URBAN PUBLIC LIBRARIES: PERFORMANCE ANALYSIS USING DYNAMIC-NETWORK-DEA

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Abstract
The aim of this work is to posit a model to evaluate the efficiency of a system of urban public libraries and to examine the impact of certain contextual variables on the level of performance. We take the System of Public Libraries in the city of Medellin (Colombia) as a case study and consider a production function which displays three main characteristics. First, it is a complete production function which spans the different activities undertaken by these institutions, not only the one that identifies it with its function as a repository of knowledge. Second, there is the production function in stages, which allows us to distinguish between the various activities controlled by management from those coproduced with users, together with the link between the two. The third is a production function which takes into account temporal interdependence relations by identifying quasi-fixed inputs that remain for the provision of the service over time. This then allows us to analyse how efficiency evolves during the period in question. Efficiency evaluation is carried out by employing a dynamic-network-DEA model and we also apply truncated bootstrap regression to estimate the effect of certain contextual variables on library efficiency. The results evidence a growing trend in the efficiency indices, with values that are slightly more favourable in the second stage of service provision than in the stage focusing on managing the cultural programme. Factors such as the level of education, population density, youthfulness, and safety are seen to positively affect library performance, particularly in the second stage vis-à-vis the public.

Keywords: public libraries, technical efficiency, dynamic efficiency, dynamic-network-DEA, two-stage performance evaluation.

JEL: D24, H41, Z11, Z18.

1. Introduction

Cultural institutions constitute an area of growing interest for efficiency evaluation studies, given their position as non-profit or public entities charged with providing a kind of public good as well as cultural goods and services. To date, most studies in this regard have focused on evaluating museums, theatres, symphony orchestras, and so on [1], although over the last few years assessing the performance of libraries has come to the fore (see the survey in Bernardo et al. [2]). Libraries are one of the most traditional cultural institutions, yet are also one of those to have undergone most changes in recent times due to the gradual diversification of their functions towards centres of cultural outreach and because they have shown themselves to be entities that are highly permeable to technical change both in terms of the resources they embrace and in their conditions regarding the provision of and accessibility to their services [3].

Library performance and management have usually been evaluated by taking their function as repositories of knowledge as a reference. Nevertheless, other functions such as providing an area for training and for cultural creation, exhibition or cooperation have been gaining ground, making this an aspect to be taken into account when assessing the performance of such institutions. The arrival of digital access to knowledge has been the main driver behind this change [4] and has triggered the diversification of the various services which libraries provide and which have ceased to centre around what was almost exclusively the custody and loan of bibliographical and

other material as well as the offer of an area for study. They now embrace a number of other activities such as training courses, access terminals to internet, activities for children, programmes for creating and exhibiting artwork, or offering an area for the local community to use [5]. These services represent library output today and generate a series of outcomes that can be described in terms of improvements in training, social integration as well as breaking down economic and social inequalities [6]. The production function that shapes the way in which these institutions are evaluated should therefore take into account libraries' overall offer of services, linking the various resources used (inputs) to the whole range of services provided (outputs).

Pinpointing which outputs are generated by public entities is an issue that has been a topic of debate in the economic literature, particularly with regard to distinguishing between outputs and outcomes. In this vein, Bradford et al. [7] identify two kinds of outputs in the public sector: services which are produced directly and those which are subject to citizen interest, and the result of which depends on many other factors. For De Bruijn [8], performance measurement should be geared towards outputs, given that outcomes are related to the final anticipated effect and are determined by a range of different factors, such that they prove difficult to measure, whilst outputs are easier to measure since they reflect the direct effects of an action. De Witte and Geys [9] point out that the provision of public services should be characterised through a two-stage production function. Therefore, while during the first stage the inputs are transformed into service potential, during the second, the observable outputs are generated through a co-production process with citizens, since they are specified in citizens' actual demand. For said authors, efficiency in the provision of the service should focus on the first stage, since this is where the service provider can control inputs and outputs, whereas during the second stage, the output depends on the decision of those who receive the service on their active participation, a decision which may be shaped by a large number of other external factors (interest, accessibility, size of the market, etc.). Førsund [10] states that outputs measure the entity's service potential, whilst outcomes reflect the service provided to consumers. In this regard, efficiency would focus on measuring the relation between inputs and outputs, while the relation between inputs and outcomes allows for the effectiveness in service provision to be evaluated (for an application, see Gómez-Zapata et al. [11]). In the two previous proposals, efficiency evaluation in the provision of the service is carried out before the service is delivered, and only takes into account the entity's willingness to provide it.

Our work thus proposes evaluating the performance of a group of urban public libraries, taking into account the whole production process, from the handling of primary input for constructing the basic cultural supply, to the final provision of the service, adopting a multifunctional as well as a dynamic perspective. To this end, as a case study we take the Medellin Public Library System (Spanish acronym - SBPM), a group of urban libraries, which have spread over the metropolitan area of the city of Medellin, Colombia, and which have gradually extended the activities they are engaged in an attempt to help improve citizens' quality of life and their level of education. We believe the SBPM to be a relevant case study for various reasons. First, it is a long-running project which commenced in 1952 with the creation of the Medellin Pilot Library for Latin-America (BPP), at the behest of UNESCO, as part of a programme designed to set up a model of public libraries in Latin-America, Africa, and India, which would provide those with the fewest resources and lowest levels of literacy with access to information and knowledge. Consistent with this mission, the SBPM has gradually established a group of affiliated libraries located mainly in places of greatest need and, as a new public endowment, engaged in providing educational and cultural services in these areas. This effort has been taken a stage further with the creation of second generation public libraries, so-called 'library parks', enhancing their role as centres of creation, exhibition, and cultural dissemination, and whose architectural design seeks

to have an impact on urban change. The second reason concerns the SBPM's governance pattern, which is geared towards formulating policies and strategies of economic and social development based on education and culture, and which is characterised by a system of integrated management of the networks of libraries that has enabled resources and processes to be optimised, and which is also committed to avant-garde architectural designs in the new buildings. This has provided a benchmark model for Latin-America that has spread to other countries and that has won various awards and has gained recognition [12]. Finally, the case study is also interesting because it is framed within the strategy of the city of Medellin, which has become a paradigmatic example of the use of culture as a tool for urban and social change [13,14,11].

Based on these premises, our aim is to gauge the performance of a group of libraries that make up the SBPM, considering three methodological perspectives. First, we aim to evaluate performance in terms of technical efficiency, bearing in mind all of the cultural services to derive from libraries' production function and not only that which concerns traditional loans of bibliographical resources and related items. We therefore consider a multi-output production function. Second, the aim is to propose a performance evaluation method that takes into account not only the capacity to offer the service but also the way in which the service is actually provided. We therefore need to consider the possibility of time interdependence and inter-reliant inputs as restrictions in how the production function operates, in the sense that the outcomes from one production stage may determine the outcomes of another, both in a dynamic sense (carry-over activities) as well as in a horizontal sense (interrelation of the production chain). With this aim in mind, we posit a dynamic-network-DEA (DNDEA) model, in line with the works of Tone and Tsutsui [15] which enables us to break down the different activities carried out by the entity into different stages without overlooking the effects which one previous activity might have on a subsequent activity. It can also determine more accurately how efficiency has changed over time by considering the time dependence relation between input and output variables. Third, we wish to gauge what impact certain social and urban variables have on the performance of libraries. Finally, we posit a regression analysis of efficiency ratios with certain external institutional and socioeconomic variables by applying truncated bootstrap regression models [16]. Our contribution therefore proposes a comprehensive evaluation method for efficiency which merges the application of the latest generation DEA models with an analysis of the effect of external variables. To the best of our knowledge, this is the first time this has been done in the field of libraries and cultural institutions. The study is also posited as an example of the evaluation of public cultural institutions in less developed countries, where applications of this nature remain scarce.

The paper is organised as follows. Section 2 offers a review of the prior literature addressing the evaluation of libraries, while section 3 analyses the production function which defines the activities undertaken by these institutions. Section 4 presents the method used and describes the data and variables involved in the case study. Section 5 sets out the main results to emerge. The work finishes with a selection of the main conclusions to come out of the study.

2. Literature review

2.1. Efficiency evaluation of cultural heritage institutions using a new generation of DEA models

Efficiency studies in the public sector are grounded on the relation between inputs consumed and outputs produced by each entity. This relation is determined by the

production function which allows each activity to be carried out. Some of the methods used to evaluate efficiency in the domain of cultural institutions require an explicit definition of the production function, as is the case of stochastic frontier studies (SFA), whereas others, such as those based on non-parametric techniques like Data Envelopment Analysis (DEA), offer greater flexibility by circumventing this requirement, which has led to them becoming widespread (see Emrouznejad and Yang [17]). Yet this does not mean that it is possible to ignore the production function when formulating an evaluation process, since the activities undertaken as well as the resources used and products obtained in each, set the tone for positing the evaluation.

Early studies using DEA applications in cultural heritage institutions were based on designing a single production process (black-box) in which a series of inputs (labour force and different versions of capital such as cultural endowment, equipment, and so on) gave rise to a series of outputs that were representative of the various functions performed by cultural institutions and which are not usually reflected in the market. Based on these premises, studies were mainly carried out into museums using simple production functions which focused on attracting visitors [18-20], or more complex instances of a multi-output nature [21,22]. However, it soon became apparent that not all of the production processes were in fact controlled by the institution management but that they were partly affected by external variables or were co-produced by the end user of the service, in other words by the consumer as well as by their interests, tastes, and determinants. Network-DEA evaluation models thus emerged where the production process is divided into various stages, the first few of which are under the manager's control and the outcomes of which become the intermediate input for the subsequent stages that are geared towards providing the final service. Under these premises, evaluations have also been carried out of the performance of museums [23], archives [24], and dance companies in the area of performing arts [25]. Likewise, numerous studies have emerged aimed at gauging what effect variables outside the production function have on cultural institutions' level of efficiency, applied to museums [23], archives [26], and cultural heritage agencies [27].

Few works, however, have addressed the dynamic perspective of efficiency. Exceptions include the studies by Pignataro [18] and Del Barrio-Tellado and Herrero-Prieto [22] who construct Malmquist indicators for productivity growth in samples of museums, and Guccio et al. [26], who apply Window-DEA to assess how efficiency evolves and what impact the introduction of websites has on the productivity of public archives in Italy. To the best of our knowledge, in the field of cultural institutions there are still no applications of DNDEA models which bring together in a single model the time dependency restrictions and production interrelation of a production function.

