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Managing advertising investments in marketing channels☆

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ABSTRACT

This paper investigates how should manufacturers optimally allocate resources to retailer-initiated (retailer) advertising through cooperative advertising programs and own (manufacturer) advertising in a bilateral monopoly. Retailer advertising stimulates immediate sales but may also harm long-term (post-advertising) demand, whereas manufacturer advertising aims at building brand equity and stimulates both immediate and long-term sales. A game-theoretic model in which a manufacturer and a retailer set pricing and advertising decisions over a two-period planning horizon is developed to account for the differences between manufacturer and retailer advertising. We characterize equilibrium solutions for four advertising scenarios for the manufacturer, ranging from no investment in any advertising activity to undertaking own advertising and supporting retailer advertising simultaneously. Comparing the two players' equilibrium strategies and profits across these scenarios, we find that manufacturers should avoid offering exclusively cooperative advertising programs to retailers. When retailer advertising positively influences long-term sales, manufacturers should offer cooperative advertising supports to retailers in addition to undertaking their own advertising. When retailer advertising negatively affects long-term sales, manufacturers can still undertake own advertising and offer cooperative advertising under certain conditions. However, if these conditions are not met, focusing exclusively on own advertising is their best advertising strategy. Retailers also prefer scenarios in which manufacturers advertise, but may choose not to participate in manufacturers' cooperative advertising programs. This leads to suboptimal outcomes if cooperative advertising programs are not enhanced by additional incentives (e.g., side payments or other services).

1. Introduction

One of the most challenging issues manufacturers encounter in the management of marketing channels is how to allocate advertising resources between national and cooperative advertising to respond to the expectations of channel partners and secure maximum profits. Manufacturers offer push advertising activities such as cooperative advertising to support retailers' advertising efforts, whereas their national advertising aims at building brand equity and stimulates immediate and long-term (post advertising) sales. Manufacturers who invest in both activities need to understand the effects of vertical interactions with their channel partners in the trade-off between national and cooperative advertising strategies. In fact, manufacturers' national or/and cooperative advertising decisions interact with the retailers' decisions in stimulating the demand. The combined outcome of channel partner choices not only can affect the demand but also their individual pricing and advertising decisions and profits.

National advertising consists of manufacturer-initiated campaigns targeted at consumers with the goal of stimulating demand at the retail level. This type of advertising is also known as pull advertising (e.g., non-price advertising in traditional media). It tends to focus on strengthening brand image, improving awareness and building brand preference (e.g., Nunes and Merrihue, 2007) and has positive long-term effects on sales even though this effect may decay over time (Jørgensen et al., 2000, 2001a; Ataman et al., 2010). Manufacturers invest substantial amounts on national advertising campaigns. For example, the top 200 advertisers spent a total of $165 billion combined on their U.S. campaigns during the dark days of Covid-19 in 2020, with Procter &

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Gamble co spending alone amounting to $4.7 billion (Ad Age Leading National Advertisers Report, 2021). Generally, manufacturers have full control over the content, timing, horizon, and budget of their national advertising campaigns.

Conversely, cooperative advertising refers to advertising support provided to trade partners such as retailers, with the goal of inducing additional local advertising efforts from these partners. Cooperative advertising is one of the many push promotional activities manufacturers use to support channel trade partners. In typical cooperative advertising programs, manufacturers reimburse a percentage of advertising expenses that retailers incur to advertise their products. The popularity of these programs has dramatically increased in recent years. In fact, in 2015, cooperative advertising spending provisions in North America were estimated at $36 billion, representing 12% of all advertising spending (Borrell Associates, 2015). This amount is growing with recent estimates reaching $70 billion (Reiffen, 2018). For example, in 2018, Ford increased its cooperative advertising funds from $135 million to roughly $270 million (Martinez, 2018). Macy's benefited from $394 million in coop ads in 2016 and Neiman Marcus coop support was about $50.1 million in 2017 (Chen, 2017).

Many arguments can be made in favor of providing cooperative advertising support to retailers. These include: (a) the belief that retailers have a better knowledge of their local markets and therefore can undertake more effective local advertising programs; (b) the affordability of local media compared to national media; and (c) the focus of retailer advertising on immediate sales stimulation (Jørgensen et al., 2001a; Herrington and Dempsey, 2005; Ataman et al., 2010). Further, an extensive academic literature supports the view that, under certain conditions, manufacturers can enhance channel profits by sharing part of their retailers' advertising expenses (see Aust and Buscher, 2014, and Jørgensen and Zaccour, 2014, for reviews).

On the downside, relying on retailer advertising via a cooperative advertising program generates some risks for manufacturers. Retail advertising activities primarily aim at stimulating immediate sales, but may also have positive or negative effects on long-term sales due to brand image deterioration or stockpiling for certain types of products (Herrington and Dempsey, 2005; Karray et al., 2021; Martín-Herrán et al., 2010; Martín-Herrán and Sigué, 2017a; He et al., 2019; Huang and Bai, 2021). In a standard cooperative advertising program, while the manufacturer sets some guidelines about the type of advertising activities that qualify for reimbursement, the decision to participate or not to participate in such a program rests with the retailer who also controls many critical aspects of his advertising activities. Therefore, it is not surprising to note that around 40% of cooperative advertising fund provisions often remains unused (Borrell Associates, 2015). The rate of unused cooperative advertising funds signals the extent of the discrepancy between the expectations of the manufacturers from their cooperative advertising programs and the level of retailer involvement in these programs. It also raises the issue of the optimal allocation of advertising resources in marketing channels. Funds allocated to cooperative advertising generally come at the expense of manufacturer advertising. As a result, when these funds are not used as planned, channel-advertising strategies are more likely to be less effective or suboptimal.

