

The issue of whether manufacturers should target their price promotions to their trade partners or final consumers is far from being resolved. Price promotions designed for trade partners such as retailers, also known as push price promotions, generally reduce the regular wholesale prices with the hope that channel partners can find it optimal to fully or partially pass on received price-cuts to consumers and stimulate retail sales, while price promotions targeted at final consumers, known as pull price promotions, directly offer price discounts or other monetary incentives to consumers, with the primary goals of attracting consumers to retail places and stimulating immediate sales. Typically, price promotions are viewed as instruments used by sellers to price discriminate between groups of consumers of varying price sensitivity (Narasimhan, 1984; Varian, 1980; Su et al., 2014). The primary belief underlying this view is that what really matters for consumers is the effective price they pay, regardless of where price cuts may originate.

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In this line, traditional economic literature has long considered that the choice between some forms of push and pull price promotions is a simple exercise that exclusively depends on the marketer's taste as both are expected to provide the same outcomes in terms of retail prices, quantities sold, and channel profits. Cash rebates and trade deals are believed to be two of such price promotions that manufacturers offer indifferently to consumers and retailers in the context of a bilateral monopoly for the same outcomes (Ault et al., 2000). However, this simplistic view of push and pull price promotions as subsidies offered by manufacturers to retailers and consumers is not always theoretically and empirically supported. For instance, Busse et al. (2006) provides with an empirical evidence that the subsidy analogy does not hold in the automobile industry. The effective price consumers paid is lower when a manufacturer offers an identical cash-back to consumers than to retailers. In such an industry where cash rebates alone represent marketing expenses in excess of \$3 billion per year (Bruce et al., 2006), a misallocation of promotional budgets can seriously damage manufacturers' profits and consumer welfare. A few theoretical works have attempted to explain why manufacturers may prefer cash rebates to trade deals. Gerstner and Hess (1991a) find that manufacturers prefer consumer rebates when there is a strong positive association between willingness to pay and redemption costs. Gerstner and Hess (1991b) again claim that manufacturers primarily use cash rebates to motivate retail participation in promotions even when promotions do not discriminate among consumers with different reservation prices. Ault et al. (2000) examine the argument that cash rebates are manufacturers' first choice because retailers do not fully passed on trade discounts to consumers. They find that even when this argument does not hold, manufacturers could still prefer consumers rebate as trade deals allow retailers to accumulate low-price inventories. Martín-Herrán et al. (2010) focus on trade promotions that reward retailers for units effectively sold and offer an integrative framework that explains why in some contexts the subsidy analogy may work, and why in some others it may be irrelevant. Martín-Herrán et al.'s theory supports the view that choosing between trade deals and cash rebates critically depends on the consumer sensitivity to both regular and promotional prices. Particularly, manufacturers are better (worse) off offering trade deals (cash rebates) when consumers are more (less) sensitive to promotions than to regular prices. This theory also claims that the subsidy analogy holds only if consumers react identically to regular and promotional prices.

Departing from cash rebates, Kumar et al. (2004) also investigate the optimal choice between push and pull price promotions in a context where an exogenously sales expansion target is set. The push price promotion in this research is a trade discount offered to retailers, while coupons are used as pull price promotions. It is demonstrated that the manufacturer may choose either trade deals or coupons depending on coupon redemption rates and the equivalence ratio of price reduction to coupon face value.

Observe that previous research on the choice between pull price and push price promotions exclusively applies on consumer promotions such as cash rebates, shelf price discounts, and coupons that have the property that they offer discounts up front at the time of purchase (see also, Demirag et al. (2011) in the presence of competition). Theories developed for this type of price promotions are hardly generalizable to promotions that offer a discount after a purchase or at the time of a second purchase. As a matter of fact, Lu and Moorthy (2007) demonstrate that coupons designed to offer deals up front with the purchase and rebates that are redeemed after the purchase do not work the same way and offer different outcomes. Particularly, they find that rebates are efficient at surplus extraction, while coupons offer more fine-tuned control over who

buys the products. There is therefore a need to further study the trade-offs between push and pull price promotions using promotions with various properties to deepen our understanding of the challenges marketers encounter in the planning of their price promotion programs.

In this paper, we develop a framework where a manufacturer in a bilateral monopoly context has the possibility of offering a trade deal or a rebate-like promotion or a combination of both at the same time. We call rebate-like promotions any promotional activities that require consumers to first purchase the product at the regular price and then benefit from a price discount for the next purchase. We particularly focus on what is known as on-package coupons (Chen et al., 2005; Dhar et al., 1996) where consumers are aware of the pledged discount and it is factored into their decisions to make the first purchase at the regular price. As a consequence, on-package coupons have the potential of directly influencing first and repeat purchases. Our current modeling effort does allow for price discrimination. Due to the slippage phenomenon, consumers who purchase the product in the first period may or may not redeem their coupons in the second period, depending on their price sensitivity, among other factors.

The current research answers the following three questions:

1. What are the conditions under which manufacturers should exclusively offer either trade deals or on-package coupons or a combination of the two promotion types?
2. How does the coupon redemption rate impact on the manufacturers' profits and, consequently, the type of consumer promotions that manufacturers are more likely to implement?
3. How do push and pull price promotions affect the sharing of channel profits?

We hope the answers to the first two questions will expand our understanding of the use of pull and push price promotions and offer useful guidelines to marketing managers to effectively plan and implement these promotional vehicles. More particularly, our distinctive contribution is to show that rebate-like promotions such as on-pack coupons, which incite consumers to make a first purchase at the regular price and a second purchase at a discounted price, are different from instant rebates and peel-off coupons and that this difference should be acknowledged in the theories that deal with manufacturers' strategic choice between push and pull price promotions.

In addition, the answer to the third question will also enrich the debate over the impact of some promotional activities on the relative profitability of channel members. Manufacturers' increasing use of price promotions has been associated to a shift in the channel's economic power from the manufacturers to the retailers (Ailawadi, 2001; Nijs et al., 2001). However, empirical and theoretical research on the relative profitability of channel members gives mixed results (Farris and Ailawadi, 1992; Martín-Herrán and Sigué, 2011; Messinger and Narasimhan, 1995; Sigué, 2008; Srinivasan et al., 2004). In this research, we explore the idea that pull and push price promotions may have different impacts on the channel members' relative profits. As a matter of fact, during manufacturers' pull price promotions, such as coupons, retailers have the option of increasing their regular prices (Bruce et al., 2006; Busse et al., 2006; Gerstner and Hess, 1991a&b). On the other hand, manufacturers fully rely on retailers to pass on to consumers the deals provided in push price promotions, which theoretically should give retailers additional leverage to increase their profits. It is not clear which of these two externalities benefits the retailers more, to the detriment of the manufacturers, in the case of trade deals and on-pack coupons.

The rest of the paper is organized as follows: First, we introduce two models: the Benchmark Model (BM) and the Full-Promotion Model (FM). Second, we derive the equilibria, identify the conditions under which the manufacturer may find it optimal to offer push and/or push price promotions, and compare the players' optimal profits in each scenario. Finally, we conclude by highlighting our contributions to advancing research and practice in this area.

2. The models

Consider a two-member channel of distribution where a manufacturer distributes a single product through an exclusive independent retailer. We build two stylized two-period models to account for various common business situations.

2.1. Benchmark Model

The benchmark model (BM) corresponds to a standard pricing game in which no price incentive is offered (Martín-Herrán et al., 2010). The manufacturer and retailer each set their regular wholesale price (w_i) and regular retail price (p_i) at period i , $i \in \{1, 2\}$. We assume that consumer demand at period i is linear and given by

$$q_i = g - p_i,$$

where g is a positive parameter representing the baseline demand. For simplicity, consumer sensitivity to regular retail price is normalized to 1. Observe that the baseline demand does not change from the first to the second period and that the players keep the same set of decision variables. As a consequence, the first-period game is basically repeated in the second period.

Denote by Π_i^M and Π_i^R the manufacturer's and retailer's profits at period i and assume a constant and positive marginal production cost (c). The two players' profit functions at period i are given by

$$\Pi_i^M = (w_i - c)q_i, \quad \Pi_i^R = (p_i - w_i)q_i, \quad i \in \{1, 2\}.$$

Let us assume that the manufacturer is the channel leader and the retailer is the follower. Channel leadership here is decided on an ad hoc basis. However, Jørgensen et al. (2001) endogenously demonstrate that manufacturer leadership is desirable for both manufacturers and retailers, while retailer leadership could be detrimental for channel profits. The following are optimal expressions for the first period in the benchmark model (superscript B):

$$p_1^B = \frac{1}{4}(c + 3g), \quad w_1^B = \frac{1}{2}(c + g), \quad q_1^B = \frac{1}{4}(g - c). \quad (1)$$

To ensure a positive demand in the first period, condition $g > c$ is needed. From now on, we assume that this condition is always satisfied.

The total manufacturer's and retailer's profits are given by

$$\Pi^M = \Pi_1^M + \delta \Pi_2^M, \quad \Pi^R = \Pi_1^R + \delta \Pi_2^R,$$

where $0 < \delta \leq 1$ and represents the discount rate applied to future profits by both the manufacturer and retailer. Smaller values of δ mean that the two channel partners are short-term oriented, while farsighted channel members consider the first-period profits to be as important as the second period and set δ near or equal to 1. As in Sigué (2008), we assume farsighted

channel members and set $\delta = 1$. This assumption is realistic in the context the low interest rates observed in recent years.

