H_∞ Observer-Based Control for Uncertain Fuzzy Systems with Application of the Quadruple-Tank Process

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***Abstract*— This paper considers the problem of designing an H_∞ observer-based controllers for continuous nonlinear systems presented by Takagi–Sugeno (T–S) model with the presence of parameter uncertainties and external disturbance. Some change of variables has been developed to linearize the bilinear terms. As a consequence, the bilinear problem conditions are converted into a set of Linear Matrix Inequalities (LMIs). Sufficient c onditions f or d esign t he o bserver a nd controller gains are deduced in terms of LMIs conditions which can be practically solved in single step. The four-tank process application is used to show the effectiveness of the proposed method, revealing a better compromise between the simplicity and the conservatism of design method, outperforming in respect to previous approaches.**

***Index Terms*— Observer-Based Control, Fuzzy Model, Uncertain Systems, Disturbance, Linear Matrix Inequality (LMIs), Quadruple Tank Process.**

I. INTRODUCTION

The fuzzy model approach is a very effective tool to develop controllers for nonlinear systems [1]. In particular, Takagi-Sugeno (T-S) systems have demonstrated the ability to accurately represent complex nonlinear systems using fuzzy sets and membership functions [2], [3]. Therefore, a great deal of attention is being paid to the controller and observer synthesis for Takagi-Sugeno systems [4], [5]. A robust H_∞ fuzzy observer-based controller design method for uncertain T–S fuzzy systems with unmeasurable premise variables has been presented in [6]. A less conservative Condition for observer-based controller of one-sided Lipschitz nonlinear systems has been proposed in [7], based on the Young's relation and several matrix decompositions. In [8] a new two-step observer-based controller design technique for large uncertainty Lipschitz nonlinear systems with more degree of freedom. In [9], [10] the authors investigated the problem of robust observer–based control for T-S fuzzy models with parametric uncertainties and disturbances. However, the bilinear matrix inequality conditions in the cited references are solved by using cone-complementary linearization or two-step approach.

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Based on these previous results, this work proposes a direct robust and a simple method to designing the H_∞ observer-based controller for T-S fuzzy systems with uncertain parameters and external disturbances by using a single step resolution procedure. Firstly, we have dealt with the control problem of a very complex system that includes gross nonlinear coupling by using some notation based on direct matrix decomposition, which permits to find the conditions directly in the form of LMIs. Secondly, the observer and the controller gains are yielded by solving some linear matrix inequalities in a single step. In the end, the proposed design method gives less conservative LMI conditions than those constituted in the literature [9] and [10]. Therefore, this paper produces some results for the H_∞ observer-based control design devoted to uncertain fuzzy bilinear models. A comparison of the results obtained with those of [10] is recommended.

The plan of this paper is structured as follows. In Sect. II, the description of the system and preliminaries are presented. In Sect. III, the robust observer-based control design are obtained through LMI approach. In Sect. IV, an example of an application of the quadruple tank process is given to show the need for such controllers. The paper draws to a close with some conclusions.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Let us consider the continuous-time Takagi-Sugeno model for nonlinear dynamical system described by:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{m} h_i(\xi(t))\{(A_i+\Delta A_i(t))x(t)+B_iu(t)\}+\omega(t) \\ y(t) &= (C+\Delta C(t))x(t) \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $y(t)\in\mathbb{R}^p$ is the output vector and $u(t)\in\mathbb{R}^r$ is the input vector. $\omega(t)\in l_2^q$ is the unknown exogenous disturbance. $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times r}$ and $C_i\in\mathbb{R}^{p\times n}$ $(i=1,\cdots,m)$, are known constant matrices. $\Delta A_i(t)$ $(i=1,\cdots,m)$ and $\Delta C(t)$ are unknown matrices represent model uncertainty.

First, we assume the following assumptions:

Assumption 1: The pairs (A_i,B_i) and (A_i,C) $(i=1,\cdots,m)$, are respectively stabilizable and detectable.

Assumption 2: Uncertain matrices $\Delta A_i(t)$ $(i=1,\cdots,m)$ and $\Delta C(t)$ are unknown matrices represent time-varying model uncertainties, which are assumed to be,

$$\Delta A_i(t)=M_iF_i(t)N_i, \quad and \quad \Delta C(t)=M_{m+1}F_{m+1}(t)N_{m+1} \tag{2}$$

with

$$F_k^T(t)F_k(t) \leqslant I, \qquad \forall k=1,2,\cdots,(m+1) \tag{3}$$

The proposed observer-based controller that we are studying in this paper is according to:

$$\hat{x}(t) = \sum_{i=1}^{m} h_i(\xi(t))\{A_i\hat{x}(t)+B_iu(t)+L_i(y(t)-C\hat{x}(t))\} \tag{4}$$

$$u(t) = -\sum_{j=1}^{m} \mu_i(\xi(t))K_j\hat{x}(t) \tag{5}$$

where $L_i \in \mathbb{R}^{n\times p}$ and $K_j \in \mathbb{R}^{x\times n}$ $(i,j=1,\cdots,m)$ are the controller and the observer gains to be resolute respectively, and $\hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x(t)$.