Despite the strategic and financial importance of advertising decisions in marketing channels, very little is known about manufacturers' decision to allocate resources to both national and cooperative advertising activities. In fact, most published works on channel advertising have focused on demonstrating the profitability of cooperative advertising arrangements in different channel configurations (e.g., Jørgensen et al., 2000, 2003; Karray, 2013; Karray and Amin, 2015; Karray et al., 2017; He et al., 2019; Zhang et al., 2020a,b, 2021). However, these works do not examine whether, despite being profitable, cooperative advertising programs are the best allocation of advertising resources for manufacturers over other alternatives. Some other works investigate who between manufacturers and retailers should take on some advertising responsibilities (e.g., Zhang et al., 2020a,b). Lastly, a few works deal with the fundamental issue of this research, which is how manufacturers should allocate their resources to own-controlled activities targeted at consumers as well as activities that aim at giving extra incentives to channel partners to perform specified tasks. These works are, however, limited to price promotional activities such as trade deals, rebates, and coupons (e.g., Gerstner and Hess, 1991a,b, ; Martín-Herrán and Sigué, 2015).

Building on this knowledge base, this research aims to deepen the understanding of the strategic issues manufacturers encounter in allocating their advertising resources to national and cooperative advertising programs. The main management challenge for manufacturers is not limited to knowing whether the addition of cooperative advertising to their current advertising arrangements is beneficial; rather, it is finding the best trade-off between these two types of advertising, which is acceptable to each channel member and generates the best possible profits. For example, in the context of a bilateral monopolistic channel, assuming that the retailer already undertakes local advertising, some of the strategic options available for the manufacturer are: undertake national advertising and support retailer advertising via a cooperative advertising program (Scenario 1), support retailer advertising via a cooperative advertising program (Scenario 2), undertake national advertising (Scenario 3), or leave the full responsibility of channel advertising to the retailer - no manufacturer advertising (Scenario 4). It is important to recall that the participation of a retailer in a cooperative advertising program is voluntary. Therefore, any full assessment of the manufacturer's advertising strategic options should take into account the retailer's preferences in order to understand a phenomenon such as the existence of unused cooperative advertising funds. Therefore, the main questions guiding this research are:

1. Which of the above four advertising scenarios should manufacturers adopt? What market conditions are conducive to the implementation of each of these scenarios?
2. Can the manufacturer's advertising choice lead to channel conflict? Put differently, can manufacturers and retailers' preferences diverge in terms of which kind of advertising strategy is undertaken by the manufacturers? Moreover, should retailers partake in cooperative advertising programs when offered by manufacturers?

To answer these questions, we develop a game-theoretic model to study the advertising and pricing decisions of a marketing channel led by a manufacturer selling through a single retailer. We study these decisions over a two-period horizon to model the differences between national and cooperative advertising with regards to how they affect sales in the short (immediate) and long (post advertising) terms. This dynamic formulation also allows us to look into the impact of the manufacturer's strategic advertising decisions on pricing considerations in the channel.

Our findings indicate that, depending on the effects of both manufacturer and retailer advertising, the manufacturer can invest both in national and cooperative advertising (Scenario 1), invest exclusively in cooperative advertising (Scenario 2), undertake only national advertising (Scenario 3), or abstain from any kind of advertising (Scenario 4). A major finding of this research is that the magnitude and nature (positive or negative) of the long-term effects of retailer advertising play an important role in the manufacturer's strategic choice. Comparing equilibrium profits from these different scenarios generates additional interesting insights. In particular, both channel members gain higher profits when the manufacturer invests in national advertising either on its own or combined it with cooperative advertising. However, the interests of the retailer may diverge from the manufacturer depending on whether retailer advertising negatively affects long-term sales. In particular, when retailer advertising negatively impacts long-term sales, the retailer prefers Scenario 3, while the manufacturer may still prefer Scenario 1 under certain conditions. In this case, suboptimal strategies and profits may occur, given that the retailer will not participate in a cooperative advertising program unless it is enhanced by side payments or the provision of additional services.

The rest of the paper is organized as follows. First, we provide a brief literature review. Second, we describe the model and discuss its assumptions. Third, we derive the equilibrium solutions. Fourth, comparisons of profits across different equilibrium solutions are performed to derive insights. Finally, we conclude and discuss the managerial and theoretic implications of this work.

2. Background literature

This article relates to works done in the following research streams in the marketing and management science literature: push and pull resource allocation, distribution of advertising responsibilities in marketing channels, and cooperative advertising. Below, we provide a brief overview of these research streams and discuss the distinctive contributions of our work.

2.1. Push and pull resource allocation

Push and pull marketing strategies have been the focus of much debate in the marketing literature. Manufacturers undertake pull marketing strategies, such as advertising and rebates to improve awareness, build brand equity, and stimulate sales (Nunes and Merrihue, 2007). They also use push marketing strategies by providing incentives to the channel intermediaries to carry the product or to promote it at the point-of-sale (Chan et al., 2017). A major difference between push and pull marketing strategies consists in the effects they have on consumers and the way they influence demand (Parment, 2008). Because they aim to build brand equity, pull marketing activities create positive long-term effects on sales even though this effect may decay over time (Jørgensen et al., 2000, 2001a; Ataman et al., 2010). Conversely, push marketing activities are directed to channel partners with the purpose to stimulate immediate sales (Jørgensen et al., 2001a; Herrington and Dempsey, 2005; Ataman et al., 2010). Depending on the activity pursued by the channel members, push strategies can either help or damage the brand's image (Herrington and Dempsey, 2005; Martín-Herrán et al., 2010; Martín-Herrán and Sigué, 2017a).

