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www.elsevier.com/locate/ijpe

PII: S0925-5273(16)30330-9
DOI: <http://dx.doi.org/10.1016/j.ijpe.2016.11.007>
Reference: PROECO6580

To appear in: *Intern. Journal of Production Economics*

Received date: 6 April 2016
Revised date: 3 October 2016
Accepted date: 10 November 2016

Cite this article as: Salma Karray, Guiomar Martín-Herrán and Simon-Pierre Sigué, Cooperative Advertising for Competing Manufacturers: The Impact of Long-Term Promotional Effects, *Intern. Journal of Production Economics* <http://dx.doi.org/10.1016/j.ijpe.2016.11.007>

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Cooperative Advertising for Competing Manufacturers: The Impact of Long-Term Promotional Effects †

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Abstract

The effectiveness of cooperative advertising programs is studied in a market where two competing manufacturers deal with an exclusive retailer and two products. Two two-stage game theoretic models are developed to analyze the long-term effects of retailer's promotions, which can be positive or negative, on the effectiveness of cooperative advertising. Closed-form equilibrium solutions are obtained and compared. We find that the level of product substitutability and the sign and magnitude of the long-term effects of retailer's promotions on sales determine whether cooperative advertising should be offered and accepted by the manufacturers and retailer. In particular, depending on the level of product substitutability, cooperative advertising can benefit both the manufacturers and retailer even when retailer's promotions negatively affects future sales. Conversely, it may not be in the interest of the manufacturers to offer cooperative advertising when the products are fairly undifferentiated regardless of the nature of the

*Research of the first author is supported by the National Sciences and Engineering Council of Canada (NSERC). Grant # 1509. The second author's research is partially supported by MEC under project ECO2014-52343-P, co-financed by FEDER funds and the COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy evaluation".

†The authors thank the editor and three anonymous reviewers for their useful comments and suggestions in improving this manuscript.

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long-term effects of promotions. Finally, the manufacturers and retailer may refuse to respectively offer or participate in cooperative advertising programs that enhance total channel profits.

Keywords: Cooperative Advertising; Supply Chain Management; Game theory; Marketing-OR interface.

1 Introduction

American businesses offered about \$36 billion in cooperative advertising programs to their vendors in 2015, which represents 12% of their total advertising costs (Borrell Associates Report 2015). These programs are promotional incentives manufacturers offer to support retailers' advertising and promotional activities for their products. Yet, the conditions under which cooperative advertising programs benefit all channel members under various channel structures are not fully known. Casual observations however show that manufacturers select the type of retailer advertising and promotions that they want to support (see examples in the Co-op Advertising Programs Sourcebook¹). For instance, some manufacturers do not sponsor their retailer advertising that can negatively affect their brand equity and sales over time. On the other hand, some retailers choose not to participate in manufacturers' cooperative advertising programs.

There is an extensive number of works that look at the benefits of cooperative advertising for channel members. Aust and Buscher (2014) and Jørgensen and Zaccour (2014) provide two comprehensive reviews of this literature. The majority of these works uses static games and investigates cooperative advertising in bilateral monopoly contexts, where a manufacturer sells her product to a single retailer. The main finding of these static models is that cooperative advertising can be effective in boosting retailer's advertising, expanding demand, and ultimately increasing profits for each channel member (e.g., Dant and Berger 1996; Jørgensen et al. 2000; Huang and Li 2001; Yue et al. 2006; Karray and Zaccour 2006; Xie and Ai 2006; Yan 2010; Kunter 2012; Yang et al. 2013; Karray 2013). In addition, a few bilateral monopoly models use differential games and consider that advertising contributes to building brand equity (goodwill) over time. In this category, research has considered promotional decisions only and found that cooperative advertising improves channel members' profits (Jørgensen et al. 2000; Jørgensen et al. 2003; Karray and Zaccour 2005; He et al. 2009). These dynamic models assume that retailer's promotional activities have positive long term effects on

¹See http://www.co-opsourcebook.com/coop_sample.htm

sales (by increasing the goodwill stock). A notable exception is the paper by Jørgensen et al. (2003), which considers that the retailer's promotional activities damage the manufacturer's brand image (goodwill). They show that cooperative advertising can be profit-improving for products with low brand image or with intermediate brand image, especially when retail promotions are not highly detrimental to the brand.

A few studies have expanded these results to the case where some competition arises in the channel. Considering a static framework, Bergen and John (1997) used a consumer-based model and showed that cooperative advertising programs can benefit a single manufacturer selling through multiple retailers. Karray and Zaccour (2007) modeled advertising decisions alone and found that cooperative advertising can lead to a prisoner's dilemma situation for manufacturers when competition arises at both levels of the channel. Other researchers have recently showed that these programs are not always effective for competing retailers (Karray and Amin 2015; Liu et al. 2014). Karray and Amin (2015) consider a channel where a manufacturer sells through two competing retailers. They show that the levels of pricing and advertising competition in the marketplace significantly affect the effectiveness of cooperative advertising in coordinating the channel. Liu et al. (2014) consider a two-manufacturers, two-retailers channel and evaluate the effectiveness of cooperative advertising assuming exogenous cooperative advertising rates. They find that these programs do not benefit the channel if they lead to a significant decrease in the channel members' unit margins.

Considering only advertising decisions, and modeling the dynamic effects of advertising in a channel with competing retailers, other papers have studied the effects of differential cooperative advertising rates (e.g., He et al. 2011; Chutani and Sethi 2012). They show that the retailers might not benefit from cooperative advertising taking into account advertising decisions in their model. Adding pricing decisions while considering a fixed total market demand, Chutani and Sethi (2014) show (numerically) that cooperative advertising does not benefit competing retailers.

These findings point to three important takeaways. First, while cooperative advertising is shown to be beneficial for firms in bilateral monopolistic channels, this result does not always hold in markets where at least one channel member faces competition. Second, it is important to model pricing decisions in addition to the retailers' marketing efforts when assessing the effectiveness of cooperative advertising. This is because these programs impact the channel members' unit margins, which in turn affects the profitability of cooperative advertising. Third, the long-term effects on demand matter, especially when retailer's promotions can harm the brand image over time and competitive interactions in the channel are modeled.

This paper builds on these three lessons from the literature. It investigates the effectiveness of cooperative advertising in a context where competing manufacturers offer cooperative advertising programs to a single retailer in a two-period planning horizon. In this context, the research questions are: Should the manufacturers offer and the retailer accept cooperative advertising programs? Could cooperative advertising arrangements **benefit the entire channel**? To answer these questions, we develop and analyze two analytical models. In the Cooperative Advertising (CA) model, each of the two manufacturers sets the wholesale price of her product in each period and determines her cooperative advertising support rate in the first period; while the retailer sets the retail prices for the two products in each period as well as his advertising efforts for each product in the first period. In the Non Cooperative Advertising (NCA) model, all the decision variables of the CA model are kept, except the cooperative advertising support rates. The first-period retail promotions are allowed to have a positive, null or a negative effect on the second-period sales to account for the long-term effects of promotions (Martín-Herrán et al. 2010; Sigué 2008).

We obtain closed-form feedback Stackelberg equilibrium solutions for two games. Comparison of equilibrium profits shows that, under some identified conditions that depend on the nature (long-term effects) of retail promotions and the degree of product substitutability, the retailer should refuse to participate in the manufacturers' cooperative advertising programs, while, in others, the manufacturers should not offer cooperative advertising programs even if they can maximize total channel profits.

The rest of the paper is organized as follows. First, we present the model **for symmetric manufacturers**, then we discuss the equilibrium solutions obtained for the NCA and CA games. Next, we compare equilibrium solutions across games to evaluate the effectiveness of the cooperative advertising program **and extend the analysis to asymmetric manufacturers**. Finally, we conclude and discuss managerial implications.

2 The model

We consider a channel where two competing manufacturers are selling substitutable products through a common retailer. The channel members make their decisions over a two-period horizon. In each period, the manufacturers are channel leaders; i.e., they simultaneously set their decisions for that period prior to the retailer. In period 1, the retailer decides of his promotions for the manufacturers' products ($a_{i1} > 0$ for $i \in \{1, 2\}$). These consist in a variety of promotional activities and can include flyers, displays, local advertising, etc. The

retailer also decides simultaneously of his consumer prices for the manufacturers' products in period 1 ($p_{i1} > 0$ for $i \in \{1, 2\}$). Each manufacturer i sets her wholesale price in period 1 ($w_{i1} > 0$). Additionally, she can choose to share the cost of the retailer's promotions for her brand in period 1 by offering a cooperative advertising program where she reimburses the retailer's promotional costs associated with her product at a rate $s_i \in (0, 1)$. In period 2, the manufacturers set their wholesale prices ($w_{i2} > 0$) and the retailer sets his retail prices for each product i ($p_{i2} > 0$).

