

The impact of foresight in a transboundary pollution game[☆]

Hassan Benchenkroun¹, Guiomar Martín-Herrán^{2,*}

Abstract

We study the impact of foresight in a transboundary pollution game; i.e. the ability of a country to control its emissions taking into account the relationship between current emissions and future levels of pollution and thus on future damages. We show that when all countries are myopic, i.e., choose the 'laissez-faire' policy, their payoffs are smaller than when all countries are farsighted, i.e., non-myopic. However, in the case where one myopic country becomes farsighted we show that the welfare impact of foresight on that country is ambiguous. Foresight may be welfare reducing for the country that acquires it. This is due to the reaction of the other farsighted countries to that country's acquisition of foresight. The country that acquires foresight reduces its emissions while the other farsighted countries extend their emissions. The overall impact on total emissions is ambiguous. Moreover, our results suggest that incentive mechanisms, that involve a very small (possibly zero) present value of transfers, can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of foresight and can be reimbursed by that country from the gains from foresight that it enjoys in the long run.

JEL Classification: C73, D90, Q59.

Keywords: myopia, differential games, transboundary pollution.

1. Introduction

It is well known that in a transboundary pollution game where emissions are a by-product of production and accumulate into a harmful stock pollutant, the non-cooperative equilibrium typically results in an over-polluted environment. Countries typically ignore the externality imposed on each other. An important feature of transboundary pollution games is that pollution emissions accumulate and therefore the action at any given moment has a lasting impact on the environment. The literature on dynamic pollution games (see Jørgensen et al. (2010) for a survey of dynamic pollution games and Bertinelli et al. (2014) and El Ouardighi et al. (2016)

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*Departamento de Economía Aplicada (Matemáticas), Universidad de Valladolid, Avda. Valle de Esgueva, 6, 47011, Valladolid, Spain. Tel.: +34 983 423330 Fax: +34 983 423299; E-mail: guiomar@eco.uva.es

¹Department of Economics & CIREQ, McGill University, Leacock Building, 855 rue Sherbrooke Ouest, Montreal, Quebec, H3A 2T7 Canada, Tel: +1 514 398 4400 Ext. 09060, E-mail: hassan.benchekroun@mcgill.ca

²IMUVA, Universidad de Valladolid

for recent contributions to this literature) typically considers from the outset that all the players are farsighted, i.e., able to have an environmental policy to control their respective emissions. In this paper we consider two types of behavior: (i) a country can be myopic and adopt a “laisser-faire” policy which amounts, in our framework to ignore the impact of its current emissions on the accumulation of pollution and (ii) a country can be farsighted and is able to control its emissions taking into account the impact of its emissions on the pollution stock.

A myopic country adopts a “laisser-faire” policy, which in a competitive market maximizes the benefit from consumption ignoring the damages caused by emissions. Using a “laisser-faire” policy can result from the inability (or unwillingness) of a government to legislate or to enforce an environmental regulation. Indeed, the authority to regulate polluting industries can require the passing of important pieces of legislation which can be a costly process. Thus, the analysis of the impact of foresight versus myopia is equivalent to the analysis of acquiring the means to regulate a polluting industry versus a “laisser-faire” policy. For example in the US, the Environmental Protection Agency gets the authority to write regulations from laws written by the US Congress, e.g. Clean Air Act or Clear Water Act. There is thus a first stage at the legislative level that authorizes the regulation agency to design and implement regulations. The “laisser-faire” scenario can be interpreted as a scenario describing a country that has not passed the legislation that authorizes regulation. We study in this paper the impact of the acquisition of foresight on the equilibrium outcome of the transboundary pollution game à la Dockner and Long (1993) or Ploeg and de Zeeuw (1992) where the instantaneous welfare of each country is given by a benefit from consumption minus the damage caused by the stock of pollution. While in the case of a local pollutant, the decision to abandon “laisser-faire” amounts to a cost benefit analysis of regulating a local industry by a single decision maker, in the context of a transboundary pollution problem, this decision has strategic ramifications that need to be taken into account. We compute a Markov Perfect Nash Equilibrium of a differential game of transboundary pollution where a subset of players are myopic.

We would like to point out that the notion of foresight used in this paper and the related literature above is distinct from the notion of foresight used in the coalition theory literature. In that literature, a farsighted player is a player that is assumed to take into account the impact of his decision to leave or join a coalition on other players’ decision to be in a coalition or not (see e.g., Diamantoudi and Sartzetakis (2002) in the case of environmental agreements within a static framework or Breton et al. (2010) and De Zeeuw (2008) within a dynamic framework³). The objective is to study the size of stable coalitions and whether large coalitions can be self-enforcing. In our paper we analyze the gains from foresight for a single player. Farsighted players in our framework are not assumed to jointly choose their emissions strategies as would a coalition of countries; they each maximize their own individual payoff while taking the strategies of the other countries as given.

In the case of a single decision maker the impact of foresight on the decision maker’s welfare is clearly positive. Indeed, a farsighted country can still choose the path chosen under myopia and therefore the acquisition of foresight will typically allow the country to attain a higher level of utility.

³For more details see Calvo and Rubio (2012), a recent survey of the literature that uses dynamic state-space games to analyze the formation of international agreements to control pollution.

In the case of several decision makers the impact of the acquisition of foresight turns out to be ambiguous, even in the limit case where the cost to acquire foresight is nil. It is in principle possible for a farsighted country to pick the same pollution path it would have chosen under myopia, however it is not necessarily true that the path chosen under myopia constitutes a best-response to the vector of strategies played by the other countries. The acquisition of foresight by a country typically induces that country to reduce its emissions compared to the case where it was myopic. The other countries respond to this reduction in emissions by increasing their emissions. In a multiplayer setup the reaction of other players (countries) is important and as it turns out can negate a goodwill gesture of a myopic country adopting a farsighted behavior (and reducing its emissions compared to the case where it was myopic). The sum of all countries' emissions may increase if one myopic player (country) became farsighted. This result is quite surprising, since myopia is generally associated with careless management of the environment and therefore one would assume that environmental quality unambiguously improves when a myopic country becomes farsighted. For a given total number of countries, the quality of the environment is not a monotonic function of the number of myopic countries. This is true for the short run only; we show that the acquisition of foresight by a country results in a decrease of the steady-state level of pollution. Thus, in the long run foresight results in a better quality of the environment.

We also examine the change in welfare from the acquisition of foresight and show that in a transboundary pollution game, contrary to a single decision maker problem, it can be negative. This can happen when the value of the damage parameter or when the stock of pollution is large enough. This is a rather pessimistic result since it is precisely in circumstances where pollution causes severe damage or when the stock of pollution is large that one would like all the countries to be farsighted and reduce their emissions. Numerical simulations reveal that, starting from an initial stock of pollution such that the present value of the gains from foresight is zero, the instantaneous welfare path of a myopic country crosses from below the path of instantaneous welfare it would enjoy if it were farsighted. Therefore, the change in instantaneous welfare from the acquisition of foresight is initially negative before turning positive. This suggests that incentive mechanisms, that involve a very small (possibly zero) present value of transfers, can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of foresight and can be reimbursed by the country from the gains from foresight that it enjoys in the long run.