The issue of optimal allocation of resources between pull and push marketing activities in distribution channels has been previously investigated in the analytical literature. However, most of the existing works focus on price-promotional activities for which a price incentive is given either to trade partners and/or to final consumers. For instance, Gerstner and Hess (1991a,b), study the trade-offs among push (trade deals), pull (consumer rebates), push-pull (combination of trade deals and consumer rebates), and retailer rebates in a bilateral monopoly. Martín-Herrán et al. (2010) investigate the conditions under which manufacturers of products such as automobiles and household appliances offer cash rebates to consumers at the time of purchase or trade deals to retailers. Martín-Herrán and Sigué (2015) consider the trade-off between trade deals and rebate-like promotions targeted at consumers such as on-pack coupons that stimulate the first and second purchases or a combination of the two promotional offers. These works offer frameworks that identify formal conditions under which manufacturers can adopt either one of these price-promotional activities.

Our paper extends this research strand to advertising. Advertising challenges in marketing channels are different from those of price promotional activities. For instance, unlike push-price promotional arrangements, which are generally linked to sales units, in a cooperative advertising program, the manufacturer rewards the retailer based on the level of advertising undertaken, not on the quantities purchased (Aust and Buscher, 2014; Jørgensen and Zaccour, 2014). The relationship between cooperative advertising support and retail sales is therefore not straightforward for all channel members. Consequently, in some cases, retailers may not be interested in participating in such programs, especially when the value of advertising that is supported is not obvious to them (Borrell Associates, 2015; Galloway, 2016). In addition, retailer participation in push-price promotions is rarely an issue. Rather, manufacturers are typically concerned about retailers passing on to consumers the attractive deals they offer. Another phenomenon specific to channel advertising decisions is the free-riding issue, which exists when both the manufacturer and the retailer simultaneously undertake advertising in the channel (Martín-Herrán and Sigué, 2017a,b, ; Ma, 2021). Each channel member tends to under-invest in advertising with the expectation of taking advantage of the partner's advertising spending. The manufacturer's optimal allocation of advertising resources must consider this externality as a factor affecting the attractiveness of the various advertising arrangements.

2.2. Allocation of advertising responsibilities in marketing channels

Another relevant stream of research includes works that investigate whether channel efficiency can be improved by finding an optimal allocation of different activities between channel members (e.g., Sigué and Chintagunta, 2009; Li et al., 2016; Zhang et al., 2020a,b; Martín-Herrán and Sigué, 2021). For instance, Sigué and Chintagunta (2009) consider the assumption of perfectly substitutable manufacturer and retailer advertising in the context of a franchisor dealing with two adjacent franchisees as they investigate advertising role allocation. It is assumed that, regardless of who performs advertising, brand advertising contributes to building the franchise goodwill, while promotional advertising is limited to stimulating current sales. These authors find that, under certain conditions, a profit-maximizing manufacturer can delegate/centralize the two types of advertising or delegate only promotional advertising to the franchisees. Zhang et al. (2020a,b) also study whether the manufacturer and/or the retailer should undertake informative advertising in the context where the manufacturer either sells exclusively through the retailer or uses both the retailer's store and a direct channel. They find that the impact of the manufacturer's encroachment on channel members' profits depends, among others, on who is responsible for advertising. Our paper differs from these previous works in many regards. Particularly, we consider that retailer and manufacturer advertising activities are not perfectly substitutable. Retailer advertising positively affects current sales and may have a positive or negative impact on second-period sales, while manufacturer advertising positively influences current and post-advertising sales. We also consider that in a conventional marketing channel, the retailer's decision to invest in local advertising is not under the control of the manufacturer. Therefore, manufacturers are more concerned about the optimal allocation of their own advertising resources, taking the retailer's advertising decisions as given. The strategic problem facing the manufacturer in this case is whether to leave the full responsibility of channel advertising to the retailer by not engaging in any advertising activity, or to share it with him either by offering cooperative advertising, undertaking national advertising, or doing both simultaneously.

2.3. Cooperative advertising

A large number of works formally investigates the profitability of cooperative advertising programs under various channel structures using game-theoretic analysis (see Aust and Buscher, 2014 and Jørgensen and Zaccour, 2014, for reviews). Commonly, these works determine the optimal cooperative advertising participation rates, and identify the conditions under which manufacturers offer such programs as incentives to boost retailer advertising and increase channel demands and profits. The main findings from this literature suggest that cooperative advertising can improve the overall channel's performance and stimulate advertising. The theoretical literature that studied cooperative advertising programs used either static or dynamic games. In static games, strategies are determined for a single period and the long-term effects of advertising are overlooked (e.g., Karray, 2013; Karray and

Amin, 2015; Zhao et al., 2016; Chaab and Rasti-Barzoki, 2016). In dynamic games, some advertising activities are generally assumed to contribute to building brand goodwill, which affects sales over time (Jørgensen et al., 2000, 2001a, 2003 ; Zhang et al., 2013; He et al., 2011; Chutani and Sethi, 2018; Huang et al., 2018; Lu et al., 2019). Many of these previous works include pricing and both manufacturer and retailer advertising decisions, but the main focus is often on determining whether or not the manufacturer should support retailer advertising given a specific channel configuration (e.g., Szmerekovsky and Zhang, 2009; Zhao et al., 2016). Regardless of whether or not the manufacturer already undertakes own advertising, these works typically examine whether adding a cooperative advertising program to a set of available activities can enhance channel profits.