The total manufacturer's and retailer's optimal profits are given by

$$\Pi^{M,B} = \frac{1}{4}(c - g)^2, \quad \Pi^{R,B} = \frac{1}{8}(c - g)^2. \quad (2)$$

The profit function in each period and for each channel member is a strictly concave function with respect to the corresponding decision variable, implying that the necessary conditions for optimality are sufficient too, and that the equilibrium is unique.

Comparing the two channel members' profits in (2), it is easy to see that the manufacturer earns twice the profit of the retailer. As expected, the Stackelberg manufacturer takes the biggest share of channel profits.

2.2. Full-Promotion Model

The Full-Promotion model (FM) assumes that the manufacturer keeps his first-period regular wholesale price (w_1^B) unchanged, that he offers both "cents-off" coupons (C) to consumers at the first-period purchase, to be redeemed at the second-period purchase, and a trade deal (d) to the retailer in the first period, and that he controls the second-period wholesale price (w_2). The trade deal in this model should be seen as a discount on the regular wholesale price, which the manufacturer offers to the retailer in the hopes that the latter will, either fully or in part, pass the savings along to consumers in the form of a retailer price promotion. On the other hand, the manufacturer's coupons targeting the customers directly could have various goals, including attracting new buyers, increasing the consumption of current buyers and retaining new and current customers. Consumers are aware that any coupons unused at the end of the second period of the game are lost.

The retailer endogenously sets a retail price (p_i) for each period i , $i \in \{1, 2\}$. The demand functions for each period of the game are given by

$$\begin{aligned} q_1 &= g - p_1^B + \alpha(p_1^B - p_1) + \theta C, \\ q_2 &= g - p_2 + \gamma(q_1 - q_1^B) + \mu q_1, \end{aligned}$$

where p_1^B is the regular retail price that maximizes the retailer's profit in the BM, α is a positive parameter representing consumer sensitivity to the promotional price differential. A retail price that is higher during the promotional period than the regular retail price has a negative impact on the first-period demand, while a promotional retail price that is lower than the regular retail price has a positive impact on the first-period demand. We assume that the consumer sensitivity to the regular retail price (here normalized to 1) may be different from the consumer sensitivity to the promotional price differential (α). We assume that $0 < \alpha \leq 1$.

The parameters θ and μ respectively represent the effectiveness of the manufacturer's coupon on the first-period demand and the coupon redemption rate at the second period of the game. The values of θ are within the interval $0 < \theta \leq \alpha$, to account for the fact that consumers are more sensitive to a price reduction that applies to the current purchase than to the promise of a price reduction that applies to the next purchase (Gilpatric, 2009). In addition, consumers are aware of the promise of the second-period price discount when they purchase the product in the first period. The parameter γ satisfies $-1 \leq \gamma \leq 1$ and represents the impact of the incremental promotional sales of the first period on second-period sales.

We assume that the second-period baseline demand, given by $g + \gamma(q_1 - q_1^B) + \mu q_1$, depends on the source of the first period's promotional incremental sales ($q_1 - q_1^B$) and on the purchases from first-period buyers who redeem their coupons (μq_1). Observe that if the promotional activity has no impact on the first-period demand, there is no promotional incremental sales, $q_1 - q_1^B = 0$. Normally, however, the first-period demand when coupons are offered should be higher than the benchmark demand due to the promotional incentive, i.e., $q_1 - q_1^B > 0$. In such a context, if $\gamma < 0$, first-period promotional sales reduce the baseline demand in the second period. This is more likely to occur when current customers buy large quantities to stockpile for future use (purchase acceleration). First-period promotional sales expand the baseline demand in the second period when they are generated by new customers who adopt the product for future use, $\gamma > 0$ (category expansion). The purchase of larger quantities by current buyers who increase their consumption during the first period and revert to their normal consumption habits in the second period has no significant effect on the second-period sales, $\gamma = 0$. On top of these potential traditional post-promotional effects, we model the coupon-induced repeat-purchase effect, which is the demand from the first-period buyers who take advantage of their first-period coupons. Such an effect does not exist when the manufacturer offers instant-rebate or peel-off coupons at the first-period purchase (e.g., Martín-Herrán et al., 2010). This coupon-induced second-period demand increase is a positive function of both the first-period demand and the redemption rate (μ). We assume that $0 \leq \mu \leq 1$, to account for the slippage phenomenon resulting from the fact that many consumers purchase products to take advantage of future discounts or rebates, but far fewer actually redeem them after their first purchase.

The manufacturer and retailer set their decision variables to maximize their discounted profits during the two periods given by

$$\begin{aligned}\Pi^M &= (w_1^B - d - c)q_1 + \delta((w_2 - c)q_2 - \mu C q_1), \\ \Pi^R &= (p_1 - w_1^B + d)q_1 + \delta(p_2 - w_2)q_2.\end{aligned}$$

The decision variables of the two channel members directly impact on the first-period demand. This creates free-riding opportunities. The manufacturer may not offer the desired coupon value, expecting the retailer to compensate with a large pass-through from the trade promotion offer, while the retailer may refrain from doing just that in the hopes that the manufacturer will offer consumers a larger coupon value.

Table 1 summarizes the decision variables, demand functions, and players' profits for the two models at each period, $i, i \in \{1, 2\}$.

Table 1: Model specifications

	Period 1	Period 2
Benchmark Model		
Manufacturer's controls	w_1	w_2
Retailer's controls	p_1	p_2
Demand functions	$q_1 = g - p_1$	$q_2 = g - p_2$
Manufacturer's profit	$\Pi_1^M = (w_1 - c)q_1$	$\Pi_2^M = (w_2 - c)q_2$
Retailer's profit	$\Pi_1^R = (p_1 - w_1)q_1$	$\Pi_2^R = (p_2 - w_2)q_2$
Full-promotion Model		
Manufacturer's controls	C, d	w_2
Retailer's controls	p_1	p_2
Demand functions	$q_1 = g - p_1^B + \alpha(p_1^B - p_1) + \theta C$	$q_2 = g - p_2 + \gamma(q_1 - q_1^B) + \mu q_1$
Manufacturer's profit	$\Pi_1^M = (w_1^B - d - c)q_1$	$\Pi_2^M = (w_2 - c)q_2 - \mu C q_1$
Retailer's profit	$\Pi_1^R = (p_1 - w_1^B + d)q_1$	$\Pi_2^R = (p_2 - w_2)q_2$

We discuss two additional assumptions below.

First, the retailer orders the exact quantity of product that she is able to sell during a given period and therefore holds no inventory. The manufacturer and retailer work together to streamline their supply chain and eradicate inventory costs. Such an integrated system allows the manufacturer to avoid retailer stockpiling during promotions and to reward the retailer for actual sales rather than quantities purchased.

Second, we assume that the production, distribution and processing costs of coupons and trade deals as well as their associated retailer promotions, are zero (e.g., Gerstner and Hess, 1991a&b; and Dhar et al., 1996). As a result, the only direct cost associated to the manufacturer's couponing campaign is the transfer payment he makes to the retailer in the second period, as consumers redeem the coupons, $\mu C q_1$. Redemption costs are believed to be the critical portion of the cost of couponing (Dhar et al., 1996). Any increase in either the redemption rate, the value of the coupon, or the number of the first-period buyers could potentially harm the manufacturer's second-period profit.

3. Equilibrium strategies

As stated above, we use the Stackelberg equilibrium concept in the Full-Promotion Model, with the sequence of moves defined as follows. The manufacturer announces his first-period decisions, and the retailer reacts to these announcements, determining her optimal first-period retail price. Then, the manufacturer announces his second-period wholesale price and the retailer reacts to that announcement, determining her optimal second-period retail price. To obtain subgame-perfect equilibria, we solve the game backwards. In other words, we first solve the retailer's problem in the second period and then substitute the retailer's optimal decisions into the manufacturer's second-period problem. Thereafter, the optimal strategies of the second period are incorporated into the players' total profits. The retailer's general problem is first solved to determine her first-period retail price, and subsequently, the manufacturer's general problem is solved to determine the optimal trade deal and coupon value.

At the equilibrium, we can demonstrate that, in each period, the conditions ensuring the strict concavity of the retailer's profit functions with respect to her decision variables are satisfied. On the other hand, the manufacturer's second-period profit function is strictly concave with

respect to his decision variable, while the total profit function may not be concave with respect to his decision variables. As a consequence, an interior equilibrium that maximizes the channel members' objective functions may be attained only for some specific values of a few model parameters. The following proposition characterizes these possibilities.