This work was inspired by related studies on the impact of myopia or naive behavior in the area of management and marketing (see, for example, Benčekroun et al. (2009) and Martín-Herrán et al. (2012), and references therein). Myopic behavior was also examined in the case of the fisheries. Sandal and Steinshamn (2004) examine the case of Cournot competition in the fisheries and allowed for the possibility that all or some players ignore the impact of their harvest on the resource dynamics. They considered the case where only one player is non-myopic and where the number of players is endogenous, and determined the condition under which a player becomes active. In contrast with Sandal and Steinshamn (2004) we find that if all players are myopic, then a player always benefits from unilaterally becoming non-myopic. This contrast can be explained by the fact that in their framework players are oligopolist in the market of output and therefore a change in the extraction of a player is met by a change (in the opposite direction) in the production of the myopic players as well as possible entry of new players. Whereas in the

context of our transboundary pollution game, the business as usual level of emissions of myopic players is not affected by the emissions of the player that switch from myopic to non-myopic behavior. Moreover in the case of transboundary pollution the number of countries involved is fixed and not endogenously determined.

The next section presents the model and gives the Markov-Perfect equilibrium of the differential game where a subset of countries is myopic. The comparison of the case where all countries are myopic to the case where all are farsighted is given in Section 3, and Section 4 gives the impact on the equilibrium outcomes of having one country changing from a myopic behavior to a farsighted behavior. Our results are summarized in Section 5.

2. Model

Consider $N + M$ countries indexed by $l = 1, \dots, N + M$. Each country produces a single consumption good, the production of which generates emission of a pollutant. The preferences of consumers and the emission-consumption trade-off functions are such that the instantaneous benefits of country l from $E_l \geq 0$, the emission rates of country l , is $AE_l - \frac{1}{2}E_l^2$. The objective of country l is to maximize the discounted sum of utility net of the environmental damage caused by the accumulated stock of pollution, P ,

$$\max_{E_l(t)} \int_0^\infty [AE_l(t) - \frac{1}{2}E_l^2(t) - \frac{s}{2}P^2(t)]e^{-rt} dt$$

where $s > 0$ is a damage parameter and $r > 0$ is the discount rate. The stock of pollution P accumulates according to

$$\dot{P}(t) = \sum_{l=1}^{N+M} E_l(t) - kP(t), \quad P(0) = P_0 \geq 0, \quad (1)$$

where $k > 0$ denotes the natural rate of decay.

We consider the case where M countries, indexed by $j = 1, \dots, M$, are myopic. The emissions of a myopic player correspond to the emissions under a business as usual scenario or a “laisser-faire” scenario in a market of a consumption good, denoted by C , the production of which generates emissions $E = C$; the utility from the consumption good is given by $AC - \frac{1}{2}C^2$. Therefore, the demand of the consumption good is given by $A - C$ and is being supplied by a competitive industry with a constant marginal cost of production assumed equal to zero without loss of generality. The “laisser-faire” policy would yield an emission $E = A$ in each myopic country. In our framework the myopic player policy amounts to a policy that does not take into account the damages from pollution and the accumulation of the stock of pollution and only maximizes the instantaneous benefit from consumption. However we would like to note that a myopic player is aware of the differential equation that governs the stock of pollution (1). The source of myopia is the inability to intervene⁴ and not the ignorance of (1)⁵.

⁴due to the lack of legislative authority or lack of political will.

⁵Throughout the paper we take N and M as exogenously given. Our analysis can serve as a preliminary step to solve a meta game where the number of myopic and farsighted players is endogenously determined (see Section 5 of the companion working paper version).

The other N countries, indexed by $i = 1, \dots, N$, are farsighted, i.e. they take into account (1) when maximizing their utility. The following vector of strategies where a myopic country chooses:

$$E_j^{M,N}(P) = A \quad (2)$$

and a non-myopic country chooses

$$E_i^{M,N}(P) = \begin{cases} A - \beta_i^{M,N} - \alpha_i^{M,N}P & \text{for } P \leq P_{M,N} \\ 0 & \text{for } P \geq P_{M,N} \end{cases} \quad (3)$$

where

$$\begin{aligned} \alpha_i^{M,N} &= \frac{1}{2(1-2N)} \left(2k + r - \sqrt{(2k+r)^2 + 4s(2N-1)} \right), \\ \beta_i^{M,N} &= \frac{A\alpha_i^{M,N}(M+N)}{k+r+\alpha_i^{M,N}(2N-1)}, \\ \mu_i^{M,N} &= -\frac{A^2 - 2A\beta_i^{M,N}(M+N) + (2N-1)(\beta_i^{M,N})^2}{2r} \end{aligned}$$

and

$$P_{M,N} = \frac{A - \beta_i^{M,N}}{\alpha_i^{M,N}},$$

constitutes a Markov-Perfect Nash equilibrium of the pollution game. This is a straightforward generalization of Dockner and Long (1993) to the case of several players (countries) and where a subset of players are myopic. Note that myopic players have a dominant strategy which is emitting at a rate A for all P , regardless of the strategies chosen by the other players. This implies that a myopic player does not need to be aware that there exists other type of players that are farsighted. On the other hand, farsighted players are assumed to know that M players are myopic. This can be motivated, for example, by the fact that the decision maker or the government in a myopic country is known to ignore the impact of its emissions on the stock of pollution and has undertaken no initiative to regulate and control the emissions of its country. In our framework, myopia yields the same outcome as ignoring the damage caused by emissions.

A farsighted country i 's value function is then

$$V_i^{M,N}(P) = -\frac{1}{2}\alpha_i^{M,N}P^2 - \beta_i^{M,N}P - \mu_i^{M,N},$$

and a myopic country j 's value function:

$$V_j^{M,N}(P) = -\frac{1}{2}\alpha_j^{M,N}P^2 - \beta_j^{M,N}P - \mu_j^{M,N},$$

where

$$\begin{aligned} \alpha_j^{M,N} &= \frac{s}{2k+r+2N\alpha_i^{M,N}}, \\ \beta_j^{M,N} &= \frac{(A(M+N) - N\beta_i^{M,N})\alpha_j^{M,N}}{k+r+N\alpha_i^{M,N}}, \\ \mu_j^{M,N} &= \frac{2(A(M+N) - N\beta_i^{M,N})\beta_j^{M,N} - A^2}{2r}. \end{aligned}$$

We would like to emphasize that even if a player is myopic he is still able to acknowledge the evolution of the stock pollution as given by (1) when computing his value function⁶. The source of myopia is the lack of power (or will) to regulate and not the ignorance of (1).

There exists an asymptotically stable steady-state pollution stock given by:

$$\begin{aligned} P_{ss}^{M,N} &= \frac{A(M+N) - N\beta_i^{M,N}}{k + N\alpha_i^{M,N}} \\ &= \frac{A(M+N)(k+r + \alpha_i^{M,N}(N-1))}{(k+r + \alpha_i^{M,N}(2N-1))(k + \alpha_i^{M,N}N)}. \end{aligned}$$

Note that $\alpha_i^{M,N}$ is positive and therefore $\beta_i^{M,N}$ and $P_{ss}^{M,N}$ are positive too.

Some preliminary remarks are worth making. We show in Appendix B that

$$\left(\beta_i^{M-1,N+1} - \beta_i^{M,N}\right) < 0 \text{ and } \left(\alpha_i^{M-1,N+1} - \alpha_i^{M,N}\right) < 0, \quad (4)$$

therefore

$$P_{M,N} < P_{M-1,N+1}. \quad (5)$$

Moreover it can be shown that $P_{M,N}$, the threshold value of the stock of pollution beyond which a farsighted country ceases production, can take negative values⁷. When $P_{M,N} < 0$, emissions of farsighted players, given by (3), are nil. This happens when the number of myopic players is large enough and the damage from emissions is large enough.