As in Liu et al. (2014), we assume the following consumer-derived demand functions, d_{ij} , for each product i in each period j and for $i, j \in \{1, 2\}$:

$$d_{i1} = \frac{1}{1 - \theta^2} [(g + a_{i1}) - \theta(g + a_{(3-i)1}) - p_{i1} + \theta p_{(3-i)1}], \quad (1)$$

$$d_{i2} = \frac{1}{1 - \theta^2} [(g + \alpha a_{i1}) - \theta(g + \alpha a_{(3-i)1}) - p_{i2} + \theta p_{(3-i)2}]. \quad (2)$$

The parameter g is positive and represents the baseline demand (accumulated brand equity) for each product **when no brand is being advertised and product substitutability is null**. We assume that both products have the same baseline demand mainly to simplify the model. It means that the two competing firms have the same original market power. **This assumption will be relaxed later to assess the generalizability of our findings**. The parameter $\theta \in (0, 1)$ denotes the degree of substitutability or the intensity of competition between the two products. In the extreme retail promotions where $\theta = 0$, the two products are independent and, consequently, their demand functions are only affected by their own promotional and/or pricing decisions. These demand functions are derived **from maximization** of the consumer utility function developed by Spence (1976) and commonly used in the marketing and economics literatures, **assuming that advertising for a product affects its baseline demand** (e.g., Ingene and Parry 2007; Cai et al. 2012; Liu et al. 2014).²

The parameter α represents the long term effect of the retailer's promotions. As in Sigué (2008) and Martín-Herrán et al. (2010), we allow this parameter to take positive or negative values to represent various retailer promotional activities. While it was originally believed that

²We adapt the utility function in Spence (1976) to consider demand in each period j ($j = 1, 2$). As in Liu et al. (2014), advertising affects consumer utility functions through its impact on the equity for the product. The demand functions are then obtained through maximization of the following consumers' utility functions:

$$U_j = \sum_{i=1,2} ([g_i + f_j(a_{ij})] d_{ij} - d_{ij}^2/2) - \theta d_{1j} d_{2j} - \sum_{i=1,2} p_{ij} d_{ij},$$

for $j = 1, 2$, $f_1(a_{i1}) = a_{i1}$ and $f_2(a_{i2}) = \alpha a_{i1}$.

retailers primarily invest in promotions to boost sales in the short run, it is now empirically proven that retail promotions also affect sales in the long run (Herrington and Demsey 2005). Retail promotions can have positive long-term effects on sales ($\alpha > 0$) if they contribute to strengthening the product's brand preference, encouraging brand switching, or generating repeat purchase. In contrast, retail promotions have negative long-term effects on sales ($\alpha < 0$) if they contribute to depreciating brand preference and quality perception of the product over time (DelVecchio et al. 2006; Sigué and Karray 2007).

Promotional costs for the manufacturers and for the retailer are convex. Convex promotional costs are common in the marketing literature (Aust and Buscher 2014; Martín-Herrán and Sigué 2011; Liu et al. 2014). They imply that marginal costs of advertising are increasing. When the manufacturer i offers a cooperative advertising program with a participation rate s_i to the retailer, the manufacturer i 's portion of the retailer's promotional expenses on her product is $s_i a_{i1}^2$, while the retailer's promotional cost for the product i is $(1 - s_i) a_{i1}^2$.

The manufacturers and retailer determine their respective decisions so as to maximize their own profits. In period 1, in case of a cooperative advertising program, manufacturer i 's profit function, M_{i1} for $i \in \{1, 2\}$, and the retailer's profit for the two products, R_1 , are given by:

$$\begin{aligned} M_{i1} &= w_{i1}d_{i1} - s_i a_{i1}^2, \\ R_1 &= \sum_{i=1,2} [(p_{i1} - w_{i1})d_{i1} - (1 - s_i) a_{i1}^2]. \end{aligned}$$

In period 2, manufacturer i 's profit function, M_{i2} for $i \in \{1, 2\}$, and the retailer's profit for both products, R_2 , are as follow:

$$\begin{aligned} M_{i2} &= w_{i2}d_{i2}, \\ R_2 &= \sum_{i=1,2} (p_{i2} - w_{i2})d_{i2}. \end{aligned}$$

The retailer keeps no stocks and orders in each period the quantities that correspond to the demand of each of the two products. This assumption allows the manufacturers to reward the retailer for effective sales rather than for quantities purchased (Martín-Herrán et al. 2010). The channel members make their decisions in the following sequence. In each period, the two manufacturers play Nash together and Stackelberg with the retailer. More specifically, in period 1, the manufacturers decide simultaneously of their wholesale prices and of their cooperative advertising rates (if any). Then the retailer chooses his first period's pricing and promotional decisions simultaneously for both manufacturers' products, knowing the manufacturers' first period decisions. This is based

on the observation that retailers are usually aware of manufacturers' pricing and cooperative advertising offers when they make their promotional and pricing decisions. In period 2, the manufacturers choose their wholesale prices simultaneously, knowing all the decisions made in period 1 by all channel members. Finally, the retailer sets simultaneously his prices in period 2 for both products knowing all decisions made in period 1 as well as the manufacturers' wholesale prices in period 2.

3 Equilibrium solutions

We obtain feedback equilibrium solutions using backwards induction for the following two games. The first game is a benchmark situation, where manufacturers withhold any cooperative advertising support to the retailer ($s_i = 0$, $i \in \{1, 2\}$). This game is hereafter denoted by NCA (no cooperative advertising). In the second game, each manufacturer i offers a cooperative advertising program at a rate $s_i \neq 0$ ($i \in \{1, 2\}$). This game is denoted by CA (cooperative advertising). For both games, we obtain analytical equilibrium solutions that depend on the model's parameters (g , α and θ). Since the solutions are long, we omit them here for clarity. The equilibrium solutions and the method for obtaining them are summarized in the Appendix. **Observe that a third game in which only one manufacturer offers a cooperative advertising can be considered. This game is not sustainable at the equilibrium when the two manufacturers are symmetric as it benefits the manufacturer who offers a cooperative advertising to the detriment of the other. The findings of this game are therefore not discussed in this paper. They are however provided in the Supplementary Appendix (available online).**

Given the complexity of the analytical solutions and the conditions specifying interior and concavity conditions, we illustrate the analytical results we obtain in figures. In all these figures, we allow the parameters α and θ to vary in acceptable ranges (as explained below), while normalizing the baseline demand parameter, g , to 1 without loss of generality. We emphasize that all results obtained and discussed hereafter are based on analytical findings not on numerical analysis and that the figures are used as a way to simplify exposition of our results.

We obtain a unique equilibrium solution for the NCA game, while two equilibrium solutions denoted by Scenario I and Scenario II are derived for the CA game (See the Appendix for details). In Figure 1 below, we characterize the feasible domains for these three equilibrium solutions. Particularly, we identify the values of the model parameters (α and θ) for which

the following conditions are verified. First, for each equilibrium, the corresponding players' decisions, margins, profits and demands are positive. Second, the manufacturers' participation rates are lower than one in the two scenarios of the CA game. Third and final, the concavity conditions ensuring that the extremum are interior maximum for the two games are satisfied.

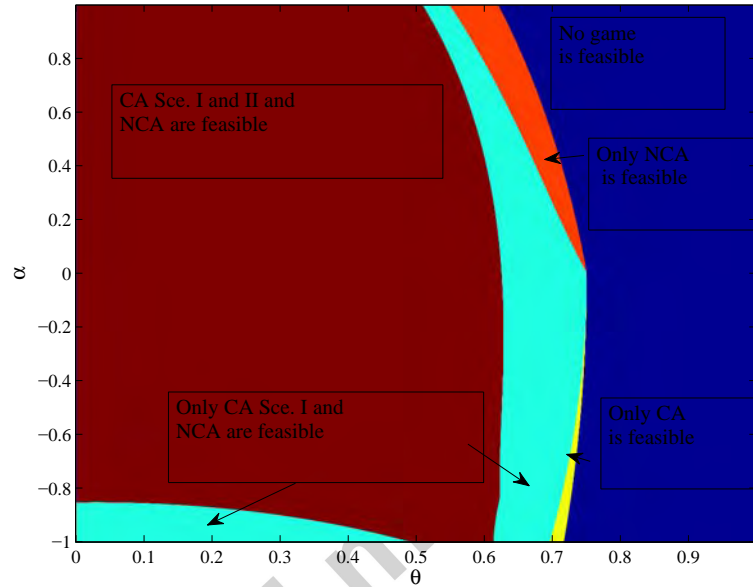


Figure 1: Feasible and unfeasible regions for both games.

Figure 1 shows the feasible and unfeasible regions for each of the equilibrium solutions of the two games. **A game is unfeasible in a region when at least one of the conditions listed above is not verified in that region given the parameters' values.** For instance, the players' decisions, margins, and profits are expected to be positive. Economically, if any of these conditions are not met, at least one player will have to leave the market due to his/her inability to cope with the market conditions. This is the case in Figure 1 for products that are highly substitutable. Due to the heavy competitive pressure, the manufacturers are forced to set very low (even negative) wholesale prices or/and offer cooperative advertising programs that cover more than the retailer's advertising expenses.