2.2. Efficiency evaluation in libraries: state of the art

Libraries are one of the main cultural institutions which, together with museums and archives, make up the classical triad of entities in the domain of cultural heritage. They have provided one of the most common fields for case studies in efficiency evaluation analyses and have progressed similarly in terms of applying methodological developments in the field. In this way, most of the early works applying the DEA technique to assess the performance of these institutions considered a single process (black box) in which a set of inputs, occasionally classed as discretional and non-discretional, were transformed into one or more outputs. As a reference, some works took a basic production function which focuses on the custody and loan of material and consultations in the various rooms, whilst others presented a production function offering a wider supply that includes carrying out training activities, terminals for internet access or providing areas for the community to use. Works in the first category include those of Chen [28], who evaluates the technical and scale efficiency of 23

university libraries in Taipei, pinpointing possible sources of inefficiency; Vitaliano [29], who assesses the performance of 184 libraries located in the state of New York, and who seeks to identify sources of inefficiency through a regression of the efficiency indices on slacks of inputs and outputs; the work of Worthington [30], who evaluates the efficiency of 168 libraries in New South Wales in Australia, taking as a measure of output an indicator of each entity's volume of circulation of materials; and finally the work of Reichmann and Sommersguter-Reichmann [31] which deals with an intercountry efficiency comparison of university libraries, mainly analysing their archival and circulation functions. Examples of works in the second category include Srakar et al. [32] and Guajardo [6] who identify library services as those related to accessing books and other materials, internet access and the use of technologies, as well as cultural programmes and participation in events.

However, other studies point out that library activities are conducted through a process involving stages, as can be seen in the work of Hammond [33]. During the first stage, the basic inputs, measured in terms of work and capital, are transformed into service potential measured in terms of the size of the collection and hours open. These outputs are converted into intermediate inputs at a second stage so as to produce observable results that are quantified as the circulation of material and consultations dealt with. Finally, users merge these results with their own basic inputs in order to obtain the outputs derived from the use of the information. Continuing with this idea, Simon et al. [34] posit a three-stage university library evaluation model: maintaining the resources required to provide the service, providing the service, and the institution's impact. Adopting a similar approach, De Witte and Geys [9] identify a production process comprising two stages: generating the service potential and the actual provision of the service. In this case, however, efficiency evaluation focuses solely on the first stage, whereas the second stage examines the impact of external factors. These approaches allow libraries' production function to be broken down, thereby enabling sources of inefficiency to be pinpointed more accurately. Nevertheless, the models used assume independence between the stages, neglecting the fact that the decisions taken in one activity may shape the results of subsequent activities. In an effort to overcome this limitation, Guccio et al. [35] apply a centralised network-DEA model following the proposals of Liang et al. [36] to evaluate national public libraries in Italy. In this case, the first stage is confined to the activity related to conserving the bibliographical resources made available to users, whilst the second stage deals with the provision of the final service. The results to emerge from this work evidence better efficiency results in the conservation stage than in the service provision stage.

Most of the works described do not include the time dimension in the efficiency analysis. In contrast, Miidla and Kikas [37] posit a study of 20 public libraries in Estonia over a period of four years, with an efficiency analysis using DEA for each year. De Carvalho et al. [38] assess the efficiency of a group of university libraries in Brazil applying DEA in two consecutive periods, while Stroobants and Bouckaert [39] employ DEA and Free Disposal Hull (FDH) to evaluate a group of local public libraries in Flanders. It should be pointed out, however, that these works follow a concept of comparative statics model when evaluating efficiency each year in an individual and independent manner. Comparisons between different periods may prove to be misleading since, in each time period, the efficiency index marks the distance to a different frontier which in each case is constructed on data observed in the same time period [40]. An alternative approach to evaluate the change in efficiency over time involves decomposing the Malmquist productivity index into the two concepts of technological progress (displacement of the efficiency frontier) and improvement in efficiency (movements of the units towards the frontier). Some studies in the field of libraries have adopted this approach [34,41]. Nevertheless, these models also evidence the same limitation by assuming that the inputs from each period are used in

full to obtain the outputs from the same period, without therefore assuming that there might be dependence relations between inputs and outputs over time, and that decisions taken on the input in one period might have an impact on the output to emerge from subsequent periods.

Our proposal seeks to overcome both restrictions (time interrelation and production interrelation) and offers analytical innovation in the efficiency study of cultural institutions and, particularly, with regard to libraries. In this way and as we said in the introduction, the analytical strategy stems from a three-fold perspective. First, it is an approach which seeks to embrace the range of new activities undertaken in libraries by designing a production function that includes the diversity of outputs. Second, efficiency evaluation is posited through an NDDEA model which allows time dependence and the various sub-processes involved in the production function that lead to the final outputs to be taken into account. Finally, we aim to estimate what impact external factors concerning institutional and socioeconomic variables from the environment might have on performance.

3. The production function in public libraries

One key question in studies aimed at efficiency evaluation in the provision of public services concerns modelling the production function and identifying the inputs and outputs involved in the service provision process. In the case of libraries, prior works identify a production function which is implemented in two stages [33,9,42,10,35]. During the first stage, the entity combines a set of work and capital resources to put together its cultural offer or service potential, whilst in the second stage this becomes an actual service, which is materialised in the demand for goods and services from the public. Outputs from the first stage are specified through variables such as opening hours, the collection available or the activities planned, whilst during the second stage the results make up the input required to obtain an output that is measured in terms of the circulation of books, consultations in the library rooms or the number of people attending the various activities scheduled. The production design is based on the notion that there are derived activities and outputs which are under the entity's control, whereas other activities, and therefore the outputs generated when carrying them out, are beyond both entity and manager. In this regard, the first stage of libraries' production function would show the library's actual cultural production, whilst the second stage would involve the active participation of citizens, who join the production process as co-producers of the service [9,42].

Certain authors identify the outputs which remain beyond the entity's direct control as outcomes whereas those controlled by the manager are what really make up the outputs to emerge from the production process. As a result, some of the studies addressing library efficiency assess these entities' performance taking only the first of the previously described stages as a reference [9,42], interpreting that efficiency is measured as a relation between controlled inputs and outputs, in that the outcomes help to gauge service efficacy, in other words, the ability to achieve objectives. In this regard, Førsund [10] states that outcomes are related to more general social objectives than services in themselves might actually reflect. In the case of libraries, these general objectives cannot be confined to the circulation of books, consultations in the various rooms or participation in scheduled activities but refer instead to achievements in the area of training, digital access, entertainment or even social change and the population's cultural identity. From this perspective, it would not seem appropriate for service efficacy to be measured through variables which are far from reflecting the entity's outcomes. A different question concerns the notion that outcomes are difficult to measure and that proxy variables are used to do so, such as the number of people

who make use of a particular service. Nevertheless, outcomes go beyond this and are affected by the environment and context in which the activity is carried out and which can shape the way outputs are become outcomes. Moreover, it would prove difficult to assume that efficiency evaluation in the provision of services is conducted before the service is actually provided, in other words, during the first stage of the production process. This would be like evaluating the efficiency of a higher education institution merely on the basis of the degree qualifications it offers, but not on the results of the students who take the degree courses or, in the case of a health sector institution, assessing efficiency based on the number of hours an operating theatre is open whilst failing to take into account the patients who have undergone surgery.

We believe that the efficiency evaluation of libraries in all instances involves considering the whole service provision process, in other words, organising the resources to make up the cultural supply at the first stage, and the explicit provision of the service, reflected through public participation, in the second stage (Fig. 1). This does not entail any break with the existing literature concerning the evaluation of public service performance, if we take into account that a distinction can be drawn between services which could be said to require active management in the search for users from those which can function with passive management. If we take the case of a health service, managers do not require any special action to attract users; in other words, they should not try to make people fall ill so that they have to resort to the service. Rather, it is the users who resort to the service when they need it. An equivalent situation could be said to exist for a garbage collection service or a fire extinction service. The manager does not need to become an arsonist to attract users; the circumstances themselves create the service provision without the active participation of the manager. In other cases, as might occur in higher education institutions, there is an incentive at the end of the service, namely, obtaining a qualification or accreditation, which guides users towards the service. However, in the case of cultural institutions such as libraries, the perception of the need for the service might not be so evident to the citizen, such that active participation from management vis-à-vis attracting users becomes vitally important. This circumstance is justified by the transformation which libraries are undergoing; changing from adopting a passive and more traditional position to one in which they are considered to be a centre of cultural dissemination, one which is more dynamic and linked to the environment.

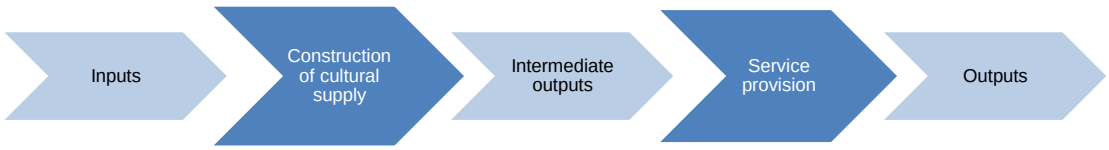

Fig. 1. Production function for public libraries. Source: authors' own

Based on these reflections, we feel that the efficiency evaluation of libraries should be considered in both the cultural supply formation stage as well as in the actual service provision stage, since the two together jointly make up the production process (Fig. 1). It is true, however, that the former allows us to measure what is merely efficiency in management and that the latter is shaped by other factors that are external to the entity. Yet it is at this stage when the service provision really occurs, since this is when users resort to it. DNDEA models thus enable the entity's overall performance to be measured in addition to showing the performance at each stage of the production process, thereby helping to identify possible inefficient behaviour. We also introduce a dynamic effect into the design of this production function by considering intertemporal effects in the decisions taken concerning inputs and outputs and by seeing how this

impacts the progress of efficiency. Finally, and in an effort to explore what effect contextual factors might have on efficiency results, there are various methodological proposals [43,16,44] that can be applied. In particular, we look at the effect of certain institutional variables and characteristics of the urban environment where the libraries are located.

4. Methodological approach and case study

4.1. Methodology

As pointed out previously, DEA has been applied on a number of different occasions to assess the performance of cultural institutions, particularly libraries [1,2]. This technique measures the performance of a set of production units by calculating their relative efficiency indices based on distances from best practice cases, drawing on known data for resources consumed as well as the goods and services obtained by each entity. In traditional models, efficiency indices are calculated by considering a single production process as a "black box", ignoring possible sub-processes that allow the final outputs to be obtained. Network-DEA (NDEA) models overcome this limitation by identifying the internal structure of the activities carried out by the entity, as well as the intermediate *links* (outputs from one activity that are included as input in a subsequent activity) between each sub-process. In this way, NDEA models provide an efficiency index for each of the activities the entity undertakes, as well as a global index, thus enabling possible sources of inefficiency to be pinpointed more easily.

Sengupta [45] and Nemoto and Goto [46] seek to model inter-temporal dependence relations in the efficiency analysis by pointing out the existence of two types of different inputs: input variables, consumed for the production of the exercise output, and capital inputs, generators of outputs in the present exercise as well as in future exercises. Capital inputs are seen as quasi-fixed resources that can grow in the medium and long term thanks to the entity's capacity to expand. This classification proves particularly relevant when evaluating cultural institutions such as museums and libraries where the compiled collections are considered as input for the system, without taking into account their permanence and development over time, which highlights the inter-temporal dimension of this resource. Dynamic efficiency evaluation models are able to incorporate inter-temporal dependence between input consumption and output production by identifying transition activities between periods. The literature provides various proposals for efficiency evaluation from a dynamic perspective [47-50]. Our work follows the model of Tone and Tsutsui [49] where the transition activities between periods are included through variables known as carry-over variables, which represent intermediate products (output from one period that is included as input in a subsequent period) similar to links between activities in NDEA models. Tone and Tsutsui [15] merge the network and dynamic approaches to construct DNDEA models which take into account the interrelations between activities and relations that can exist between the various periods. DNDEA models have previously been applied in different areas of study: Kawaguchi et al. [51] assess the efficiency of a group of hospitals in Japan, Fukuyama and Weber [52] evaluate the performance of a group of Japanese banks, Villa and Lozano [53] measure the efficiency over time of NBA basketball teams, and Moreno and Lozano [54] use DNDEA to measure efficiency in public service provision.