This is illustrated through the case where $M = N = 1$ for which we have

$$P_{11} > 0 \Leftrightarrow s < \underline{s} \equiv (3k + 2r)(k + r).$$

When $s \geq \underline{s}$, in the case where $M = N = 1$, the equilibrium emission strategies are given by $E_i^{1,1}(P) = 0$ and $E_j^{1,1}(P) = A$. When the damage parameter is large enough the farsighted country emits zero for all stocks of pollution. This does not happen if the myopic country becomes farsighted, i.e. when $M = 0$ and $N = 2$. In that case, since both are farsighted, both restrict their emissions as the stock increases, leaving for any value of s an interval of the stock of pollution where emissions are positive. This is true when $M = 0$ for all $N > 0$: we have $P_{0,N} > 0$. In the case where $M > 1$ a myopic country always emits at level A . When s is large enough the accumulation of the stock at rate A renders any marginal emissions too costly for the farsighted country.

3. Foresight vs. Myopia

In this section we compare countries' payoffs and emission strategies under two scenarios: (i) all countries are farsighted and (ii) all countries are myopic. More precisely, we compare the outcomes of the case where $(M, N) = (0, N)$ to the case where $(M, N) = (N, 0)$.

$$\Delta_0(P_0) \equiv V_i^{0,N}(P_0) - V_j^{N,0}(P_0) = \frac{2(X_1P_0^2 + X_2P_0 + X_3)}{(2N-1)(2k+r)(k+r)(r+\Gamma)^2},$$

⁶We give in Appendix A the details of the computation of such value function.

⁷Note that $P_{M,N}$ is simply a real number and not a stock per say and as such can take negative values.

for all $P_0 > 0$, where

$$\begin{aligned}
X_1 &= r(k+r) [k^2(2k+r)^2 + 2s(2N-1)(3k^2 + kr + (2N-1)s) \\
&\quad - k(k(2k+r) + 2(2N-1)s)\Gamma], \\
X_2 &= 2ANr [-k(k+r)(2k+r)^2 - (2N-1)s(2k^2 + 4kr + r^2 - 2(2N-1)s) \\
&\quad + (k(k+r)(2k+r) + (2N-1)rs)\Gamma], \\
X_3 &= A^2N^2 [r(k+r)(2k+r)^2 - 2s(2N-1)(kr - 2(2N-1)s) \\
&\quad - r((k+r)(2k+r) - 2(2N-1)s)\Gamma],
\end{aligned}$$

with

$$\Gamma = \sqrt{(2k+r)^2 + 4s(2N-1)}.$$

It can be proved that the discriminant of the quadratic polynomial $X_1P_0^2 + X_2P_0 + X_3$ is always negative, and hence, the sign of this polynomial coincides with the sign of its independent term, X_3 . Tedious but straightforward calculations have shown that X_3 is always positive, and therefore,

$$\Delta_0(P_0) > 0.$$

Thus for any initial stock of pollution, it is always true that countries' welfare is larger when they are all farsighted than when they are all myopic.

As far as the emissions are concerned,

$$E_i^{0,N}(P) - E_j^{N,0}(P) = \begin{cases} -\beta_i^{0,N} - \alpha_i^{0,N}P & \text{for } P \leq P_{0,N} \\ -A & \text{for } P \geq P_{0,N} \end{cases}$$

because $\alpha_i^{0,N} > 0, \beta_i^{0,N} > 0$, then

$$E_i^{0,N}(P) - E_j^{N,0}(P) < 0 \quad \text{for any value of } P \geq 0,$$

and thus independently of the initial stock of pollution, the individual emissions are larger when all countries are myopic than when there are all farsighted.

4. Unilateral acquisition of foresight

Suppose we have M myopic countries and N farsighted countries and that one myopic country, say player m , acquires foresight. We determine the impact of this acquisition of foresight on all countries' emission strategies, and on country m 's welfare.

4.1. Impact on emissions

4.1.1. Individual countries' emissions

Country m 's emission strategy changes from A to $E_i^{M-1,N+1}(P)$ with $A - E_i^{M-1,N+1}(P) > 0$ for all $P \geq 0$. Moreover since $(E_i^{M-1,N+1})'(P) < 0$, the larger the stock of pollution, the larger the reduction of emissions of country m .

While the emission strategy of the remaining $M - 1$ myopic countries is unchanged, clearly the equilibrium emission strategies of the N farsighted countries, change from $E_i^{M,N}(P)$ to $E_i^{M-1,N+1}(P)$.

$$E_i^{M,N}(P) - E_i^{M-1,N+1}(P) = \left(\beta_i^{M-1,N+1} - \beta_i^{M,N} \right) + \left(\alpha_i^{M-1,N+1} - \alpha_i^{M,N} \right) P.$$

From (4) and (5) we have

$$E_i^{M,N}(P) - E_i^{M-1,N+1}(P) < 0$$

for all $P < P_{M-1,N+1}$ and $E_i^{M,N}(P) - E_i^{M-1,N+1}(P) = 0$ for $P > P_{M-1,N+1}$.

Thus, farsighted countries react by increasing their emissions. The larger the stock of pollution, the larger the increase in emissions.⁸

4.1.2. Total emissions

The impact on total emissions is not a priori clear since country m reduces its emissions while the farsighted countries react by increasing their emissions.

We determine the impact of country m 's acquisition of foresight on total emissions. Let $E_W^{M,N}(P)$ denote total emissions when there are M myopic countries, N farsighted countries and when the stock of pollution is P . We have:

$$E_W^{M,N}(P) = MA + NE_i^{M,N}(P)$$

and

$$\begin{aligned} E_W^{M,N}(P) - E_W^{M-1,N+1}(P) &= \left(MA + NE_i^{M,N}(P) \right) \\ &\quad - \left((M-1)A + (N+1)E_i^{M-1,N+1}(P) \right) \end{aligned}$$

that is,

$$E_W^{M,N}(P) - E_W^{M-1,N+1}(P) = A - E_i^{M-1,N+1}(P) + N \left(E_i^{M,N}(P) - E_i^{M-1,N+1}(P) \right).$$

While $A - E_i^{M-1,N+1}(P) > 0$ we have $N \left(E_i^{M,N}(P) - E_i^{M-1,N+1}(P) \right) < 0$. The net effect on total emissions is unclear. We show that the direction of the change in total emissions is in fact ambiguous.

This result is quite surprising, since myopia is generally associated with careless management of the environment. In a multiplayer setup the reaction of other players (countries) is important and as it turns out can outweigh a goodwill gesture of a myopic player (country) adopting a farsighted behavior.

We illustrate this possibility through the simple case $M = N = 1$. The same type of qualitative results could be obtained for the general case $M, N > 1$. We first describe the different possible outcomes graphically and then give a more precise statement of the results in Proposition 1. In Lemma 4 in Appendix C, we show that $2\alpha^{0,2} > \alpha^{1,1}$ and therefore the slope

⁸Following the same arguments, it can be easily proved that the farsighted countries would increase their emissions if more than one myopic country acquires foresight.

of the sum of emissions' rules when $M = 0$ and $N = 2$ is steeper than the slope of the sum of emissions' rules when $M = N = 1$.

