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The effects of decision timing for pricing and marketing efforts in a supply chain with competing manufacturers

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Abstract

This paper investigates the impact of decision timing for pricing and marketing efforts in a supply chain led by competing manufacturers. We develop and solve six games to consider the scenarios (games) where prices and marketing efforts (ME) are decided simultaneously, and when they are not (i.e., ME is set either before or after prices). We examine these three scenarios for the benchmark case of a bilateral monopolistic channel, then extend the analysis to a supply chain with competing manufacturers. We identify the optimal decision timing by comparing equilibrium profits and strategies across games in each supply chain setup. We find that a monopolistic manufacturer always prefers that prices and ME be decided simultaneously. However, this result does not hold when product competition is taken into account. The optimal decision timing for competing manufacturers depends on the retailer's and manufacturers' ME effectiveness levels as well as on competition intensity. Specifically, when ME are not very effective, a simultaneous decision scenario is preferred because it provides the advantage of higher profit margins or sales. However, for highly effective ME, manufacturers prefer to decouple ME and pricing decisions. The retailer's optimal scenario is either to make all decisions simultaneously or to choose prices prior to ME. This means that supply chain firms can face conflict due to the decision timing for prices and ME.

Keywords: OR in marketing; marketing efforts and pricing; decision timing; competition; game theory

1. Introduction

A large analytical literature in marketing and operations research examines optimal pricing and marketing effort (ME) decisions in the supply chain. ME include a variety of nonprice demand-stimulating activities undertaken by any supply chain firm such as sales effort, advertising, nonprice promotions, and so on. Research in this field often relies on the assumption that each firm decides

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[Correction added on December 09, 2021, after first online publication: The copyright line was changed.]

on its pricing and ME simultaneously (e.g., Karray and Zaccour, 2006, 2007; Yue et al., 2006; He et al., 2009; Szmerekovsky and Zhang, 2009; Xie and Wei, 2009; Ahmadi-Javid and Hoseinpour, 2011; SeyedEsfahani et al., 2011; Kunter, 2012). A few scholars argue that there is a discrepancy in the timing of these decisions and assume that ME and prices are set at different stages instead of simultaneously by each channel member (Agrawal, 1996; Banerjee and Bandyopadhyay, 2003; Soberman and Parker, 2006; Karray and Martín-Herrán, 2008; Draganska et al., 2009; Karray, 2013; Karray and Martín-Herrán, 2019). How relevant is the assumption about the timing of these decisions while optimizing supply chain members' strategies? Does it impact their profitability? How is such impact affected by competition between manufacturers? This paper aims to answer these questions through an analytical study.

The issue of decision timing is relevant because it affects the information set available to the players at the time they make their decisions. In supply chains led by the manufacturer(s), the retailer observes the manufacturer's announced decisions before making his own. Therefore, depending on which information is announced to the retailer (ME, pricing, or both), the retailer will react by choosing different levels of ME and pricing, which will then impact the demands, revenues, and profits of all supply chain firms.

In practice, we can observe differences in decision timing practices. In many instances, pricing agreements between manufacturers and retailers are established before decisions are made about ME. In such cases, ME, including retail local advertising, nonprice promotions, and manufacturer promotions, are decided given the pricing contract in the supply chain. For example, manufacturers whose brands benefit from high levels of consumer loyalty, such as Proctor and Gamble, usually avoid frequent price adjustments that could damage their brand image and adopt, instead, a strategy of everyday low pricing, or EDLP (Raju et al., 1990). Further, some industries such as food, grocery, and ornaments set the same wholesale price for the entire selling season because of the stability of their production processes (Maiti and Giri, 2017). In such cases, manufacturers fix wholesale prices for an extended period and retailers keep the price unchanged (Kopalle et al., 1996). This EDLP strategy has been adopted by many retailers such as Walmart and Trader Joe's in the United States and Tesco in the United Kingdom (Tang et al., 2014). In these channels, ME, such as manufacturers' consumer promotions, local advertising (e.g., in retail flyers or local publications), and in-store promotional activities (e.g., displays, features, merchandising, and social media marketing activities), are decided on an ongoing basis and do not necessitate a long-term budget commitment from the manufacturer and retailer. In addition to common examples of such practices for consumer products (CPG), the automotive industry often adopts this decision-making approach as prices of new products are announced before different rebate and promotional offers are announced from both manufacturers and retailers.

In other cases, the retailer and the manufacturer may not have long-term pricing agreements. For instance, when the relationship between the supply chain members is not long-standing or economic conditions are unstable, manufacturers and retailers may choose flexibility by disengaging from any long-term pricing commitments (Karray, 2013). For example, the supply chain members may decide on different ME such as advertising before pricing when a high–low pricing strategy is implemented or when long-term contractual agreements with media agencies are established. In fact, in some industries, national advertising campaigns in traditional media outlets are set for a longer period than are prices and thereby decided at an earlier stage. Evidence from the CPG industry shows that some manufacturers fix their ME budgets for the quarter or year when drafting their marketing plan, while their prices to retailers are decided more often. Further, in the electronics industry, prices are frequently changed to take advantage of technological innovations or seasonal changes, while advertising campaigns are setup front (Maiti and Giri, 2017).

In the marketing literature, a few studies have examined the issue of pricing and ME decision timing (Kadiyali et al., 2001; Rao, 2009). In his book about marketing decisions, Rao (2009, p. 120) notes that "the possible difference in the periodicity of decision-making regarding price versus other decisions, such as marketing efforts [is a] tricky issue." Empirical research does not provide a clear explanation of why such a discrepancy might exist. This means that it could be due to various factors such as managerial practice as well as commitments with media agencies or channel partners. Different choices of timing for pricing and ME could also be due to differing marketing objectives (e.g., encourage short-term sales vs. build brand equity).

In the operations literature, the issue of how decision timing can impact supply chain members' profitability has been seldom investigated. Recently, a few papers have focused on the optimal timing of product pricing in dual channels (e.g., Liu et al., 2018; Matsui, 2020; Yan et al., 2020). To our knowledge, only Karray (2013) has attempted to investigate how ME and pricing decision timing can impact retailers' and manufacturers' profitability in a duopolistic conventional channel. Like Karray (2013), we study the optimal timing of prices and ME, but differ from that earlier work in three ways. First, in addition to a duopolistic channel, we extend our analysis for the first time to model product (manufacturer) competition. Insights derived from such analysis can shed light on how competition affects the timing of pricing and market efforts. Second, we model demand using a consumer utility approach, while their model relies on an aggregate demand formulation. Our utility-based model allows for better representation of competitive interactions among products. It captures competition as it relates to product substitutability as opposed to most aggregate models, which represent competition between products through cross-price effects (Lus and Muriel, 2009; Huang et al., 2013). Third and finally, while Karray (2013) models ME cost sharing mechanisms (cooperative advertising) between manufacturers and retailers, we omit such contracts from our model to isolate the effect of decision timing choice on the profitability of manufacturers, retailers and the entire channel.

This paper aims to identify the optimal timing of pricing and ME decisions. In a supply chain led by manufacturers, we examine different scenarios where these decisions are made simultaneously or sequentially. We first develop an understanding of this problem for a benchmark scenario without competition (duopolistic supply chain), then extend our analysis to model manufacturer (product) competition. The main research questions we address are as follows:

- What is the optimal decision timing for pricing and ME for manufacturers, retailers, and the supply chain?
- How does product competition impact optimal decision timing?

In order to answer these questions, we develop a game-theoretic model using a utility-based demand function. We solve for equilibrium prices and ME for three scenarios reflecting different timings of these decisions: (a) when they are made simultaneously; (b) when prices are decided prior to ME; and (c) when prices are chosen after ME. We obtain these results for the case of a supply chain with no competition and for the case with manufacturer competition. In each supply chain setup, we compare equilibrium profits among the three timing scenarios to identify the optimal outcome for the manufacturer(s), retailer, and entire supply chain.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 describes the model. Section 4 presents the equilibrium solutions. Section 5 compares results and presents the optimal timing scenario in each supply chain setup. Finally, Section 6 concludes and discusses future research avenues.

2. Literature review

In the game-theoretic literature, whenever a sequential non-cooperative game is played, the information set available to the players (decision makers) at the time of making their decisions ultimately determines their equilibrium solutions. For example, in the familiar Stackelberg duopoly model, the leader commits to certain decisions (quantity, price, etc.). After observing the leader's choice, the follower optimally makes his/her decisions (von Stackelberg, 1934). In a supply chain context, this can translate into the issue of channel leadership in Stackelberg games (e.g., Choi, 1991; Lee and Staelin, 1997; Jørgensen et al., 2001). In such games, the supply chain leaders announce their decisions first and the followers make their choices knowing the leaders' decisions. Different scenarios of channel leadership then imply different information sets available to each firm when making their decisions, which ultimately affects the equilibrium outputs of the game.

This paper does not examine the issue of channel leadership but rather focuses on how a supply chain leader's choice to announce different decisions at various times can affect equilibrium outcomes. Particularly, we focus on a manufacturer leadership setup where the retailer observes each manufacturer's announced decision before making his own. Depending on which information is announced to the retailer (ME, pricing, or both), they will react by choosing different levels of ME and pricing. The decision timing chosen by the manufacturer then affects all firms' equilibrium strategies, thereby their demand, revenues, and profits as well.

This problem has received very little attention in the supply chain and marketing literature, with only few works examining the optimal timing of different decisions, given a specific leadership structure between manufacturers and retailers. For example, assuming manufacturer leadership, a few studies have investigated the optimal timing of manufacturers' wholesale and direct prices in dual channels and found that such timing has a significant impact on profits (Liu et al., 2018; Matsui, 2020; Yan et al., 2020). Focusing on quality and sales efforts decisions, other works show that the timing of these decisions can impact the profitability of the supply chain firms (Gurnani et al., 2007; Liu et al., 2018).

Applications of decision timing to problems that involve both pricing and ME decisions are scarce. Notably, Karray (2013) investigates the optimal timing of pricing and ME for a bilateral monopolistic supply chain where a coordination contract is implemented. This study shows that the timing of pricing and ME decisions can significantly affect the strategic outcomes of each supply chain firm. Using an aggregate demand function that takes into account pricing and ME decisions, the main findings in Karray (2013) suggest that making pricing and ME decisions simultaneously is optimal only for high enough levels of the manufacturer's ME. For very highly effective ME by both firms, sequential play of ME and pricing allows supply chain members to implement equilibrium strategies and achieve maximum profits that would not be achieved with simultaneous decision-making. Recently, Karray and Mart ín-Herrán (2019) explore the issue of pricing and ME timing in the context of a store brand introduction for a duopolistic supply chain. They find that the manufacturer can counter the harmful impact of the retailer's store brand by changing the timing of their pricing and ME following the private label entry. These studies show that the order in which pricing and ME are set is relevant as it directly impacts the supply chain firms' profitability.

To conclude, the theoretical literature about decision timing has focused on pricing issues and highlighted the significant impact that price timing can have on the strategies and profitability of supply chain firms. A similar result has been found when the decision timing of variables other than pricing are considered. The literature that modeled both ME and pricing decisions has mostly assumed these variables are set simultaneously. In this study, we challenge this assumption and extend previous knowledge by identifying the optimal timing for these decisions and the alleviating impact of product competition.

3. Model

We start by discussing the benchmark model for a bilateral monopolistic supply chain, then present the extended model that includes manufacturer (product) competition.

3.1. Bilateral monopolistic supply chain

In this case, the supply chain is formed by a manufacturer selling his product through an independent retailer. The manufacturer makes the following decisions: his wholesale price (w) and ME (a_m). The retailer sets his price to consumers, p ($p > w$), and his ME (a_r). ME in our configuration includes a variety of nonprice marketing activities aimed at stimulating sales such as consumer promotions, features, displays, contests, sweepstakes, and local media ads for products sold in the store (Reid et al., 2005; Kalra and Shi, 2010).

The demand functions are derived from the maximization problem of a representative consumer with a quadratic and strictly concave utility function (Singh and Vives, 1984), which is given by $U = gq - q^2/2 - pq$, where q represents the demand function and g is the base utility of the product. This formulation has been commonly used in the marketing and economics literatures (e.g., Samuelson, 1974; Spence, 1976; Ingene and Parry, 2007; Cai et al., 2012; Liu et al., 2014; Karray et al., 2017; Karray and Martín-Herrán, 2019). It exhibits the classical economic properties that the marginal utility for a product diminishes as the consumption of the product increases (Samuelson, 1974).

The expression g represents the expanded base utility of the product such as $g = v + \alpha a_m + \beta a_r$. It consists of a baseline utility (v) increased by the ME undertaken by both manufacturer and retailer. The effects of ME on utility are modeled through the positive parameters α and β for the manufacturer and retailer, respectively.

Maximization of the representative consumer utility with regard to q leads to the following demand function: $q = v - p + \alpha a_m + \beta a_r$. Note that this demand is linear in price and ME. Before we write the profit functions and problems for each supply chain firm, we make the following few assumptions. First, for simplicity, we assume null production and distribution costs. Second, the ME costs of the manufacturer and retailer are assumed quadratic to represent increasing marginal

Table 1
Notation

w_i	Wholesale price of manufacturer i, $w_i > 0$
p_i	Retail price for product i, $p_i > w_i$
a_{ri}	Retailer's advertising effort for product i, $a_{ri} > 0$
a_{mi}	Manufacturer i's advertising effort, $a_{mi} > 0$
q_i	Demand for product i, $q_i > 0$
M_i	Manufacturer i's profit, $M_i > 0$
R	Retailer's profit, $R > 0$
T	Total supply chain profit, $T = R + M_1 + M_2$
v	Baseline utility parameter, $v > 0$
α	Effect of the manufacturers' advertising effort on utility, $\alpha > 0$
β	Effect of the retailer's advertising effort on utility, $\beta > 0$
γ	Competition between products, $\gamma \in (0, 1)$

costs of ME. Third, other supply chain decisions such as inventory are assumed exogenous to the model. These assumptions are commonly used in the theoretical supply chain literature about ME (e.g., Ingene and Parry, 2007; Cai et al., 2012).

Finally, the profit maximization problems of the manufacturer (M) and of the retailer (R) are given by

$$\max_{w,a_m} M = wq - a_m^2, \qquad \max_{p,a_r} R = (p - w)q - a_r^2.$$

3.2. *Supply chain with manufacturer competition*

We extend the previous model to a supply chain where two manufacturers compete by offering different products through the same retailer. Each manufacturer i ($i = 1, 2$) sets his wholesale price (w_i) and ME (a_{mi}), while the retailer sets the retail prices, p_i ($p_i > w_i$), and his ME for each product (a_{ri}). A summary of all notations used in the model is presented in Table 1.

The utility function of a representative consumer in this case is affected by the competition between the two products and is given by

$$U = \sum_{i=1,2} (g_i q_i - q_i^2/2 - p_i q_i) - \gamma q_1 q_2,$$

where q_i represents the demand function for product i, and $g_i = v + \alpha a_{mi} + \beta a_{ri}$, $i = 1, 2$.

This linear-quadratic utility formulation exhibits the following classical economic properties. First, the representative consumer's utility of owning a product decreases as the consumption of the substitute product increases. Second, the marginal utility for a product diminishes as the consumption of the product increases. Third, the value of using multiple substitutable products is less than the sum of the separate values of using each product on its own (Samuelson, 1974).

The expression g_i represents the expanded base utility of product i. It consists of the baseline utility (v) increased by the positive ME effects undertaken for the product. For simplicity, we assume that the baseline utility of consuming each product is same, meaning that the manufacturers' products are similar in all other aspects. For simplicity and without loss of generality, we fix $v = 1$ in the rest of the paper. We also assume that the manufacturers' ME have similar effects on the consumer utility function. Finally, the parameter $\gamma \in (0, 1)$ represents substitutability between the manufacturers' products (product competition), with higher values of γ indicating more intense competition between products and *vice versa*.

