Copula-based analysis of multivariate dependence patterns between dimensions of poverty in Europe

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Abstract

It is widely recognised that poverty is a multidimensional phenomenon involving not only income, but also other aspects such as education or health. In this multidimensional setting, analysing the dependence between dimensions becomes an important issue, since a high degree of dependence could exacerbate poverty. In this paper, we propose measuring the multivariate dependence between the dimensions of poverty in Europe using copula-based methods. This approach focuses on the positions of individuals across dimensions, allowing for other types of dependence beyond linear correlation. In particular, we analyse how orthant dependence between the dimensions of the AROPE rate has evolved in the EU-28 countries between 2008 and 2014 by applying non-parametric estimates of multivariate copula-based generalisations of Spearman's rank correlation coefficient. We find a general increase in the dependence between dimensions, regardless of the coefficient used.

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Moreover, countries with higher AROPE rates also tend to experiment more dependence between its dimensions.

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1 Introduction

There is a widespread agreement that poverty is a multidimensional phenomenon involving not only low incomes, but also deprivations in other dimensions such as education, health or labour; see, for instance, Sen (1985, 1987). Because of that, attention has been increasingly focused on multidimensional approaches to the analysis of poverty, to the point where the European Union (EU), for example, has adopted a multidimensional poverty and social exclusion index as a tool to monitor and implement effective poverty-reduction policies in the framework of the Europe 2020 Strategy. The index at hand, namely the AROPE (At Risk Of Poverty or social Exclusion) rate, is based on three measures: relative income poverty, material deprivation and work intensity. Also, the United Nation Development Program (UNDP) adopted, in 2010, the Multidimensional Poverty Index (MPI), which is based on the Alkire and Foster (2011) proposal. This index also considers three dimensions: education, health and standard of living. Based on these indices (or any other multivariate indicator), several authors examined the incidence and intensity of multidimensional poverty in developed and non-developed countries; see, for instance, the contributions of Nolan and Whelan (2011), Whelan et al. (2014), Alkire and Apablaza (2016), White (2017) and Atkinson et al. (2017), in the European context.

However, many of the multidimensional poverty indices, especially some of the most widely used, such as the AROPE rate and the MPI, are not sufficiently sensitive to the possible interrelation between the dimensions of poverty. Therefore, they could miss an important part of the picture; see Duclos and Tiberti (2016). In this context, several authors argue that incorporating those relationships can be relevant, since higher dependence means higher concentration of deprivations and this could make overall poverty worse; see, for instance, Atkinson and Bourguignon (1982), Bourguignon and Chakravarty (2003), Duclos et al. (2006), Seth (2013) and Ferreira and Lugo (2013). In spite of its relevance, the problem of measuring the dependence between dimensions of poverty has been scarcely addressed in the literature and this is the scope of this paper. Noticeably, as we face a problem of studying dependence in a multivariate context, special care is required, since the step from two dimensions to three (or more) dimensions is not so obvious. Actually, as Durante et al. (2014) show, some bivariate dependence properties are not preserved in higher dimensions.

In this framework, we propose complementing the analysis based on poverty indices by measuring the multivariate dependence among poverty dimensions using copula-based methods. The copula approach focuses on the positions of the individuals across dimensions, rather than on the specific values that those dimensions attain for such individuals. This approach has several advantages. First, it enables the decomposition of the joint distribution function of all dimensions into its univariate marginals and the dependence structure, which is captured by the copula. Nevertheless, as Genest and Nešlehová (2007) point out, the copula alone does not characterize the dependence in the discrete case. Second, copulas allow building scaled-free measures of dependence that capture other types of dependence beyond linear correlation. Actually, the well-known Spearman's rho and other related measures of bivariate association can be expressed in terms of copulas. Third, the copula approach facilitates the construction of multivariate generalisations of bivariate association coefficients, although the generalisation is not unique in some cases (see Section 2). Furthermore, dominance tests are also possible to establish copula-based orderings of dependence; see Decancq (2012) and the references therein. This would allow to rank pairs of multivariate distributions and perform full comparisons between two societies. However, as Decancq (2014) points out, this ordering could be 'indecisive' in many cases, meaning that the societies cannot be ranked with respect to the dependence between the poverty dimensions considered. To overcome this drawback, one may prefer using copula-based dependence measures that can rank the distributions being compared. This is the approach we adopt in this paper.

Applications of copula-based methods in welfare economics in a bivariate setting date back to Dardanoni and Lambert (2001), Quinn (2007) and Bø et al. (2012); see also the recent contribution of Aaberge et al. (2018). In a multidimensional framework, the first contribution employing copula-based methods in welfare economics is Decancq (2014). He analysed the temporal evolution of well-being in Russia by means of a multivariate Kendall's tau and a multivariate version of Spearman's rho applied to the dimensions included in the Human Development Index (HDI). Pérez and Prieto (2015) extended Decancq's results by considering other multivariate versions of Spearman's rho to study how the dependence between the dimensions of the AROPE rate has evolved in Spain over the period 2009-2013. Also, Pérez and Prieto-Alaiz (2016a) analysed the multivariate dependence between the dimensions of the HDI using data from 187 countries and three copula-based measures of multivariate association: Spearman's footrule, Gini's gamma and Spearman's rho.

The contribution of this paper is twofold. First, we consider multivariate extensions of Spearman's rho proposed by Nelsen (1996, 2002), which allow to capture some types of dependence which are essential in poverty analysis, namely those based on orthant dependence. Particularly useful is the coefficient based on lower orthant dependence, as it could measure the propensity of being simultaneously low-ranked in all dimensions of poverty. We also consider the generalizations of these coefficients to possibly non-continuous multivariate distributions proposed by Quessy (2009) and Mesfioui and Quessy (2010). Second, we apply these coefficients to perform cross-country and temporal comparisons of the multivariate dependence between the dimensions of the AROPE rate in the EU-28 countries over the period 2008-2014. As far as we know, this is the first time that these copula-based measures are applied in the European context.

The data we use comes from the EU-Statistics on Income and Living Conditions (EU-SILC) survey, which is the EU reference source for comparative statistics on income distribution and social inclusion at the European level. Our analysis complements the information on the incidence of multidimensional poverty, given by the AROPE rate, with information on the degree of multivariate dependence between its dimensions. In particular, we find that, in most EU countries, there has been an increase in the dependence between poverty dimensions over the period analysed. Noticeably, the highest increase corresponds to Spain, one of the countries most severely hit by the last economic crisis. Moreover, over all the years considered, the maximal dependence is generally found in the lower part of the joint distribution. These results imply that small values of income, no-material deprivation and work intensity tend to occur together, and this is more likely in 2014 than in 2008. We also detect strong dependence in the upper orthant of the joint distribution, suggesting that, after the crisis, most EU countries have become more polarised. Finally, we find that countries with higher AROPE rates also tend to experiment more dependence between its dimensions.

The rest of the paper is organised as follows. Section 2 summarises the basic properties of copulas and describes orthant dependence concepts. It also introduces copula-based multivariate versions of Spearman's rho coefficient and discusses how to estimate them non-parametrically using the empirical copula. New properties of the estimators considered are also included. Section 3 illustrates the use of these tools to measure how the dependence between the three indicators of the AROPE rate has evolved in the EU-28 countries over the period 2008-2014. Section 4 concludes the paper with a summary of the main results.

2 Methodology

2.1 Copulas and orthant dependence

Copulas are joint distribution functions whose one-dimensional margins are uniform on $\mathbf{I}$ =[0, 1]. More precisely, a $d-$dimensional copula C is a multivariate distribution function $C : \mathbf{I}^d \to \mathbf{I}$ defined for every $\boldsymbol{u} = (u_1, \ldots, u_d) \in \boldsymbol{I}^d$ as $C(\boldsymbol{u}) = p(\boldsymbol{U} \leq \boldsymbol{u}) = p(U_1 \leq u_1, \ldots, U_d \leq u_d)$, where U_i is $U(0, 1)$, for $i = 1, ..., d$.[1] The importance of copulas in statistics relies on the Sklar's theorem (Sklar, 1959). This theorem establishes that, if $\mathbf{X} = (X_1, ..., X_d)$ is a $d-$dimensional random vector with joint distribution function $F(\boldsymbol{x}) = F(x_1, ..., x_d) = p(X_1 \leq x_1, ..., X_d \leq x_d)$ and univariate marginal distribution functions $F_i(x_i) = p(X_i \leq x_i)$, for $i = 1, ..., d$, then there exists a copula C such that, for all $\boldsymbol{x} = (x_1, \ldots, x_d) \in R^d$, F can be represented as

$$F(\boldsymbol{x}) = C(F_1(x_1), ..., F_d(x_d)). \tag{1}$$

Hence, copulas are functions that join or "couple" multivariate distribution functions to their one-dimensional marginal distribution functions. If the margins $F_1, ..., F_d$ are all continuous, the copula C in (1) is unique; otherwise C is uniquely determined on $RanF_1 \times ... \times RanF_d$. Conversely, if C is a $d-$copula and $F_1, ..., F_d$ are univariate distribution functions, the function F defined in (1) is a joint distribution function with margins $F_1, ..., F_d$. Throughout this section, we generally assume that $F_1, ..., F_d$ are all continuous, although some issues arising when dealing with possibly non-continuous variables will be duly pointed out.

In a multidimensional poverty setting, the random vector $\mathbf{X}$ represents the relevant d dimensions of poverty for a population and the transformed variables $U_i = F_i(X_i)$, with $i = 1, \ldots, d$, attach to each individual in the population its relative position in all dimensions. For instance, an individual with position vector $(1, \ldots, 1)$ will be top-ranked in all dimensions. Each random

[1] An equivalent definition of a multivariate copula can be found in Nelsen (2006, p. 45)

variable U_i is $U(0,1)$ and the joint distribution of the vector $\boldsymbol{U} = (U_1, ..., U_d)$ is the copula C defined above. Therefore, for a given real vector $\boldsymbol{u} \in \mathbf{I}^d$, the value $C(\boldsymbol{u})$ represents the proportion of individuals in the population with positions outranked by $\boldsymbol{u}$. For instance, $C(0.25, ..., 0.25)$ will represent the probability that a randomly selected individual is simultaneously in the 1^{st} quartile ("low ranked") in all dimensions, i.e., in our setting, it will be the probability that he/she is simultaneously "poor" in all dimensions.

Any copula C satisfies the Fréchet-Hoeffding bounds inequality

$$W(\boldsymbol{u}) \leq C(\boldsymbol{u}) \leq M(\boldsymbol{u}),$$

for every $\boldsymbol{u} \in \boldsymbol{I}^d$, where $W(\boldsymbol{u}) = \max(u_1 + \cdots + u_d - d + 1, 0)$ and $M(\boldsymbol{u}) = \min(u_1, \ldots, u_d)$. M is always a copula and represents maximal dependence, i.e. the case when each of the random variables $X_1, ..., X_d$ is almost surely a strictly increasing function of any of the others (the outcomes in all dimensions are ordered in the same way). W is only a copula if $d = 2$, in which case it represents perfect negative dependence. Another important copula is the independent copula, defined as $\Pi(\boldsymbol{u}) = u_1 \times \cdots \times u_d$, which accounts for the case where the variables $X_1, \ldots, X_d$ are independent.

Finally, if $\boldsymbol{U} = (U_1, ..., U_d)$ is a random vector of variables $U(0,1)$ whose joint distribution function is the copula C, the survival function $\overline{C} : \boldsymbol{I}^d \to \boldsymbol{I}$ is defined as:

$$\overline{C}(\boldsymbol{u}) = p(\boldsymbol{U} > \boldsymbol{u}) = p(U_1 > u_1, \ldots, U_d > u_d).$$

In our setting, for instance, $\overline{C}(0.75, ..., 0.75)$ will represent the probability that a randomly selected individual is simultaneously in the 4^{th} quartile ("top ranked") in all dimensions, i.e., the probability that he/she is simultaneously "rich" in all dimensions. In general, $\overline{C}$ is not a copula. Moreover, if $U_1, ..., U_d$ are independent random variables, then its survival function is $\overline{\Pi}(\boldsymbol{u}) = (1 - u_1) \times \cdots \times (1 - u_d)$. For a comprehensive review of copulas, see Nelsen (2006).

In this paper, we use copulas to study measures of multivariate association derived from multivariate dependence concepts. The notions of dependence in the multivariate case can be defined in different ways. The one we handle in this paper is orthant dependence and it is defined as follows (Nelsen, 2006):

- $\mathbf{X}$ is positively *lower orthant dependent* (PLOD) if $C(\boldsymbol{u}) \geq \Pi(\boldsymbol{u})$, for each $\boldsymbol{u} \in \mathbf{I}^d$, that is, if the probability that the variables $X_1, ..., X_d$ are simultaneously small is at least as great as it would be were they independent.

- $\mathbf{X}$ is positively *upper orthant dependent* (PUOD) if $\overline{C}(\boldsymbol{u}) \geq \overline{\Pi}(\boldsymbol{u})$, for each $\boldsymbol{u} \in \mathbf{I}^d$, that is, if the probability that the variables $X_1, ..., X_d$ are simultaneously large is at least as great as it would be were they independent.

- $\mathbf{X}$ is positively *orthant dependent* (POD) if both inequalities hold.

The corresponding negative concepts (NLOD, NUOD and NOD) are defined by reversing the sense of the inequalities above. For $d = 2$, PLOD and PUOD are the same and reduce to POD. Obviously, the same reduction occurs with the analogous negative concepts. For poverty analysis, lower orthant dependece will be the more relevant concept.

