Additivity on Nonlinear Stem Taper Functions: A Case for Corsican Pine in Northern Spain

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Abstract: A system of additive equations was developed to predict whole-tree volume and the different components of Corsican pine. In this work, the nonlinear seemingly unrelated regression (NSUR) approach, which guarantees additivity in nonlinear equations, was evaluated. The effect of bark thickness on the accuracy of the results for all tree components was also assessed. Data for 351 trees, ranging in age from 10 to 72 years, were collected from 65 public and private sites. The volume estimates show average biases that range in absolute values from 2.19 to 31.02 dm^3 for whole-tree, from 1.41 to 27.31 dm^3 for wood, and from 1.05 to 16.52 dm^3 for bark volume components. Errors in volume predictions were relatively small, representing less than 3% of the average observed wood volume and less than 6% of the average observed bark volume. This research showed that satisfactory predictions can be obtained from forcing additivity using NSUR approach with a minimal number of easily measurable tree variables, such as dbh and total height. FOR. SCI. 59(4):464–471.

Keywords: volume equations, additivity, stem form, bark thickness, Seemingly Unrelated Regression, Corsican pine

Rapidly rising costs of conventional fuels have caused a renewed interest in the use of wood and bark residues for fuel. Accurate predictions involving wood and bark products classified by merchantable sizes are a matter of interest for forest managers. Volume prediction to any merchantable limit has been achieved by several methods, but the most widely used is to define an equation describing the stem taper. Integration of the taper equation from the ground to any height provides an estimate of the merchantable volume to that height. Stem form and the variation of taper have been widely studied through the development of taper functions by many forest researchers throughout the world (e.g., Kozak 1988, Newnham 1992, Daquitaine et al. 1999, Bi et al. 2010). To develop a taper function, pairs of data of diameter and height along the stem are required. In most cases, stem taper functions are referenced to dbh and predict the diameter inside bark (Garber and Maguire 2003, Sharma and Zhang 2004, Calama and Montero 2006), although rarely they predict the diameter over bark (Rojo et al. 2005, Trincado and Burkhart 2006, Crecente-Campo et al. 2009, Sevillano-Marco et al. 2009) and bark thickness (Laasasenaho et al. 2005).

Taper equations have been developed in Spain since the 1970s. A considerable number of taper equations have been developed for particular regions and species, most of them for softwoods (e.g., Rojo et al. 2005, Crecente-Campo et al. 2009, Sevillano-Marco et al. 2009) but also some for hardwoods (e.g., Barrio-Anta et al. 2007, Rodríguez et al. 2010).

European black pine (*Pinus nigra* Arnold) forests, comprising a variety of subspecies, generally occupy medium and high mountain zones, often on substrates rich in magnesium. Populations of black pine extend over more than 3.5 million ha from western North Africa through southern Europe to Asia Minor. Black pine is one of the major species used for afforestation of arid and rocky terrain in the sub-Mediterranean region (Isajev et al. 2004). Corsican pine (*Pinus nigra* subsp. *laricio* [Poir.] Maire) is an important subspecies for sandy soils and drier coastal areas. This conifer is native to Corsica in France and Calabria and to Sicily in southern Italy, and it is one of the main tree species used in plantations in the Basque Country (northern Spain) because of its extremely fast growth and its mechanical properties for lumber. According to the Spanish government (Ministerio de Agricultura 2007), in the Basque Country, there are approximately 325,000 ha of forest, of which 12,728 are occupied by Corsican pine. In addition, Corsican pine annual harvest (23,300 m^3) represents 4% of the total harvested volume of wood in the Basque Country (Ministerio de Agricultura 2007).

At present, a stem taper function for Corsican pine plantations is available for France (Meredieu et al. 1999). This equation predicts the diameter over bark along the stem, but it cannot estimate the volume of the bark, which is becoming increasingly important for enhancing forest bioenergy. This model is incorporated on the Capsis (AMAP 2012) PNN module, a distance-independent tree growth model for pure even-aged stands of black pine (Dreyfus 1993).

In the present study, an additive system of taper equations for Corsican pine is developed. To conduct this study,

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a data set from northern Spain is used. This system requires values for only two variables for each tree: dbh and total height (th). Three interrelated equations constitute the system: a stem taper function to predict diameter over bark, a taper equation to predict diameter inside bark, and a bark curve to predict double-bark thickness from the ground to any height of the tree.