For the purpose of illustration we consider a numerical example where $A = 10, k = 0.1$ and $r = 0.1$. We cover different cases that can arise as the value of s changes in Figures 1 to 3. Throughout Figures 1, 2, 3 the solid line corresponds to $E_W^{0,2}(P)$ and the dashed curve to $E_W^{1,1}(P)$.

In Figure 1a $s = 0.03$: the acquisition of foresight by the myopic country results in a decrease of total emissions for all $P \geq 0$.

In Figure 1b $s = 0.07$, Figure 2a $s = 0.085$ and Figure 2b, $s = 0.1$: the impact of the acquisition of foresight by the myopic country has an ambiguous impact on total emissions.

In Figure 3, $s = 0.3$: the acquisition of foresight by the myopic results in a decrease of total emissions for all $P \geq 0$.

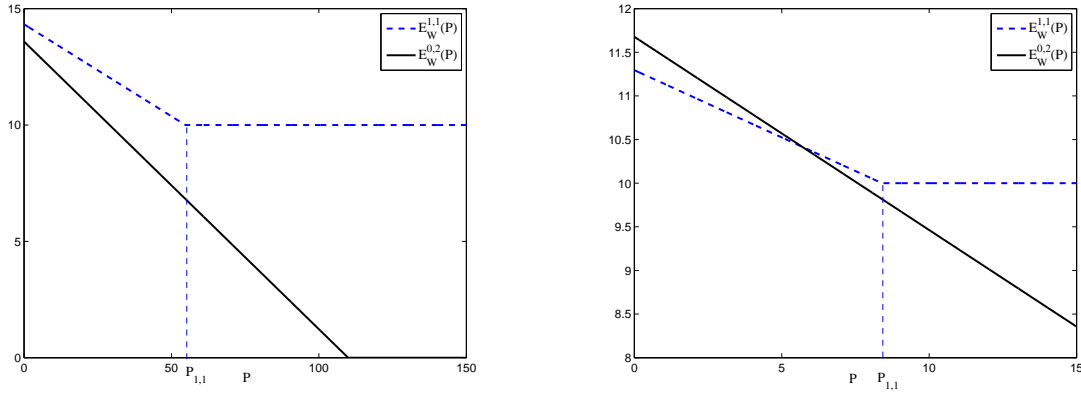


Figure 1: Comparison total emissions: $s = 0.03$ (left); $s = 0.07$ (right)

Proposition 1. Let $\underline{s} = (3k + 2r)(k + r)$ and $\bar{s} = (5k + 4r)(k + r) > \underline{s}$. There exists $\hat{s} < \underline{s}$ such that

(i) for $s \leq \hat{s}$ we have

$$E_W^{0,2}(P) < E_W^{1,1}(P) \text{ for all } P > 0;$$

(ii) for $\hat{s} < s < \bar{s}$ we have that there exists a unique $\tilde{P} > 0$ such that

$$E_W^{0,2}(P) > E_W^{1,1}(P) \Leftrightarrow P < \tilde{P};$$

(iii) for $s \geq \bar{s}$ we have

$$E_W^{0,2}(P) < E_W^{1,1}(P) \text{ for all } P \geq 0.$$

Proof. See Appendix C. ■

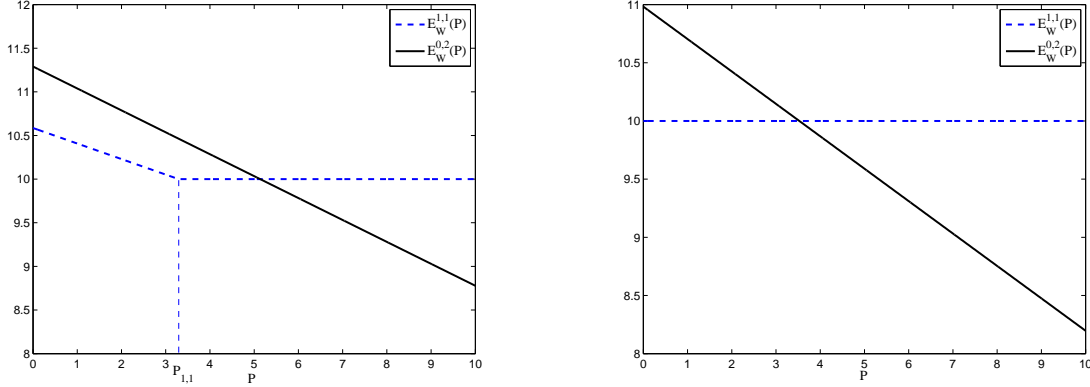


Figure 2: Comparison total emissions: $s = 0.085$ (left); $s = 0.1$ (right)

Part ii) of Proposition 1 shows an interesting and rather surprising possibility. It states that, holding the total number of countries fixed, the total level of emissions is not a monotonic function of the number of myopic countries.

We note that the threshold \tilde{P} in Proposition 1 ii) can be larger or smaller than $P_{1,1}$. Numerical simulations show that there exists a \tilde{s} such that $\tilde{P} < P_{1,1}$ iff $s < \tilde{s}$ with the following two possibilities $\hat{s} < \tilde{s} < \underline{s}$ or $\hat{s} < \underline{s} < \tilde{s}$, depending on the values of k and r . In the latter case for all $s < \underline{s}$ we have $\tilde{P} < P_{1,1}$.

The possibility that a unilateral environmentally friendly policy may backfire and result in an increase in global emissions was established in the seminal paper, Hoel (1991). However the motivation of the increase in emissions is different from the one in our framework. Indeed, in Hoel (1991) a unilateral reduction of emissions may weaken the threat point of a bargaining game over emission reductions and thereby result in an agreement with more emissions. In the non-cooperative game considered in Hoel (1991), a unilateral emission reduction results in a decrease of global emissions. In contrast, in our model, in the non-cooperative game a unilateral emission reduction may result in an increase of global emissions.

Proposition 2. *Suppose there are two countries, one farsighted and one myopic. The long-run level of the stock of pollution would decrease if the myopic country becomes farsighted:*

$$P_{SS}^{1,1} > P_{SS}^{0,2}.$$

Proof. See Appendix D. ■

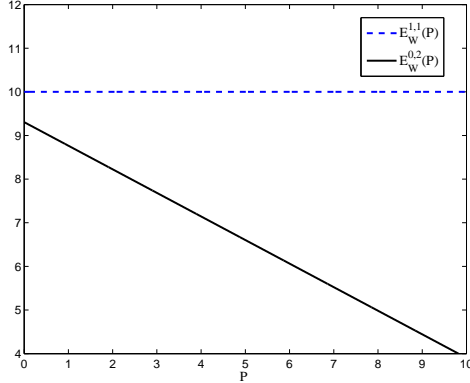


Figure 3: Comparison total emissions: $s = 0.3$

By means of numerical simulations we have obtained that the result of Proposition 2 holds for all $M, N \geq 1$:

$$P_{ss}^{M,N} > P_{ss}^{M-1,N+1}.$$

Thus the acquisition of foresight by a single country may result in an increase of overall pollution emissions, however it always results in a decrease of the steady-state stock of pollution.