Maximization of the representative consumer utility with regard to q_i leads to the following demand functions:

$$q_i = \frac{1}{(1-\gamma^2)}[(1-\gamma) - p_i + \gamma p_j + \alpha a_{mi} + \beta a_{ri} - \gamma(\alpha a_{mj} + \beta a_{rj})], \quad i, j = 1, 2, i \neq j. \tag{1}$$

Note that these demand functions are linear in prices and ME. The advantage of using this utility-based formulation instead of an aggregate demand function is that modeling consumer utility provides us with meaningful interpretations of the model parameters, especially of the substitutability parameter γ. In our model, the market size, price, ME, and cross-sensitivity parameters are all related and depend on the product substitutability parameter. In fact, as products become more substitutable (γ increases), demand sensitivity to prices and ME increases while the overall market size decreases. In aggregate linear models, the impact of each decision variable on demand is commonly represented by a single independent parameter (e.g., $q_i = x - z_1 p_i + z_2 p_j + y_1 a_{mi} + y_2 a_{ri} - y_3 a_{mj} - y_4 a_{rj}$). Therefore, in such models, an increase in product substitutability does not affect the demand sensitivity to the product's own price, ME, nor the total market size, which does not accurately represent common observations of how consumers react to product differentiation (e.g., Talluri and Van Ryzin, 2006, pp. 395–396).[1]

We further assume that both manufacturers have similar costs and formulate the profit maximization problems of the manufacturers (M_i) and retailer (R) as follows:

$$\max_{w_i, a_{mi}} M_i = w_i q_i - a_{mi}^2, \quad i = 1, 2, \qquad \max_{p_1, p_2, a_{r1}, a_{r2}} R = \sum_{i=1,2} \left[(p_i - w_i) q_i - a_{ri}^2 \right].$$

4. Games

To study the effects of different decision timings on the supply chain members' strategies and profits, we consider three scenarios (games). In each of these games, we assume that the manufacturer(s) is (are) leaders, while the retailer is a follower. We also assume that the competing manufacturers make their decisions simultaneously (play Nash). These assumptions are common in industries where companies use similar marketing planning practices and in most supply chains (Sudhir, 2001; Sethuraman, 2009; Ingene et al., 2012). This means that the manufacturer(s) always announce their decisions first and the retailer reacts to the manufacturers' decisions and chooses his own. We also assume that the retailer reacts by making the same kind of decision(s) (pricing, ME, or both) as those announced by the manufacturers (Karray, 2013; Karray and Martín-Herrán, 2019). This is to reflect the practice that in manufacturer-led channels, retailers usually set their retail prices after observing the manufacturers' wholesale price. They also set only their ME after learning about the

[1] For more discussion on the value of utility-based demand formulations, see Lus and Muriel (2009).

manufacturers' ME in an effort to coordinate promotional activities in the channel and to avoid unnecessary duplication of efforts.

Given these assumptions, we focus on the three games below. The following is a description of each in the context of manufacturer competition:

- Simultaneous decision-making in both price and ME (S). Game S is played in two stages. First, manufacturers play Nash and each sets his ME and pricing decisions simultaneously (w_i and a_{mi}, $i = 1, 2$). Knowing the manufacturers' decisions, the retailer then reacts by also setting both his ME and pricing strategies (p_i and a_{ri}, $i = 1, 2$).
- Sequential decision-making where ME are set before prices (MP). Game MP is played in four stages. First, manufacturers play Nash and each decides on his ME strategies. Second, the retailer sets his relevant ME decisions, knowing the manufacturers' ME levels. Third, the manufacturers play Nash and set their wholesale prices, knowing the retailer's ME decisions and their own. Fourth, the retailer decides on his retail prices, knowing the manufacturers' ME and wholesale prices as well as his ME levels.
- Sequential decision-making where ME are chosen after prices (PM). This game is also played in four stages. First, manufacturers play Nash and announce their wholesale prices. Second, the retailer sets his prices knowing the manufacturers' prices. Third, the manufacturers play Nash and set their ME strategies knowing the retailer's prices and their own. Finally, the retailer sets his ME levels, knowing all previously announced decisions.

5. Equilibrium solutions

We start by discussing the equilibrium solution for the case of the benchmark duopolistic channel. Then, we present the equilibrium for the case of a supply chain with competing manufacturers.

5.1. Bilateral monopolistic supply chain

We solve each of the three games (S, MP, and PM) described in the previous section by backward induction. We provide here a brief description of the procedure for solving these games. In the simultaneous scenario (S), the game is played in two stages. We start by solving the retailer's problem in ME and price, then use the obtained reaction functions to write the manufacturer's profit and solve his problem in his wholesale price and ME.

In the sequential game where ME is decided prior to pricing (MP), the game is played in four stages. We start by solving the retailer's pricing problem to get his price then use it to write the manufacturer's problem. The latter is solved to get the wholesale price. We then use the obtained expressions of both wholesale and retail prices to write the retailer's problem and solve it in his ME. Finally, the solution, along with all other pricing reactions functions, are injected in the manufacturer's problem, which is then solved to obtain the equilibrium ME strategy for the manufacturer.

A four-stage game is also played in the case where prices are decided prior to ME (PM). In this game, we start by solving the retailer's problem in ME, then use the obtained reaction function to write the manufacturer's problem. The latter is solved to get the manufacturer's ME. The obtained expression is then used to write the retailer's problem and solve it in the retail price. Finally, the obtained retail price along with all other ME reaction functions obtained in previous stages are injected in the manufacturer's problem, which is solved to get the equilibrium wholesale price. The analytical expressions of the equilibrium solution for each game is obtained by Karray and Martín-Herrán (2019) whose focus was the effects of store brand introduction. We refer to the proof in Karray and Martí n-Herrán (2019) and report the equilibrium solution in Appendix A for clarity.

We characterize the interior equilibrium conditions to check that (1) the obtained equilibrium solutions in each game verify the positivity conditions for all prices, ME, demand, margins ,and profits, and (2) the concavity conditions ensuring that the extrema are interior maxima are satisfied. We denote by each game's feasible region the parameter space in α and β that satisfies all positivity and concavity conditions in that game. The necessary conditions for interior equilibrium solutions for all games simultaneously (feasibility conditions) are given by $\beta \in (0, 2)$ and $\alpha \in (0, \sqrt{8 - 2\beta^2})$ (see Appendix A, Table A1). These conditions have to be verified when comparing equilibrium solutions obtained in these three games. Next, we present the sensitivity analysis of the equilibrium solution in each game to changes in the model parameters α and β.

Proposition 1. *In the case of a bilateral supply chain, sensitivity analysis of equilibrium solutions in games S, MP, and PM, given interior equilibrium conditions are as follows:*

$$\frac{\partial x}{\partial \alpha}, \frac{\partial x}{\partial \beta} > 0, \quad \forall x \in \{w, p, a_m, a_r, q, M, R\}.$$

Proof. The derivation of most of the signs of the derivatives is straightforward, given the interior equilibrium conditions. ■

This proposition shows that the equilibrium prices, ME, demand, and profit increase with higher ME effectiveness both by the retailer and manufacturer regardless of when they choose their prices and ME. This is mainly because ME boost consumer utility, which allows the retailer and manufacturer to charge higher prices and provides them with the incentive to invest more in ME. Despite the increase in prices, ME that are more effective ultimately expand demand, leading to increased revenues and profits for both firms. Finally, this proposition indicates that choosing different timings of pricing and ME does not change the sensitivity of manufacturer's and retailer's equilibrium outputs to ME effects.

5.2. Supply chain with manufacturer competition

To differentiate between results for games with and without competition, we use the superscript C to denote games and equilibrium solutions for the competitive case. Similar to the benchmark model, we use backward induction to solve each of the three games for the supply chain with competing manufacturers. However, in this case, manufacturers play Nash so each manufacturer's problem is solved in the relevant decision variable(s) simultaneously with the competing manufacturer. The analytical expressions of the equilibrium solution for each game are presented in the next proposition.

Table 2
Equilibrium solutions for the supply chain with manufacturer competition

	S^C	MP^C	PM^C
w	$\frac{(\beta^2+4(\gamma-1))(\beta^2-4(\gamma+1))}{\Phi}$	$\frac{4(2-\gamma)(\gamma^2-1)(\beta^2(\beta^2+24\gamma^2-32)+16(1-\gamma^2)(\gamma^2-4)^2)}{\Omega}$	$\frac{4(4-\beta^2-4\gamma)(\gamma^2-1)^2}{\Delta}$
a_m	$\frac{\alpha}{\beta}a_r$	$\frac{16\alpha(2-\gamma)(1-\gamma^2)[\beta^2-2(\gamma^4-6\gamma^2+8)]}{\Omega}$	$\frac{\alpha}{2(1-\gamma^2)}w$
p	$\frac{\beta^4-2(\gamma+5)\beta^2+8(3-2\gamma^2+\gamma)}{\Phi}$	$\frac{(2\gamma-3)}{2(\gamma-1)}w$	$\frac{2(\gamma+1)(\Psi-2(\gamma+1)(\gamma-1)^2((\beta^2-4)^2-16\gamma^2))}{\Delta(\beta^2-4(1+\gamma))}$
a_r	$\frac{\beta(4-\beta^2)}{\Phi}$	$\frac{\beta}{4(2-\gamma)(1-\gamma^2)}w$	$\frac{\beta\Psi}{\Delta(\beta^2-4(1+\gamma))}$
q	$\frac{2}{\beta}a_r$	$\frac{1}{2(1-\gamma^2)}w$	$\frac{2}{\beta}a_r$

Proposition 2. *The equilibrium solution for a supply chain with manufacturer competition in games S^C, MP^C, and PM^C are included in Table 2, where*

$$\Phi = \alpha^2\left(\beta^2 - 4\right) - 2\beta^2\left(2\gamma - \beta^2 + 8\right) - 16(\gamma + 1)(\gamma - 2),$$

$$\Omega = \left[\beta^2-4(\gamma-2)^2(\gamma+1)\right]^2\left[\beta^2+4(\gamma-1)(\gamma+2)^2\right]-16\alpha^2\left(\gamma^2-1\right)(\gamma-2)\left[\beta^2-2\left(\gamma^2-4\right)\left(\gamma^2-2\right)\right],$$

$$\Psi = \alpha^2\left[\beta^2(2\gamma^2 + 6 - \beta^2) + 8\left(\gamma^2 - 1\right)\right] + 4(\beta^2 - 4)(1 - \gamma^2)^2,$$

$$\Delta = 8\gamma\left(1 - \gamma^2\right)\left[\alpha^2 + 2\left(\gamma^2 - 1\right)\right] - 4\gamma^2\beta^2\alpha^2 + \left(\beta^2 - 4\right)[\beta^2\alpha^2 - 8\left(\gamma^2 - 1\right)^2].$$

Proof. See Appendix B. ■

The details of the solution methodology and expressions of the reaction functions and second-order conditions are included in Appendix B. We characterize the interior equilibrium conditions to check that (1) the obtained equilibrium solutions in each game verify the positivity conditions for all prices, ME, demands, margins, and profits, and (2) the concavity conditions ensuring that the extrema are interior maxima are satisfied (see Appendix B). These conditions are long, so we omit them here for ease of presentation. We denote by each game's feasible region the parameter space in α, β, and γ that satisfies all positivity and concavity conditions in that game.

Next, we study the sensitivity of equilibrium solutions to changes in the model parameters in each game, namely to the competition level (γ) and to the manufacturers' (α) and retailer's ME effects (β). The results are presented in Propositions 3–5. Note that these propositions do not report all the sensitivity analyses we have conducted. Whenever we find that the signs of our analyses can either be positive or negative depending on the values of the parameters, we identify the analytical conditions for the positivity of these expressions. We omit these conditions here because, given their complexity, no analytical insight can be derived. The proof for Propositions 3–5 is included in Appendix B.

Proposition 3. *In the case of a supply chain with competing manufacturers, sensitivity analysis of equilibrium solutions in each game to α given interior equilibrium conditions are as follows:*

$$\textit{Game } S^C \textit{and } MP^C : \frac{\partial x}{\partial \alpha} > 0,\ \forall x \in \{w, p, a_m, a_r, q, R\}; \quad \textit{Game } PM^C : \frac{\partial x}{\partial \alpha} > 0,\ \forall x \in \{p, a_r, q, R\}.$$

The signs of all other expressions can either be positive or negative depending on the values of the model's parameters.

Proof. See Appendix B. ■

The results in these propositions indicate that, at equilibrium, in all scenarios (S^C, MP^C, and PM^C), higher levels of manufacturers' ME effects (α) stimulate ME at both levels of the supply chain. The only exception is in the PM^C game, where a_m can react positively or negatively to changes in α depending on the model parameters' values. This first result indicates that, everything else being the same, a change in the manufacturers' ME effect can result in higher or lower investments in a_m depending solely on the decision timing implemented in the supply chain. In particular, when manufacturers set their prices prior to ME, they should not always increase their ME as consumers value their efforts more.

Note also that, in all games, higher levels of α boost not only the ME of the retailer, manufacturer, or both but also increase demand, therefore the retail revenue and ultimately his profit. As the manufacturers' ME effects increase, the retailer benefits from charging higher prices, even when the manufacturers are also charging a higher wholesale price (in the S^C and MP^C games). However, manufacturers do not always benefit when their ME are more effective. In the S^C and MP^C games, manufacturers' revenues increase with higher levels of α but their profitability does not always improve since their ME costs are higher. In the PM^C game, an increase in the effectiveness of manufacturers' ME may not benefit them either because they need to charge lower wholesale prices, invest more in ME or both.

Compared to the bilateral supply chain case (Proposition 1), these results show that product competition significantly changes the impact that ME have on the manufacturers' profitability in all games. While a monopolistic manufacturer gains from higher effectiveness levels of his ME, such an effect is not sustained when he is facing competition. Further, in the PM^C game, both manufacturers' strategies (w and a_m) do not always increase with higher levels of α as is the case for the bilateral monopoly.

Proposition 4. *In the case of a supply chain with competing manufacturers, sensitivity analysis of equilibrium solutions in each game to β given interior equilibrium conditions are as follows:*

$$\text{Game } S^C: \frac{\partial x}{\partial \beta} > 0,\ \forall x \in \{p, a_m, a_r, q, R\}; \quad \text{Game } MP^C: \frac{\partial a_m}{\partial \beta} > 0; \quad \text{Game } PM^C: \frac{\partial q}{\partial \beta} > 0.$$

The signs of all other expressions can either be positive or negative depending on the values of the model's parameters.

Proof. See Appendix B. ■

Looking at how changes in the retail ME effectiveness (β) affect equilibrium solutions in the different games, we find a positive impact of β on a_m in both the S^C and MP^C games but a mitigated effect in the PM^C game. The retailer's ME increase with his effectiveness level in the S^C game, but can either increase or decrease in both the MP^C and PM^C games.

Comparing these results to the ones reported in Proposition 3, note that, at equilibrium, the manufacturers' ME react in a similar way qualitatively to a change in the retailer's ME effects than to their own. However, the retailer's ME sensitivity to changes in β is different. These differences can be explained by looking at the effect of β on prices, demand, and retail profit. In the S^C game, the retailer benefits from higher levels of β even if he has to charge lower prices and/or gain lower margins. In this game, the increase in demand is driven by the higher levels of ME at both levels of the supply chain. However, in the sequential games (MP^C and PM^C), an increase in β does not necessarily expand the retailer's and/or the manufacturers' ME and demand. It may even increase prices, which explains why the retailer may not benefit from higher β. These findings show that the retailer's ME effects have a complex effect on strategies at equilibrium, especially when ME decisions are decoupled from pricing, in which cases the retailer reacts to each decision type separately.

Finally, compared to the bilateral supply chain case (Proposition 1), these results show that product competition significantly changes the impact that ME have on the pricing, ME strategies, and profitability of the supply chain firms, especially in the nonsimultaneous games. In fact, while both manufacturer and retail prices increase with higher levels of β in the bilateral monopoly case, they could decrease or increase in the competitive case. Further, in game PM^C, the manufacturers' ME do not always increase with higher retail ME effects as is the case in the bilateral monopoly channel.

Proposition 5. *In the case of a supply chain with competing manufacturers, sensitivity analysis of equilibrium solutions in game S^C to γ given interior equilibrium conditions are as follows:*

$$\frac{\partial w}{\partial \gamma}, \frac{\partial M}{\partial \gamma} < 0, \quad \frac{\partial x}{\partial \gamma} > 0 \Leftrightarrow \gamma < \frac{1}{8}\left(4 - \beta^2\right), \ \forall x \in \{a_m, a_r, q\}. \tag{2}$$

The signs of all other expressions in game S^C as well as in the other two games can either be positive or negative depending on the values of the model's parameters.

Proof. See Appendix B. ■

The results in this proposition address the sensitivity of the equilibrium solutions to changes in γ in the S^C game. In this case, the manufacturers' wholesale prices and profit decrease with higher levels of competition. This means that when manufacturers make all their decisions simultaneously, higher competition leads to a price war, which damages their profits. The effect of competition on manufacturers' ME depends on the competition level and on the retailer's ME effectiveness as shown in condition (2). Namely, for a given level of β, both manufacturers' and retailer's ME increase with higher levels of competition when γ is low enough and decrease otherwise. This means that, in the S^C game, the manufacturers and retailer should invest more in ME for more substitutable products only when the competition is not too high. Alternatively, cutting down on ME investments as competition increases should be adopted when the products are competing more closely. Further, for retail ME that are highly effective, ME at both levels of the supply chain are likely to decrease with γ and *vice versa*, indicating that the impact of competition on strategies is highly intertwined with ME effects.

Note that we cannot determine a definite sign for the effect of γ on retail price and profit. Contrary to the usual economic belief derived from pricing models, when ME effects are taken into account, higher competition levels do not necessarily decrease prices to consumers. The retailer may find it optimal to increase his price instead and either invest more in ME to boost demand and revenues and/or cut the price to consumers to expand demand further.

Finally, this proposition reports only the sensitivity of equilibrium strategies to γ in the S^C game and does not discuss results for the MP^C and PM^C games. This is because each equilibrium strategy in these games can either increase or decrease with γ given a set of complex conditions on the parameters. While these conditions are analytically intractable, we can deduce that manufacturers who decouple their ME and pricing decisions should not necessarily decrease their prices when faced with higher competitive pressures. Also, they do not always lose from intensified competition. This is an important result showing again that the timing of ME and pricing decisions plays an important role in how manufacturers and their retailers adjust their strategies to important market conditions.