Signify that $e(t)=x(t)-\hat{x}(t)$ error estimate and identifying a vector $\eta(t)=\left[\begin{array}{cc} x(t) & e(t) \end{array}\right]^T$, subsequently the augmented system can be addressed as,

$$\dot{\eta}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} h_i(\xi(t))h_j(\xi(t)) \left[\begin{array}{cc} A_i-B_iK_j & B_iK_j \\ \Delta A_i - L_i\Delta C & A_i - L_iC \end{array}\right]\eta(t) + \left[\begin{array}{c} I_n \\ I_n \end{array}\right]\omega(t) \tag{6}$$

Next, we prove that for any $\omega(t) \in L_2[0,\infty)$ of Takagi-Sugeno fuzzy parametric uncertain system (6) are considered if the following two provisions are fulfilled:

- The closed loop system (6) is robustly asymptotically stable when $\omega(t)=0$.
- Let in $\mu>0$ to be a fixed, under zero-initial conditions, the following performance index H_∞ holds:

$$\Im(\eta(t),\omega(t)) = \int_0^\infty \left[\mu^{-1}\eta(t)^T\eta(t) - \mu\omega(t)^T\omega(t)\right]dt < 0 \tag{7}$$

The following lemmas are very helpful for our establishment in this paper.

Lemma 1: [11] $\forall\,\sigma$ positive constant, and the real matrices M,N, and $F \in \mathbb{R}$ with appropriate dimension, such that $F^T(t)F(t) \leqslant I$ the following inequality holds:

$$MFN + N^TF^TM^T \leqslant \frac{1}{\sigma}MM^T + \sigma N^TN \tag{8}$$

Lemma 2: [12] For matrices R, P, V, and Z with appropriate dimensions and scalar ζ. Inequality,

$$R + Z^TP^T + PZ < 0 \tag{9}$$

is satisfied if the following condition holds:

$$\left[\begin{array}{cc} R & \zeta P + Z^TV^T \\ * & -\zeta V - \zeta V^T \end{array}\right] < 0 \tag{10}$$

III. Robust Observer-Based Control Design

In this section, our goal is to focus on the stability analysis problem for T-S Fuzzy systems (1) in the presence of parametric uncertainties and disturbance.

Theorem 1: For given scalars ζ, μ and positive scalars $\varepsilon_k > 0$ $(k=1,\cdots,m)$, the robust observer-based system (6) is asymptotically stable with the performance attenuation-μ in (7), if there exist symmetric positive-definite matrices $P_1 \in \mathbb{R}^{n\times n}$, $P_2 \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{m\times m}$, $Q_{1j} \in \mathbb{R}^{m\times n}$, $Q_{2i} \in \mathbb{R}^{n\times p}$, $(i,j=1,\cdots,m)$ satisfying the following LMIs:

$$\left[\begin{array}{cc} \Psi_{ij} & \Sigma \\ * & \Omega \end{array}\right] < 0,\ if\ i=j \tag{11}$$

$$\frac{1}{2}\left[\begin{array}{cc} \Psi_{ij} & \Sigma \\ * & \Omega \end{array}\right] + \frac{1}{2}\left[\begin{array}{cc} \Psi_{ij} & \Sigma \\ * & \Omega \end{array}\right] < 0,\ if\ i>j \tag{12}$$

for $i,j=1,\cdots,m$, where,

$$\begin{aligned} \Psi_{ij}^{(1,1)} &= P_1A_i^T + A_iP_1 - Q_{1j}^TB_i^T - B_iQ_{1j} + \rho_1N_1^TN_1 \\ &\quad +\rho_2N_2^TN_2, \end{aligned}$$

$$\Psi_{ij}^{(1,2)} = B_iQ_{1j},$$

$$\Psi_{ij}^{(2,2)} = A_i^TP_2 + P_2A_i - C^TQ_{2i}^T - Q_{2i}^TC,$$

$$\Sigma = \left[\begin{array}{cccccc} P_1 & \zeta(P_1B_i - B_iV) - Q_{1j}^T & P_1M_1 & 0 & I & 0 \\ P_2 & Q_{1j}^T & P_2M_1 & Q_{2i}M_2 & 0 & I \end{array}\right],$$

and

$$\Omega = diag\left\{-\mu I, -\zeta V - \zeta V^T, -\sigma_1 I, -\sigma_2 I, -\mu I, -\mu I\right\},$$

The observer-based control gains are given by $K_j = V^{-1}Q_{1j}$ and $L_i = P_2^{-1}Q_{2i}$.