4.2. Impact on the payoffs

We now examine the impact of the acquisition of foresight on countries' payoff. We can use the payoffs determined in Section 2. We evaluate the sign of the following difference of welfare

$$\Delta(P_0) = V_j^{M,N}(P_0) - V_i^{M-1,N+1}(P_0),$$

where we recall that $V_j^{M,N}(P_0)$ denotes the value function of a representative myopic country (player j) when there are M myopic countries and N farsighted countries; and $V_i^{M-1,N+1}(P_0)$ denotes the value function of a representative non-myopic country (player i) when there are $M - 1$ myopic countries and $N + 1$ farsighted countries. The sign of this difference $\Delta(P_0)$ will typically depend on the model parameters, in particular, the initial level of the stock of pollution and the damage parameter, s .

We have run numerical simulations to compare the difference for two different scenarios concerning the initial value of the stock of pollution. We have fixed different values of the discount rate r , $r \in \{0.1, 0.5, 1\}$ and different values of the regeneration rate k , $k \in \{0.1, \dots, 0.9\}$ and analyzed the difference $\Delta(P_0)$ as a function of the damage cost parameter s . For the

numerical simulations we have assumed $N \in \{2, \dots, 10\}$. The results below are true even in the case where $N = 1$. However they involve evaluating the value functions at corner solutions. Presenting the results for corner solutions would burden the notation and presentation without adding insight. We have opted to give the results in the case of interior solutions only.

The main findings are summarized below.

- **Result 1:** The case where $P_0 = 0$.

For any k, r and N fixed there exists $\tilde{s} > 0$ such that

$$\Delta(0) = V_j^{M,N}(0) - V_i^{M-1,N+1}(0) > 0 \text{ if } s > \tilde{s}.$$

The sign of the difference above is independent of the value of M , and then, result 1 remains valid for any value of $M \geq 1$.

We have *ceteris paribus*:

- \tilde{s} decreases as N increases.
- \tilde{s} increases as k increases.
- \tilde{s} increases as r increases.

Under this assumption, for any N, k, r and s such that $V_j^{M,N}(0) - V_i^{M-1,N+1}(0) > 0$, then it is better for a myopic country (belonging to a group of M symmetric myopic countries) to remain myopic than to be in the non-myopic group.

While the level of the initial stock is a priori arbitrary, a particular level of stock $P_0 = P_{ss}^{M,N}$ is a natural candidate to investigate.

- **Result 2:** The case where $P_0 = P_{ss}^{M,N}$

For any k, r and N fixed there exists $s^+ > 0$ such that

$$\Delta(P_{ss}^{M,N}) = V_j^{M,N}(P_{ss}^{M,N}) - V_i^{M-1,N+1}(P_{ss}^{M,N}) > 0 \text{ if } s > s^+. \quad (6)$$

The sign of the difference above is independent of the value of M , and then, result 2 remains valid for any value of $M \geq 1$.

We have, *ceteris paribus*:

- s^+ decreases as N increases.
- s^+ increases as k increases.
- s^+ increases as r increases.

The main implication of results 1 and 2 is that the unilateral acquisition of foresight can be welfare reducing for the country acquiring foresight. This is explained by the fact that a farsighted country pollutes less than when if it were myopic, and the fact that the other (farsighted) countries react by expanding their emissions. The overall impact of foresight may

well be negative. This disincentive to acquire foresight exists when the damage parameter is large enough.⁹

Remark 1. *From results 1 and 2 we have that the range of the damage parameter for which a myopic country has a positive incentive to acquire foresight is smaller the larger the number of countries N . Under a larger competition it is less likely that a myopic country has an incentive to acquire foresight.*

Remark 2. *The role played by the discount rate is not straightforward. The smaller the discount rate the smaller the range of the damage parameter under which a myopic country has a positive incentive to acquire foresight. This can be surprising since a more patient country would in principle value the information about the future more and would therefore gain more from foresight. This intuition does not carry over in a game theoretic setting. A smaller discount rate implies that the competitors of the myopic country value the future more, are more mindful of the long-run impact of emissions and its damage. This can eliminate the incentive of a myopic country to acquire foresight.*

Moreover the numerical simulations suggest that

$$s^+ > \tilde{s}$$

therefore we have

	$s < \tilde{s} < s^+$	$\tilde{s} < s < s^+$	$\tilde{s} < s^+ < s$
$\Delta(0)$	-	+	+
$\Delta(P_{ss}^{M,N})$	-	-	+

If s is large enough no country has an incentive to become farsighted¹⁰.

For $s \in (\tilde{s}, s^+)$ the sign of the gains from foresight depends on P_0 . The numerical simulations suggest that, if $\tilde{s} < s < s^+$ there exists a $\bar{P} \in (0, P_{ss}^{M,N})$ such that $\Delta(\bar{P}) = 0$ and $\Delta(P) < 0$ iff $P > \bar{P}$, a myopic country has an incentive to acquire foresight iff $P_0 > \bar{P} \in (0, P_{ss}^{M,N})$. Moreover, as the stock of pollution converges to the steady state, the incentive to acquire foresight remains positive.

We can also infer from this analysis that for \bar{P} such that $\Delta(\bar{P}) = 0$ the impact of the acquisition of foresight on the 'instantaneous' welfare of a country is not uniformly positive.

⁹We have checked the robustness of all these results when more than one myopic player acquires foresight. The sensitivity of the threshold \tilde{s} and s^+ to changes in the parameters is qualitatively similar, except for the number of foresight players, N . Both thresholds \tilde{s} and s^+ increase as N increases when more than one myopic player acquires foresight. This result is interesting and shows that the sensitivity of the threshold to the number of farsighted players N depends on the number of myopic players that acquire foresight simultaneously. Therefore, it is not clear in a game that, with more players, the set of parameters where a myopic player would gain from foresight shrinks.

¹⁰To compare threshold values \hat{s} , \underline{s} and \bar{s} in Proposition 1 (where $M = N = 1$) to the thresholds \tilde{s} and s^+ (from Results 1 and 2) we ran numerical simulations with $r \in \{0.1, 0.5, 1\}$ and $k \in \{0.1, \dots, 0.9\}$. In all the simulations we obtained $\tilde{s} < s^+ < \hat{s} < \underline{s} < \bar{s}$.

This is true from the fact that at the steady state the incentive to acquire foresight is strictly positive. Since the overall impact of foresight is nil, the impact of foresight in the short-run must be negative.

This is illustrated through an example. Consider the case where $N = 10$, $M = 1$, $A = 1$, $k = 0.1$, $r = 0.1$ we then have $\tilde{s} = 0.00635334$ and $s^+ = 0.0079026$. Let fix, for instance, $s = \frac{\tilde{s} + s^+}{2}$, then $\bar{P} = 18.9502$.

We consider the time path of the instantaneous welfare, i.e., for the myopic country

$$W_j^{M,N}(t) = \frac{A^2}{2} - \frac{s}{2} P_{M,N}^2(t),$$

and for a farsighted country

$$W_i^{M,N}(t) = A E_i^{M,N}(P_{M,N}(t)) - \frac{1}{2} \left(E_i^{M,N}(P_{M,N}(t)) \right)^2 - \frac{s}{2} P_{M,N}^2(t)$$

for $P_0 = \bar{P}$ and $P_{M,N}(t)$ the equilibrium path of the stock of pollution when there are M myopic countries and N farsighted countries. We can notice from Figure 4 that the instantaneous welfare path of a myopic country, $W_j^{M,N}(t)$, crosses from below the path of instantaneous welfare it would enjoy if it were farsighted, $W_i^{M-1,N+1}(t)$. Therefore, the change in instantaneous welfare from the acquisition of foresight is initially negative before turning positive. This suggests that incentive mechanisms can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of foresight and can in principle be reimbursed by the country from the gains from foresight that it enjoys in the long run. A very small net present value of transfers (possibly zero) can be enough to induce a myopic country to acquire foresight; facilitating access to credit can be, for example, an important tool to entice a country to acquire foresight.