In this framework, the differences $[C(\boldsymbol{u})-\Pi(\boldsymbol{u})]$ and $[\overline{C}(\boldsymbol{u})-\overline{\Pi}(\boldsymbol{u})]$ can be regarded as measures of "local" lower and upper orthant dependence, respectively; see Nelsen (1996). Accordingly, the copula-based measures of multivariate association to be introduced in next Section are based on these differences.

2.2 Copula-based multivariate extentions of Spearman's rho

One of the best-known measures of association between two random variables X_1 and X_2 is Spearman's rank correlation coefficient, also known as Spearman's rho (ρ_S). This measure, which is the correlation coefficient of the transformed random variables $F_1(X_1)$ and $F_2(X_2)$, can be expressed in terms of their copula C as follows (Nelsen, 1991):

$$\rho_S = 12 \int_{\mathbf{I}^2} C(u_1, u_2) du_1 du_2 - 3 = 12 \int_{\mathbf{I}^2} u_1 u_2 dC(u_1, u_2) - 3. \tag{2}$$

When we move to a multivariate setting, several extensions of Spearman's rho can be found in the literature. The first copula-based generalisation of bivariate Spearman's rho, due to Wolff (1980) and Nelsen (1996), is a multivariate extension of the left-hand side expression in equation (2) and is defined as:

$$\rho_d^- = \frac{2^d(d+1)}{2^d - (d+1)} \int_{\mathbf{I}^d} [C(\boldsymbol{u}) - \Pi(\boldsymbol{u})] d\Pi(\boldsymbol{u}) = \frac{(d+1)}{2^d - (d+1)} \left[2^d \int_{\mathbf{I}^d} C(\boldsymbol{u}) d\Pi(\boldsymbol{u}) - 1 \right]. \tag{3}$$

Following Nelsen (1996), ρ_d^- can be regarded as a multivariate measure of average lower orthant dependence. In fact, ρ_d^- assesses, to some extent, the similarity between our multivariate data $\mathbf{X}$ (represented by its copula C) and the situation of independence (represented by copula Π) in the lower orthant.

In a similar fashion, Nelsen (1996) defined a second generalisation of Spearman's rho, derived from average upper orthant dependence. This measure, which is a multivariate extension of the right-hand side expression in equation (2), is given by:

$$\rho_d^+ = \frac{2^d(d+1)}{2^d - (d+1)} \int_{\mathbf{I}^d} [\overline{C}(\boldsymbol{u}) - \overline{\Pi}(\boldsymbol{u})] d\Pi(\boldsymbol{u}) = \frac{(d+1)}{2^d - (d+1)} \left[2^d \int_{\mathbf{I}^d} \Pi(\boldsymbol{u}) dC(\boldsymbol{u}) - 1 \right]. \tag{4}$$

From this expression, ρ_d^+ could be thought of as the normalised average difference between $\overline{C}$ – representing the behaviour of our data in the upper orthant – and $\overline{\Pi}$ – representing independence in such orthant.

The third copula-based multivariate version of Spearman's rho, due to Nelsen (2002), is the average of the two generalizations described above, namely:

$$\rho_d = \frac{\rho_d^- + \rho_d^+}{2} = \frac{(d+1)}{2^d - (d+1)}\left[2^{d-1}\left(\int_{\mathbf{I}^d} C(\boldsymbol{u})d\Pi(\boldsymbol{u}) + \int_{\mathbf{I}^d} \Pi(\boldsymbol{u})dC(\boldsymbol{u})\right) - 1\right]. \tag{5}$$

This coefficient ρ_d is further discussed in Dolati and Úbeda-Flores (2006) as an example of Average Orthant Dependence (AOD) measure of multivariate concordance. See also Taylor (2007).

When the distribution of $\mathbf{X}$ is radially symmetric, it follows that $\rho_d^- = \rho_d^+ = \rho_d$. Moreover, if $\mathbf{X}$ is PLOD (NLOD) then $\rho_d^- \geq 0$ ($\rho_d^- \leq 0$); if $\mathbf{X}$ is PUOD (NUOD) then $\rho_d^+ \geq 0$ ($\rho_d^+ \leq 0$); and if $\mathbf{X}$ is POD (NOD) then $\rho_d \geq 0$ ($\rho_d \leq 0$). Furthermore, when the copula of $\mathbf{X}$ is the upper bound M, the three measures defined above attain their maximum value, 1, and they all become zero when the components of $\mathbf{X}$ are independent ($C = \Pi$). A lower bound for the three of them is $[2^d - (d+1)!]/\{d![2^d - (d+1)]\}$; see Nelsen (1996).

Noticeably, for $d = 2$, the three coefficients above, ρ_2^-, ρ_2^+ and ρ_2, reduce to bivariate Spearman's rho defined in (2). Furthermore, in the trivariate case ($d = 3$), ρ_3 becomes the average of the three pairwise Spearman's rho coefficients, that is:

$$\rho_3 = \frac{\rho_3^- + \rho_3^+}{2} = \frac{\rho_{12} + \rho_{13} + \rho_{23}}{3}, \tag{6}$$

where ρ_{ik} denotes the pairwise Spearman's rho coefficient for the bivariate random variable (X_i, X_k), with $1 \leq i < k \leq 3$; see Nelsen (1996). Moreover, in the trivariate case, Nelsen and Úbeda-Flores (2012) and García et al. (2013) develop other copula-based coefficients of dependence that include, as particular cases, ρ_3^- and ρ_3^+; see Appendix 2.

The advantage of ρ_d^- and ρ_d^+ is that they are capable of revealing some forms of dependence that ρ_d fails to detect. See, for instance, Example 1 in Nelsen and Úbeda-Flores (2012) where $\rho_3 = 0$, presumably indicating no dependence, whereas ρ_3^+ and ρ_3^- are different from 0, indicating some degree of upper and lower average orthant dependence, respectively. See also Example 2 in

Nelsen (1996).

The dependence measures described so far are developed for continuous variables. However, when ties can occur with non null probability, many of the desirable properties of these measures may fail to hold. As Genest and Nešlehová (2007) point out, the use of copulas when the marginals are non-continuous is subject to caution, because some of the properties do not carry over from the continuous to the non-continuous case, due to the lack of uniqueness of Sklar's representation (1). In turn, copula-based concordance measures such as Spearman's rho are margin-dependent. In this context, Quessy (2009) and Mesfioui and Quessy (2010) have proposed tie-corrected versions of the multivariate Spearman's coefficients in (3)-(4) and (5), respectively. These coefficients are suitable for non-continuous variables and can be written (Genest et al., 2013) as follows:

$$\rho_d^{-\maltese} = \frac{(d+1)}{2^d-(d+1)}\left[2^d E\left(\prod_{i=1}^{d}(1-\widetilde{F}_i(X_i))\right)-1\right], \tag{7}$$

$$\rho_d^{+\maltese} = \frac{(d+1)}{2^d-(d+1)}\left[2^d E\left(\prod_{i=1}^{d}\widetilde{F}_i(X_i)\right)-1\right], \tag{8}$$

$$\rho_d^{\maltese} = \frac{\rho_d^{-\maltese}+\rho_d^{+\maltese}}{2}, \tag{9}$$

where, for all $i \in \{1, ..., d\}$ and $x \in \mathbb{R}$,

$$\widetilde{F}_i(x) = \frac{1}{2}\{\Pr(X_i < x) + \Pr(X_i \leq x)\}.$$

If all the components of $\mathbf{X}$ are continuous, one would have $\widetilde{F}_i = F_i$ for all i and the coefficients in (7)-(9) will reduce to those in (3)-(5). Moreover, the former inherit some of the properties of the latter. For instance, they all become 0 in the case of multivariate independence and attain their maximum value under the copula M, although their values are smaller than 1 under perfect association when the probability of ties is positive for one or more of the variables; see Quessy (2009).

2.3 Non-parametric estimation

In practice, the copula C is unknown and the coefficients described in Section 2.2 must be estimated from the data. Therefore, empirical versions of these coefficients are required. Let $\mathbf{X}_1, ..., \mathbf{X}_n$ be a sample of n serially independent random vectors from the d-dimensional continuous vector $\mathbf{X}$ with associated copula C, where $\mathbf{X}_j = (X_{1j}, ..., X_{dj})$ for $j = 1, ..., n$. The copula C can be estimated non-parametrically by the empirical copula $\widetilde{C}_n$ defined as:

$$\widetilde{C}_n(\boldsymbol{u}) = \frac{1}{n} \sum_{j=1}^{n} \prod_{i=1}^{d} \mathbf{1}_{\{\widetilde{U}_{ij} \leq u_i\}}, \text{ for } \boldsymbol{u} = (u_1, ..., u_d) \in \mathbf{I}^d, \tag{10}$$

where $\mathbf{1}_A$ denotes the indicator function on a set A and $\widetilde{U}_{ij}$ are the transformed data to $[0, 1]$ by scaling ranks, i.e.

$$\widetilde{U}_{ij} = R_{ij}/n, \tag{11}$$

where R_{ij} denotes the *rank* of X_{ij} among $\{X_{i1}, ..., X_{in}\}$, with $i = 1, ..., d$ and $j = 1, ..., n$.

Statistical inference for ρ_d^- and ρ_d^+ based on the empirical copula is discussed in Schmid and Schmidt (2007) and Schmid et al. (2010). In particular, these authors propose estimating nonparametrically the coefficients ρ_d^- and ρ_d^+ by replacing the copula C in (3) and (4), respectively, with the empirical copula in (10). However, Pérez and Prieto-Alaiz (2016b) show that the resultant statistics are not proper estimators of their population counterparts, since they can take values out of the parameter space. The modifications proposed by Blumentritt and Schmid (2014) and Bedo and Ong (2014) have still some drawbacks, as they fail to achieve the maximum value 1 for maximal dependence and take a narrower range of values than they should. To overcome these problems, Pérez and Prieto-Alaiz (2016b) propose alternative feasible nonparametric estimators of ρ_d^- and ρ_d^+, based on the results in Joe (1990), which are given by the following expressions, respectively:

$$\widehat{\rho_d^-} = \frac{\frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\widetilde{\overline{U}}_{ij} - \left(\frac{n+1}{2n}\right)^d}{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^d - \left(\frac{n+1}{2n}\right)^d}, \tag{12}$$

$$\widehat{\rho_d^+} = \frac{\frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\widetilde{U}_{ij} - \left(\frac{n+1}{2n}\right)^d}{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^d - \left(\frac{n+1}{2n}\right)^d}, \tag{13}$$

where $\widetilde{\overline{U}}_{ij} = \overline{R}_{ij}/n$ and $\overline{R}_{ij} = n + 1 - R_{ij}$. By construction, both $\widehat{\rho_d^-}$ and $\widehat{\rho_d^+}$ achieve their maximum value 1 for maximal dependence and they become 0 in the case of independence. Moreover, these estimators share the same asymptotic normal distribution as those in Schmid and Schmidt (2007). Nonetheless, the asymptotic variances cannot be explicitly evaluated for the majority of known copulas (not even for $d = 2$) but, as shown in Schmid and Schmidt (2007), they can consistently be estimated by nonparametric bootstrap methods. Therefore, in the empirical application (Section 3), bootstrap methods will be applied to estimate their standards errors and perform statistical inference.

To estimate the coefficient ρ_d in (5), we propose the following plug-in estimator

$$\widehat{\rho}_d = \frac{\widehat{\rho_d^-} + \widehat{\rho_d^+}}{2}, \tag{14}$$

where $\widehat{\rho_d^-}$ and $\widehat{\rho_d^+}$ are the estimators in (12) and (13), respectively. Noticeably, this estimator coincides with the estimator of ρ_d proposed by Dolati and Úbeda-Flores (2006) in the framework of AOD measures of multivariate concordance; see Proposition 1 in Appendix 1.

For the bidimensional case ($d = 2$), all the estimators above, namely $\widehat{\rho_2^-}$, $\widehat{\rho_2^+}$ and $\widehat{\rho}_2$, coincide with the well-known sample version of bivariate Spearman's rho. In the trivariate case ($d = 3$), the estimators $\widehat{\rho}_3^{+}$ and $\widehat{\rho}_3^{-}$ reduce to:

$$\widehat{\rho}_3^{+} = \frac{8}{n(n-1)(n+1)^2}\sum_{j=1}^{n} R_{1j}R_{2j}R_{3j} - \frac{n+1}{n-1}, \tag{15}$$

$$\widehat{\rho}_3^{-} = \frac{8}{n(n-1)(n+1)^2}\sum_{j=1}^{n} \overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j} - \frac{n+1}{n-1}. \tag{16}$$

Moreover, it can be shown (see Proposition 2 in Appendix 1) that property (6) continues to hold for the corresponding empirical coefficients, that is

$$\widehat{\rho}_3 = \frac{\widehat{\rho}_3^{-} + \widehat{\rho}_3^{+}}{2} = \frac{\widehat{\rho}_{12} + \widehat{\rho}_{13} + \widehat{\rho}_{23}}{3}, \tag{17}$$

where $\widehat{\rho}_{ik}$ denotes the bivariate sample Spearman's rho for the pair (X_i, X_k), with $1 \leq i < k \leq 3$. Hence, in the trivariate case, the sample version of the coefficient ρ_3 can be easily computed as the average of their corresponding pairwise sample coefficients.