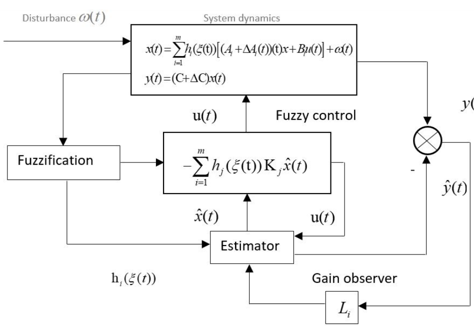

Fig. 1. The structural scheme of the fuzzy system

Proof: The following Lyapunov function can be associated with system dynamics (6):

$$V(x(t),e(t)) = x^T(t)P_1x(t) + e^T(t)P_2e(t), \qquad P_1>0,\ P_2>0$$

Then, the derivative of $V(x(t),e(t))$ further the trajectories of the error dynamics (6), we get,

$$\begin{aligned} \dot{V}(x(t),e(t)) &= \dot{x}^T(t)P_1x(t) + x^T(t)P_1\dot{x}(t) + \dot{e}^T(t)P_2e(t) \\ &+ e^T(t)P_2\dot{e}(t) \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} h_i(\xi(t))h_j(\xi(t))\,[2P_1(A_i - B_iK_j \\ &+ \Delta A_i)x(t) + 2x^T(t)((\Delta A_i - L_i\Delta C)^TP_2 \\ &+ P_1B_iK_j)e(t) + 2e^T(t)P_2(A_i - L_iC)e(t)] \\ &+ 2\omega^T(t)P_1x(t) + 2\omega^T(t)P_2x(t) \end{aligned} \tag{13}$$

Under zero initial conditions, the Lyapunov function fulfills $V(0)=0$ and $V(\infty)\geqslant 0$ that lead to:

$$\Im = \int_0^\infty \left[\dot{V}(\eta(t)) + \mu^{-1}\eta(t)^T\eta(t) - \mu\omega(t)^TQ_d\omega(t)\right]dt - V(\infty)$$

$$\Im \leqslant \int_0^\infty \left[\dot{V}(\eta(t)) + \mu^{-1}\eta(t)^T\eta(t) - \mu\omega(t)^T Q_d \omega(t)\right] dt \tag{14}$$

with the above analysis, to satisfy the attenuation level given in (7), the following conditions should be fulfilled:

$$\dot{V}(\eta)(t)) + \mu^{-1}\eta(t)^T\eta(t) - \mu\omega(t)^T\omega(t) < 0 \tag{15}$$

Including (13) and (14), then $\Im(\eta(t),\omega(t))$ become:

$$\Im(\eta(t),\omega(t)) = \phi^T\Upsilon\phi \tag{16}$$

where

$$\phi^T = \begin{bmatrix} x^T(t) & e^T(t) & \omega^T(t) \end{bmatrix}, \tag{17}$$

and

$$\begin{aligned}\Upsilon &= \sum_{i,j=1}^{m} h_i(\xi(t))h_j(\xi(t)) \left(\begin{bmatrix} \Phi_{ij}^{(1,1)} & 0 & P_1 \\ * & \Phi_{ij}^{(2,2)} & P_2 \\ * & * & -\mu I \end{bmatrix} \right. \\ &+ \begin{bmatrix} -P_1B_iK_j - K^TB^TP_1 & P_1B_iK_j & 0 \\ K^TB^TP_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &\left. + \underbrace{\begin{bmatrix} P_1\Delta A_i + \Delta A_i^T P_1 & (\Delta A_i - L_i\Delta C)^T P_2 & 0 \\ P_2(\Delta A_i - L_i\Delta C) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Upsilon_\Delta} \right).\end{aligned} \tag{18}$$

where

$$\Phi_{ij}^{(1,1)} = P_1A_i + A_i^TP_1 + \mu^{-1}I,$$

$$\Phi_{ij}^{(2,2)} = \Psi_{ij}^{(2,2)} + \mu^{-1}I.$$

In Υ presented by (18), we get the presence of bilinear terms $P_1B_iK_j$. This problem of bilinearities has been studied by many researchers [8], [9], [10] to linearize it. These works, however, based on the cone-complementary linearization algorithm to transform the presence of the coupling terms $P_1B_iK_j$ into a problem optimization of LMI or two-step design approaches. In this paper, we will establish an LMI condition based in one step, which is more practical and easy to apply to the methods proposed by [8], [9], [10]. Thus, to realize this, by setting a nonsingular matrix V and giving $K_j = V^{-1}Q_{1j}$, it can be specified that,