6. Optimal timing of pricing and marketing effort decisions

We compare equilibrium solutions obtained for the benchmark case of a bilateral monopolistic supply chain. Then, we extend the analysis to the case of a supply chain with competing manufacturers. Finally, we compare these results to assess the effects of manufacturer competition.

6.1. Benchmark case: bilateral monopolistic supply chain

For a bilateral monopolistic chain, we compare equilibrium strategies and profits across the different games. The results are stated in the following propositions.

Proposition 6. *For a bilateral monopolistic supply chain, comparisons of equilibrium strategies in games S, PM and MP lead to the following results:*

$$x^{MP} < x^S \; \forall x \in \{p, a_m, a_r, q\}, \quad w^{MP} \pm w^S,$$
$$w^{PM} < w^S, \quad x^{PM} \pm x^S \; \forall x \in \{p, a_m, a_r, q\},$$
$$x^{PM} \pm x^{MP} \; \forall x \in \{w, p, a_m, a_r, q\},$$

with the sign $\pm$ meaning that the comparison can lead to positive or negative results depending on the values of the model's parameters.

Proof. See Appendix C for proof and for analytical conditions. ■

This proposition shows that the decision timing chosen by the manufacturer greatly influences the prices charged to consumers and retailers. It also largely impacts investments in ME as well as sales units. The nature of this influence depends on the scale of ME effectiveness.

For low levels of ME effectiveness, the game where ME are chosen prior to prices (MP) leads to the lowest levels of ME both by the manufacturer and retailer, as well as to the lowest retail prices and sales. However, compared to the other two games, the manufacturer charges the highest wholesale price in the MP game in order to compensate for the lowest level of sales units, which explains his low investment in ME. Therefore, the retailer may gain the lowest revenue in this game, which in turn explains why he invests the lowest level of ME in the MP game. Alternatively, the highest levels for prices, sales, and ME investments are achieved in game PM or S. In these games, the negative effects of high prices on demand are compensated for by the higher ME investments, which ultimately boosts demand. Finally, when prices and ME are decided simultaneously by each channel member, the levels of prices, ME, and sales are moderate to high at equilibrium.

These findings can be explained as follows. When prices are announced prior to ME, the manufacturer commits to the lowest wholesale price and the retailer opportunistically gains a high margin. The retailer's high price and margin then encourage both parties to boost their ME decisions in the next stage in order to increase demand. However, when the decision about ME is made prior to prices, the manufacturer commits to a low level of ME, which in turn also leads the retailer to follow suit and limit his ME investment. This forces the retailer to lower his price in order to increase demand. It also leads the manufacturer to charge a high wholesale price to boost his revenue. When the decisions about prices and ME are made simultaneously, the channel members do not have to commit to low levels of ME (as in the MP game) or prices (as in the PM game), and hence choose moderate to high levels for these decisions.

These results do not hold for highly effective ME. In this case, both supply chain firms invest the least in ME in the PM game. This results in low sales, which in turn explains the low retail price in the PM game. Further, for highly effective ME, the manufacturer charges his highest wholesale price in the MP game but commits to lower ME investments than in the other two games. This forces the retailer to lower his price in order to increase demand. Finally, when prices and ME are decided simultaneously by each channel member, the levels of prices and ME are moderate at equilibrium.

Next, we derive results for pairwise comparisons of equilibrium profits for the three games (S, MP, and PM) before identifying the preferred game for the manufacturer, retailer and total supply chain.

Proposition 7. *For a bilateral monopolistic supply chain, pairwise comparisons of equilibrium profits in games S, MP, and PM lead to the following results:*

$$\begin{aligned} M^S &> \max(M^{MP}, M^{PM}), \quad M^{MP} \pm M^{PM}, \\ R^S &> R^{MP}, \quad R^S \pm R^{PM}, \quad R^{MP} \pm R^{PM}, \\ T^S &> T^{MP}, \quad T^S \pm T^{PM}, \quad T^{MP} \pm T^{PM}, \end{aligned}$$

with the sign $\pm$ meaning that the comparison can lead to positive or negative results depending on the values of the model's parameters.

Proof. See Appendix C for proof and for analytical conditions. ■

Proposition 7 shows results of pairwise comparisons of profits for the manufacturer, retailer, and total channel. These comparisons are useful to understand the optimality of each game when only one other decision timing is available/possible for the supply chain.

First, when the manufacturer can choose between game S and MP, comparisons of equilibrium profits indicate that both the manufacturer's and retailer's profits are higher in game S than in MP for all parameters' values. Consequently, game S also yields higher total channel profit than does game MP. This result is driven by a better profit margin for the retailer, and by higher ME and sales in game S for both firms.

Second, when the manufacturer can choose between game S and PM, game S is preferred mainly because of his higher profit margin. However, the retailer does not always agree; depending on the

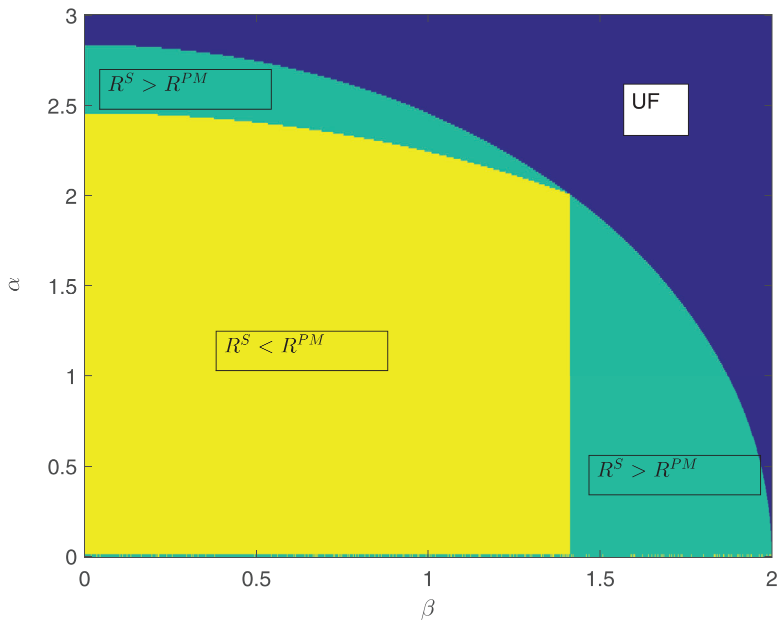

Fig. 1. Comparison of the retailer's profits in games S and PM (bilateral supply chain).

values of the ME effects, α and β, the retailer may find either game S or PM optimal. As we can see in Fig. 1, the retailer prefers game S for high levels of α and/or β and game PM otherwise.[2] This is because the retailer gains a lower margin and spends more on ME but gains higher sales in game S. Therefore, the retailer is playing suboptimally whenever the ME effect levels are not too high, leading to a channel conflict in this situation. In most cases, the retailer's preferred game is the one that also provides the highest total supply chain profit. This means that, as the leader, the manufacturer should consider adopting the decision timing that optimizes the total channel profits and redistributing it accordingly, for example, by implementing a profit-sharing mechanism.

Third, when the manufacturer can choose between the MP and PM games, the preferences of the manufacturer, retailer, and total supply chain depend on the values of α and β. Figure 2 shows that the manufacturer prefers game MP to PM only for low levels of β combined with high enough α. For all other values of α and β, he gains more profit by playing according to PM rather than to MP. This is because, under these conditions, the manufacturer gains a higher margin but lower sales in MP than in PM (Proposition 6). Figure 2 also shows that the retailer's profit is higher in PM than in MP in most parameters' domain due to a lower wholesale price and higher sales. When α is very high, the retailer prefers the MP game, which requires a lower ME investment. Comparing the total supply chain profits in MP and PM indicates similar results to the retailer. Therefore, the manufacturer and retailer may disagree on which decision timing serves best their interests, and the manufacturer should consider choosing the timing that optimizes the total supply chain profit rather than his own and redistributing it accordingly.

[2]In all figures, "UF" denotes the region where the feasibility conditions are not satisfied and the games cannot be compared.

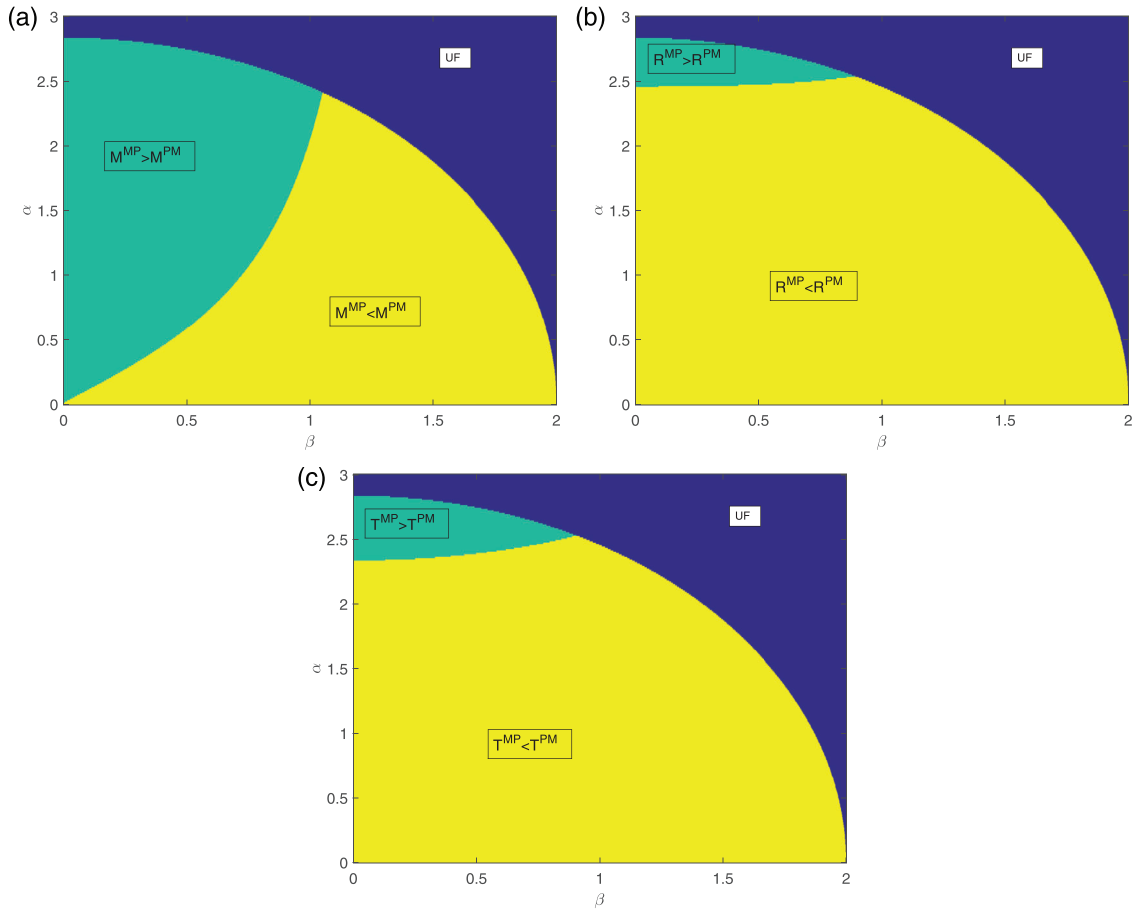

Fig. 2. Comparison of profits in games MP and PM (bilateral supply chain).

In the next proposition, we discuss the optimal game for each firm and for the total supply chain by comparing equilibrium profits from the three games simultaneously.

Proposition 8. *For a bilateral monopolistic supply chain, the optimal decision timing for price and ME is as follows:*

- *Game S is the manufacturer's optimal game.*
- *The retailer may prefer either game S or PM depending on the values of α and β.*
- *The retailer's preferred game leads to the highest total channel profit.*

Proof. Straightforward from Proposition 7. ■

In order to discuss more clearly the results reported in this proposition, we graphically represent these results by plotting the profit comparisons across the three games for the manufacturer and retailer in Fig. 3, left and right, respectively. We do not plot the comparison of the total supply chain profits because it leads to similar results to those for the retailer.

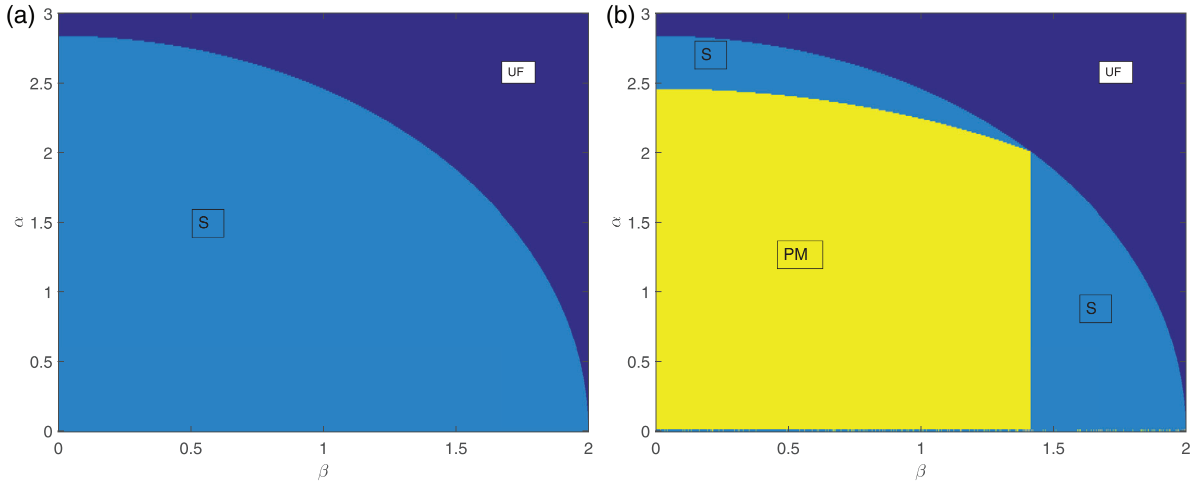

Fig. 3. Optimal game for the manufacturer (left) and for the retailer (right) (bilateral supply chain).

Proposition 8 shows that whenever all three games can be played (i.e., are feasible), as the supply chain leader, an opportunistic independent manufacturer should choose his ME and pricing decisions simultaneously (Fig. 3, left). This result can be explained by the fact that in game S, the manufacturer does not need to precommit to a low level of the decision that is chosen first (price in game PM or ME in game MP). He also does not need to increase the decision chosen last in order to compensate for the lost sales ensued by either a high retail price or low ME investments. When prices and ME decisions are chosen simultaneously, the absence of precommitments protects demand from extreme variations in prices and ME at the retail level, therefore leading to optimal profit levels for the manufacturer.

Proposition 8 also shows that the retailer's interest is not always aligned with the manufacturer's optimal game. In fact, game S is optimal for the retailer only for certain values of the ME effects parameters, α and β. Figure 3, right, shows that when α and/or β are high, the retailer's profit is highest when the manufacturer plays the S game. However, for lower levels of ME effectiveness, the retailer prefers game PM to S. This is because, in this case, higher retail margins and sales units are earned by the retailer in game PM, which results in a larger retail revenue. Since the retailer also invests more ME in game PM than in game S, his additional ME costs are offset by the increase in his revenue only when ME are largely effective. Finally, looking at the total channel profit across the three games, Proposition 8 indicates a similar result for the total channel's profit as for that of the retailer. This means that the retailer gains more profit than the manufacturer would lose if the latter switches its decision timing from S to PM. Therefore, the manufacturer should consider choosing the suboptimal game PM and redistribute the additional total profit.

For the benchmark scenario (bilateral monopoly), our results can be compared to those obtained in Karray (2013) who also studied the optimal decision timing for pricing and ME in a duopolistic supply chain led by the manufacturer. The results in Karray (2013) were different from the ones in this paper. While we find that the S game is predominately preferred by the manufacturer, it is the optimal manufacturer's game only for low values of retail ME effects (β) in Karray (2013). The dissimilarities in these results are mainly due to different modeling of the problem as they considered a coordinated supply chain through cooperative advertising. In fact, channel coordination can alleviate the decision timing impact on equilibrium strategies. For example, we find that the wholesale price is lower when chosen prior to ME (in the PM game) than when these decisions are announced simultaneously, while Karray (2013) shows different results. These discrepancies can be explained by the effects of cooperative programs, which are commonly found to inflate ME spending and prices in duopolistic channels (Jørgensen and Zaccour, 2014). Such programs can then mask the full impact that decision timing has on optimal profits.

6.2. Supply chain with manufacturer competition

In the case of a supply chain with competing manufacturers (products), we derive results for pairwise comparisons of equilibrium profits and strategies for the three games (S^C, MP^C, and PM^C) before identifying the preferred game for the manufacturers, retailer, and total supply chain. For simplicity and to obtain analytical results, we restrict our analysis to the case where $\alpha = \beta$. Then, we relax this assumption in Section 6.2.1 and explore results using a numerical method.