Proposition 1. *The following two possibilities can arise:*

1. *If $\theta - \alpha\mu = 0$, then the manufacturer's total profit, Π^M , is a concave function of the manufacturer's first-period decision variables (C and d). Therefore, the manufacturer's problem admits an interior solution.*
2. *If $\theta - \alpha\mu \neq 0$, then the manufacturer's total profit, Π^M , is not a concave function of the manufacturer's first-period decision variables (C and d). Therefore, the manufacturer's problem does not admit an interior solution and the solution is attained at the boundary, i.e., at $C = 0$ or $d = 0$.*

Proof. The first minor of the Hessian matrix of function Π^M with respect to the manufacturer's first-period decision variables, C and d , always takes negative values for any value of $\alpha \in (0, 1]$, $\mu \in [0, 1]$ and $\gamma \in [-1, 1]$. The determinant of the Hessian matrix is zero for the first case described in the statement of the proposition and negative for the second case. Therefore, the quadratic form associated with the Hessian matrix is negative semidefined and undefined in the first and second cases, respectively. As a result, we can conclude that in the first case, Π^M is a concave function with respect to the manufacturer's first-period decision variables, C and d , and that an optimal interior solution can be attained; while in the second case, the candidate to be extremum is a saddle point. The maximum is attained at the boundaries. ■

It is easy to see that Case 1 in Proposition 1 holds when the redemption rate, μ , is zero only when coupons are not effective on the first-period demand ($\theta = 0$). This means that a necessary condition to simultaneously offer both a trade deal and coupons is to have a positive coupon redemption rate, otherwise the manufacturer only offers either a trade deal or coupons if the two promotional activities only impact on the first period-demand. Thus, in addition of the effect of both trade deals and coupons on the first purchase, the manufacturer feels the need to use these two types of promotional activities at the same time to particularly stimulate repeat-purchase, which translates in an increase of the second-period demand.

3.1. Trade deal or Coupons

The following two propositions characterize the equilibrium solutions for the scenarios in which the manufacturer offers either only a trade deal to the retailer or coupons to consumers. The simultaneous offering of the two types of promotion is not a feasible option in this case.

Proposition 2. *Assume $\theta - \alpha\mu \neq 0$; then, the Stackelberg equilibrium strategies for the FM*

when $C = 0$ and $\mu = 0$ are as follows:

$$d^{C=0} = \frac{(g-c)(\gamma^2\alpha^2(8+\gamma(4-\gamma))+8\alpha(4+\gamma^2)-128)}{128\alpha(8-\gamma^2\alpha)}, \quad (3)$$

$$p_1^{C=0} = c + \frac{(g-c)(24-\alpha(\gamma(\gamma+12)-72)-12\gamma^2\alpha^2)}{16\alpha(8-\gamma^2\alpha)}, \quad (4)$$

$$w_2^{C=0} = c + \frac{(g-c)(\gamma\alpha(\gamma(\gamma-4)+24)-24\gamma+128)}{32(8-\gamma^2\alpha)},$$

$$p_2^{C=0} = c + \frac{3(g-c)(\gamma\alpha(\gamma(\gamma-4)+24)-24\gamma+128)}{64(8-\gamma^2\alpha)}, \quad (5)$$

and the total manufacturer's and retailer's optimal profits are

$$\Pi^{M,C=0} = \frac{(g-c)^2\Omega_1}{1024\alpha(8-\gamma^2\alpha)}, \quad (6)$$

$$\Pi^{R,C=0} = \frac{(g-c)^2\Omega_2}{4096\alpha(8-\gamma^2\alpha)^2}, \quad (7)$$

where

$$\Omega_1 = 64 + 16\alpha[88 + \gamma(\gamma - 20)] + \alpha^2[576(1 + \gamma) + \gamma^2((\gamma - 4)^2 - 144)],$$

$$\Omega_2 = 2(\alpha\gamma^2 - 8)[3\alpha((\gamma - 2)^2 - 12) - 8][8 + \alpha((\gamma - 2)^2 + 20)] + \alpha[128 + 24\gamma(\alpha - 1) + \alpha\gamma^2(\gamma - 4)]^2.$$

Proof. See the Appendix. ■

Remark 1. If coupons are not offered ($C = 0$) as it is the case in Proposition 2, the first and second-period demand functions are the same as in the trade-promotion model in Martín-Herrán et al. (2010).

Observe in this scenario that when coupons are not offered ($C = 0$), the redemption rate is zero ($\mu = 0$). Obviously, without coupons, consumers have nothing to redeem. The analysis of the equilibrium solution in the first scenario ($C = 0$ and $\mu = 0$) allows for some observations. For any value of $\alpha \in (0, 1]$ and $\gamma \in [-1, 1]$, the following inequalities apply:

$$p_1^{C=0} > c, \quad w_2^{C=0} > c, \quad p_2^{C=0} > c, \quad \Pi^{M,C=0} > 0, \quad \Pi^{R,C=0} > 0.$$

However, to ensure that the trade deal $d^{C=0}$ is positive, the following condition must be satisfied:

$$\alpha \in (0.618931, 1], \quad \gamma \in (\underline{\gamma}, 1], \quad (8)$$

where $\underline{\gamma}$ is the smallest root of the following polynomial equation in the variable γ :

$$\alpha^2(\gamma - 4)\gamma^3 - 8\alpha(1 + \alpha)\gamma^2 - 64\alpha\gamma + 128(1 - \alpha) = 0.$$

We proved numerically, as illustrated in Table 2, that $\underline{\gamma}$ decreases as α increases.

Table 2: Effect of α on $\underline{\gamma}$.

α	$\underline{\gamma}$
0.618931	1
0.62	0.996115
0.75	0.584068
1	0

The manufacturer's decision to exclusively offer a trade deal in this scenario depends on both the consumer sensitivity to the promotional price differential and the incremental promotional sales effect. Condition (8) states that when the consumer sensitivity to a promotional price differential takes relatively high values ($\alpha \in (0.618931, 1]$), the nature and the magnitude of the impact of the incremental promotional sales on the second-period demand do matter. If $0.618931 \leq \alpha < 1$, meaning that the consumer sensitivity to the promotional price differential is lower than the consumer sensitivity to regular prices, but higher than the specified threshold, then manufacturers should only offer trade deals to retailers if the expected impact of such promotional activities on post-promotional sales is positive, i.e., trade deals contribute to expanding the baseline demand for the product. In any case, manufacturers should not offer trade deals to retailers if the consumer sensitivity to the promotional price differential does not reach the threshold of $\alpha = 0.618931$. Figure 1 displays in gray, the area in the parameter space where the manufacturer finds it optimal to offer the retailer a trade deal.

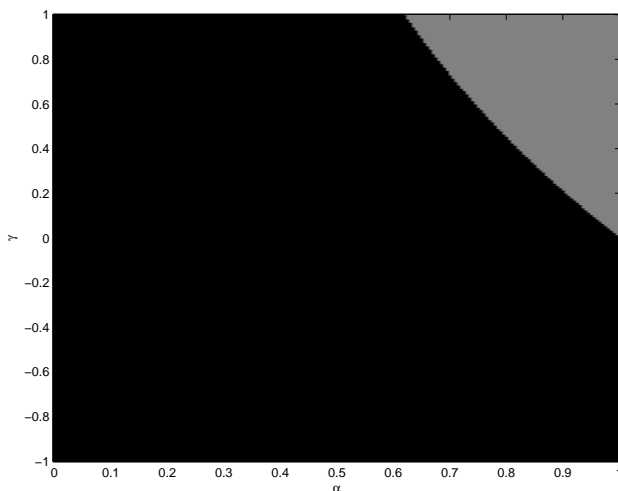


Figure 1: Parameter region where $d^{C=0} > 0$: Gray area

Proposition 3. *Assume $\theta - \alpha\mu \neq 0$; then, the Stackelberg equilibrium strategies for the FM*

when $d = 0$ are as follows:

$$C^{d=0} = \frac{(g-c)\Sigma_1}{64\theta[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]}, \quad (9)$$

$$p_1^{d=0} = c + \frac{(g-c)\Sigma_2}{64\alpha[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]}, \quad (10)$$

$$w_2^{d=0} = c + \frac{(g-c)\Sigma_3}{16[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]},$$

$$p_2^{d=0} = c + \frac{3(g-c)\Sigma_3}{32[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]}, \quad (11)$$

and the total manufacturer's and retailer's optimal profits are

$$\Pi^{M,d=0} = \frac{(g-c)^2\Sigma_4}{512\theta[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]}, \quad (12)$$

$$\Pi^{R,d=0} = \frac{(g-c)^2\Sigma_5}{1024\alpha[(\theta + \alpha\mu)(\gamma + \mu)^2 - 16\mu]^2}, \quad (13)$$

where

$$\begin{aligned} \Sigma_1 &= 16\theta(\gamma(\gamma - 8) - (4 + \mu)^2) + \mu[128 - 8\alpha(\mu(\mu - 8) + 4\gamma(\mu - 2) + 3\gamma^2 - 16) \\ &\quad + \alpha^2(\gamma + \mu)^2(\gamma^2 + \gamma(\mu - 4) - 4(\mu + 2))], \\ \Sigma_2 &= 16\theta(\gamma(6\alpha\mu + \mu - 4) + 3\alpha\mu^2 - 4\mu + (1 + 3\alpha)\gamma^2 - 8) \\ &\quad + \mu[\alpha^2(40 + \gamma^2 + \gamma(\mu - 4) - 4\mu)(\gamma + \mu)^2 - 8\alpha(72 + 2\gamma^2 + \gamma(\mu - 12) - \mu(\mu + 12)) - 64], \\ \Sigma_3 &= \mu[\gamma(24 - \alpha(\mu^2 + 8(1 - \mu))) + 4\alpha\mu^2 - 8(1 + \alpha)\mu - \alpha\gamma^2(\gamma + 2(\mu - 2)) - 128] - 16\theta(\gamma + \mu), \\ \Sigma_4 &= 64\theta(\theta + \alpha\mu)(\gamma^2 + \gamma(\mu - 4) - 4(\mu + 1)) - 32\theta\mu(\gamma^2 + 4(\mu + 10) - \gamma(12 + \mu)) \\ &\quad - \mu^2(\alpha(\gamma^2 + \gamma(\mu - 4) - 4(\mu + 2)) - 8)^2, \\ \Sigma_5 &= 2[(2\theta + \mu\alpha)(\gamma^2 + \gamma(\mu - 4) - 4(\mu + 2)) - 8\mu][16\theta(\gamma^2(1 + \alpha) + \gamma(\mu(2\alpha + 1) - 4) + \mu(\alpha\mu - 4) - 8) \\ &\quad + \mu(\alpha^2(\gamma + \mu)^2(8 + \gamma^2 + \gamma(\mu - 4) - 4\mu) - 64 - 8\alpha(8 + 2\gamma^2 + \gamma(\mu - 12) - \mu(\mu + 12))) \\ &\quad + \alpha[\alpha\gamma^2\mu(\gamma + 2(\mu - 2)) + 4\mu(32 + 4\theta + 2(1 + \alpha)\mu - \alpha\mu^2) + \gamma(16\theta + \mu(\alpha(\mu^2 + (1 - \mu)) - 24))]^2. \end{aligned}$$