The work by Szmerekovsky and Zhang (2009) is directly related to ours. It investigates whether the manufacturer should share the costs of retailer advertising or undertake national advertising and lower the wholesale price in a bilateral monopoly. These authors demonstrate that the optimal arrangement for the manufacturer is to undertake national advertising and offer a lower wholesale price to the retailer. Cooperative advertising is not beneficial, whether local advertising is more or less effective in stimulating local demand than the retail price. Zhao et al. (2016) complement this work and show that the manufacturer benefits from providing cooperative advertising to the retailer when price elasticity is large enough. Otherwise, the manufacturer should undertake only national advertising. These two works differ from ours in two major ways. First, they use a static game and overlook the long-term effect of both manufacturer and retailer advertising, while we use a two-period game and model the differences between the long-term effect of manufacturer advertising and retailer advertising. Second, they use a multiplicatively separable demand function in advertising and price. In the first, local and national advertising increase demand in a multiplicative way, while in the second, the effects of these two types of advertising are separated. Conversely, we use a well-known linear demand function that allows us to consider the individual effects of each of the two players' decision variables on consumer demand.

In summary, this article adds to the literature on the optimal allocation of push and pull marketing resources in marketing channels. We demonstrate that even when cooperative advertising contributes to improving channel profits, as established in the literature, manufacturers still need to find the optimal trade-off between this activity and their own advertising programs and pricing decisions. As a result, manufacturers may be better off undertaking exclusively their own advertising program rather than supporting retailer advertising activities or combining own advertising with a cooperative advertising program. In addition, retailers may not participate in cooperative advertising programs to take advantage of manufacturers' advertising and pricing decisions.

3. Model

Consider a bilateral monopoly in which a manufacturer sells a product to a retailer, who then sells the same product to consumers in a two-period planning horizon. We assume that the two channel members operate in a market where competition does not exist or has no consequence on vertical interactions between channel members. This is a common assumption in the marketing channel literature, which helps to focus on the impacts of vertical interactions on pricing and advertising decisions (e.g., Martín-Herrán et al., 2010; Karray, 2013; Malekian and Rasti-Barzoki, 2019).

Let w_i and p_i represent, respectively, the wholesale and retail prices in period i ($i = 1, 2$). Let a_m and a_r be, respectively, the manufacturer's and the retailer's advertising efforts in the first period of the game. The variable t denotes the manufacturer's cooperative advertising support rate. All notations are summarized in Table 1. For parsimony, we consider that both the manufacturer and retailer adopt a pulsing advertising schedule and only advertise in the first period to benefit from the long-term advertising effects in the second period. Pulsing advertising schedules where advertisers alternate between high and zero levels of advertising are considered a cost-saving practice, especially when advertising effects slowly diminish over time (Mesak and Ellis, 2009; Martín-Herrán and Sigué, 2017b). An extension to a continuous advertising schedule where both channel members advertise in the second period is possible, but it would make it difficult to assess the long-term effects of the first-period advertising strategies, which are critical for this research.

Table 1
Notation used in the paper.

i	Index for periods, $i = 1, 2$
w_i	Manufacturer's wholesale price in period i, $w_i > 0$
p_i	Retailer's price in period i, $p_i > w_i$
a_r	Retailer's advertising effort in period 1, $a_r > 0$
a_m	Manufacturer's advertising effort in period 1, $a_m > 0$
t	Manufacturer's cooperative advertising rate in period 1, $t \in (0, 1)$
d_i	Demand in period i, $d_i > 0$
g	Baseline demand in each period, $g > 0$
β	Effect of retailer's advertising on period 1 demand, $\beta \in (0, 1)$
α	Effect of manufacturer's advertising on period 1 demand, $\alpha \in (0, 1)$
δ	Effect of retailer's advertising on period 2 demand, $\delta \in (-1, 1)$
c_r	Cost parameter for the retailer's advertising, $c_r > 0$
c_m	Cost parameter for the manufacturer's advertising, $c_m > 0$
M_i	Manufacturer's profit in period i, $M_i > 0$
M	Manufacturer's total profit in both periods, $M = M_1 + M_2$
R_i	Retailer's profit in period i, $R_i > 0$
R	Retailer's total profit in both periods, $R = R_1 + R_2$
Scenario 1	The manufacturer invests both in national and cooperative advertising.
Scenario 2	The manufacturer invests in cooperative advertising.
Scenario 3	The manufacturer invests in national advertising.
Scenario 4	The manufacturer does not advertise.

Insert Table 1 about here.

Our model is based on several assumptions. First, for convenience and tractability, we assume the following linear demand functions in periods 1 and 2, respectively: $d_1 = g - p_1 + \beta a_r + \alpha a_m$ and $d_2 = g - p_2 + \delta a_r + \alpha^2 a_m$. The use of linear demand functions is common in the marketing channels literature (Martín-Herrán et al., 2010; Martín-Herrán and Sigué, 2017a; Assarzadegan and Hejazi, 2021). The positive parameter g accounts for the baseline demand, β and δ stand for the first-period and second-period effects of retailer advertising, respectively, while α denotes the effect of manufacturer advertising in the first period. Our model incorporates the levels of manufacturer and retailer advertising effectiveness on consumer utility (and demand) in each period. In a real business context, advertising effectiveness levels (i.e., α, β and δ in our model) depend on several factors including the nature of the product sold, factors internal to each channel member, consumer behavior and competition.