Materials and Methods

Study Area and Data Collection

Data from 65 public and private forests in the Basque Country region in northern Spain, situated between the longitude coordinates 03°26′49″ W and 01°46′37″ W and the latitude coordinates 43°26′56″ N and 42°37′19″ N, were used. A total of 351 Corsican pines were selected for destructive sampling according to the protocol of Garber and Maguire (2003). Before felling, three attributes were recorded for each *j*th tree: dbh, dbh_j (to the nearest 0.1 cm); total tree height, h_j (to the nearest 0.01 m); and height to the lowest living branch, hlb_j (to the nearest 0.01 m). Each *j*th tree was felled, minimizing stem breakage. A tape measure was stretched along the bole from the base to the tree top. Total height from the base of the stump to the top of the tree was recorded with a laser hypsometer. The tree was divided into sections, and thin disks were removed for diameter measurements at the following *i*th heights (h_i): ground level, 80 cm above the ground level, breast height, and a height midway between every other whorl pair above breast height (1–2 m intervals). For each disk, height (h_i, to the nearest 0.01 m), diameter over bark (diameter outside $bark_{ij}$, to the nearest 0.1 cm), and diameter inside bark (diameter inside $bark_{ij}$, to the nearest 0.1 cm) were measured in two opposite positions, and double-bark thickness (dbt_{ij}) was estimated according to this formula: dbt_{ij} = diameter outside $bark_{ij}$ − diameter inside $bark_{ij}$. A total of 3,397 disks were cut, averaging 3–17 disks per tree (Table 1).

Table 1. Mean, maximum, minimum, and SD for tree characteristics.

Variable	Mean	Maximum	Minimum	SD
disks · tree^{-1}	8.5	17.0	3.0	2.7
dbh (cm)	22.9	52.4	8.5	8.1
dibh (cm)	19.4	45.4	7.5	7.0
dbtbh (cm)	9.0	1.0	1.4	3.5
dob (cm)	20.8	54.6	7.9	8.2
dib (cm)	17.9	45.6	7.1	7.0
dbt (cm)	2.9	9.8	0.6	1.5
h (m)	14.4	29.8	4.1	5.8
hcb (m)	6.1	18.2	0.5	4.3
hd = h · dbh^{-1}	63.1	110.4	32.0	15.5
age (yr)	33.2	72.0	10.0	15.4
v (m^3)	0.379	2.568	0.013	0.422
ibv (m^3)	0.290	2.063	0.010	0.330
bv (m^3)	0.089	0.606	0.003	0.094

Sample size (n) = 351 trees; dibh, diameter inside bark at breast height; dbtbh, double-bark thickness at breast height; h, total height; hcb, height to the crown base; hd = h · dbh^{-1}, slenderness coefficient; v, over bark volume; ibv, inner bark volume; bv, bark volume.

Volume Component Estimation: Stem Taper Function

Three tree components (w = wood; b = bark; and wt = whole tree) were considered. To obtain an accurate volume prediction for each component, several methods can be used, usually involving the use of volume ratio and stem taper equations. In this case, a stem taper function analysis was carried out. A taper equation describes the mathematical relation between tree height and the stem diameter at that height. It is thus possible to calculate the diameter, for any component, at any arbitrary height. The model Stud described by Daquitaine et al. (1999) was selected for evaluation because it is robust against heteroscedasticity, and it is flexible to capture the variations in the stem form (Lizarralde 2008, Cabanillas 2010, Rodríguez et al. 2010, Fonweban et al. 2012). This is a variable-exponent taper equation that describes the stem shape with a changing exponent from ground to top. This model is basically an allometric function with the following formulation: $d_{ijk} = u(h_{ij}/h)^q$, where d_{ijk} is the *i*th diameter measurement of the *k*th component (diameter outside bark, diameter inside bark, and dbt) for the *j*th tree at any height (h_{ij}), u is an exponential function that describes the butt region, and q is the exponent term describing the tree form. As suggested in several studies, it is advantageous to incorporate some stem form surrogate into taper models, typically a diameter-height ratio, height-diameter ratio, height to the crown base (hcb), or crown ratio (Kozak 1988, Newnham 1992). We evaluated which of these variables were correlated with the estimated parameters of the model, and we expanded the original model by incorporating some of them. In any case, height to the crown base (hcb) was not significant, so the model only included height-diameter ratio. The mathematical formulation of the Stud model is as follows (Equation 1)