As we can see, in the biggest area of the parameter space, the equilibrium solutions of the NCA and CA (Scenarios I or II) games are simultaneously feasible (dark red-colored region). In this large area, the players can play any of the three equilibria. There are two areas where only one of the two games can be played alone. The NCA can be played alone

when the products are substantially substitutable ($\theta > 0.6$) and retail promotions heavily increase long-term sales (light red-colored region). Scenario I of the CA game can be played alone when the products are highly substitutable ($\theta > 0.7$) and retail promotions heavily damage long-term sales (yellow-colored region in Figure 1). There are also two other areas in which only Scenario I of the CA game and the NCA game are feasible (green-colored regions). These areas are characterized by either low levels of product substitutability ($\theta < 0.45$) and very damaging long-term effects of retail promotions ($\alpha < -0.82$), or relatively high levels of product substitutability ($\theta \in (0.57, 0.72)$).

Observe that for the CA game, Scenario I is feasible in a larger area than Scenario II, which means that in some areas, only Scenario I can be played, and in others the two scenarios will be considered. **Figure 2 displays the players' preferences with respect to these two scenarios.**

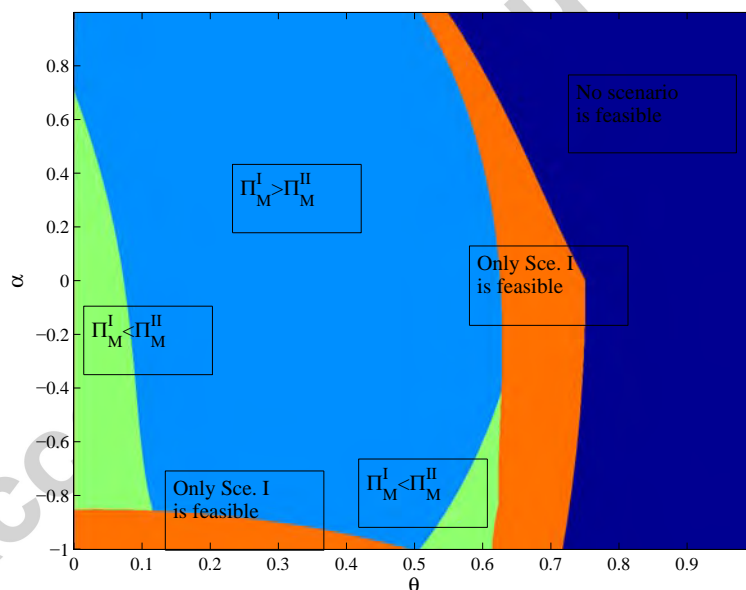


Figure 2: Comparison of optimal total manufacturer's profits under the different scenarios in the CA game.

Symmetric manufacturers always select the same scenario. The real equilibrium selection issue in Figure 2 is therefore between the manufacturers and the retailer as their preferences diverge in many areas (see discussion in Section 6.2 of the Appendix). However, given that the manufacturers are leaders in their respective channels, they will always announce first the strategies that best serve

their interests. Once the announcement is made, the retailer will have no choice but to play the strategy that optimizes his profit given the announcements made by the manufacturers. This means that, at any time, the manufacturers' preferred scenario will always be selected. Hereafter, we consider the manufacturers' chosen equilibrium in the CA game and only focus on whether or not the CA game (Scenario I or II) improves channel profits compared to the NCA game.

4 Effectiveness cooperative advertising programs

To assess the effectiveness of the cooperative advertising program, we compare the equilibrium profits obtained in the NCA and CA games for the manufacturers, retailer, and the total channel over the two periods. Profit comparisons are conducted in the feasible domain for both the NCA and CA games (see Figures 3-6).

4.1 Effects of cooperative advertising programs on the manufacturers' profits

We now examine whether or not two competing profit-maximizing manufacturers should offer cooperative advertising programs to their common retailer. To address this issue, we compare the manufacturers' optimal profits in the CA (Scenarios I and II) and NCA games. The findings of our analysis are summarized in Figure 3.

Figure 3 shows that in most areas of the feasible parameter space, the manufacturers are better off when they offer cooperative advertising programs to the retailer than when they do not (light blue-colored area). Thus, within this feasible domain, competing manufacturers may well offer cooperative advertising programs to a single retailer whether the retailer's promotion positively or negatively affects the long-term sales of their respective products.

Two notable exceptions exist where manufacturers find it optimal not to offer cooperative advertising programs (both green-colored areas). In the first case, the retailer promotions significantly damage sales in the next period ($\alpha < -0.82$) and the products are highly differentiated ($\theta < 0.15$). This is because for highly differentiated products, manufacturers act as quasi monopolists and use cooperative advertising offerings to substantially increase wholesale prices to alleviate the negative effects of retailer promotions on the second-period sales. As a consequence, the retailer also increases first-period retail prices, although its margins decrease. These players' strategies negatively affect the first-period demands and manufacturers' profits, which cannot be compensated in the second period (see Figures 7-9 in the Appendix

for detailed analyses of demands and players' strategies). In the second case, the intensity of competition is relatively high ($\theta \in (0.56, 0.68)$) regardless of the nature of the long-term effects of retail promotions. Cooperative advertising programs intensify price competition and manufacturers have no choice but to decrease their first-period wholesale prices. The first-period demands increase, but the loss in the first-period margins for manufacturers is detrimental to their profits.

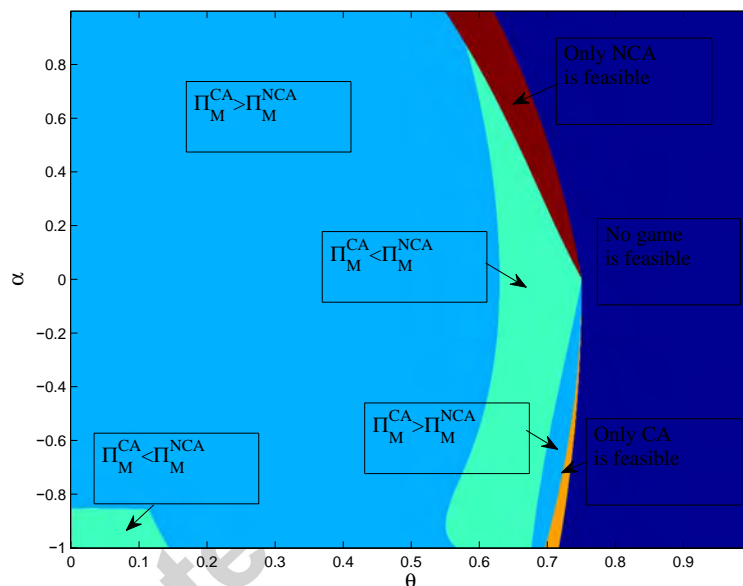


Figure 3: Comparison of manufacturers' optimal profits in the different games.

4.2 Effects of cooperative advertising programs on the retailer's profits

To assess the acceptability of the manufacturers' proposed cooperative advertising programs for the retailer, we compare his optimal profits in the CA and NCA games. The results are illustrated in Figure 4.

As we can see, cooperative advertising improves the retailer's profit in most parts of the feasibility domain as illustrated in Figure 4 (light blue-colored area). There are however two exceptions where cooperative advertising does not benefit the retailer (green-colored regions). In these particular areas, either the retailer's promotion significantly damages sales in the next period ($\alpha < -0.82$) and the intensity of competition between products is limited ($\theta < 0.45$),

or the intensity of competition is relatively high ($\theta \in (0.68, 0.7)$) and the retailer's promotions heavily damages long-term sales. As previously discussed in the case of the manufacturers, for relatively differentiated products, the first-period demands decrease when manufacturers support retail promotions that substantially harm long-term sales. In addition, while the retailer increases his first-period prices in the CA game, his margins decrease. The combination of smaller first-period demands and margins in the CA game hurts the overall profits of the retailer.

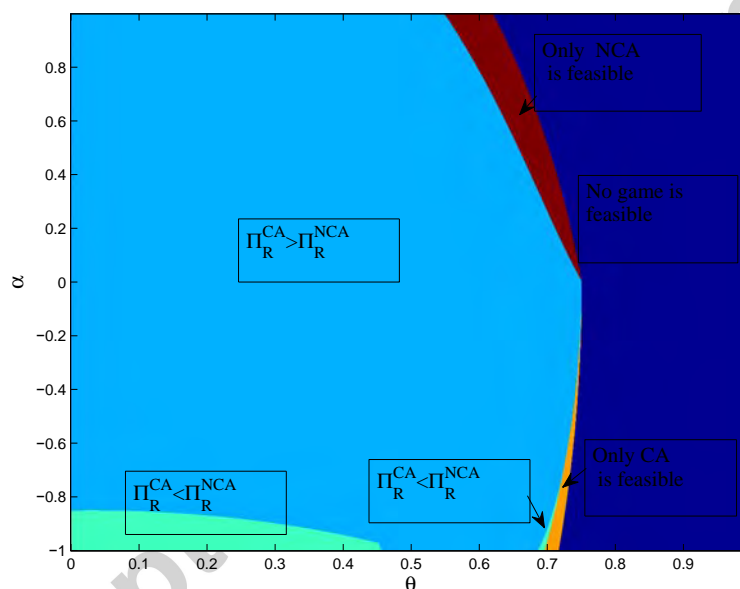


Figure 4: Comparison of retailer's optimal profits in the different games.

