Abstract

The problem of model predictive control (MPC) under parametric uncertainties for a class of nonlinear systems is addressed. An adaptive identifier is used to estimate the parameters and the state variables simultaneously. The algorithm proposed guarantees the convergence of parameters and the state variables to their true value. The task is posed as an adaptive model predictive control problem in which the controller is required to steer the system to the system setpoint that optimizes a user-specified objective function.

The technique of adaptive model predictive control is developed for two broad classes of systems. The first class of system considered is a class of uncertain nonlinear systems with input to state stability property. Using a generalization of the set-based adaptive estimation technique, the estimates of the parameters and state are updated to guarantee convergence to a neighborhood of their true value.

The second involves a method of determining appropriate excitation conditions for nonlinear systems. Since the identification of the true cost surface is paramount to the success of the integration scheme, novel parameter estimation techniques with better convergence properties are developed. The estimation routine allows exact reconstruction of the systems unknown parameters in finite-time. The applicability of the identifier to improve upon the performance of existing adaptive controllers is demonstrated. Then, an adaptive nonlinear model predictive controller strategy is integrated to this estimation algorithm in which robustness features are incorporated to account for the effect of the model uncertainty.

To study the practical applicability of the developed method, the estimation of state variables and unknown parameters in a stirred tank process has been performed. The results of the experimental application demonstrate the ability of the proposed techniques to estimate the state variables and parameters of an uncertain practical system.

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Chapter 1
Introduction

The problem of parameter and state estimation of a class of nonlinear systems is addressed. An adaptive identifier and optimal control problem are used to estimate the parameters and control system states variables simultaneously. The proposed method is derived using a new formulation. An algorithm is developed to update these sets using the available information. The algorithm proposed guarantees the convergence of parameters and the state variables to their true value.

The technique of estimation is applied to two broad classes of systems. The first involves a class of continuous time nonlinear systems subject to bounded state and input systems with constant unknown parameters. Using the proposed set-based adaptive estimation, we can steer system optimally to the origin. The formulation provides robustness to parameter estimation error. The parameter uncertainty set and the uncertainty associated with an auxiliary variable is updated such that the set is guaranteed to contain the unknown true values. The second class of system considered is a class of nonlinear systems with ISS-CLF stability condition. Using a generalization of the set-based adaptive estimation technique proposed, the estimates of the parameters and state are updated to guarantee convergence to a neighborhood of their true value. To study the practical applicability of the developed method, the estimation of state variables and parameters in a stirred tank process has been performed. The results demonstrate the ability of the proposed techniques to estimate the state variables and parameters of the uncertain system.

1.1 Introduction

Effective monitoring of a process is possible only when accurate information on the state variables and parameters of the process are available. Example of process state variables are concentrations of the reacting species in a reactor, temperature and molecular weight distribution in a polymerization process. These variables uniquely define the states of the process and in many cases may directly/indirectly define the final product quality. Rate of heat production in a reactor, overall heat coefficient in jacketed reactors and specific growth rate in bioreactors are the examples of process parameters. Information on the parameters of a process provides a better understanding of the process dynamics and also allow for the development of an accurate and representative models of process.

In practice, due to inadequacy of available sensors or operational limitations, some of the essential process state variables cannot be measured frequently. In addition important process parameters may have to be estimated from available measurements. In such cases, estimates of the inaccessible, but essential, state variables and parameters of the process are usually obtained by employing state and parameter estimation methods. Many techniques exist for the estimation of states for a variety of classes of dynamical systems that can achieve accurate state estimates in a variety of conditions. However, these techniques rely on the knowledge of the system parameters. Uncertainty in the model parameters for instance can generate (possibly large) bias in the estimation of the unmeasured state variables. In cases where large uncertainties of the process parameters exist, it is imperative to use techniques that are able to combine state observation with parameter estimation.

The motivation for this research arises from the need to develop reliable state and parameter estimation methods that are capable of providing continuous and accurate estimates of inaccessible state variables and parameters of a nonlinear process in a presence of exogenous disturbance and running the system to the origin which is frequently encountered in practice.

1.2 Organization of the Dissertation

Chapter 2: Chapter 2 is divided into two parts. First, the technical preliminaries required to develop the parameter and state estimation methodology proposed in Chapter 3 are introduced. The topics include Persistence of Excitation (PE), Lyapunov Stability, Projection Algorithm, Observability, State Observers and Adaptive identifiers. The second section contains a review of the past and recent works in the field of real time optimization and model predictive control of nonlinear systems.

Chapter 3: In this chapter, the adaptive estimation method derived to solve the problem of ISS stable systems. Using a set-based adaptive estimation, the estimates for the parameters and the state variables are updated to guarantee convergence. A simulation example is used to illustrate the developed procedure and ascertain the theoretical results.

Chapter 4: In this chapter, we consider the problem of parameter identification and state estimation of a continuous-time nonlinear system subject to unknown parametric uncertainty. The formulation is developed to provide robustness to parameter estimation error. The uncertainty associated with an auxiliary variable defined for state estimation is updated such that the set is guaranteed to contain the unknown true values. A simulation example is used to illustrate the developed procedure and ascertain the theoretical results. After convergence of the variables to the true values, a model based predictive control is defined to run the system simultaneously to the origin.

Chapter 5: Based on the results in Chapter 4, the estimation technique is applied to a mixing tank problem. The developed method is used to estimate state and parameters of the experimental process. The estimation routine employed guarantees convergence of state and parameters to their true values.

Chapter 6: A summary of the design procedure given in Chapter 3 and 4 is provided, and conclusions are drawn based on the investigations of Chapters 3, 4 and 5. Suggestions for directions of future work are given.

Chapter 2
Literature Review

The design methodology for simultaneous parameter and state estimation and optimal control of class of a nonlinear systems is largely developed from the concepts of linear system theory, parameter identifiers, projection algorithm and adaptive predictive control. In this chapter, these concepts are briefly introduced for the understanding of this thesis work. The detailed discussion regarding the relationships between the concepts are discussed in Chapter 3. This chapter also summarizes the recent and early works by researchers active in robust adaptive predictive control techniques that are of importance in relation to this thesis.

2.1 Technical Preliminaries

2.1.1 Persistence of Excitation

The concept of persistent excitation (PE), when it arose in the 1960s in the context of system identification. The term PE was coined to express the property of the input signal to the plant that guarantees that all the modes of the plant are excited. In the late 1970s, it became clear that the concept of PE also played an important role in the convergence of the controller parameters to their desired values. Recent work on robustness of the adaptive systems in the presence of bounded disturbance, time-varying parameters, and un-modeled dynamics of the plant revealed that the concept of PE is also intimately related to speed of convergence on the parameters to their final values, as well as the bounds on the magnitudes of the parameter errors. In both linear and nonlinear adaptive systems, parameter convergence is related to the satisfaction of persistence of excitation condition, which can be defined in the continuous time as follows.

Definition 2.1.1: [Krstic et. al, 1995] A vector function ϕ : is said to be persistently exciting if there exist positive constants α_1, α_2 and T_0 such that

$$\alpha_1 I \geq \int_t^{t+T_0} \phi(\tau)\phi(\tau)^T \geq \alpha_2 I, \forall t \geq 0 \tag{1}$$

Although the matrix $\phi(\tau)\phi(\tau)^T$ may be singular at every instant τ , the PE condition requires that ϕ span a entire n_ϕ dimensional space as τ varies from t to t+T0, that is, integral of matrix $\phi(\tau)\phi(\tau)^T$ should attain full rank over any interval of some length T_0 or in other words, (1) requires that $\phi(t)$ varies such that the integral of the matrix $\phi(\tau)$ is uniformly positive definite over any time interval $[t, t + T0]$. The properties of PE signals as well as various other equivalent definitions and interpretations are given in the literature. In adaptive linear systems, the PE condition is converted to the sufficient richness (SR) condition on the reference input signal. Necessary and sufficient conditions for parameter convergence are then developed in terms of the reference signal. A popular result implies that exponential convergence is achieved whenever the reference signal contains enough frequencies, i.e., whenever the spectral density of the signal is nonzero in at least n_θ points, where n_θ is the number of unknown parameters in the adaptive scheme. Otherwise, convergence to a characterizable subspace of the parameter space is achieved. Despite the fact that the theory of parameter convergence for linear systems is well established, very few results are available for nonlinear systems. This is mainly because the familiar tools in linear adaptive control cannot be directly extended to nonlinear systems. In most of the available results, stability and performance properties are proved by assuming that a vector function, which depends on closed-loop signals is persistently exciting. However, the means of verifying this PE condition a priori for a given nonlinear system remains an open problem, in general. In [Lin and Kanellakopoulos, 1998], a procedure is provided for determining a priori whether or not a specific reference signal is sufficiently rich for a specific output feedback nonlinear system, and hence whether or not parameter estimates will converge. Nevertheless, the main result in [Lin and Kanellakopoulos, 1998] is that the presence of nonlinearities in the plant usually reduces the SR condition requirement on the reference signal and thus enhances parameter convergence.

2.1.2 Lyapunov Stability

Lyapunov stability analysis plays an important role in the stability analysis of dynamical systems described by ordinary differential equations. This technique is very useful and convenient in practice because the stability of the system can be determined directly from the differential equations describing the system. In other words, the Lyapunov method enables one to determine the nature of stability of an equilibrium point of the system without explicitly integrating the ordinary differential equations. In addition, the Lyapunov analysis is applicable to continuous-time and discrete-time systems, linear and nonlinear systems, time-invariant and time-varying systems. From the classical theory of mechanics, a vibratory system is stable if its total energy is continually decreasing until an equilibrium state is reached. A physical example that illustrates this concept is a simple pendulum in which the equations of motion described by the forces acting on the system, vanish at steady state [Khalil, 2002]. The method of Lyapunov, is based on the following behavior. If the system has an asymptotically stable equilibrium state, then the stored energy of the system decays with increasing time until it finally reaches its minimum value at the equilibrium state. For a general system, however it is not simple to describe its dynamics through an energy function. To overcome this difficulty, the Lyapunov function which acts as a fictitious energy function, was introduced [Ogata, 1987].

Lyapunov stability analysis plays an important role in the stability analysis of dynamical systems described by ordinary differential equations. The Lyapunov function, denoted by V (.), is a scalar, positive definite function. It is generally assumed to be continuous with continuous partial derivatives. When taken along the systems trajectory, the time derivative of the Lyapunov function is negative definite or negative semidefinite. These desired properties of the Lyapunov function can be formally stated in the stability theorem described by [Khalil, 2002] for a non-autonomous system.

Theorem 2.1.2.1. [Khalil, 2002] Consider the non-autonomous system

$$\dot{x}(t) = f(t, x(t)) \tag{2}$$

where $f : [0, \infty) \times D \to R^n$ is piecewise continuous in t and locally Lipschitz in x(t) on $[0, \infty) \times D$, system (2) at t = 0 and $D = \{x(t) \in R^n | \; ||x(t)|| < r\}$. Let $V : [0, \infty) \times D \to R$ be a continuously differentiable function such that,

$$\alpha_1(||x(t)||) \leq V(t, x(t)) \leq \alpha_2(||x(t)||) \tag{3}$$

$$\dot{V}(t, x(t)) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x(t)) \leq 0 \tag{4}$$

$$\int_t^{t+\epsilon} V(\tau, \phi(\tau, t, x(t)))d\tau \leq -\lambda V(t, x(t)), \quad 0 < \lambda < 1 \tag{5}$$

$\forall t \geq 0, \forall x(t) \in D$, for some ϵ, where $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ are class K functions defined in $[0, r)$ and $\phi(\tau, t, x(t))$ is the solution of the system that starts at $(t, x(t))$. Then, the origin is uniformly asymptotically stable.

If all the assumptions hold globally and $\alpha_1(\cdot)$ belongs to class k_∞ then the origin is globally uniformly asymptotically stable.

Now that the stability considerations based on Lyapunov theory are defined, the next step consists of finding a convenient Lyapunov function to design the adaptive updating laws, such that Theorem 2.1.2.1 is satisfied.