Second, retailer and manufacturer advertising are not perfectly substitutable. Consequently, the two channel members' advertising costs as well as their impacts on sales are differentiated. In particular, retailer advertising always positively impacts the first-period sales ($\beta \in (0, 1)$), but its second-period effects can be either positive or negative ($\delta \in (-1, 1)$). This is in line with works which consider that while retailer advertising mainly aims at boosting short-term sales, its long-term effects are still controversial due to the fact that it can either damage the manufacturer's brand image or create stockpiling for certain products, resulting in decreased post-advertising sales (e.g., Jørgensen et al., 2003; Herrington and Dempsey, 2005; Karray et al., 2017, 2021; Martín-Herrán and Sigué, 2017a; He et al., 2019; Huang and Bai, 2021). On the other hand, manufacturer advertising is known to contribute to building a lasting brand image that decays over time (e.g., Aust and Buscher, 2014; Huang and Bai, 2021). We therefore assume that the effect of the first-period manufacturer advertising ($\alpha \in (0, 1)$) decays in the second period (α^2). As a result, the effective baseline demand of the second period is given by: $g + \delta a_r + \alpha^2 a_m$. It can be larger or smaller than the original baseline demand (g) depending on whether the combined effect of the first-period advertising of the retailer and the manufacturer in the second period is positive or negative. This combined effect could be negative either when retailer advertising heavily damages the manufacturer brand image or contributes significantly to stockpiling in the first period.

Third, we assume convex costs for retailer advertising, $c_r a_r^2$, and manufacturer advertising, $c_m a_m^2$ (e.g., Martín-Herrán and Sigué, 2017b). The positive parameters c_m and c_r denote the manufacturer's and the retailer's cost parameters, respectively. In addition, when the manufacturer offers a cooperative advertising program to the retailer, the channel members' costs associated with the program are, respectively: $tc_r a_r^2$ and $(1-t)c_r a_r^2$. These assumptions are used to ensure that higher advertising levels lead to higher costs and that marginal costs of advertising activities are increasing (e.g., Malekian and Morteza, 2019; Xie et al., 2020).

Fourth, we consider that each channel member acts so as to maximize his total profit over both periods, which is a mere addition of the profits obtained in each period ($M = M_1 + M_2$, $R = R_1 + R_2$). Such an assumption is realistic in the current context of low interest rates, especially when the planning horizon is short (e.g., Martín-Herrán and Sigué, 2017a). Each firm's profit function in each period is given by sales revenues minus the advertising cost paid in that period if any. The manufacturer's first-period and second-period profits are: $M_1 = w_1 d_1 - c_m a_m^2 - c_r t a_r^2$ and $M_2 = w_2 d_2$, while those of the retailer are: $R_1 = (p_1 - w_1)d_1 - (1-t)c_r a_r^2$ and $R_2 = (p_2 - w_2)d_2$.

Finally, for simplicity, we have normalized the price effect on demand to one, while the different effects of the two types of advertising cannot exceed one in absolute values. This implies that the price effect on demand is expected to be higher than the effects of either of the two types of advertising on demand. This is a standard assumption in the marketing literature (e.g., Martín-Herrán and Sigué, 2017a). The values of the parameters α, β, and δ are not set up-front to allow us to consider situations where the relative importance of national and local advertising varies.

4. Equilibrium solutions

We use the Stackelberg equilibrium concept to derive the equilibria for the four models. Consistent with the usual practice in the literature, we consider that the manufacturer and retailer are, respectively, the Stackelberg leader and follower. Role allocation in the channel is generally done on an ad hoc basis, but the manufacturer's leadership is known to formally benefit all channel members (Jørgensen et al., 2001b). Consequently, the sequence of moves of the players is as follows. The manufacturer announces her first-period wholesale price and advertising decisions in Stage 1. The retailer reacts to the manufacturer's announcement in Stage 2 and sets his first-period retail price and local advertising decisions. In Stage 3, the manufacturer announces her second-period wholesale price given the first-period decisions. Finally, the retailer reacts to the previously announced strategies in Stage 4 and sets his second-period pricing decision.

Given this sequence of play, we obtain subgame-perfect equilibria by deriving optimal solutions using backwards induction. Essentially, the retailer's second-period optimal decision in Stage 4 are obtained first. Then, the manufacturer's optimal second-period wholesale price is derived after the introduction of the retailer's optimal decision in Stage 4 into the manufacturer's problem. Third, the optimal decisions obtained in Stages 3 and 4 are incorporated into the retailer's first-period problem in Stage 2 to derive the retailer's optimal first-period decisions. Finally, the manufacturer's first-period decisions in Stage 1 are derived knowing the players' strategies in all other stages.

The derivation of the equilibrium strategies is provided in Appendix A. Proposition 1 below summarizes the main finding.

Proposition 1. *Considering that the retailer always advertises in the first period ($a_r > 0$), at the equilibrium, the manufacturer can adopt any of the following four scenarios:*

- Scenario 1: $a_m > 0$, $t > 0$.
- Scenario 2: $a_m = 0$, $t > 0$.
- Scenario 3: $a_m > 0$, $t = 0$.
- Scenario 4: $a_m = 0$, $t = 0$.

Scenario 1 corresponds to the situation where the manufacturer advertises and provides cooperative advertising support to the retailer. Scenario 2 is optimal when the manufacturer does not advertise, but provides cooperative advertising support to the retailer. Scenario 3 refers to the situation where the manufacturer advertises and offers no cooperative advertising support program. Scenario 4 corresponds to the situation where the manufacturer does not undertake advertising and offers no advertising support to the retailer. In this case, the retailer is left with the full responsibility of advertising the manufacturer's product in the local market. An analysis of the feasibility (positivity and concavity) conditions of these four scenarios helps to better understand this optimal solution. The optimal decisions and profits derived in Appendix A all depend on the model parameters, including g, α, β, δ, c_m, and c_r. Some conditions need to be imposed on these parameters to obtain non-negative profits, positive pricing decisions, and non-negative advertising decisions in the first period. These conditions are requirements for sustainable and good business practices. Non-negative profits ensure that the two channel partners can minimally break-even. Positive prices are expected in any unsubsidized business relationship. Also, non-negative advertising in the first period is consistent with business practice. In the case of this work, the manufacturer may or may not undertake advertising and/or support retailer advertising.