From the same numerical simulations above we have obtained that the acquisition of foresight by country m results in an increase of all the other countries' payoffs. This is not straightforward because a 'friendly' unilateral action by the country that reduces its emissions (in this case the myopic country acquiring foresight) may be met by an excessive increase in emissions of the other farsighted countries that could end up reducing all countries' welfare. All the numerical simulations we carried point to an increase of all the other countries' welfare.

5. Concluding remarks

In this paper we have studied the impact of the ability to forecast the effect of current emissions on future levels of pollution and thus on future damages.

When all countries are farsighted their payoffs are larger than when all countries are myopic. However, we have shown that, in the case where one myopic country becomes farsighted, the welfare impact of foresight on that country is ambiguous. This is due to the reaction of the other farsighted countries to that country's acquisition of foresight. The country that acquires foresight reduces its emissions while the other farsighted countries extend their emissions. Global emissions may increase in the short run. In contrast, the sum of all countries' emissions at the steady state is a decreasing function of the number of non-myopic countries. In the long run foresight results in a better quality of the environment.

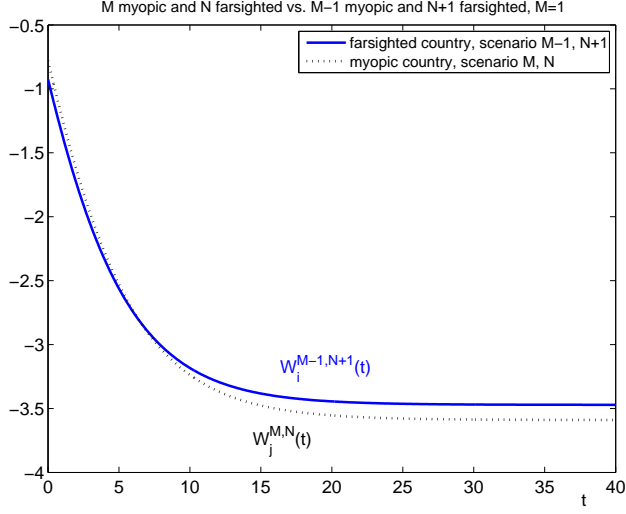


Figure 4: Comparison instantaneous welfare

The analysis of welfare shows that foresight may not benefit the country that acquires it when pollution causes severe damage or when the stock of pollution is beyond a certain threshold. Numerical simulations reveal that, starting from an initial stock of pollution such that the present value of the gains from foresight is zero, the change in instantaneous welfare from the acquisition of foresight is initially negative before turning positive. Incentive mechanisms can play an important role in inducing a country to adopt a farsighted behavior and diminishing the number of myopic countries. These incentives would compensate the myopic country for the short-run losses incurred from the acquisition of foresight and can in principle be reimbursed by the country from the gains from foresight that it enjoys in the long run. Access to credit may result in more countries being farsighted.

Appendix A: Computation of a myopic country j 's value function

The value function of the myopic player j , $V_j^{M,N}(P)$ is characterized as follows. In view of the linear-quadratic structure of the differential game, we assume a linear-quadratic value function form,

$$V_j^{M,N}(P) = -\frac{1}{2}\alpha_j^{M,N}P^2 - \beta_j^{M,N}P - \mu_j^{M,N}.$$

To obtain the coefficients $\alpha_j^{M,N}, \beta_j^{M,N}, \mu_j^{M,N}$ we use the equation

$$rV_j^{M,N}(P) = AE_j^{M,N}(P) - \frac{1}{2}\left(E_j^{M,N}(P)\right)^2 - \frac{s}{2}P^2 + \frac{dV_j^{M,N}(P)}{dP} \left[\sum_{l=1}^{N+M} E_l^{M,N}(P) - kP \right] \quad (7)$$

that must be satisfied for all $P \geq 0$.

This equation is obtained by differentiating totally with respect to time in

$$V_j^{M,N}(P, t) = \int_t^\infty \left[AE_j^{M,N}(z) - \frac{1}{2}\left(E_j^{M,N}(z)\right)^2 - \frac{s}{2}P^2(z) \right] e^{-r(z-t)} dz.$$

$$\begin{aligned} \frac{dV_j^{M,N}(P)}{dP} \dot{P} &= - \left(AE_j^{M,N}(P) - \frac{1}{2}\left(E_j^{M,N}(P)\right)^2 - \frac{s}{2}P^2 \right) \\ &+ r \int_t^\infty \left[AE_j^{M,N}(z) - \frac{1}{2}\left(E_j^{M,N}(z)\right)^2 - \frac{s}{2}P^2(z) \right] e^{-r(z-t)} dz. \end{aligned}$$

The coefficients of $V_j^{M,N}(P)$ then can be identified after substituting in (7) the optimal choice of the myopic player's emissions given in (2) as well as the optimal choice of the farsighted player's emissions given in (3).

Appendix B: Proof of $\alpha_i^{M-1, N+1} - \alpha_i^{M, N} < 0$, $\beta_i^{M-1, N+1} - \beta_i^{M, N} < 0$

$$\alpha_i^{M, N} = f(k, r, s, N) = \frac{1}{2(1-2N)} \left(2k + r - \sqrt{(2k+r)^2 + 4s(2N-1)} \right),$$

can be rewritten as

$$\alpha_i^{M, N} = f(k, r, s, N) = \frac{2s}{2k+r + \sqrt{(2k+r)^2 + 4s(2N-1)}}.$$

$f(k, r, s, N)$ is clearly decreasing in N ; the larger N , the smaller $f(k, r, s, N)$ and therefore $\alpha_i^{M-1, N+1} - \alpha_i^{M, N} < 0$.

Coefficient

$$\beta_i^{M, N} = \frac{A\alpha_i^{M, N}(M+N)}{k+r + \alpha_i^{M, N}(2N-1)}$$

after some manipulations and once the expression of $\alpha_i^{M, N}$ has been replaced reads:

$$\beta_i^{M, N} = \frac{A(M+N)2s}{(k+r) \left(2k+r + \sqrt{(2k+r)^2 + 4s(2N-1)} \right) + (2N-1)2s}.$$

With $M+N$ fixed, $\beta_i^{M, N}$ is a decreasing function of N , and therefore

$$\left(\beta_i^{M-1, N+1} - \beta_i^{M, N} \right) < 0.$$

Appendix C: Proof of Proposition 1

Lemma 1. $E_W^{0,2}(0) \underset{\leq}{\geq} A$ iff $s \underset{\leq}{\geq} \bar{s} = (5k + 4r)(k + r)$.

Proof.

$$\begin{aligned} E_W^{0,2}(0) > A &\Leftrightarrow E_W^{0,2}(0) = 2E_i^{0,2}(0) > A \Leftrightarrow 2(A - \beta_i^{0,2}) > A \Leftrightarrow A - 2\beta_i^{0,2} > 0 \\ &\Leftrightarrow A - \frac{A8s}{(k+r)(2k+r) + 6s + (k+r)\sqrt{(2k+r)^2 + 12s}} > 0 \\ &\Leftrightarrow (k+r)(2k+r) - 2s + (k+r)\sqrt{(2k+r)^2 + 12s} > 0. \end{aligned}$$

If $(k+r)(2k+r) - 2s > 0$ (i.e. $s < \frac{(k+r)(2k+r)}{2}$), then $E_W^{0,2}(0) > A$.