In order to estimate the tie-corrected generalizations of multivariate Spearman's rho coefficients in (7)-(9), Genest et al. (2013) propose the following rank-based estimators:

$$\widehat{\rho}_d^{-\maltese} = \frac{(d+1)}{2^d-(d+1)}\left[2^d\frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\left(\frac{2n+1}{2n} - \frac{\widetilde{R}_{ij}}{n}\right) - 1\right], \tag{18}$$

$$\widehat{\rho}_d^{+\maltese} = \frac{(d+1)}{2^d-(d+1)}\left[2^d\frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\left(\frac{\widetilde{R}_{ij}}{n} - \frac{1}{2n}\right) - 1\right], \tag{19}$$

$$\widehat{\rho}_d^{\maltese} = \frac{\widehat{\rho}_d^{-\maltese} + \widehat{\rho}_d^{+\maltese}}{2}, \tag{20}$$

where $\widetilde{R}_{ij}$ is the mid-rank of X_{ij} among $\{X_{i1}, ..., X_{in}\}$, with $i = 1, ..., d$ and $j = 1, ..., n$. Genest et al. (2013) show that these estimators are asymptotically normally distributed and provide expressions of their limiting variances, thereby correcting errors in the asymptotic variance formulas derived in Quessy (2009) for $\widehat{\rho}_d^{-\maltese}$ and $\widehat{\rho}_d^{+\maltese}$ and Mesfioui and Quessy (2010) for $\widehat{\rho}_d^{\maltese}$. Nevertheless, the asymptotic variances are complex and hence, in practice, they will be estimated by bootstrap methods.

3 Empirical application

As we said in the Introduction, multidimensional poverty depends not only on the proportion of individuals deprived in each dimension but also on the degree of interdependence between dimensions, since higher dependence means higher concentration of deprivations and this could make overall poverty worse. In this context, we propose complementing the information given by traditional multidimensional poverty indices with measures of multivariate dependence between poverty dimensions. In particular, we apply the copula-based coefficients described in Section 2 to measure the evolution of the dependence between the dimensions of poverty in the EU-28 countries over the period 2008-2014.

3.1 Data and variables

The data we use comes from the EU-SILC survey, which is the key reference for data on income and living conditions in the EU. In particular, we use the cross-sectional surveys of all years of the period 2008-2014.

The dimensions of poverty we consider are those included in the AROPE rate, namely income, material needs and work intensity. The selection of these dimensions is based on the relevance of the AROPE rate in the European context, as it is the headline indicator to monitor and implement effective poverty-reduction policies in the framework of the Europe 2020 Strategy. In fact, one of the Europe 2020 headline targets established by the European Commission is to reduce, by 20 million, the number of people at risk of poverty and social exclusion.[2]

[2]Despite the importance of the AROPE rate from the public policy perspective, the choice of the dimensions involved in its calculation is not exempt of criticism; see, for instance, the discussion in Nolan and Whelan (2011, ch.11).

The three measures characterising the three dimensions of the AROPE rate are defined as follows. The measure of income is the equivalised disposable income, which is calculated as the total income of the household, after taxes and other deductions, divided by the equivalised household size.[3] The work intensity of a household is the ratio of the total number of months that all working-age household members have worked during the income reference year and the total number of months they could have theoretically worked during the same period.[4] Material deprivation is originally defined as the enforced lack in a number of essential items, namely: 1) the capacity of facing unexpected expenses; 2) one-week annual holiday away from home; 3) a meal involving meat, chicken or fish every second day; 4) an adequately warm dwelling; 5) a washing machine; 6) a colour television; 7) a telephone; 8) a car; 9) the capacity to pay their rent, mortgage or utility bills. For ease of interpretation we transform this variable into a variable that indicates the number of no-deprivations out of the nine possible, so that the new variable takes the following values: 0 (having all the 9 possible deprivations), 1 (having eight out of the nine aforementioned deprivations), ..., 9 (having no deprivations). Thus, high values of the three variables considered (equivalised disposable income, work intensity, and number of no-deprivations) convey lower chance to be poor, while low values of each variable convey higher chance to be poor.

The unit of analysis is the household. We only work with subsamples of households for which we have complete information for all the three variables. In particular, in these subsamples, households composed only of children, of students aged 18-24 and/or people aged 60 or more are excluded, due to their missing values in the work intensity variable.[5] In these subsamples, the sample sizes range from 2270 households (Cyprus, 2009) to 14773 households (Italy, 2008).

[3]The equivalised household size is defined according to the modified OECD scale, which gives a weight of 1 to the first adult, 0.5 to other household members aged 14 or over and 0.3 to household members aged less than 14.

[4]Eurostat considers that a working-age person is a person aged 18-59 years, excluding also the students aged 18-24 years.

[5]The representation of each country in the whole cross-country sample does not change when going from the full sample to the restricted one.

As we explained in Section 2, copula-based methods requires ranking the households in each dimension. In doing so, ties could arise in one or multiple variables. In our case, for example, the work intensity and material deprivation variables are of non-continuous nature, thus leading to a considerable number of ties. The problem of having ties in a copula-based framework was already mentioned in Section 2, where it was remarked that, in the presence of ties, the copula in (1) is no longer unique. Therefore, the values of the copula-based multivariate extensions of Spearman's rho can vary widely even based on the same joint distribution. Different alternatives to deal with ties can be found in the literature; see, for example, Quessy (2009), Mesfioui and Quessy (2010), Genest et al. (2013) and Decancq (2014). In this paper, we focus on two of these alternatives in order to analyse how robust our results are to the method used. On one hand, we compute the tie-corrected estimators of the multivariate extensions of Spearman's rho defined in (18)-(20), as proposed by Genest et al. (2013). On the other hand, following Decancq (2014) we break the ties using additional information from other secondary variables so that we eventually get, for each variable, unique ranks, $\{1, 2, \ldots, n\}$, and hence the coefficients defined in (12)-(14) can be directly applied to these ranks; see below. We are aware that it is unclear the effect of using additional secondary variables on the concordance properties of the original variables. In spite of that, we will see later that both approaches lead to very similar conclusions regarding the evolution of the dependence between poverty dimensions in Europe.

To start with, we will explain in detail how we use additional information to break the ties. Firstly, when a tie occurs in work intensity, households are ranked according to two secondary ranking variables measuring the intensity in both education and health of the household. The intensity of education is the sum of the highest ISCED (International Standard Classification of Education) level attained by all members of the household that are not currently in education divided by the highest possible value of this sum. The health intensity indicator is constructed in a similar way as the sum of the values of the self-assessed health indicator of all members of the household divided by the highest possible value of this sum. The choice of these secondary variables is not arbitrary. Both the relationships between educational and labour market outcomes and between health and labor market attainments are well documented in the literature; see, for example, Nickell (1979), Mincer (1991), Wolbers (2000), Farber (2004) and Riddell and Song (2011), regarding the former and Chirikos (1993), Ettner et al. (1997), Currie and Madrian (1999), Pelkowski and Berger (2004) and García Gómez and López Nicolás (2006), regarding the latter. As secondary ranking variable for material deprivation, we use the burden of the housing cost. An overburden of the housing cost can be seen as an indicator of financial stress (Whelan and Maître, 2012; Deidda, 2015) and as an indicator of vulnerability (Brandolini et al., 2010). We use both a dummy variable taking the value 1 if the housing cost is a burden for the household and the value of the housing cost itself. Thus, households for which the housing cost is a burden are assigned worse positions than those for which it is not. If a tie still exists for those households for which the housing cost is a burden they are ranked using the value of the housing cost. That is, the higher is the housing cost the worse is the position of the household. Both in the case of work intensity and material deprivation, if ties still exist after ranking households according to the secondary variables, the ties are broken at random. Thus, after this procedure, households are eventually assigned unique ranks, $\{1, 2, \ldots, n\}$, for each variable and the estimators $\hat{\rho}_d^-$, $\hat{\rho}_d^+$ and $\hat{\rho}_d$ in (12)-(14) can be computed using these ranks.

3.2 A primer look at the transformed data

In this section we show some examples of multivariate association in our data. To illustrate cross-country comparisons, Figure 1 represents the unique ranks described above, rescaled to [0, 1] as defined in (11), for the three dimensions of the AROPE rate in Bulgaria and Romania in 2008. As we can see, the points are not uniformly distributed over the unit cube, indicating departure from independence. Actually, in both countries we observe a positive association, as the points tend to concentrate around the main diagonal of the cube, that is, the three variables tend to be jointly large or small together. Moreover, both plots are denser around the vertexes (0, 0, 0) and (1, 1, 1), but in Bulgaria the concentration is higher around the former than around the latter, suggesting that dependence in the lower orthant is higher than in the upper orthant. The contrary occurs in Romania, where there is a higher concentration of observations around the vertex (1, 1, 1), suggesting that upper orthant dependence is higher than lower orthant dependence. As a matter of fact, these patterns are properly captured by the coefficients $\hat{\rho}_3^-$ and $\hat{\rho}_3^+$, which in the case of Bulgaria will fulfil the condition $\hat{\rho}_3^- > \hat{\rho}_3^+$, while they will behave the other way round in Romania.

< Insert Figure 1 here >

To illustrate temporal comparisons, Figure 2 displays two scatter plots representing the scaled ranks for Spain in 2008 and 2014. As we can see, there has been an increase in the multivariate dependence between dimensions of the AROPE rate in Spain over this period, as the concentration of the observations around the main diagonal is higher in 2014 than in 2008. Moreover, in both years, the concentration of points in the lower orthant seems to be higher than in the upper orthant. Hence, we would expect $\hat{\rho}_3^- > \hat{\rho}_3^+$, being both coefficients higher in 2014 than in 2008.

< Insert Figure 2 here >

To complement this graphical analysis, we have split the unit cube $[0,1]^3$ in 64 boxes of the same size and we have computed (see Table 1) the observed relative frequencies in the four boxes along the main diagonal for the same countries and time periods represented in Figures 1 and 2. The four boxes are denoted as $\{\mathbf{u} \leq 0.25,\ 0.25 < \mathbf{u} \leq 0.5,\ 0.5 < \mathbf{u} \leq 0.75,\ \mathbf{u} > 0.75\}$, where $\mathbf{u} \leq 0.25$ denotes the component-wise inequality, i.e. $u_i \leq 0.25$ for $i = 1, 2, 3$, and so this first box records the share of households being simultaneously in the 1^{st} quartile (low-ranked) in all dimensions. The other three boxes are defined similarly.

< Insert Table 1 here >

If the three variables were independent, the proportion of points in each box would be the same and equal to 1.56%. However, in all the examples in Table 1, there is a larger proportion of points concentrated around the main diagonal implying departure from independence. Furthermore, in all cases, the frequencies are higher in the extreme boxes, suggesting positive orthant dependence, in agreement with the patterns displayed in Figures 1 and 2.

3.3 Estimation results

In this section, we analyse the evolution of the multivariate dependence between poverty dimensions in the EU-28 countries over the period 2008-2014 using both the non-parametric estimators in (12)-(14) applied to the unique ranks as explained in Section 3.1, and the tie-corrected estimators in (18)-(20). As we pointed out in Section 2.3, the asymptotic variances of these estimators are complex. Therefore, we rely on a nonparametric bootstrap method to compute the bootstrap standard errors as the sample standard deviation of 1000 bootstrapped point estimates of the coefficients.

Figure 3 displays, for the EU-28 countries and over the whole period analysed, the evolution of the values of $\hat{\rho}_3^-$ (in Panel A) and $\hat{\rho}_3^+$ (in Panel B) together with the 95% standard confidence intervals using the bootstrap standard errors.[6] Figure 4 displays similar results for the tie-corrected estimators $\widehat{\rho_3^-}^{\maltese}$ (in Panel A) and $\widehat{\rho_3^+}^{\maltese}$ (in Panel B).

< Insert Figure 3 here >

< Insert Figure 4 here >

Several conclusions emerge from these figures. First, the patterns of the evolution of dependence over the period analysed are very similar whether we use the continuous (Figure 3) or tie-corrected (Figure 4) versions of the coefficients, although the former seem to have slightly

[6]We have also computed the 95% bootstrap percentile confidence intervals obtaining very similar results not displayed here to save space. The results are available upon request.

larger values than the latter. Second, all the coefficients are always positive, indicating a positive multivariate association between poverty dimensions both in the lower and in the upper orthant. This means that low (high) values of income tend to occur with low (high) values of the other two poverty dimensions. Third, Figure 3 shows that, regardless of the year and the country, the value of $\hat{\rho}_3^-$ (Panel A) is greater than that of $\hat{\rho}_3^+$ (Panel B), except for the case of Romania, and the same result holds for the tie-corrected versions of the coefficients (Figure 4). This means that average lower orthant dependence tends to be higher than average upper orthant dependence, that is, the probability of being simultaneously low-ranked in all poverty dimensions tends to be higher than the probability of being simultaneously high-ranked in all dimensions. Fourth, there are different cross-country profiles in the evolution of multivariate dependence. For instance, in Spain there is a clear increasing trend in the multivariate dependence between dimensions of poverty in both the lower and the upper orthant over the period analysed. An increasing trend is also found in other countries such as Cyprus, Denmark, Italy or The Netherlands. However, no decreasing trend shows up in any country. On the other hand, in some countries such as Greece and the UK, there is not a clear trend, but the dependence in 2014 is clearly higher than in 2008, since the corresponding confidence intervals do not overlap. However, in countries like Austria, Germany or Sweden, there is a considerable overlap in the confidence intervals for these two years and thus we cannot give meaningful conclusions on the variation of the dependence coefficients.