$$d_{ijk} = (1 + \theta_3 \cdot e^{-\theta_4 \cdot (h_{ij}/h_j)}) \cdot \theta_5 \cdot (1 - (h_{ij}/h_j))^{\theta_1 + \theta_2(1 - (h_{ij}/h_j))} \qquad (1)$$

where $\theta_1 = \theta_{10} + \theta_{11} \cdot (h_j/dbh_j)$; $\theta_5 = \theta_{50} + \theta_{51} \cdot Z_j$, d_{ijk} is the *i*th diameter measurement of the *k*th component (diameter outside bark, diameter inside bark, and dbt) for the *j*th tree at any height (h_{ij}), Z_j (cm) is the reference breast height measure (dbh, dibh, or dbtbh for diameter outside bark, diameter inside bark, and dbt predictions, respectively) for the *j*th tree, and θ_i represents the parameters to estimate; θ_{10} and θ_{11} control the upper part of the curve, θ_2 controls the middle part, θ_3 is the size of the buttress, θ_4 is the length of the stem affected by the buttress, and θ_{50} and θ_{51} explain the diameter at ground level without the buttress.

Diameter predictions based on stem taper functions, for different tree components, depend on the reference measurement at breast height (i.e., diameter inside bark predictions depend on the diameter inside bark at breast height). Moreover, in standing trees, diameter inside bark is generally calculated by subtracting the double-bark thickness (dbt) on the diameter over bark (diameter outside bark; i.e., diameter inside bark = diameter outside bark − dbt). Therefore, wood and bark volume estimations are based on bark-thickness measurements, but bark-thickness measurements are time-consuming and often imprecise, compromising subsequent predictions. For this reason, in this article, two ways for fitting the different stem taper functions for each tree component are evaluated: case 1, in which, for each component, the taper equation depends on its reference breast height measurement (dbh, dibh, or dbtbh for diameter outside bark, diameter inside bark, and dbt predictions, respectively) and case 2, in which, for all components, the taper functions only depend on the dbh. Thus, case 1 is a priori a more accurate method but also more expensive, whereas case 2 may represent significant cost savings in the measurement of standing trees.

Because the database contains multiple observations for each tree, it is reasonable to expect that the observations within each tree are spatially correlated, which violates the assumption of independent error terms. Spatial autocorrelation measures the degree of how a phenomenon of interest is correlated to itself in space (Cliff and Ord 1973). Positive spatial autocorrelation ($\rho_i > 0$) indicates that similar values appear close to each other or as a cluster in space. Negative spatial autocorrelation ($\rho_i < 0$) indicates that neighboring values are dissimilar or, equivalently, that similar values are dispersed. Null spatial autocorrelation indicates that the spatial pattern is random. To overcome possible autocorrelation, for each tree component, the error term was modeled using a continuous autoregressive error structure (Gregoire et al. 1995). Models were fitted using the SAS/ETS MODEL procedure (SAS Institute, Inc. 2010). To check randomness, autocorrelation plots were evaluated by computing autocorrelations for residual values at varying spatial lags.

Procedures for Additivity on Nonlinear Taper Equations

The property of additivity assures that regression functions are consistent with each other. That is, if one tree component is part of another component, it is logical to expect the estimate of the part not to exceed the estimate of the whole. In addition, if a component is defined as the sum of two subcomponents, its regression estimate should equal the sum of the regression estimates of the two subcomponents. The problem of forcing additivity on a set of linear tree biomass functions has been discussed since the 1970s (Kozak 1970, Chiyenda and Kozak 1984, Cunia and Briggs 1985, Parresol 1999). According to Parresol (2001), there are only two procedures to force additivity in nonlinear equations: a simple combination approach and a nonlinear joint-generalized regression with parameter restrictions. In the first procedure, the over bark taper equation is defined as the sum of the separately calculated inside bark taper equation and a double-bark function. The second procedure or nonlinear seemingly unrelated regression (NSUR) approach is more general and flexible than the first procedure (Parresol 2001). In this case, the errors in different equations might be correlated and the efficiency of the estimation might be improved by taking these cross-equation correlations into account, using the NSUR procedure. As for ordinary least squares, the NSUR method assumes that all the regressors are independent variables, but NSUR uses the correlations among the errors in different equations to improve the regression estimates (SAS Institute, Inc. 2010). Therefore, a set of nonlinear regression functions are specified such that each component regression contains its own independent variables, the total tree regression is a function of all independent variables used, and the additivity is ensured by setting constraints on the regression coefficients. These two procedures have been successfully used with different forest species (Carvalho and Parresol 2003, Bi et al. 2010, Ruiz-Peinado et al. 2011).