In order to include in our evaluation model of urban public libraries in Medellin the intertemporal aspects linked to the use of inputs as well as the internal structure of activities in the service provision, we posit a DNDEA model following the work of Tone and Tsutsui [15]. It is a non-radial SBM (slacks-based-measure) model which does not therefore require proportional changes in inputs and outputs, and which offers a

measure of strong efficiency by including in the efficiency indices the information relative to the slacks. This model calculates an efficiency index for each period, for each activity and also for each activity in each period by observing four kinds of different variables: inputs, outputs, links, and carry-overs. The inputs indicate the resources consumed by each entity to produce its own outputs, whilst the links show the output part of an activity that is used as input in a subsequent activity. Carry-overs are variables that take into account interdependence between the input consumption and output production of an activity in consecutive time periods.

Following Tone and Tsutsui [15], we assume a set composed of n DMUs, in our case libraries, that carry out K activities, over the whole cultural production process. For all of these activities, libraries consume m_k inputs and produce r_k outputs in each activity k, over T time periods. Additionally, each library produces p_{kh} intermediate outputs (links) in each activity k, which are incorporated as inputs in the subsequent production process h activity. Finally, each activity k of the production process has q_k carry-overs which show the links between this activity in period t and period t+1. We call x_{ijk}^t $(i = 1, \dots, m_k;\ j = 1, \dots, n;\ k = 1, \dots, K; t = 1, \dots T)$ the observed input i consumed by DMU j in activity k, in period t; y_{ijk}^t $(i = 1, \dots r_k;\ j = 1, \dots, n;\ k = 1, \dots, K; t = 1, \dots T)$ the observed output i produced by the library j in activity k, in period t; $z_{j(k,h)_l}^{\alpha,t}(j = 1, \dots, n; l = 1, \dots, p_{kh}; t = 1, \dots T, \alpha = fix\ or\ free\ links)$ the observed link l produced by DMU j in activity k and consumed in activity h, in period t; and $\omega_{jkc}^{\alpha,t}(j = 1, \dots, n; c = 1, \dots, q_k;\ k = 1, \dots, K; t = 1, \dots T - 1;\ \alpha = fix\ or\ free\ carry - overs)$ the observed carry-over c of DMU j, in activity k, from period t to period $t + 1$.

We can evaluate the overall efficiency of each library by solving the following optimisation problem:

$$\rho_o^* = \min \frac{\sum_{t=1}^{T} W^t \left[\sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + p_{(k,h)}^{in} + q_k^{bad}} \left(\sum_{i=1}^{m_k} \frac{s_{ik}^{t-}}{x_{iok}^t} + \sum_{l=1}^{p_{(k,h)}^{in}} \frac{s_{(k,h)l}^{in,t-}}{z_{o(k,h)l}^{in,t}} + \sum_{c=1}^{q_k^{bad}} \frac{s_{kc}^{bad,t-}}{\omega_{okc}^{bad,t}} \right) \right] \right]}{\sum_{t=1}^{T} W^t \left[\sum_{k=1}^{K} w_k \left[1 - \frac{1}{r_k + p_{(k,h)}^{out} + q_k^{good}} \left(\sum_{i=1}^{r_k} \frac{s_{ik}^{t+}}{y_{iok}^t} + \sum_{l=1}^{p_{(k,h)}^{out}} \frac{s_{(k,h)l}^{out,t+}}{z_{o(k,h)l}^{out,t}} + \sum_{c=1}^{q_k^{good}} \frac{s_{kc}^{good,t+}}{\omega_{okc}^{good,t}} \right) \right] \right]}$$

subject to:

$$\begin{aligned} x_{iok}^t &= \sum_{j=1}^{n} x_{ijk}^t \lambda_{jk}^t + s_{ik}^{t-} \qquad (\forall i, \forall k, \forall t) \\ y_{iok}^t &= \sum_{j=1}^{n} y_{ijk}^t \lambda_{jk}^t - s_{ik}^{t+} \qquad (\forall i, \forall k, \forall t) \\ \lambda_{jk}^t &\geq 0,\ \ s_{ik}^{t-} \geq 0, \qquad s_{ik}^{t+} \geq 0, \qquad (\forall i, \forall k, \forall t) \end{aligned} \tag{1}$$

$$\sum_{j=1}^{n} z_{j(k,h)l}^{\propto,t} \lambda_{jk}^t = \sum_{j=1}^{n} z_{j(k,h)l}^{\propto,t} \lambda_{jh}^t \qquad \left(l = 1, \dots, p_{(k,h)}^{\propto}, \forall (k,h), \forall t\right) \tag{2}$$

$$\sum_{j=1}^{n} \omega_{jkc}^{\propto,t} \lambda_{jk}^t = \sum_{j=1}^{n} \omega_{jkc}^{\propto,t} \lambda_{jk}^{t+1} \qquad (c = 1, \dots, q_k^{\propto}, \forall k, t = 1, \dots, T - 1) \tag{3}$$

It is a non-oriented model which takes account of excesses in inputs as well as defects in outputs, where W^t represents the weight assigned to each period t (being $\sum_{t=1}^{T} W^t = 1$) and w_k the weight assigned to each activity in the production process ($\sum_{k=1}^{K} w_k = 1$). The first group of restrictions (1) refers to the inputs consumed and outputs produced, being s_{ik}^{t-} and s_{ik}^{t+} the slacks for the inputs and for the outputs, respectively, and λ_{jk}^{t} the intensity corresponding to activity k of unit j for period t. Restrictions (2) and (3) refer to the links between activities and the carry-over between different time periods. In our application, we assume the equal weighting hypothesis, both for the stages of the production function as well as for the time periods considered. We also assume that the links between stages may be fixed or free and that the carry-overs are fixed and non-discretional. The model presented assumes constant returns to scale. Applying the program provides an index of global efficiency for each library which takes the value of 1 for efficient units and the value <1 for those which operate below the optimum level. The model also calculates the efficiency indices for each activity of the production function, for each time period and for each activity in each time period[1].

As a final point concerning methodological aspects, our work seeks to determine what effect some variables that are external to the entities and related to the socioeconomic features of the environment might have on the efficiency results. We adopt the approach suggested by Simar and Wilson [16] —based on truncated regression and bootstrapping— to explain differences in the efficiency indices caused by the conditioning effect of external variables in accordance with the following specification:

$$\theta_k = \beta\, x_k + \varepsilon_k$$

where θ_k represents the set of efficient scores from the model of the previous stage, $\varepsilon_k \sim N(0, \sigma_\varepsilon^2)$ is a vector of error terms, and β is a vector of parameters for the series of independent and environmental variables x_k. Following the first algorithm in Simar and Wilson ([16]: 41-42) entails the following three steps:

i. We apply maximum likelihood to estimate a β and σ_ε in the truncated regression of the efficiency scores previously obtained on a set of covariates z, using the subset of libraries with scores below one $(\widehat{\mathrm{DEA}}_\mathrm{l} < 1)$

ii. We loop over the following three steps L times to obtain a set of bootstrapped estimates of the parameters β and σ_ε; namely, $\mathrm{B} = \left[\left(\hat{\beta}'^{\mathrm{b}}, \hat{\sigma}_\varepsilon^{\mathrm{b}}\right)\right]_{\mathrm{b}=1}^{\mathrm{L}}$

   For each library with $(\widehat{\mathrm{DEA}}_\mathrm{l} < 1)$, we draw $\varepsilon_\mathrm{l}^{\mathrm{b}}$ from the following normal distribution: $\mathrm{N}(0, \hat{\sigma}_\varepsilon^2)$ right – Truncated at point $\left(1 - \hat{\beta}' \mathrm{z}_\mathrm{l}\right)$

   We compute $\mathrm{DEA}_\mathrm{l}^{\mathrm{b}} = \hat{\beta} \mathrm{z}'_\mathrm{l} + \varepsilon_\mathrm{l}^{\mathrm{b}}$, again for the library for which $(\widehat{\mathrm{DEA}}_\mathrm{l} < 1)$ and estimate $\hat{\beta}^{\mathrm{b}}$ and $\hat{\sigma}_\varepsilon^{\mathrm{b}}$ by truncated regression and maximum likelihood using the artificial competitiveness scores computed in the previous step as the dependent variable.

iii. Finally, we use values in B and the original estimates to build a confidence interval for parameters β and σ_ε.

We specifically follow this first algorithm in Simar and Wilson [16], since it allows for the introduction of efficiency ratios based on non-radial distances [55], such as the

[1] For more details concerning how the various efficiency indices are defined as well as information concerning the technical features of the model, see Tone and Tsutsui [15]. The calculations for the empirical application studied were carried out with the DEA-SOLVER-PRO version 14.0 program.

dynamic-network SBM model used in the research. Consistency in the application of this method depends on complying with the condition of separability [16,56,57], which implies that the contextual variables only affect the efficiency distribution but not the production possibilities. We feel that this condition is met in our case study, libraries, since it does seem logical for the provisions thereof to be different depending on the environment in which they are located. We also assume this condition is complied with, in that our sample size makes it virtually impossible to reject the null hypothesis of separability when applying the test proposed by Daraio et al. [57]. Nevertheless, in order to support the robustness of the results, different econometric estimates have been tested, such as Tobit and fractional logit regression (FLR) models, which are pertinent for dependent variables bounded between 0 and 1, which is the case of our efficiency index [58,59].

4.2. Data and variables

Data for our study were provided by the SBPM and are related to the 25 libraries that make up the group for the period between 2015 and 2018. One of the libraries was excluded from the study because the main facilities were closed during the period considered, such that the final sample consists of 24 libraries[2]. The SBPM was set up in 2006 to ensure free access to culture, information and entertainment, fostering active citizen participation, and seeking to provide a tool to help citizens move towards respect and peaceful coexistence. This goal points libraries towards a function that is predominantly one of use as opposed to collecting and conservation[3]. The system is run as an institutional unit in terms of its objectives, activities, and availability of resources, which ensures the consistency required for our study. However, it should be pointed out that even though all the libraries operate in the same manner, the way in which they emerged leads to two differing presentation formats. On the one hand, there is a group consisting of the first libraries to join the system and that were already functioning, and which were later joined by other smaller and medium sized libraries, making up the group of “proximity libraries”, and which became urban reference points as access points for the use and generation of knowledge. The second group contains the so-called “park libraries” and responds to a new concept of library, seen as a centre of cultural development, thereby widening and boosting the complementary functions as poles of projection and cultural creation. These libraries are housed in architecturally unique buildings that have an important urban and visual impact and that attract users.