Proof. See the Appendix. ■

Remark 2. The second-period demand in this scenario generalizes that in Martín-Herrán et al. (2010) to include the coupon redemption rate (μ) and the benchmark retail price (p_1^B). If the coupon redemption rate is set to zero and $\alpha = 1$, this scenario becomes identical to the consumer-promotion model in Martín-Herrán et al. (2010).

In analyzing this equilibrium solution, we found that the retailer's profit, $\Pi^{R,d=0}$, is positive for any value of $\alpha \in (0, 1]$ and $\gamma \in [-1, 1]$ if either θ or μ is different from zero. Any positive impact of the couponing campaign on overall demand benefits the retailer as she does not support the cost of this activity. The manufacturer's optimal profit, $\Pi^{M,d=0}$, is always positive,

and $p_1^{d=0}, w_2^{d=0}, p_2^{d=0}$ are always greater than the unit cost, c , for any value of the parameters in the defined subsets, except for very low values of μ (for example, $\mu = 0.1$) and extreme values of γ (i.e., $\gamma = -1$ or $\gamma = 1$). In these cases, the following conditions must be added: ($0 < \alpha < \bar{\alpha}$ and $0 \leq \theta \leq \alpha$) or ($\bar{\alpha} < \alpha < 2$ and $0 \leq \theta \leq \bar{\theta}(\alpha)$).

There are additional conditions on parameters α, μ, γ and θ that ensure that the coupon value to consumers, $C^{d=0}$, is positive. We refrain from writing these conditions here because they are given by huge expressions³. Numerical simulations allow us to illustrate in Figures 2 and 3 how some of the model parameters affect the manufacturer's decision to offer consumer coupons. In Figure 2, let us set $\mu = 0.25$ and $\gamma \in \{-1, -0.5, 0.5, 1\}$ and, in Figure 3, $\gamma = 0$ and $\mu \in \{0.25, 0.5, 0.75, 1\}$.

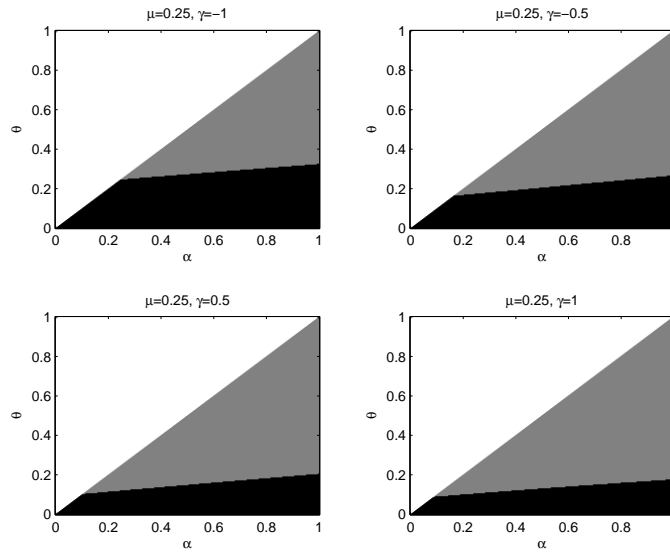


Figure 2: Parameter region where $C^{d=0} > 0$: Gray area. $\mu = 0.25; \gamma \in \{-1, -0.5, 0.5, 1\}$.

Figure 2 shows that the gray area in the parameter space where the manufacturer finds it optimal to offer coupons to consumers expands with the increase of the incremental promotional sales effect on the second-period demand when the redemption rate is hold constant. As expected, the manufacturer is more likely to offer coupons to consumers when they attract new buyers and contribute to expanding the baseline demand in subsequent periods. Figure 3, on the other hand, supports that when the effect of the incremental promotional sales effect is hold constant, the area where the manufacturer offers coupons to consumers contracts with the increase in the redemption rate, which means that coupons are less attractive when they are heavily redeemed. As a result, manufacturers are more likely to offer coupons to consumers when they contribute to expanding the second-period demand by attracting new buyers, but when the coupons are not actually redeemed in large numbers. The explanation behind this

³Conjecture 1 in the Appendix collects the results of the numerical simulations carried out to qualitatively characterize the area of the parameter space for which the coupon value to consumers, $C^{d=0}$, is positive.

finding is that couponing campaign costs increase as the number of consumers who redeem their coupons in the second period increases, making this promotional activity very costly and less desirable for manufacturers.

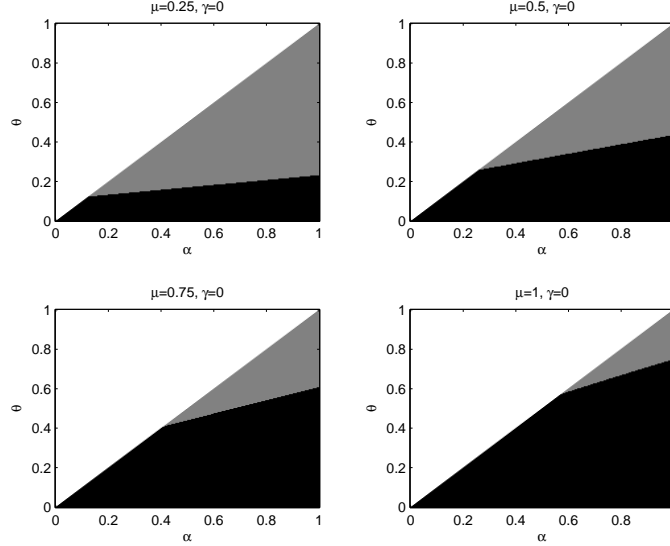


Figure 3: Parameter region where $C^{d=0} > 0$: Gray area. $\gamma = 0; \mu \in \{0.25, 0.5, 0.75, 1\}$.

Proposition 4. $d^{C=0}$ and $C^{d=0}$ cannot be simultaneously positive.

Proof. The trade deal in the scenario $C = 0, \mu = 0, d^{C=0}$, is positive if condition (8) is satisfied. It is easy to see that for any value of the parameter space if $\mu = 0$, the coupon in the scenario $d = 0, C^{d=0}$, cannot take positive values. ■

Proposition 4 deals with Case 2 in Proposition 1 1 and supports the view that, in this particular scenario where, $\theta - \alpha\mu \neq 0$, the manufacturer can only provide either of the two promotional activities: $C = 0, d > 0$ or $d = 0, C > 0$, but not the two at the same time. As a result, the manufacturer's choice of any of these promotional activities is done on an ad hoc basis given that their profits cannot be endogenously compared.

Proposition 5. The manufacturer's profit for the scenario $d = 0, \Pi^{M,d=0}$, is always greater for μ positive than for μ equal to zero.

Proof. The manufacturer's profit for scenario $d = 0$ is given by (12). Setting $\mu = 0$ in this expression and by means of numerical simulations, it can be shown that for the range of the parameters for which $C^{d=0} > 0$, the result in the proposition holds. ■

Given the above-discussed conditions under which the manufacturer offers coupons, Proposition 5 claims that the manufacturer earns more profit with coupons when the coupon redemption rate is positive. In other words, when offering coupons is a feasible option, it is in the manufacturer's best interest to have some level of coupon redemption. This finding suggests that, despite the fact that coupon redemption drives the costs of the couponing campaign up, its positive

effect on stimulating repeat purchase in the second period cannot be ignored. The coupon-induced repeat-purchase effect contributes to enhancing the appeal of on-pack coupons. As a result, on-pack coupons are more attractive than peel-off coupons or instant rebates (Martín-Herrán et al., 2010), which only affect the first purchase and do not have any direct effect on subsequent purchases. Further numerical analysis reveal that manufacturers are more likely to earn more profit when coupon redemption (slippage) is lower (higher) than otherwise. As the second-period demand increase via coupon-induced repeat-purchase is achieved by increasing the total cost of the couponing campaign, on-pack coupons should be more beneficial to the manufacturer only when coupons are redeemed by nonregular buyers. Alternatively, if coupons are redeemed by many regular buyers in the second period, the manufacturer's profitability from the couponing campaign is reduced as these buyers would have anyway purchased the product at the regular price⁴.