If the errors in the different equations are correlated (the normal situation for biomass and volume equations), the NSUR procedure is preferable to the simple combination approach (Parresol 2001, Bi et al. 2010) because seemingly unrelated regression takes into account the contemporaneous correlations, which results in lower variance. Only the NSUR procedure was evaluated in this work, and it was fitted using the SAS/ETS MODEL procedure (SAS Institute, Inc. 2010).

Model Evaluation and Validation

Two NSUR fits were performed (case 1 and case 2). The estimates of each tree component after the two different fits were compared using both numerical and graphical analyses (Huang et al. 2003). Two statistical criteria obtained from the residuals were examined: root mean square error (RMSE; Equation 2) and the coefficient of determination for nonlinear regression (pseudo-R^2; Equation 3).

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (Y_{ik} - \bar{Y}_i)^2}{n - p}} \tag{2}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (Y_{ik} - \hat{Y}_{ik})^2}{\sum_{i=1}^{n} (Y_{ik} - \bar{Y}_i)^2} \tag{3}$$

where Y_{ik} and $\hat{Y}_{ik}$ are the ith observed and predicted values of diameter for component k, $\bar{Y}_k$ is the mean of n observed values for the same component, and p is the number of model parameters.

The taper functions were also assessed using box plots representing diameter residuals (diameter outside bark, diameter inside bark, and dbt) by relative height along the stem (i.e., 5, 15, 25, and so on, up to 95%). The same was done for volume residuals in each tree component by diameter class. These graphs, drawn by relative height or dbh classes, are important to show areas or tree size classes for which the functions provide especially poor or good predictions (Kozak and Smith 1993, Crecente-Campo et al. 2009).

Because an independent sample was not available to test the quality of predictions, an n-way cross-validation (i.e., a first-order jackknife approach) of each model was performed, estimating the residual for one tree by excluding that tree every time. This was then repeated for all trees in the data set (351 times) and the RMSE of the estimate and the model efficiency (MEF) (equivalent to the R^2 of the fitting phase) were calculated from these residuals. Although this approach is not a real method of model validation (Huang et al. 2003), it has been used as an additional criterion to select the best model (Myers 1990) while waiting for a new independent data set to assess the true quality of the predictions.

Results

The models were initially fitted without expanding the error terms to account for autocorrelation, and, consequently, strong autocorrelation among all models was observed. A first-order continuous autoregressive error structure was required to model the inherent autocorrelation of the hierarchical data. A similar trend in residuals of the taper model as a function of the distance between the measurements along the stem within the same tree was apparent in all of the models analyzed. Autocorrelation plots of the residuals obtained from each tree component in case 2 are shown in Figure 1.

All the estimated parameters were significant at $P < 0.05$ (Table 2). The spatial autocorrelation coefficients (ρ_i) for each fit were positive and highly significant, indicating strong spatial autocorrelation in the dependent variables. With examination of the six combinations of the residuals (two fits and three tree components), in all cases the assumption of the homoscedasticity of the residues was satisfied, i.e., their variances do not vary with the effects being modeled (Figure 2). Only in the estimation of "dbt" (bottom row) and especially in case 2 (right column) we found a slight heteroscedasticity in the residual, but all cases indicated a random pattern of residuals around 0. All the models provided good data fits (Table 3), although, in the case of wood and whole-tree components, they explained more than