Proposition 9. *For a supply chain with competing manufacturers, comparison of equilibrium profits in games S^C and MP^C leads to the following results:*

- $M^{MP^C} > M^{S^C}$ *for* $\gamma > 0.3$. *Otherwise,* M^{MP^C} *can be higher or lower than* M^{S^C}.
- $R^{MP^C} < R^{S^C}$.
- $T^{MP^C} < T^{S^C}$.

Proof. See Appendix D for proof and for analytical conditions. ■

The first result in this proposition indicates that the manufacturers' profit is higher in game MP^C than in S^C for high product competition levels (γ). Otherwise, manufacturers may prefer either game S^C or MP^C. Therefore, manufacturers' preference between games S^C and MP^C depends mainly on the level of competition in the market. This result can be explained by the fact that, in game S^C, the manufacturers have to make price concessions for high levels of product competition, which hurts their profit margins. This in turn restricts the manufacturers' investments in ME, and ultimately lowers their demand for their products and results in lower profits (Proposition 5). Further, comparison of equilibrium strategies shows that ME are higher in game S^C than MP^C in most parameters' domain, while prices can be higher or lower. Therefore, when competition is high, manufacturers should pre-commit to lower levels of ME as their cost savings will compensate for the lost revenue due to a low profit margin. However, when products are not close competitors, this precommitment may not be profitable as higher levels of ME may benefit both brands.

This proposition also shows that game S^C provides higher profits to the retailer than does game MP^C. This is mainly because of the higher ME levels and retail profit margin in the S^C game, which lead to revenue gains. Despite higher ME costs in game S^C than in MP^C, the retailer's ME cost savings are enough to provide him with a higher profit. Finally, the total channel's profit is higher in game S^C than in MP^C. Since the manufacturers are the supply chain leaders, when they act opportunistically, they would choose the game that provides them with maximum profits. In this case, the optimal decision timing for the manufacturers does not always provide the retailer with maximum profit. This is especially true when the competition level between the manufacturers' products is high enough. In this case, conflict can arise among the supply chain firms, which can be alleviated by the manufacturers' choice of their suboptimal game along with a profit reallocation contract.

Proposition 10. *For a supply chain with competing manufacturers, comparison of equilibrium profits in games S^C and PM^C shows that the manufacturers', retailer's and total channel's profits may be higher or lower in game S^C than in PM^C depending on the values of the model parameters.*

Proof. See Appendix D for proof and for analytical conditions. ■

This proposition shows that the retailer and manufacturers may prefer game S^C or PM^C depending on the model parameters. Further, the analytical conditions indicate that, for any given value of the ME effect (α), the manufacturers prefer game S^C for low enough competition levels (γ), and game PM^C otherwise (see Appendix D). This result is mainly driven by the effect of decision timing on wholesale prices and ME. In fact, the manufacturers charge higher prices in game S^C than in PM^C when α is low enough. The gain in profit margin is sufficient to compensate for any increase in ME costs or loss in unit sales. However, for high levels of α, the manufacturers have to charge a lower price in game S^C than in PM^C. Further, ME levels are higher in game S^C when α is high (Proposition 2), which increases manufacturers' costs and leads to lower profits. Therefore, when the ME are highly effective, the additional ME investment required in game S^C does not boost demand significantly enough and game PM^C becomes more profitable for the manufacturer.

For the retailer, we find a different result. The S^C game provides higher retail profit than does PM^C when either ME are highly effective or when they are low but competition between the manufacturers' products is intense. In such cases, a larger profit margin and higher ME levels in game S^C provide the retailer with sales gains that are significant enough to boost his revenues despite the increase in his ME costs. Alternatively, low levels of ME effectiveness lead to lower ME levels and ultimately lower sales in game S^C than in PM^C. Because of the lower retail revenue in game S^C for low levels of α, the PM^C game becomes more profitable in this case.

Comparing the manufacturers' and retailer's preferences, note that the retailer gains more profit in game S^C whenever the manufacturers prefer PM^C (high β). Therefore, for most values of α and β, the retailer's and manufacturers' preferences diverge. The only case where their interests are aligned is when α is high. Therefore, the decision timing choices between S^C and PM^C can lead to conflict among the supply chain firms, which can be alleviated by the manufacturers' choice of their suboptimal game along with a profit reallocation contract.

Proposition 11. *For a supply chain with competing manufacturers, comparison of equilibrium profits in games MP^C and PM^C shows that the retailer's profit in game MP^C is lower than in game PM^C, while the manufacturers' and total channel's profits may be higher or lower in MP^C than in PM^C depending on the values of the model parameters.*

Proof. See Appendix D for proof and for analytical conditions. ■

This proposition shows that comparison of the manufacturers' profits obtained in games MP^C and PM^C depends on the model parameters. The analytical results indicate that the manufacturers prefer game MP^C to PM^C for very large values of α or very low values of γ (see Appendix D). This is mainly because, in most cases, manufacturers charge a higher wholesale price in game MP^C than in PM^C. They also invest less (more) in ME for low (high) enough values of α. In which case, the retailer reacts to the low manufacturers' ME investment by also lowering both his ME and price, which brings down his profit margin and sales, and ultimately decreases his revenue.

Therefore, when ME do not have a large influence on consumers, manufacturers prefer game PM^C. Despite the fact that manufacturers' profit margin is lower in PM^C, the retailer's profit margin is better, which induces the retailer to invest more in ME and reduce their prices to mitigate any negative effects on demand. Alternatively, manufacturers prefer game MP^C when their ME is effective enough because they can use their higher profit margin to increase their ME spending. They also prefer game MP^C when ME are highly effective because of the positive impact on revenues. The retailer prefers game PM^C to MP^C because he can gain higher profit margins in game PM^C due to the manufacturers' early commitment to lower wholesale prices.

Finally, conflict between the manufacturers and retailer may arise due to diverging preferences for decision timing. In fact, comparison of the total supply chain's profits shows that game MP^C yields lower profits overall than game PM^C in most parameters domain. Therefore, the total supply chain's profit comparisons are mostly aligned to those of the retailer, and channel conflict can be solved if the manufacturers choose their suboptimal game while implementing a profit reallocation contract.

Next, we discuss the optimal game for each firm and for the total supply chain by comparing equilibrium profits from the three games simultaneously.

Proposition 12. *For a supply chain with competing manufacturers, the most profitable game could be S^C, MP^C, or PM^C for the manufacturers and either S^C or PM^C for the retailer and the total supply chain depending on the values of the model parameters.*

Proof. See Appendix D for proof and for analytical conditions. ■

This proposition shows that any of the three games can be optimal for the manufacturers while either game MP^C or PM^C is optimal for the retailer as well as for the entire system. To gain more insights, we explore the analytical conditions for each case.

For the competing manufacturers, game S^C is optimal when γ is low enough (see Appendix D). In such cases, the manufacturers prefer to set all their decisions simultaneously rather than committing to either prices or ME separately. The explanation for this result is different when looking at the two alternative games. For low levels of α, compared to game PM^C, the manufacturers prefer S^C mostly because it allows them to gain a higher margin, while investing less in ME. On the contrary, game S^C is preferred by manufacturers over MP^C because of the higher ME and sales it entails. However, the advantages of game S^C over the other two games do not hold when γ becomes large.

For high levels of competition, the optimal game for the manufacturers depends on the level of ME effectiveness. For low enough values of α, the manufacturers' low investments in ME stimulate demand enough to boost revenues and result in cost savings, which ultimately makes game MP^C optimal for manufacturers. Alternatively, for high enough values of α, game MP^C is only optimal if γ is also high. Otherwise, game PM^C provides maximum manufacturers' profits. This is mainly because it provides higher unit margins compared to S^C, and greater ME and sales compared to MP^C.

On the retailer's side, either game PM^C or game S^C can be optimal. The retailer prefers game PM^C for low enough values of both α and γ. In such cases, PM^C benefits the retailer over the other two games through higher margins coupled with demand stimulation by investing more in ME. Alternatively, the retailer's optimal game is S^C for high values of α and/or γ. This is because lower ME investments can stimulate demand enough while benefiting the retailer through cost savings. Note that game MP^C is not optimal for the retailer mainly because it either results in a lower retail margin or demand due to insufficient ME spending. Finally, the results for the retailer are almost opposite to those obtained for the manufacturers.

Comparisons of total supply chain profits across the three games yields a similar finding than the one reported for the retailer. In fact, for most values of the model parameters, the optimal game for the total supply chain profit is the same than for the retailer. Recall that in almost all cases, the manufacturers' and retailer's interests are not aligned. As the supply chain leaders, opportunistic independent manufacturers who can choose among any of these three scenarios for decision timing may then choose a suboptimal timing strategy, as their preferred game will not lead to the highest supply chain profit. The manufacturers may then consider choosing the suboptimal sequence of move and redistribute the additional total profit.

6.2.1. Numerical analysis for the full model with manufacturer competition

We now extend our results in Section 6.2 to the case where $\alpha \neq \beta$. We adopt a similar approach and derive results for pairwise comparisons of equilibrium profits and strategies for the three games (S^C, MP^C, and PM^C) before identifying the preferred game for the manufacturers, retailer, and total supply chain. Numerical analyses are conducted to identify how the equilibrium profits compare in the three games for five different values of $\gamma \in (0, 1)$, $\alpha \in (0, 3)$ and $\beta \in (0, 2)$. The numerical analysis considers a mesh of 0.001 for each parameter, which means 1000 different values of γ, 3000 values of α, and 2000 values of β. This leads to a numerical analysis of the profit functions for 6 billion value combinations of parameters α, β, and γ. The profit comparisons are exclusively conducted in areas of the parameter space where all three games are feasible.

Because strategy comparisons do not lead to straightforward results (signs depend on the model's parameters), we do not include them and focus on profit comparisons instead. Our numerical analysis shows similar qualitative results to those obtained in Propositions 9–12 in the case of $\alpha = \beta$. For ease of illustration, we include Figs. E1–E3 to show results of pairwise profit comparisons between games. We focus here on the numerical results presenting the optimal game for each supply chain member as well as for the entire channel.

Our numerical analysis extends the result in Proposition 12 to the case of $\alpha \neq \beta$, and shows that any of the three games can be optimal for the manufacturers and either game S^C or PM^C is optimal for the retailer and the total supply chain. To gain more insights, we showcase the results in Fig. 4 by plotting the profit comparisons across the three games for the manufacturers (left) and retailer (right), respectively. We do not plot the comparison of the total supply chain profit because it leads to similar results to those for the retailer in Fig. 4, right. The results shown in this figure are for $\gamma = 0.2$ without loss of generality as they do not change qualitatively for other values of γ.

For the competing manufacturers, game S^C is optimal when α and/or β are not too high (Fig. 4, left). However, the advantages of game S^C over the other two games do not hold when either α or β become large. For high enough values of α, the manufacturers' low investments in ME stimulate demand enough to boost revenues and result in cost savings, which ultimately makes game MP^C optimal for manufacturers. Alternatively, for high enough values of β, game PM^C provides

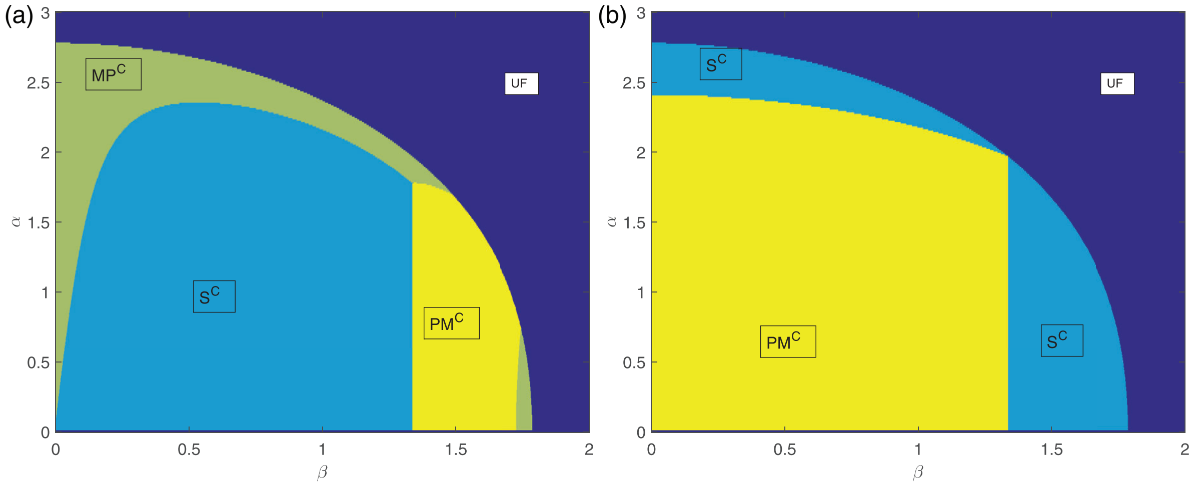

Fig. 4. Optimal game for manufacturers (left) and for the retailer (right) for $\gamma = 0.2$ (competitive supply chain).

maximum manufacturers' profits. This is mainly because it provides higher unit margins compared to S^C as well as greater ME and sales compared to MP^C.

On the retailer's side, Fig. 4, right, shows almost an opposite result to the one for the manufacturers. In fact, the retailer prefers game PM^C for low enough values of α and β. In such cases, PM^C benefits the retailer over the other two games through higher margins coupled with demand stimulation by investing more in ME. Further, Fig. 4, right, shows that the retailer's optimal game is S^C for high values of α or β . This is because lower ME investments can stimulate demand enough while benefiting the retailer through cost savings. Game MP^C is not optimal for the retailer mainly because it either results in a lower retail margin or lower demand due to insufficient ME spending.

Comparisons of total supply chain profits across the three games yields a similar finding than the one reported for the retailer. This indicates that the manufacturers may benefit from choosing a suboptimal sequence of move and redistribute the additional total profit.

7. Conclusions

This paper investigates the impact of decision timing for pricing and ME in supply chains. Different from the existing literature, we focus on a supply chain with competing manufacturers selling through a common retailer. We develop and solve six games where different decision timings are considered: (1) prices and ME are decided simultaneously, (2) ME is set before prices, and (3) ME is chosen after prices. We first examine these three scenarios for the benchmark case of a bilateral monopolistic channel. Then, we extend the analysis to consider competing manufacturers. We use a utility-based demand to model the effects of ME and prices, then solve for equilibrium strategies in each of the six games.

Comparisons of equilibrium profits across games provide important new results. For a bilateral monopolistic supply chain, we find that the manufacturer prefers simultaneous decision-making for price and ME. The retailer prefers the same timing but only for high levels of ME effects. Otherwise, he mostly prefers a scenario when prices are decided at an earlier stage than ME to benefit from the manufacturer's wholesale price concessions. Therefore, the retailer and manufacturer can face conflicting interests in most cases only because of how the manufacturer, as the leader, decides the timing of pricing and ME. Looking at the optimal total supply chain profit across scenarios, we find that, in most cases, it is highest in the timing scenario that is preferred by the retailer. Therefore, the manufacturer should consider choosing a suboptimal timing where he precommits to low prices before ME are determined in order to benefit the entire system. These results differ from previous studies that investigated the impact of different timings for pricing and ME in a similar supply chain setup (Karray, 2013).

Comparisons of equilibrium profits across games for a supply chain with manufacturer (product) competition show different results from the bilateral case. First, modeling product competition can significantly alter the manufacturers' optimal decision timing choice. Indeed, in the bilateral case, the manufacturer prefers to set pricing and ME simultaneously, while either one of the three games can be optimal for competing manufacturers. The explanation for this result is as follows. In a bilateral channel, when the ME and pricing decisions are decoupled, the monopolistic manufacturer has to pre-commit to either a low price (in the PM^C game) or ME (in the MP^C game), depending on ME effectiveness levels. When we account for competing manufacturers, the latter may precommit to higher levels of either price or ME in order to effectively compete in the market, depending on the intensity of product substitution and the effectiveness of the different ME. Second, for the retailer, we note that simultaneous decision-making is more profitable than when ME are decided prior to prices whether there is product competition in the channel. This is mainly because the retailer prefers that the manufacturer invests high levels of ME, which a precommitment would prohibit. Third, regardless of product competition, different preferences for pricing and ME can lead to channel conflict. However, the presence of product competition increases the opportunity for conflict in the supply chain. In fact, the profits of the retailer and manufacturer(s) are aligned in a larger portion of the parameter domain in the bilateral case than in the competitive case.

These results contribute to the literature in different ways. First, from a modeling perspective, our findings indicate that whenever ME and prices are modeled in the supply chain, assumptions about the timing of these decisions can greatly affect equilibrium outcomes. Therefore, a clear justification of such assumptions needs to be provided by the modeler by referring to either managerial practice or other constraints prohibiting the supply chain leader from choosing a different decision timing scenario. Second, from a managerial perspective, in supply chains where manufacturers can freely choose their decision timing for pricing and ME, managers should account for the influence of their decision-making process, and carefully assess the impact of any price or ME commitments to their channel partners as such contracts can affect their and the other firms' profitability. They should also consider choosing a different timing from their preferred one along with a profit-sharing mechanism in order to benefit the entire channel. In practice, this means that supply chain contracts that require manufacturers' commitment to either a low price (e.g., EDLP contracts) or low ME may not be beneficial to manufacturers, especially when ME are not highly effective.