Table 1 presents the 24 libraries evaluated (16 proximity libraries and eight park libraries), whose location in the city of Medellin can be seen in Fig. 2. In this regard, it should be pointed out that 21 of these units are located in disadvantaged areas, which are of a lower socioeconomic level and which have traditionally suffered periods of violence and high levels of crime. This highlights their role as institutions geared towards promoting the transformation of the social fabric. Furthermore, these libraries are located throughout the city’s urban area and in the main towns in adjacent rural areas, thus allowing the greatest number of people possible to gain access to the services, regardless of their socioeconomic position.

[2] Two of the libraries (numbers 23 and 24) commenced their activity in 2015. Given the limited number of entities available, we decided not to exclude them from the sample. We estimated the records for the first full year based on the data for the period during which they were operating.

[3] It should be pointed out that the *Biblioteca Pública Piloto de Medellín para América-Latina* shares both functions in that it is charged with the task of preserving and maintaining the heritage holdings, thanks to its position as the entity that manages the Legal Depository in Colombia.

Table 1. Libraries belonging the SBPM

Foundation	Library	Commune	Strata(*)
1952	*1 – Biblioteca Pública Piloto de Medellín para América Latina*	*Laureles-Estadio*	4
1958	*2 – Biblioteca Filial San Antonio de Prado*	*Corregimiento de San Antonio de Prado*	2
1977	*3 – Biblioteca Tren de Papel Carlos Castro Saavedra*	*Castilla*	2
1980	*4 – Biblioteca San Javier La Loma*	*San Javier*	2
1985	*5 – Biblioteca La Floresta*	*La América*	4
1986	*6 – Biblioteca Juan Zuleta Ferrer*	*Aranjuez*	2
1987	*7 – Biblioteca Popular Nº 2*	*Popular*	2
1990	*8 – Biblioteca Fernando Gómez Martínez*	*Robledo*	2
1991	*9 – Biblioteca Santa Elena*	*Corregimiento de Santa Elena*	2
1994	*10 – Biblioteca Palmitas*	*Corregimiento de San Sebastián de Palmitas*	2
1994	*11 – Biblioteca Centro Occidental*	*San Javier*	2
1995	*12 – Biblioteca Santa Cruz*	*Santa Cruz*	2
1996	*13 – Biblioteca Granizal*	*Popular*	2
1998	*14 – Biblioteca El Limonar*	*Corregimiento de San Antonio de Prado*	2
2006	*15 – Parque Biblioteca Presbítero José Luis Arroyave*	*San Javier*	2
2007	*16 – Parque Biblioteca León de Greiff, La Ladera*	*Villahermosa*	2
2007	*17 – Parque Biblioteca Tomas Carrasquilla, La Quintana*	*Robledo*	2
2007	*18 – Parque Biblioteca Belén*	*Belén*	4
2008	*19 – Parque Biblioteca Fernando Botero*	*Corregimiento de San Cristóbal*	2
2011	*20 – Parque Biblioteca José Horacio Betancur*	*Corregimiento de San Antonio de Prado*	2
2011	*21 – Parque Biblioteca Manuel Mejía Vallejo*	*Guayabal*	3
2012	*22 – Parque Biblioteca Gabriel García Márquez*	*Doce de Octubre*	2
2015	*23 – Biblioteca Altavista*	*Corregimiento de Altavista*	2
2015	*24 – Biblioteca Ávila*	*Buenos Aires*	2

(*) Socioeconomic stratification of urban areas, in accordance with the National Department of Statistics in Colombia, which classifies into groups the residential dwellings that receive and have access to public services as follows: 1. Low-low, 2. Low, 3. Medium-low, 4. Medium, 5. Medium-high, and 6. High.

Source: authors' own based on SBPM statistics

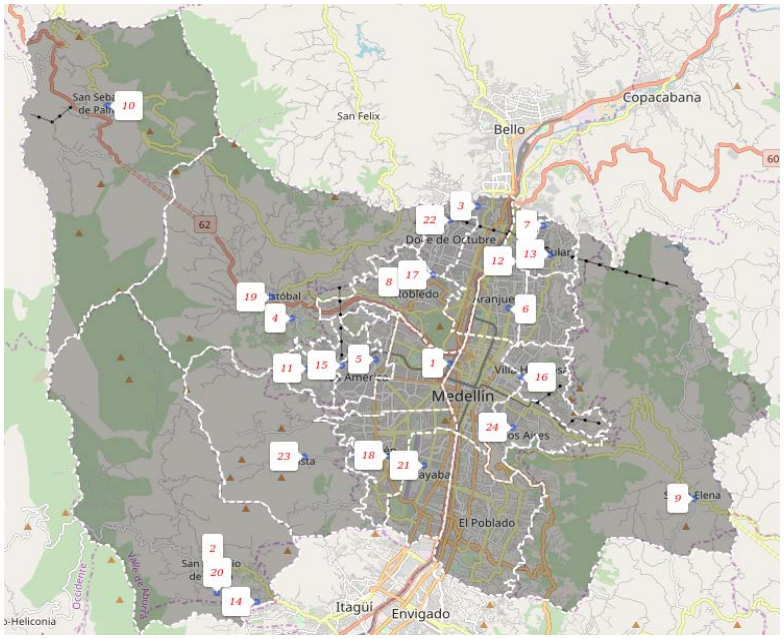

Fig. 2. Geographical location of the SBPM libraries. Source: authors' own

Taking into account these particularities, we sought to pinpoint the presence of unusual observations which might impact the frontier of best performances. Taking the works of Banker et al. [60] and Banker and Gifford [61] as a reference, we calculate the super-efficiency indices for a traditional SBM model, and find that there is no entity or group of entities that systematically obtain high values over the period analysed, such that all of the entities which make up our sample are retained.

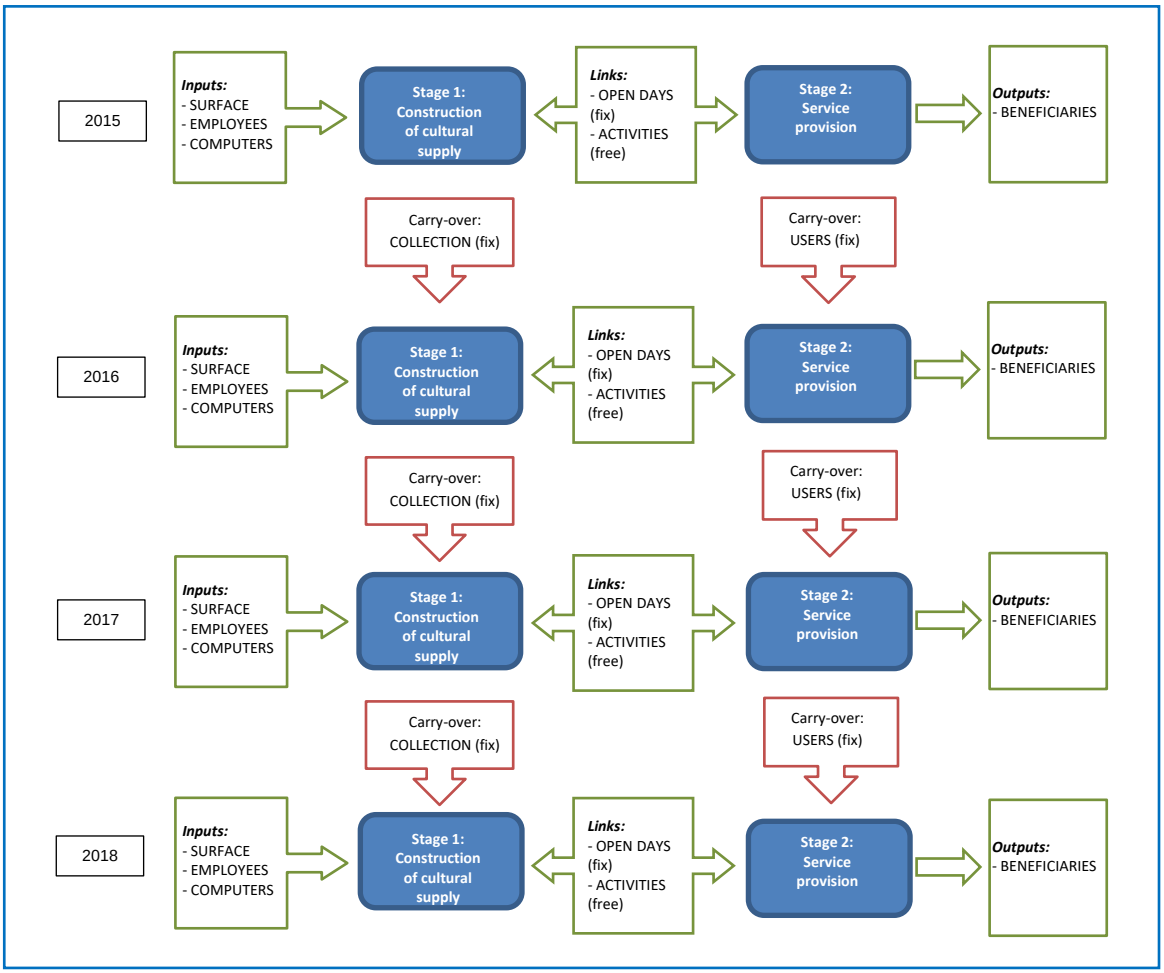

Fig. 3. Input/output structure of the performance evaluation model for libraries. Source: authors' own

Fig. 3 specifically shows the SBPM evaluation model. Library activities follow a production function design that is structured in two stages, as can be seen in Fig. 1, throughout the four-year period. The first stage deals with the group of tasks that generate the institution's cultural supply. In this activity, libraries consume labour and capital resources to make a varied range of services available to users. In our case, the variables that are representative of the inputs are the space where the activity takes place (SURFACE), the entity's staff (EMPLOYEES), computer equipment (COMPUTERS), and the collection of bibliographical material and other related material (COLLECTION). We interpret the three first variables as belonging to the category of input variables, by generating expenses that are consumed during each period (staff costs, facility maintenance costs, and amortisations through the use of facilities and equipment). Nevertheless, the variable which identifies each institution's collection represents a quasi-fixed input [46,62] since it is not consumed in a single exercise but remains, and grows over time, enabling a link to be established between the various periods for the training provided by the cultural offer. From this perspective,

we include the variables SURFACE, EMPLOYEES, and COMPUTERS as inputs for the first stage of libraries' production process, whilst COLLECTION is involved in the process as a carry-over; in other words, an intertemporal resource that can affect efficiency outcomes over time. Bearing in mind that new incorporations to the collection are determined outside each entity, we describe this variable as a fixed or non-discretional carry-over. Given these resources, entities organise and establish the cultural supply, which we represent through two variables related to the availability of this programme: number of days the libraries are open to the public (OPENDAYS), and number of activities organised[4] (ACTIVITIES). These variables act as links between the two stages of the production process by functioning as outputs in the first stage and as inputs in the second. We assume that the first of the links may be considered fixed, in that the opening hours are decided by a higher authority and cannot be handled freely by the managers. In contrast, the ACTIVITIES link belongs to the "free" category, since each library freely decides and organises the group of activities to be scheduled for each period.