2.1.3 Projection Algorithm

It is important to mention that, in general, the parameters that characterize a system, have a physical meaning and are bounded above and/or below. For this reason, it is desired to constrain the parameter estimates to lie inside a bounded set. An effective method for keeping the parameter estimates within some defined bounds is to use a projection algorithm. In many practical problems where θ represents the parameters of a physical plant, we may have some a priori knowledge as to where θ is located in R^n. This knowledge usually comes in terms of upper or lower bounds for the elements of θ or in terms of a well defined subset of R^n, etc. Using this a priori information, adaptive laws can be designed that are constrained to search for estimates of θ in the set where θ is located. Intuitively such a procedure may improve the convergence and reduce the time taken in convergence when initial values of the parameter is chosen to be far away from the unknown θ . In [Krstic et al., 1995], a projection operator is defined for the general convex parameter set Π.

Consider a convex set $\Pi_\epsilon = \{\hat{\theta} \in R^p | P(\hat{\theta}) \leq \epsilon\}$, , where the convex function $P : R^p \to R$ is assumed to be smooth. The set Π_ϵ is the union of the set $\Pi = \{\hat{\theta} \in R^p | P(\hat{\theta}) \leq 0\}$ and a boundary around it. The interior of Π is denoted by $\dot{\Pi}$, and $\nabla_{\hat{\theta}} P$ represents an outward normal vector at $\hat{\theta} \in \partial\Pi$. The projection operator is defined as follows

$$proj(\tau) = \begin{cases} \tau \text{ if } \hat{\theta} \in \dot{\Pi} \text{ or } \nabla_{\hat{\theta}} P^T \tau \leq 0; \\ (I - c(\hat{\theta})\Gamma \frac{\nabla_{\hat{\theta} P \nabla \hat{\theta} P^T}}{\hat{\theta} P^T \nabla \hat{\theta} P})\tau \text{ if } \hat{\theta} \in \Pi \backslash \dot{\Pi} \text{ and } \nabla_{\hat{\theta}} P^T \tau > 0;. \end{cases} \tag{6}$$

$$c(\hat{\theta}) = min\{1, \frac{P(\hat{\theta})}{\epsilon}\}$$

Here, Γ belongs to G of all positive definite symmetric $p \times p$ matrices and τ is the vector of nominal update laws that is, in the absence of the projection algorithm the update law $\dot{\hat{\theta}} = \tau$.

The properties of the projection operator, $Proj\{\tau, \hat{\theta}, \Gamma\}$, are given by

1. The mapping $Proj : R^p \times \Pi_\epsilon \times G \rightarrow R^p$ is locally lipschitz in its arguments $\tau, \hat{\theta}, \Gamma$.
2. $Proj\{\tau\}^T \Gamma^{-1} Proj\{\tau\} \leq \tau^T \Gamma^{-1} \tau, \forall \hat{\theta} \in \Pi_\epsilon$.
3. Let $\Gamma(t)\tau(t)$ be continuously differentiable and

$$\hat{\theta} = Proj\{\tau\}, \hat{\theta}(t)(0) \in \Pi_\epsilon$$

Then, on its domain of the definition, the solution $\hat{\theta}(t)$ remains in Π_ϵ.

The adaptive laws with the above projection modification retain all the properties established in the absence of the projection and guarantee that $\hat{\theta} \in \Pi_\epsilon, \forall t \geq 0$ provided $\hat{\theta}(0) \in \Pi_\epsilon$.

2.1.4 Observability

Consider a continuous time linear system of the form

$$\dot{x} = Ax + Bu \tag{7}$$

$$y = Cx \tag{8}$$

where $x \in R^n$ is a state vector, $u \in R^n$ is the control input, $y \in R^n$ are the outputs, and matrices, A,B and C are of appropriate dimensions. Observability is a property of dynamical system, first introduced by [Kalman, 1960]. This property is meant to express the availability of measurement data with respect to ones ability to reconstruct or make inferences regarding the values of unmeasured state variables.

Definition 2.1.4.1: A linear continuous time system given by (7,8) is observable if for any initial state x_0 and some final time t, the initial state x_0 can be uniquely determined by knowledge of the inputs u and outputs y for all time t.

In other words, observability is related to the problem of determining the value of the state vector knowing only the output y over some interval of time. This is a question of determining when the mapping of the state into the output associates a unique state with every output that can occur. If a system is observable, then its initial state can be determined. If the initial state is known, then values of the states at any time can be calculated. Hence, observability implies that values of the state at any time are fully reconstructible as long as the inputs and outputs are known exactly. Observability can be checked by a matrix rank test performed on the systems observability matrix.

Theorem 2.1.4.1: The continuous time LTI system (7,8) is observable if and only if the observability matrix is defined by

$$O(C,A) = [C^T, (CA)^T, \cdots, (CA^{(n-1)})^T]^T \tag{9}$$

is of rank n.

The concept of observability is central to the design of state observers and state estimators, which are discussed in the next section.

2.1.5 State Observers

Many nonlinear control design and adaptive system techniques assume state feedback; this implies that all the state variables are measured and are available for feedback. In practice, this is not always true, either for economic or technical reasons, such as sensor failures. In most cases,only a subset of the state variables are available for measurement. Intuitively, we want to use the measured states or outputs of the system and extend the state-dependent techniques to output-dependent techniques for system design. The idea is similar to what has been widely applied in LTI systems, i.e., build an observer that yields asymptotic estimates of the system state based on the output of the system, and then update the control/ adaptation law using on-line estimation of the unmeasured states. In control theory, a state observer is a dynamical system whose outputs are the estimates of the state variables of the system [Ioannau and Sun, 1996]. The main criterion that observers must satisfy is that the estimation error $\tilde{x}(t) = (x(t) - \hat{x}(t))$ tends to zero in the limit as $t \to \infty$ where $\hat{x}(t)$ is the estimate of the state x(t) at time t. If the dynamics of the plant give rise to a linear time-invariant system, then there exists an estimator of the form

$$\hat{x}(t) = A\hat{x}(t) + L(y - \hat{y}) + Bu \tag{10}$$

$$\hat{y}(t) = C\hat{x}(t) + Du \tag{11}$$

which guarantees convergence of the state estimation error to zero, provided that the plant is observable. The observer given by Eqs. (2.5a) and (2.5b) is referred to as a Luenberger observer [Ioannau and Sun, 1996]. The matrix L is designed so that the matrix $(A - LC)$ is stable, which ensures the stability of the observers error dynamics. In fact, the eigenvalues of $(A - LC)$, and, therefore, the rate of convergence of $\tilde{x}(t)$ to zero can be arbitrarily chosen by designing L appropriately. Therefore, it follows that $\hat{x}(t) \to x(t)$ exponentially fast as $t \to \infty$, with a rate that depends on the matrix $(A - LC)$. This result is valid for any matrix A and any initial condition $x(0)$ as long as (C, A) is an observable pair.

2.1.6 Adaptive Identifiers

The adaptive identifiers represent a class of real time parameter estimation schemes that are used to estimates (typically) slow time-varying parameters of dynamical systems. Under suitable conditions, these identifiers can guarantee convergence of the estimated parameters to the unknown parameter values. The design of such scheme includes the selection of plant input so that a certain signal vector, is PE. Adaptive identifier designs are natural extension of observer design for linear time invariant (LTI) systems with unknown parameters. When the parameters of the system are unknown, an adaptive identifier is designed to estimate the parameters of the dynamical system. This was first accomplished in [Kreisselmeier, 1977; Kudva and Narendra, 1973]. Traditionally, an adaptive identifiers consists of a state prediction subject to parameter estimations and a parameter update law. Different representations have been discussed in detail for LTI systems [Ioannau and Sun, 1996; Narendra and Annaswamy, 1989; Sastry and Bodson, 1989]. Basic methods used to design adaptive laws include Lyapunov-based design, gradient methods, and recursive least squares methods. Subsequently alternative techniques have been generalized to the design of adaptive observers for nonlinear systems, linear time-varying systems and systems with disturbances. Adaptive laws only become parameter identifiers if the input signal u has to be chosen to be sufficiently rich so that the regressor vector ϕ is PE.

2.2 Parameter Estimation in Nonlinear Systems

State Estimators are deterministic/stochastic dynamic systems that are used to reconstruct the inaccessible but important process state variables, from available measured variables. The problem of state estimation in chemical processes has been studied extensively since the mid 1970s. In particular, the extended Kalman Filter (EKF) has been used widely for state estimation [Bastin and Dochain, 1991]. The design of an EKF is based on the linear approximation of a nonlinear process model. It is generally recognized that, the linearization at each time step can introduce large errors and even cause divergence of the filter [Wan and Van Der Merwe, 2000]. These concerns are especially acute in complex industrial set-ups [Wilson et al., 1998]. Although higher order Kalman filters exist, they are more difficult to implement and prone to instability. Due to the complex nonlinear behavior of many chemical and biochemical processes, reliable state estimation should be based on nonlinear models that can capture the complex nonlinear behavior. Furthermore, several studies have found that linear state estimators are inadequate for many nonlinear processes [Valluri and Soroush, 1996] and [Tatiraju and Soroush, 1997], motivating the use of nonlinear observers/estimators. The Luenberger observer is well established method of estimating the state variables of a known observable system using input-output data,that can be adjusted to handle to estimate the state of a linear time-invariant system with unknown parameters as well. The structure of the observer as the adaptive laws for updating its parameters has to be chosen judiciously for this purpose. This was accomplished in [Kudva and Narendra, 1973; Luders and Narendra, 1974; Narendra and Annaswamy, 1989]. In 1977, an alternate method of generating the estimates of the states and the parameters of the plant was suggested [Kreisselmeier, 1977] where the adaptive algorithms ensured faster rate of convergence of the parameters estimates under certain conditions. When the system further depends on some unknown parameters, the observer design has to be modified so that both state variables and parameters can be estimated, leading to so-called adaptive observers. Various results in that respect can be found, going back to [Luders and Narendra, 1974], and [Kreisselmeier, 1977] for linear systems, or [Marino, 1990]) for nonlinear ones, but nonlinearities depending only on input/output. Recently an alternative result as adaptive observer has been designed on adaptive observation for linear time-varying systems [Zhang, 2002],which guarantees global exponential convergence for noise-free systems. The adaptive observer proposed provides robustness in the presence of modeling and measurement noises. In the paper [Adetola and

Guay, 2008], the authors considered a system with exogeneous disturbances and showed that parameter convergence can be guaranteed under certain conditions of persistency of excitation condition. The authors proposed a novel set-based adaptive estimation with an appropriate adaptation law for the unknown parameters. The proof of the convergence of the estimates to their true values is achieved using Lyapunov theories.

2.3 Real-time Optimization

One of the key challenges in the process industry is how to best operate the plant under different conditions such as feed compositions, production rates, energy availability, feed and product prices that changes all the time. Real-time optimization (RTO), which refers to the online economic optimization of a process plant, is a widely employed technology to meet this challenge. RTO attempts to optimize process performance (usually measured in terms of profit or operating cost) thereby enabling companies to push the profitability of their processes to their true potential as operating conditions change. The popular RTO is based on the assumption that model and disturbance transients can be neglected if the optimization execution time interval is long enough to allow the process to reach and maintain steady-state. A typical RTO system includes components for steady-state detection, data acquisition and validation, process model updating, optimization calculations and optimal operating policies transfer to advanced controllers.

2.3.1 Model Predictive Control

Model predictive control (MPC) or receding horizon control (RHC) is a family of control that utilizes a process model along with cost factors and optimum target operating point to calculate process control moves that drives the plant to the most economic constraints while ensuring stable operation. The control technique has proven to be extremely successful in the process industry. Linear (and nonlinear) model predictive control remains the industry standard with increasing number of reported applications and significant improvements in technical capability [Camacho and Bordons, 1995]. Consider the time-invariant nonlinear system of the form

$$\dot{x} = f(x(t), u(t)) \tag{12}$$

subject to the pointwise state and input constraints $x(t) \in X \subset R^{n_x}$ and $u(t) \in U \subset R^{n_u}$, respectively. The vector field $f : R^{n_x} \times R^{n_u} \to R^{n_x}$ satisfies $f(0,0) = 0$, the set U is compact, X is connected and $(0,0) \in (X, U)$.