This work can be extended in many ways. First, future research can explore different model assumptions (e.g., multiplicative demand functions) or additional operational variables (e.g., inventory), with which timing can also affect profits. Second, our findings show that accounting for manufacturer competition significantly changes the impact of decision timing for channel members. Future works can add retail competition to our model. Finally, we focus our analysis on manufacturer-led channels where the retailer reacts by setting the same type of decision (ME, price, or both) that the manufacturers announce. An interesting extension would be to consider other leadership scenarios in the channel.

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Appendix A: Equilibrium solution for the bilateral supply chain

Table A1
Equilibrium solutions in the bilateral monopolistic supply chain

	S	MP	PM
w	$\frac{4-\beta^2}{8-\alpha^2-2\beta^2}$	$\frac{8(16-\beta^2)}{(\beta^2-16)^2-32\alpha^2}$	$\frac{4}{8-\alpha^2\beta^2}$
a_m	$\frac{\alpha}{8-\alpha^2-2\beta^2}$	$\frac{32\alpha}{(\beta^2-16)^2-32\alpha^2}$	$\frac{2\alpha}{8-\alpha^2\beta^2}$
p	$\frac{6-\beta^2}{8-\alpha^2-2\beta^2}$	$\frac{12(16-\beta^2)}{(\beta^2-16)^2-32\alpha^2}$	$\frac{2[8-(\beta^2-2)(\alpha^2+2)]}{(8-\alpha^2\beta^2)(4-\beta^2)}$
a_r	$\frac{\beta}{8-\alpha^2-2\beta^2}$	$\frac{\beta(16-\beta^2)}{(\beta^2-16)^2-32\alpha^2}$	$\frac{\beta[4+\alpha^2(2-\beta^2)]}{(8-\alpha^2\beta^2)(4-\beta^2)}$
q	$\frac{2}{8-\alpha^2-2\beta^2}$	$\frac{4(16-\beta^2)}{(\beta^2-16)^2-32\alpha^2}$	$\frac{2[4+\alpha^2(2-\beta^2)]}{(8-\alpha^2\beta^2)(4-\beta^2)}$
M	$\frac{1}{8-\alpha^2-2\beta^2}$	$\frac{32}{(\beta^2-16)^2-32\alpha^2}$	$\frac{4}{(8-\alpha^2\beta^2)(4-\beta^2)}$
R	$\frac{4-\beta^2}{(8-\alpha^2-2\beta^2)^2}$	$\frac{(16-\beta^2)^3}{[(\beta^2-16)^2-32\alpha^2]^2}$	$\frac{[4+\alpha^2(2-\beta^2)]^2}{(8-\alpha^2\beta^2)^2(4-\beta^2)}$

Appendix B: Equilibrium solution for the supply chain with manufacturer competition

We obtain feedback equilibrium solutions using backward induction for the three games for the supply chain with manufacturer competition. The first game is S^C, where the manufacturers decide simultaneously on marketing efforts (ME) and pricing. In the second game, that is, MP^C, the manufacturers decide on ME prior to pricing. In the third game (PM^C), the manufacturers decide on pricing prior to ME. In each of these three games, the retailer is the follower; he makes the same decision(s) as the manufacturers in the previous stage of the game.

B.1. The S^C game

In S^C, the game is played in two stages. First, we consider the retailer's problem in the second stage given by

$$\max_{p_i,a_{ri}} R = \sum_{i=1,2} \left[(p_i - w_i)q_i - a_{ri}^2\right], \tag{B1}$$

where q_i is given by (1). Solving the following first-order equilibrium conditions:

$$\frac{\partial R}{\partial p_i} = \frac{\partial R}{\partial a_{ri}} = 0, \quad i = 1, 2,$$

yields the reaction functions to the manufacturers decision variables, that is,

$$p_i = \frac{2\left(4-\beta^2-4\gamma^2\right)(a_{mi}\alpha+v+w_i) - 2\gamma\beta^2(a_{mj}\alpha+v-w_j)-\left(4-\beta^2\right)\beta^2 w_i}{\left(4-\beta^2\right)^2 - 16\gamma^2}, \tag{B2}$$

$$a_{ri} = \frac{\beta\left(4-\beta^2\right)(a_{mi}\alpha+v-w_i)-4\beta\gamma(a_{mj}\alpha+v-w_j)}{\left(4-\beta^2\right)^2 - 16\gamma^2}, \quad i,j=1,2,\ i\neq j. \tag{B3}$$

The strict concavity of the retailer's profit function with respect to his decision variables p_i and a_{r_i}, $i = 1, 2$, is ensured if the quadratic form associated with the Hessian matrix ($\mathcal{H}$) is negative definite. The entries of this matrix considering the variables order p_1, p_2, a_{r1}, a_{r2} are

$$h_{11} = h_{22} = -\frac{2}{1-\gamma^2},\ h_{33} = h_{44} = -2,\ h_{12} = h_{21} = \frac{2\gamma}{1-\gamma^2},\ h_{13} = h_{31} = h_{24} = h_{42} = \frac{\beta}{1-\gamma^2},$$
$$h_{14} = h_{41} = h_{23} = h_{32} = -\frac{\beta\gamma}{1-\gamma^2}, \quad h_{34} = h_{43} = 0.$$

The quadratic form associated with the Hessian matrix is negative definite if and only if (iff) the following four conditions are satisfied:

$$h_{11} < 0, \quad h_{11}h_{22} - h_{12}h_{21} > 0,$$
$$h_{11}h_{22}h_{33} + h_{12}h_{23}h_{31} + h_{13}h_{21}h_{32} - h_{11}h_{23}h_{32} - h_{12}h_{21}h_{33} - h_{13}h_{22}h_{31} < 0,$$
$$Det[\mathcal{H}] > 0.$$

The first two conditions are satisfied for any $\gamma \in (0, 1)$ ($h_{11}h_{22} - h_{12}h_{21} = 4$).
The third and fourth conditions read, respectively, $\frac{2(\beta^2-4+4\gamma^2)}{(1-\gamma^2)^2} < 0$ and $\frac{(4-\beta)^2-16\gamma^2}{(1-\gamma^2)^2} > 0$.
Therefore, we can easily conclude that if the following conditions are satisfied:

$$4-\beta^2-4\gamma^2 > 0, \quad \left(4-\beta^2\right)^2 - 16\gamma^2 > 0, \tag{B4}$$

then, the retailer's profit function is a strictly concave function in the decision variables p_i and a_{r_i}.

We then insert the retailer's reaction functions into the manufacturers' optimization problems. The manufacturer i's optimization problem is given by

$$\max_{w_i, a_{mi}} M_i = w_i q_i - a_{m_i}^2, \tag{B5}$$

where q_i is given by (1) and p_i and a_{ri} have been replaced by (B2) and (B3), respectively. The manufacturers play a Nash game and decide their price and ME simultaneously. The manufacturers' first-order optimality conditions for a Nash game are given by

$$\frac{\partial M_i}{\partial w_i} = \frac{\partial M_i}{\partial a_{mi}} = 0, \quad i = 1, 2.$$

Solving these equations, the equilibrium wholesale prices and ME given in Table 2 are obtained.

The strict concavity of manufacturer i's profit function with respect to his decision variables w_i and a_{mi} is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix considering the variables order w_i, a_{mi} are

$$j_{11} = -\frac{4(4-\beta^2)}{(4-\beta^2)^2-16\gamma^2}, \quad j_{22} = -2, \quad j_{12} = j_{21} = \frac{2\alpha(4-\beta^2)}{(4-\beta^2)^2-16\gamma^2}.$$

The quadratic form associated with the Hessian matrix is negative definite iff the following two conditions are satisfied:

$$j_{22} < 0, \quad j_{11}j_{22} - j_{12}j_{21} > 0.$$

The first condition is always satisfied, and the second condition reads:

$$\frac{4\left(4-\beta^2\right)[2(\left(4-\beta^2\right)^2-16\gamma^2)-\alpha^2\left(4-\beta^2\right)]}{[(4-\beta^2)^2-16\gamma^2]^2} > 0.$$

Therefore, the condition

$$\left(4-\beta^2\right)[2(\left(4-\beta^2\right)^2-16\gamma^2)-\alpha^2\left(4-\beta^2\right)] > 0 \tag{B6}$$

ensures that manufacturer i's profit is a strictly concave function in the decision variables w_i and a_{mi}.

Next, we insert the equilibrium manufacturers' expressions into the retailer's reaction functions to obtain the equilibrium retail prices and ME of the retailer as functions of the model's parameter values. The equilibrium of game S^C exists iff conditions in (B4) and (B6) are satisfied. These conditions ensure that all strategies, demands, retailer's margins, manufacturers' profits and retailer's profits are positive.

B.2. The MP^C game

In MP^C, the game is played in four stages. To solve the game backward, we start with stage 4 and solve the retailer's problem in

$$\max_{p_i} R = \sum_{i=1,2} \left[(p_i - w_i)q_i - a_{ri}^2\right], \tag{B7}$$

where q_i is given by (1). We then solve the first-order conditions, $\frac{\partial R}{\partial p_i} = 0$, $i = 1, 2$, which provide the retailer's price reaction functions to his ME and to the manufacturers' decision variables, namely:

$$p_i = \frac{1}{2}(\alpha a_{mi} + \beta a_{ri} + v + w_i), \quad i = 1, 2. \tag{B8}$$

The strict concavity of retailer's profit function with respect to his decision variables $p_i, i = 1, 2$ is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix are:

$$h_{11} = h_{22} = -\frac{2}{1-\gamma^2}, \quad h_{12} = h_{21} = \frac{2\gamma}{1-\gamma^2}.$$

The second condition reads: $\frac{4}{1-\gamma^2} > 0$, and therefore, the two conditions are always satisfied for any value of $\gamma \in (0, 1)$. Therefore, the retailer's profit is a strictly concave function in the decision variables $p_i, i = 1, 2$ for any values of the model's parameter.

In stage 3, we insert these reaction functions into the manufacturer i's pricing problem to get

$$\max_{w_i} M_i = w_i q_i - a_{mi}^2, \tag{B9}$$

where q_i is given by (1) and $p_i, i = 1, 2$ have been replaced by their expressions in (B8). The manufacturers play a Nash game and the first-order optimality conditions $\frac{\partial M_i}{\partial w_i} = 0, i = 1, 2$ are solved to get the manufacturers' wholesale prices as functions of ME a_{mi} and a_{ri} such as

$$w_i = \frac{2v+\left(2-\gamma^2\right)(a_{mi}\alpha+a_{ri}\beta)-\gamma\left(a_{mj}\alpha+a_{rj}\beta+v(1+\gamma)\right)}{4-\gamma^2}, \quad i, j = 1, 2, i \neq j. \tag{B10}$$

Each manufacturer i's profit is a strictly concave function in the decision variable w_i for any values of the model's parameter ($\frac{\partial^2 M_i}{\partial w_i^2} = -\frac{1}{1-\gamma^2} < 0$). The manufacturers' price reaction functions are then inserted into the retailer's price reaction functions in (B8) and into the retailer's profit function.

In stage 2, we solve the retailer's ME problem given by

$$\max_{a_{ri}} R = \sum_{i=1,2} \left[(p_i - w_i)q_i - a_{ri}^2\right], \tag{B11}$$

where q_i is given by (1) and $p_i, w_i, i = 1, 2$ have been replaced by their expressions in (B8) and (B10).

The strict concavity of the retailer's profit function with respect to his decision variables a_{r1} and a_{r2} is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix are

$$j_{11} = j_{22} = -\frac{\left(3\gamma^2-4\right)\beta^2 + 4\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}{2(4-\gamma^2)^2(1-\gamma^2)}, \quad j_{12} = j_{21} = -\frac{\beta^2\gamma^3}{2(4-\gamma^2)(1-\gamma^2)}.$$

The quadratic form associated with the Hessian matrix is negative definite iff the following two conditions are satisfied:

$$j_{11} < 0, \quad j_{11}j_{22} - j_{12}j_{21} > 0.$$

The second condition reads:

$$\frac{\beta^4 + 8\left(3\gamma^2-4\right)\beta^2 + 16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}{4(4-\gamma^2)^2(1-\gamma^2)} > 0.$$

Therefore, the conditions

$$\left(3\gamma^2-4\right)\beta^2+4\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)>0, \tag{B12}$$

$$\beta^4+8\left(3\gamma^2-4\right)\beta^2+16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)>0 \tag{B13}$$

ensure that the retailer's profit is a strictly concave function in the decision variables a_{ri}, $i=1,2$.

We solve the retailer's first-order conditions given by $\frac{\partial R}{\partial a_{ri}}=0$. The solution gives the retailer's ME strategies as functions of the manufacturers' ME decision variables

$$a_{ri}=-\frac{\beta\left(\alpha a_{mi}\left(\beta^2+12\gamma^2-16\right)+4a_{mj}\alpha\gamma^3+v\left(\beta^2+4(\gamma-1)(\gamma+2)^2\right)\right)}{\beta^4+8\left(3\gamma^2-4\right)\beta^2+16\left(\gamma^2-4\right)^2\left(1-\gamma^2\right)},\quad i,j=1,2,i\neq j, \tag{B14}$$

which are then inserted in the pricing reaction functions of the manufacturers and of the retailer.

In stage 1, these functions are placed in the manufacturer i's advertising problem in

$$\max_{a_{mi}} M_i=w_iq_i-a_{mi}^2, \tag{B15}$$

where q_i is given by (1) and p_i, w_i, a_{ri}, $i=1,2$ have been replaced by their expressions in (B8), (B10), and (B14).

The first-order optimality conditions for the Nash between the manufacturers ($\frac{\partial M_i}{\partial a_{mi}}=0$, $i=1,2$) give the equilibrium solutions for a_{m_i}.

If $\frac{\partial^2 M_i}{\partial a_{mi}^2}<0$, then the manufacturer i's profit is a strictly concave function in the decision variable a_{mi}. We have

$$\frac{\partial^2 M_i}{\partial a_{mi}^2}=-2+\frac{64\alpha^2(1-\gamma^2)(\beta^2-2(\gamma^4-6\gamma^2+8))^2}{\left(\beta^4+8\left(3\gamma^2-4\right)\beta^2+16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\right)^2},$$

and an easy manipulation of this last expression allows us to conclude that the condition

$$\left(\beta^4+8\left(3\gamma^2-4\right)\beta^2+16\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)\right)^2-32\alpha^2\left(1-\gamma^2\right)\left(\beta^2-2\left(\gamma^4-6\gamma^2+8\right)\right)^2>0, \tag{B16}$$

implies that the manufacturer i's profit is a strictly concave function in the decision variable a_{mi}.

The equilibrium solutions for a_{ri}, $i=1,2$, are then next inserted in the manufacturer's and retailer's reaction functions to obtain the equilibrium retail and wholesale prices, and the ME of the retailer as functions of the model's parameters.

The equilibrium of game MP^C exists iff conditions in (B12), (B13), and (B16) are satisfied. With the help of the "Reduce" command of Mathematica 11.0, which solves equations or inequalities for variables and eliminates quantifiers, these conditions ensure that all strategies, demands, retailer's margins as well as the manufacturers' and retailer's profits are positive.