$$P_1B_iK_j = (P_1B_i - B_iV)V^{-1}Q_{1j} + B_iQ_{1j} \tag{19}$$

Then, with the help of the Eq. (19), we can conclude that inequality Υ is equivalent to:

$$\begin{aligned}\Upsilon &= \overbrace{\Upsilon_1 + \Upsilon_\Delta}^{R} \\ &+ \mathscr{H}_e\left\{ \underbrace{\begin{bmatrix} (P_1B_i - B_iV) \\ 0 \\ 0 \end{bmatrix}}_{P} V^{-1} \underbrace{\begin{bmatrix} -Q_{1j} & 0 & Q_{1j} \end{bmatrix}}_{Z} \right\} < 0\end{aligned} \tag{20}$$

where

$$\Upsilon_1 = \begin{bmatrix} \hat{\Phi}_{ij}^{(1,1)} & B_iQ_{1j} & P_1 \\ * & \Phi_{ij}^{(2,2)} & P_2 \\ * & * & -\mu I \end{bmatrix} \tag{21}$$

and

$$\hat{\Phi}_{ij}^{(1,1)} = \mathscr{H}_e(P_1A_i - B_iQ_{1j}) + \mu^{-1}I.$$

By Lemma 2, we can use the following matrix condition of inequality to check (20):

$$\Psi = \begin{bmatrix} \Upsilon_1 + \Upsilon_\Delta & \Upsilon_2 \\ * & -\zeta V - \zeta V^T \end{bmatrix} < 0 \tag{22}$$

where

$$\Upsilon_2 = \zeta P + Z^TV^T = \begin{bmatrix} \zeta(P_1B_i - B_iV) - Q_{1j}^T \\ Q_{1j}^T \end{bmatrix}$$

At this point, we must linear the last inequality, namely, by taking the uncertain terms apart, using Lemma 1, we have reformulated the yielded matrix to the sum of the matrix with the uncertainties and other matrix without the uncertainties, we obtain,

$$\begin{aligned}\Psi &= \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ * & -\zeta V - \zeta V^T \end{bmatrix} \\ &+ \begin{bmatrix} P_1M_1 \\ P_2M_1 \\ 0 \\ 0 \end{bmatrix} F_1(t) \begin{bmatrix} N_1 & 0 & 0 & 0 \end{bmatrix} \\ &+ \left\{ \begin{bmatrix} P_1M_1 \\ P_2M_1 \\ 0 \\ 0 \end{bmatrix} F_1(t) \begin{bmatrix} N_1 & 0 & 0 & 0 \end{bmatrix} \right\}^T \\ &+ \begin{bmatrix} 0 \\ -Q_{2i}M_2 \\ 0 \\ 0 \end{bmatrix} F_2(t) \begin{bmatrix} N_2 & 0 & 0 & 0 \end{bmatrix} \\ &+ \left\{ \begin{bmatrix} 0 \\ -Q_{2i}M_2 \\ 0 \\ 0 \end{bmatrix} F_2(t) \begin{bmatrix} N_2 & 0 & 0 & 0 \end{bmatrix} \right\}^T\end{aligned}$$

Appling Lemma 1 and using Eq. (3), we have the inequality:

$$\begin{aligned}\Psi &\leqslant \widetilde{\Psi} = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ * & -\zeta V - \zeta V^T \end{bmatrix} \\ &+ \sigma_1X_1X_1^T + \sigma_1^{-1}X_2X_2^T + \sigma_2X_3X_3^T + \sigma_2^{-1}X_4X_4^T\end{aligned} \tag{23}$$

for any $\sigma_1 > 0$ and $\sigma_2 > 0$, such that,

$$\begin{aligned}X_1^T &= \begin{bmatrix} N_1 & 0 & 0 & 0 \end{bmatrix}, \\ X_2^T &= \begin{bmatrix} M_1^TP_1 & M_1^TP_2 & 0 & 0 \end{bmatrix}, \\ X_3^T &= \begin{bmatrix} N_2 & 0 & 0 & 0 \end{bmatrix}, \\ X_2^T &= \begin{bmatrix} 0 & M_2^TQ_{2i} & 0 & 0 \end{bmatrix}.\end{aligned}$$

Based on the Schur complement, inequality (11) can be achieved by inequality (23). Then, for the rest of the proof, we can see on [13], which complete the proof.

IV. Application of the Quadruple Tank Process

A. Process description

A quadruple tank process studied in [10] is used in this section to show the performances of the proposed approach. The quadruple-tank process model described in Figure 1 has been developed in [14]. Moreover, the process is composed of four identical cylindrical tanks 1-4, and a reservoir 5, which constitutes the base of the system. Two pumps are used to transfer water from a reservoir into four overhead tanks. These two pumps are aimed to give a well-defined flow by rotation.