If $(k+r)(2k+r) - 2s < 0$ (i.e. $s > \frac{(k+r)(2k+r)}{2}$), then

$$E_W^{0,2}(0) > A \Leftrightarrow 4s((5k + 4r)(k + r) - s) > 0.$$

It can be easily proved that

$$\frac{(k+r)(2k+r)}{2} < (5k + 4r)(k + r).$$

Therefore, we can conclude that

$$E_W^{0,2}(0) > A \Leftrightarrow s < \bar{s} = (5k + 4r)(k + r).$$

■

Lemma 2. $P_{1,1} \underset{\leq}{\geq} 0$ iff $s \underset{\leq}{\geq} \underline{s} = (3k + 2r)(k + r)$

Proof.

$$\begin{aligned} P_{1,1} = \frac{A - \beta_i^{1,1}}{\alpha_i^{1,1}} > 0 &\Leftrightarrow A - \beta_i^{1,1} > 0 \Leftrightarrow \frac{A(4k + 3r - \sqrt{(2k+r)^2 + 4s})}{r + \sqrt{(2k+r)^2 + 4s}} > 0 \\ &\Leftrightarrow 4k + 3r - \sqrt{(2k+r)^2 + 4s} > 0 \\ &\Leftrightarrow (3k + 2r)(k + r) - s > 0 \\ &\Leftrightarrow s < \underline{s} = (3k + 2r)(k + r). \end{aligned} \tag{8}$$

■

Lemma 3. There exists $\hat{s} (< \underline{s})$ such that $E_W^{1,1}(0) - E_W^{0,2}(0) = 2\beta^{0,2} - \beta^{1,1} \underset{\leq}{\geq} 0$ iff $s \underset{\leq}{\geq} \hat{s}$.

Proof. We first show that there exists a unique root \hat{s} to $-\beta_i^{1,1} + 2\beta_i^{0,2} = 0$.

Let us denote

$$\begin{aligned}
h(s) &= -\beta_i^{1,1} + 2\beta_i^{0,2} \\
&= -\frac{4As}{(k+r)\left(2k+r+\sqrt{(2k+r)^2+4s}\right)+2s} \\
&\quad + \frac{8As}{(k+r)\left(2k+r+\sqrt{(2k+r)^2+12s}\right)+6s}.
\end{aligned}$$

Let us note that

$$h(0) = 0, \quad h'(0) = \frac{2A}{(k+r)(2k+r)} > 0, \quad \lim_{s \rightarrow \infty} h(s) = -\frac{2A}{3} < 0.$$

Therefore, when s is small enough we have $h > 0$, while for s big enough we have $h < 0$. We want to prove that function h decreases monotonically and therefore, there exists a unique \hat{s} such that

$$h(s) = -\beta_i^{1,1} + 2\beta_i^{0,2} > 0 \Leftrightarrow s < \hat{s}. \quad (9)$$

After some manipulations we have:

$$\begin{aligned}
h(s) &= -\beta_i^{1,1} + 2\beta_i^{0,2} > 0 \\
\Leftrightarrow g(s) &= -\frac{2s}{k+r} + 2k+r - \sqrt{(2k+r)^2+12s} + 2\sqrt{(2k+r)^2+4s} > 0.
\end{aligned}$$

If we prove that $g'(s) < 0$ for $s > 0$, then h decreases monotonically.

$$\begin{aligned}
g'(s) &= -\frac{2}{k+r} - \frac{6}{\sqrt{(2k+r)^2+12s}} + \frac{4}{\sqrt{(2k+r)^2+4s}}. \\
\lim_{s \rightarrow 0} g'(s) &= -2 \left[\frac{1}{k+r} + \frac{1}{2k+r} \right] < 0; \quad \lim_{s \rightarrow \infty} g'(s) = -\frac{2}{k+r} < 0.
\end{aligned}$$

Let see that the maximum value of $g'(s)$ is negative. The possible extrema of $g'(s)$ are the roots of

$$g''(s) = 0.$$

$$g''(s) = -2 \left[-18 \left((2k+r)^2 + 12s \right)^{-3/2} + 4 \left((2k+r)^2 + 4s \right)^{-3/2} \right].$$

The unique possible root of $g''(s) = 0$ is given by:

$$\check{s} = (2k+r)^2 \frac{2^{-2/3} - 9^{-2/3}}{4(3 \times 9^{-2/3} - 2^{-2/3})}.$$

$$g'(\check{s}) = -2 \left[\frac{1}{k+r} + \frac{3}{(2k+r)\sqrt{1+12a}} - \frac{2}{(2k+r)\sqrt{1+4a}} \right],$$

where

$$a = \frac{2^{-2/3} - 9^{-2/3}}{4(3 \times 9^{-2/3} - 2^{-2/3})} \simeq 1.5727$$

Therefore,

$$\begin{aligned} g'(\check{s}) &< -2 \left[\frac{1}{2k+r} + \frac{3}{(2k+r)\sqrt{1+12a}} - \frac{2}{(2k+r)\sqrt{1+4a}} \right] \\ &= -\frac{2}{2k+r} \left[1 + \frac{3}{\sqrt{1+12a}} - \frac{2}{\sqrt{1+4a}} \right] \\ &\simeq -\frac{2}{2k+r} 0.9323 < 0. \end{aligned}$$

The value of $g'(s)$ evaluated at its extremum is negative, so it is $g'(s)$ for all $s > 0$.

Let us argue by contradiction and assume that there exists s^* such that $g'(s^*) \geq 0$. Then, necessarily g' possesses at least a maximum point \underline{s} and the value of g' at this point is positive. So that, under the hypothesis that there exists s^* such that $g'(s^*) > 0$, there exists s^{**} such that $g''(s^{**}) = 0$ and $g'(s^{**}) \geq 0$. Since we have proved that there is at most one zero (\check{s}) of $g''(s) = 0$, necessarily $s^{**} = \check{s}$. But we have computed that $g'(\check{s}) < 0$, in contradiction with $g'(s^{**}) > 0$. So that the hypothesis that there exists s^* such that $g'(s^*) \geq 0$ is false, and $g'(s) < 0$ for all s .

We secondly prove that $\hat{s} < \underline{s}$.

From (9) we know that there exists \hat{s} such that $E_W^{0,2}(0) = E_W^{1,1}(0) > A$.

For convenience we use the notation $E_{W, s=\hat{s}}^{M,N}(0)$ to indicate that the value of $E_W^{M,N}(0)$ is evaluated at $s = \hat{s}$, and we have:

$$E_{W, s=\hat{s}}^{0,2}(0) = E_{W, s=\hat{s}}^{1,1}(0) > A.$$

Moreover, $P_{1,1} = 0$ when $s = \underline{s}$, that is

$$E_{W, s=\underline{s}}^{1,1}(0) = A.$$

Therefore we have

$$E_{W, s=\hat{s}}^{1,1}(0) > E_{W, s=\underline{s}}^{1,1}(0),$$

or

$$E_{i, s=\hat{s}}^{1,1}(0) > E_{i, s=\underline{s}}^{1,1}(0) = 0.$$

From the fact that $E_W^{1,1}(0)$ (or $E_i^{1,1}(0)$) is a decreasing function of s we have that $\hat{s} < \underline{s}$. ■

Lemma 4. $2\alpha^{0,2} - \alpha^{1,1} > 0$.