MPC algorithms optimize the future plant behaviour and satisfy the given constraints by solving the following finite horizon open loop optimal control problem:

$$min_{u^p} \quad J = \int_t^{t+T} L(x^p(\tau), u^p(\tau))d\tau + W(x^p(t+T)) \tag{13}$$

$$\text{s.t.} \quad \dot{x}^p = f(x^p(\tau), u^p(\tau)), x^p(t) = x_t$$

$$x^p(\tau) \in X, u(\tau) \in U$$

$$x^p(t+T) \in X_f$$

where $(.)^p$ denotes the predicted variables (internal to the controller). The stage cost $L(x^p, u^p)$ is a semi-definite positive function. The terminal penalty $W(x^p(t+T))$ and terminal constraint $x^p(t+T) \in X_f$ are included for stability considerations.

At each time step, the solution to the optimization problem is found over a certain prediction horizon, T, using the current state of the plant or its estimate as the initial state. The optimization yields an optimal control sequence and the first control action is implemented on the plant until the results of the next update are available.

Model predictive control is part of the multi-level hierarchy of control structure. Using a numerical optimization scheme as an integral part of the structure allows great flexibility, especially concerning the incorporation of constraints. Though such optimization over a finite horizon does not guarantee stability and performance, considerable research has been devoted to address these shortcomings. Linear MPC theory and related issues such as closed-loop stability and online computation have been well studied and characterized [Magni, 2001, Mayne, et. al, 2000]. Over the past few years, nonlinear model predictive control (NMPC) schemes with some favorable properties have been developed. The theory related to stability of state and output feedback NMPC have reached a point of relative maturity, see for example [Chen and Allgower 1998, Findeisen et. al. 2003] for review.

2.3.2 Closed-loop Stability of NMPC Based on Nominal Model

A general sufficient condition for closed-loop stability of MPC based on nominal models is given below [Findeisen et. al. 2003].

Criterion 2.1 The terminal penalty function $W : X_f \to R_{\geq 0}$ and terminal set X_f are such that there exists a local feedback $k_f : X_f \to U$ satisfying

1. $0 \in X_f \subset X, X_f$ is closed
2. $W(x)$ is positive semi-definite and continuous with respect to $x \in R^{n_x}$
3. $k_f(x) \in U, \forall x \in X_f$
4. X_f is positively invariant under k_f
5. $L(x, k_f(x)) + \frac{\partial W}{\partial x} f(x, k_f(x)) \leq 0, \forall x \in X_f$

The conditions are primarily concerned with the selection of terminal region X_f and terminal penalty term $W(\cdot)$. Condition 5 requires $W(\cdot)$ to be a control Lyapunov function, over the (local) domain X_f , and dissipates energy at a rate $L(x, k_f(.))$. This criterion, which is able to re-cast many of the available MPC frameworks with guaranteed stability, provides a means of checking whether a given MPC scheme guarantees stability a-priori. Stability is proven by showing strict decrease of the optimal cost function J^*, which is a Lyapunov function for the closed-loop system. The domain of attraction for the controller is the set where the optimization problem is feasible.

2.3.3 NMPC for Uncertain Systems

The quality of the model used in MPC is crucial to the performance of the controller. The assumption that the prediction model is identical to the actual model is unrealistic. Although, due to the receding horizon policy, a standard implementation of MPC using a nominal model of the system dynamics exhibits nominal robustness to sufficiently small disturbances [Camacho and Bordons, 1995], such marginal robustness guarantee may be unacceptable in practical applications. Present model/plant mismatch and disturbances must be accounted for in the computation of the control law to achieve robust stability.

One way to cope with uncertainty in the system model is to employ robust MPC methods, which explicitly account for systems uncertainties. Since robust controllers (in general) cannot learn changes in the plant, their performance is limited by the quality of the model plus the uncertainty description initially available. On the other hand, adaptive control has the potential to improve system performance as it updates the model online based on measurement data. However, practical applications of adaptive controllers are limited by the conflicting objective of parameter estimation and control. This could lead to a worse transient performance than a non-adaptive controller when poor estimates are used. Moreover, the controller may induce large transient oscillations in an effort to improve the estimation quality.

2.3.4 Robust Model Predictive Control

Robust techniques have been employed in MPC designs to reduce the sensitivity of the controller to uncertainty. Consider the following uncertain system

$$\dot{x} = f(x, u, \nu) \tag{14}$$

where $\nu(t) \in D$ represents any arbitrary bounded uncertainty or disturbance signal. Many robust MPC techniques have been proposed to stabilize the uncertain system for all possible realization of the disturbance $\nu(t) \in D$. These include approaches based on nominal prediction [Magni et al 2003] and those based on min-max or worst-case techniques [Adetola and guay 2008].

The nominal based approach in [Marruedo et. sl. 2002] uses global Lipschitz constants to compute 1_1 worst-case upper bound on the distance between a solution of the actual uncertain model and the nominal model. These bounds are then used to redefine the terminal region and constraints in a way that guarantees robust feasibility of the closed-loop system. The controller proposed in [Limon et. al. 2005] is based on the concept of reachable sets. The approach uses a local procedure to approximate the sets that contain the predicted evolution of the uncertain system for all possible uncertainties. Then, a dual mode MPC strategy is proposed to robustly stabilize the system. The methods based on nominal prediction have similar computational complexity with standard NMPC but exhibit a higher level of conservatism.

A well embraced method for reducing conservatism in open loop min-max scheme and improve performance is to introduce some form of feedback in the prediction [Lee and Yu, 1997]. This can be achieved by parameterizing the control sequence in terms of the systems state. Unfortunately, such optimization with respect to closed-loop strategies is intractable (in most cases) since the problem size grows exponentially with the size of the problem data. In general, robust MPC is designed to meet control specifications for the " worst case" uncertainty. This approach may not always achieve optimal performance, in particular, if the worst case scenario rarely exists. Other approaches, such as adaptive control may yield a better performance.

2.3.5 Adaptive Model Predictive Control

Adaptive MPC is an attractive way to handle static uncertainties that can be expressed in the form of constant unknown model parameters. While a few results are available for linear adaptive MPC [Mayne et. al., 2000], only a small amount of progress has been made in developing adaptive NMPC schemes. Consider the parameter-affine nonlinear system of the form

$$\dot{x} = f(x, u, \theta), f(x, u) + g(x, u)\theta, \tag{15}$$

$$= f(x) + g_1(x)u + g_2(x)\theta \tag{16}$$

The result in [Adetola and Guay 2004], implements a certainty equivalence nominal-model MPC feedback to stabilize this system subject to an input constraint $u \in U$. Assuming the availability of the state vector $\dot{x}$, the identifier guarantees parameter convergence when an excitation condition is satisfied. It is only by assumption that the true system trajectory remains bounded during the identification phase. Moreover, there is no mechanism to enhance the satisfaction of the PE condition and thereby decrease the identification period.

In general, the design of adaptive nonlinear MPC schemes is very challenging because the " separation principle assumption" widely employed in linear control theory is not applicable to a general class of nonlinear systems, in particular in the presence of constraints. Moreover, it is difficult to guarantee state constraints satisfaction in the presence of an adaptive mechanism. A true adaptive nonlinear MPC algorithm must address the issue of robustness to model uncertainty while updating the systems parameters.

Recent work [DeHaan and Guay, 2007] has focused on the use of adaptation to improve upon the performance of robust MPC for constrained nonlinear systems. A set-valued description of the parametric uncertainty is directly adapted online to reduce the conservativeness of the robust MPC solutions, especially with respect to the design of the terminal penalty and constraints. The parameterization of the feedback MPC policy in terms of the uncertainty set and the underlying min-max feedback MPC used in the study make the controllers computation very challenging. The result can be viewed as a conceptual result that focus on performance improvement rather than implementation. The idea of coupling set-based identification with robust control calculations was extended in [DeHaan et. al. 2007] to a less computationally complex robust-MPC framework.

2.4 Summary

Concepts and principles of parameter and state estimation are reviewed in this chapter. An overview of recent developments in parameter and state estimation of systems has been presented.

Also, we have reviewed principle of model predictive control strategy and talked on robust model predictive control and adaptive model predictive control strategies.

An interesting problem is presented when, in addition to unknown parameters, states of the system are also unknown. The following chapters present adaptive model predictive approaches, which are applicable to a class of nonlinear system.

Chapter 3
Adaptive Receding Horizon Control
Of Input Constrained Nonlinear Systems

In this chapter a method for adaptive receding horizon control of nonlinear systems is introduced. Asymptotically stability and optimality in run of the closed loop systems in the presence of parametric uncertainty is obtained employing input to state stabilizing Lyuapunov control functions.

3.1 Introduction

Receding Horizon Control (RHC), usually called Model Predictive Control (MPC) has long been preferred tool for advanced control applications. The relative ease with which constrained can be incorporated has attracted great deal of interest both in academia and industry. They arise in a range of classical, as well as certain emerging, engineering applications [Qin and Badgwell, 2003] . Despite significant advances in this area still there is an obstacle that an accurate knowledge of the model is instrumental. As a result, its application remains constrained to processes with well-established model dynamics. However, in general, most physical systems possesses parameter uncertainties and unmeasurable parameters [Krstic et. al., 1995] and mechanisms to upgrade the unknowns or uncertainty parameters are highly appealing [Adetola and Guay, 2004].

The main goal of this chapter is to report an online adaptive integrated parameter estimation and RHC control method for input constrained nonlinear systems. To date, very few adaptive nonlinear RHC schemes are developed for nonlinear systems [Peter and Guay 2007, Adetola and Guay, 2004].

In the present work, we report a stable adaptive receding horizon scheme for parametric uncertain systems. The adaptive receding horizon control scheme developed in this report is based on the knowledge of an ISS-control lyapunov function (ISS-CLF) for the nonlinear system. The structure of the chapter is as follows. First, we review some technical background and then introduce and formulate the problem, thereby an Adaptive RHC+CLF problem is constructed in Section 3. However, the problem can not be solve until a reliable estimation strategy be used which is stated in Section 4. Finally we prove asymptomatically stability of the proposed scheme.

3.2 Mathematical Background

Consider the function $W : D \times R^+ \to R$. Assume $0 \in D$ and $W(x,t)$ is continuous and has continuous partial derivatives to all its arguments.

Definition 3.2.1: W is said to be positive definite in D if

$$W(0,t) = 0, \quad \forall t \in R^+$$

$$W(x,t) > 0, \quad \forall x \neq 0, x \in D.$$

Definition 3.2.2: $W(x,t)$ is said to be decrescent in D if there exist a positive definite function V(x) such that

$$|W(x,t)| \leq V(x), \quad \forall x \in D$$

Definition 3.2.3: $W(x,t)$ is radically unbounded if

$$|W(x,t)| \to \infty \quad as \quad |x| \to \infty \quad \text{uniformly in t}$$

Definition 3.2.4: Consider the system

$$\dot{x} = f(x,t),$$

$f : D \times R^+ \to R^n$ Where f is point-wise continuous in t on $D \times [0, \infty]$. $x = 0 \in D$ is an equilibrium point if

$$f(0,t) = 0, \forall t \leq t_0.$$

Theorem 3.2.1: If in a neighborhood D of x=0 there exists function $W(\cdot,\cdot), D \times [0,\infty) \to R$ such that

i) $W(\cdot,\cdot)$ is positive definite, and,

ii) The derivation of $W(\cdot,\cdot)$ along any solution of system f(x,t) is negative semi definite in D, then,

This equilibrium point is stable and $W(\cdot,\cdot)$ is called a lyapunov function.

Moreover, If $W(\cdot,\cdot)$ is also decrescent then the origin is uniformly stable.

Theorem 3.2.2: If in a neighborhood D of x=0 there exists function $W(\cdot,\cdot), D\times[0,\infty)\to R$ such that

i) $W(\cdot,\cdot)$ is positive definite and decrescent, and,

ii) The derivation of $W(\cdot,\cdot)$ is negative definite in D, then,

The equilibrium state is uniformly asymptotically stable.

Theorem 3.2.3: If there exist function $W(\cdot,\cdot), D\times[0,\infty)\to R$ such that

i) $W(\cdot,\cdot)$ is positive definite, decrescent and radically unbounded, and,

ii) The derivation of $W(\cdot,\cdot)$ is negative definite for $\forall x\in R^n$, then

The equilibrium state is uniformly asymptotically stable.