To get a better insight regarding the change in multivariate dependence between 2008 and 2014, Table 2 reports point estimates (with standard errors) for these two years and for $\hat{\rho}_3^-$, $\hat{\rho}_3^+$ and $\hat{\rho}_3$. In columns 3, 6 and 9, we also display the results of a two-independent sample t-test with unequal variances, calculated using bootstrap standard errors. In particular, we perform a one-sided test to determine if the increases or decreases in the value of the coefficients between 2008 and 2014 are statistically significant. The corresponding p-value (in parentheses) is computed assuming asymptotic normality of the t-statistic. Table 3 displays the same results for $\widehat{\rho_3^-}^{✠}$,

$\widehat{\rho}_3^{+\maltese}$ and $\widehat{\rho}_3^{\maltese}$. Interestingly, in most EU-28, we find a significant increase in all the coefficients over the period analysed. Thus, we can say that there has been a general increase in the multivariate orthant dependence between dimensions of poverty in the EU over the period 2008-2014. Moreover, this increase is found both in the lower and in the upper orthant, which means that, over the period analysed, there has been a general increase in both the probability of being simultaneously low-ranked and the probability of being simultaneously high-ranked in all dimensions of poverty. Noticeably, the highest increase in both the lower and upper orthant dependence is found in Spain, one of the countries most hardly hit by the economic crisis. Another country severely affected by the crisis, namely Greece, also experienced a substantial increase in these two types of dependence.

< Insert Table 2 here >

< Insert Table 3 here >

To complement the analysis of three-dimensional dependence, we have also analysed all possible pairwise relationships between the dimensions of the AROPE rate. The results are displayed in Tables 4 and 5. The first feature that is worth pointing out is that the bivariate coefficients share many of the properties of the trivariate coefficients. In particular, in all the countries and for both years, all of them are positive and, in most of the countries, they are larger in 2014 than in 2008, with the differences being statistically significant at 5% in most cases. Additionally, these tables reveal that, in general, the dependence tends to be higher between income and the other two dimensions than between work intensity and no-material deprivation.

< Insert Table 4 here >

< Insert Table 5 here >

Finally, as we said in the Introduction, quantifying the dependence between the dimensions of the AROPE rate provides a useful complement to the information given by this indicator. In this context, we wonder whether those countries with higher AROPE rates are also countries with high levels of dependence between its dimensions. To address this issue, Panel A of Figure 5 depicts two scatter plots showing the relationship between the AROPE rate and the coefficient $\hat{\rho}_3^-$ for the EU-28 countries in the years 2008 and 2014.[7] Panel B of the same figure displays the same results for the coefficient $\widehat{\rho_3}^{-\maltese}$. In all graphs, the horizontal and vertical reference lines represent the corresponding values for the whole EU-28. We focus on $\hat{\rho}_3^-$ and $\widehat{\rho_3}^{-\maltese}$ because they measure lower orthant dependence, which is the key point in poverty analysis. The main features from these figures are the following: a) there is a positive relationship between the AROPE rate and lower orthant dependence, that is, countries with high incidence of multidimensional poverty tend to experience also a high degree of multivariate dependence between its dimensions in the lower orthant; b) those countries with either very low or very high values of both $\hat{\rho}_3^-$ and $\widehat{\rho_3}^{-\maltese}$ in 2008 have converged, over the period analysed, to the situation of the majority of the EU-28 countries; c) in the EU-28 as a whole (see the reference lines), there has been an increase in the AROPE rate accompanied with an increase in the multivariate dependence between its dimensions.

< Insert Figure 5 here >

4 Conclusions

This paper proposes to measure the dependence between dimensions of poverty using copula-based multivariate generalisations of Spearman's rho. Two of these coefficients, namely ρ_d^- for continuous data and $\rho_d^{-\maltese}$ for possibly non-continuous data, turn out to be essential in poverty analysis as they enable to measure the dependence between the poverty dimensions in the

[7] The AROPE rate is calculated here as the proportion of households in our sample that are poor in at least one of the three dimensions considered.

lower orthant of the joint distribution. Hence, they capture the propensity of a household to be simultaneously low-ranked in all dimensions.

Our empirical application provides a more comprehensive picture on how multidimensional poverty has evolved in the EU-28 countries over the period 2008-2014, by complementing the information about the incidence of poverty with measures of the multivariate dependence between its dimensions. In particular, we use multivariate generalisations of Spearman's rho to assess multivariate dependence and we consider, as variables characterising poverty, those included in the AROPE rate: income, material needs and work intensity. The nature of the last two variables entails the presence of ties when ranking the households according to such variables. To address this problem, we adopt two different approaches, namely the use of estimators for the continuous case after breaking the ties using additional information and the use of tie-corrected estimators for possibly discontinuous data. Interestingly, the results obtained keep robust to the approach used.

Our first conclusion is that, for all the EU-28 countries and all the years considered, there is a positive multivariate association between poverty dimensions, regardless of the coefficient used. Moreover, this dependence has noticeably increased in Europe between 2008 and 2014 and for most of the countries this increase is statistically significant and it is especially remarkable in those countries most hardly hit by the economic crisis like Spain and Greece. Another important conclusion is that, in the vast majority of European countries, the maximal dependence is found in the lower orthant. Therefore, small values of the three poverty dimensions tend to occur together and this simultaneous concentration of small values of income, no-material deprivations and work intensity is more likely to occur in 2014 than in 2008. Finally, we detect a positive relationship between the incidence of multidimensional poverty, measured by the AROPE rate, and the dependence between its dimensions. This means that countries with a high poverty incidence tend to experiment also a higher degree of dependence between the dimensions of poverty.

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Table 1. Share of households in the main diagonal of the unit cube $[\mathbf{0}, \mathbf{1}]^{\mathbf{3}}$

	$\mathbf{u} \leq 0.25$	$0.25 < \mathbf{u} \leq 0.5$	$0.5 < \mathbf{u} \leq 0.75$	$\mathbf{u} > 0.75$	Total
Bulgaria (2008)	11.06%	3.43%	3.30%	7.28%	25.07%
Romania (2008)	5.80%	2.34%	2.57%	9.10%	19.81%
Spain (2008)	7.23%	2.28%	2.35%	3.29%	15.15%
Spain (2014)	8.14%	2.77%	2.32%	5.27%	18.5%

Table 2. Coefficients of trivariate dependence between the dimensions of the AROPE rate

	$\hat{\rho}_3^-$			$\hat{\rho}_3^+$			$\hat{\rho}_3$		
	2008	2014	t-test	2008	2014	t-test	2008	2014	t-test
Austria	0.373	0.412	2.458	0.338	0.372	2.262	0.355	0.392	2.442
	(0.011)	(0.011)	(0.007)	(0.011)	(0.010)	(0.012)	(0.011)	(0.010)	(0.007)
Belgium	0.501	0.558	4.221	0.436	0.484	3.549	0.468	0.521	4.012
	(0.010)	(0.010)	(0.000)	(0.009)	(0.010)	(0.000)	(0.009)	(0.009)	(0.000)
Bulgaria	0.572	0.553	-1.167	0.528	0.529	0.046	0.550	0.541	-0.575
	(0.011)	(0.011)	(0.122)	(0.012)	(0.011)	(0.482)	(0.011)	(0.011)	(0.283)
Cyprus	0.383	0.446	3.535	0.379	0.437	3.326	0.381	0.441	3.559
	(0.014)	(0.011)	(0.000)	(0.013)	(0.011)	(0.000)	(0.013)	(0.011)	(0.000)
Czech Republic	0.394	0.421	2.097	0.370	0.383	1.065	0.382	0.402	1.653
	(0.008)	(0.010)	(0.018)	(0.008)	(0.009)	(0.143)	(0.007)	(0.009)	(0.049)
Germany	0.437	0.447	0.928	0.397	0.401	0.397	0.417	0.424	0.692
	(0.008)	(0.008)	(0.177)	(0.007)	(0.008)	(0.346)	(0.007)	(0.007)	(0.245)
Denmark	0.276	0.374	5.911	0.229	0.325	6.381	0.252	0.349	6.340
	(0.012)	(0.012)	(0.000)	(0.010)	(0.011)	(0.000)	(0.011)	(0.011)	(0.000)
Estonia	0.425	0.441	0.966	0.392	0.410	1.175	0.408	0.425	1.105
	(0.012)	(0.011)	(0.167)	(0.011)	(0.010)	(0.120)	(0.011)	(0.010)	(0.134)
Greece	0.412	0.520	8.573	0.404	0.508	8.134	0.408	0.514	8.646
	(0.010)	(0.008)	(0.000)	(0.010)	(0.008)	(0.000)	(0.009)	(0.008)	(0.000)
Spain	0.342	0.499	15.792	0.314	0.465	15.909	0.328	0.482	16.372
	(0.007)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)
Finland	0.378	0.410	2.769	0.330	0.353	2.144	0.354	0.381	2.557
	(0.008)	(0.008)	(0.003)	(0.008)	(0.008)	(0.016)	(0.007)	(0.008)	(0.005)
France	0.412	0.441	2.596	0.373	0.395	2.014	0.393	0.418	2.387
	(0.008)	(0.008)	(0.005)	(0.008)	(0.008)	(0.022)	(0.008)	(0.008)	(0.008)
Croatia	NA	0.502	NA	NA	0.497	NA	NA	0.500	NA
	NA	(0.011)	NA	NA	(0.011)	NA	NA	(0.011)	NA
Hungary	0.449	0.525	6.980	0.434	0.509	6.429	0.442	0.517	6.966
	(0.008)	(0.007)	(0.000)	(0.009)	(0.008)	(0.000)	(0.008)	(0.007)	(0.000)
Ireland	0.546	0.562	1.144	0.491	0.545	3.748	0.519	0.554	2.579
	(0.010)	(0.009)	(0.126)	(0.011)	(0.010)	(0.000)	(0.010)	(0.009)	(0.005)
Italy	0.384	0.443	7.168	0.355	0.407	6.621	0.369	0.425	7.136
	(0.006)	(0.006)	(0.000)	(0.005)	(0.006)	(0.000)	(0.006)	(0.006)	(0.000)
Lithuania	0.444	0.506	3.821	0.402	0.483	5.239	0.423	0.494	4.673
	(0.012)	(0.011)	(0.000)	(0.011)	(0.011)	(0.000)	(0.011)	(0.011)	(0.000)
Luxembourg	0.382	0.377	-0.277	0.339	0.329	-0.525	0.360	0.353	-0.414
	(0.013)	(0.013)	(0.391)	(0.013)	(0.013)	(0.300)	(0.012)	(0.012)	(0.339)
Latvia	0.475	0.477	0.102	0.445	0.460	0.976	0.460	0.469	0.552
	(0.012)	(0.011)	(0.459)	(0.011)	(0.010)	(0.165)	(0.011)	(0.010)	(0.290)
Malta	0.491	0.481	-0.590	0.486	0.453	-1.874	0.488	0.467	-1.268
	(0.013)	(0.012)	(0.277)	(0.013)	(0.012)	(0.030)	(0.013)	(0.011)	(0.102)
Netherlands	0.272	0.387	9.497	0.232	0.331	8.949	0.252	0.359	9.575
	(0.009)	(0.008)	(0.000)	(0.008)	(0.008)	(0.000)	(0.008)	(0.008)	(0.000)
Poland	0.438	0.472	3.868	0.434	0.454	2.082	0.436	0.463	3.067
	(0.006)	(0.007)	(0.000)	(0.006)	(0.007)	(0.019)	(0.006)	(0.006)	(0.001)
Portugal	0.426	0.486	3.830	0.417	0.469	3.237	0.421	0.477	3.652
	(0.012)	(0.010)	(0.000)	(0.013)	(0.010)	(0.001)	(0.012)	(0.009)	(0.000)
Romania	0.439	0.419	-1.481	0.478	0.448	-2.246	0.458	0.433	-1.931
	(0.009)	(0.010)	(0.069)	(0.009)	(0.010)	(0.012)	(0.009)	(0.009)	(0.027)
Sweden	0.357	0.375	1.166	0.315	0.316	0.066	0.336	0.345	0.660
	(0.010)	(0.012)	(0.122)	(0.009)	(0.011)	(0.474)	(0.009)	(0.011)	(0.255)
Slovenia	0.410	0.463	4.922	0.395	0.437	4.019	0.402	0.450	4.664
	(0.008)	(0.008)	(0.000)	(0.007)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)
Slovak Republic	0.408	0.451	2.802	0.387	0.412	1.745	0.397	0.432	2.368
	(0.011)	(0.011)	(0.003)	(0.011)	(0.010)	(0.040)	(0.010)	(0.010)	(0.009)
United Kingdom	0.423	0.522	8.377	0.377	0.482	8.835	0.400	0.502	8.914
	(0.009)	(0.007)	(0.000)	(0.009)	(0.008)	(0.000)	(0.009)	(0.007)	(0.000)

Note: Standard errors for the coefficients and p-values for the one-side t-test are displayed in parentheses.