Proof.

$$\begin{aligned} 2\alpha_i^{0,2} - \alpha_i^{1,1} > 0 &\Leftrightarrow 2s \left[\frac{2}{2k+r+\sqrt{(2k+r)^2+12s}} - \frac{1}{2k+r+\sqrt{(2k+r)^2+4s}} \right] > 0 \\ &\Leftrightarrow 2k+r+2\sqrt{(2k+r)^2+4s} - \sqrt{(2k+r)^2+12s} > 0. \end{aligned}$$

Because $2\sqrt{(2k+r)^2+4s} - \sqrt{(2k+r)^2+12s}$ is positive, therefore, $2\alpha_i^{0,2} - \alpha_i^{1,1} > 0$. ■

Proof of Proposition 1

If $s \leq \hat{s}$, then, from Lemmas 1 and 3, $A < E_W^{0,2}(0) \leq E_W^{1,1}(0)$ with $E_W^{0,2}(0) = E_W^{1,1}(0)$ only for $s = \hat{s}$. This along with $2\alpha^{0,2} - \alpha^{1,1} > 0$ (from Lemma 4) yields (i).

If $\hat{s} < s < \bar{s}$ we distinguish two cases:

a) if $\hat{s} < s < \underline{s}$, then from Lemma 3, $E_W^{0,2}(0) > E_W^{1,1}(0) > A$ and therefore since from Lemma 4, $2\alpha^{0,2} > \alpha^{1,1} > 0$ and at $P_{0,2}$ we have $E_W^{0,2}(P_{0,2}) = 0 < E_W^{1,1}(P_{0,2}) = A$, there exists a unique \tilde{P} such that

$$E_W^{0,2}(P) > E_W^{1,1}(P) \Leftrightarrow P < \tilde{P}.$$

b) if $\underline{s} \leq s < \bar{s}$. The same arguments invoked in a) apply to obtain (ii). The only difference with case a) is that from Lemma 1, $E_W^{1,1}(P) = E_W^{1,1}(0) = A$ for all $P \geq 0$.

If $\underline{s} < \bar{s} \leq s$ then $E_W^{0,2}(0) < A$ and $E_W^{1,1}(P) = 0$ for all P since from Lemma 2, $P_{1,1} \leq 0$. From the fact that $E_W^{0,2}(\cdot)$ is a strictly decreasing function of P we conclude (iii).

Appendix D: Proof of Proposition 2

For $s < \underline{s}$

$$\begin{aligned} P_{SS}^{1,1} > P_{SS}^{0,2} &\Leftrightarrow -8As(-8k - 7r + \sqrt{(2k+r)^2 + 12s}) > 0 \\ &\Leftrightarrow -8k - 7r + \sqrt{(2k+r)^2 + 12s} < 0 \\ &\Leftrightarrow -12((5k+4r)(k+r) - s) < 0. \end{aligned}$$

Because we are assuming $s < \underline{s} < \bar{s} = (5k+4r)(k+r)$, then, $P_{SS}^{1,1} > P_{SS}^{0,2}$.

For $\underline{s} < s < \bar{s}$

$$\begin{aligned} P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} &\Leftrightarrow 4k^2 - 12s + 5kr + k\sqrt{(2k+r)^2 + 12s} < 0 \\ &\Leftrightarrow -12(k^4 + 3k^3r + k^2(2r^2 - 9s) - 10krs + 12s^2) < 0 \\ &\Leftrightarrow (s - s_1)(s - s_2) > 0, \end{aligned}$$

where

$$s_{1,2} = \frac{1}{24}k \left(9k + 10r \pm \sqrt{33k^2 + 36kr + 4r^2} \right), \quad (10)$$

$0 < s_1 < s_2$.

Therefore,

$$P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} \Leftrightarrow s < s_1 \text{ or } s > s_2.$$

It can be proved that $\underline{s} > s_2$, and then, $s > \underline{s}$ implies $s > s_2$, and $P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2}$.

For $s > \bar{s}$ it can be shown that $P_{SS}^{0,2} > P_{SS}^{1,1}$. Indeed we have

$$P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2} \Leftrightarrow s < s_1 \text{ or } s > s_2,$$

where s_1, s_2 are given in (10).

It can be proved that $\bar{s} > s_2$, and then, $s > \bar{s}$ implies $s > s_2$, and $P_{SS}^{1,1} = \frac{A}{k} > P_{SS}^{0,2}$.

References:

- [1] Benchekroun, H., Martín-Herrán, G., Taboubi, S., 2009. Could myopic pricing be a strategic choice in marketing channels? A game theoretic analysis. *Journal of Economic Dynamics & Control* 33, 1699–1718.
- [2] Bertinelli, L., Camacho, C., Zou, B., 2014. Carbon capture and storage and transboundary pollution: A differential game approach. *European Journal of Operational Research* 237, 721–728.
- [3] Breton, M., Sbragia, L., Zaccour, G., 2010. A dynamic model for international environmental agreements. *Environmental and Resource Economics* 45, 25–48.
- [4] Calvo, E., Rubio, S., 2012. Dynamic models of international environmental agreements: A differential game approach. *International Review of Environmental and Resource Economics* 6, 289–339.
- [5] Diamantondi, E., Sartzetakis, E.S., 2002. International environmental agreements - the role of foresight. *Core Discussion Paper* 2002/6.
- [6] Dockner, E., Long, N.V., 1993. International pollution control: Cooperative versus non-cooperative strategies. *Journal of Environmental Economics and Management* 24, 13–29.
- [7] El Ouardighi, F., Sim, J., Kim, B., 2016. Pollution accumulation and abatement policy in a supply chain. *European Journal of Operational Research* 248, 982–996.
- [8] Hoel, M., 1991. Global environmental problems: The effects of unilateral actions taken by one country. *Journal of Environmental Economics and Management* 20, 55–70.
- [9] Jørgensen, S., Martín-Herrán, G., Zaccour, G., 2010. Dynamic games in the economics and management of pollution. *Environmental Modeling and Assessment* 15, 433–467.
- [10] Martín-Herrán, G., Taboubi, S., Zaccour, G., 2012. Dual role of price and myopia in a marketing channel. *European Journal of Operational Research* 219, 284–295.
- [11] Ploeg, F., de Zeeuw, A.J., 1992. International aspects of pollution control. *Environmental and Resource Economics* 2(2), 117–139.
- [12] Sandal, L.K., and S. I. Steinshamn, 2004. Dynamic Cournot-competitive harvesting of a common pool resource. *Journal of Economic Dynamics & Control* 28, 1781–1799.
- [13] Zeeuw, A.J. de, 2008. Dynamics effects on the stability of international environmental agreements. *Journal of Environmental Economics and Management* 554, 163–174.

Legends:

Figure 1a: Comparison total emissions: $s = 0.03$

Figure 1b: Comparison total emissions: $s = 0.07$

Figure 2a: Comparison total emissions: $s = 0.085$

Figure 2b: Comparison total emissions: $s = 0.1$

Figure 3: Comparison total emissions: $s = 0.3$

Figure 4: Comparison instantaneous welfare