3.2. Trade deal and/or coupons

We turn to the case where the manufacturer can simultaneously offer a trade deal to the retailer and coupons to consumers. The following proposition characterizes the equilibrium solution for this particular case.

Proposition 6. *Assume $\theta - \alpha\mu = 0$; then, the Stackelberg equilibrium strategies for the FM are as follows:*

$$d + \mu C = \frac{(g - c)\Gamma_1}{128\alpha(8 - \alpha(\gamma + \mu)^2)}, \quad (14)$$

$$p_1 = c + \mu C + \frac{(g - c)\Gamma_2}{16\alpha(8 - \alpha(\gamma + \mu)^2)}, \quad (15)$$

$$w_2 = c + \frac{(g - c)\Gamma_3}{32(8 - \alpha(\gamma + \mu)^2)},$$

$$p_2 = c + \frac{3(g - c)\Gamma_3}{64(8 - \alpha(\gamma + \mu)^2)}, \quad (16)$$

and the total manufacturer's and retailer's optimal profits are:

$$\Pi^M = \frac{(g - c)^2\Gamma_4}{1024\alpha(8 - \alpha(\gamma + \mu)^2)}, \quad (17)$$

$$\Pi^R = \frac{(g - c)^2\Gamma_5}{4096(8 - \alpha(\gamma + \mu)^2)^2}, \quad (18)$$

⁴As pointed out one of the reviewers, it should be kept in mind that, the redemption rate plays a dual role in this research. It impacts on the second-period demand and, at the same time, reduces the manufacturer's profitability by increasing promotional costs.

where

$$\begin{aligned}
\Gamma_1 &= 8\alpha(16+\gamma^2+4\gamma(\mu+2)+\mu(3\mu+8))-\alpha^2(\gamma+\mu)^2(\gamma^2+\gamma(\mu-4)-4(\mu+2))-128, \\
\Gamma_2 &= \alpha(4(18-\mu(3+\mu))-\gamma(5\mu+12)-\gamma^2)-12\alpha^2(\gamma+\mu)^2+24, \\
\Gamma_3 &= \alpha\gamma^2(\gamma+2(\mu-2))+8(1+3\alpha)\mu-4\alpha\mu^2+\gamma(\alpha(24+\mu(\mu-8))-24)+128), \\
\Gamma_4 &= \alpha^2[\gamma^4+2\gamma(\mu-4)(\gamma^2-4(18+\mu))+\gamma^2(\mu^2-16(\mu+8))+16(\mu^2+36(\mu+1))] \\
&\quad +16\alpha(24+(\gamma-4)(\gamma-3\mu-16))+64, \\
\Gamma_5 &= \alpha[\gamma(\alpha(24+\mu(\mu-8))-24)+\alpha\gamma^3+2\alpha\gamma^2(\mu-2)+8\mu(3\alpha+1)-4\alpha\mu^2+128]^2 \\
&\quad +2(\alpha(\gamma+\mu)^2-8)(\alpha(24+\gamma^2+\gamma(\mu-4)-4\mu)+8)(3\alpha(\gamma^2+\gamma(\mu-4)-4(2+\mu))-8).
\end{aligned} \tag{19}$$

Proof. See the Appendix. ■

As expression (14) shows, at the equilibrium, the manufacturer has an infinite number of possibilities for his decision variables, C and d , including setting the variables at the boundary, i.e., at $C = 0$ or $d = 0$. Any combination of C and d that satisfies (14) is optimal and leads to the manufacturer's maximized profit in (18).

The expression for $d + \mu C$ given in (14) is positive if the model parameters belong to one of the following two subsets:

$$\begin{aligned}
\alpha &\in (0.375345, 0.618931], \quad \mu \in (\underline{\mu}, 1], \quad \gamma \in (\underline{\gamma}, 1], \\
\alpha &\in (0.618931, 1], \quad \mu \in [0, 1], \quad \gamma \in (\underline{\gamma}, 1],
\end{aligned}$$

where bounds $\underline{\mu}$, $\underline{\gamma}$ are, respectively, the highest, the lowest, root of the following two polynomials in variables μ , γ , respectively:

$$\begin{aligned}
3\alpha^2\mu^3 + \alpha(24 + 17\alpha)\mu^2 + \alpha(96 + 25\alpha)\mu + \alpha(200 + 11\alpha) - 128 &= 0, \\
\alpha^2\gamma^4 + \alpha^2(3\mu - 4)\gamma^3 + (3\alpha^2\mu(\mu - 4) - 8\alpha(1 + \alpha))\gamma^2 + (\alpha^2\mu^2(\mu - 12) - 16\alpha\mu(\alpha + 2) - 64\alpha)\gamma \\
+ 128(1 - \alpha) - 8\alpha\mu(3\mu + 8) - 4\alpha^2\mu^2(\mu + 2) &= 0.
\end{aligned}$$

The manufacturer's decision to offer the two types of promotions critically depends on three parameters, namely, the consumer sensitivity to the promotional price differential, the incremental promotional sales effect, and the coupon redemption rate. For instance, if the consumer sensitivity to the promotional price differential is very small ($\alpha \leq 0.375345$), then the manufacturer finds it optimal not to offer a trade deal and/or coupons, regardless of the value of the two other parameters. For relatively small values of α ($0.375345 < \alpha \leq 0.618931$), both the incremental promotional sales effect and the coupon redemption rate have to reach certain specified thresholds before the manufacturer can offer both promotions. Figure 4 below illustrates how the area where the manufacturer can offer the two types of promotions change at different levels of the parameter values. The message from Figure 4 is that the manufacturer can offer simultaneously the two types of promotions as long as consumers are price sensitive enough to justify the offering of trade deal to the retailer and coupon offering to consumers can generate sufficient repeat purchase to complement the traditional first period's incremental sales effect on the second-period demand. Particularly, low redemption rates mainly go well with first-period promotional activities that expand the baseline demand via the attraction of new buyers, while with relative high redemption rates, the manufacturer can still find it optimal to

offer the two promotions when the first-period incremental promotional sales negatively impact on the second-period demand. The rationale is that when consumer sensitivity to promotional price differential is high, the retailer has an incentive to reduce the retail price as a consequence of the trade deal offered by the manufacturer, consumers buy more in the first period, and even with a high redemption rate, the overall revenue increase during the two periods can compensate the related couponing cost supported in the second period.

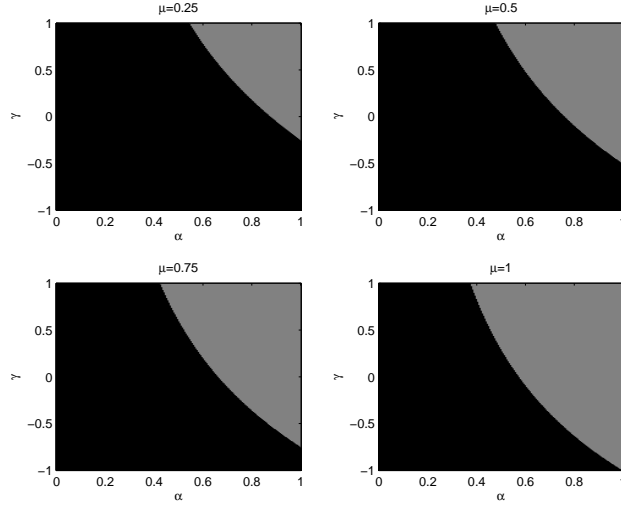


Figure 4: Parameter region where $d + \mu C > 0$: Gray area. $\theta = \alpha\mu$, $\mu \in \{0.25, 0.5, 0.75, 1\}$.

Remark 3. *Let us note that from (14), the following expressions are the manufacturer's boundary promotional strategies:*

$$d = 0, \quad \mu \neq 0, \quad C = \frac{(g - c)\Gamma_1}{128\alpha\mu(8 - \alpha(\gamma + \mu)^2)}, \quad (20)$$

$$C = 0, \quad \mu = 0, \quad d = \frac{(g - c)[\alpha^2\gamma^2(8 + 4\gamma - \gamma^2) + 8\alpha(4 + \gamma)^2 - 128]}{128\alpha(8 - \alpha\gamma^2)}, \quad (21)$$

where Γ_1 is given in (19).

This remark highlights the fact that although the manufacturer has the option of offering the two promotions at the same time, nothing prevents him from focusing on only one of them to achieve the maximized profit in (18). In such a context, one of the two boundary solutions in (21) and (22) will be chosen.

On the other hand, Equation (15) establishes that, except for $\mu = 0$, the optimal first-period retail price, p_1 , is a strictly increasing function of the coupon offered to consumers, C . In other words, as expected, the retailer acts opportunistically and increases her first-period retail price as the coupon value given to consumers increases. The retailer also increases the first-period retail price in anticipation of the number of first-period buyers who redeem their coupons in the second period to fully take advantage of the coupon offering. Overall, however, it can be

demonstrated that, under certain conditions, the first-period retail price, p_1 , in this scenario remains lower than the benchmark first-period retail price⁵, p_1^B . This price can even be lower if $C = 0$, because of the impact of the manufacturer’s trade deal. Linking the retail price to the value of coupons offered to consumers reduces the effectiveness of both the couponing campaign and the trade deal that the manufacturer simultaneously offer directly to the retailer, especially when consumers are very sensitive to promotional price differentials.