The second stage of libraries' production function therefore corresponds to providing users with the service. To achieve this, each entity first has a list of registered users such that the greater the number of registered users, the greater the capacity to attract visitors and consumers to use the services. From this perspective, the register of users (USERS) is included as additional input in the service provision stage, which is also seen as carry-over since it is not consumed in each exercise but is maintained over time. It is also fixed and is non-discretional, in the sense that the entity has no capacity to take decisions with regard to its size. Finally, based on the cultural supply scheduled by the institution, and taking into account that users and other consumers will be using it, we specify the service provision, in other words the output of the activity that we will be measuring through the total number of the institution's beneficiaries during each period (BENEFICIARIES). This variable includes book loans, consultations in the various rooms, ICT users, people attending the other scheduled activities and those who use the libraries' various areas.

The descriptive statistics of the variables used are shown in Table 2. Together with the variables of the DNDEA model, we present some variables that describe the environment and which will serve as a reference to assess the degree to which they determine libraries' efficiency levels. Some are institutional in nature, such as how old the library is (YEARS), whilst others are related to the size of the area: inhabitants (POPULATION) and surface area (OUTSIDE). Finally, some of the variables are socioeconomic, such as educational attainment (EDUCATION), youth indices (YOUTH, INFANCY), and safety conditions (SAFETY). See specifications in Table 2.

Table 2. Descriptive statistics

Variables		Min.	Max.	Mean	St. Dev.
Production Function					
SURFACE	Library surface area (m^2)	56	7500	2294.39	2481.64
EMPLOYEES	Library staff (no. of employees)	1	58	12.74	12.31
COMPUTERS	No. of computers available to the public	0	79	26.25	18.25
OPENDAYS	No. of days open to the public	145	364	314.39	42.44
ACTIVITIES	No. of other library services: reading, training, and cultural activities	150	6673	1019.47	807.42
COLLECTION	No. of bibliographic materials for loan and consultation	1109	322959	29638.30	59708.84

[4] Amongst these, we distinguish between reading activities (reading clubs and workshops, bibliographical promotion), training activities (writing works, digital literacy, SBPM research) and cultural management activities (art exhibitions, performing arts shows, discussions, cultural cooperation, and so on).

Variables		Min.	Max.	Mean	St. Dev.
BENEFICIARIES	Total beneficiaries of library services (circulation and other activities)	7332	534944	114444.74	108402.38
USERS	No. of people registered as SBPM users	81	85369	9733.55	16795.38
Environmental variables					
POPULATION	Neighbourhood population where the library is located	1716.67	235184.00	51254.93	80356.42
EDUCATION	Average number of people in the commune with postgraduate studies	0.00	0.09	0.01	0.02
YOUTH	Percentage of population under 20 years old in the commune	12.50	35.01	27.59	6.41
YEARS	Years the library has been in operation since its foundation	4.00	67.00	24.00	16.57
INFANCY	No. of children under five years old per woman of child-bearing age in the commune	10.41	32.69	23.16	5.36
SAFETY	Household perception of freedom to move, and safety in the commune	1.59	2.20	1.77	0.13
OUTSIDE	Size of the commune	2.20	70.46	20.41	23.90

Source: authors' own based on SBPM and Medellin city council

5. Results

Table 3 shows the descriptive statistics of the efficiency indices estimated in accordance with the DNDEA model. The model provides an overall efficiency index for the study period (2015-2018), an efficiency index for each of the years assessed and an efficiency index for each stage in each year. The results show an overall level of efficiency for the period analysed of 70.52%, which may be deemed acceptable for the group of libraries in the system. In addition, an increasing trend may be seen over time, which could be understood as stemming from the learning effect. Only two libraries are efficient globally; in other words, for all the years in the study and in all of their activities[5], although the number of efficient entities in individual terms in each year or at each production stage has risen over time[6].

Table 3: Overall and period efficiency

		Overall Score	2015	2016	2017	2018	MI(*)
Overall	Efficiency rate	0.7052	0.5669	0.726	0.8100	0.7827	1.6496
	Max	1	1	1	1	1	
	Min	0.4077	0.3184	0.3731	0.42	0.471	
	St. Dev	0.1591	0.1935	0.196	0.1948	0.1915	
	No. of efficient libraries	2	3	6	10	8	
Stage 1	Average efficiency rate	0.7163	0.5271	0.7176	0.8134	0.8074	2.517
	Max	1	1	1	1	1	
	Min	0.2435	0.0867	0.2165	0.2398	0.2516	
	St. Dev	0.2286	0.3284	0.2654	0.2602	0.2807	
	No. of efficient libraries	4	6	9	13	13	
Stage 2	Average efficiency rate	0.7607	0.6427	0.7608	0.8369	0.8024	1.346
	Max	1	1	1	1	1	
	Min	0.2904	0.2665	0.2993	0.2754	0.3202	
	St. Dev	0.2013	0.2703	0.2312	0.2265	0.2372	
	No. of efficient libraries	4	6	9	12	11	

(*) Accumulated productivity growth by Malmquist Index for the period 2015-2018

Source: authors' own

[5] These libraries are the *Biblioteca Pública Piloto de Medellín para América Latina* and the *Biblioteca Altavista*.

[6] The results may evidence certain limitations due to the sample size and may lead to lower discriminatory power in the efficiency indices. The appendix offers various tests, with the conclusion being that the results of the research may be deemed robust and feasible.

The interest which urban public libraries in Medellin arouses amongst users is reflected through the results by stages, which show better mean efficiency values for the service provision stage (76.07%) than for the first stage that involves creating the cultural supply (71.63%). Nevertheless, the progression over time reveals that first-stage efficiency grows at a faster pace than the second, with the two rates becoming virtually equal for the final year of the study. This is borne out by the analysis of the Malmquist indices (Table 3), which confirm a mean growth of 64% in system productivity, while productivity over the period rose by 151% in the first stage and by 34% in the second. Such a significant difference in favour of the first management stage reflects the substantial effort being made to put together a cultural supply for libraries that is increasingly diverse and appealing, and is not confined solely to activities involving consultation and loaning bibliographical material.

The distribution of the efficiency indices in the library system (Fig. 4) displays very similar trends, although perhaps this is more constrained and selective in the sample of the efficient units for the overall index than for the partial indices by stage. Once again, mean efficiency levels are slightly higher in service provision than in management. Nevertheless, in order to test whether the same units are achieving the same level of performance, we estimate various rank correlation coefficients among the ordinal results of the efficiency indices (Table 4). This reveals a positive and significant correlation between the global index range and the indices in the first and second stage, which are higher in the latter, although there is no confirmed relation between the ranges of the stages. This leads us to think that the efficient units may differ in the two stages.

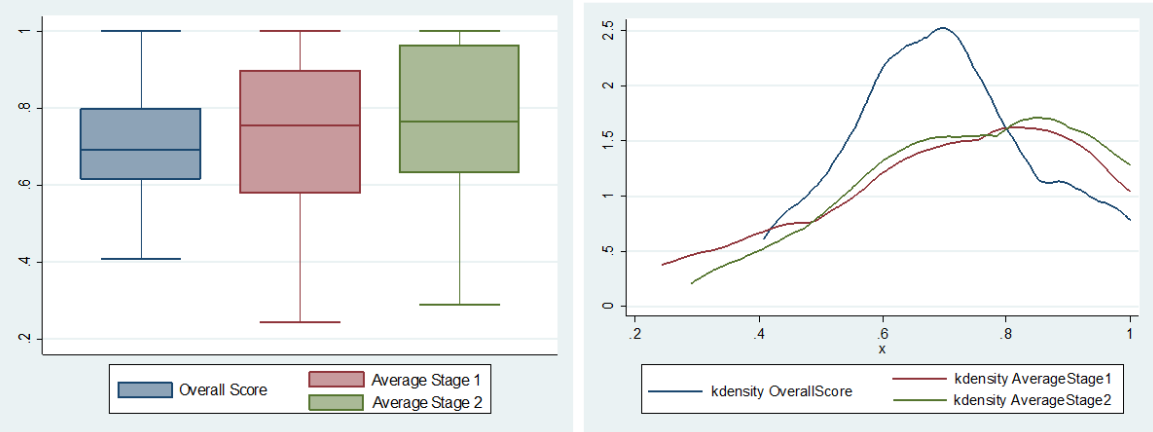

Fig. 4. Efficiency ratio distribution: boxplot analysis and density functions. Source: authors' own

Table 4. Correlation analysis of efficiency ratios

		Correlation coefficients		
		Overall Score	**Average Stage 1**	**Average Stage 2**
Pearson's coefficient	Overall Score	1.0000		
	Average Stage 1	0.5184***	1.0000	
	Average Stage 2	0.7906***	-0.0846	1.000
Spearman's Rho	Overall Score	1.0000		
	Average Stage 1	0.5404***	1.0000	
	Average Stage 2	0.7535***	-0.0162	1.000
Kendall's tau	Overall Score	1.0000		
	Average Stage 1	0.3743	1.0000	
	Average Stage 2	0.6019***	-0.0185	1.000

* p-value < 0.1; ** p-value < 0.05; *** p-value < 0.01
Source: authors' own

Indeed, this is related to the two models of facilities pinpointed in our sample (see Section 4.2): proximity libraries and so-called park libraries. Although they all share aims and resources, the former exhibit a more conventional format vis-à-vis their spaces and respond more faithfully to the classical notion of what a library is; whilst the latter, which tend to have been opened more recently, display greater visibility thanks to a more modern and avant-garde architectural style, added to which they also tend to offer a wider range of activities. As a result, testing whether there are differences in terms of the performance of the two kinds of institution would seem appropriate. Table 5 shows the mean efficiency values in each of these categories. As can be seen, there are no major differences between the two models as regards mean overall efficiency, although differences do emerge in the values of each of the stages into which these entities' production function is structured: traditional libraries evidence substantially better performance in the management stage, whereas park libraries prove to be more efficient when providing the service and when attracting users.

We used a Student t test of equal means (Table 5) to ascertain whether there were any significant differences in the efficiency results for the two kinds of library. We found that there was no substantial evidence to distinguish between the two in terms of overall performance, although differences did emerge for the ratios at each production stage. This once again bears out the idea that the group of efficient units is different in each activity, and that proximity libraries are more efficient at the management stage, probably because they specialise in compiling and loaning, whereas the park libraries are more efficient when it comes to attracting users, which is almost certainly due to their attractive facilities, but also because they strive to offer a more diversified and appealing cultural offer. This finding underpins libraries' tendency to emerge as new centres of leisure for citizens. In addition to the activities found in the classical library model as places which house and preserve different types of cultural material (books, journals music, films), they now offer other cultural activities plus a wide array of entertainment that is closely linked to the immediate environment, and are places which are committed to technological progress and cater to a wide range of interests.

Table 5. Average efficiency values for Proximity Libraries and Park Libraries

	Proximity Libraries	***Park Libraries***	***T Test (p < 0.05)***
Overall efficiency			
Mean	0.70	0.72	0.7664
Variance	0.03	0.02	
Average efficiency stage 1			
Mean	0.82	0.51	0.0003**
Variance	0.02	0.05	
Average efficiency stage 2			
Mean	0.68	0.92	0.0032**
Variance	0.04	0.01	

* p-value < 0.1; ** p-value < 0.05; *** p-value < 0.01
Source: authors' own

Fig. 5 shows the location of the libraries that display the best practices over the period analysed. The first striking result is the substantial increase in globally efficient units, which is found to be true for both kinds of specialisation and which is spread over a much wider urban spectrum. It is also possible to see how other entities, all of which are proximity libraries, achieve optimal results in the management stage, whereas others achieve optimal results when attracting users, with all of these basically being park libraries.