Definition 3.2.5 : A continuous function $\alpha(r)$ defined over $r\in[0,a]$ is said to belong to class K if it is strictly increasing and $\alpha(0)=0$. It belongs to class K_∞ if $a=\infty$ and $\alpha(r)\to\infty$ as $r\to\infty$.

Definition 3.2.6 : A continuous function $\beta:[0,a)\times[0,\infty)\to[0,\infty)$ is said to belong to class KL if for any fixed s, the mapping $\beta(r,s)$ belong to class K respect to r, and for any fixed r, the mapping $\beta(r,s)$ is decreasing respect to s, and $\beta(r,s)\to 0$ as $s\to\infty$.

Consider the system

$$\dot{x}=f(t,x,u)$$

where $f:[0,\infty]\times R^n\times R^m\to R^n$ is piecewise continuous in t and locally litschitz in x and u. The input $u(t)$ is piecewise continuous function of t for all $t\geq 0$.

Definition 3.2.7: The system defined above is said to be input-to-state stable if there exist a KL function β and a class K function γ such that or any initial state $x(t_0)$ and any bounded input $u(t)$, the solution $x(t)$ exists for all $t\geq t_0$ and satisfies

$$|x(t)|\leq\beta(|x(t_0),t-t_0|)+\gamma(sup_{\tau\in(t_0,t)}(|u(\tau)|))$$

3.3 Problem Description

We consider the nonlinear system,

$$\dot{x}=f(x)+P(x)d+g(x)u,\quad x\in R^n,u\in R^m \tag{1}$$

where $f(x), F(x)$ and $g(x)$ are smooth. For simplicity we consider $f(0) = 0, F(0) = 0$ so that $x = 0$ is the equilibrium of the uncontrolled system.

Definition 3.1: A smooth positive definite radically unbounded function $V : R^n \to R_+$ is called an ISS-control Lyapunov function (ISS-CLF) for eq(1) if there exist class K functions α_1, α_2 and a class K_∞ function ρ such that $\alpha_1(\|x\|) \leq V \leq \alpha_2(\|x\|)$ and the following holds for all $x \neq 0$ and all $d \in R^r$

$$\|x\| \geq \rho(\|d\|)$$
$$\inf_{u \in R^m}\{\frac{\partial V}{\partial x}[f(x) + P(x)d + g(x)u]\} < 0. \tag{2}$$

Note 3.1: The existence of an ISS-CLF guarantees that the nonlinear systems eq(1) is input to state stable with respect to the disturbance input d.

In this chapter we consider nonlinear systems of the form

$$\dot{x} = f(x) + P(x)\theta + g(x)u, \tag{3}$$

where $x \in R^n$ and $u \in R^m$ are systems measurable states and control inputs respectively, $\theta \in R^p$ is the vector of unknown constant parameters. $f(x) : R^n \to R^m$ is a smooth vector function, $P(x) : R^n \to R^{m \times p}$ and $g(x) : R^n \to R^{m \times m}$ are smooth matrix valued functions.

3.4 Adaptive MPC

Computation of optimal input control action in model predictive control greatly relies on the knowledge of parameter estimates, however, since there is no guarantee that the parameter estimations approach their true value, the existence of offset is inevitable. The scheme presented here ensures the robust stability of the controller to the estimation errors; Fist, we develop a receding horizon controller based on the known ISS-CLF function to stabilize the nonlinear system, and then an adaptive estimation procedure is developed to estimate the parameters. Combination of the two schemes globally asymptotically stabilizes the origin of the closed loop system. We define the unknown parameter estimation error

$$\tilde{\theta} = \theta - \hat{\theta}$$

where $\hat{\theta}$ are the assumed known parameter estimates and use the substitution of $P(x)\theta = P(x)\hat{\theta} + P(x)\tilde{\theta}$ to rewrite the eq(3) as,

$$\dot{x} = f(x) + P(x)\hat{\theta} + g(x)u + P(x)\tilde{\theta}, \tag{4}$$

For this system it is supposed that an ISS-CLF function, say V, is known. The problem of determining an ISS-CLF for constrained systems has become the focus of research recently and considerable progress has been reported in [Primbs, 1999]. Based on the available ISS-CLF V we can introduce the following pointwise min-norm problem,

$$\min \quad u^T u \tag{5}$$

$$\text{subject to} \quad \dot{V} \leq -\hat{\sigma}(x(t)) \tag{6}$$

with $\hat{\sigma}(x(t)) > 0$.

Since the input is unbounded, the stability constraint may make the problem infeasible for an arbitrary choice of $-\hat{\sigma}(x(t)$ therefore its value must be properly chosen to avoid infeasibility. We propose to accomplish this by solving the following optimization problem in u and ψ:

$$\min \quad u^T u + \lambda\psi^2 \tag{7}$$

$$\text{subject to} \quad \dot{V} \leq -\sigma(x(t)) + \psi \tag{8}$$

$$\psi \geq 0 \tag{9}$$

$$-\sigma(x(t)) + \psi \leq 0 \tag{10}$$

and set,

$$\hat{\sigma}(x(t)) = \sigma(x(t)) - \psi \tag{11}$$

with $\lambda > 0$ a design parameter to be chosen. The other design parameter is σ which can be chosen with lesser restrictions. We choose it based on the Sontag's unconstrained formula,

$$\sigma = \sqrt{a^2 + q(x)\|b\|^2} \tag{12}$$

where $a = \frac{\partial V}{\partial x} f + \frac{\partial V}{\partial x} P(x)\hat{\theta} + \|\frac{\partial V}{\partial x} P\|\rho^{-1}(\|x\|), b = \frac{\partial V}{\partial x} g$ and $q(x)$ is some positive definite function.

The new re-formulated problem can be looked at as a point-wise min-norm problem in which the objective function contains the penalty term $\lambda\psi^2$, which is used to soften the constraints and avoid the infeasibility. For each arbitrarily large but finite λ, this problem is always feasible due to the existence of the ISS-CLF function V, (see eq (2)).

Lemma 3.4.1.: Let $(u^*; \psi^*)$ be the optimal solution of the problem (5)-(6) for any given state $x(t)$; then u^* is also the optimal solution of (7)-(8) with $\hat{\sigma}(x(t)) = \sigma(x(t)) - \psi^*$.
Proof: The proof is similar to that of Lemma 7.3.1. in [Primbs, 1999].
The implication of Lemma 1 is that allows us to always refer to the point-wise min-norm problem (5)-(6), even though the problem (7)-(8) is effectively solved in order to obtain a feasible solution. By determining a feasible $\hat{\sigma}$ for the constrained point-wise min-norm problem, we extend formulations for an Adaptive RHC+CLF scheme, given by,

$$\min_u J = \int_t^{t+T} q(x(\tau), u(\tau))d\tau \tag{13}$$

subject to

$$\dot{x} = f(x) + P(x)\bar{\theta} + g(x)u \tag{14}$$

$$\bar{\theta} = \hat{\theta} \tag{15}$$

$$V(x(t+T)) \leq V(x^{\text{iss}}(t+T)) \tag{16}$$

$$\dot{V} \leq -\hat{\sigma}(x(t)) \tag{17}$$

where $x^{\text{iss}}(t+T)$ is the state prediction for the model subject to the min-norm based controller problem starting at the current state $x(t)$ with the current parameter estimation $\hat{\theta}(t)$. In RHC+CLF formulation the unknown parameter $\bar{\theta}$ in eq (15) replaces with the last estimated parameter value. The optimizer computes the required control moves over prediction horizon T. $u(k|k)$is implemented on the plant form the time step k to $k+1$ and then new estimates of the unknown parameter $\hat{\theta}(k)$ is obtained from the parameter update law. The prediction and control horizons are shifted forward by one step and a new optimization problem is solved at time $k+1$ and the procedure repeats by the end of the control horizon.
Notice that, in the formulation (13)-(17), $\hat{\theta}$ is supposed to be constant over the interval [t,t+T] and therefore the error in the actual system model would appear as $P(x)\tilde{\theta}(t)$. In the

remainder we study stability of RHC+CLF scheme to design parameter update law $\dot{\hat{\theta}}$.

3.5 Estimation of Uncertainty

Let $x^*(\tau)$ be the state trajectory resulting from the proposed RHC+CLF scheme. Consider the function,

$$W = \frac{1}{T}\int_t^{t+T} V(x^*(\tau))d\tau \tag{18}$$

This function is positive definite and radically unbounded if V be positive definite and radically unbounded. Differentiating W respect to t we get,

$$\dot{W} = \frac{1}{T}(V(x^*(t+T)) - V(x(t)))$$

Following the constraint eq(17) and eq(18) we can write

$$\dot{W} \leq -\frac{1}{T}\int_t^{t+T} \hat{\sigma}(x^{\text{iss}}(\tau)). \tag{19}$$

Note that, in the context of the RHC-CLF, over the interval $[t, t+T]$ the parameter estimates are considered as constants which produce the measurable error term $\frac{\partial V}{\partial x}P(x^{\text{iss}}(\theta))\tilde{\theta}(t)$ along the trajectories of the nominal system, and therefore, employing some sort of certainty equivalence thinking, we can rewrite it as,

$$\dot{W} \leq -\frac{1}{T}\int_t^{t+T} \hat{\sigma}(x^{\text{iss}}(\tau)) + \frac{1}{T}\int_t^{t+T} \frac{\partial V}{\partial x}P(x^{\text{iss}}(\tau))d\tau\tilde{\theta}(t). \tag{20}$$

In the following we employ this function to provide a state prediction routine and a parameter update law. The predicted states, x_p using $\hat{\theta}$ are generated by the dynamical system,

$$\dot{x}_p = f(x) + P(x)\hat{\theta} + g(x)u + K(x - x_p). \tag{21}$$

Defining the prediction error by $e = x - x_p$ we can write the prediction error dynamic as,

$$\dot{e} = P(x)\tilde{\theta} - Ke. \tag{22}$$

We augment the Lyapunov function as,

$$V_1 = W(x) + \frac{1}{2}e^T e + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{23}$$

whose derivative using the eq (20) is,

$$\dot{V}_1 \leq -\frac{1}{T}\int_t^{t+T} \hat{\sigma}(x^{\text{iss}}(\tau))d\tau + \frac{1}{T}\int_t^{t+T} \frac{\partial V}{\partial x}P(x^{\text{iss}}(\tau))d\tau\tilde{\theta} + e^T(P(x)\tilde{\theta} - Ke) - \dot{\hat{\theta}}^T\Gamma\tilde{\theta}, \quad (24)$$

Let

$$\Psi = \Gamma P(x)^T e - \Gamma(\frac{1}{T}\int_t^{t+T} \frac{\partial V}{\partial x}P(x^{\text{iss}}(\tau))d\tau)^T \quad (25)$$

where $\Gamma = \Gamma^T > 0$ is a tuning parameter to control the rate of the adaptation of the parameters. To produce bounded parameter estimates we employ the parameter projection law as defined in chapter 2 given by,

$$\dot{\hat{\theta}} = \text{Proj}\{\hat{\theta}, \Psi\} \quad (26)$$

Using this projection algorithm we take,

$$\dot{V}_1 \leq -\frac{1}{T}\int_t^{t+T} \hat{\sigma}(x^{\text{iss}}(\tau))d\tau - \frac{1}{2}e^T Ke \quad (27)$$

which is semi-definite with respect to e, x and $\tilde{\theta}$.

Theorem 3.5.1: (Lasalle-Yoshizawa's theorem)[Khalil, 2002] Let $V(x)$ be a continuously differentiable function of the states x such that,

i) $V(x)$ is positive definite

ii) $V(x)$ is radically unbounded

iii) $\dot{V}(x) \leq -R(x)$, where R(x) is positive semi-definite.

Then, all the solutions of the system satisfy

$$\lim_{t\to\infty} R(x(t)) = 0$$

And if W(x) is positive definite, then the equilibrium x=0 is globally asymptotically stable.