B.3. The PM^C game

In PM^C, the equilibrium solution is played by solving a four-stage game. We follow a similar approach as for MP^C. To solve the game backward, we start with stage 4 and solve the retailer's ME problem given by

$$\max_{a_{ri}} R = \sum_{i=1,2} (p_i - w_i)q_i - a_{ri}^2, \tag{B17}$$

where q_i is given by (1). We then solve the following first-order conditions: $\frac{\partial R}{\partial a_{ri}} = 0$, $i = 1, 2$, which provides the retailer's advertising reaction functions to his pricing and to the manufacturers' decision variables, namely:

$$a_{ri} = \frac{\beta(p_i - \gamma(p_j - w_j) - w_i)}{2(1-\gamma^2)}, \quad i, j = 1, 2,\ i \neq j. \tag{B18}$$

The retailer's profit is a strictly concave function in the decision variables a_{ri} for any values of the model's parameter (the quadratic form associated with the Hessian matrix ($h_{11} = h_{22} = -2$, $h_{12} = h_{21} = 0$) is always negative definite). In stage 3, we insert these reaction functions into the manufacturer i's advertising problem to get

$$\max_{a_{mi}} M_i = w_i q_i - a_{mi}^2, \tag{B19}$$

where q_i is given by (1), and a_{ri}, $i = 1, 2$ have been replaced by their expressions in (B18). We then solve the manufacturers' first-order optimality conditions for a Nash game $\frac{\partial M_i}{\partial a_{ri}} = 0$, $i = 1, 2$, to get the manufacturers' ME as functions of pricing strategies p_i and w_i such that

$$a_{mi} = \frac{\alpha w_i}{2(1-\gamma^2)}, \quad i = 1, 2. \tag{B20}$$

Each manufacturer i's profit is a strictly concave function in the decision variable a_{mi} for any values of the model's parameters ($\frac{\partial^2 M_i}{\partial a_{mi}^2} = -2 < 0$). The expression in (B20) is then inserted into the retailer's ME reaction functions in (B18) and into the retailer's profit function. Now we solve the retailer's pricing problem in stage 2 of the game given by

$$\max_{p_i} R = \sum_{i=1,2} (p_i - w_i)q_i - a_{ri}^2, \tag{B21}$$

with q_i given in (1). Next, we solve the retailer's first-order conditions ($\frac{\partial R}{\partial p_i} = 0$, $i = 1, 2$). The solution gives the retailer's prices as function of the manufacturers' wholesale prices

$$\begin{aligned} p_i = {} & \frac{1}{(1-\gamma^2)\left((\beta^2-8)\beta^2 + 16(1-\gamma^2)\right)} \left\{\gamma\beta^2 w_j\left(2(1-\gamma^2) - \alpha^2\right) + w_i\left[\alpha^2\left(4(1-\gamma^2) - \beta^2\right)\right.\right. \\ & \left.\left. + \left(1-\gamma^2\right)\left((\beta^2-6)\beta^2 + 8(1-\gamma^2)\right)\right] + 2(1-\gamma)(\gamma+1)^2 v\left(4(1-\gamma) - \beta^2\right)\right\}. \end{aligned} \tag{B22}$$

The strict concavity of the retailer's profit function with respect to his decision variables p_1 and p_2 is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix are

$$j_{11} = j_{22} = \frac{\beta^2(1+\gamma^2) - 4(1-\gamma^2)}{2(1-\gamma^2)^2}, \quad j_{12} = j_{21} = \frac{\gamma(2(1-\gamma^2) - \beta^2)}{(1-\gamma^2)^2}.$$

The quadratic form associated with the Hessian matrix is negative definite iff the following two conditions are satisfied:

$$j_{11} < 0, \quad j_{11}j_{22} - j_{12}j_{21} > 0.$$

The second condition reads:

$$\frac{(\beta^2-8)\beta^2 + 16(1-\gamma^2)}{4(1-\gamma^2)^2} > 0.$$

Therefore, the conditions

$$\left(\gamma^2+1\right)\beta^2 - 4\left(1-\gamma^2\right) < 0, \quad (\beta^2-8)\beta^2 + 16(1-\gamma^2) > 0 \tag{B23}$$

ensure that the retailer's profit is a strictly concave function in the decision variable p_i, $i = 1, 2$.

The expression in (B21) is then inserted in the ME reaction functions of the manufacturers in (B20) and of the retailer in (B18). Next, these functions are placed in the manufacturer i's pricing problem in stage 1, which is

$$\max_{w_i} M_i = w_i q_i - a_{mi}^2, \tag{B24}$$

where q_i is given by (1), and a_{ri}, a_{mi}, p_i, $i = 1, 2$ have been replaced by their expressions in (B18), (B20), and (B22).

The first-order optimality conditions for a Nash equilibrium between the manufacturers ($\frac{\partial M_i}{\partial w_i} = 0$, $i = 1, 2$) give the equilibrium solutions for w_i, $i = 1, 2$ in the PM^C scenario.

If $\frac{\partial^2 M_i}{\partial w_i^2} < 0$, then the manufacturer i's profit is a strictly concave function in the decision variable w_i. We have

$$\frac{\partial^2 M_i}{\partial w_i^2} = -\frac{\alpha^2\beta^4 - 4\beta^2\left(\alpha^2\left(\gamma^2+1\right) + 2\left(1-\gamma^2\right)^2\right) + 32\left(1-\gamma^2\right)^2}{2(1-\gamma^2)^2((\beta^2-8)\beta^2 + 16(1-\gamma^2))}.$$

Therefore, the condition

$$-\frac{\alpha^2\beta^4 - 4\beta^2\left(\alpha^2\left(\gamma^2+1\right) + 2\left(1-\gamma^2\right)^2\right) + 32\left(1-\gamma^2\right)^2}{(\beta^2-8)\beta^2 + 16(1-\gamma^2)} < 0 \tag{B25}$$

ensures that the manufacturer i's profit is a strictly concave function in the decision variable w_i.

Finally, the equilibrium wholesale prices are inserted in the manufacturers' and retailer's reaction functions to obtain the equilibrium retail prices, and the retailer's ME as functions of the model's parameters.

The equilibrium of game PM^C exists iff conditions in (B23) and (B25) are satisfied. With the help of the "Reduce" command of Mathematica 11.0, these conditions ensure that all strategies, demands, retailer's margins as well as manufacturers' and retailer's profits are positive.

B.4. Sensitivity analyses of equilibrium solutions in the S^C, MP^C, and PM^C games

B.4.1. Proof of Proposition 3. Recall that the equilibrium in the S^C game exists iff conditions in (B4) and (B6) are satisfied. These two conditions also ensure that all strategies, demands, retailer's margins as well as manufacturers' and retailer's profits are positive:

$$\frac{\partial w}{\partial \alpha} = \frac{2\alpha v(4-\beta^2)((4-\beta^2)^2-16\gamma^2)}{\left(\alpha^2\left(\beta^2-4\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)^2}.$$

Condition (B4) implies $4-\beta^2<0$, and therefore, $\frac{\partial w}{\partial \alpha}>0$ iff $(4-\beta^2)^2-16\gamma^2>0$. The second condition in (B4) implies that last inequality is satisfied:

$$\frac{\partial p}{\partial \alpha} = \frac{2\alpha\left(4-\beta^2\right)g\left(\beta^4-2(\gamma+5)\beta^2-8\left(2\gamma^2-\gamma-3\right)\right)}{\left(\alpha^2\left(\beta^2-4\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)^2}.$$

Condition (B4) implies $4-\beta^2>0$, and hence, $\frac{\partial p}{\partial \alpha}>0$ iff $\beta^4-2(\gamma+5)\beta^2-8(2\gamma^2-\gamma-3)>0$. This last inequality is ensured if the retail price is positive:

$$\frac{\partial a_m}{\partial \alpha} = \frac{\left(4-\beta^2\right)v\left(\alpha^2\left(4-\beta^2\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)}{\left(\alpha^2\left(\beta^2-4\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)^2}.$$

Condition (B4) implies $4-\beta^2>0$, and therefore, $\frac{\partial a_m}{\partial \alpha}>0$ iff $\alpha^2(4-\beta^2)+2(\beta^2+2\gamma-4)(\beta^2-4(\gamma+1))>0$. It can be analytically proved that conditions (B4) and (B6) imply that last inequality is satisfied:

$$\frac{\partial a_r}{\partial \alpha} = \frac{2\alpha\beta\left(\beta^2-4\right)^2 v}{\left(\alpha^2\left(\beta^2-4\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)^2}>0,$$

$$\frac{\partial q}{\partial \alpha} = \frac{4\alpha\left(\beta^2-4\right)^2 v}{\left(\alpha^2\left(\beta^2-4\right)+2\left(\beta^2+2\gamma-4\right)\left(\beta^2-4(\gamma+1)\right)\right)^2}>0.$$

Using Mathematica 11.1, we show that the retailer's optimal profits increase as α increases.

Let us recall that the equilibrium in the MP^C game exists iff conditions in (B12), (B13), and (B16) are satisfied. Additionally, these three conditions ensure that all strategies, demands, retailer's margins, as well as manufacturers' and retailer's profits are positive:

$$\frac{\partial w}{\partial \alpha} = \frac{128b(2-\gamma)^2(1-\gamma^2)^2 vZ_1}{Z_2},$$
$$\frac{\partial p}{\partial \alpha} = \frac{64\alpha(2-\gamma)^2(1-\gamma)(\gamma+1)^2(3-2\gamma)vZ_1}{Z_2},$$
$$\frac{\partial a_r}{\partial \alpha} = \frac{32\alpha\beta(2-\gamma)(1-\gamma^2)vZ_1}{Z_2},$$
$$\frac{\partial q}{\partial \alpha} = \frac{64\alpha(2-\gamma)^2(1-\gamma^2)vZ_1}{Z_2},$$

where

$$Z_1 = \left(\beta^4+8(3\gamma^2-4)\beta^2-16(\gamma^2-4)^2(\gamma^2-1)\right)\left(2(\gamma^4-6\gamma^2+8)-\beta^2\right),$$
$$Z_2 = \Big[\left(\beta^2-4(\gamma-2)^2(\gamma+1)\right)^2\left(\beta^2-4(1-\gamma)(\gamma+2)^2\right)$$
$$-\ 16\alpha^2(2-\gamma)(1-\gamma^2)\left(\beta^2-2(\gamma^4-6\gamma^2+8)\right)\Big]^2 > 0.$$

Both factors in Z_1 are positive under conditions (B12), (B13), and (B16):

$$\frac{\partial a_m}{\partial \alpha} = \frac{16(2-\gamma)(1-\gamma^2)v\left(2(\gamma^4-6\gamma^2+8)-\beta^2\right)Z_3}{Z_2},$$

where

$$Z_3 = 16\alpha^2(\gamma-2)(1-\gamma^2)\left(\beta^2-2(\gamma^4-6\gamma^2+8)\right)$$
$$-\left(\beta^2-4(\gamma-2)^2(\gamma+1)\right)^2\left(\beta^2+4(\gamma-1)(\gamma+2)^2\right).$$

The sign of $\frac{\partial a_m}{\partial \alpha}$ coincides with the sign of $(2(\gamma^4-6\gamma^2+8)-\beta^2)Z_3$. The first factor is positive to ensure that the advertising a_m is positive. Therefore, $\frac{\partial a_m}{\partial \alpha} > 0$ iff $Z_3 > 0$. With Mathematica 11.0, we analytically proved that conditions (B12), (B13), and (B16) imply that last inequality is satisfied.

We also show that the retailer's optimal profits increase as α increases.

Let us recall that the equilibrium in the PM^C game exists iff conditions in (B23) and (B25) are satisfied. Additionally, these conditions ensure that all strategies, demands, retailer's margins, and manufacturers and retailer's profits are positive:

$$\frac{\partial p}{\partial \alpha} = \frac{8\alpha\left(1-\gamma^2\right)^2 v\left(\beta^6-\left(4\gamma^2-6\gamma+6\right)\beta^4-8\gamma\left(3\gamma^2-4\gamma+1\right)\beta^2-32(\gamma-1)^3(\gamma+1)\right)}{Z_5},$$

where

$$Z_5 = \left(\alpha^2\beta^4 - 4\beta^2\left(\alpha^2(\gamma^2+1)+2(\gamma^2-1)^2\right)+8(1-\gamma^2)\left((\alpha^2-2)\gamma+2\gamma^3-4\gamma^2+4\right)\right)^2 > 0.$$

We show that the last factor in the numerator of $\frac{\partial p}{\partial \alpha}$ is positive under conditions (B23) and (B25):

$$\frac{\partial a_r}{\partial \alpha} = \frac{8\alpha\beta(1-\gamma^2)^2 v(\beta^2+4\gamma-4)^2}{Z_5} > 0,$$

$$\frac{\partial q}{\partial \alpha} = \frac{16\alpha(1-\gamma^2)^2 v(\beta^2+4\gamma-4)^2}{Z_5} > 0,$$

$$\frac{\partial q}{\partial \beta} = \frac{4\beta v Z_6}{(\beta^2-4(\gamma+1))^2 Z_5},$$

where

$$\begin{aligned} Z_6 = {} & \alpha^4(\beta^8 - 4(\gamma^2+3)\beta^6 + 8(\gamma^4+7)\beta^4 + 128(\gamma^2-1)\beta^2 + 64(\gamma^6 - 3\gamma^2+2)) \\ & + 8\alpha^2(\gamma^2-1)^2\left(32\gamma^4 - \beta^2(\beta^2-4)^2 + 4(\beta^4-4\beta^2-8)\gamma^2\right) \\ & + 32(\gamma^2-1)^4\left((\beta^2-4)^2+8\gamma^2\right). \end{aligned}$$

Conditions (B23) and (B25) imply $Z_6 > 0$.

We also show that the retailer's optimal profits increase as α increases.

B.4.2. Proof of Proposition 4. For game S^C we get

$$\frac{\partial a_r}{\partial \beta} = \frac{v\left((\beta^2-4)^2(-\alpha^2+2\beta^2+8)+16(3\beta^2-4)\gamma^2+4\gamma(\beta^2-4)^2\right)}{\left(\alpha^2(\beta^2-4)+2(\beta^2+2\gamma-4)(\beta^2-4(\gamma+1))\right)^2}.$$

Therefore, $\frac{\partial a_r}{\partial \beta} > 0$ iff $(\beta^2-4)^2(-\alpha^2+2\beta^2+8)+16(3\beta^2-4)\gamma^2+4\gamma(\beta^2-4)^2 > 0$. We proved that conditions (B4) and (B6) imply that the last inequality is satisfied:

$$\frac{\partial a_m}{\partial \beta} = \frac{4\alpha\beta v\left((\beta^2-4)^2+8\gamma^2\right)}{\left(\alpha^2(\beta^2-4)+2(\beta^2+2\gamma-4)(\beta^2-4(\gamma+1))\right)^2} > 0,$$

$$\frac{\partial q}{\partial \beta} = \frac{8\beta v\left((\beta^2-4)^2+8\gamma^2\right)}{\left(\alpha^2(\beta^2-4)+2(\beta^2+2\gamma-4)(\beta^2-4(\gamma+1))\right)^2} > 0.$$

Using Mathematica 11.1, we show that the retailer's optimal profits increase as β increases.

Concerning game MP^C we have

$$\frac{\partial a_m}{\partial \beta} = \frac{64\alpha\beta(2-\gamma)\big(1-\gamma^2\big)v\big(4(\gamma-2)^2(\gamma+1)-\beta^2\big)Z_4}{Z_2},$$

where

$$Z_4 = \big(\beta^4+\big(\gamma^2(\gamma(2-3\gamma)+24)-32\big)\beta^2-4(\gamma-2)(\gamma+2)\big((\gamma(\gamma(\gamma+7)-2)-20)\gamma^2+16\big)\big).$$

The sign of $\frac{\partial a_m}{\partial \beta}$ coincides with the sign of $(4(\gamma-2)^2(\gamma+1)-\beta^2)Z_4$. The first factor is positive to ensure that the retailer's profits are positive. Therefore, $\frac{\partial a_m}{\partial \beta} > 0$ iff $Z_4 > 0$. With Mathematica 11.0, we analytically proved that conditions (B12), (B13), and (B16) imply that the last inequality is satisfied.

B.4.3. Proof of Proposition 5. For game S^C, one has

$$\frac{\partial w}{\partial \gamma} = \frac{4\big(4-\beta^2\big)v\Big(8\gamma\big(\alpha^2+\beta^2-4\big)-\big(\beta^2-4\big)^2-16\gamma^2\Big)}{\big(\alpha^2\big(\beta^2-4\big)+2\big(\beta^2+2\gamma-4\big)\big(\beta^2-4(\gamma+1)\big)\big)^2}.$$

Condition (B4) implies $4-\beta^2 > 0$, and therefore, $\frac{\partial w}{\partial \gamma} < 0$ iff $8\gamma(\alpha^2+\beta^2-4)-(\beta^2-4)^2-16\gamma^2 < 0$. We analytically proved that conditions (B4) and (B6) imply that the last inequality is satisfied:

$$\frac{\partial a_r}{\partial \gamma} = \frac{4\beta\big(4-\beta^2\big)v\big(\beta^2+8\gamma-4\big)}{\big(\alpha^2\big(\beta^2-4\big)+2\big(\beta^2+2\gamma-4\big)\big(\beta^2-4(\gamma+1)\big)\big)^2},$$

$$\frac{\partial a_m}{\partial \gamma} = \frac{4\alpha\big(4-\beta^2\big)v\big(\beta^2+8\gamma-4\big)}{\big(\alpha^2\big(\beta^2-4\big)+2\big(\beta^2+2\gamma-4\big)\big(\beta^2-4(\gamma+1)\big)\big)^2},$$

$$\frac{\partial q}{\partial \gamma} = \frac{8\big(4-\beta^2\big)v\big(\beta^2+8\gamma-4\big)}{\big(\alpha^2\big(\beta^2-4\big)+2\big(\beta^2+2\gamma-4\big)\big(\beta^2-4(\gamma+1)\big)\big)^2}.$$

Condition (B4) implies $4-\beta^2 > 0$, and therefore, $\frac{\partial a_r}{\partial \gamma} > 0$, $\frac{\partial a_m}{\partial \gamma} > 0$ and $\frac{\partial q}{\partial \gamma} > 0$ iff $\beta^2+8\gamma-4 > 0$.

Using Mathematica 11.1, we show that the manufacturer's optimal profits decrease as γ increases.