These findings support the view that cooperative advertising programs in such a configuration benefit the retailer in the feasible domain when the retail promotion does not impact, moderately damages, or stimulates long-term sales regardless of the level of competition between the two products. The level of competition does however determine whether or not cooperative advertising is a profit-enhancing activity for the retailer when his advertising activities can significantly harm future sales. Particularly, the retailer may not welcome cooperative advertising programs from the manufacturers if their products are highly differentiated or are very competitive, depending on the ranges of the model parameters.

Summarizing, Figure 5 combines the findings in Figures 3 and 4. In Figure 5, Regions R_1 , R_2 , R_3 and R_4 indicate the following:

- Region 1 (R_1): $\Pi_R^{CA} > \Pi_R^{NCA}$ and $\Pi_M^{CA} > \Pi_M^{NCA}$,
- Region 2 (R_2): $\Pi_R^{CA} < \Pi_R^{NCA}$ and $\Pi_M^{CA} < \Pi_M^{NCA}$,
- Region 3 (R_3): $\Pi_R^{CA} < \Pi_R^{NCA}$ and $\Pi_M^{CA} > \Pi_M^{NCA}$,
- Region 4 (R_4): $\Pi_R^{CA} > \Pi_R^{NCA}$ and $\Pi_M^{CA} < \Pi_M^{NCA}$.

One can easily see that the manufacturers' and retailer's preferences will diverge for some values of parameters θ and α (regions R_3 and R_4 in Figure 5). The manufacturers could be interested in implementing the CA game, while the retailer prefers the NCA game (region R_3). An example is when the products are not very differentiated ($\theta \in (0.15, 0.45)$) and the retailer's promotion heavily damages long-term sales ($\alpha < -0.82$). In such cases, manufacturers may well offer cooperative advertising programs, but the retailer will not participate in such programs as they do not serve his best interests. Particularly, the margins the retailer receives in the CA game are smaller than in the NCA in addition to the fact that the first-period demands are also smaller.

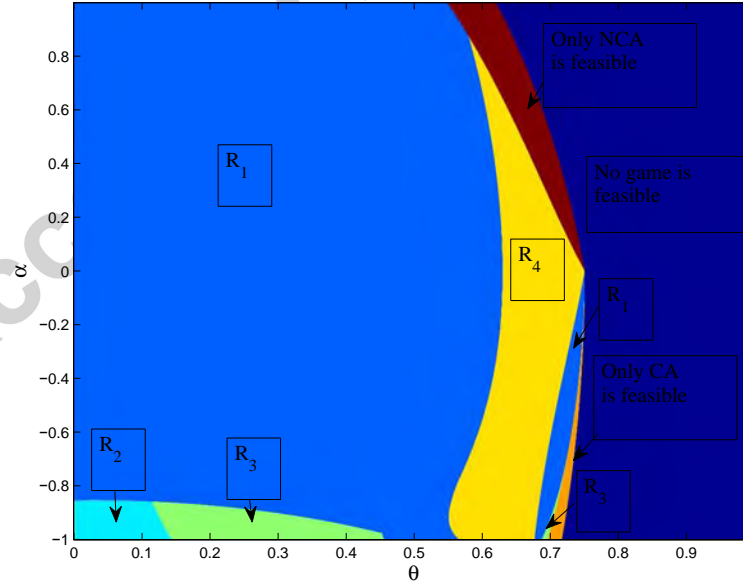


Figure 5: Comparison of manufacturers' and retailer's optimal profits in the different games.

On the other hand, there is another region in the feasible domain in which the manufacturer would prefer playing the NCA game, while the retailer will prefer the CA game (region R_4).

In this region, the intensity of competition is relatively high ($\theta \in (0.56, 0.68)$) regardless of the nature of the long-term effects of the retailer's promotion. Remember that, within the feasible domain, regardless of the intensity of competition between the two products, the retailer always welcomes cooperative advertising when his promotions can increase long-term sales. From the manufacturers' perspective, however, supporting retailer's promotions that can expand next period demand when the products are highly substitutable may erode margins and negatively affect profits.

4.3 Effects of cooperative advertising programs on the total channel's profits

We now assess the impact of cooperative advertising programs for the entire channel by comparing the total channel profits in the CA and NCA games. Figure 6 offers an illustration of our findings.

Figure 6 shows that cooperative advertising creates channel surplus in most areas of the feasible domain, except when retailer's promotions significantly damage sales in the next period ($\alpha < -0.82$) and the intensity of competition between products is limited ($\theta < 0.4$) (green-colored area). In this particular area, cooperative advertising is not worth undertaking as it does reduce the profits available for channel members.

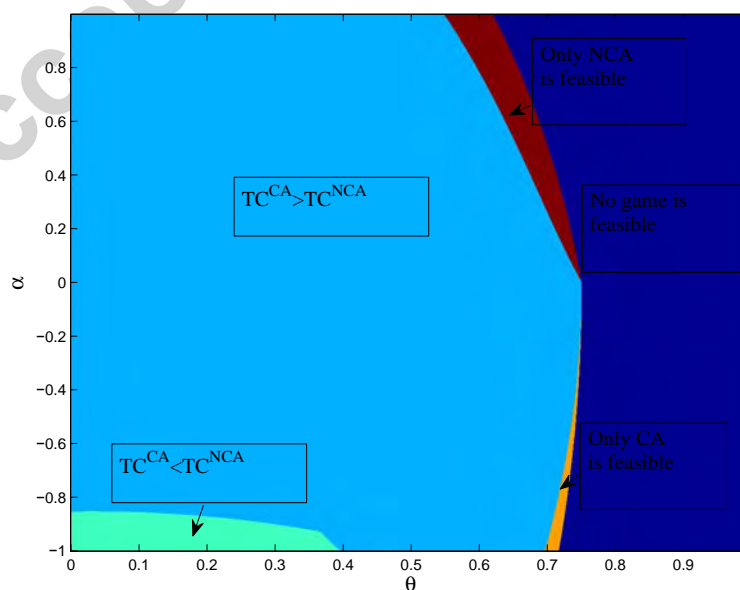


Figure 6: Comparison of total channel's optimal profits in the different games.

Contrasting Figures 5 and 6, one can see that when channel members act so as to maximize their individual profits, the decisions to offer or accept a cooperative advertising program may deviate from those that maximize the overall channel profits. Profit-maximizing manufacturers may consider offering cooperative programs for some specific values of the parameters within this area ($\theta \in (0.15, 0.4)$) that generate net losses for the retailer and for the entire channel (region R_3 in Figure 5). On the other hand, in all other areas within the feasible domain, cooperative advertising programs create channel surpluses, but again they may not align the interests of all channel members. For instance, when the intensity of competition is relatively high ($\theta \in (0.56, 0.68)$), the manufacturers find it optimal not to offer cooperative advertising programs at the expense of the retailer who mainly benefits from these programs (region R_4 in Figure 5).

In such a context, the retailer would have to offer side payments to manufacturers to convince them to offer cooperative advertising. Side payments are possible because channel surpluses generated by cooperative advertising programs are collected by the retailer. In the automotive industry, some manufacturers have already adopted this type of side payments. For example, Hyundai charges its dealers an average of \$500 on each new-vehicle invoice to pay for its advertising activities. On the other hand, in areas where cooperative advertising programs lead to losses for the retailer that are higher than the manufacturers' gains, the cooperative advertising program will not be accepted by the retailer, and therefore cannot be implemented.

4.4 Effectiveness of cooperative advertising for asymmetric manufacturers

Until now, we have assumed perfectly symmetric manufacturers. In this section, we relax this assumption and expand our analysis to the case where the manufacturers have different baseline demands (g_1 and g_2). Our analysis is summarized as follows.

We solve both the NCA and CA games (see Supplementary Appendix). We obtain an analytical solution for the NCA game for $g_1 \neq g_2$, while we could not get a closed-form equilibrium solution for the CA game. So, we assume $g_1 > g_2$, such that $g_2 = x.g_1$, where x is a positive percentage. Then, we solve the CA game numerically for each value of $x \in (0.3, 0.5, 0.7, 0.9)$ and for all values of $\theta \in (0, 1)$ and $\alpha \in (-1, 1)$. After identifying the feasible domains in each of the two games and for each numerical scenario, we compare equilibrium solutions in the CA and

NCA games.

These numerical analyses suggest that the manufacturer with the larger baseline demand (Manufacturer 1) and the retailer prefer the CA game to the NCA game for all considered values of x . However, the manufacturer with the smaller baseline demand (Manufacturer 2) may prefer either the CA or the NCA game depending on the values of the parameters x , α , and θ . These findings suggest that even when the manufacturers' market powers are different, they may as well find it optimal to support retailer advertising.

5 Conclusion

This paper studies cooperative advertising arrangements in a two-period planning horizon in a context where two competing manufacturers sell two products to a single retailer, who then promotes and offers these products to consumers. The retailer can undertake any type of promotions, including those that may have either positive or negative effects on sales in the post-promotional period. The specific questions addressed in this research are: Should the manufacturers offer and the retailer accept cooperative advertising programs in such a configuration? Could cooperative advertising arrangements **benefit the entire channel**? To answer these questions, we developed and analyzed two analytical models. For each model, the manufacturers play a Nash game between them and a Stackelberg game, in leadership positions, with the retailer.