We carry out numerical simulations to identify the feasibility conditions with respect to the key advertising parameters. This is achieved by normalizing the parameters g, c_m and c_r to 1. The assumption of an identical cost structure for both channel members is explained by the fact that they are more likely to use similar media or advertising agencies in the context of a bilateral monopoly. We then fix three different values of $\beta \in \{0.25, 0.5, 0.75\}$ and vary $\alpha \in (0, 1)$ and $\delta \in (-1, 1)$. For each of the three values of β, we numerically evaluate the positivity and concavity conditions that define the feasibility conditions of each scenario. These conditions are evaluated in a grid of (0, 1) for α and (−1, 1) for δ with a mesh of 0.000 5 for α and 0.001 for δ. For each of the three numerical scenarios of β we evaluate the results for 2,000 value combinations of parameters α and δ. In total, this means that our numerical analysis for each value of β was performed for 4 million combinations of numerical values. Fig. 1 displays the feasibility conditions of the four scenarios. A scenario is unfeasible (UF) in a region when at least one of these feasibility conditions is not met.

A few remarks can be made regarding Fig. 1. First, everything else being equal, the feasibility conditions of the four scenarios depends on the effects of manufacturer advertising (α) and both the first-period (β) and second-period (δ) effects of retailer advertising. Second, all four scenarios are feasible when the second-period effect of retailer advertising is positive. This means the manufacturer may adopt any of the four scenarios if, in addition to boosting first-period sales, the first-period retailer advertising contributes to expanding second-period sales. This result applies regardless of the magnitude of the first-period effect of retailer advertising and the effect of manufacturer advertising. Third, when the second-period effect of retailer advertising is negative, meaning that retailer advertising harms the second-period sales, the magnitude of the first-period effect of retailer advertising and the effect of manufacturer advertising both matter. Particularly, the four scenarios are more likely to be feasible in the context where retailer advertising substantially stimulates first-period sales, despite its negative effect on second-period sales. Otherwise, some of the conditions on channel members' strategies and profits, which are necessary to ensure a healthy business, cannot be met because the sales gained from the first period cannot make up for the lost second-period sales. Fourth, the feasibility

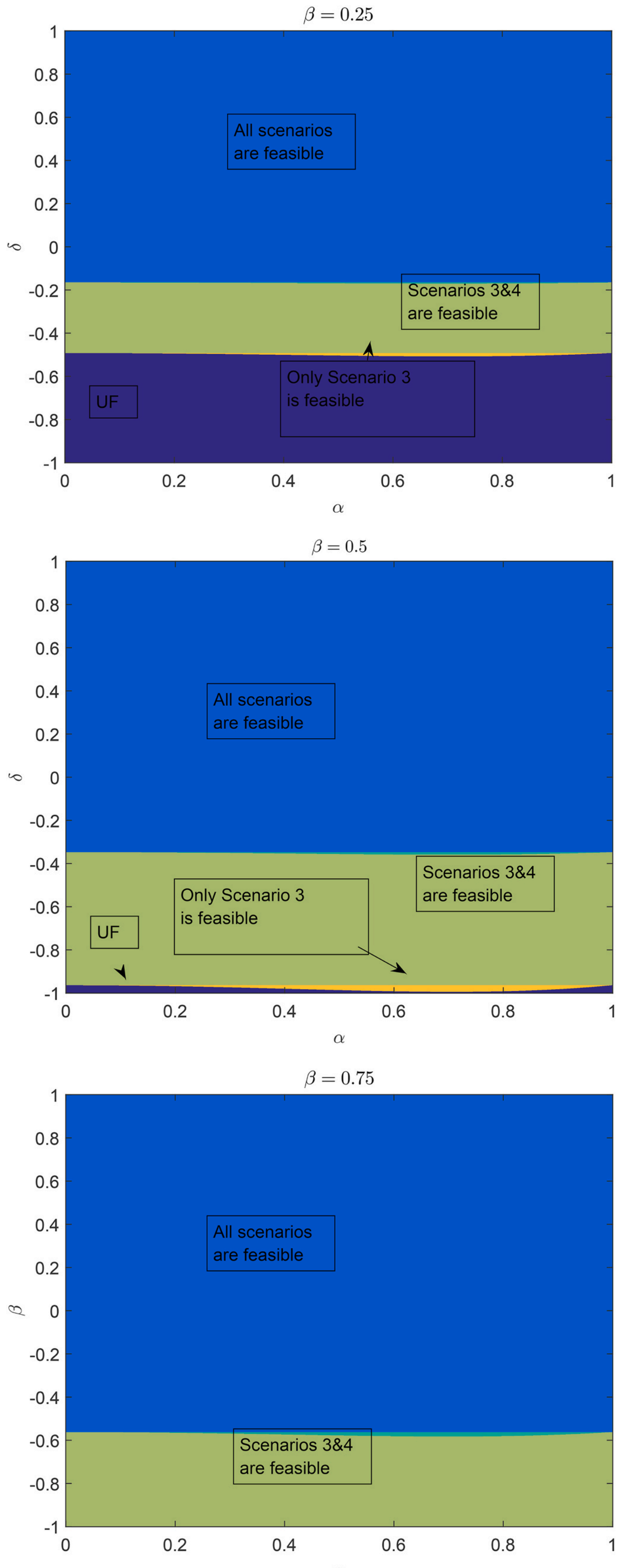

Fig. 1. Feasibility regions for the different scenarios. $\beta = 0.25$ (left), $\beta = 0.5$ (middle), $\beta = 075$ (right).