Appendix C: Comparison across games for the bilateral supply chain

C.1. Proof of Proposition 6

Direct comparison of the retail price, the manufacturer's and retailer's advertising as well as the demand at equilibrium for games MP and S show that under the feasibility conditions:

$$x^{MP} < x^S, \quad \forall x \in \{p, a_m, a_r, q\}.$$

The feasibility condition also implies that

$$w^{PM} < w^S.$$

With the help of Mathematica 11.0, we have deduced the following conditions characterizing when the comparison of equilibrium solutions leads to positive or negative results:

$$p^S < p^{PM} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad 0 < \alpha < \sqrt{\frac{4+4\beta^2-\beta^4}{2}},$$

$$p^S > p^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta < \sqrt{2} & \text{and} \quad \sqrt{\frac{4+4\beta^2-\beta^4}{2}} < \alpha < \sqrt{8-2\beta^2}, \\ & \text{or} \\ \sqrt{2} < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8-2\beta^2}. \end{cases}$$

$$a_r^S < a_r^{PM} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad 0 < \alpha < \sqrt{6-\beta^2},$$

$$a_r^S > a_r^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta < \sqrt{2} & \text{and} \quad \sqrt{6-\beta^2} < \alpha < \sqrt{8-2\beta^2}, \\ & \text{or} \\ \sqrt{2} < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8-2\beta^2}. \end{cases}$$

$$a_m^S < a_m^{PM} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad 0 < \alpha < 2,$$

$$a_m^S > a_m^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta < \sqrt{2} & \text{and} \quad 2 < \alpha < \sqrt{8-2\beta^2}, \\ & \text{or} \\ \sqrt{2} < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8-2\beta^2}. \end{cases}$$

$$q^S < q^{PM} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad 0 < \alpha < \sqrt{6-\beta^2},$$

$$q^S > q^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta < \sqrt{2} & \text{and} \quad \sqrt{6-\beta^2} < \alpha < \sqrt{8-2\beta^2}, \\ & \text{or} \\ \sqrt{2} < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8-2\beta^2}. \end{cases}$$

$$w^S < w^{MP} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad 0 < \alpha < \frac{\sqrt{64-20\beta^2+\beta^4}}{2\sqrt{6}},$$

$$w^S > w^{MP} \quad \text{iff} \quad 0 < \beta < \sqrt{2} \quad \text{and} \quad \frac{\sqrt{64-20\beta^2+\beta^4}}{2\sqrt{6}} < \alpha < \sqrt{8-2\beta^2}.$$

$$a_m^{MP} < a_m^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta \leq 0.9949 & \text{and} \quad 0 < \alpha < 0.25\sqrt{\frac{\beta^4-32\beta^2+128}{2-\beta^2}}, \\ & \text{or} \\ 0.9949 < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8-2\beta^2}. \end{cases}$$

$$a_m^{MP} > a_m^{PM} \quad \text{iff} \quad 0 < \beta < 0.9949 \quad \text{and} \quad 0.25\sqrt{\frac{\beta^4 - 32\beta^2 + 128}{2 - \beta^2}} < \alpha < \sqrt{8 - 2\beta^2},$$

$$w^{MP} < w^{PM} \quad \text{iff} \quad 1.0887 < \beta < 1.9002 \quad \text{and} \quad 0.7070\sqrt{\frac{\beta^4 - 16\beta^2}{\beta^4 - 16\beta^2 + 16}} < \alpha < \sqrt{8 - 2\beta^2},$$

$$w^{MP} > w^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta \leq 1.08871 & \text{and} \quad 0 < \alpha < \sqrt{8 - 2\beta^2}, \\ & \text{or} \\ 1.08871 < \beta < 1.90021 & \text{and} \quad 0 < \alpha < 0.7070\sqrt{\frac{\beta^4 - 16\beta^2}{\beta^4 - 16\beta^2 + 16}}, \\ & \text{or} \\ 1.90021 \leq \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8 - 2\beta^2}. \end{cases}$$

$$q^{MP} < q^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta \leq 0.8747 & \text{and} \quad 0 < \alpha < g_{1\beta}, \\ & \text{or} \\ 0.8747 < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8 - 2\beta^2}. \end{cases}$$

$$q^{MP} > q^{PM} \quad \text{iff} \quad 0 < \beta < 0.8747 \quad \text{and} \quad g_{1\beta} < \alpha < \sqrt{8 - 2\beta^2},$$

where $g_{1\beta}$ denotes the exact second root of a fourth-order polynomial equation $g_1(x) = 0$ which coefficients depend on β as follows:

$$g_1(x) = 192\beta^2 - 12\beta^4 + (384 - 192\beta^2 - 6\beta^4 + \beta^6)x^2 + 32(\beta^2 - 2)x^4.$$

$$a_r^{MP} < a_r^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta \leq 0.7154 & \text{and} \quad 0 < \alpha < g_{2\beta}, \\ & \text{or} \\ 0.7154 < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8 - 2\beta^2}. \end{cases}$$

$$a_r^{MP} > a_r^{PM} \quad \text{iff} \quad 0 < \beta < 0.7154 \quad \text{and} \quad g_{2\beta} < \alpha < \sqrt{8 - 2\beta^2},$$

where $g_{2\beta}$ denotes the exact second root of a fourth-order polynomial equation $g_2(x) = 0$ which coefficients depend on β as follows:

$$g_2(x) = 256 + 16\beta^2 - 2\beta^4 + (192 - 128\beta^2 + 7\beta^4)x^2 + 16(\beta^2 - 2)x^4.$$

$$p^{MP} < p^{PM} \quad \text{iff} \quad \begin{cases} 0 < \beta \leq 1.00399 & \text{and} \quad 0 < \alpha < g_{3\beta}, \\ & \text{or} \\ 1.00399 < \beta < 2 & \text{and} \quad 0 < \alpha < \sqrt{8 - 2\beta^2}. \end{cases}$$

$$p^{MP} > p^{PM} \quad \text{iff} \quad 0 < \beta < 1.00399 \quad \text{and} \quad g_{3\beta} < \alpha < \sqrt{8 - 2\beta^2},$$

where $g_{3\beta}$ denotes the exact second root of a fourth-order polynomial equation $g_3(x) = 0$ which coefficients depend on β as follows:

$$g_3(x) = 64\beta^2 + 28\beta^4 - 2\beta^6 + (128 + 128\beta^2 - 86\beta^4 + 5\beta^6)x^2 + 32(\beta^2 - 2)x^4.$$

C.2. Proof of Proposition 7

C.2.1. Comparison of equilibrium profits in games S and MP

- Manufacturer's profits:

$$M^S - M^{MP} = \frac{\beta^2(\beta^2 + 32)g^2}{(\alpha^2 + 2\beta^2 - 8)\left(32\alpha^2 - (\beta^2 - 16)^2\right)}.$$

Under the feasibility conditions the denominator is positive, and hence, $M^S > M^{MP}$.

- Retailer's profits:

$$R^S - R^{MP} = \frac{\beta^2 g^2\left(\alpha^4(\beta^4 - 48\beta^2 - 256) + (\beta^2 - 16)^2(\beta^2 - 4)(4\alpha^2 + 3(\beta^2 - 16))\right)}{(\alpha^2 + 2\beta^2 - 8)^2\left((\beta^2 - 16)^2 - 32\alpha^2\right)^2}.$$

The sign of the difference coincides with the sign of the numerator. This expression is always positive under the feasibility conditions. Therefore, $R^S > R^{MP}$.

- Total channel's profits:

$$T^S - T^{MP}$$
$$= \frac{\beta^2 g^2\left(\alpha^4(\beta^4 - 16\beta^2 + 768) + \alpha^2(3\beta^6 - 80\beta^4 + 4096\beta^2 - 20480) + (\beta^2 - 4)(\beta^2 - 112)(\beta^2 - 16)^2\right)}{(\alpha^2 + 2\beta^2 - 8)^2\left((\beta^2 - 16)^2 - 32\alpha^2\right)^2}.$$

The sign of the difference coincides with the sign of the numerator. The Reduce command in Mathematica 11.0 analytically shows that this expression is always positive under the feasibility conditions. Therefore, $T^S > T^{MP}$.

C.2.2. Comparison of equilibrium profits in games S and MP

- Manufacturer's profits:

$$M^S - M^{PM} = -\frac{\alpha^2(\beta^2 - 2)^2 g^2}{(\beta^2 - 4)(\alpha^2 + 2\beta^2 - 8)(\alpha^2\beta^2 - 8)}.$$

The three factors in the denominator are negative from the feasibility conditions. Therefore, $M^S > M^{PM}$.

- Retailer's profits:

$$R^S - R^{PM} = \frac{\alpha^2(\beta^2-2)g^2(\alpha^2+\beta^2-6)\left(\alpha^4(\beta^2-2)+\alpha^2(3\beta^4-16\beta^2+12)-16(\beta^2-4)\right)}{(\beta^2-4)(\alpha^2+2\beta^2-8)^2(\alpha^2\beta^2-8)^2}.$$

Because under the feasibility conditions $\beta^2 - 4 < 0$, the sign of the difference is the opposite to the sign of the numerator. With the help of the "Reduce" command from Mathematica 11.0, we analytically prove that under the feasibility conditions the following applies:

$$R^S - R^{PM} > 0 \quad \text{iff}\left(0 < \beta < \sqrt{2},\ \sqrt{6-\beta^2} < \alpha < \sqrt{8-2\beta^2}\right)\text{or}$$
$$\left(\sqrt{2} < \beta < 2,\ 0 < \alpha < \sqrt{8-2\beta^2}\right),$$
$$R^S - R^{PM} < 0 \quad \text{iff } 0 < \beta < \sqrt{2}, \quad 0 < \alpha < \sqrt{6-\beta^2}.$$

- Total channel's profits:

$$T^S - T^{PM} = \frac{\alpha^2(\beta^2-2)g^2\left(\alpha^6(\beta^2-2)+\alpha^4(\beta^2-6)(3\beta^2-4)+\alpha^2(\beta^6-22\beta^4+84\beta^2-24)+64(\beta^2-4)\right)}{(\beta^2-4)(\alpha^2+2\beta^2-8)^2(\alpha^2\beta^2-8)^2}.$$

Because under the feasibility conditions $\beta^2 - 4 < 0$, the sign of the difference is the opposite to the sign of the numerator. The "Reduce" command from Mathematica 11.0 analytically shows that under the feasibility conditions the following applies:

$$T^S - T^{PM} > 0 \quad \text{iff}\left(0 < \beta < \sqrt{2}, \quad f_{1\beta} < \alpha < \sqrt{8-2\beta^2}\right)\text{or}\left(\sqrt{2} < \beta < 2,\ 0 < \alpha < \sqrt{8-2\beta^2}\right),$$
$$T^S - T^{PM} < 0 \quad \text{iff } 0 < \beta < \sqrt{2}, \quad 0 < \alpha < f_{1\beta},$$

where $f_{1\beta}$ denotes the exact third root of a sixth-order polynomial equation $f_1(x) = 0$ which coefficients depend on β as follows:

$$f_1(x) = -256 + 64\beta^2 + (-24 + 84\beta^2 - 22\beta^4 + \beta^6)x^2 + (24 - 22\beta^2 + 3\beta^4)x^4 + (-2 + \beta^2)x^6.$$

C.2.3. Comparison of equilibrium profits in games S and MP

- Manufacturer's profits:

$$M^{MP} - M^{PM} = \frac{4g^2\left(8\alpha^2(\beta^2-2)^2 - \beta^2(\beta^2+32)\right)}{(\beta^2-4)(\alpha^2\beta^2-8)\left((\beta^2-16)^2-32\alpha^2\right)}.$$

The first and second factors in the denominator are negative, while the third factor is positive. Therefore, the product of the three factors is positive. Therefore, the sign of the difference is similar to the sign of the following expression $8\alpha^2(\beta^2-2)^2-\beta^2(\beta^2+32)$. The "Reduce" command from Mathematica 11.0 analytically shows that under the feasibility conditions the following applies:

$$M^{MP}-M^{PM}>0 \quad \text{iff}\, 0<\beta\leq 1.07838, \quad \frac{1}{2\sqrt{2}}\frac{\beta}{2-\beta^2}\sqrt{32+\beta^2}<\alpha<\frac{1}{4\sqrt{2}}(16-\beta^2),$$

$$M^{MP}-M^{PM}<0 \quad \text{iff}\begin{cases} 0<\beta\leq 1.07838, \quad 0<\alpha<\frac{1}{2\sqrt{2}}\frac{\beta}{2-\beta^2}\sqrt{32+\beta^2} \\ \qquad \text{or} \\ 1.07838<\beta<2, \quad 0<\alpha<\frac{2\sqrt{2}}{\beta}. \end{cases}$$

- Retailer's profits:

$$R^{MP}-R^{PM}=\frac{4g^2 Num_1}{(\beta^2-4)(\alpha^2\beta^2-8)^2\left((\beta^2-16)^2-32\alpha^2\right)^2},$$

$$\begin{aligned}Num_1 =&\,16\alpha^4\big(\alpha^2(\beta^2-2)(16\alpha^2(\beta^2-2)-\beta^6+34\beta^4-320\beta^2+384)+256\big)\\ &+(\beta^2-16)^2\big(\alpha^4\beta^2(-4\beta^4+81\beta^2-160)+2\alpha^2(\beta^6-6\beta^4-192\beta^2+384)-12\beta^2(\beta^2-16)\big).\end{aligned}$$

From the feasibility conditions, the denominator of the difference is negative. Therefore, the sign of the difference is opposite to the sign of the numerator. The "Reduce" command from Mathematica 11.0 analytically shows that under the feasibility conditions the following applies:

$$R^{MP}-R^{PM}>0 \quad \text{iff}\, 0<\beta\leq 1.07838, \quad f_{2\beta}<\alpha<\frac{1}{4\sqrt{2}}(4-\beta^2),$$

$$R^{MP}-R^{PM}<0 \quad \text{iff}\begin{cases} 0<\beta\leq 1.07838, \quad 0<\alpha<f_{2\beta} \\ \qquad \text{or} \\ 1.07838<\beta<2, \quad 0<\alpha<\frac{2\sqrt{2}}{\beta}, \end{cases}$$

where $f_{2\beta}$ denotes the exact third root of a eighth-order polynomial equation $f_2(x)=0$, the coefficients of which depend on β as follows:

$$\begin{aligned}f_2(x)=&\,49,152\beta^2-9216\beta^4+576\beta^6-12\beta^8+(196,608-122,880\beta^2+9984\beta^4+512\beta^6-76\beta^8\\ &+2\beta^{10})x^2+(4096-40,960\beta^2+25,856\beta^4-3776\beta^6+209\beta^8-4\beta^{10})x^4\\ &+(-12,288+16,384\beta^2-6208\beta^4+576\beta^6-16\beta^8)x^6+(1024-1024\beta^2+256\beta^4)x^8.\end{aligned}$$

- Total channel's profits:

$$T^{MP} - T^{PM} = \frac{4g^2 Num_2}{\left(\beta^2-4\right)\left(\alpha^2\beta^2-8\right)^2\left(\left(\beta^2-16\right)^2-32\alpha^2\right)^2},$$

where

$$\begin{aligned}Num_2 = {} & \alpha^2\left(4\alpha^2+1\right)\beta^{10} - \left(16\alpha^6+79\alpha^4+140\alpha^2+4\right)\beta^8 + 64\left(5\alpha^6-10\alpha^4+56\alpha^2+9\right)\beta^6 \\ & +1024\alpha^2\left(\alpha^6-12\alpha^4+12\alpha^2+128\right) + 64\left(4\alpha^8-81\alpha^6+308\alpha^4-364\alpha^2-240\right)\beta^4 \\ & -1024\left(\alpha^8-15\alpha^6+40\alpha^4+56\alpha^2-112\right)\beta^2.\end{aligned}$$

From the feasibility conditions, the denominator of the difference is negative. Therefore, the sign of the difference is opposite to the sign of the numerator. The "Reduce" command from Mathematica 11.0 analytically shows that under the feasibility conditions, the following applies:

$$T^{MP} - T^{PM} > 0 \quad \text{iff}\, 0 < \beta \le 1.07838, \quad f_{3\beta} < \alpha < \frac{1}{4\sqrt{2}}(4-\beta^2),$$

$$T^{MP} - T^{PM} < 0 \quad \text{iff}\begin{cases} 0 < \beta \le 1.07838, \quad 0 < \alpha < f_{3\beta} \\ \qquad\qquad \text{or} \\ 1.07838 < \beta < 2, \quad 0 < \alpha < \frac{2\sqrt{2}}{\beta}, \end{cases}$$

where $f_{3\beta}$ denotes the exact third root of a eighth-order polynomial equation $f_3(x) = 0$ which coefficients depend on β as follows:

$$\begin{aligned}f_3(x) = {} & 114,688\beta^2 - 15,360\beta^4 + 576\beta^6 - 4\beta^8 \\ & +(131,072 - 57,344\beta^2 - 23,296\beta^4 + 3584\beta^6 - 140\beta^8 + \beta^{10})x^2 \\ & +(12,288 - 40,960\beta^2 + 19,712\beta^4 - 640\beta^6 - 79\beta^8 + 4\beta^{10})x^4 \\ & +(-12,288 + 15,360\beta^2 - 5184\beta^4 + 320\beta^6 - 16\beta^8)x^6 + (1024 - 1024\beta^2 + 256\beta^4)x^8.\end{aligned}$$

Appendix D: Comparison across games for the supply chain with manufacturer competition: case $\alpha = \beta$

All the results in this Appendix have been analytically proved with the help of the "Reduce" command of Mathematica 11.0.