Comparing the optimal profits of the players from the two models, we identified the conditions under which the manufacturers should offer and the retailer should accept cooperative advertising programs. These conditions depend both on the long-term effects of retail promotions and on the intensity of competition between the two products. Particularly, within the feasible domain, the retailer always welcomes cooperative advertising programs regardless of the degree of product substitutability when promotions do not heavily damage future sales, while, depending on the degree of product substitutability, the manufacturers may or may not offer cooperative advertising programs regardless of the long-term effects of retail promotions. We also find that channel members' decisions with respect to cooperative advertising may not **benefit the total channel** by aligning their interests with the objective of the entire channel. In some areas of the feasible domain, the manufacturers' or retailer's decisions may not lead to the maximization of the total channel profits.

The findings of this research add to the current cooperative advertising literature in several

ways. First, we extend the work by Jørgensen et al. (2003), which examined for the first time whether cooperative advertising is worth implementing in a bilateral monopoly where retail promotions can damage the manufacturer's brand image. Their proposed theory holds that the manufacturer should offer cooperative advertising either when the initial brand image is small or when it is intermediate and retail promotions do not heavily damage the manufacturer's brand image. Our findings extend these results and show that when competition is considered at the manufacturers' level, the intensity of product substitutability matters and the manufacturers can successfully implement cooperative advertising programs even if retail promotions heavily damage their future sales.

Second, the analysis of competition between manufacturers also changes previous findings of dynamic cooperative advertising in bilateral monopoly settings where retail promotions are assumed to produce positive carryover effects (e.g., Jørgensen et al. 2000). In this research, the retailer's advertising carryover effect positively impacts the level of support the manufacturer offers to the retailer. Our findings support the view that it may not be in the interest of the manufacturers to support retail promotions when their products are fairly substitutable whether retail promotions stimulate future sales or not. Manufacturers' reliance on retail promotions in such a competitive environment will give incentives to the retailer to act opportunistically and ask for the biggest share of the channel surplus. This is consistent with what Ailawadi (2001) calls a retail extortion view of trade promotions as it is proven here that if cooperative advertising programs are offered in these conditions, they will exclusively benefit the retailer at the expense of the manufacturers.

Finally, based on the principle of individual rationality, it is known that cooperative advertising arrangements are implemented only if they improve or, at least, do not damage the profits of any party involved compared to the situation where they are not offered. We have proven here that the decisions to offer and participate in a cooperative advertising program based on individual rationality may not serve the interest of the channel taken as an entity. Particularly, the manufacturers and retailer may refuse to respectively offer and participate in cooperative advertising programs that enhance total channel profits. This indicates that **reaping the benefits of** cooperative advertising in the configuration studied in this paper may require either the use of additional side payment mechanisms that can better align the interest of channel members, or to control some market parameters. For instance, manufacturers may ask the retailer to share the cooperative advertising surplus via various payments. Some auto makers ask for the retailer's contributions to advertising, while also offering cooperative advertising programs. The manufacturers may also prevent the retailer from engaging in

promotional activities that can damage their brands. In fact, many manufacturers set specific eligibility criteria for their cooperative advertising programs. The manufacturers may also invest in marketing efforts that aim at increasing the differentiation of their products. This can reduce their reliance on retail promotions and better align channel members' interests.

This research has some limitations. We have used simplified models to derive meaningful results that can enhance our understanding of the use of cooperative advertising in channels. Even with these simplified models, we obtained complex analytical findings that we could only graphically illustrate. Future researchers can relax some of our assumptions to study other relevant aspects of the problem overlooked here. For example, the manufacturers can directly invest in advertising to influence the degree of substitutability of their products. The retailer can also advertise and the manufacturers can support retail promotions in the second period. Competition can be added at the retail level. **Finally, demand uncertainty could be added as in some recent works (e.g., Tsao 2015).** All these issues deserve further research.

6 Appendix

We obtain feedback equilibrium solutions using backwards induction for two games. The first game is a benchmark situation, denoted hereafter by the superscript *NCA*, where manufacturers withhold cooperative advertising support to the retailer ($s_i = 0, i = 1, 2$). In the second game, the manufacturers offer cooperative advertising at a rate $s_i \neq 0 (i = 1, 2)$. This game is denoted hereafter by the superscript *CA*. Next, we present the method for obtaining the equilibrium for each game.

6.1 Benchmark game: No cooperative advertising

Proposition 1 *The equilibrium strategies for the benchmark game where no cooperative advertising is implemented in the channel are given by*

$$p_2^{NCA} = \frac{(3-2\theta)^2(2-\theta)[g(3+\alpha+4\theta) - \alpha w_1^{NCA}]}{2[\alpha^2(\theta-3) + (2-\theta)^2(3-2\theta)(3+4\theta)]}, \quad (3)$$

$$w_2^{NCA} = \frac{(3-2\theta)(2-\theta)(1-\theta)[g(3+\alpha+4\theta) - \alpha w_1^{NCA}]}{\alpha^2(\theta-3) + (2-\theta)^2(3-2\theta)(3+4\theta)}, \quad (4)$$

$$a_1^{NCA} = \frac{g[\alpha(3-\theta) + (2-\theta)^2(3-2\theta)] - (2-\theta)^2(3-2\theta)w_1^{NCA}}{\alpha^2(\theta-3) + (2-\theta)^2(3-2\theta)(3+4\theta)}, \quad (5)$$

$$p_1^{NCA} = \frac{g[\alpha(1-\alpha)(3-\theta) + 4(2-\theta)^2(1+\theta)(3-2\theta)] - w_1^{NCA}[\alpha^2(3-\theta) - 2(2-\theta)^2(3-2\theta)(1+2\theta)]}{2[\alpha^2(\theta-3) + (2-\theta)^2(3-2\theta)(3+4\theta)]}, \quad (6)$$

$$w_1^{NCA} = g \frac{Numw_1^{NCA}}{Denw_1^{NCA}}, \quad (7)$$

where $p_j^{NCA} = p_{1j}^{NCA} = p_{2j}^{NCA}$, $w_j^{NCA} = w_{1j}^{NCA} = w_{2j}^{NCA}$, $j = 1, 2$, and $a_1^{NCA} = a_{11}^{NCA} = a_{21}^{NCA}$ and $Numw_1^{NCA}$ and $Denw_1^{NCA}$ are long expressions that depend on the model's parameters g , α and θ .

Proof. *Fourth stage:* At this stage of the game, the retailer chooses the second-period prices, p_{12} and p_{22} , in order to maximize his second-period profits. Therefore, the retailer's problem can be written as

$$\max_{p_{12}, p_{22}} R_2, \quad (8)$$

where

$$R_2 = \sum_{i=1,2} (p_{i2} - w_{i2})d_{i2}$$

denotes the retailer's second-period profits and d_{i2} is the second-period demand function for product i given in (1) and (2) for the first and second products, respectively.

The solution to problem (8) gives the retailer's reaction functions, that is, p_{12} and p_{22} , as functions of the wholesale prices, w_{12} and w_{22} , and of the retailer's advertising in the first period, a_{11} and a_{21} .

The retailer's second-period profit is a strictly concave function of his decision variables in this period, p_{12} and p_{22} . From the first-order optimality conditions for the problem in (8), the following expressions can be derived:

$$p_{12} = \frac{1}{2}(g + w_{12} + \alpha a_{11}), \quad (9)$$

$$p_{22} = \frac{1}{2}(g + w_{22} + \alpha a_{21}). \quad (10)$$

Third stage: At this stage of the game, the manufacturers play a Nash game between them and choose their second-period wholesale prices, w_{12} and w_{22} , in order to maximize their second-period profits. Therefore, the problem that manufacturer i is facing can be written as:

$$\max_{w_{i2}} M_{i2}, \quad (11)$$

where

$$M_{i2} = w_{i2}d_{i2},$$

denotes manufacturer i 's second-period profit and d_{i2} is the demand function in this period, defined in (1) and (2) for the first and second products, respectively. At this stage of the game, the manufacturers know the retailer's pricing reaction functions derived in Stage 4, and incorporate this information when deciding their optimal pricing strategies. We then replace the retail prices by the obtained reaction functions in (9) and (10) in the manufacturers' objective functions in (11).

The solution to the manufacturers' problems gives us the wholesale prices, w_{12} and w_{22} , as functions of the retailer's advertising in the first period, a_{11} and a_{21} .