Lemma 3.5.1: (Barbalat Lemma) [Khalil, 2002] Let $\Phi : R \to R$ be a uniformly continuous function on $[0, \infty]$. Suppose that $\lim_{\tau\to\infty}\Phi(\tau)d\tau$ exists and is finite. Then, $\Phi(t) \to 0$, as $t \to \infty$

By Lasalle-Yoshizawa's theorem, we conclude that in (27) $e, \tilde{\theta}$ and x are bounded, and based on Barbalat Lemma, e and x converge to the origin. The main result of the report is summarized in the following Theorem 3.5.2.

Theorem 3.5.2: The Adaptive RHC+CLF scheme (13)-(17)and the adaptive law (26) globally asymptotically stabilize the origin of the system (3).

3.6 Simulation Examples

For illustration of the effectiveness of the proposed approach a simulation test has been done on an modified example taken from [Primbs, 1999]. The purpose is to asymptotically stabilize the Van der Pol system given by,

$$\dot{x}_1 = x_2 \tag{28}$$

$$\dot{x}_2 = -x_1(\frac{\pi}{2} + arctan(5x_1)) - \frac{5(0.5 + x_1^2)}{2(1 + 25x_1^2)} + u \tag{29}$$

where x_1 and x_2 are the states, u is the control input. The constructed ISS-CLF function is

$$V^{iss} = \frac{1}{2}x_1^2 + \frac{1}{2}(x_1 + x_2)^2 \tag{30}$$

We construct the minimization function,

$$\mathrm{J} = \int_t^{t+T} (x_1^2(t) + u^2(t))dt \tag{31}$$

s.t.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1(\frac{\pi}{2} + arctan(5x_1)) - \frac{5(0.5 + x_1^2)}{2(1 + 25x_1^2)} + u$$

$$V(x(t + T)) \leq V(x^{iss}(t + T)$$

$$T = 3, dt = 1, x_0 = [3, -2]^T$$

Results of the simulation is shown in the Figure 1. It is clear that the developed ISS-CLF algorithm can stabilized the system easily.

In the next step we modify the system and consider stabilization of an amplifier with unknown parameters. Consider the system defined by,

$$\dot{x}_1 = x_2 \tag{32}$$

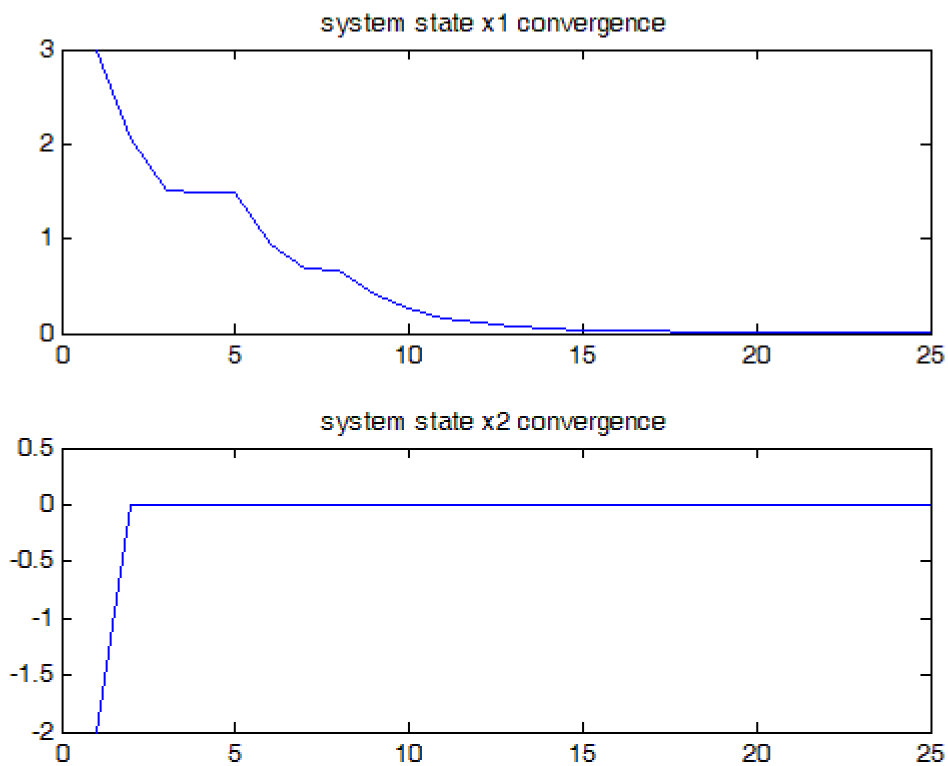

Figure 1: (a) convergence of the system states to the origin

$$\dot{x}_2 = -x_1(\frac{\pi}{2} + arctan(5x_1)) - \frac{\theta(0.5 + x_1^2)}{2(1 + 25x_1^2)} + u \tag{33}$$

where x_1 and x_2 are the states, u is the control input, and θ is the unknown parameter. We take the real value of $\theta = 5$ and try to develop an estimation algorithm to converge to this value.

$$\text{J} = \int_t^{t+T} (x_1^2(t) + u^2(t))dt \tag{34}$$

s.t.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1(\frac{\pi}{2} + arctan(5 * x_1)) - \frac{\bar{\theta}(0.5 + x_1^2)}{2(1 + 25x_1^2)} + u$$

$$\bar{\theta} = \hat{\theta}$$

$$V(x(t+T)) \leq V(x^{iss}(t+T)$$

The estimation algorithm is as follows,

$$\dot{\hat{\theta}} = -\Gamma \frac{(0.5 + x_1^2)(x_2 - x_2p)}{2 + 50x_1^2} \tag{35}$$

$$+ \int_T (x_1^{\text{iss}}(\tau) + x_2^{\text{iss}}(\tau)) \frac{0.5 + x_1^{\text{iss}}(\tau)^2}{2 + 50x_1^{\text{iss}}(\tau)^2} d\tau$$

$$\dot{x}_1 = x_2 + K_1(x_1 - x_1p)$$

$$\dot{x}_2 = -x_1(\frac{\pi}{2} + arctan(5 * x_1)) - \frac{\bar{\theta}(0.5 + x_1^2)}{2(1 + 25x_1^2)} + u + K_2(x_2 - x_2p)$$

where the parameters are as follows,

$$K_1 = K_2 = 2, T = 0.6, x(0) = [3, -2]^T, x_p(0) = [0, 0]^T, \tag{36}$$

$$\theta(0) = 0, \theta_{\text{real}} = 5 \tag{37}$$

Results of the simulation shown in Figure 2 and Figure 3 prove the efficiency of the method in estimation of the parameters of the system and also in optimal running of the system to the origin.

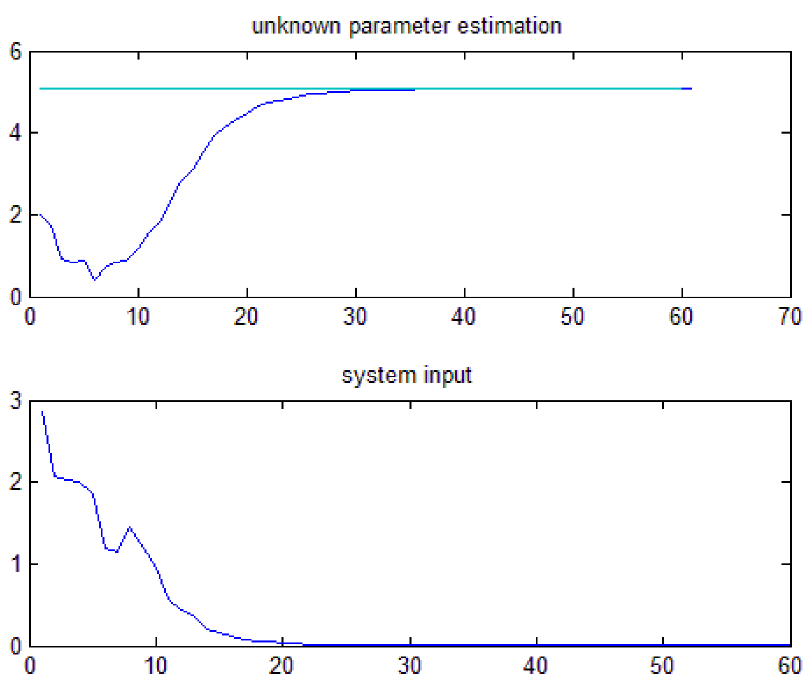

Figure 2: (a) shows convergence of the parameters estimation and (b) is the system input

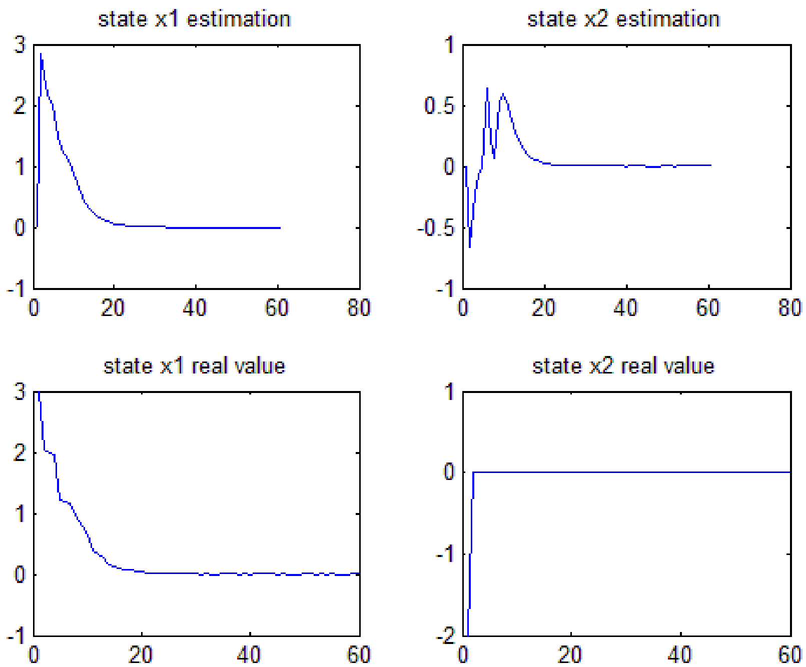

Figure 3: the system state estimations and corresponding real values

3.7 Summary

In this chapter we introduced a method for adaptive receding horizon control of nonlinear systems . Asymptotically stability and optimality of the closed loop systems in the presence of parametric uncertainty has been shown based on input-to-state stabilizing Lyuapunov control functions. At the end, we used two simulation to prove the efficiency of the proposed method.

Chapter 4
Passivity Based Parameter Estimation in Adaptive Control of Nonlinear Systems

In this chapter a method for adaptive parameter estimation of nonlinear systems with imperfect state measurement is introduced. Globally uniform boundedness of all system signals in the presence of parametric uncertainty is obtained through stability analysis.

4.1 Introduction

There are control problems whereby the reference trajectory is not known a priori but depends on the unknown parameters of the system dynamics. The controller finds the operating set-points which optimizes a performance or cost function and tries to run the system to that point. The uncertainty associated with the function makes it necessary to use some sort of adaptation and perturbation to search for the optimal operating condition. However, the main challenges with adaptive control approaches lies with the ability to recover the true unknown values of the parameters. In most approaches exact reconstruction of the unknown parameters in finite time (FT) is obtained provided a given persistence of excitation (PE) condition is satisfied. A common approach to ensuring a PE condition in adaptive control is to introduce a perturbation signal as the reference input or to add it to the target set point or trajectory, however, this constant PE in many cases deteriorates the desired tracking or regulation performance. Recently a finite time parameter estimation method is developed in [Adetola and Guay, 2008] which introduces such a PE signal and remove it when the parameters are assumed to have converged. In this chapter, we remove this assumption and consider problems where only a part of the state and just the scalar plant output is available for measurement. For these systems, we build exponentially convergent nonlinear observers and replace the unmeasured states by their estimates. Then, develop a model predictive control strategy to run the system to the origin and study stabilizing property of the optimization problem in the next chapter.

4.2 Mathematical Background

Before starting the main algorithm we briefly review some basic passivity results. The concept of passivity has been used principally in network synthesis and became a fundamental feedback control concept in Popov [Popov 1966].

Consider the system

$$\dot{x} = P(x, u) \tag{1}$$

where $x \in R^n$ is the state and $u \in R^m$ is the input the system.