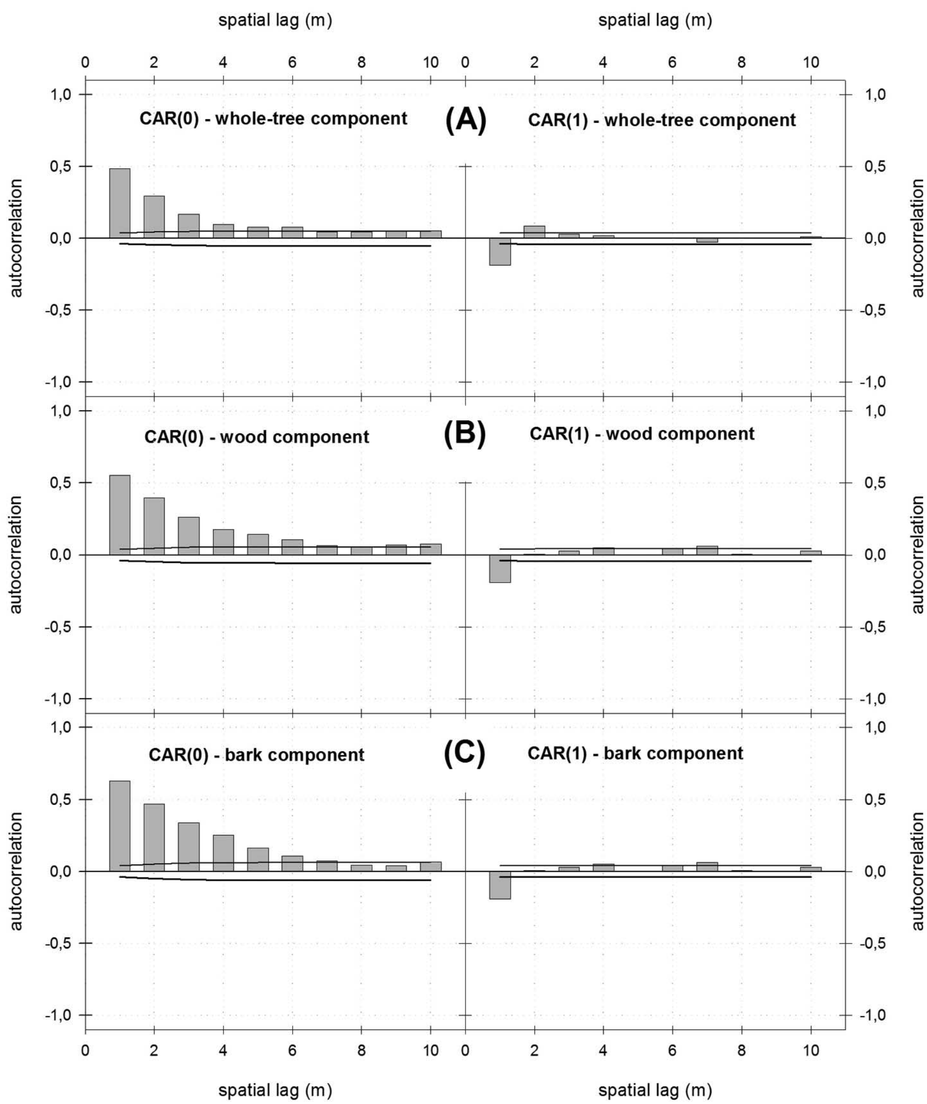

Figure 1. Autocorrelation plots for additive stem diameter prediction on the tree components (A. whole-tree; B. wood; C. bark), fitted without considering the autocorrelation parameters (left column) and using a first-order continuous autoregressive error structure (right column) for residuals obtained in case 2. Continuous lines represent the 95%-confidence region.

Table 2. Parameter estimates for the models analyzed.

Case	ED	θ_{10}	θ_{11}	θ_2	θ_3	θ_4	θ_{50}	θ_{51}	ρ_1
1	dib	0.78869	0.00207	0.35332	0.17814	19.62196	1.81916	0.91106	0.88854
		(0.0267)	(0.0004)	(0.0379)	(0.0099)	(1.8871)	(0.1183)	(0.0100)	(0.0107)
	dbt	0.29868	−0.00709	1.35200	1.79852	8.58929	0.47981	0.40563	0.25390
		(0.0582)	(0.0008)	(0.2512)	(0.2039)	(0.3205)	(0.0295)	(0.0343)	(0.0220)
2	dib	0.76486	0.00203	0.31441	0.16922	20.14156	1.48395	0.78923	0.89064
		(0.0260)	(0.0004)	(0.0366)	(0.0098)	(2.0439)	(0.1223)	(0.0085)	(0.0101)
	dbt	0.65230	−0.00822	1.20140	1.38421	10.39340	0.82077	0.06728	0.76061
		(0.1574)	(0.0025)	(0.3012)	(0.1520)	(0.5586)	(0.0536)	(0.0053)	(0.0145)

Approximate SEs appear in parentheses. ED, estimated diameter (dib, diameter inside bark; dbt, double-bark thickness); θ_i, parameters to estimate; ρ_1, spatial autocorrelation parameter.