The manufacturers' second-period profit functions are strictly concave in their decision variables in this period, w_{12} and w_{22} . From the first-order conditions for each manufacturer i 's problem in (11), we get:

$$w_{12} = \frac{\alpha[(2 - \theta^2)a_{11} - \theta a_{21}] + g(2 - \theta - \theta^2)}{4 - \theta^2}, \quad (12)$$

$$w_{22} = \frac{\alpha[(2 - \theta^2)a_{21} - \theta a_{11}] + g(2 - \theta - \theta^2)}{4 - \theta^2}. \quad (13)$$

Replacing these expressions into the retailer's reaction functions in (9) and (10), we obtain the second-period retail prices as a function of the retailer's advertising in the first period (a_{11} and a_{21}) as follows:

$$p_{12} = \frac{1}{2(4 - \theta^2)}[g(2 + \theta)(3 - 2\theta) + \alpha(2(3 - \theta^2)a_{11} - \theta a_{21})], \quad (14)$$

$$p_{22} = \frac{1}{2(4 - \theta^2)}[g(2 + \theta)(3 - 2\theta) + \alpha(2(3 - \theta^2)a_{21} - \theta a_{11})]. \quad (15)$$

The second-period retailer's and manufacturers' optimal profits are obtained replacing the expressions (12), (13), (14) and (15). They are given by

$$R_2 = \frac{(36 - 35\theta^2 + 3\theta^4)[(g + \alpha a_{11})^2 + (g + \alpha a_{21})^2] + 4\theta(6 - 9\theta^2 + \theta^4)(g + \alpha a_{11})(g + \alpha a_{21})}{4(4 - \theta^2)^2(1 - \theta^2)(9 - 4\theta^2)}, \quad (16)$$

$$M_{12} = \frac{3[\theta(g + \alpha a_{21}) - (3 - 2\theta^2)(g + \alpha a_{11})][6\theta(g + \alpha a_{21}) - (9 - 8\theta^2)(g + \alpha a_{11})]}{8(9 - 4\theta^2)^2(1 - \theta^2)}, \quad (17)$$

$$M_{22} = \frac{3[\theta(g + \alpha a_{11}) - (3 - 2\theta^2)(g + \alpha a_{21})][6\theta(g + \alpha a_{11}) - (9 - 8\theta^2)(g + \alpha a_{21})]}{8(9 - 4\theta^2)^2(1 - \theta^2)}. \quad (18)$$

Second stage: In the first period, the retailer chooses the retail prices, p_{11} and p_{21} and his promotional efforts, a_{11} and a_{21} , in order to maximize his total profits, R , given by the discounted sum of his first and second period profits as follows:

$$R = R_1 + t_R R_2,$$

where $0 < t_R \leq 1$ is the retailer's discount rate. We assume it is equal to 1 for simplicity and without loss of generality.

Taking into account the second-period retailer's profits (given by (16)), the retailer's total profit is:

$$R = \sum_{i=1,2} \left[(p_{i1} - w_{i1}) \frac{a_{i1} + g - p_{i1} - \theta(a_{(3-i)1} + g) + \theta p_{(3-i)1} - a_{i1}^2}{1 - \theta^2} \right] + \frac{(36 - 35\theta^2 + 3\theta^4) [(g + \alpha a_{11})^2 + (g + \alpha a_{21})^2] + 4\theta(6 - 9\theta^2 + \theta^4)(g + \alpha a_{11})(g + \alpha a_{21})}{4(4 - \theta^2)^2(1 - \theta^2)(9 - 4\theta^2)}.$$

It can be easily proved that the retailer's total profit in the first and second periods, R , is a concave function in the retailer's first-period decision variables, p_{11} , p_{21} , and a_{11} , a_{21} if and only if the following two conditions are satisfied:

$$(4 - \theta^2)^2(27 - 48\theta^2 + 16\theta^4) - \alpha^2(36 - 35\theta^2 + 3\theta^4) > 0,$$

$$\alpha^2(216 - 306\theta^2 + 28\theta^4) - \alpha^4(9 - \theta^2) - (4 - \theta^2)^2(81 - 180\theta^2 + 64\theta^4) < 0.$$

The conditions above ensure the concavity of the function R . Next, we maximize R with respect to the first-period retailer's decisions, p_{11} , p_{21} , a_{11} and a_{21} , to get the optimal reaction functions, i.e., the first-period retail prices, p_{11} and p_{21} , and the retailer's promotional efforts, a_{11} and a_{21} , as functions of the first-period wholesale prices, w_{11} and w_{21} :

$$p_{i1} = \frac{g\Gamma_1 + \Gamma_2 w_{i1} + \theta\Gamma_3 w_{(3-i)1}}{2[\alpha^2(216 - 306\theta^2 + 28\theta^4) - \alpha^4(9 - \theta^2) - (4 - \theta^2)^2(9 - 4\theta^2)(9 - 16\theta^2)]}, \quad (19)$$

$$a_{i1} = \frac{g\Lambda_1 + \Lambda_2 w_{i1} + \theta\Gamma_3 w_{(3-i)1}}{\alpha^2(216 - 306\theta^2 + 28\theta^4) - \alpha^4(9 - \theta^2) - (4 - \theta^2)^2(9 - 4\theta^2)(9 - 16\theta^2)}, \quad (20)$$

where

$$\Gamma_1 = (1 - \alpha)\alpha^3(9 - \theta^2) - \alpha(3 - \theta)(2 + \theta)^2(3 + 2\theta)(3 - 4\theta) - 4(1 + \theta)(9 - 4\theta^2)(3 - 4\theta)(4 - \theta^2)^2 + \alpha^2(252 - 48\theta - 293\theta^2 + 5\theta^3 + 26\theta^4),$$

$$\Gamma_2 = \alpha^2(180 - 319\theta^2 + 30\theta^4) - \alpha^4(9 - \theta^2) - 2(4 - \theta^2)^2(9 - 4\theta^2)(3 - 8\theta^2),$$

$$\Gamma_3 = \alpha^2(48 - 5\theta^2) - 4(4 - \theta^2)^2(9 - 4\theta^2),$$

$$\Lambda_1 = -\alpha^3(9 - \theta^2) - \alpha(3 - \theta)(2 + \theta)^2(3 + 2\theta)(3 - 4\theta) - (9 - 4\theta^2)(3 - 4\theta)(4 - \theta^2)^2 + \alpha^2(2 - \theta)^2(3 + \theta)(3 - 2\theta),$$

$$\Lambda_2 = 3(4 - \theta^2)^2(9 - 4\theta^2) - \alpha^2(36 + 13\theta^2 - 2\theta^4).$$

First stage: The manufacturers take into account the retailer's reaction functions in the first period when playing the Nash game between them. In period 1, manufacturer i chooses her wholesale price, w_{i1} , in order to maximize her total profits, M , during the first and second periods:

$$M_i = M_{i1} + t_M M_{i2},$$

where $0 < t_M \leq 1$ is the manufacturers' discount rate. We assume it is equal to 1 for simplicity and without loss of generality as in the case of the retailer.

Taking into account the second-period manufacturers' profits (given by (17) and (18)), manufacturer i 's total profits during the first and second periods are as follows:

$$M_i = w_{i1} \frac{a_{i1} + g - p_{i1} - \theta(a_{(3-i)1} + g) + \theta p_{(3-i)1}}{1 - \theta^2} + \frac{3[\theta(g + \alpha a_{(3-i)1}) - (3 - 2\theta^2)(g + \alpha a_{i1})][6\theta(g + \alpha a_{(3-i)1}) - (9 - 8\theta^2)(g + \alpha a_{i1})]}{8(9 - 4\theta^2)^2(1 - \theta^2)}.$$

It can be easily proved that manufacturer i 's total profits in the first and second periods, M_i , $i = 1, 2$ is a concave function in her first-period decision variable, w_{i1} , if and only if the following condition is satisfied:

$$\Delta > 0$$

where

$$\begin{aligned} \Delta &= 4\alpha^8(9 - \theta^2)^2 + 48(9 - 4\theta^2)^2(4 - \theta^2)^4(9 - 25\theta^2 + 16\theta^4) \\ &+ \alpha^6(-18144 + 19908\theta^2 - 4325\theta^4 + 367\theta^6 - 12\theta^8) \\ &- \alpha^2(4 - \theta^2)^2(186624 - 536220\theta^2 + 494253\theta^4 - 154226\theta^6 + 10752\theta^8) \\ &+ \alpha^4(357696 - 777600\theta^2 + 528948\theta^4 - 111750\theta^6 + 10757\theta^8 - 424\theta^{10}). \end{aligned}$$

Replacing the first-period retailer's reaction functions obtained in Stage 2 in manufacturer i 's total profit, M_i , and taking into account the above condition to ensure the concavity of M_i , we then obtain the manufacturers' optimality conditions as follows:

$$\frac{\partial M_1}{\partial w_{11}} = 0; \quad \frac{\partial M_2}{\partial w_{21}} = 0,$$

and the first-period manufacturers' strategies can be derived. We get

$$w_{11}^{\text{NCA}} = w_{21}^{\text{NCA}} = w_1^{\text{NCA}}, \quad (21)$$

where w_1^{NCA} is given by (7).