Definition 4.2.1:The System (1) is said to be passive if there exist a continuous nonnegative storage function $S : R^n \times R_+ \to R_+$ which satisfies $S(0, t) = 0, \forall t \geq 0$ such that for all $u \in C^0, x(0) \in R^n, t \geq t_0 \geq 0$,

$$\int_{t_0}^{t} y^T(\sigma)u(\sigma)d\sigma \geq S(x(t), t) - S(x(t_0), t_0). \tag{2}$$

Definition 4.2.2: The System (7) is said to be strictly passive if there exist a continuous nonnegative storage function $S : R^n \times R_+ \to R_+$ which satisfies $S(0, t) = 0, \forall t \geq 0$ and a positive definite function (dissipation rate) $\psi : R^n \to R_+$, such that for all $u \in C^0, x(0) \in R^n, t \geq t_0 \geq 0$

$$\int_{t_0}^{t} y^T(\sigma)u(\sigma)d\sigma \geq S(x(t), t) - S(x(t_0), t_0) + \int_{t_0}^{t} \psi(x(\sigma))d\sigma. \tag{3}$$

Lemma 4.2.1: Suppose the System (7) is strictly passive. If S is positive definite, radically unbounded, and decresecent, then for $u \equiv 0$ the equilibrium $x = 0$ of (7) is globally uniformly asymptotically stable.

4.3 Problem Description

The considered system is the following nonlinear parametric affine system

$$\dot{x} = f(x, y) + F(x, u)^T\theta, \tag{4}$$

$$y = h(x, u), \tag{5}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^r$ is the output, and $\theta \in R^p$ is the unknown parameter vector to be identified which lies within an initially known compact set Ω_θ. Also, it is supposed that $f(x, y), F(x, u)$ and $h(x, u)$ are smooth matrix valued functions in x as,

$$f(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ \vdots \\ f_n(x,y) \end{bmatrix}, F(x,y) = \begin{bmatrix} F_1(x,y) \\ F_2(x,y) \\ \vdots \\ F_n(x,y) \end{bmatrix}. \tag{6}$$

Also $f(0,t) = 0$ and $h(0,t) = 0$ for all $t \geq 0$.

The main goal of this note is to provide the true estimates of the plant parameters in FT while preserving the properties of the controlled closed-loop system and therefore it is assumed that there is a known bounded control law to, upon the control objective, (robustly) stabilize the plant and/or to force the output to track a reference signal.

To prepare for the parameter identification procedure to be presented in the next section, we consider simplicity in the equation (5) as $h(x,u) = e_1^T x$ and rewrite the system as,

$$\dot{x} = f(x,y) + F(x,u)^T \theta, \tag{7}$$

$$y = x_1 = e_1^T x, \tag{8}$$

however, emphasize that it implies no restriction on the whole parameter estimation procedure.

4.4 Infinite Time Parameter Identification

Denoting the state predictor for (7) as $\hat{x}$ we define

$$\dot{\hat{x}} = f(x,y) + F(x,u)^T \hat{\theta} + A(x - \hat{x}), \tag{9}$$

where $\hat{\theta}$ is a parameter estimate generated via the update law to be developed and the state estimation error will be $\varepsilon = x - \hat{x}$ with the dynamic governed by,

$$\dot{\varepsilon} = A\varepsilon. \tag{10}$$

We introduce the filters

$$\dot{\bar{\Omega}}_0 = -A(\bar{\Omega}_0 + y) - f(x,y), \tag{11}$$

$$\dot{\bar{\Omega}} = -A\bar{\Omega} + F(x,u), \tag{12}$$

and denote the output estimation as,

$$\hat{y} = \bar{\Omega}_0 + \bar{\Omega}^T \hat{\theta} \tag{13}$$

which result in output estimator error,

$$\epsilon = y + \bar{\Omega}_0 - \bar{\Omega}^T \hat{\theta} \tag{14}$$

Substituting (11) and (12) into (14) we get,

$$\epsilon = \bar{\Omega}^T \tilde{\theta} + \tilde{\epsilon} \tag{15}$$

where $\tilde{\epsilon}$ is governed by,

$$\dot{\tilde{\epsilon}} = -A\tilde{\epsilon} + \varepsilon. \tag{16}$$

We define the update law for $\tilde{\theta}$ as,

$$\dot{\hat{\theta}} = \text{Proj}\{\Gamma \frac{\bar{\Omega}\epsilon}{1 + \nu |\bar{\Omega}|^2}\}. \tag{17}$$

$$\Gamma = \Gamma^T \geq 0, \nu > 0,$$

Lemma 4.4.1: Consider the operator D defined as,

$$D : \tilde{\theta} \rightarrow \bar{\Omega}\epsilon \tag{18}$$

$$\dot{\bar{\Omega}} = -A\bar{\Omega} + F(x, u), \tag{19}$$

$$\epsilon = \bar{\Omega}^T \tilde{\theta} + \tilde{\epsilon} \tag{20}$$

$$\dot{\tilde{\epsilon}} = -A\tilde{\epsilon} + \varepsilon. \tag{21}$$

$$\dot{\hat{\theta}} = \text{Proj}\{\Gamma \frac{\bar{\Omega}\epsilon}{1 + \nu \|\bar{\Omega}\|^2}\}, \Gamma = \Gamma^T \geq 0, \nu > 0 \tag{22}$$

This system is strictly passive. Hence, the equilibrium $\epsilon = 0$ and $\tilde{\theta} = 0$ are globally uniformly stable.

Proof: Using (15) we will have,

$$\int_0^t (\bar{\Omega}\epsilon)^T \tilde{\theta} d\tau = \int_0^t \epsilon^T \bar{\Omega}^T \tilde{\theta} d\tau = \int_0^t \epsilon^T (\epsilon - \tilde{\epsilon}) d\tau \tag{23}$$

$$= \int_0^t (\|\epsilon - \frac{1}{2}\tilde{\epsilon}\| - \frac{1}{4}\|\tilde{\epsilon}\|^2) d\tau \geq -\frac{1}{4}\int_0^t \|\tilde{\epsilon}\|^2 d\tau$$

on the other hand from using (16) and (10) by letting $c = \underline{\lambda}(A)$ we have,

$$\frac{1}{2}\frac{d}{dt}(\tilde{\epsilon}^2 + \frac{1}{c}\|\varepsilon\|_P^2) \leq -c\tilde{\epsilon}^2 + \varepsilon\tilde{\epsilon} - \frac{1}{2c}\|\varepsilon\|^2 \tag{24}$$

$$\leq -\frac{c}{2}\tilde{\epsilon}^2 - \frac{c}{2}(\tilde{\epsilon} - \frac{1}{c}\varepsilon)^2 + \frac{1}{2c}\varepsilon^2 - \frac{1}{2c}\|\varepsilon\|^2$$

$$\leq -\frac{c}{2}\tilde{\epsilon}^2 - \frac{1}{2c}\|\varepsilon\|^2.$$

Using this and (10) we can deduce,

$$\frac{d}{dt}(\frac{1}{2}\|\tilde{\epsilon}\|^2) \leq -c_0\|\tilde{\epsilon}\|^2 \tag{25}$$

with suitable choice of c_0. Integrating this equation we come to,

$$-\frac{1}{4}\int_0^t \|\tilde{\epsilon}\|^2 d\tau \geq \frac{1}{2c_0}\|\tilde{\epsilon}(t)\|^2 - \frac{1}{2c_0}\|\tilde{\epsilon}(0)\|^2 + \frac{3}{4}\int_0^t \|\tilde{\epsilon}\|^2 d\tau \tag{26}$$

and substituting (26) in (23) we obtain,

$$\int_0^t (\bar{\Omega}\epsilon)^T \tilde{\theta} d\tau \geq \frac{1}{2c_0}\|\tilde{\epsilon}(t)\|^2 - \frac{1}{2c_0}\|\tilde{\epsilon}(0)\|^2 + \frac{3}{4}\int_0^t \|\tilde{\epsilon}\|^2 d\tau \tag{27}$$

which proves that the operator (18) is strictly passive.

Remark 4.4.1: Using the globally uniformly stability of ϵ and $\tilde{\theta}$, based on the defined update law we can show that $\dot{\hat{\theta}}$ is bounded on its maximal interval of exitance.

Theorem 4.4.1: All the signals in the closed loop adaptive system consisting of the plant (7)-(8), filters (11)-(12) and the update law (17) are globally uniformly bounded and in particular, $\lim_{t\to\infty}\epsilon = \lim_{t\to\infty}\dot{\hat{\theta}} = 0$. and, $\lim_{t\to\infty}(\hat{\theta} - \theta) = 0$.

Proof: Combining (11), (12) and (14), we get,

$$\dot{\epsilon} = A_0\epsilon + F(x,u)^T\tilde{\theta} - \bar{\Omega}^T\dot{\hat{\theta}} \tag{28}$$

Becuase of the boundness of all the signals, $\dot{\epsilon}$ is bounded. Since ϵ is asymptotically stable, hence,

$$\lim_{t\to\infty}\int_0^t \dot{\epsilon}(\tau)d\tau = \lim_{t\to\infty}\epsilon(t) - \epsilon(0) = -\epsilon(0) < \infty.$$

So, by Barbalat's lemma we have $\dot{\epsilon} \to 0$. Now, since, $\dot{\hat{\theta}}$ is globally uniformly stable, we conclude that $F(x,u)^T\tilde{\theta}(t) \to 0$, and if we provide the richness condition:

$$\frac{1}{T}\int_t^{t+T} F(x,u)F(x,u)^T \geq c_0 I, \quad c_0 > 0 \tag{29}$$

then, the parameter error $\tilde{\theta}$ will converge to zero asymptotically.

Remark 4.2.2: The benefit of using this special formulation is that in construction of the filters we dont need just to measure the whole state x.

Remark 4.4.3: The result in Theorem 4.4.1 is independent of the control input u.

4.4 Finite Time Parameter Identification

In this section we are going to formulate a finite time parameter identification algorithm.

Lemma 4.4.1: Consider again the equation (16) and(15),

$$\epsilon = \bar{\Omega}^T\tilde{\theta} + \tilde{\epsilon}$$

$$\dot{\tilde{\epsilon}} = -A\tilde{\epsilon} + \varepsilon$$

Define,

$$\mu = \varepsilon - \epsilon\tilde{\theta} \tag{30}$$

$$\dot{\mu} = -A\mu, \mu(0) = \epsilon(0) \tag{31}$$

and matrixes,

$$\dot{Q} = \epsilon^T\epsilon, Q(0) = 0 \tag{32}$$

$$\dot{C} = \epsilon^T(\epsilon\hat{\theta} + \varepsilon - \mu) \tag{33}$$

Suppose there exists a time t_c and a constant $c_1 > 0$ such that $Q(t_c)$ is invertible i.e.

$$Q(t_c) = \int_{t_0}^{t} \epsilon(\tau)^T\epsilon(\tau)d\tau > c_1 I$$

Then,

$$\theta = Q(t)^{-1}C(t), \forall t \geq t_0 \tag{34}$$

Proof: consider,

$$Q(t)\theta = \int \epsilon^T(\tau)\epsilon(\tau)[\hat{\theta}(\tau) + \tilde{\theta}(\tau)]d\tau \tag{35}$$

Using the fact that $\epsilon - \mu = \epsilon\tilde{\theta}$, it follows that,

$$\theta = Q(t)^{-1}\int C(\tau)d\tau = Q(t)^{-1}C(t).$$

Remark 4.4.1: The result obtained here is independent of the control u and parameter identifier $\dot{\hat{\theta}}$ used for parameter estimation, Hence we use the nominal estimate θ^0 with no parameter adaption. So if

$$\theta^c = Q(t_c)^{-1}C(t_c)$$

The finite time identifier is,

$$\hat{\theta}^c = \begin{cases} \theta^0 & \text{if } \; t < t_c \\ \theta_c & \text{if } \; t > t_c \end{cases}$$

The benefit of using this algorithm is that we can predict when PE condition of - invertibility of Q - is satisfied and to turn off the identification machine after that.

4.6 Supporting Example

In this section we consider an example of identification in a nonlinear system to show the efficiency of the proposed algorithm.