Table 3. Tie-corrected coefficients of trivariate dependence between the dimensions of the AROPE rate

	$\widehat{\rho_3^{-\maltese}}$			$\widehat{\rho_3^{+\maltese}}$			$\widehat{\rho_3^{\maltese}}$		
	2008	2014	t-test	2008	2014	t-test	2008	2014	t-test
Austria	0.346	0.374	1.836	0.302	0.325	1.750	0.324	0.349	1.842
	(0.011)	(0.010)	(0.033)	(0.010)	(0.009)	(0.040)	(0.010)	(0.009)	(0.033)
Belgium	0.484	0.535	3.899	0.408	0.458	4.022	0.446	0.497	4.049
	(0.010)	(0.009)	(0.000)	(0.009)	(0.009)	(0.000)	(0.009)	(0.009)	(0.000)
Bulgaria	0.545	0.518	-1.688	0.488	0.474	-0.927	0.517	0.496	-1.355
	(0.011)	(0.011)	(0.046)	(0.011)	(0.011)	(0.177)	(0.011)	(0.011)	(0.088)
Cyprus	0.368	0.434	3.766	0.360	0.418	3.550	0.364	0.426	3.781
	(0.014)	(0.011)	(0.000)	(0.012)	(0.011)	(0.000)	(0.013)	(0.010)	(0.000)
Czech Republic	0.355	0.374	1.547	0.321	0.321	-0.029	0.338	0.347	0.841
	(0.008)	(0.009)	(0.061)	(0.007)	(0.008)	(0.488)	(0.007)	(0.008)	(0.200)
Germany	0.426	0.424	-0.185	0.373	0.367	-0.681	0.399	0.395	-0.427
	(0.007)	(0.008)	(0.427)	(0.006)	(0.007)	(0.248)	(0.007)	(0.007)	(0.335)
Denmark	0.243	0.337	6.143	0.198	0.285	7.132	0.220	0.311	6.735
	(0.011)	(0.011)	(0.000)	(0.008)	(0.009)	(0.000)	(0.009)	(0.010)	(0.000)
Estonia	0.398	0.419	1.307	0.346	0.373	1.938	0.372	0.396	1.647
	(0.012)	(0.011)	(0.096)	(0.010)	(0.010)	(0.026)	(0.011)	(0.010)	(0.050)
Greece	0.392	0.510	9.514	0.379	0.496	9.755	0.386	0.503	9.946
	(0.010)	(0.008)	(0.000)	(0.009)	(0.008)	(0.000)	(0.009)	(0.008)	(0.000)
Spain	0.357	0.508	15.976	0.331	0.472	16.020	0.344	0.490	16.454
	(0.007)	(0.006)	(0.000)	(0.006)	(0.006)	(0.000)	(0.006)	(0.006)	(0.000)
Finland	0.357	0.384	2.468	0.305	0.322	1.807	0.331	0.353	2.221
	(0.008)	(0.008)	(0.007)	(0.006)	(0.006)	(0.035)	(0.007)	(0.007)	(0.013)
France	0.372	0.395	2.085	0.322	0.338	1.674	0.347	0.367	1.941
	(0.008)	(0.008)	(0.019)	(0.007)	(0.007)	(0.047)	(0.007)	(0.007)	(0.026)
Croatia	NA	0.499	NA	NA	0.479	NA	NA	0.500	NA
	NA	(0.010)	NA	NA	(0.010)	NA	NA	(0.010)	NA
Hungary	0.420	0.485	5.931	0.388	0.447	5.387	0.404	0.466	5.855
	(0.008)	(0.007)	(0.000)	(0.008)	(0.007)	(0.000)	(0.008)	(0.007)	(0.000)
Ireland	0.538	0.549	0.824	0.477	0.525	3.456	0.508	0.537	2.222
	(0.010)	(0.009)	(0.205)	(0.010)	(0.009)	(0.000)	(0.010)	(0.009)	(0.013)
Italy	0.389	0.439	6.261	0.355	0.399	6.057	0.372	0.419	6.349
	(0.005)	(0.006)	(0.000)	(0.005)	(0.005)	(0.000)	(0.005)	(0.005)	(0.000)
Lithuania	0.408	0.481	4.519	0.355	0.437	5.769	0.381	0.459	5.254
	(0.011)	(0.011)	(0.000)	(0.010)	(0.010)	(0.000)	(0.010)	(0.011)	(0.000)
Luxembourg	0.341	0.335	-0.399	0.305	0.296	-0.612	0.323	0.315	-0.507
	(0.012)	(0.012)	(0.345)	(0.010)	(0.010)	(0.270)	(0.011)	(0.011)	(0.306)
Latvia	0.433	0.458	1.606	0.382	0.421	2.731	0.407	0.439	2.204
	(0.012)	(0.011)	(0.054)	(0.010)	(0.010)	(0.003)	(0.011)	(0.010)	(0.014)
Malta	0.485	0.470	-0.858	0.464	0.430	-2.069	0.474	0.450	-1.491
	(0.013)	(0.012)	(0.195)	(0.012)	(0.011)	(0.019)	(0.012)	(0.011)	(0.068)
Netherlands	0.269	0.341	9.085	0.230	0.314	9.353	0.250	0.343	9.415
	(0.008)	(0.008)	(0.000)	(0.006)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)
Poland	0.413	0.451	4.237	0.394	0.419	2.902	0.404	0.435	3.697
	(0.006)	(0.007)	(0.000)	(0.006)	(0.006)	(0.002)	(0.006)	(0.006)	(0.000)
Portugal	0.392	0.458	4.343	0.358	0.424	4.471	0.375	0.441	4.548
	(0.012)	(0.010)	(0.000)	(0.012)	(0.009)	(0.000)	(0.011)	(0.009)	(0.000)
Romania	0.379	0.361	-1.436	0.397	0.367	-2.432	0.388	0.364	-1.978
	(0.009)	(0.009)	(0.075)	(0.008)	(0.009)	(0.008)	(0.008)	(0.009)	(0.024)
Sweden	0.305	0.332	1.927	0.259	0.274	1.351	0.282	0.303	1.708
	(0.009)	(0.011)	(0.027)	(0.007)	(0.009)	(0.088)	(0.008)	(0.009)	(0.044)
Slovenia	0.352	0.427	7.051	0.316	0.380	6.853	0.334	0.404	7.196
	(0.008)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)	(0.007)	(0.007)	(0.000)
Slovak Republic	0.364	0.405	2.655	0.327	0.355	2.133	0.346	0.380	2.482
	(0.011)	(0.011)	(0.004)	(0.009)	(0.009)	(0.016)	(0.010)	(0.010)	(0.007)
United Kingdom	0.401	0.495	8.122	0.346	0.444	9.378	0.373	0.469	8.955
	(0.009)	(0.007)	(0.000)	(0.008)	(0.007)	(0.000)	(0.008)	(0.007)	(0.000)

Note: Standard errors for the coefficients and p-values for the one-side t-test are displayed in parentheses.

Table 4. Coefficients of pairwise dependence between the dimensions of the AROPE rate

	$\hat{\rho}_{income,work}$			$\hat{\rho}_{income,no-deprivation}$			$\hat{\rho}_{work,no-deprivation}$		
	2008	2014	t-test	2008	2014	t-test	2008	2014	t-test
Austria	0.429	0.467	1.990	0.388	0.419	1.571	0.249	0.289	1.906
	(0.014)	(0.013)	(0.023)	(0.014)	(0.014)	(0.058)	(0.015)	(0.014)	(0.028)
Belgium	0.570	0.618	3.127	0.506	0.568	3.877	0.329	0.378	2.492
	(0.011)	(0.010)	(0.001)	(0.011)	(0.011)	(0.000)	(0.014)	(0.014)	(0.006)
Bulgaria	0.571	0.552	-0.975	0.643	0.568	-4.237	0.435	0.502	3.180
	(0.014)	(0.014)	(0.165)	(0.012)	(0.013)	(0.000)	(0.015)	(0.014)	(0.001)
Cyprus	0.465	0.489	1.052	0.469	0.513	2.164	0.209	0.322	4.412
	(0.018)	(0.014)	(0.146)	(0.016)	(0.013)	(0.015)	(0.020)	(0.017)	(0.000)
Czech Republic	0.431	0.459	1.776	0.427	0.439	0.789	0.290	0.307	1.077
	(0.010)	(0.012)	(0.038)	(0.010)	(0.012)	(0.215)	(0.010)	(0.013)	(0.141)
Germany	0.487	0.468	-1.502	0.467	0.504	2.948	0.297	0.301	0.297
	(0.009)	(0.009)	(0.067)	(0.009)	(0.009)	(0.002)	(0.010)	(0.011)	(0.383)
Denmark	0.327	0.427	4.838	0.284	0.397	5.501	0.145	0.224	3.607
	(0.015)	(0.014)	(0.000)	(0.014)	(0.015)	(0.000)	(0.015)	(0.016)	(0.000)
Estonia	0.471	0.493	1.124	0.439	0.411	-1.379	0.315	0.372	2.700
	(0.014)	(0.013)	(0.131)	(0.015)	(0.014)	(0.084)	(0.016)	(0.014)	(0.003)
Greece	0.462	0.538	4.708	0.505	0.647	10.481	0.257	0.358	5.390
	(0.013)	(0.010)	(0.000)	(0.011)	(0.008)	(0.000)	(0.014)	(0.013)	(0.000)
Spain	0.510	0.560	4.314	0.294	0.503	16.637	0.179	0.382	15.018
	(0.008)	(0.008)	(0.000)	(0.010)	(0.008)	(0.000)	(0.010)	(0.009)	(0.000)
Finland	0.459	0.506	3.535	0.386	0.378	-0.602	0.217	0.260	2.730
	(0.010)	(0.009)	(0.000)	(0.010)	(0.010)	(0.274)	(0.011)	(0.011)	(0.003)
France	0.403	0.419	1.122	0.498	0.515	1.330	0.277	0.320	2.858
	(0.011)	(0.010)	(0.131)	(0.009)	(0.009)	(0.092)	(0.011)	(0.010)	(0.002)
Croatia	NA	0.638	NA	NA	0.496	NA	NA	0.366	NA
	NA	(0.011)	NA	NA	(0.014)	NA	NA	(0.015)	NA
Hungary	0.514	0.559	3.357	0.474	0.578	7.602	0.337	0.414	4.931
	(0.010)	(0.009)	(0.000)	(0.011)	(0.009)	(0.000)	(0.012)	(0.010)	(0.000)
Ireland	0.610	0.663	3.348	0.528	0.529	0.079	0.418	0.468	2.520
	(0.012)	(0.010)	(0.000)	(0.013)	(0.013)	(0.468)	(0.015)	(0.013)	(0.006)
Italy	0.505	0.506	0.159	0.356	0.432	7.524	0.248	0.337	7.862
	(0.007)	(0.007)	(0.437)	(0.007)	(0.007)	(0.000)	(0.008)	(0.008)	(0.000)
Lithuania	0.530	0.578	2.602	0.404	0.490	4.116	0.335	0.415	3.660
	(0.014)	(0.013)	(0.005)	(0.015)	(0.014)	(0.000)	(0.016)	(0.015)	(0.000)
Luxembourg	0.448	0.454	0.254	0.490	0.460	-1.449	0.144	0.146	0.096
	(0.016)	(0.016)	(0.400)	(0.014)	(0.015)	(0.074)	(0.019)	(0.019)	(0.462)
Latvia	0.524	0.571	2.564	0.494	0.487	-0.365	0.364	0.348	-0.729
	(0.014)	(0.012)	(0.005)	(0.013)	(0.012)	(0.358)	(0.015)	(0.015)	(0.233)
Malta	0.651	0.646	-0.302	0.475	0.429	-2.021	0.340	0.326	-0.537
	(0.013)	(0.011)	(0.381)	(0.017)	(0.015)	(0.022)	(0.019)	(0.017)	(0.296)
Netherlands	0.374	0.478	7.154	0.283	0.371	5.747	0.099	0.227	7.888
	(0.011)	(0.010)	(0.000)	(0.011)	(0.011)	(0.000)	(0.011)	(0.011)	(0.000)
Poland	0.449	0.527	7.085	0.494	0.490	-0.346	0.365	0.372	0.537
	(0.007)	(0.008)	(0.000)	(0.007)	(0.008)	(0.365)	(0.009)	(0.009)	(0.295)
Portugal	0.478	0.546	3.514	0.497	0.521	1.233	0.289	0.365	3.446
	(0.015)	(0.012)	(0.000)	(0.015)	(0.012)	(0.109)	(0.018)	(0.013)	(0.000)
Romania	0.518	0.498	-1.231	0.515	0.459	-3.392	0.342	0.343	0.079
	(0.011)	(0.012)	(0.109)	(0.011)	(0.012)	(0.000)	(0.013)	(0.014)	(0.468)
Sweden	0.402	0.444	2.244	0.361	0.356	-0.268	0.245	0.237	-0.426
	(0.012)	(0.014)	(0.012)	(0.012)	(0.015)	(0.394)	(0.013)	(0.015)	(0.335)
Slovenia	0.495	0.559	5.004	0.410	0.449	2.749	0.302	0.342	2.738
	(0.009)	(0.009)	(0.000)	(0.010)	(0.010)	(0.003)	(0.010)	(0.011)	(0.003)
Slovak Republic	0.420	0.498	4.130	0.451	0.443	-0.461	0.320	0.354	1.694
	(0.013)	(0.013)	(0.000)	(0.013)	(0.013)	(0.322)	(0.014)	(0.014)	(0.045)
United Kingdom	0.506	0.571	4.720	0.420	0.529	7.390	0.274	0.405	7.782
	(0.010)	(0.009)	(0.000)	(0.011)	(0.009)	(0.000)	(0.013)	(0.011)	(0.000)

Note: Standard errors for the coefficients and p-values for the one-side t-test are displayed in parentheses.