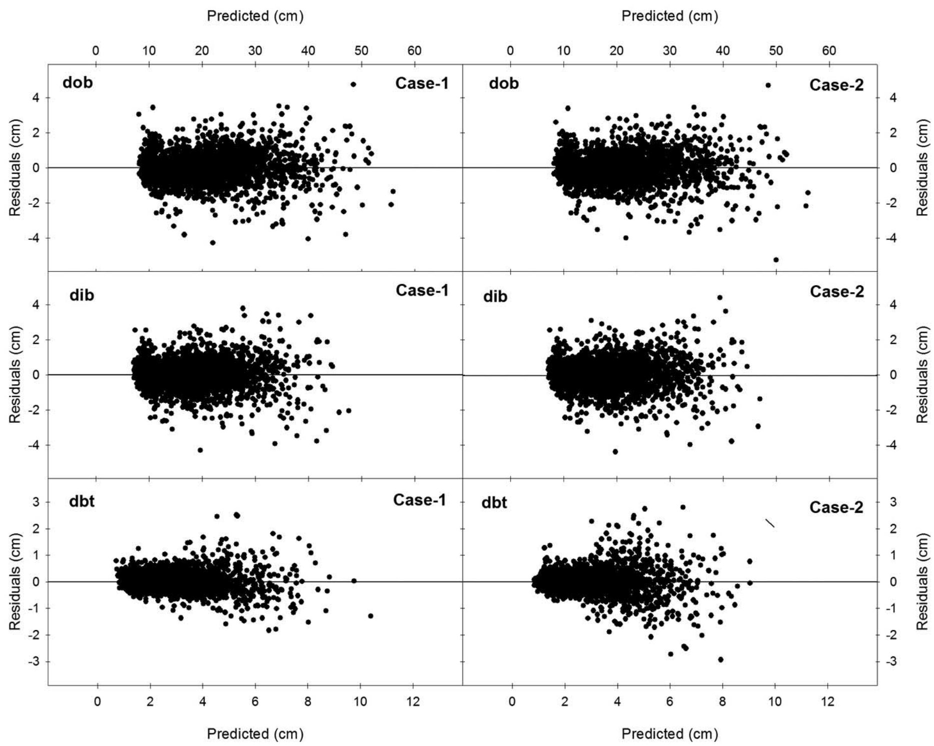

Figure 2. Scatterplots of residuals versus predicted for the diameter outside bark (dob; top row), diameter inside bark (dib; middle row), and dbt (bottom row) in each different case (case 1 in the left column and case 2 in the right column).

Table 3. Goodness-of-fit statistics of the models analyzed.

	Fitting phase						Cross-validation phase					
	RMSE			R^2			RMSE			R^2		
Case	dob	dib	dbt	dob	dib	dbt	dob	dib	dbt	dob	dib	dbt
1	0.8548	0.8276	0.3736	0.9890	0.9859	0.9348	1.1064	1.0184	0.3884	0.9816	0.9786	0.9295
2	0.8470	0.8357	0.4550	0.9892	0.9856	0.9033	1.1141	1.1122	0.6129	0.9813	0.9745	0.8245

dob, diameter over bark; dib, diameter inside bark; dbt, double-bark thickness.

98% of the total variance of the diameter, whereas the bark component explained more than 90% of the total variance of the dbt. As expected, taper functions depending on dbh for all tree components and, in particular, in the dbt model, produced worse predictions than taper equations, depending on their reference breast height measurement. They enhance these differences in cross-validation, for which differences between cases are large, both in terms of RMSE (case 1 = 0.3884; case 2 = 0.6129) and MEF (case 1 = 0.9295; case 2 = 0.8245). The biggest RMSE was found in the whole-tree prediction (RMSE = 0.855), whereas in the other components, it was 0.836 and 0.455 for diameter inside bark and dbt, respectively. In all fits, RMSE obtained in the cross-validation phase (Table 3) was, on average, 1.253 times higher, ranging from 1.041 to 1.384 times, than those obtained in the fitting phase.