Consider the system,

$$\dot{x}_1^2 = x_2 + x_1^2\theta \tag{36}$$

$$\dot{x}_2 = u \tag{37}$$

with the parameters,

$$x_0 = [0, 10]^T, \theta_0 = 0, \theta_{real} = 5$$

$$N = 8, dt = 0.1$$

The results of simulation show the efficiency of the proposed method. Figure 1 shows convergence of the parameters estimation and the system input. It is clear that the system states converges to the real values. Figure 2 shows the system state estimations and corresponding real values. Estimation of the system state can track the real states of the system very well.

4.7 Summary

In this chapter a method for adaptive parameter estimation of nonlinear systems was introduced. Globally uniform boundedness of all system signals in the presence of parametric

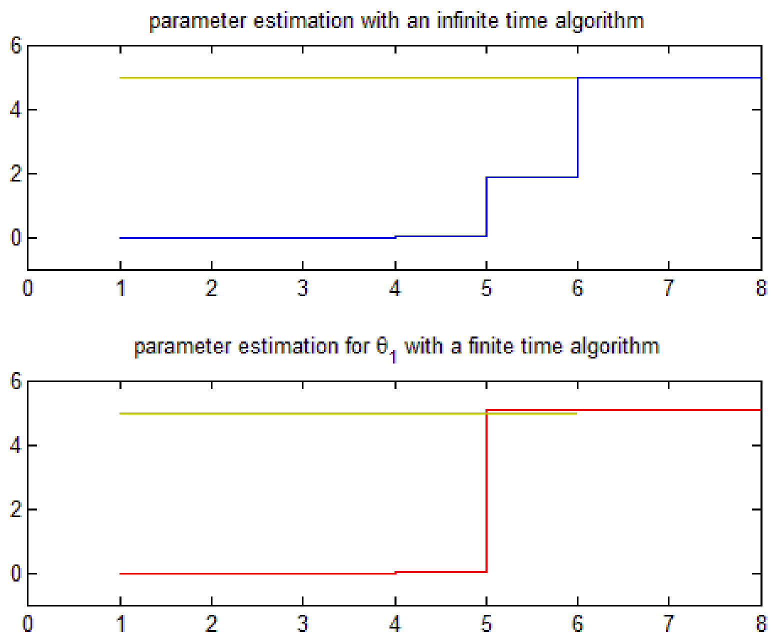

Figure 1: (a) shows convergence of the parameters estimation and (b) is the system input

uncertainty was obtained through stability analysis. In the next chapter results of this chapter will be used to build up a model predictive control scheme to steer the system to the origin according to the estimated values of the parameters.

Chapter 5
Adaptive Predictive Control of Nonlinear Systems: An Application to a CSTR system

In this chapter a method for adaptive predictive control of nonlinear systems is presented. This method is based on finite time identification algorithm derived in the last chapter. The efficiency of the proposed scheme is examined on an example of the CSTR system.

5.1 Introduction

There are control problems whereby the reference trajectory is not known a priori but depends on the unknown parameters of the system dynamics. The controller finds the operating set-points that optimize a performance or cost function. The uncertainty associated with the function makes it necessary to use some sort of adaptation and perturbation to search for the optimal operating condition. One of the main challenges with model based or adaptive control approaches is the ability to recover the true unknown values of the parameters. In most approaches exact reconstruction of the unknown parameters in finite time (FT) is obtained provided a given persistence of excitation (PE) condition is satisfied. A common approach to ensuring a PE condition in adaptive control is to introduce a perturbation signal as the reference input or to add it to the target set point or trajectory, however, this constant PE in many cases deteriorates the desired tracking or regulation performance. In last chapter we introduced a finite time parameter estimation method which introduces such a PE signal and remove it when the parameters are assumed to have converged.

In the present chapter , predictive control of uncertain nonlinear systems subject to state and input constraint is considered. We develop a new adaptive predictive control scheme that enjoy all the desired properties without becoming computationally prohibitive. The scheme is based on two main ideas. First, by working on a suitable estimation scheme, it is shown how to obtain a finite time estimation algorithm. Second, by use of the predictive control strategy, the system is running to the desired stable point. Stability of the predictive control algorithm is enforced by constraining the terminal state to belong to the estimated stability region.

5.2 Problem Statement

The system considered is the following nonlinear parameter affine system

$$\dot{x} = f(x, y) + F(x, u)^T \theta, \tag{1}$$

$$y = h(x, u), \tag{2}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^r$ is the output, and $\theta \in R^p$ is the unknown parameter vector to be identified which lies within an initially known compact set Ω_θ. Also, it is supposed that $f(x, y)$ and $F(x, y)$ are smooth matrix valued function.

5.3 Parameter Identification Algorithm

In the last chapter a finite time identification algorithm has been developed that we review here.

Denoting the state predictor for (1) as $\hat{x}$ we define

$$\dot{\hat{x}} = f(x, y) + F(x, u)^T \hat{\theta} + A(x - \hat{x}), \tag{3}$$

where $\hat{\theta}$ is a parameter estimate generated via the update law to be developed and the state estimation error will be $\varepsilon = x - \hat{x}$ with the dynamic governed by,

$$\dot{\varepsilon} = A\varepsilon. \tag{4}$$

We introduce the filters

$$\dot{\bar{\Omega}}_0 = -A(\bar{\Omega}_0 + y) - f(x, y), \tag{5}$$

$$\dot{\bar{\Omega}} = -A\bar{\Omega} + F(x, u), \tag{6}$$

and denote the output estimation as,

$$\hat{y} = \bar{\Omega}_0 + \bar{\Omega}^T \hat{\theta} \tag{7}$$

which result in output estimator error,

$$\epsilon = y + \bar{\Omega}_0 - \bar{\Omega}^T \hat{\theta} \tag{8}$$

Substituting (5) and (6) into (8) we get,

$$\epsilon = \bar{\Omega}^T \tilde{\theta} + \tilde{\epsilon} \tag{9}$$

where $\tilde{\epsilon}$ is governed by,

$$\dot{\tilde{\epsilon}} = -A\tilde{\epsilon} + \varepsilon. \tag{10}$$

Now consider

$$\mu = \varepsilon - \epsilon\tilde{\theta} \tag{11}$$

$$\dot{\mu} = -A\mu, \mu(0) = \epsilon(0) \tag{12}$$

and matrixes,

$$\dot{Q} = \epsilon^T \epsilon, Q(0) = 0 \tag{13}$$

$$\dot{C} = \epsilon^T(\epsilon\hat{\theta} + \varepsilon - \mu) \tag{14}$$

Suppose there exists a time t_c and a constant $c_1 > 0$ such that $Q(t_c)$ is invertible i.e.

$$Q(t_c) = \int_{t_0}^{t} \epsilon(\tau)^T \epsilon(\tau) d\tau > c_1 I$$

Then,

$$\theta = Q(t)^{-1}C(t), \forall t \geq t_0 \tag{15}$$

Remark 4.4.1: The result obtained here is independent of the control u and parameter identifier $\dot{\hat{\theta}}$ used for parameter estimation, Hence we use the nominal estimate θ^0 with no parameter adaption. So if

$$\theta^c = Q(t_c)^{-1}C(t_c)$$

The finite time identifier is,

The benefit of using this algorithm is that we can predict when PE condition - invertibility of Q - is satisfied and to turn off the identification machine after that.

5.4 Adaptive Predictive Control Scheme

The goal in this section is to design a MPC law to be implemented using the standard nonlinear model predictive control techniques, with takes into account constraint imposed on the process input, output and state variables.

The model predictive feedback is defined as,

$$u = k_{\text{mpc}}(x, \hat{\theta}) = u^*(\cdot) \tag{16}$$

$$u^*(\cdot) \equiv \text{argmin}_{u_{[0,T]}} J(\cdot), \tag{17}$$

$$J(\cdot) = \int_0^T L(t, \bar{x}, u) d\tau + W(\bar{x}) \tag{18}$$

$$\dot{\bar{x}} = f(\bar{x}, y) + F(\bar{x}, y)\theta_c, \quad \bar{x}(0) = x \tag{19}$$

$$\theta_c = \hat{\theta}(\tau) \tag{20}$$

$$u(\tau) \in U, \bar{x}(\tau) \in X, \quad \forall \tau \in [0, T]$$

$$\bar{x}(T) \in X_f$$

5.4.1 Closed loop Robust Stability:

Robust stability is guaranteed if predicted state at terminal time belong to a robustly invariant set for all possible uncertainties. For computation of this invariant set, we let the linearized dynamics of the system as

$$x(k+1) = A(\theta_c)x(k) + B(\theta_c)u(k), \quad x(t) = \bar{x}, k \geq t. \tag{21}$$

Under the assumption of the stabilizablity of the pair (A,B), we consider the control law,

$$u = K^{LQ}(\theta_c)x \tag{22}$$

where $K^{LQ}(\theta_c) = (R + B^T(\theta_c)PB(\theta_c))^{-1}B^T(\theta_c)PA(\theta_c)$ and P is the unique positive solution of the algebraic Reccati equation

$$P(\theta_c) = A^T(\theta_c)P(\theta_c)A(\theta_c) + Q - A^T(\theta_c)P(\theta_c)B(\theta_c)(R + B^T(\theta_c)P(\theta_c)B(\theta_c))^{-1}B(\theta_c)^TP(\theta_c)A(\theta_c). \tag{23}$$

Hence, the linearized system $A_{cl}(\theta_c) := A(\theta_c) + B(\theta_c)K(\theta_c)$ is stable. We let a positive matrix $\tilde{Q}$, and γ be a real positive scalar such that $\gamma \leq \lambda_{\text{min}}\tilde{Q}$. Let $\Pi(\tilde{\theta})$ be the unique symmetric positive definite solution of the Lyapunov equation,

$$A_{cl}^T(\theta_c)\Pi(\tilde{\theta})A_{cl}(\theta_c) - \Pi(\tilde{\theta}) + \tilde{Q} = 0. \tag{24}$$

Then, there exist a constant $c \in (0, \infty)$ satisfying a neighborhood $X_f(\tilde{\theta})$ of the origin of the form $X_f(\tilde{\theta}) = \{x \in R^n, x^T\Pi(\tilde{\theta})x \leq c\}$.

Criterion 5.4.1: For any, θ_1 and $\theta_2 \in \Theta_c$, with $\|\theta_1\| \leq \|\theta_2\|$, we have $X_f(\theta_1) \leq X_f(\theta_2)$.

Theorem 5.4.1: Let $X_0(\Theta)$ denotes the set of initial states for which proposed MPC has a solution. Assuming criteria 1 is satisfied, then for the closed loop system state, x originating from any $x_0 \in X_0$ feasibly approaches the origin as $t \to \infty$.
Proof: Proof is very similar to the proof 4.2 in [Mayne and Michalska, 1990].

5.5 CSTR Dynamics

In this section, the new estimation and also adaptive MPC algorithm is applied to the highly nonlinear model of a continuous stirred tank reactor (CSTR). Assuming the liquid volume, the CSTR for an exothermic, irreversible reaction, $A \to B$, is described by the following dynamic model, based on a component balance for reactant balance for A and a energy balance,

$$\dot{C}_A = \frac{q}{V}(C_{af} - C_A) - k_0 exp(-\frac{E}{RT})C_A, \tag{25}$$

$$\dot{T} = \frac{q}{V}(T_f - T) + \frac{(-\triangle H)}{\rho C_p} k_0 exp(-\frac{E}{RT})C_A + \frac{UA}{V\rho C_p}(T_c - T) \tag{26}$$

where C_A is the concentration of A in the reactor, T is the reactor temperature, and T_c is the temperature of the coolant stream. The constraints are $280K \leq T_c \leq 370K, 280K \leq T \leq 370K, 0 \leq C_A \leq 1mol/I$. The objective is to regulate C_A and T by manipulating T_c. The nominal operating conditions corresponding to the unstable equilibrium $C_A^{eq} = 0.5mol/I, T^{eq} = 350K, T_c^{eq} = 300$ are: $q = 100\text{I/min}, T_f = 350\text{K}, V = 100I, \rho = 1000\text{g/I}, C_p = 0.239\text{J/gK}, \triangle H = -5{\times}10^4\text{J/mol}, E/R = 8750\text{K}, k_0 = 7.2{\times}10^{10}\text{min}^{-1}, UA = 5{\times}10^4 J/\text{min}K$. The nonlinear discrete time model of the system can be obtained by using the state vector $x = [C_A - C_A^{eq}, T - T^{eq}]^T$ and the input $u = T_c - T_c^{eq}$ and discretzing with the sampling time $\triangle t = 0.01\text{min}$. Let $Q = \text{diag}(1/0.5, 1/350)$, and $R = 1/300$, and $\tilde{Q} = 0.05I, \lambda = 0.01$, yielding $c = 0.0915$.
In the simulation, we have considered $\theta_1 := k_0$ and $\theta_1 := k_0\triangle H$ as uncertain parameters with the starting values $\theta_1(0) = 6 \times 10^{10}, \theta_2(0) = 5 \times 6 \times 10^{14}$. Convergence of the parameters with finite time algorithm and un-definite time algorithms are shown in the figure (1) and (2), respectively. Superiority of the finite time algorithm is that we can turn off the parameter

estimation machine after a definite time. Using the finite time algorithm we have controlled the SCTR system employing the proposed adaptive predictive scheme. Results, shown in figure (3) indicates the convergence of the parameters to the origin.