Table 5. Tie-corrected coefficients of pairwise dependence between the dimensions of the AROPE rate

	$\hat{\rho}^{\maltese}_{income,work}$			$\hat{\rho}^{\maltese}_{income,no-deprivation}$			$\hat{\rho}^{\maltese}_{work,no-deprivation}$		
	2008	2014	t-test	2008	2014	t-test	2008	2014	t-test
Austria	0.382	0.426	2.282	0.386	0.394	0.473	0.205	0.229	1.296
	(0.014)	(0.013)	(0.011)	(0.012)	(0.011)	(0.318)	(0.014)	(0.013)	(0.097)
Belgium	0.528	0.576	3.014	0.486	0.539	3.848	0.323	0.374	2.927
	(0.012)	(0.011)	(0.001)	(0.010)	(0.010)	(0.000)	(0.012)	(0.012)	(0.002)
Bulgaria	0.510	0.474	-1.836	0.656	0.585	-4.191	0.383	0.428	2.135
	(0.014)	(0.014)	(0.033)	(0.012)	(0.012)	(0.000)	(0.015)	(0.014)	(0.016)
Cyprus	0.381	0.424	1.839	0.520	0.554	1.847	0.191	0.300	4.490
	(0.018)	(0.015)	(0.033)	(0.014)	(0.012)	(0.032)	(0.019)	(0.016)	(0.000)
Czech Republic	0.347	0.356	0.538	0.449	0.467	1.313	0.218	0.219	0.067
	(0.010)	(0.012)	(0.295)	(0.009)	(0.011)	(0.095)	(0.009)	(0.011)	(0.473)
Germany	0.446	0.420	-1.974	0.499	0.515	1.523	0.252	0.250	-0.186
	(0.009)	(0.009)	(0.024)	(0.007)	(0.008)	(0.064)	(0.010)	(0.010)	(0.426)
Denmark	0.245	0.352	5.448	0.272	0.376	6.338	0.144	0.205	3.524
	(0.014)	(0.014)	(0.000)	(0.011)	(0.012)	(0.000)	(0.012)	(0.013)	(0.000)
Estonia	0.396	0.424	1.435	0.458	0.444	-0.774	0.261	0.320	2.958
	(0.014)	(0.013)	(0.076)	(0.013)	(0.012)	(0.220)	(0.015)	(0.013)	(0.002)
Greece	0.374	0.494	7.325	0.556	0.673	9.682	0.227	0.343	6.524
	(0.013)	(0.010)	(0.000)	(0.010)	(0.007)	(0.000)	(0.013)	(0.012)	(0.000)
Spain	0.444	0.518	6.188	0.377	0.550	16.084	0.210	0.402	15.733
	(0.009)	(0.008)	(0.000)	(0.008)	(0.007)	(0.000)	(0.009)	(0.008)	(0.000)
Finland	0.390	0.440	3.738	0.392	0.377	-1.277	0.211	0.240	2.244
	(0.009)	(0.009)	(0.000)	(0.008)	(0.008)	(0.101)	(0.009)	(0.009)	(0.012)
France	0.342	0.352	0.665	0.475	0.485	0.831	0.223	0.263	2.866
	(0.011)	(0.010)	(0.253)	(0.008)	(0.008)	(0.203)	(0.010)	(0.010)	(0.002)
Croatia	NA	0.595	NA	NA	0.525	NA	NA	0.347	NA
	NA	(0.011)	NA	NA	(0.013)	NA	NA	(0.015)	NA
Hungary	0.439	0.468	2.160	0.489	0.592	7.887	0.285	0.339	3.489
	(0.010)	(0.009)	(0.015)	(0.010)	(0.008)	(0.000)	(0.011)	(0.010)	(0.000)
Ireland	0.574	0.640	4.055	0.533	0.521	-0.661	0.417	0.449	1.747
	(0.013)	(0.010)	(0.000)	(0.011)	(0.012)	(0.254)	(0.013)	(0.013)	(0.040)
Italy	0.456	0.462	0.599	0.422	0.484	7.058	0.238	0.310	6.942
	(0.007)	(0.007)	(0.275)	(0.006)	(0.006)	(0.000)	(0.007)	(0.008)	(0.000)
Lithuania	0.423	0.498	3.931	0.435	0.515	4.145	0.285	0.365	3.823
	(0.014)	(0.013)	(0.000)	(0.014)	(0.013)	(0.000)	(0.015)	(0.015)	(0.000)
Luxembourg	0.348	0.364	0.724	0.468	0.412	-3.246	0.154	0.170	0.803
	(0.016)	(0.016)	(0.234)	(0.012)	(0.012)	(0.001)	(0.014)	(0.015)	(0.211)
Latvia	0.425	0.500	3.964	0.518	0.519	0.033	0.278	0.299	1.033
	(0.014)	(0.012)	(0.000)	(0.012)	(0.012)	(0.487)	(0.015)	(0.014)	(0.151)
Malta	0.612	0.582	-1.705	0.493	0.458	-1.626	0.318	0.309	-0.359
	(0.013)	(0.012)	(0.044)	(0.016)	(0.014)	(0.052)	(0.018)	(0.016)	(0.360)
Netherlands	0.355	0.464	7.408	0.281	0.344	5.543	0.112	0.220	8.217
	(0.011)	(0.010)	(0.000)	(0.008)	(0.008)	(0.000)	(0.009)	(0.009)	(0.000)
Poland	0.385	0.451	5.868	0.514	0.525	1.053	0.312	0.329	1.393
	(0.008)	(0.008)	(0.000)	(0.007)	(0.008)	(0.146)	(0.008)	(0.009)	(0.082)
Portugal	0.374	0.455	4.125	0.523	0.549	1.519	0.229	0.319	4.334
	(0.016)	(0.012)	(0.000)	(0.014)	(0.011)	(0.064)	(0.016)	(0.013)	(0.000)
Romania	0.377	0.350	-1.675	0.553	0.497	-3.575	0.234	0.246	0.650
	(0.011)	(0.012)	(0.047)	(0.010)	(0.012)	(0.000)	(0.012)	(0.013)	(0.258)
Sweden	0.350	0.383	1.870	0.307	0.316	0.637	0.189	0.210	1.303
	(0.012)	(0.014)	(0.031)	(0.010)	(0.011)	(0.262)	(0.011)	(0.012)	(0.096)
Slovenia	0.369	0.463	7.173	0.420	0.465	3.561	0.213	0.282	4.985
	(0.010)	(0.009)	(0.000)	(0.009)	(0.009)	(0.000)	(0.010)	(0.010)	(0.000)
Slovak Republic	0.337	0.429	4.949	0.454	0.441	-0.702	0.246	0.270	1.314
	(0.013)	(0.013)	(0.000)	(0.013)	(0.013)	(0.241)	(0.013)	(0.014)	(0.094)
United Kingdom	0.456	0.517	4.417	0.411	0.519	8.394	0.253	0.372	7.726
	(0.011)	(0.009)	(0.000)	(0.010)	(0.009)	(0.000)	(0.011)	(0.010)	(0.000)

Note: Standard errors for the coefficients and p-values for the one-side t-test are displayed in parentheses.

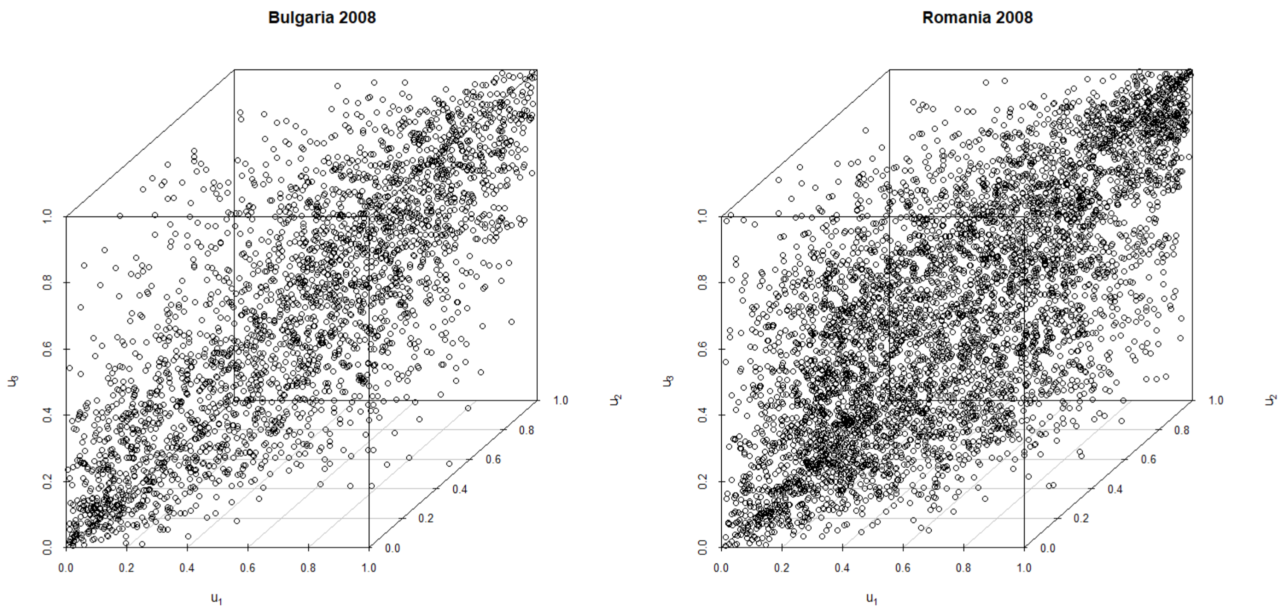


Figure 1: Scatter plots of scaled ranks for Bulgaria (2008) and Romania (2008)

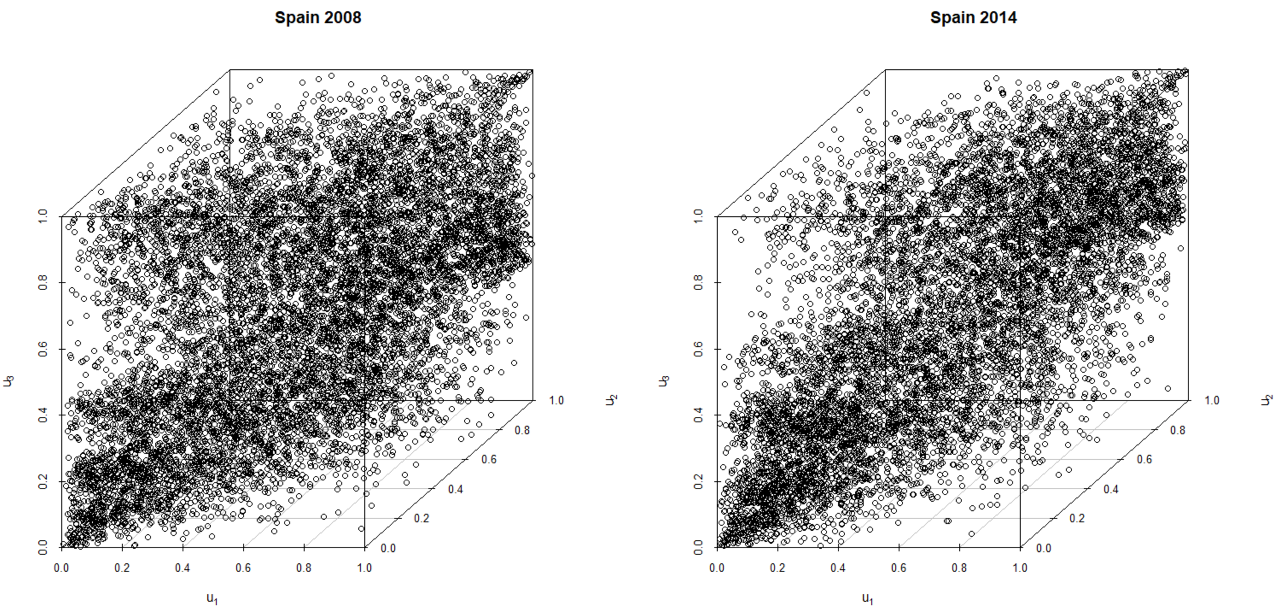
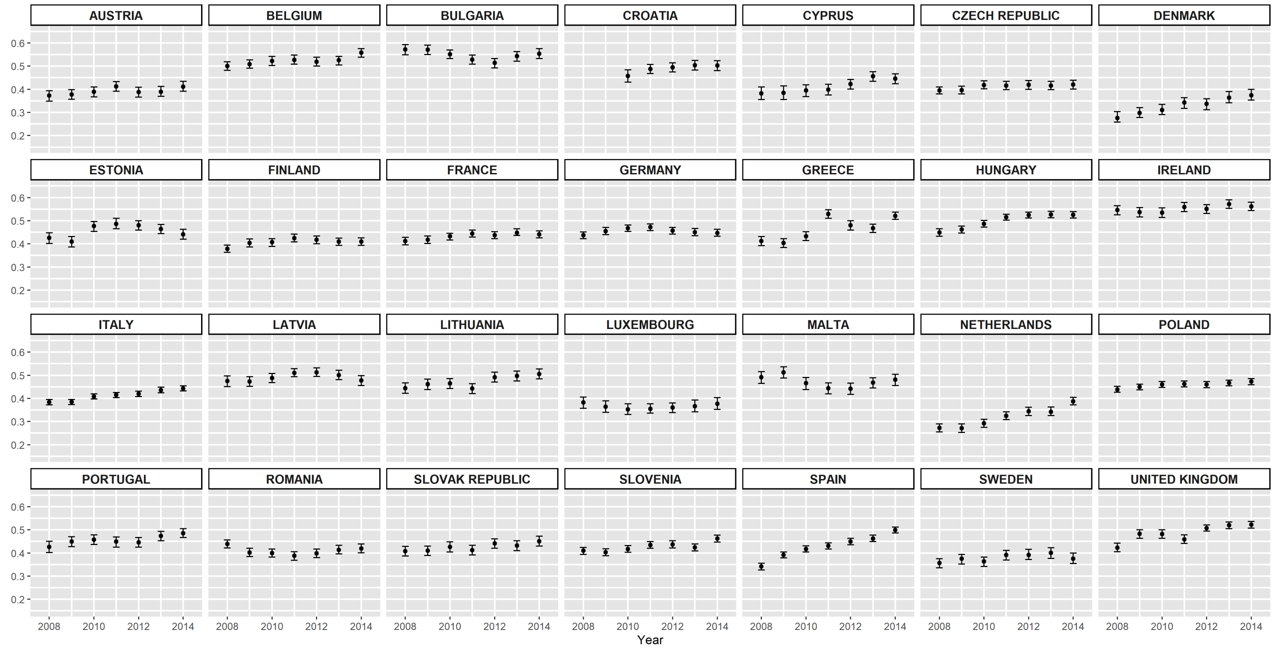