Under the hypothesis $\alpha = \beta$, it can be shown that all three games are feasible iff one of the following conditions is satisfied:

$$0 < \alpha < 2\sqrt{2/3} = 1.633, \quad 0 < \gamma < \frac{\sqrt{32-20\alpha^2+3\alpha^4}}{4\sqrt{2}},$$

$$0 < \gamma < 1, \quad 0 < \alpha < \sqrt{2/3}\sqrt{5-\sqrt{1+24\gamma^2}}.$$

D.1. Proof of Proposition 9

Under the feasibility conditions, the manufacturer's, retailer's, and total supply chain's profits for games S^C and MP^C are compared as follows.

- Manufacturer's profits: $M^{MP^C} > M^{S^C}$ for $\gamma > 0.3896$. Otherwise, M^{MP^C} can be higher or lower than M^{S^C}. In particular, for $\gamma < 0.3896$ and $0 < \alpha < 2\sqrt{2/3}$,

$$M^{S^C} > M^{MP^C} \quad \text{iff} \quad 0 < \gamma < \widetilde{\gamma}(\alpha),$$
$$M^{S^C} < M^{MP^C} \quad \text{iff} \quad \widetilde{\gamma}(\alpha) < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

  where $\tilde{\gamma}(\alpha)$ represents the exact fifth root of a tenth polynomial equation $h_1(x) = 0$, where the coefficients of the polynomial depend on α (we refrain from writing the expression of polynomial $h_1(x)$ because it is a long expression). Furthermore, $\widetilde{\gamma}(\alpha)$ decreases as α increases and hence, $\widetilde{\gamma}(\alpha) < \widetilde{\gamma}(0) \simeq 0.3896$.
- Retailer's profits: $R^{MP^C} < R^{S^C}$.
- Total supply chain's profits: $T^{MP^C} < T^{S^C}$.

D.2. Proof of Proposition 10

Under the feasibility conditions, the manufacturer's, retailer's, and total supply chain's profits for games S^C and PM^C are compared as follows.

- Manufacturers' profits:
  - $M^{S^C} > M^{PM^C}$ if one of the following conditions is satisfied:

$$0 < \alpha < \sqrt{2}, \quad 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$
$$\sqrt{2} < \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \widehat{\gamma}(\alpha);$$

  - $M^{S^C} < M^{PM^C}$ if one of the following conditions is satisfied:

$$0 < \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$
$$\alpha = \sqrt{2}, \quad 0 < \gamma < \frac{1}{2\sqrt{2}},$$
$$\sqrt{2} < \alpha < 2\sqrt{2/3}, \quad \widehat{\gamma}(\alpha) < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

where $\widehat{\gamma}(\alpha)$ represents the exact third root of a ninth polynomial equation $h_2(x) = 0$, where the coefficients of the polynomial depend on α (we refrain from writing the expression of polynomial $h_2(x)$ because it is a long expression). Furthermore, $\hat{\gamma}(\alpha)$ is an inverted U-shape function of α, which is zero for $\alpha = \sqrt{2}$ and $\alpha = 2\sqrt{2/3}$ and takes a maximum at $\alpha \simeq 1.585$.

- Retailer's profits:
  * $R^{S^C} > R^{PM^C}$ if one of the following conditions is satisfied:

$$0 < \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$\sqrt{2} \le \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}};$$

  * $R^{S^C} < R^{PM^C}$ if $0 < \alpha < \sqrt{2}, \quad 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}}$.
- Total supply chain's profits:
  * $T^{S^C} > T^{PM^C}$ if one of the following conditions are satisfied:

$$0 < \alpha < 0.289, \text{ and either } \overline{\gamma}_2(\alpha) < \gamma < \overline{\gamma}_1(\alpha) \text{ or } \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$0.289 \le \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$\sqrt{2} \le \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}};$$

  * $T^{S^C} < T^{PM^C}$ if one of the following conditions are satisfied:

$$0 < \alpha < 0.289, \quad \text{and either} \quad 0 < \gamma < \overline{\gamma}_2(\alpha) \quad \text{or} \quad \overline{\gamma}_1(\alpha) < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

$$\alpha = 0.289, \quad \text{and either} \ 0 < \gamma < \overline{\gamma}_2(\alpha) \ \text{ or } \ \overline{\gamma}_2(\alpha) < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

$$0.289 < \alpha < \sqrt{2} \quad 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

where $\overline{\gamma}_1(\alpha)$ and $\overline{\gamma}_2(\alpha)$ represent the exact sixth and fifth roots of a ninth polynomial equation $h_3(x) = 0$, where the coefficients of the polynomial depend on α (we refrain from writing the expression of polynomial $h_3(x)$ because it is a long expression). Furthermore, $\overline{\gamma}_1(\alpha)$ is a decreasing and strictly concave function of α, while $\overline{\gamma}_2(\alpha)$ is an increasing and strictly convex function of α. Both functions take values between 0 and 1, and $\overline{\gamma}_2(\alpha) < \overline{\gamma}_1(\alpha)$ for $0 < \alpha < 0.289$.

D.3. Proof of Proposition 11

Under the feasibility conditions, the manufacturer's, retailer's and total supply chain's profits for games S^C and PM^C are compared as follows:

- Manufacturers' profits:
- $M^{MP^C} > M^{PM^C}$ if $0 < \alpha < 1.43$, $\underline{\gamma}(\alpha) < \gamma < \frac{\sqrt{32-20\alpha^2+3\alpha^4}}{4\sqrt{2}}$, where $\underline{\gamma}(\alpha)$ represents the exact 5th root of a 25th polynomial equation $h_4(x) = 0$, where the coefficients of the polynomial depend on α (we refrain from writing the expression of polynomial $h_4(x)$ because it is a long expression). Furthermore, $\underline{\gamma}(\alpha)$ is an increasing and strictly convex function of α. For $0 < \alpha < 1.43$, function $\underline{\gamma}(\alpha)$ is upper bounded by 0.34.
- Retailer's profits $R^{MP^C} < R^{PM^C}$.
- Total supply chain's profits: $T^{MP^C} < T^{PM^C}$ for $\gamma < 0.901$ or $\alpha > 0.142$.

D.4. Proof of Proposition 12

Under the feasibility conditions the following applies:

- The most profitable game for the manufacturers is
  * S^C if one of the following conditions is satisfied:

$$0 < \alpha \le 1.27, \quad 0 < \gamma < \widetilde{\gamma}(\alpha),$$

$$1.27 < \alpha < \sqrt{2}, \quad 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

$$\sqrt{2} < \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \widehat{\gamma}(\alpha).$$

  * MP^C if one of the following conditions is satisfied:

$$0 < \alpha \le 1.27, \quad \widetilde{\gamma}(\alpha) < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$1.27 < \alpha < 1.43, \quad \underline{\gamma}(\alpha) < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}}.$$

  * PM^C if one of the following conditions is satisfied:

$$1.27 < \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \underline{\gamma}(\alpha),$$

$$\alpha = \sqrt{2}, \quad 0 < \gamma < \underline{\gamma}(\alpha),$$

$$\sqrt{2} < \alpha < 1.43, \quad \widehat{\gamma}(\alpha) < \gamma < \underline{\gamma}(\alpha),$$

$$1.43 < \alpha < 2\sqrt{2/3}, \quad \widehat{\gamma}(\alpha) < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}}.$$

- The most profitable game for the retailer is
  - S^C if one of the following conditions is satisfied:

$$0 < \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$\sqrt{2} \leq \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}}.$$

  - PM^C iff $0 < \alpha < \sqrt{2},\ 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}}$.
- The most profitable game for the total supply chain is
  - S^C if one of the following conditions is satisfied:

$$0 < \alpha < 0.289, \text{ and either } \overline{\gamma}_2(\alpha) < \gamma < \overline{\gamma}_1(\alpha) \text{ or } \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$0.289 \leq \alpha < \sqrt{2}, \quad \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}} < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}},$$

$$\sqrt{2} \leq \alpha < 2\sqrt{2/3}, \quad 0 < \gamma < \frac{\sqrt{32 - 20\alpha^2 + 3\alpha^4}}{4\sqrt{2}}.$$

  - PM^C if one of the following conditions is satisfied:

$$0 < \alpha < 0.289, \quad \text{and either } \ 0 < \gamma < \overline{\gamma}_2(\alpha) \ \text{ or } \ \overline{\gamma}_1(\alpha) < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

$$\alpha = 0.289, \quad \text{and either} \quad 0 < \gamma < \overline{\gamma}_2(\alpha) \ \text{ or } \ \overline{\gamma}_2(\alpha) < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}},$$

$$0.289 < \alpha < \sqrt{2}, \quad 0 < \gamma < \frac{\sqrt{\frac{8-6\alpha^2+\alpha^4}{4+\alpha^2}}}{\sqrt{2}}.$$

Appendix E: Comparison across games for the supply chain with manufacturer competition

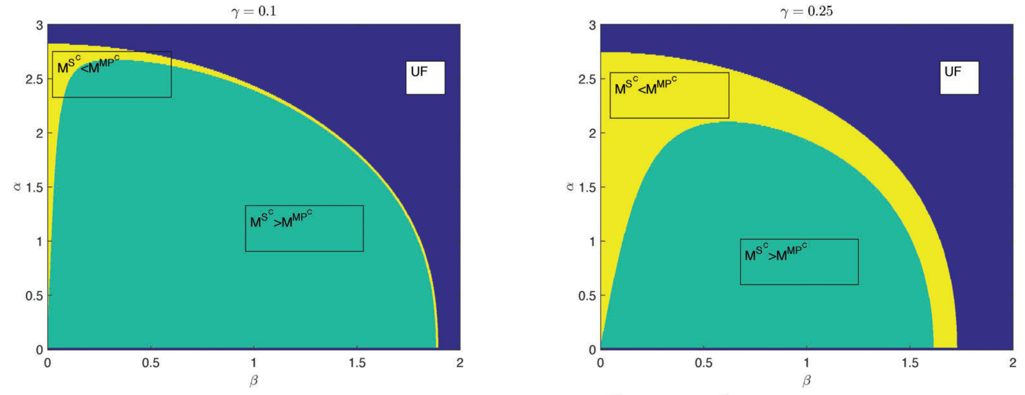

Fig. E1. Comparison of manufacturers' profits in games S^C and MP^C (competitive supply chain).

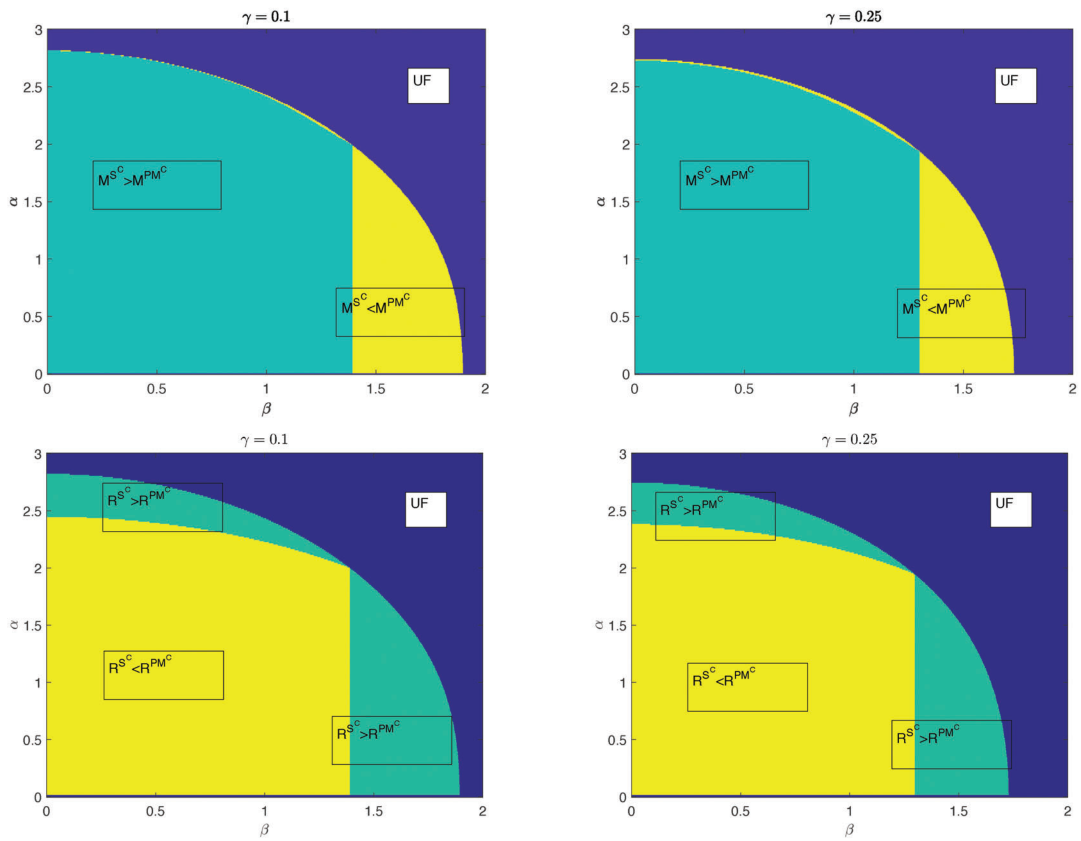

Fig. E2. Comparison of profits in games S^C and PM^C (competitive supply chain).

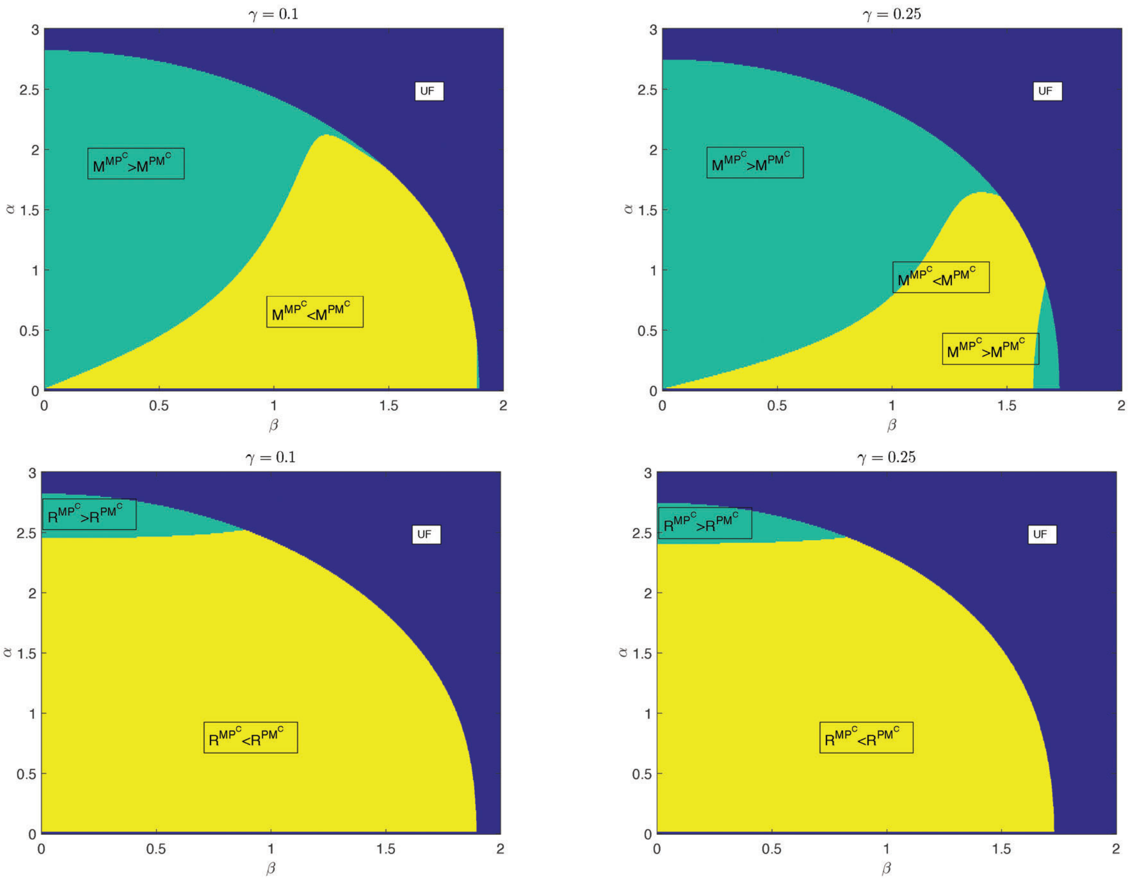

Fig. E3. Comparison of profits in games MP^C and PM^C (competitive supply chain).