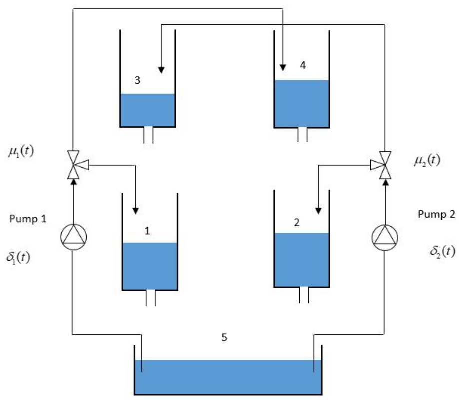

Fig. 2. The Quadruple Tank Process Scheme

B. Modeling and main result

The continuous–time nonlinear dynamical system of four tanks is represented by the following equations:

$$\begin{aligned}
\dot{x}_1(t) &= -\frac{a_{p1}}{A_{p1}}\sqrt{2gx_1(t)} + \frac{a_{p3}}{A_{p1}}\sqrt{2gx_3(t)} + \frac{\mu_1(t)k_{\delta 1}}{A_{p1}}\delta_1(t) \\
\dot{x}_2(t) &= -\frac{a_{p2}}{A_{s2}}\sqrt{2gx_2(t)} + \frac{a_{p4}}{A_{p2}}\sqrt{2gx_4(t)} + \frac{\mu_2(t)k_{\delta 2}}{A_{p2}}\delta_2(t) \\
\dot{x}_3(t) &= -\frac{a_{p3}}{A_{p3}}\sqrt{2gx_3(t)} + \frac{(1-\mu_2(t))k_{\delta 2}}{A_{p3}}\delta_2(t) \\
\dot{x}_4(t) &= -\frac{a_{s4}}{A_{p4}}\sqrt{2gx_3(t)} + \frac{(1-\mu_1(t))k_{\delta 1}}{A_{p4}}\delta_1(t)
\end{aligned} \tag{24}$$

where $x_k(t)$ is the liquid level in the tank-k; A_{pk} is the cross sectional area of the tank-k; a_{pk} is the cross-sectional area of the outlet hole of the tank–k, g is acceleration of gravity, $\mu_j(t) \in [0,\ 1]$ is time-varying valve flow proportion; $\delta_j(t)$ is the voltage control signal of Pump-j, with the corresponding coefficient $k_{\delta j}$. Moreover, these Pumps-j are designed to give a well determined movement of rotation. The description of the parameters of tank process model, comment on TABLE I. The measured voltage (V) output signal $y(t) \in \mathbb{R}^p$, are assumed proportional to level measurement (cm) of tank 1 and tank 2.

Remark 1. The valves can be used to introduce disturbances and uncertainty parameters. These external unmeasured disturbance flows can either complete or incomplete the level of four tanks. Process variations include uncertainties in the valve and loss of level in the tanks. The objective of this study is to design a robust controller that makes the system stable without losing the level in such tanks.

We note that nonlinearity terms $F_k(t) = \sqrt{x_k(t)}$ of the liquid level in the tank–k $\forall k = 1, \cdots, 4$ is specified in two systems based on successive sector rules as follows:

If $x_k(t)$ is M_1 **Then** $F_k(t) = C_{Fz1}x_k(t)$
If $x_k(t)$ is M_2 **Then** $F_k(t) = C_{Fz2}x_k(t)$

Thus, the TS-Fuzzy model is proposed as:

$$F_k(t) = (\xi_1(x_k)C_{Fz1} + \xi_2(x_k)C_{Fz2})x_k(t) \tag{25}$$

Note that the membership functions–MFs are given by the equation:

$$\xi_j(x_i) = \frac{\varpi_j(x_k)}{\sum_{j=1}^{2}\varpi_j(x_k)}, \qquad j = 1,2,\ k = 1,\cdots,4 \tag{26}$$

and fulfilled the properties:

$$\sum_{j=1}^{2}\xi_j(x_k) = 1,\ \ \xi_j(x_k) \in [0,\ 1], \qquad j = 1,2,\ k = 1,\cdots,4 \tag{27}$$

where

$$\varpi_j(x_k) = \frac{1}{\left[1 + \left|\dfrac{x_k - c_j}{a_j}\right|\right]^{2b_j}}, \qquad j = 1,2,\ k = 1,\cdots,4 \tag{28}$$

We found the results mentioned in Table II of the fuzzy parameters similar to [10].

Remark 2. Using the same fuzzy structure and nonlinear function characteristics, then the proposed rules (25) are reducing to calculate only $\xi_j(x_1)$, $j = 1,2$. This has reduced the number of member functions from 8 to 2.