Taking into account the symmetric results in (21), (19) and (20), these expressions imply $p_{11}^{\text{NCA}} = p_{21}^{\text{NCA}} = p_1^{\text{NCA}}$ and $a_{11}^{\text{NCA}} = a_{21}^{\text{NCA}} = a_1^{\text{NCA}}$ and can be simplified as (6) and (5). Finally, substituting $a_{11}^{\text{NCA}} = a_{21}^{\text{NCA}} = a_1^{\text{NCA}}$ by (5) in (12), (13), (14), (15), the expressions in (4) and (3) are easily deduced. ■

6.2 The cooperative advertising game

Proposition 2 *The equilibrium strategies for the cooperative advertising game where a cooperative advertising is implemented in the channel is given by the following two solutions.*

$$p_2^{CA} = \frac{(2-\theta)(3-2\theta)[\alpha w_1^{CA} - g(3+\alpha+4\theta-4(1+\theta)s_1^{CA})]}{2[\alpha^2 - (2-\theta)^2(3+4\theta-4(1+\theta)s_1^{CA})]}, \quad (22)$$

$$w_2^{CA} = \frac{(2-\theta)(1-\theta)[\alpha w_1^{CA} - g(3+\alpha+4\theta-4(1+\theta)s_1^{CA})]}{\alpha^2 - (2-\theta)^2(3+4\theta-4(1+\theta)s_1^{CA})}, \quad (23)$$

$$a_1^{CA} = \frac{(2-\theta)^2 w_1^{CA} - [\alpha + (2-\theta)^2]g}{\alpha^2 - (2-\theta)^2(3+4\theta-4(1+\theta)s_1^{CA})}, \quad (24)$$

$$p_1^{CA} = \frac{[\alpha^2 - 2(2-\theta)^2(1+2\theta-2(1+\theta)s_1^{CA})]w_1^{CA} - [\alpha(1-\alpha) + 4(2-\theta)^2(1+\theta)(1-s_1^{CA})]g}{2[\alpha^2 - (2-\theta)^2(3+4\theta-4(1+\theta)s_1^{CA})]}, \quad (25)$$

$$w_1^{CA} = \frac{Numw_1^{CA}}{Denw_1^{CA}}, \quad (26)$$

where $Numw_1^{CA}$ and $Denw_1^{CA}$ are long expressions that depend on the model's parameters g , α and θ and on the manufacturers' participation rate s_1^{CA} .

Furthermore, the two equilibria of the game, denoted by a superscript I or II, correspond to the two possible expressions for the cooperative advertising support rate as follows:

$$(s_1^{CA})^I = \frac{(2+\theta)^2(3-4\theta) - \alpha^2}{4(2+\theta)^2(1-\theta)}, \quad (27)$$

$$(s_1^{CA})^{II} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}. \quad (28)$$

$(s_1^{CA})^{II}$ requires $B^2 - 4AC \geq 0$, where

$$A = 16(2-\theta)^3(1-\theta^2)(2+\theta)[3(4-\theta^2) + \alpha(\theta(1-4\theta) + 10)],$$

$$B = 4(2-\theta)^2(1-\theta)[(1+\theta)(8\alpha^3 - 13(4-\theta^2)^2) + 2\alpha^2\theta(4+3\theta(2+\theta)) - \alpha(2+\theta)(102 + \theta(55 - 109\theta - 28\theta^2 + 32\theta^3))],$$

$$C = -\alpha^5(2-\theta) + 12(2-\theta)^4(2+\theta)^2(1-\theta^2) - \alpha^4(12 + \theta(-20 + 7\theta + 2\theta^2)) + \alpha(2-\theta)^2(1-\theta)(2+\theta)(126 + \theta(57 + 2\theta(89 + 16\theta - 32\theta^2))) + \alpha^2(2-\theta)^2(20 - \theta(24 + \theta(27 - 18\theta - 20\theta^2))) - \alpha^3(60 - \theta(76 + \theta(63 - 2\theta(53 - 17\theta)))).$$

Proof. It can be immediately shown that the fourth and third stages are identical to those in the proof of Proposition 1 and therefore, they are omitted here.

Second stage: In the first period, the retailer chooses the retail prices, p_{11} and p_{21} and his promotional efforts, a_{11} and a_{21} , in order to maximize his total profits over the first and second periods:

$$R = R_1 + R_2,$$

where the discount rate applied to future profits has been fixed to 1.

Taking into account the second-period retailer's profit (given by (16)), the retailer's total profit during the first and second periods is as follows:

$$R = \sum_{i=1,2} \left[(p_{i1} - w_{i1}) \frac{a_{i1} + g - p_{i1} - \theta(a_{(3-i)1} + g) + \theta p_{(3-i)1}}{1 - \theta^2} - (1 - s_i) a_{i1}^2 \right] \\ + \frac{(36 - 35\theta^2 + 3\theta^4)((g + \alpha a_{11})^2 + (g + \alpha a_{21})^2) + 4\theta(6 - 9\theta^2 + \theta^4)(g + \alpha a_{11})(g + \alpha a_{21})}{4(4 - \theta^2)^2(1 - \theta^2)(9 - 4\theta^2)}.$$

It can be easily proved that the retailer's total profit function, R , is concave in his first-period decision variables, p_{11}, p_{21} , and a_{11}, a_{21} if and only if the following conditions are satisfied:

$$(4 - \theta^2)^2(3 - 4\theta^2) - \alpha^2(4 - 3\theta^2) - 4(4 - \theta^2)^2(1 - \theta^2)s_{11} > 0, \quad (29)$$

$$\alpha^4 + (4 - \theta^2)^2(9 - 16\theta^2) - \alpha^2(24 - 26\theta^2) + 16s_{11}s_{21}(4 - \theta^2)^2(1 - \theta^2) \\ 4(s_{11} + s_{21})(\alpha^2(4 - 3\theta^2) - (4 - \theta^2)^2(3 - 4\theta^2)) \geq 0. \quad (30)$$

Taking into account the above conditions, and maximizing R with respect to the retailer's first-period decisions p_{11}, p_{21} , and a_{11}, a_{21} , we get the optimal retail prices and promotional efforts as functions of the first-period wholesale prices, w_{11} and w_{21} :

$$p_{i1} = \frac{g\Omega_1(s_{11}, s_{21}) + \Omega_2(s_{11}, s_{21})w_{i1} + \Omega_3(s_{11}, s_{21})w_{(3-i)1}}{2\Xi(s_{11}, s_{21})}, \quad (31)$$

$$a_{i1} = \frac{g\Phi_1(s_{11}, s_{21}) + \Phi_2(s_{11}, s_{21})w_{i1} + \Phi_3(s_{11}, s_{21})w_{(3-i)1}}{\Xi(s_{11}, s_{21})}, \quad (32)$$

where

$$\Omega_1(s_{11}, s_{21}) = 4s_{21}(1 - \theta)[(4 - \theta^2)^2 + (2 + \theta)^2(\alpha - 4s_{11}(2 - \theta)^2(1 + \theta))] - 4(3 - \theta - 4\theta^2)(4 - \theta^2)^2 \\ - 4(s_{11} + s_{21})(\alpha^2(4 - 3\theta^2) - (4 - \theta^2)^2(3 - 4\theta^2)) + (1 - \alpha)\alpha^3 - \alpha(3 - 4\theta)(2 + \theta)^2 \\ + \alpha^2(28 - 4\theta - 25\theta^2),$$

$$\Omega_2(s_{11}, s_{21}) = \alpha^2(20 - 27\theta^2) - \alpha^4 - 2(4 - \theta^2)^2(3 - 8\theta^2 + 2s_{21}(1 + 4s_{11}(1 - \theta^2))) \\ - 4(s_{11} + s_{21})(\alpha^2(4 - 3\theta^2) - (4 - \theta^2)^2(3 - 4\theta^2)),$$

$$\Omega_3(s_{11}, s_{21}) = 4\theta(\alpha^2 - (4 - \theta^2)^2(1 - s_{21})),$$

$$\Phi_1(s_{11}, s_{21}) = (\alpha + (2 - \theta)^2)(\alpha^2 + 4s_{21}(1 - \theta)(2 + \theta)^2 - (2 + \theta)^2(3 - 4\theta)),$$

$$\Phi_2(s_{11}, s_{21}) = 3(4 - \theta^2)^2 - \alpha^2(4 + \theta^2) + 4s_{21}(4 - \theta^2)^2,$$

$$\Phi_3(s_{11}, s_{21}) = 4\theta[\alpha^2 - (4 + \theta^2) + s_{21}(4 - \theta^2)^2],$$

$$\Xi(s_{11}, s_{21}) = \alpha^2(24 - 26\theta^2) - \alpha^4 - (4 - \theta^2)^2(9 - 16\theta^2) - 16s_{11}s_{21}(4 - \theta^2)^2(1 - \theta^2) \\ - 4(s_{11} + s_{21})(\alpha^2(4 - 3\theta^2) - (4 - \theta^2)^2(3 - 4\theta^2)).$$

First stage: The manufacturers take into account the retailer's reaction functions in the first period and play Nash to choose the wholesale prices in period 1, w_{11} and w_{21} . Manufacturer i maximizes her total profit M_i given by the sum of her profits obtained in periods 1 and 2 such as:

$$M_i = M_{i1} + M_{i2},$$

where the manufacturers' discount rate has been fixed again at 1.