The manufacturer’s and the retailer’s optimal profits are positive, and $w_2 > c$ and $p_2 > c$ for any value of $\alpha \in (0, 1]$, $\mu \in [0, 1]$ and $\gamma \in [-1, 1]$. Also, conditions on the model parameters ensuring $p_1 > c$ can be derived. The next proposition assesses the impact of the coupon redemption rate on the manufacturer’s optimal profit.

Proposition 7. *The manufacturer’s profit for any optimal strategy given in expression (17) is always higher for μ positive than for μ equal to zero.*

Proof. Replace $\mu = 0$ in (17) and, for the range of the parameters we have fixed, the comparison of this new optimal profit and of the profit in (17) gives the stated result. ■

Observe that when $\mu = 0$, the manufacturer can only offer a trade deal. Therefore, Proposition 7 has two major implications. First, it means that on-pack coupons should be preferred to promotional activities that can impact consumer behavior at the time of the first purchase but that do not induce repeat purchases, like peel-off coupons or instant rebates do. Second, when boundary strategies are considered, as in (20) and (21), the manufacturer is better off adopting the strategy in (20). In other words, in this given area of the parameter space, the manufacturer finds it optimal to offer coupons to consumers instead of a rebate to the retailer, providing that the coupon redemption rate is positive. The rationale is that a trade deal offered only to the retailer acts as a subsidy that the retailer may choose to either partly or fully pass on to consumers to reduce the price paid for their first purchase, while on-pack coupons generate additional sales at the regular price or at an even higher promotional price, and also generate additional coupon-induced repurchases among the first-period buyers.

3.3. Channel profit sharing

We now investigate how the channel profit is allocated between the two channel members. Specifically, we answer the question of who, between the manufacturers and the retailers, earn the most profits in the three scenarios studied above.

Proposition 8. *For scenario $C = 0$, the manufacturer’s profit, $\Pi^{M,C=0}$, is always greater than the retailer’s profit, $\Pi^{R,C=0}$.*

Proof. A comparison of expressions (6) and (7) gives the result. ■

This scenario offers the possibility to the retailer to fully or partially pocket the trade deal monies received from the manufacturer and increase her unit margin. Proposition 8 claims the manufacturer still earns more profit than the retailer. Remember that the manufacturer’s decision to offer trade deals critically depends on a condition that inversely links two parameters:

⁵We thank one of the reviewers for asking us to investigate how promotional activities impact on the double marginalisation issue. Our analysis indicates that, under some identified conditions, promotional activities reduce the retail price and increase the overall channel profits compared to the benchmark scenario.

consumer sensitivity to the promotional price differential and the incremental promotional sales effect. In such a context, it is also in the retailer's best interest to partially or fully pass on to consumers the price discount offered by the manufacturer to maximize either the first-period or second-period sales. As a result, the manufacturer's trade deal offering enhances the profits of the two channel members, but does not dramatically contribute to a shift of channel profits from the manufacturer to the retailer.

Proposition 9. *For scenario $d = 0$, the manufacturer's profit, $\Pi^{M,d=0}$, can be greater or lesser than the retailer's profit, $\Pi^{R,d=0}$, depending on the values of the parameters. Specifically, numerical simulations for $\mu \in \{0.1, 0.25, 0.5, 1\}$, $\gamma \in \{-1, -0.5, 0, 0.5, 1\}$, $\alpha \in (0, 1]$ and $\theta \in [0, \alpha]$, allow us to conclude that in the following cases:*

- $\mu \in \{0.1, 0.25\}, \gamma \in \{-1, -0.5, 0, 0.5, 1\}$;
- $\mu = 0.5, \gamma \in \{0.5, 1\}$;
- $\mu = 1, \gamma = 1$;

$$\Pi^{R,d=0} > \Pi^{M,d=0} \Leftrightarrow \bar{\theta}(\alpha) < \theta < \alpha,$$

where

- for γ and μ fixed, $\bar{\theta}(\alpha)$ increases as α increases;
- for γ and α fixed, $\bar{\theta}(\alpha)$ increases as μ increases.

We offer in Figures 5 and 6 an illustration of how μ and γ affect the sharing of channel members' profits in the parameter space. In Figure 5, $\mu = 0.25$ and $\gamma \in \{-1, -0.5, 0, 0.5, 1\}$, while in Figure 6, $\mu = 0.75$ and $\gamma \in \{-1, -0.5, 0, 0.5, 1\}$.

There exist four different areas in Figures 5 and 6: the black area where $C^{d=0}$ is not positive, the light gray area where the manufacturer earns more profit than the retailer, the dark gray area where the retailer earns more profit than the manufacturer, and the white area where the condition $\theta \leq \alpha$ is not satisfied. Figure 5 shows that when the redemption rate is relatively small, the area where the retailer earns more profit than the manufacturer grows with the effect of the incremental promotional sales effect. This finding holds regardless of whether the incremental promotional sales effect is positive or negative. Figure 6 demonstrates that when the redemption rate is high enough, the manufacturer is always better off than the retailer.

This scenario offers the possibility to the retailer to increase her first-period price to take advantage of the coupon offering and increase her unit margin. Our assessment of the double marginalization problem in this context indicates that, in some areas of the parameter space, for example, when $\mu = 0.25$ and $\gamma \in \{0, 0.5, 1\}$, depending on the values of θ and α , the offering of coupons reduces the total channel profits and increases retail prices compared to the benchmark model. As a rational manufacturer offers coupons to consumers only if they improve his benchmark profit, the overall impact of coupons, in such a context, on the retailer's profit can only be negative. As a result, the manufacturer's relative share of channel profits increases. This is possible because, although the retailer can secure higher unit margins by increasing retail prices, the impact of such opportunistic behavior can significantly reduce the

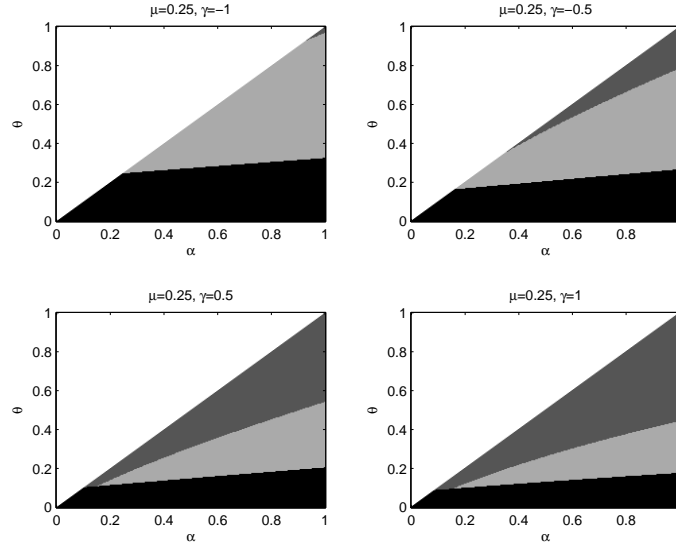


Figure 5: Parameter region where $C^{d=0} > 0$ and $\Pi^{M,d=0} < \Pi^{R,d=0}$: Dark gray area. $\mu = 0.25; \gamma \in \{-1, -0.5, 0.5, 1\}$.

total demand over the two periods, damaging her own overall profitability⁶. Situations where the retailer deliberately reduces retail prices to support the manufacturer’s couponing campaign are more likely to benefit the two channel members via a substantial increase of the overall two-period demand, especially when consumers are very sensitive to both promotional price differentials and coupons. Therefore, the increase of the retailer’s relative share of profit when the manufacturer offers on-pack coupons should be attributed to the combined promotional efforts of the two channel partners and their second-period effects.

Proposition 10. *For the scenario $\theta - \alpha\mu = 0$, numerical simulations for $\mu \in \{0.1, 0.25, 0.5, 1\}$, $\gamma \in \{-1, -0.5, 0, 0.5, 1\}$, $\alpha \in (0, 1]$ and $\theta \in [0, \alpha]$, allow us to conclude that always $\Pi^R < \Pi^M$.*

⁶Further analyses indicate that an increase of μ simultaneously increases the second-period wholesale and retail prices, regardless of the value of other parameters, when $\gamma \leq 0$. On the other hand, when $\gamma > 0$, an increase of μ can simultaneously either increase or decrease the second-period prices.

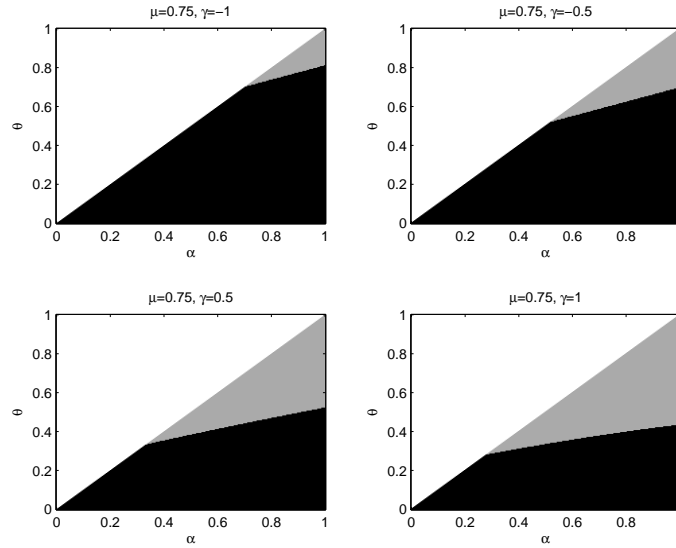


Figure 6: Parameter region where $C^{d=0} > 0$ and $\Pi^{M,d=0} > \Pi^{R,d=0}$: Light gray area. $\mu = 0.75; \gamma \in \{-1, -0.5, 0.5, 1\}$.