TABLE I
NOMINAL PARAMETERS OF TANK PROCESS.

Parameters	Concept (Unit)	Values
A_{p1},A_{p3}	Areas of tanks 1.3 (m^2)	$2,8 \times 10^{-3}$
A_{p2},A_{p4}	Areas of tanks 2,4 (m^2)	3.2×10^{-3}
a_{p1}, a_{p3}	Areas of outlet in tanks 1,3 (m^2)	7.1×10^{-6}
a_{p2}, a_{p4}	Areas of outlet in tanks 2,4 (m^2)	5.7×10^{-6}
$k_{\delta i}$	Coefficient of pump-i, $i = 1,2$ ($ml\ V^{-1}s^{-1}$)	$3.33, 3.35$
x_i	The liquid level in tank-i (m)	
μ_i	The value scaling of flow at valve-i	
δ_i	The voltage control signal at pump-i (V)	

Time-varying parameters $\mu_i(t)$, $i = 1,2$, is given as follows:

$$\begin{aligned}
\theta_1(t) &= \frac{\mu_1(t)}{2}, & \theta_2(t) &= \frac{(1-\mu_1(t))}{2}, \\
\theta_3(t) &= \frac{\mu_2(t)}{2}, & \theta_4(t) &= \frac{(1-\mu_2(t))}{2}.
\end{aligned} \tag{29}$$

then $\sum_{k=1}^{4}\theta_k(t) = 1$.

The optimal value of the disturbance attenuation level from the pump to the tanks will be contained in the system, assuming that disturbance is presented in the following form:

TABLE II
PARAMETERS OF MEMBERSHIP FUNCTIONS.

Parameters	Concept	Values
a_1	The width of MFs	0.0021
a_2		0.3078
b_1		0.7219
b_2		5.3137
c_1	The center of MFs	–0.1656
c_2		0.3155
C_{Fz1}	The coefficient of fuzzy set in region M_1	2.389×10^3
C_{Fz2}	The coefficient of fuzzy set in region M_2	0.8149

$$\omega(t) = 10^{-2}\times[2.2sin(30\pi t)\ 1.5cos(20\pi t)\cdots \\ -4.8sin(26\pi t)\ 5.7sin(31\pi t)]^T$$

The nominal constant matrices are given as follows:

$$A_i = \begin{bmatrix} \frac{-a_{p1}\sqrt{2g}C_{Fzj}}{A_{p1}} & 0 & \frac{a_{p3}\sqrt{2g}C_{Fzj}}{A_{p1}} & 0 \\ 0 & \frac{-a_{p2}\sqrt{2g}C_{Fzj}}{A_{p2}} & 0 & \frac{a_{p4}\sqrt{2g}C_{Fzj}}{A_{p2}} \\ 0 & 0 & \frac{-a_{p3}\sqrt{2g}C_{Fzj}}{A_{p3}} & 0 \\ 0 & 0 & 0 & \frac{-a_{p4}\sqrt{2g}C_{Fzj}}{A_{p4}} \end{bmatrix}$$

If $i=1,\cdots,4$ **Then** $j=1$, **And** $i=5,\cdots,m$ **Then** $j=2$, with $m=8$.

$$B_{1,5} = \begin{bmatrix} \frac{2k_{\delta 1}}{A_{p1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{2,6} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{2k_{\delta 2}}{A_{p2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_{3,7} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{2k_{\delta 2}}{A_{p3}} \\ 0 & 0 \end{bmatrix}, \quad B_{4,8} = \begin{bmatrix} 0 & 0 & 0 & \frac{2k_{\delta 1}}{A_{p4}} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

We consider the uncertain model given by the model validation approach to compensate for the disparity and the relative approximation of the error between system states and measurement outputs.

The uncertainty matrices of the above system are taken,
$M_i = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}^T$,
$N_i = \eta_i \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}^T$,
$M_{m+1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$,
$M_{m+1} = \eta_{m+1} \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$,
for $i=1,\cdots,8$, and $m=8$.

For physical or economic reasons of an acceptable experimental model, under which only the liquid level in tanks 1,2 can measure, the other dynamic states will be estimated, using observer-based control study in this paper.

C. Simulation results

We have taken $\eta_i \in [1,150]$, $i=1,\cdots,8.$, $m=8$, $\sigma_1 = 1$, $\sigma_2 = 1$ and $\zeta = 10^{-6}$. The responses of volume in such tanks and their estimates starting from the initial conditions $x(0) = \begin{bmatrix} 0.35 & 0.15 & 0.42 & 0.32 \end{bmatrix}^T$, and $\hat{x}(0) = \begin{bmatrix} 0.22 & 0.02 & 0.23 & 0.11 \end{bmatrix}^T$.