conditions of Scenarios 1 and 2 imply the fulfillment of those of Scenarios 3 and 4, which tolerate more harmful retailer advertising in the second period. In the case of Scenario 3, the manufacturer mitigates the negative effects of retailer advertising in the second period by further investing in the first period, while in Scenario 4, the manufacturer leaves the full responsibility of advertising to the retailer who cannot afford heavy advertising investment without direct or indirect support. Conversely, the cooperative advertising support provided to the retailer in Scenarios 1 and 2 stimulates more retailer advertising in the first period and, consequently, amplifies its damaging effect in the second period. Scenario 2 is the worst of all as the manufacturer has no other advertising medium to mitigate the negative effect of retailer advertising in the second period.

5. Selecting an advertising arrangement

Given that the four advertising scenarios discussed above are all possible equilibrium solutions depending on the area in the parameter space, we now examine their profitability for both the manufacturer and the retailer. In particular, we compare the equilibrium profits of the two channel partners across the four scenarios to identify the conditions under which they are more likely to be implemented. The comparison of the manufacturer's profits across scenarios is analytically done and the results are presented in Propositions 2 and 3. Regarding the retailer, the results of the profit comparisons of Scenarios 1 and 2, and Scenarios 3 and 4, are also obtained analytically and stated in Propositions 4 and 5, respectively. Analytical results could not be obtained for the other combinations. The proofs are reported in Appendix B. Figs. 2 and 3 are generated from the numerical comparisons of the manufacturer's and the retailer's profits across Scenarios 1 and 4, primarily for illustrative purposes. These comparisons are carried out using the same numerical approach described in the preceding section and under the same parameter values previously discussed.[1]

Proposition 2. *The manufacturer prefers Scenario 1, in which she undertakes her own advertising and offers a cooperative advertising support, to Scenarios 2 and 3 where she performs exclusively one of the two advertising activities.*

Proposition 2 and Fig. 2 support the view that whenever Scenario 1 is feasible, it provides the manufacturer with the largest profit. In other words, the manufacturer obtains the best outcome when she simultaneously invests in own advertising and offers cooperative advertising support to the retailer. This is because, in Scenario 1, both the manufacturer and the retailer invest more heavily in advertising than in any other scenario in order to reach the highest demands. In fact, in terms of advertising expenditures, Scenario 1 alleviates the weaknesses of the other three scenarios. It allows the manufacturer to not only invest directly in advertising as in Scenario 3, but also to give extra incentive to the retailer to increase his advertising as in Scenario 2. This, in turn, mitigates the free-riding issue that is more severe in Scenario 3. In fact, although the manufacturer provides more generous support for retailer advertising in Scenario 2, when retailer advertising increases second-period sales, this support is not sufficient to replace manufacturer advertising in Scenario 1. The first-period wholesale and retail prices in Scenario 1 are higher than in all other scenarios due to the higher demand, resulting from increased advertising from both the manufacturer and retailer. [2]

Next, the comparison of the manufacturer's profits in Scenarios 3 and 4 leads to the following proposition.

[1] The results in this section were obtained for $g = c_m = c_r = 1$. We obtain similar results when we use different alternative values for c_m and c_r.

[2] Comparison of equilibrium strategies across the four scenarios has been done using the same numerical approach described above.