Figure 2: Scatter plots of scaled ranks for Spain (2008 and 2014)

Panel A: lower orthant dependence



Panel B: upper orthant dependence

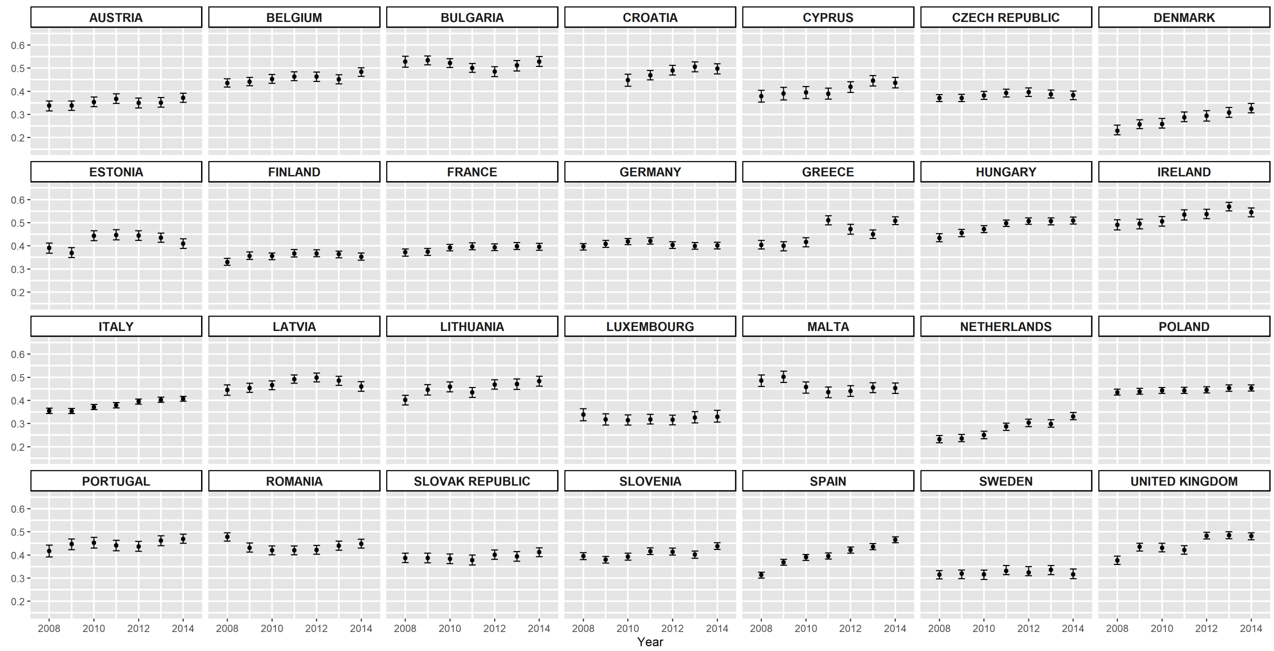
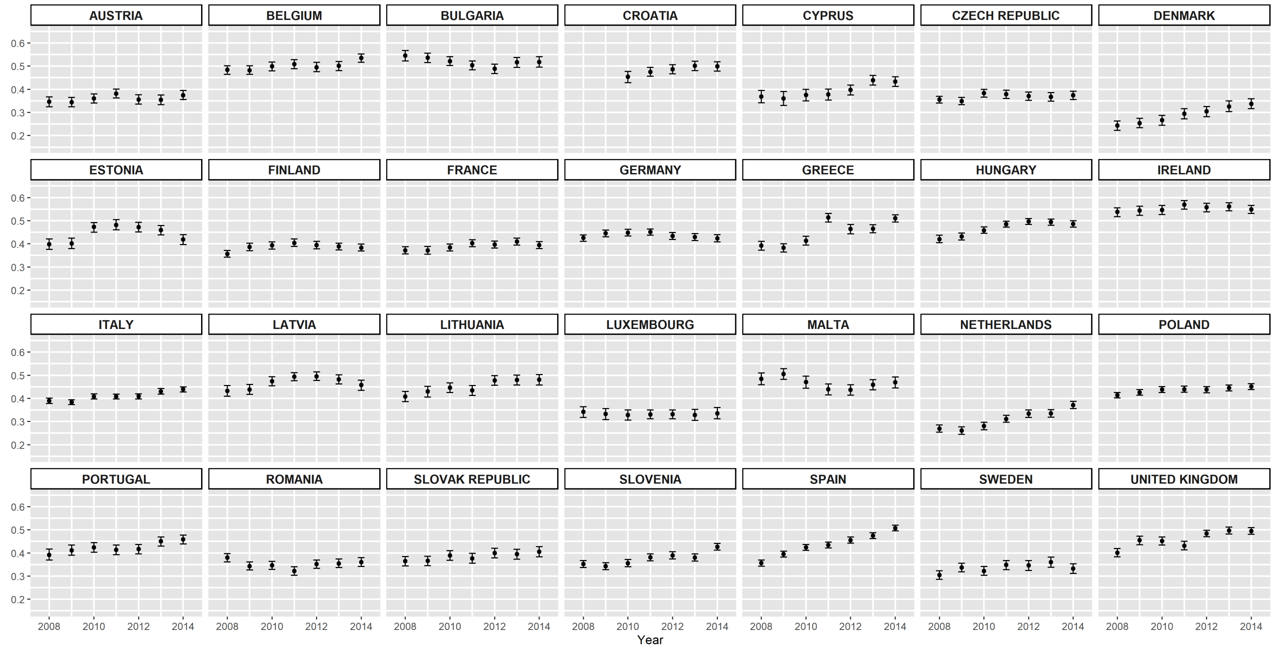


Figure 3: Evolution of $\hat{\rho}_3^-$ (Panel A) and $\hat{\rho}_3^+$ (Panel B) and their bootstrap standard 95% confidence intervals for EU-28 over the period 2008-2014.

Panel A: lower orthant dependence



Panel B: upper orthant dependence

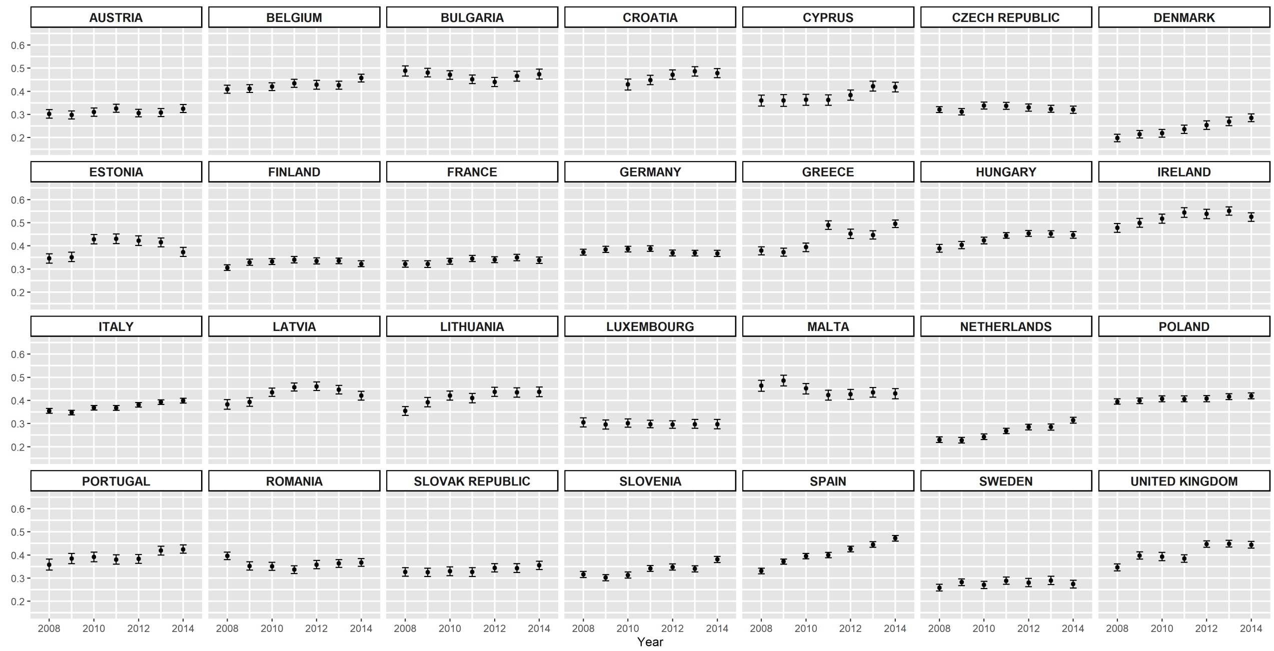
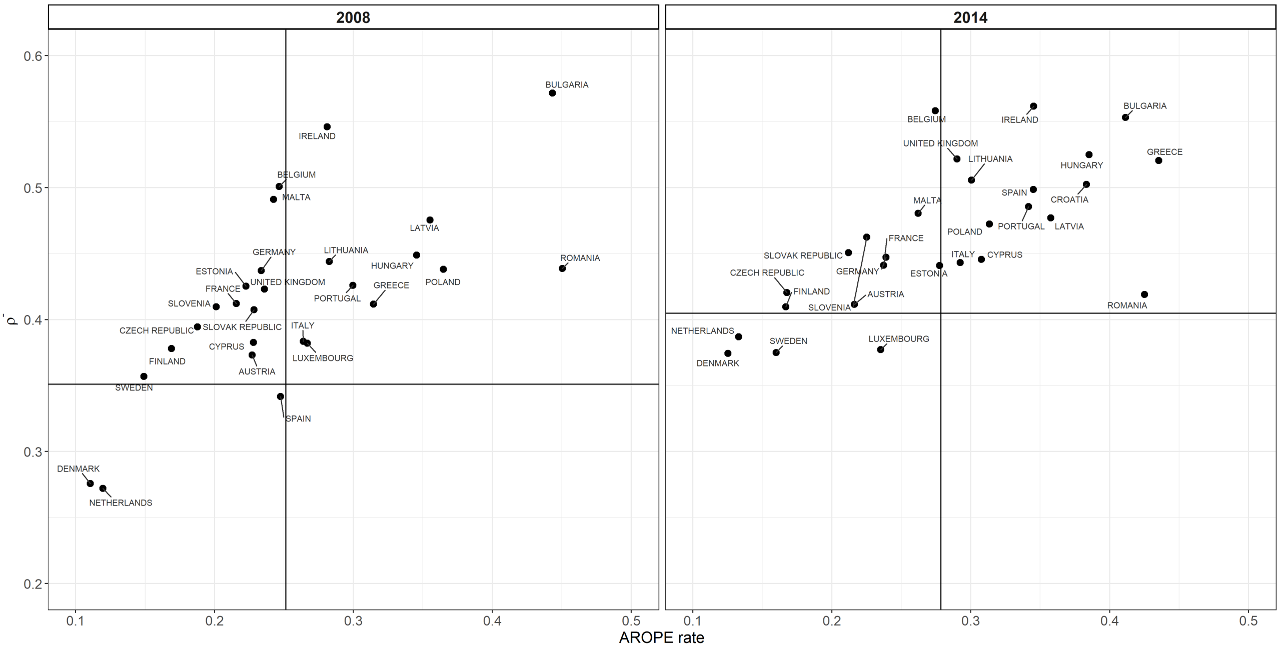


Figure 4: Evolution of $\widehat{\rho_3^{-\maltese}}$ (Panel A) and $\widehat{\rho_3^{+\maltese}}$ (Panel B) and their bootstrap standard 95% confidence intervals for EU-28 over the period 2008-2014

Panel A



Panel B

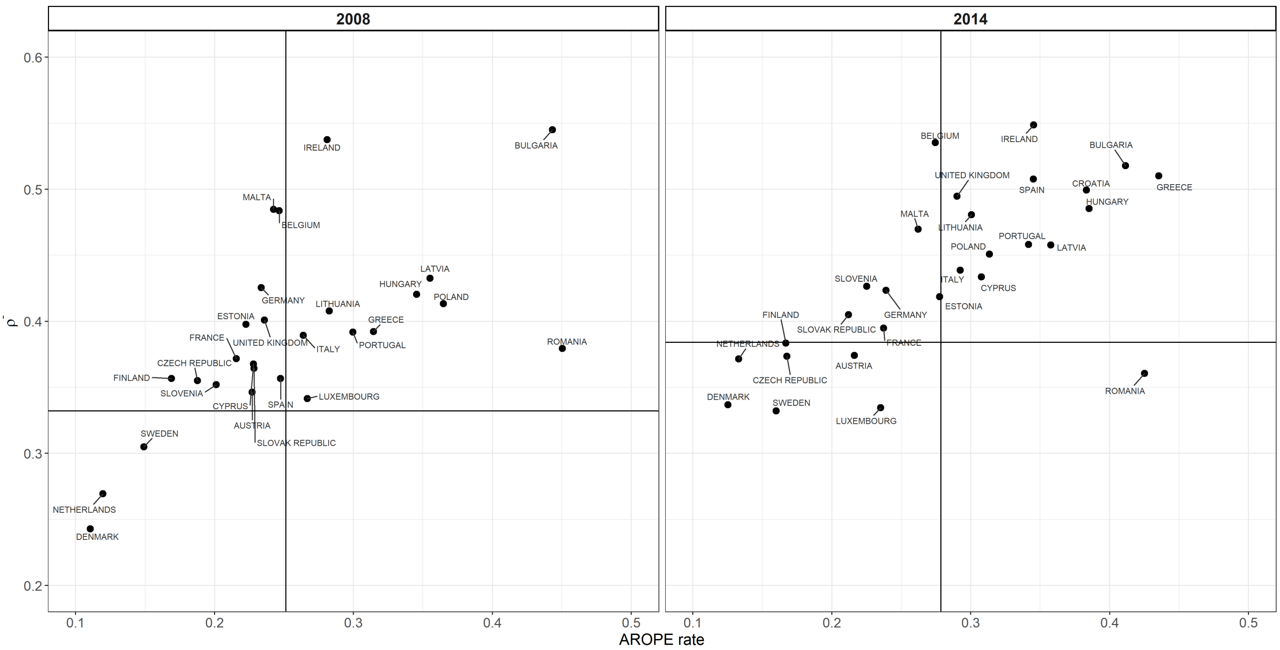


Figure 5: Relationship between AROPE rate and $\hat{\rho}_3^-$ (Panel A) and between AROPE rate and $\widehat{\rho_3^-}^{\maltese}$ (Panel B) for EU-28 and years 2008 (left) and 2014 (right).