Then, the evolution of the volume in the corresponding tanks to the measured states (x_1,x_2) shown in Figure3, and the evaluation of the unmeasured states (x_3,x_4) shown in Figure 4, respectively. It can be witnessed that the observer estimates adequately the states under the effect of model uncertainty and disturbance.

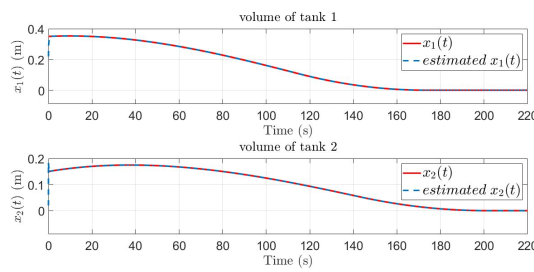

Fig. 3. Evolution of measurable states and its estimation

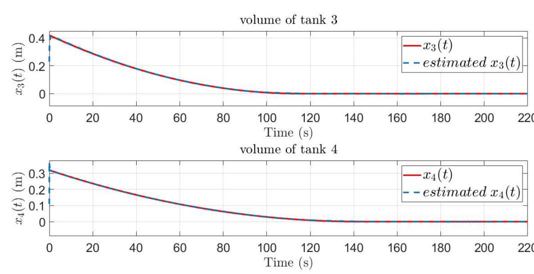

Fig. 4. Evolution of unmeasurable states and its estimation

Using toolbox and the solver [16], the fuzzy controller gains are given by:

$K_1 = V^{-1}Q_{11} =$
$$10^{-12}\times\begin{bmatrix} -4.1017 & -0.1492 & 6.6598 & 0.2456 \\ -2.9752 & -0.1469 & 8.1761 & 0.2511 \end{bmatrix}$$

$K_2 = V^{-1}Q_{12} =$
$$10^{-12}\times\begin{bmatrix} -2.2439 & -0.1478 & 7.7373 & 0.2415 \\ -2.5663 & -0.1619 & 8.1427 & 0.2510 \end{bmatrix}$$

$\cdots$

$K_8 = V^{-1}Q_{18} =$
$$10^{-11}\times\begin{bmatrix} 1.2946 & 7.5198 & -2.9512 & -0.1198 \\ 1.0082 & 6.3198 & -2.1725 & -0.1005 \end{bmatrix}$$

and observer gains:

$$L_1 = P_2^{-1} Q_{21} =$$
$$10^3 \times \begin{bmatrix} -0.2027 & -0.8860 & -0.2946 & -1.4192 \\ 0.3503 & 1.5258 & 0.5237 & 2.4438 \end{bmatrix}$$
$$L_2 = P_2^{-1} Q_{22} =$$
$$10^3 \times \begin{bmatrix} -0.2027 & -0.8859 & -0.2946 & -1.4191 \\ 0.3503 & 1.5258 & 0.5237 & 2.4437 \end{bmatrix}$$
$$\cdots$$
$$L_8 = P_2^{-1} Q_{28} =$$
$$\begin{bmatrix} 28.1322 & 122.1351 & 42.1789 & 195.4160 \\ 130.5573 & 568.9715 & 195.7768 & 910.3424 \end{bmatrix}$$

Based on the results of the simulation, the controller values and the observer gains, we can see that the proposed design conditions in Theorems 1 of this paper are much less conservative than in Theorems 1 of [10]. With our results, we can conclude on the ability to reduce the errors to about zero of the tank process, even with the existence of parameter uncertainties and perturbations.

We consider the distribution rate of flows in the tanks placed in the position of the regulating valve $\mu_1(t)$ and $\mu_2(t)$, that will be adjusted according to the following rules:

If $t_{sim}(p) = 0, \cdots, 20$ **Then** $\mu_1(t) + \mu_2(t) \in [1,2]$, **MP**

If $t_{sim}(p) = 20, \cdots, 120$ **Then** $\mu_1(t) + \mu_2(t) \in [0,1]$, **NMP**

If $t_{sim}(p) = 120, \cdots, 220$ **Then** $\mu_1(t) + \mu_2(t) \in [1,2]$, **MP**

If $t_{sim}(p) = 220, \cdots, 300$ **Then** $\mu_1(t) + \mu_2(t) \in [0,1]$, **NMP**

where **MP** is minimum phase setting [15], and **NMP** is no– minimum phase setting. In the end, by using the setting rules, the estimation error of the volume in such tanks is given in Figure 5.

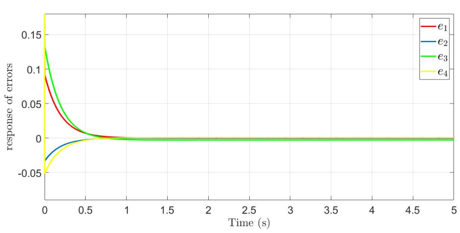

Fig. 5. Evolution of the estimation errors of such tanks

Finally, Figure 6 gives the stabilized control signal of the observer-based controller design for the quadruple-tank process laboratory system.