The detailed error analysis on diameter and volume predictions showed a similar trend in all evaluated fits (Figure

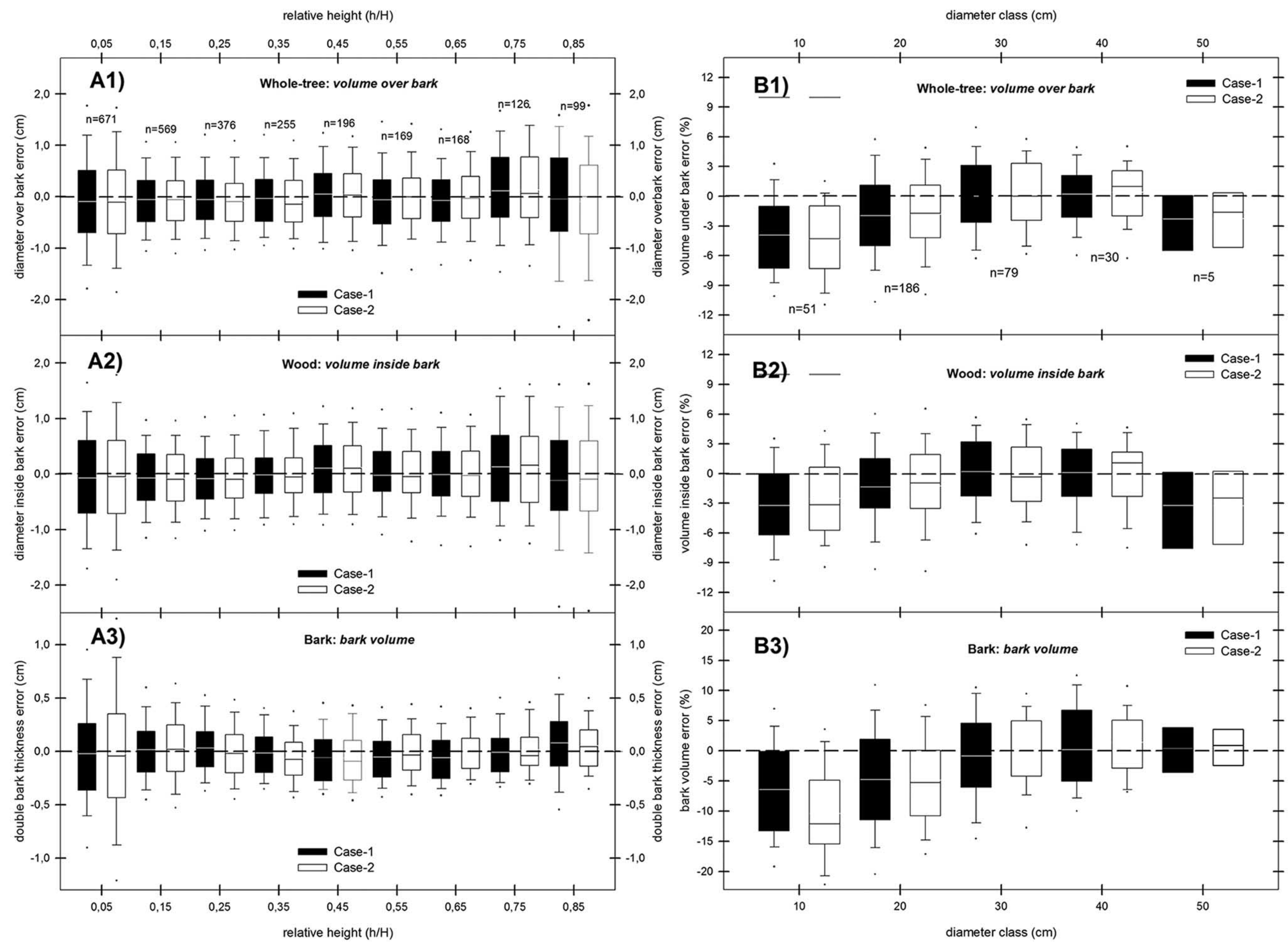

Figure 3. A. Box plots of diameter residuals (*y*-axis, cm) against relative stem height classes (*x*-axis, percentage). B. Box plots of volume residuals (*y*-axis, m^3) against dbh classes (*x*-axis, cm) for the two analyzed cases (case 1 in black and case 2 in white) on the three tree components analyzed (1 = diameter over bark, 2 = diameter inside bark, and 3 = dbt). The boxes represent the interquartile ranges. The maximum and minimum diameter over bark prediction errors are represented by points, and the 5 and 95% are represented, respectively, by the upper and lower small horizontal lines crossing the vertical lines. The number of disks for each relative stem height class is represented by *n*.

3). The box plots of diameter residuals (diameter outside bark, diameter inside bark, and dbt) against relative height classes (Figure 3A) and residuals of the percentage volume (whole tree, wood, and bark) against diameter classes (Figure 3B) did not show any clear systematic trend that could show a deficient behavior of the models.