Appendix 1

Proposition 1. *The plug-in estimator of ρ_d defined in (14) in Section 2.3, namely*

$$\widehat{\rho}_d = \frac{\widehat{\rho_d^-} + \widehat{\rho}_d^+}{2}, \tag{A1}$$

where $\widehat{\rho_d^-}$ and $\widehat{\rho}_d^+$ are the estimators in (12) and (13), respectively, coincides with the estimator of ρ_d proposed by Dolati and Úbeda-Flores (2006) in the framework of AOD measures of multivariate concordance.

Proof. The estimator of ρ_d proposed by Dolati and Úbeda-Flores (2006), that will be denoted as $\widehat{\rho}_d^{DUF}$, is as follows (see Example 5.1 in that paper):

$$\widehat{\rho}_d^{DUF} = \frac{\frac{1}{n}\sum_{j=1}^{n}\left[C'(\frac{R_{1j}}{n+1},\cdots,\frac{R_{dj}}{n+1}) + \overline{C}'(\frac{R_{1j}}{n+1},\cdots,\frac{R_{dj}}{n+1})\right] - a_{d,n}}{b_{d,n} - a_{d,n}}, \tag{A2}$$

where

$$C'(\frac{R_{1j}}{n+1},\cdots,\frac{R_{dj}}{n+1}) = \prod_{i=1}^{d}\frac{R_{ij}}{n+1},\ \overline{C}'(\frac{R_{1j}}{n+1},\cdots,\frac{R_{dj}}{n+1}) = \prod_{i=1}^{d}(1-\frac{R_{ij}}{n+1}), \tag{A3}$$

and

$$a_{d,n} = \frac{1}{2^{d-1}},\ b_{d,n} = \frac{1}{n}\sum_{j=1}^{n}(\frac{j}{n+1})^d + \frac{1}{n}\sum_{j=1}^{n}(1-\frac{j}{n+1})^d. \tag{A4}$$

Putting back (A3) and (A4) in (A2), the following expression comes up:

$$\widehat{\rho}_d^{DUF} = \frac{\frac{1}{n}\frac{1}{(n+1)^d}\sum_{j=1}^{n}\left[\prod_{i=1}^{d}R_{ij} + \prod_{i=1}^{d}(n+1-R_{ij})\right] - \frac{1}{2^{d-1}}}{\frac{1}{n}\frac{1}{(n+1)^d}\left[\sum_{j=1}^{n}j^d + \sum_{j=1}^{n}(n+1-j)^d\right] - \frac{1}{2^{d-1}}}. \tag{A5}$$

Now, multiplying both the numerator and the denominator of (A5) by $(n+1)^d$ and taking into account that $\sum_{j=1}^{n} j^d = \sum_{j=1}^{n}(n+1-j)^d$, we have:

$$\widehat{\rho}_d^{DUF} = \frac{\dfrac{1}{n}\sum_{j=1}^{n}\left(\prod_{i=1}^{d} R_{ij} + \prod_{i=1}^{d} \overline{R}_{ij}\right) - \dfrac{(n+1)^d}{2^{d-1}}}{\dfrac{2}{n}\sum_{j=1}^{n} j^d - \dfrac{(n+1)^d}{2^{d-1}}}, \tag{A6}$$

where $\overline{R}_{ij} = n+1-R_{ij}$. On the other hand, replacing $\widehat{\rho_d^-}$ and $\widehat{\rho_d^+}$ in (A1) by their expressions in (12) and (13), respectively, the expression in (A6) comes up. Hence, it turns out that $\widehat{\rho}_d = \widehat{\rho}_d^{DUF}$ and the result is proven. □

Proposition 2. *In the trivariate case ($d = 3$), the plug-in estimator of ρ_3 defined in (14) can be computed as the average of their corresponding pairwise sample coefficients, that is,*

$$\widehat{\rho}_3 = \frac{\widehat{\rho}_{12} + \widehat{\rho}_{13} + \widehat{\rho}_{23}}{3}, \tag{A7}$$

where $\widehat{\rho}_{ik}$ denotes the bivariate sample Spearman's rho for the pair (X_i, X_k), with $1 \leq i < k \leq 3$.

Proof. First, from equations (14)-(16), we obtain the following expression for $\widehat{\rho}_3$:

$$\widehat{\rho}_3 = \frac{\widehat{\rho_3^-} + \widehat{\rho_3^+}}{2} = \frac{1}{2}\left[\frac{8}{n(n-1)(n+1)^2}\sum_{j=1}^{n}(R_{1j}R_{2j}R_{3j} + \overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j}) - 2\frac{n+1}{n-1}\right]. \tag{A8}$$

Now, taking into account that $\overline{R}_{ij} = n+1-R_{ij}$, we have

$$\overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j} = (n+1)^3 - (n+1)^2\sum_{i=1}^{3} R_{ij} + (n+1)(R_{1j}R_{2j} + R_{1j}R_{3j} + R_{2j}R_{3j}) - R_{1j}R_{2j}R_{3j},$$

and so the sumation in (A8) becomes

$$\sum_{j=1}^{n}(R_{1j}R_{2j}R_{3j}+\overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j}) = n(n+1)^3-(n+1)^2\sum_{i=1}^{3}\sum_{j=1}^{n} R_{ij}+(n+1)\sum_{j=1}^{n}(R_{1j}R_{2j}+R_{1j}R_{3j}+R_{2j}R_{3j}).$$

Now, since $\sum_{j=1}^{n} R_{ij} = 1 + 2 + ... + n = \frac{n(n+1)}{2}$, the expression above becomes

$$\sum_{j=1}^{n}(R_{1j}R_{2j}R_{3j} + \overline{R}_{1j}\overline{R}_{2j}\overline{R}_{3j}) = -\frac{1}{2}n(n+1)^3 + (n+1)\sum_{j=1}^{n}(R_{1j}R_{2j} + R_{1j}R_{3j} + R_{2j}R_{3j}). \quad \text{(A9)}$$

Putting back (A9) in (A8), it turns out that

$$\widehat{\rho}_3 = \frac{4}{n(n^2-1)}\left(\sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j}\right) - \frac{3(n+1)}{(n-1)}. \quad \text{(A10)}$$

On the other hand, the average of the pairwise sample Spearman's coefficients is

$$\begin{aligned}\frac{\widehat{\rho}_{12} + \widehat{\rho}_{13} + \widehat{\rho}_{23}}{3} &= \frac{1}{3}\left[\frac{12}{n(n^2-1)}\left(\sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j}\right) - \frac{9(n+1)}{n-1}\right] = \\ &= \frac{4}{n(n^2-1)}\left(\sum_{j=1}^{n} R_{1j}R_{2j} + \sum_{j=1}^{n} R_{1j}R_{3j} + \sum_{j=1}^{n} R_{2j}R_{3j}\right) - \frac{3(n+1)}{(n-1)}.\end{aligned}$$

Hence, comparing this last equation with (A10), the result in (A7) comes up. □

Appendix 2

This appendix includes a further discussion on other copula-based generalizations of Spearman's rho for the three-dimensional continuous case, developed by Nelsen and Úbeda-Flores (2012) and García et al. (2013), that include, as particular cases, the coefficients ρ_3^- and ρ_3^+ defined in Section 2. We also include a concise discussion on the empirical results based on the estimated values of these new coefficients. To our knowledge, there are no tie-corrected versions of these coefficients for possibly non-continous data.

The coefficients of directional trivariate dependence, proposed by Nelsen and Úbeda-Flores (2012), allow to identify dependence undetected by ρ_3^-, ρ_3^+ and ρ_3, when these take values near zero, and are defined as follows. Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, with $\alpha_i \in \{-1, 1\}$, denote the eight directions corresponding to the eight corners of the cube $\mathbf{I}^3$ in which we could measure dependence in trivariate distributions. For each direction α, a directional ρ-coefficient is defined as:

$$\rho_3^\alpha = \frac{\alpha_1\alpha_2\rho_{12} + \alpha_1\alpha_3\rho_{13} + \alpha_2\alpha_3\rho_{23}}{3} + \alpha_1\alpha_2\alpha_3\frac{\rho_3^+ - \rho_3^-}{2}. \tag{A11}$$

Noticeably, $\rho_3^{(1,1,1)} = \rho_3^+$ and $\rho_3^{(-1,-1,-1)} = \rho_3^-$. Roughly speaking, these directional coefficients try to capture how far from independence our data are around any "corner" of the cube $\mathbf{I}^3$. For instance, the coefficient $\rho_3^{(-1,-1,1)}$ will capture the propensity of the observations to be concentrated around the vertex $(0,0,1)$ in the unit cube $\mathbf{I}^3$, i.e. the propensity that small values of X_1 and X_2 tend to occur with large values of X_3, whereas the coefficient $\rho_3^{(-1,-1,-1)} = \rho_3^-$ will capture the propensity of the ranked observations to be concentrated around the corner $(0,0,0)$ of the unit cube $\mathbf{I}^3$, i.e. the propensity of being simultaneously low-ranked in all dimensions.

The largest of the eight directional ρ-coefficients defined in (A11), denoted as ρ_3^{max}, was introduced by García et al. (2013) as an index of maximal dependence. This index can be alternatively calculated using the three pairwise Spearman's rho coefficients and the three common

3-dimensional versions of Spearman's rho, as follows:

$$\rho_3^{\max} = \frac{2}{3}\max\{\rho_{12}, \rho_{13}, \rho_{23}, 3\rho_3\} - \min\{\rho_3^+, \rho_3^-\}. \tag{A12}$$

It is worth noting that $0 \leq \rho_3^{\max} \leq 1$. Actually, $\rho_3^{\max}$ attains its maximum value, 1, when $C = M$ and it becomes 0 when $C = \Pi$. Moreover, if ρ_{12}, ρ_{13} and ρ_{23} are all positive, then $\rho_3^{\max}$ is equal to either ρ_3^- or ρ_3^+; for a discussion on other properties of $\rho_3^{\max}$, see García et al. (2013).

Regarding statistical inference, García et al. (2013) construct plug-in estimators for the coefficients ρ_3^α and $\rho_3^{\max}$ by replacing in (A11) and (A12), respectively, the bivariate and trivariate Spearman's coefficients by their empirical counterparts in equations (15)-(17). The resulting estimators are asymptotically normally distributed. As expected, the estimators $\widehat{\rho_3^+}$ and $\widehat{\rho_3^-}$ in (15)-(16) come up as particular cases of the estimated directional coefficient $\widehat{\rho_3^\alpha}$ for $\alpha = (1, 1, 1)$ and $\alpha = (-1, -1, -1)$, respectively.

To illustrate the application of these coefficients, the maximal dependence coefficient, $\widehat{\rho_3^{\max}}$, has been computed for the EU-28 countries and for the two years 2008 and 2014. Since all the bivariate coefficients are positive (see Tables 4 and 5), the maximum of the eight coefficients of directional dependence, $\widehat{\rho_3^{\max}}$, should be equal to either $\widehat{\rho_3^+}$ or $\widehat{\rho_3^-}$. Noticeably, in our case, $\widehat{\rho_3^{\max}} = \widehat{\rho_3^-}$ in both years and in all countries (except in Romania, where $\widehat{\rho_3^{\max}} = \widehat{\rho_3^+}$), indicating that the positions of the households in the three dimensions of poverty tend to be aligned around the corner $(0, 0, 0)$. In terms of poverty analysis, this fact is particularly relevant because this means that, in all countries (except in Romania), there is a general strong tendency of the three poverty dimensions (income, no-material deprivation and work intensity) to take low values together, so that households tend to be simultaneously low ranked in all dimensions and this could make overall poverty worse.