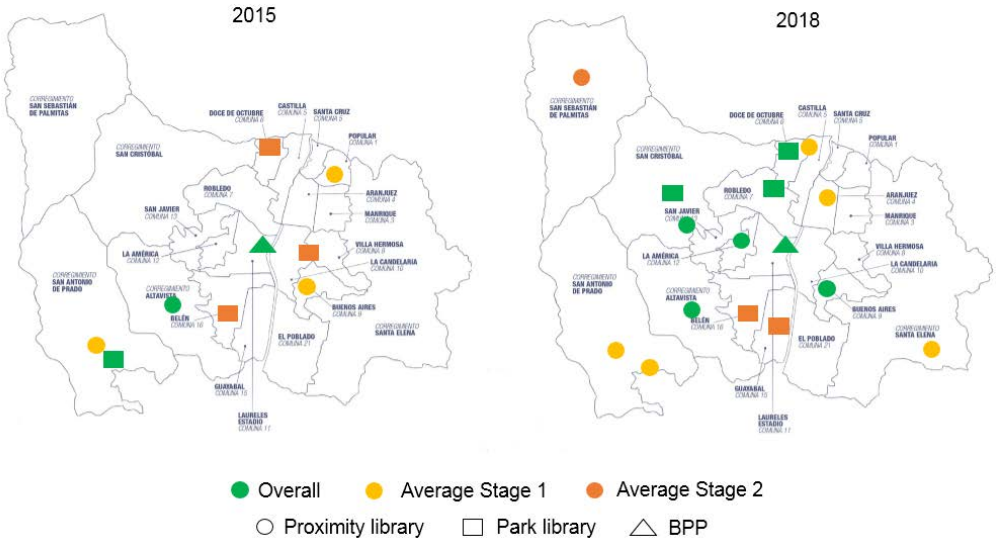

Fig. 5. Evolution of best practice libraries and location. Source: authors' own

To conclude, we sought to pinpoint some variables that are representative of the environment in which libraries conduct their activities in an effort to gauge whether the efficiency outcomes might be shaped by such a context. As pointed out in section 4.2, the variables used aim to describe the size of the location (surface area and population of the commune), the socioeconomic features (population with postgraduate studies, different youth indices and conditions of safety and freedom), as well as certain institutional indicators such as how old the libraries are. Following Simar and Wilson [16], we posit a truncated bootstrap regression (algorithm 1) taking the overall efficiency outcomes and efficiency by stages of the DNDEA model as a reference. This model is relevant because it allows us to account for serial correlation between the DEA-based efficiency scores, and it has been widely applied in second-stage regression models to estimate the influence of external factors on the efficiency ratios of several institutions [23,26,63]. The appropriateness of the method depends also on the compliance with the condition of separability between the inputs-outputs space and the contextual variables although, as already pointed out, given the sample size it is virtually impossible to reject non-separability. Nonetheless, in order to reinforce the robustness of the results, other econometric models have also been applied that prove to be appropriate to the case study of a dependent variable bounded in an interval between 0 and 1, such as the efficiency ratios [58,59]. In this vein, Tobit and FLR estimation models are applied, the results of which are presented in Table 6[7]. Models are statistically consistent and the values obtained confirm the robustness of the results since they show no significant differences, in addition to which all the signs of the regressors remain stable, such that some results even reinforce the explanatory meaning of certain variables.

Overall, the results show that the contextual variables have a significant impact on second-stage efficiency whilst, in contrast, they have hardly effect on the first. This is consistent with our initial approach when we pointed out that the first stage of the process is controlled by the entity, whilst the second takes place in conjunction with the users. As regards the variables that describe the population, we find a significant and

[7] Different previous tests have also been carried out to find the most suitable and explanatory model. Several iterations have been developed, testing and removing possible multicollinearity problems, and the usual validation tests have been performed. The coefficients obtained in the various tests are stable, thereby evidencing the robustness of our estimates. The estimations performed were carried out using *Stata* software.

positive relation between the presence of a young population, who display high levels of educational attainment, and efficiency in attracting visitors. In the same vein, Horrigan [3] finds that the highest rate of users in libraries is to be found among young well-qualified people, while Japzon and Gong [64] and Glorieux et al. [65] report a positive relation between library use and education. In a similar vein, van Eijck and Bargeman [66] report a higher level of cultural participation linked to high levels of education. In contrast, the results show an inverse relation between efficiency in attracting visitors and the presence of a population under the age of five. As regards the features of the communes of each library, the highest efficiency levels when it comes to attracting users are positively related to the communes with the largest population and the best perception of safety, and are negatively related to those which are largest in terms of surface area. These results are consistent with some previous studies [64] which report greater levels of demand for the service per user when libraries cater to smaller areas with better spatial accessibility. Finally, also evident is a significant relation between the age of the library and the efficiency results, although in the opposite sense depending on the stage taken as a reference. Data show positive results from experience in terms of creating cultural offer, whilst in contrast, the older libraries experience greater difficulties in attracting a higher number of visitors. One explanation for this lies in the fact that park libraries were founded more recently and, as has previously been evidenced, prove to be more efficient at attracting users given their capacity and the activities they engage in.

Table 6. Regression analysis between efficiency rates and external variables

Variable	Overall score			Score stage 1			Score stage 2		
	SW	Tobit	FLR	SW	Tobit	FLR	SW	Tobit	FLR
POPULATION	0.227 (0.112)**	0.229 (0.106)**	1.227 (0.503)**	0.329 (0.168)*	0.164 (0.206)	1.390 (0.722)*	0.286 (0.147)**	0.221 (0.135)	1.807 (0.861)**
EDUCATION	0.644 (0.319)**	0.844 (0.169)***	5.056 (1.074)***	0.659 (0.422)	0.396 (0.320)	3.972 (1.834)**	0.829 (0.447)**	0.878 (0.213)***	5.892 (1.96)***
YOUTH	1.034 (0.482)**	1.157 (0.434)**	5.957 (2.242)***	0.778 (0.627)	1.173 (0.840)	5.898 (2.895)**	1.983 (0.643)***	1.082 (0.550)**	8.496 (3.584)**
YEARS	-0.220 (0.098)**	-0.238 (0.087)**	-1.069 (0.468)**	0.977 (0.185)***	0.404 (0.166)**	2.753 (1.035)***	-0.625 (0.135)***	-0.536 (0.109)***	-3.408 (0.686)***
INFANCY	-0.782 (0.515)	-0.796 (0.473)	-4.069 (2.440)*	-0.281 (0.679)	-0.835 (0.923)	-3.632 (3.206)	-1.783 (0.649)***	-0.833 (0.602)	-7.046 (3.684)*
SAFETY	0.355 (0.171)**	0.391 (0.162)**	1.873 (0.705)***	0.194 (0.230)	0.304 (0.312)	1.917 (1.320)	0.454 (0.208)**	0.361 (0.205)*	2.194 (0.901)**
OUTSIDE	-0.360 (0.203)*	-0.388 (0.189)*	-1.990 (0.759)***	-0.238 (0.274)	-0.382 (0.368)	-2.235 (1.308)*	-0.577 (0.249)**	-0.354 (0.240)	-2.752 (1.060)***
Constant	0.372 (0.158)**	0.273 (0.121)**	-1.352 (0.544)**	-0.238 (0.274)	0.195 (0.228)	-2.461 (1.161)**	0.505 (0.198)**	0.492 (0.151)***	-0.264 (0,691)
Chi Wald squared	22.34***	27.16***	48.53***	34.89***	11.59	53.02***	34.25***	26.55***	75.21***
Sigma	0.094 (0.014)***	0.092 (0.013)***		0.122 (0.020)***	0.179 (0.026)***		0.105 (0.017)***	0.117 (0.017)***	
Pseudo R2		-1.7163	0.0718		11.8967	0.1006		-5.6827	0.1479

* p-value < 0.1; ** p-value < 0.05; *** p-value < 0.01. Standard errors are shown in parentheses. Regression models: SM, Simar-Wilson model (5,000 bootstrap replicas); Tobit regression model; FLR, Fractional logic regression model

Source: authors' own

6. Conclusions

Our work posits a proposal to assess the performance of a group of urban public libraries in the city of Medellin, Colombia, over a period of four years (2015-2018). Previous studies into efficiency evaluation in cultural institutions have on some occasions included a time dimension. Yet, up until now, in no cases have they considered the possibility of dependence interrelations between inputs and outputs over time. The main contribution our work makes is to apply a DNDEA model to evaluate performance in cultural institutions where, in addition to considering links between the different activities undertaken by the institution with interrelated inputs, we also include the possible relations between periods. Determining these time relations stems from the distinction between inputs consumed over the period, and others that project their effects beyond the study period, behaving as quasi-fixed capital that can be increased over time, and thus helping to obtain the output dynamically. This is the case of the system of libraries studied herein, where input variables such as the bibliographical collection or the register of users cannot be treated in the same way as other resources consumed in each period, such as staff or equipment costs since, whereas the latter represent an entry flow that is renewed each year, the former remain over time, and their maintenance or growth is linked to the institution's very *raison d'être*. Ignoring such a difference between inputs when evaluating the service provision process might lead to obtaining biased outcomes.

The results of our study offer acceptable levels of overall efficiency when considering all of the activities and periods evaluated. The development over time points to a growing trend in the efficiency indices, in an effect that be seen as sustained learning. The results in the two stages into which we structure the activity in the entities (creation of the cultural supply under management control and service provision vis-à-vis attracting beneficiaries) seem to evidence in general terms that the second service provision function is slightly more efficient than the first, which is the mere provision thereof. Nevertheless, there is a highly significant rise in productivity in management activities over the period analysed, which reflects the substantial efforts made by managers to put together a cultural offer for libraries that is increasingly diverse and attractive.

Although all of the libraries in the SBPM apply the same management criteria, it is possible to identify two kinds of institution that display different behaviours. On the one hand, there are libraries which reflect the more traditional concept as places where collections are housed and made available to the public, and which specialise in activities mostly related to promoting reading. On the other hand, there is another group of units that are more closely akin to the concept of a cultural centre where, in addition to the usual functions that libraries engage in, meeting and leisure areas are made available where training activities, artistic events and cooperation with cultural agents from the surrounding area can take place. The results of our study evidence that the latter kind, the park libraries, are more efficient at attracting the public when compared to the proximity libraries which, although they have the capacity to provide basic services at a lower cost, their infrastructure and spaces are limited such that they are less able to attract users.