Taking into account the second-period manufacturers' profits (given by (17) and (18)), manufacturer i 's total profit earned in both periods is

$$M_i = w_{i1} \frac{a_{i1} + g - p_{i1} - \theta(a_{(3-i)1} + g) + \theta p_{(3-i)1} - s_{i1} a_{i1}^2}{1 - \theta^2} + \frac{3[\theta(g + \alpha a_{(3-i)1}) - (3 - 2\theta^2)(g + \alpha a_{i1})][6\theta(g + \alpha a_{(3-i)1}) - (9 - 8\theta^2)(g + \alpha a_{i1})]}{8(9 - 4\theta^2)^2(1 - \theta^2)}.$$

With the help of Mathematica 10.1 we can deduce the conditions ensuring that the manufacturers' total profit, $M_i, i = 1, 2$, is a concave function in the manufacturer's first-period decision variable, w_{i1} . We refrain from writing these expressions because of their complexity.

Replacing the first-period retailer's reaction functions obtained in Stage 2 in M_i , and taking into account the conditions that ensure the concavity of M_i in her choice variable in the first-period (w_{i1}), the optimality conditions for the manufacturers' problems are obtained as follows

$$\frac{\partial M_1}{\partial w_{11}} = 0; \quad \frac{\partial M_2}{\partial w_{21}} = 0, \quad \frac{\partial M_1}{\partial s_{11}} = 0; \quad \frac{\partial M_2}{\partial s_{21}} = 0.$$

Given the symmetric structure of the game for the manufacturers, the above system of four equations simplifies as a system of two equations. From the first equation one can get the first-period wholesale price as a function of s_1 as shown in (26). Substituting this last expression into the other optimality condition, a non-linear equation in variable s_1 is obtained which admits different solutions. We removed the solutions that imply a null promotional investment and three possibilities for the first-period manufacturers' strategies are derived. One of these solutions is removed because it does not satisfy the concavity conditions in (29) and (30). Finally, two possibilities for s_1 remain feasible and their expressions are reported in (27) and (28). The remainder optimal expressions (22), (23), (24), (25) can easily be obtained taking into account the symmetric manufacturers' first-period optimal strategies. We then get two optimal solutions for the CA game (Scenario I and Scenario II). ■

As we can see in Figure 1, both solutions (Scenarios I and II) are feasible in the red region of Figure 1. In order to identify the equilibrium solution in this region, we proceed to comparing the total profits obtained over both periods by each channel member and by the

total channel. Analytical results are graphically represented similarly to Figures 1 to 6. We omit these figures for ease of illustration.

We compare the optimal manufacturers' profits obtained in Scenarios I and II and find that the manufacturers prefer Scenario I in most of the feasible domain where both scenarios are feasible. Scenario II is preferred by manufacturers whenever product substitutability is very low and the long term effect of the retailer's promotions are not too high. It is also preferred when the market is characterized by high substitutability ($0.5 < \theta < 0.6$) and α is very low. This is the solution used in the paper.

We also compare the optimal retailer's profit obtained in Scenarios I and II and find that the retailer prefers the solution in Scenario II in most of the feasible domain where both scenarios are feasible. An exception is noted in the region where product substitutability (θ) levels are very high, in which case Scenario I is preferred by the retailer.

Given these dissimilar preferences for Scenarios I and II by the manufacturers and the retailer, we study conditions on the model parameters for which the channel members have similar and dissimilar preferences for Scenarios I and II. We find that the retailer and manufacturers will prefer different solutions in two cases. In the first case, the manufacturers prefer Scenario I, while the retailer prefers Scenario II for intermediate levels of θ and a wide range of α . In the second case, the manufacturers prefer Scenario II, while the retailer prefers Scenario I for high enough levels of θ and very low negative values of α .

Finally, we compare optimal total channel profits in Scenarios I and II in the CA game. The results show that Scenario I (II) is optimal for low (high) levels of θ whenever both Scenarios are feasible. These comparisons show that while the manufacturers are the channel leaders and therefore will implement their preferred equilibrium solution opportunistically, the total channel profit might suffer from such a strategy. In particular, the total channel profit can be higher when the retailer's preferred equilibrium is implemented, mainly when the market is characterized by low enough substitutability levels (low θ). This suggests that the CA game can lead to Pareto-inefficient results. This is a new finding that has not been observed so far in the literature studying cooperative advertising using static games with no manufacturing competition.

6.3 Comparisons of the optimal decision variables, retailer's optimal margins and demands in the different games

In all feasible domain where both games are feasible, we find $p_1^{CA} > p_1^{NCA}$ and $a_1^{CA} > a_1^{NCA}$. The analytical results obtained from comparing the other equilibrium outputs are reported in

Figures 7-9.

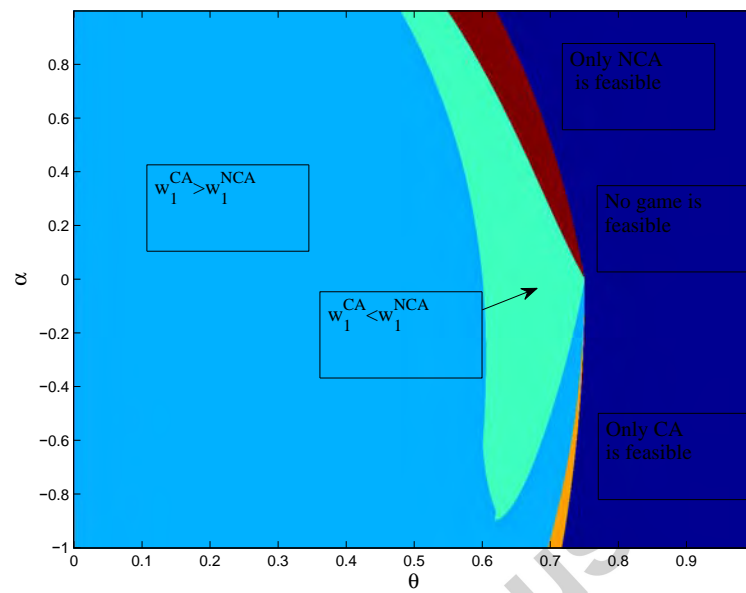


Figure 7: Comparison of first-period optimal wholesale prices in the different games.

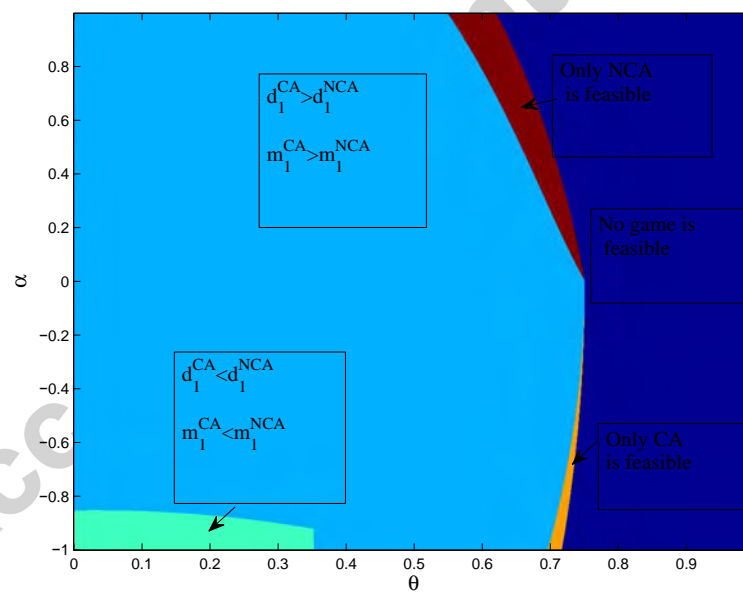


Figure 8: Comparison of retailer's first-period optimal margins and first-period demands in the different games.

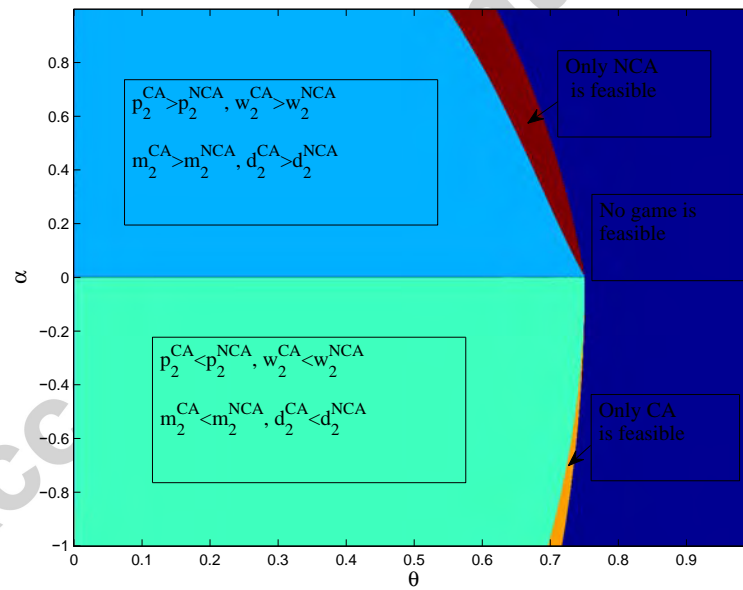


Figure 9: Comparison of second-period optimal retail and wholesale prices, retailer's margins and demands in the different games.

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