All models estimated tree component volumes well across the sampled diameter classes. It should be noted that the precision tends to decrease (higher variability) with diameter increment, i.e., for the largest trees. The volume estimates show average biases, in absolute values, that range from 2.19 to 31.02 dm^3 for whole-tree, from 1.41 to 27.31 dm^3 for wood, and from 1.05 to 16.52 dm^3 for bark volume components. In all cases, the volume was overestimated in small trees (dbh $<$ 25 cm) and was underestimated in the largest trees (dbh $>$ 45 cm). In both cases, these biases were small, representing less than 3% of the average observed wood volume and less than 6% of the average observed bark volume.

Discussion

Additive stem taper functions are not common worldwide. This type of function improves stem estimation and allows better decisions in operational forestry in a wide array of management objectives (bioenergy, timber, and others). Different additive stem taper functions were fitted for black pine trees in the Basque Country (northern Spain). All models were fitted by using a first-order continuous autoregressive error structure to deal with the problem of autocorrelation associated with the use of repeated measures within an individual tree. According to West et al. (1984), although accounting for autocorrelation does not improve the predictive ability of the model, it prevents underestimation of the covariance matrix of the parameters and improves interpretation of the statistical properties.

The Stud model was found to represent stem shape quite accurately, especially in the high-volume butt region. According to Cao et al. (1980) this is an important feature in most variable exponent models. The model evaluated captures the variations of the tree accurately for the three tree components; bark, wood, and whole-tree. Finally, the numerical integration of the taper function yields a reliable mean prediction for the different volumes of a particular tree.

In terms of accuracy, the proposed additive volume system demonstrated RMSE values (approximately 0.85 for diameter outside bark, 0.83 for diameter inside bark, and 0.40 for dbt) similar in range to those obtained for other coniferous studies: 0.85 for diameter outside bark in radiata pine (Sevillano-Marco et al. 2009), 1.09 for diameter inside bark in lodgepole pine (Garber and Maguire 2003), 1.08 for diameter inside bark in Ponderosa pine (Garber and Maguire 2003), and 0.77 (ranging from 0.68 to 0.86) for diameter inside bark in Jack pine (Sharma and Zhang 2004). For typical species in the Mediterranean area, the RMSE obtained ranged from 1.47 to 1.71; 1.710 for diameter inside bark in stone pine (Calama and Montero 2006), 1.511 for diameter outside bark in Aleppo pine (Cabanillas 2010), and 1.479 for diameter outside bark in maritime pine (Rojo et al. 2005). These large RMSE values are probably due to the high variability of bark thickness, which is primarily controlled by environmental and genetic factors. Despite the practical importance of bark thickness (e.g., bioenergy assessment and nutrient studies), models predicting it at different relative stem heights are not common. For example, Laasasenaho et al. (2005) obtained an error in bark volume that varied from 5.27 to 7.79 dm^3 for Norway spruce, and they proposed combining the bark model with existing stem curve models to calculate stem volume, both under and over bark, for any arbitrary portion of the stem.

The pattern of the plots of d residuals (inside and over bark) against relative height classes (Figure 3) is similar for other species (Garber and Maguire 2003, Sharma and Zhang 2004, Rojo et al. 2005, Crecente-Campo et al. 2009). The model tends to underestimate the diameters in the lower and upper sections, whereas the midsection diameters are overestimated. In addition, for relative heights between 0 and 10% and 75 and 85%, both models showed larger SEs of the estimates than at other height intervals. These relative height classes may be associated with stem butt swell, in which it is common to find them in other stem taper functions (Jiang et al. 2005), and small diameter near the top of the tree where they have low relevance for the subsequent estimation of the volume. Because stem analysis was usually not performed at top diameter values smaller than 7 cm and few measurements existed in the top sections, these results should be considered carefully. However, because the latter part of the stem is the least valuable and the part that accumulates least volume, these results do not have a great impact on the overall performance and applied use of the models. For sections closer to the ground, both models provided good estimates. Accurate predictions of diameters of these sections are important because the base log is particularly important from a commercial point of view. All these statistics and plots show no clear advantage of case 1 (for each component the taper equation depends on its reference breast height measurement) against the other. However, case 2 (for all components the taper functions only depend on the dbh) represents a significant time reduction in data collection.

In summary, this additive system appropriately described the data and accurately predicted the volume for the three tree components along the stem, showing an appropriate distribution of the residuals and providing more than acceptable goodness-of-fit statistics.

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