We have also sought to test whether efficiency outcomes are shaped by the environment in which libraries carry out their activities. The results show that it is precisely in the service provision stage where dependence relations between efficiency and external variables emerge, whilst the cultural supply creation stage generally seems to remain unaffected by such effects. Consistent with some previous studies, we find a positive relation between the number of inhabitants in the communes and their safety conditions and the level of efficiency in attracting users. In contrast, a library serving a larger area is found to have a negative effect on efficiency due to the difficulties involved in providing the service to a large number of people. As regards

population characteristics, results show greater levels of efficiency in the second stage linked to the presence of a young and highly educated population, whilst the effect is the opposite when there are more youngsters under the age of five. Finally, the institution's age is seen to have contradictory effects on performance. Although experience seems to improve the outcomes from the first management stage, it triggers the opposite effect when it comes to service provision, where public preference for new and more modern facilities that are adapted to new demands is evident.

The results of this research are of interest not only because of their usefulness for library managers and those responsible for local cultural policy but also because they show how institutions such as libraries have shifted towards performing functions that are becoming increasingly diversified, and towards providing amenities that contribute to social and urban change. The methodological contribution is also innovative in the scientific field of evaluating cultural institution efficiency, and may give rise to other comparative studies, particularly with regard to less developed countries, where applications of this kind remain scarce.

Our study is subject to certain limitations which in turn also offer opportunities for future research. The total number of libraries is a methodological restriction for the dynamic analysis, because considering several years means the number of variables increases in the evaluation model and that discriminatory power is thus lost. This is why the Appendix provides a comparative analysis of the black-box, NDEA, and DNDEA models in order to test the robustness of the results. In addition, when examining the impact of the contextual variables, it would be advisable to explore and obtain data for smaller geographical units such as districts, which would allow for a more accurate analysis of the determinants of efficiency in libraries. Likewise, it would also prove interesting to conduct satisfaction surveys or public evaluation questionnaires in the surrounding area in order to compare efficiency outcomes. Finally, changes in the profile of libraries from traditional to cultural centres might lead us to consider a faster obsolescence of library assets, also driven by a change in demand. However, our results reflect significant increases in productivity in the first stage of management, where those responsible are striving to build a cultural supply that reaches beyond the traditional activities of loan and maintenance of the reading collection. It would, however, be interesting to ask ourselves at what cost this is achieved, such that fresh lines of enquiry should posit the application of allocative efficiency models.

APPENDIX: Robustness tests on the results

The model put forward to evaluate the SBPM, and which has eight relevant variables in the production function and a relatively limited sample size (24 libraries), might suggest the existence of certain problems related to the discriminatory power of the results. Some authors [66-69] who have studied the problem of dimensionality associated to DEA models point to a loss of discriminatory power as the number of variables increases. In the case of conventional models, the generally accepted rule is that the number of observations should exceed three times the total number of inputs and outputs [69]. Nevertheless, Kao [68] points out that NDEA models exhibit greater discriminatory power than conventional models because different sub-units are created when breaking down the production process into different stages, which might be considered as expanding the sample. In the case of DNDEA models, Avkiran [69] proposes applying an equivalent rule to that of the black-box models in order to achieve an acceptable level of discrimination. Given the impossibility of expanding our sample, we posit two further tests that allow us to evaluate the robustness of the outcomes.

Firstly, and following Avkiran [69], we calculate the efficiency indices for the conventional SBM model (black box) which does not take into account either the links between stages or the links between periods (carry-over). We also calculate the efficiency indices with an NDEA model for each year, where the links between activities are taken into consideration but where the links between time periods are not. In both cases, we opted to include the variable COLLECTIONS as primary input of the production function. In doing so, our aim is to test whether the entities at the frontier remain the same despite the changes in the models' discriminatory power. Table A1 shows the comparative results of the three models. As can be seen, the discriminatory power of efficient units based on NDEA is greater than that of DNDEA and black-box models. Nevertheless, the units located at the frontier remain the same when changing the model, and increase in number as the discriminatory power decreases, thus strengthening the stability of the results. NDEA efficiency ratios over the period 2015-2018 evidence a certain degree of stability, unlike in the DNDEA model where there is a clear increase in the efficiency indices. This difference might point to the existence of a certain bias in the static models to study the efficiency dynamic, by failing to take into account the difference between inputs consumed in the exercise and those which, far from being consumed, remain over time in the form of permanent capital, as occurs with the bibliographical collection and the register of users in the case of libraries.

Secondly, we have sought to test the stability of the results by positing different DNDEA-SBM models in which we exclude one of the inputs in each case. We find that no new frontiers are generated, with new efficient entities in each new model [70,69]. In our case, the two optimal entities remain at the frontier of efficient performance in all cases. Moreover, the correlation coefficients between the new efficiency indices obtained after each extraction and the efficiency index of the original model are above 0.98 in all cases.

Table A1. Results comparison from SBM, NDEA, and DNDEA models

Models		2015	2016	2017	2018
SBM	Average	0.6729	0.6052	0.7066	0.704
	Max	1	1	1	1
	Min	0.282	0.2158	0.3349	0.36
	Efficient libraries	1,2,14,15,18,19,23,24	2,5,15,18,24	1,5,7,18,19, 24	1,2,5,18,19,22,24
NDEA	Average	0.5156	0.4424	0.5005	0.491
	Max	1	1	1	1
	Min	0.2141	0.097	0.1756	0.1566
	Efficient libraries	1,20,23	23,24	2,18,19,23	19,23,24
DNDEA	Average	0.5669	0.726	0.81	0.7827
	Max	1	1	1	1
	Min	0.3184	0.3731	0.42	0.471
	Efficient libraries	1,20,23	1,2,19,20,23,24	1,2,7,10,11,18,19,20, 23,24	1,4,5,17,19,22,23,24

Source: authors' own

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References

[1] Fernández-Blanco, V., Herrero-Prieto, L., and Prieto-Rodríguez, J. (2013). Performance of cultural heritage institutions, In: I. Rizzo and A. Mignosa, eds. *Handbook on economics of cultural heritage*. Cheltenham: Edward Elgar Publishing Ltd, 470-490.

[2] Bernardo, M., de Souza, M., Moreira, R. and Rodrigues, L. (2020). University library performance management: applying zero-sum gains DEA models to resource allocation. *Socio-Economic Planning Sciences*. https://doi.org/10.1016/j.seps.2020.100808

[3] Horrigan, J. (2015). "Libraries at the Crossroads", Pew Research Center, Available at: http://www.pewinternet.org/2015/09/15/2015/Libraries-at-crossroads/

[4] Aabø, S. (2005). The role and value of public libraries in the age of digital technologies. *Journal of Librarianship and Information Science,* 37(4), 205-211. https://doi.org/10.1177%2F0961000605057855

[5] Fujiwara, D., Lawton, R. and Mourato, S. (2019). More than a good book: contingent valuation of public library services in England. *Journal of Cultural Economics*, 43(4), 639-666. https://doi.org/10.1007/s10824-019-09369-w

[6] Guajardo, S. (2018). Special District Libraries and Operating Costs: An Application of Data Envelopment Analysis (DEA) with Discretionary and Non-Discretionary Inputs. *Journal of Library Administration*, 58(3), 241-263. https://doi.org/10.1080/01930826.2018.1436758

[7] Bradford, D., Malt, R. and Oates, W. (1965). The rising cost of local public services: some evidence and reflections. *National Tax Journal*, 22(2), 185-202.

[8] De Bruijn, H. (2002). Performance measurement in the public sector: strategies to cope with the risk of performance measurement. *International Journal of Public Sector Management*, 15(7), 578-594. https://doi.org/10.1108/09513550210448607

[9] De Witte, K. and Geys, B. (2011). Evaluating efficient public good provision: Theory and evidence from a generalised conditional efficiency model for public libraries. *Journal of Urban Economics*, 69(3), 319-327. https://doi.org/10.1016/j.jue.2010.12.002

[10] Førsund, F. (2017). Measuring effectiveness of production in the public sector. *Omega,* 73, 93-103. https://doi.org/10.1016/j.omega.2016.12.007

[11] Gómez-Zapata, J., Herrero-Prieto, L. and Rodríguez-Prado, B. (2020). Does music soothe the soul? Evaluating the impact of a music training programme in Medellin, Colombia. *Journal of Cultural Economics*. https://doi.org/10.1007/s10824-020-09387-z

[12] Peña-Gallego, L. (2011). Las bibliotecas públicas de Medellín como motor de cambio social y urbano de la ciudad. *BiD: textos universitaris de biblioteconomia i documentació,* 27. https://dx.doi.org/10.1344/105.000001798

[13] Espinal-Monsalve, N. and Gómez-Zapata, J. (2019). La cultura como elemento de transformación social y urbana de la ciudad de Medellín – Colombia. *2nd Spanish-Chinese Seminar on Cultural Economics*, Shanghai (2018) Mimeo, and *Commentary on Cultural Industry in China, 27*, 226-246 (Chinese version).

[14] Gómez-Zapata, J., Espinal-Monsalve, N. and Herrero-Prieto, L. (2018). Economic valuation of museums as public club goods: Why build loyalty in cultural heritage consumption?. *Journal of Cultural Heritage,* 30, 190-198. https://doi.org/10.1016/j.culher.2017.09.010

[15] Tone, K. and Tsutsui, M. (2014). Dynamic DEA with network structure: A slacks-based measure approach. *Omega*, 42(1), 124-131. https://doi.org/10.1016/j.omega.2013.04.002

[16] Simar, L. and Wilson, P. (2007). Estimation and inference in two-stage, semiparametric models of production processes. *Journal of Econometrics*, 136(1), 31-64. https://doi.org/10.1016/j.jeconom.2005.07.009

[17] Emrouznejad, A. and Yang, G. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978-2016. *Socio-Economic Planning Sciences*, 61, 4-8. https://doi.org/10.1016/j.seps.2017.01.008

[18] Pignataro, G. (2002). Measuring the efficiency of museums: a case study in Sicily, In: I. Rizzo and R. Towse, eds. *The economics of heritage. a study in the political economy of culture in Sicily*. Cheltenham: Edward Elgar, 65-78.