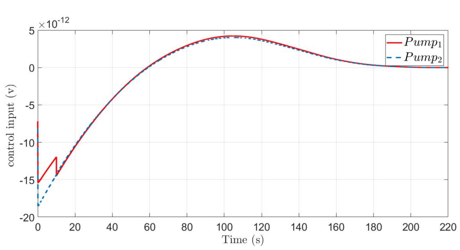

Fig. 6. Evolution of the pump signal control

V. CONCLUSIONS

A direct robust and a simple method to designing an H_∞ observer-based controller for T-S systems with time-varying parameter uncertainties and admissible external disturbances is studied in this paper. By using the Lyapunov function, sufficient conditions for designing simultaneously the controller and the observer gains are achieved in a single step by solving a set of LMIs. This method gives a less conservative than the other control methods considered in the literature. An application of the model of a laboratory process with four interconnected tanks has been developed to demonstrate the effectiveness of the proposed methodology. As a future study, we aim to adapting and expanding our approach for delayed systems, however, the H_∞ observer-based control design problem for time-delay systems.

REFERENCES

[1] T. Takagi, & M. Sugeno, Fuzzy identification of systems and its applications to modeling and control. IEEE transactions on systems, man, and cybernetics, (1), 116-132, 1985.

[2] H. Moodi, & M. Farrokhi, On observer-based controller design for Sugeno systems with unmeasurable premise variables. ISA transactions, 53(2), 305-316, 2014.

[3] W. B. Xie, C. C. Lim, J. Zhang, & L. Huang, New approaches to observer design and stability analysis for T–S fuzzy system with multiplicative noise. Journal of the Franklin Institute, 354(2), 887-901, 2017.

[4] A. Golabi, M. Beheshti, & M. H. Asemani, H_∞ robust fuzzy dynamic observer-based controller for uncertain Takagi–Sugeno fuzzy systems. IET control theory & applications, 6(10), 1434-1444, 2012.

[5] H. K. Lam, H. Li, & H. Liu, Stability analysis and control synthesis for fuzzy-observer-based controller of nonlinear systems: a fuzzy-model-based control approach. IET Control Theory & Applications, 7(5), 663-672, 2013.

[6] M. H. Asemani, & V. J. Majd, A robust H_∞ observer-based controller design for uncertain T–S fuzzy systems with unknown premise variables via LMI. Fuzzy Sets and Systems, 212, 21-40, 2013.

[7] S. Ahmad, M. Rehan, & K. S. Hong, Observer-based robust control of one-sided Lipschitz nonlinear systems. ISA transactions, 65, 230-240, 2016.

[8] A. Zemouche, R. Rajamani, H. Kheloufi, & F. Bedouhene, Robust observer-based stabilization of Lipschitz nonlinear uncertain systems via LMIs-discussions and new design procedure. International journal of robust and nonlinear control, 27(11), 1915-1939, 2017.

[9] V. L. B. Tuan, & A. El Hajjaji, Robust observer-based control for TS fuzzy models application to vehicle lateral dynamics. In 2018 26th Mediterranean Conference on Control and Automation (MED) (pp. 1-6). IEEE, 2018.

[10] V. L. B. Tuan, A. El Hajjaji, & G. Naami, Robust TS-Fuzzy observer-based control for Quadruple-Tank system. In 2019 12th Asian Control Conference (ASCC) (pp. 307-312). IEEE, 2019.

[11] C. H. Lien, Robust observer-based control of systems with state perturbations via LMI approach. IEEE Transactions on Automatic Control, 49(8), 1365-1370, 2004.

[12] X.H. Chang, L. Zhang, & J. H. Park., Robust static output feedback H_∞ control for uncertain fuzzy systems. Fuzzy Sets Syst. 273, C, 87–104, 2015.

[13] K. Tanaka, & H. O. Wang, Fuzzy control systems design and analysis: a linear matrix inequality approach. John Wiley & Sons, 2004.

[14] K. H. Johansson, & J. L. R. Nunes, A multivariable laboratory process with an adjustable zero. In Proceedings of the 1998 American Control Conference. ACC (IEEE Cat. No. 98CH36207) (Vol. 4, pp. 2045-2049). IEEE, 1998.

[15] K. H. Johansson, Relay feedback and multivariable control (Doctoral dissertation, Lund University), 1997.

[16] L. J. Yalmip, A toolbox for modeling and optimization in MATLAB. Proceedings of the IEEE Computer-Aided Control System Design Conference, Taipei, Taiwan, pp. 284-289, 2004.