Recall that in this scenario, the manufacturer may offer both a trade deal to the retailer and coupons to consumers. The retailer can therefore fully or partially pocket the trade deal monies received from the manufacturer and/or increase retail price to take advantage of the manufacturer's promotional programs. The lion's share of the channel profits goes to the manufacturer, except in the very unlikely case where the effect of the incremental promotional sales effect and the coupon redemption rate are equal to 1 and consumers are very sensitive to promotional price differentials. This condition also implies that consumers are equally sensitive to coupons, regular prices and temporary price changes. As a result, although the retailer can increase her unit margins in this scenario, the increase of the overall demand for the two periods due to the manufacturer's promotional activities is not enough to transform the retailer to the biggest earner. As previously discussed, the situation is even worse when the total channel profit is reduced compared to the benchmark profit, as this occurs at the retailer's expense.

4. Conclusion and discussion

This paper studies a bilateral monopoly channel in which the manufacturer has the possibility of offering a trade deal to the retailer and/or on-pack coupons to consumers. Previous works have studied optimal promotional choices in situations where manufacturers offer trade deals or/and consumer promotional activities that directly and exclusively affect the first purchase. This paper extends this research stream by considering a consumer promotional activity (on-pack coupons) that stimulates both the first and repeat purchases. Table 3 summarizes the main findings of this research.

Table 3: Main findings

Two scenarios	$\theta - \alpha\mu = 0$	$\theta - \alpha\mu \neq 0$	
		$C = 0, \mu = 0$	$d = 0$
Promotion choice	trade deal and/or coupons	trade deal	coupons
Redemption rate impact	positive		positive
Manufacturer's preference	both or coupons at boundaries	trade deal	coupons
Biggest share of channel profit	manufacturer	manufacturer	retailer or manufacturer

The manufacturer has two opposite options: (1) exclusively use either one of the two promotional vehicles; or (2) offer various combinations of the two promotions that satisfy a specified condition, including setting the optimal promotions at the boundaries. Whether the manufacturer only offers coupons to consumers or offers both a trade deal and coupons, the manufacturer earns more profits when coupons are redeemed than when they are not redeemed. The manufacturer may have no choice but to offer only either a trade deal or coupons; however, when the two types of promotions can be offered simultaneously, the optimal levels of the two promotional vehicles are negatively and linearly related. In this particular configuration, however the manufacturer would prefer the exclusive offering of coupons to the exclusive offering of a trade deal. The retailer is more likely to earn more profit than the manufacturer only when the manufacturer solely offers on-pack coupons to consumers and the retailer supports the manufacturer's couponing campaign by also discounting the retail price to significantly increase the overall demand over the two periods. In the two other scenarios, the manufacturer still takes the lion's share of the channel profits, as in the benchmark model where a price promotion is not offered.

These findings have three major managerial and theoretical implications. First, the second-period redemption rate, which is inherent to on-pack coupons, plays a critical role in the manufacturer's decision to simultaneously offer a trade deal and coupons. When the redemption rate is zero, the manufacturer exclusively offers a trade deal to the retailer. Thus, a push-pull price promotion strategy that uses on-pack coupons to attract consumers is only feasible if it can stimulate second purchases. In other words, the manufacturers' pull price promotional activities, such as instant rebates (as in Martín-Herrán et al. (2010)) and peel-off promotions, which offer discounts up front to all buyers at the time of the first purchase and do not directly stimulate a second purchase should not be combined with push price promotions such as trade discounts.

Gerstner and Hess (1995) report a similar finding and stress the requirement of complementarity between pull and push price promotions when they have to be undertaken together. They support the view that pull price promotions should be combined to push price promotions to improve channel coordination when they are targeted to price-conscious consumers and made less accessible to price-insensitive consumers. Note however that, although our model can price discriminate, in the sense that price-insensitive consumers may not redeem their coupons, price discrimination is not a necessary condition for combining coupons and trade deals. Manufacturers can still find optimal to offer both trade deals and coupons even if all consumers redeem their coupons at the second period.

Second, the decision to offer either trade deal or on-pack coupons is not always based on the comparison of generated profits. Manufacturers can select either of them on an ad hoc basis using heterogeneous factors, overlooked in this work. On the other hand, when manufacturers really do have the option to offer either trade deals or pull price promotions that can stimulate first and second purchases, they make more profits by offering pull price promotions. Martín-Herrán et al. (2010) instead found that manufacturers are better (worse) off offering trade deals (cash rebates) when consumers are more (less) sensitive to promotions than to regular prices. Trade deals offered only to retailers and cash rebates to consumers act as price subsidies for the first-period purchases. Observe that our current setup goes beyond the impact of a temporary price incentive on the first-period purchases. As a matter of fact, not only do on-pack coupons stimulate first-period purchases at the regular price, on the basis of the promised discount, but they also encourage second purchases, because some coupon holders will redeem them to benefit from the promised discounts. Manufacturers incur promotional costs only when a consumer purchases a second unit. As a result, the overall impact of on-pack coupons on manufacturers' profits is far higher than any single trade deal program for retailers, where manufacturers incur promotional costs up front for each unit sold. In addition, the slippage phenomenon associated with on-pack coupons enhances their appeal for manufacturers (Yang et al., 2009). Chen et al. (2005) observed that, in some instances, the slippage is large enough to encourage sellers to offer the product for free after a rebate. Their Google search on the key words "free after rebate" generated 1,400,000 results. We performed a similar Google search and found around 14,800,000 results, which indicates a growing interest in promotional vehicles such as on-pack coupons that allow manufacturers to take advantage of slippage.

Finally, based on the comparison of channel members' total profits, the conventional wisdom stating that manufacturers' price promotions shift channel profits from the manufacturers to the retailers does not hold when trade deals are offered⁷. Conversely, when manufacturers exclusively offer on-pack coupons or other price-promotion activities that directly affect both first and repeat purchases, retailers can earn more profits than manufacturers under some identified conditions. As a result, our research adds to the current debate by showing that the on-pack coupon redemption rate as well as the incremental promotional sales effects are key for identifying when retailers are more likely to earn more profits than manufacturers. Particularly, retailers can get the lion's share of channel profits when the redemption rate is relatively small (large

⁷Martín-Herrán and Sigué (2011) demonstrate that, in some cases, the use of the share of the incremental promotional profit leads to different results. The manufacturer can still earn more total channel profit, while the retailer takes the lion's share of the incremental promotional profit due to the original structural advantage of the former in the benchmark model.

slippage) and the couponing campaign significantly expands the baseline demand. In such a context, both the manufacturer and retailer has mutual incentives to significantly invest in demand expansion. As a result, the retailer supports the manufacturer's couponing campaign by reducing retail prices. Contrary to the belief that retailers take advantage of manufacturers' promotional activities to increase their margins, the increase of retailers' share of channel profits, in such a context, should be attributed to the increase of the overall demand. It is not achieved at the expense of manufacturers' profits, which also increase. This is consistent with previous papers that support the view that positive long-term effects of promotions benefit both retailers and manufacturers (e.g. Ailawadi, 2001; Sigué, 2008).

Our theoretical setup is based on various limiting assumptions, including keeping the benchmark wholesale price unchanged during the first period of the game, restricting the game to two periods, using a linear demand function, preventing the retailer from stockpiling when the manufacturer offers a trade deal and disregarding horizontal competition. Some of these assumptions can be relaxed in future research to allow for the examination of more realistic scenarios and the use of other modeling approaches such as the Hotelling model to study horizontal interactions between competing retailers.

Appendix A. Proof of Proposition 2

The game is played in four stages and is solved backwards. We next describe and solve the problem the players are facing at each stage of the game.

Stage 4: At this stage of the game, the retailer chooses the second-period price, p_2 , in order to maximize her second-period profits. Therefore, the retailer's problem can be written as:

$$\max_{p_2} \Pi_2^R, \quad (\text{A.1})$$

where

$$\Pi_2^R = (p_2 - w_2)q_2,$$

denotes the retailer's second-period profits and q_2 is the second-period demand function given by

$$q_2 = g - p_2 + \gamma(q_1 - q_1^B) + \mu q_1 = g - p_2 + \gamma(g - p_1^B + \alpha(p_1^B - p_1) + \theta C - q_1^B) + \mu(g - p_1^B + \alpha(p_1^B - p_1) + \theta C). \quad (\text{A.2})$$

In the last equality, the expression of the demand function in the first period has been replaced. Recall that by p_1^B , w_1^B and q_1^B we denote the first-period wholesale, the regular retail price and the demand in the benchmark model, which are given in (1).

The solution to problem (A.1) gives us the retailer's reaction function, that is, p_2 , as a function of the wholesale price, w_2 , of the coupon offered by the manufacturer to consumers, C , and of the retail price in the first period, p_1 .