[19] Del Barrio-Tellado, M., Herrero-Prieto, L. and Sanz, J. (2009). Measuring the efficiency of heritage institutions: a case study of a regional system of museums in Spain. *Journal of Cultural Heritage,* 10(2), 258-268. https://doi.org/10.1016/j.culher.2008.08.012

[20] Taheri, H. and Ansari. S. (2013). Measuring the relative efficiency of cultural-historical museums in Tehran: DEA approach. *Journal of Cultural Heritage*, 14(5), 431-438. https://doi.org/10.1016/j.culher.2012.10.006

[21] Mairesse, F. and Vanden-Eeckaut, P. (2002). Museum assessment and FDH technology: Towards a Global Approach. *Journal of Cultural Economics*, 26(4), 261-286. https://doi.org/10.1023/A:1019970325060

[22] Del Barrio-Tellado, M. and Herrero-Prieto, L. (2014). Evaluating the efficiency of museums using multiple outputs: evidence from a regional system of museums in Spain. *International Journal of Cultural Policy*, 20(2), 221-238. https://doi.org/10.1080/10286632.2013.764290

[23] Del Barrio-Tellado, M. and Herrero-Prieto, L. (2019). Modelling museum efficiency in producing inter-reliant outputs. *Journal of Cultural Economics,* 43(3), 485-512. https://doi.org/10.1007%2Fs10824-019-09347-2

[24] Guccio, C., Martorana, M., Mazza, I., Pignataro, G. and Rizzo, I. (2020). An assessment of the performance of Italian public historical archives: Preservation

vs utilization. *Journal of Policy Modelling*. https://doi.org/10.1016/j.jpolmod.2019.12.006

[25] Del Barrio-Tellado, M., Herrero-Prieto, L. and Murray, C. (2020). Audience success or art for art's sake? Efficiency evaluation of dance companies in the United States. *Nonprofit Management and Leadership*. https://doi.org/10.1002/nml.21411

[26] Guccio, C., Martorana, M., Mazza, I. and Rizzo, I. (2020). Back to the Future: Does the use of information and communication technology enhance the performance of public historical archives?. *Journal of Cultural Economics*. https://doi.org/10.1007/s10824-020-09385-1

[27] Finocchiaro-Castro, M., Guccio, C. and Rizzo, I. (2011). Public intervention on heritage conservation and determinants of heritage authorities' performance: a semi-parametric analysis. *International Tax and Public Finance*, 18(1), 1-16. https://doi.org/10.1007/s10797-010-9132-1

[28] Chen, T. (1997). A measurement of the resource utilization efficiency of university libraries. *International Journal of Production Economics*, 53(1), 71-80. https://doi.org/10.1016/S0925-5273(97)00102-3

[29] Vitaliano, D. (1998). Assessing Public Library Efficiency Using Data Envelopment Analysis. *Annals of Public and Cooperative Economics,* 69(1), 107-122. https://doi.org/10.1111/1467-8292.00075

[30] Worthington, A. (1999). Performance Indicators and Efficiency Measurement in Public Libraries. *Australian Economic Review*, 32(1), 31-42. https://doi.org/10.1111/1467-8462.00091

[31] Reichmann, G. and Sommersguter-Reichmann, M. (2006). University library benchmarking: An international comparison using DEA. *International Journal of Production Economics*, 100(1), 131-147. https://doi.org/10.1016/j.ijpe.2004.10.007

[32] Srakar, A., Kodrič-Dačić, E., Koman, K. and Kavaš, D. (2017). Efficiency of Slovenian Public General Libraries: A Data Envelopment Analysis Approach. *Lex localis – Journal of Local Self-Government*, 15(3), 559-581. https://doi.org/10.4335/15.3.559-581(2017)

[33] Hammond, C. (2002). Efficiency in the provision of public services: a data envelopment analysis of UK public library systems. *Applied Economics*, 34(5), 649-657. https://doi.org/10.1080/00036840110053252

[34] Simon, J., Simon, C. and Arias, A. (2011) Changes in productivity of Spanish university libraries. *Omega*, 39(5), 578-588. https://doi.org/10.1016/j.omega.2010.12.003

[35] Guccio, C., Mignosa, A. and Rizzo, I. (2018). Are public state libraries efficient? An empirical assessment using network Data Envelopment Analysis. *Socio-Economic Planning Sciences*, 64, 78-91. https://doi.org/10.1016/j.seps.2018.01.001

[36] Liang, L., Cook, W. and Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics*, 55(7), 643-653. https://doi.org/10.1002/nav.20308

[37] Miidla, P. and Kikas, K. (2009). The efficiency of Estonian central public libraries. *Performance Measurement and Metrics*, 10(1), 49-58. https://doi.org/10.1108/14678040910949684

[38] De Carvalho, F., Jorge, M., Filgueiras, M., Russo, M. and Oliveira de Sá, N. (2012), Library performance management in Rio de Janeiro, Brazil: Applying DEA to a sample of university libraries in 2006-2007. *Library Management*, 33(4-5), 297-306. https://doi.org/10.1108/01435121211242335

[39] Stroobants, J. and Bouckaert, G. (2014). Benchmarking local public libraries using non-parametric frontier methods: A case study of Flanders. *Library & Information Science Research*, 36(3-4), 211-224. http://dx.doi.org/10.1016/j.lisr.2014.06.002

[40] Emrouznejad, A. and Thanassoulis, E. (2005). A mathematical model for dynamic efficiency using data envelopment analysis. *Applied Mathematics and Computation*, 160(2), 363-378. https://doi.org/10.1016/j.amc.2003.09.026

[41] Vrabková, I. (2018). Models of static and dynamic technical efficiency of municipal libraries in the Czech Republic. *Socio-Economic Planning Sciences*, 68, 100646. https://doi.org/10.1016/j.seps.2018.09.001

[42] De Witte, K. and Geys, B. (2013). Citizen coproduction and efficient public good provision: Theory and evidence from local public libraries. *European Journal of Operational Research*, 224(3), 592-602. https://doi.org/10.1016/j.ejor.2012.09.002

[43] Daraio, C. and Simar, L. (2005). Introducing Environmental Variables in Nonparametric Frontier Models: a Probabilistic Approach. *Journal of Productivity Analysis*, 24(1), 93-121. https://doi.org/10.1007/s11123-005-3042-8

[44] Banker, R. and Natarajan, R. (2008). Evaluating Contextual Variables Affecting Productivity Using Data Envelopment Analysis. *Operations Research, 56*(1), 48-58. https://doi.org/10.1287/opre.1070.0460

[45] Sengupta, J. (1996). Dynamic data envelopment analysis. *International Journal of Systems Science*, 27(3), 277-284. https://doi.org/10.1080/00207729608929214

[46] Nemoto, J. and Goto, M. (1999). Dynamic data envelopment analysis: modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economics Letters*, 64(1), 51-56. https://doi.org/10.1016/S0165-1765(99)00070-1

[47] Bogetoft, P., Färe, R., Grosskopf, S., Hayes, K. and Taylor, L. (2009). Dynamic network DEA: An illustration. *Journal of the Operations Research Society of Japan*, 52(2), 147-162. https://doi.org/10.15807/jorsj.52.147

[48] Chen, C. (2009). A network-DEA model with new efficiency measures to incorporate the dynamic effect in production networks. *European Journal of Operational Research*, 194(3), 687-699. https://doi.org/10.1016/j.ejor.2007.12.025

[49] Tone, K. and Tsutsui M. (2010). Dynamic DEA: A slacks-based measure approach. *Omega*, 38(3-4), 145-156. https://doi.org/10.1016/j.omega.2009.07.003

[50] Kao, C. (2013). Dynamic data envelopment analysis: A relational analysis. *European Journal of Operational Research*, 227(2), 325-330, https://doi.org/10.1016/j.ejor.2012.12.012

[51] Kawaguchi, H., Tone, K. and Tsutsui, M. (2014). Estimation of the efficiency of Japanese hospitals using a dynamic and network data envelopment analysis model. *Health Care Management Science*, 17(2), 101-112, https://doi.org/10.1007/s10729-013-9248-9

[52] Fukuyama, H. and Weber, W. (2015). Measuring Japanese bank performance: a dynamic network DEA approach. *Journal of Productivity Analysis*, 44(3), 249-264. https://doi.org/10.1007/s11123-014-0403-1

[53] Villa, G. and Lozano, S. (2018). Dynamic Network DEA approach to basketball games efficiency. *Journal of the Operational Research Society*, 69(11), 1738-1750. https://doi.org/10.1080/01605682.2017.1409158

[54] Moreno, P. and Lozano, S. (2018). Super SBI Dynamic Network DEA approach to measuring efficiency in the provision of public services. *International Transactions in Operational Research*, 25(2), 715-735. https://doi.org/10.1111/itor.12257

[55] Reig-Martínez, E., Gómez-Limón, J. and Picazo-Tadeo, A. (2011). Ranking farms with a composite indicator of sustainability. *Agricultural Economics*, 42(5), 561-575. https://doi.org/10.1111/j.1574-0862.2011.00536.x

[56] Simar, L. and Wilson, P. (2011). Two-stage DEA: caveat emptor. *Journal of Productivity Analysis*, 36(2), 205-218. https://doi.org/10.1007/s11123-011-0230-6

[57] Daraio, C., Simar, L. and Wilson, P. (2018). Central limit theorems for conditional efficiency measures and tests of the 'separability' condition in non-parametric, two-stage models of production. *The Econometrics Journal*, 21(2), 170-191. https://doi.org/10.1111/ectj.12103

[58] Ramalho, E., Ramalho, J. and Henriques, P. (2010) Fractional regression models for second stage DEA efficiency analyses. *Journal of Productivity Analysis*, 34, 239-255. https://doi.org/10.1007/s11123-010-0184-0

[59] Hayashi, M. (2017). Using DEA to analyze the efficiency of welfare offices and influencing factors: the case of Japan's municipal public assistance programs, In K. Tone, ed. *Advances in DEA Theory and Applications: With Extensions to Forecasting Models*. Hoboken: John Wiley & Sons, 300-314. https://doi.org/10.1002/9781118946688.ch21

[60] Banker, R., Das, S. and Datar, S. (1989). Analysis of cost variances for management control in hospitals. *Res Governmental Nonprofit Account,* 5, 269-291.

[61] Banker, R. and Gifford, J. (1988).*A relative efficiency model for the evaluation of public health nurse productivity*. Pittsburgh: Mimeo, Carnegie Mellon University.

[62] Fallah-Fini, S., Triantis, K. and Johnson, A. (2014). Reviewing the literature on non-parametric dynamic efficiency measurement: state-of-the-art. *Journal of Productivity Analysis*, 41(1), 51-67. https://doi.org/10.1007/s11123-013-0349-8

[63] Finocchiaro Castro, M. and Guccio, C. (2014). Searching for the source of technical inefficiency in Italian judicial districts: an empirical investigation. *European Journal of Law and Economics*, 38, 369-391. https://doi.org/10.1007/s10657-012-9329-0

[64] Japzon, A. and Gong, H. (2005). A Neighborhood Analysis of Public Library Use in New York City. *Library Quarterly*, 75(4), 446-463. https://doi.org/10.1086/502786

[65] Glorieux, I., Kuppens, T. and Vandebroeck, D. (2007). Mind the gap: Societal limits to public library effectiveness, *Library & Information Science Research*, 29(2), 188-208. https://doi.org/10.1016/j.lisr.2007.03.003

[66] van Eijck, K. and Bargeman, B. (2004). The changing impact of social background on lifestyle: "Culturalization" instead of individualization?. *Poetics*, 32, 447-469. https://doi.org/10.1016/j.poetic.2004.09.001

[67] Dyson, R., Allen, R., Camanho, A., Podinovski, V., Sarrico, C. and Shale, E. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research*, 132(2), 245-259. https://doi.org/10.1016/S0377-2217(00)00149-1

[68] Kao, C. (2009). Efficiency decomposition in network data envelopment analysis: A relational model. *European Journal of Operational Research*, 192(3), 949-962. https://doi.org/10.1016/j.ejor.2007.10.008

[69] Avkiran, N. (2015). An illustration of dynamic network DEA in commercial banking including robustness tests. *Omega*, 55, 141-150. https://doi.org/10.1016/j.omega.2014.07.002

[70] Avkiran, N. (2007). Stability and integrity tests in Data Envelopment Analysis. *Socio-Economic Planning Sciences*, 41(3), 224-234. https://doi.org/10.1016/j.seps.2006.06.001