

CRM Series in Mathematical Physics

Şengül Kuru
Javier Negro
Luis M. Nieto *Editors*

Integrability, Supersymmetry and Coherent States

A Volume in Honour of Professor
Véronique Hussin

 Springer

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The 6th International Workshop on *New Challenges in Quantum Mechanics: Integrability and Supersymmetry* was organized in Valladolid (Spain) on June 27–30, 2017, in honor of Prof. Véronique Hussin. The conference was very successful: it attracted more than 60 researchers from different countries, and about 45 talks were delivered.

Prof. Hussin is an internationally recognized expert in many branches of mathematical physics. Her career started in her native city of Liège (Belgium); afterward she moved to Montréal, where she has stayed ever since at Université de Montréal and at CRM (Centre de Recherches Mathématiques). However, Véronique Hussin has had numerous stays in different research centers collaborating with many colleagues and friends. In particular, we want to mention her regular visits to Durham University (England), CINVESTAV (Mexico), and the University of Valladolid (Spain) where, besides her scientific activity, due to her open character, she made many friends. She has made remarkable contributions to several branches of mathematical physics, like coherent and squeezed states, supersymmetry, sigma models, nonlinear equations, and other topics that show her wide range of interests.

The workshop program paid special attention to the topics in which Prof. Hussin has been working. Based on this conference, but open to scientists in these areas, we considered a good opportunity to edit a contributed book published by Springer Nature, under the title “Integrability, Supersymmetry and Coherent States”. The book consists in a part devoted to review papers on the three main topics, whose authors are well-known experts in their fields and where the objective is to take a personal approach on some attractive aspects of these subjects. A second part is

Special thanks are due to the University of Valladolid, where the meeting in honor of Prof. Véronique Hussin *6th International Workshop on New Challenges in Quantum Mechanics: Integrability and Supersymmetry* was held.

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made up of a list of contributions related with these topics. All the papers have
satisfied a strict refereeing process by anonymous specialists before publishing. 25
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It is a pleasure, for all of us, to dedicate this volume to her. 27

Ankara, Turkey
Valladolid, Spain
Valladolid, Spain
March 31, 2019

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