Real-World Versus Risk-Neutral Measures in the Estimation of an Interest Rate Model with Stochastic Volatility

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Abstract In this paper, we consider a jump-diffusion two-factor model which stochastic volatility to obtain the yield curves efficiently. As this is a jump-diffusion model, the estimation of the market prices of risk is not possible unless a closed form solution is known for the model. Then, we obtain some results that allow us to estimate all the risk-neutral functions, which are necessary to obtain the yield curves, directly from data in the markets. As the market prices of risk are included in the risk-neutral functions, they can also be obtained. Finally, we use US Treasury Bill data, a nonparametric approach, numerical differentiation and Monte Carlo simulation approach to obtain the yield curves. Then, we show the advantages of considering the volatility as second stochastic factor and our approach in an interest rate model.

Keywords Interest rates · Stochastic volatility · Jump-diffusion · Stochastic processes · Nonparametric · Estimation · Numerical differentiation

1 Introduction

Traditionally, the financial literature assumes that interest rates move continuously and they are modelled as diffusion processes, as in and so on. However, more recent studies have showed that interest rates contained unexpected discontinuous changes, see for example Jumps in interest rates are, probably, due to different market phenomena such as surprises or shocks in foreign exchange markets. Moreover, when pricing and hedging financial derivatives jump-diffusion models are very important, since ignoring jumps can produce hedging and pricing risks, see.

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It is widely known that one-factor interest rate models are very attractive for practitioners because its simplicity and computational convenience. However, these models have also unrealistic properties. First, they cannot generate all the yield curve shapes and changes that we can find in the markets. Second, the changes over infinitesimal periods of any two interest-rate dependent variables will be perfectly correlated. Finally, as shows, none of their analyzed one-factor models captures the interest rate dynamics adequately. Therefore, we consider that at least two factors are necessary to model the term structure of interest rates. In fact, the number of factor must be a compromise between numerical efficient implementation and the capability of the model to fit data.

The main goal of this paper is twofold. We consider that the volatility is an important stochastic factor which can help to obtain the yield curves accurately. Then we show some results to estimate all the functions of this model even when a closed form solution is not known.

The rest of the paper is organized as follows. Section shows a two-factor interest rate jump-diffusion model. In Sect., we propose some results in order to estimate all the functions of the model efficiently. Finally, in Sect. we show the supremacy of considering the volatility as stochastic and the estimation of the whole functions of the model directly from data in the market using interest rate data from US Federal Reserve.