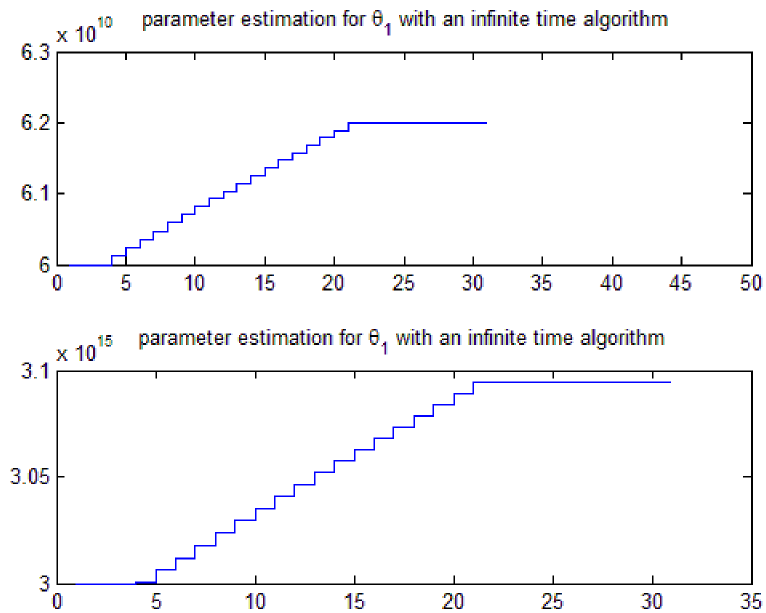

Figure 1: convergence of the parameters estimation in (a) infinite time algorithm (b) finite time algorithm

5.6 Summary

In this chapter a method for adaptive predictive control of nonlinear systems was presented. This method is based on finite time identification algorithm derived in the last chapter. The efficiency of the proposed scheme was examined on an example of the CSTR systems.

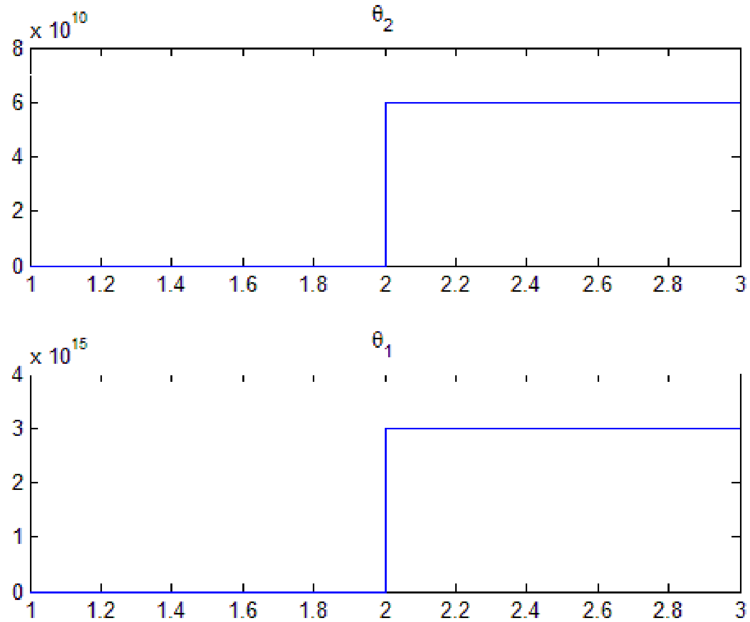

Figure 2: convergence in the parameters estimations for $\theta_0 = [4 \times 10^{10}, 5 \times 9 \times 10^{17}]$

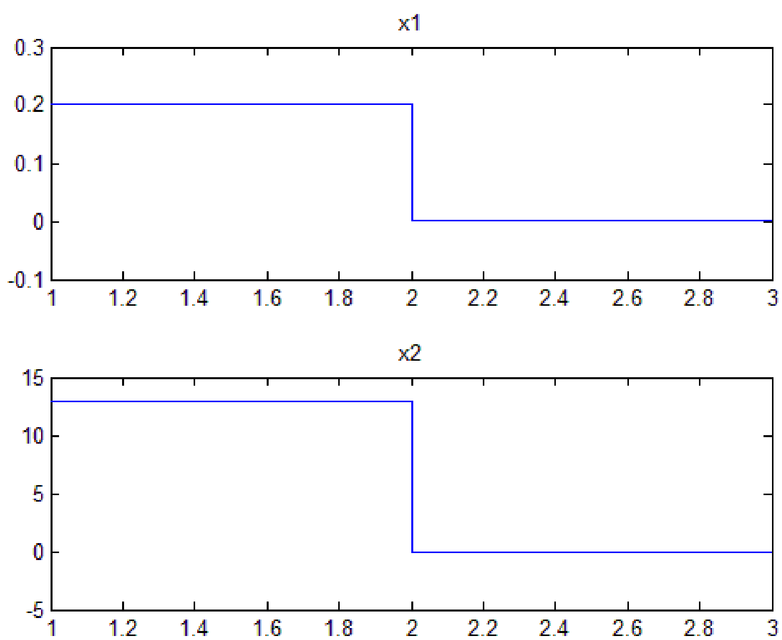

Figure 3: convergence in the state estimations for x_0=[0.2,13]

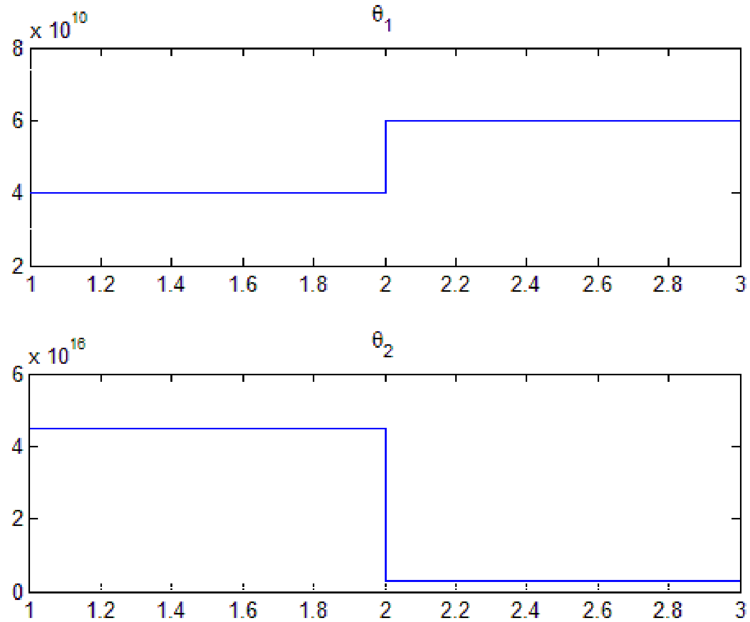

Figure 4: convergence in the state estimations for $\theta_0 = [-4 \times 10^{10}, 5 \times 9 \times 10^{17}]$

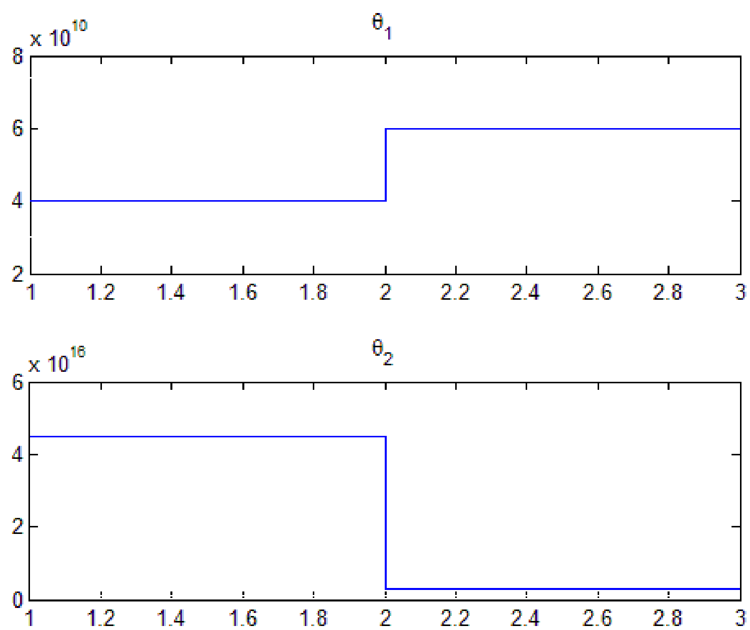

Figure 5: convergence in the state estimations for x_0=[-0.2,13]

Concluding Remarks

Adaptive mode predictive control of a class of uncertain nonlinear systems is considered which comprises known simultaneous parameter estimation and state estimation. The problem of estimation has been divided into three broader steps. Estimation is performed by using adaptive law for estimating the parameters. The second step consists in developing techniques and conditions under which one can guarantee convergence of the state and parameter estimates to their unknown true value. The techniques proposed in this thesis exploits a Lyapunov stability criterion to guarantee boundedness of the estimates and a set-update algorithm to guarantee containment of the unknown parameter values in a computable uncertainty set. The third step is model predictive control which employs the state estimation and parameter estimation uncertainty sets. The main contributions of the work are: 1) A set-based technique for estimating unknown state variables in the presence of unknown bounded disturbance. 2) Estimation of unknown parameters using set-based technique. 3) development of a model predictive approach for the unknown parametric uncertain nonlinear system 4) Application of the proposed methodology to a practical problem of CSTR.

A set-based adaptive estimation technique is proposed for simultaneous state estimation and parameter identification of a class of continuous-time nonlinear systems. The set-based adaptive identifier for parameters is used to estimate the parameters along with an uncertainty set that is guaranteed to contain the true value of the parameters. Simultaneously an auxiliary variable is used to estimate the unmeasured state variables. The method guarantees convergence of the parameter estimation error to zero and determines the unknown state of the system in the presence of unknown bounded disturbances. The estimation and identification algorithms have been implemented to a simulation example.

Removing auxiliary perturbation signals when convergence is achieved and thereby reducing the computational burden largely is always present in RTO domain, however, The benefits of finite-time identification procedure is the possibility of obtaining optimal systems excitation without injecting any auxiliary perturbation signal.

The model predictive control scheme is developed in two ways. First, based on generalized stabilizing criteria, which can be satisfied by a Lyapunov based approaches and contractive constraints methods which is integrated to the developed estimation algorithm. The second method, is based on the input to state stability property of the system and uses control Lypapunov functions according to Sontag's Formulation.

The model predictive control of a simple mixing tank CSTR. The parameter and state estimation scheme and the developed model predictive control approach for the unknown parameters is applied to this practical example. It is demonstrated that the proposed method estimates the state of the system as well as the parameters accurately and can run the system to the origin.

Future research works should be dedicated to practical application of the proposed algorithms to industrial systems. There is lots practical usage of the proposed algorithms in fuel cell control in automotive application. Also, study of effects of unknown parameters in optimization of power output in wind turbine is an open field.

Besides, there would be desirable to do theoretical studies on the stability properties of the method with respect to the bounded unknown disturbances and unmodeled dynamics. Another interesting problem would be to find a systematic method for estimating the true value of the time-varying parameters.