The retailer's second-period profits is a strictly concave function of her decision variable in this period, p_2 . From the first-order optimality condition for the problem in (A.1), the following expression can be derived:

$$p_2 = \frac{1}{2}(w_2 + g + (\gamma + \mu)(g - p_1^B + \alpha(p_1^B - p_1) + \theta C) - \gamma q_1^B). \quad (\text{A.3})$$

Stage 3: At this stage of the game the manufacturer chooses the second-period wholesale price, w_2 , in order to maximize his second-period profits. Therefore, the problem the manufacturer is facing can be written as:

$$\max_{w_2} \Pi_2^M, \quad (\text{A.4})$$

where

$$\Pi_2^M = (w_2 - c)q_2,$$

denotes the manufacturer's second-period profits and q_2 is the demand function in this period defined in (A.2). At this stage of the game, the manufacturer knows the retailer's pricing reaction function derived in Stage 4, and therefore, incorporates this information when deciding his optimal pricing strategy. Therefore, the reaction function in (A.3) has to be replaced in the manufacturer's objective function in (A.4).

The solution to this problem gives us the wholesale price, w_2 , as a function of the retail price in the first period, p_1 and the coupon offered by the manufacturer to consumers, C .

The manufacturer's second-period profits is a strictly concave function of his decision variable in this period, w_2 . From the optimality first-order condition for the problem expressed in (A.4) we get

$$w_2 = \frac{1}{2}(c + g + (\gamma + \mu)(g - p_1^B + \alpha(p_1^B - p_1) + \theta C) - \gamma q_1^B). \quad (\text{A.5})$$

Replacing this expression into the retailer's reaction function in (A.3) we obtain the second-period retail price as a function of the first-period retail price, p_1 and the coupon offered by the manufacturer to consumers, C :

$$p_2 = \frac{1}{4}(c + 3g + 3(\gamma + \mu)(g - p_1^B + \alpha(p_1^B - p_1) + \theta C) - \gamma q_1^B). \quad (\text{A.6})$$

The second-period retailer's and manufacturer's optimal profits are obtained replacing the expressions (A.5) and (A.6) and they are given by

$$\Pi_2^R = \frac{1}{256} ((g - c)(4 + \mu - \alpha(\gamma + \mu)) + 4(\gamma + \mu)(\alpha g - \alpha p_1 + \theta C))^2, \quad (\text{A.7})$$

$$\begin{aligned} \Pi_2^M &= \frac{1}{128} ((g - c)(4 + \mu - \alpha(\gamma + \mu)) + 4(\gamma + \mu)(\alpha g - \alpha p_1 + \theta C))^2 \\ &\quad - \mu C \left(g + \frac{1}{4}(c + 3g)(\alpha - 1) - \alpha p_1 + \theta C \right), \end{aligned} \quad (\text{A.8})$$

where p_1^B , w_1^B and q_1^B have been replaced by their values given in (1).

Stage 2: Moving to the first period the retailer chooses the retail price, p_1 , in order to maximize her total profits during the two periods:

$$\Pi^R = \Pi_1^R + \delta \Pi_2^R,$$

where $0 < \delta \leq 1$ is the discount rate applied to future profits and it has been fixed at 1 without loss of generality.

Taking into account the second-period retailer's profits (given by (A.7)) and the optimal retail and transfer price for this period computed in the previous stage ((A.6) and (A.5)), the retailer's total profits are as follows:

$$\begin{aligned}\Pi^R &= (p_1 - w_1^B + d)(g - p_1^B + \alpha(p_1^B - p_1) + \theta C) \\ &+ \frac{1}{256} ((g - c)(4 + \mu - \alpha(\gamma + \mu)) + 4(\gamma + \mu)(\alpha g - \alpha p_1 + \theta C))^2.\end{aligned}$$

It can be easily proved that the retailer's total profits is a strictly concave function in the retailer's first-period decision variable, p_1 , because $\alpha(\gamma + \mu)^2 - 16 < 0$ for the range of parameters we are considering.

Maximizing with respect to the first-period retail price gives the following optimal reaction function, i.e., the first-period retail price, p_1 , as a function of the deal offered by the manufacturer to the retailer, d , and the coupon offered by the manufacturer to consumers, C :

$$\begin{aligned}p_1 &= \frac{1}{4\alpha(16 - \alpha(\gamma + \mu)^2)} [32(g - p_1^B + \alpha(p_1^B + w_1^B) - \alpha d + \theta C) - 4(\gamma + \mu)^2\alpha(g\alpha + \theta C) \\ &- (g - c)\alpha(\gamma + \mu)(4 + \mu - \alpha(\gamma + \mu))].\end{aligned}\quad (\text{A.9})$$

Stage 1: At this stage of the game, the manufacturer chooses the deal, d , offered to the retailer and the coupon, C , offered to consumers in order to maximize his total profits:

$$\Pi^M = \Pi_1^M + \delta\Pi_2^M,$$

where future profits are discounted at rate δ that is assumed to be equal to 1.

Taking into account the second-period manufacturer's profits (given by (A.8)) and the optimal retail and transfer price for this period computed in the previous stages ((A.6) and (A.5)), the manufacturer's total profits are as follows:

$$\begin{aligned}\Pi^M &= (w_1^B - d - c)(g - p_1^B + \alpha(p_1^B - p_1) + \theta C) \\ &+ \frac{1}{128} ((g - c)(4 + \mu - \alpha(\gamma + \mu)) + 4(\gamma + \mu)(\alpha g - \alpha p_1 + \theta C))^2 \\ &- \mu C \left(g + \frac{1}{4}(c + 3g)(\alpha - 1) - \alpha p_1 + \theta C \right).\end{aligned}$$

Replacing the retailer's reaction function given in (A.9), the manufacturer's total profits to be maximized read:

$$\begin{aligned}\Pi^M &= \frac{1}{4(16 - \alpha(\gamma + \mu)^2)^2} \left\{ 2[4(\alpha d + \theta C)(\gamma + \mu) + (g - c)(8 + (\alpha - 1)\gamma + (\alpha + 1)\mu)]^2 \right. \\ &- \mu C(16 - \alpha(\gamma + \mu)^2) [32(\alpha d + \theta C) + (g - c)(8 - \alpha(\gamma^2 + \gamma(\mu - 4) - 4(\mu + 2)))] \left. \right\} \\ &+ (w_1^B - c - d) \left\{ g + p_1^B(\alpha - 1) + \theta C - \frac{1}{4(16 - \alpha(\gamma + \mu)^2)} [4[(\alpha c + \theta C)(8 - \alpha(\gamma + \mu)^2) - 8\alpha(d - c)] \right. \\ &+ (g - c)[8 - 3\alpha^2(\gamma + \mu)^2 + \alpha(40 - (\gamma + \mu)(4 + \mu))] \left. \right\}.\end{aligned}$$

As established in Proposition 1, if $\theta - \alpha\mu \neq 0$, then the maximum of function Π^M could only be attained at the boundaries.

First, in Proposition 2 we focus on the boundary solution $C = 0$. Maximizing function Π^M with respect to the deal offered by the manufacturer to the retailer, d , taking into account that $C = 0$, and hence, $\mu = 0$, the optimal expression $d^{C=0}$ given in (3) is obtained.

Replacing this expression in the retailer's reaction function given in (A.9), we obtain the optimal retail price for the first period.

The final expressions of the optimal pricing strategies and the manufacturer's and retailer's profits given in the statement of Proposition 2 can be obtained once p_1^B, w_1^B and q_1^B have been substituted by their values in (1).

Appendix B. Proof of Proposition 3

The derivation of the equilibrium strategies in Proposition 3 follows the same steps as in Proposition 2. Both proofs share Stages 4, 3 and 2. At Stage 1 now we focus on the boundary solution $d = 0$. Maximizing function Π^M with respect to the coupon offered by the manufacturer to consumers, C , taking into account that $d = 0$, the optimal expression $C^{d=0}$ given in (9) is obtained. Once this expression is known, the final expressions of the optimal pricing strategies and the manufacturer's and retailer's profits given in the statement of Proposition 3 can be easily derived.

Appendix C. Necessary and sufficient conditions for $C^{d=0} > 0$

Conjecture 1. For $\mu \in (0, 1]$ and $\gamma \in [-1, 1]$,

$$C^{d=0} > 0 \Leftrightarrow \underline{\alpha} < \alpha \leq 1 \text{ and } \underline{\theta}(\alpha) < \theta \leq \alpha. \quad (\text{C.1})$$

- For γ fixed, $\underline{\alpha}$ increases as μ increases.
- For μ fixed, $\underline{\alpha}$ decreases as γ increases.
- For γ and μ fixed, $\underline{\theta}(\alpha)$ increases as α increases.
- For μ and α fixed, $\underline{\theta}(\alpha)$ decreases as γ increases.
- For γ and α fixed, $\underline{\theta}(\alpha)$ increases as μ increases.

Appendix D. Proof of Proposition 6

The derivation of the equilibrium strategies in Proposition 6 follows the same steps as in Proposition 2. Both proofs share Stages 4, 3 and 2. At Stage 1 now assume $\theta - \alpha\mu = 0$ and from Proposition 1, Π^M is a concave function of the manufacturer's first-period decision variables (C and d) and, therefore, the manufacturer's problem admits an interior solution.

Maximizing function Π^M with respect to the deal offered by the manufacturer to the retailer, d , and with respect to the coupon offered by the manufacturer to consumers, C , the expression in (14) that establishes the relationship between the two manufacturer's decision variables at the optimal levels is obtained.

The final expressions of the optimal pricing strategies and the manufacturer's and retailer's profits given in the statement of Proposition 6 can be easily derived taking into account (14).

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