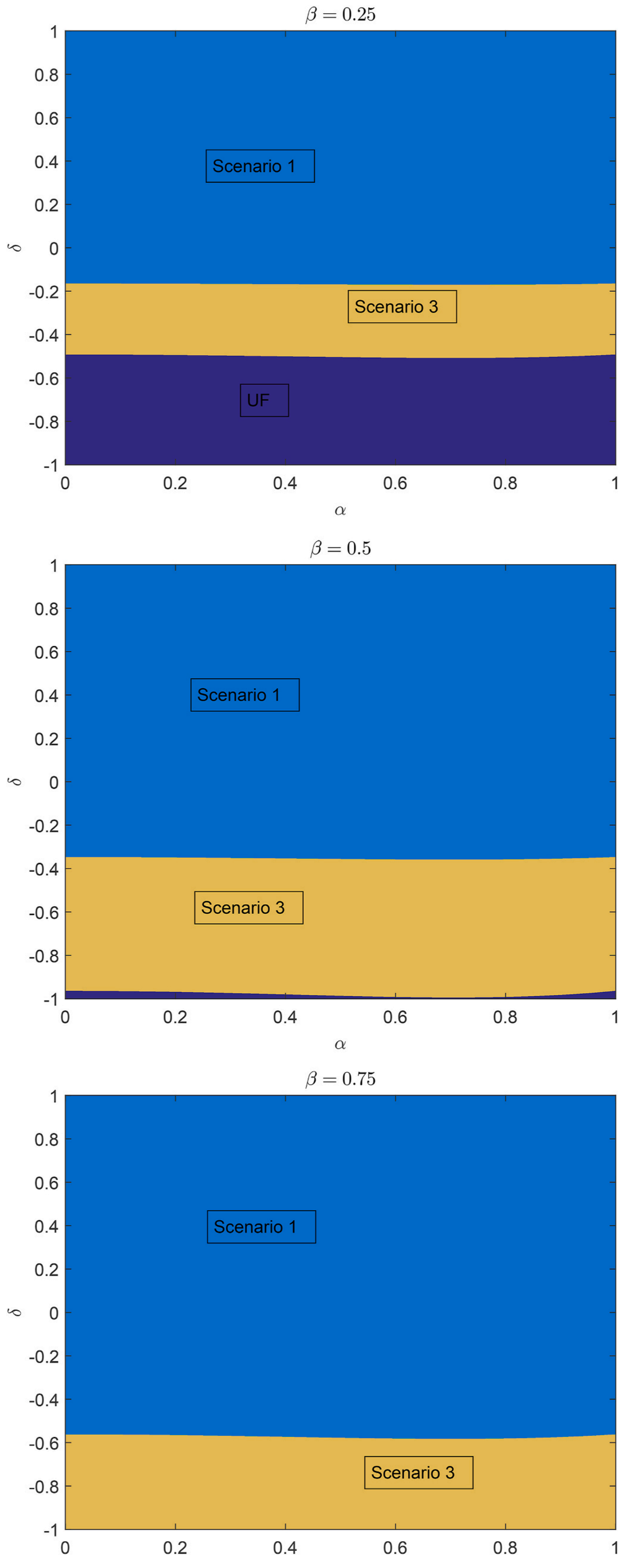

Fig. 2. Comparison of the manufacturer's profits across scenarios. $\beta = 0.25$ (left), $\beta = 0.5$ (middle), $\beta = 075$ (right).

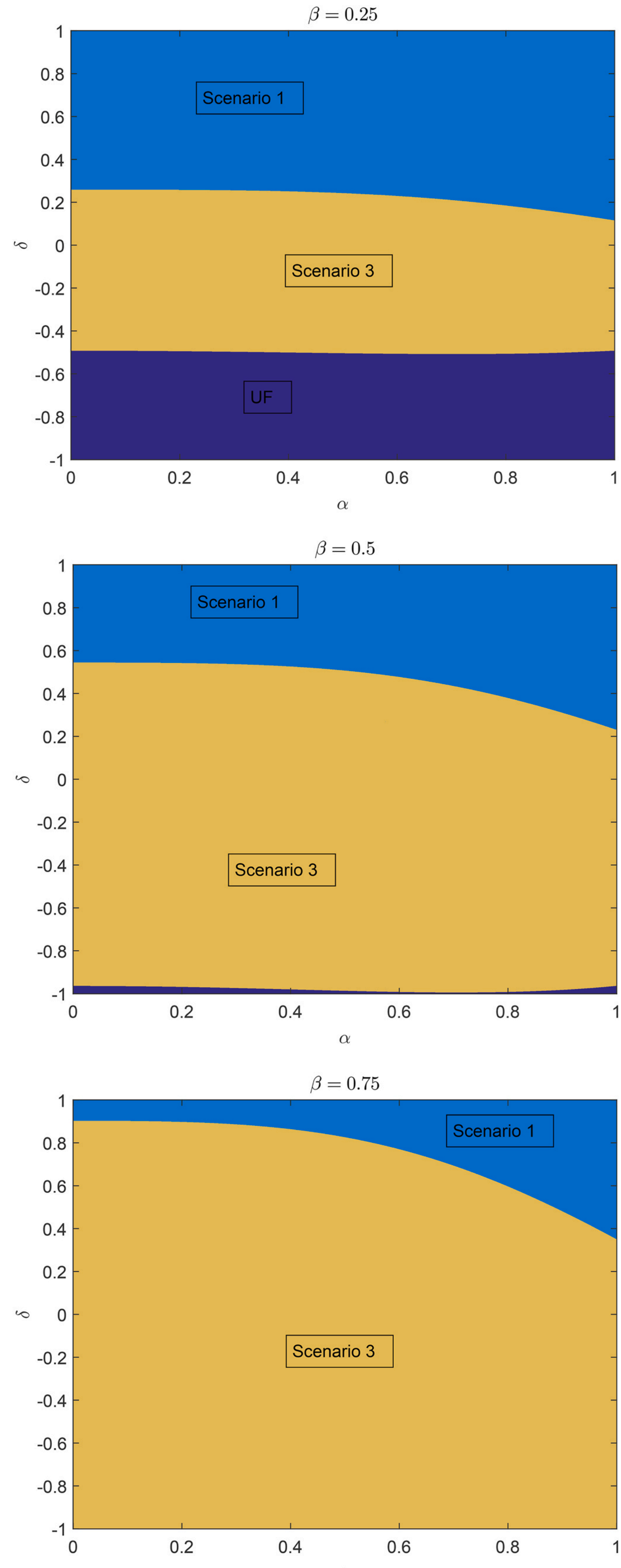

Fig. 3. Comparison of the retailer's profits across scenarios. $\beta = 0.25$ (left), $\beta = 0.5$ (middle), $\beta = 075$ (right).

Proposition 3. *The manufacturer prefers Scenario 3, in which she undertakes her own advertising, to Scenario 4 where the responsibility for the channel's advertising is left entirely to the retailer.*

Based on this proposition and Fig. 2, it is obvious that Scenario 3 is the manufacturer's second best advertising scenario in contexts where Scenario 1 is not feasible. This is more likely to occur in situations where the retailer's first-period advertising has a relatively large negative impact on the second-period sales. Overall, the predominance of Scenario 3 over Scenario 4 is not only due to the addition of manufacturer advertising in Scenario 3, which helps to increase the first-period demand, but also to the fact that it allows the two channel members to charge higher prices, which results in higher profits. In contexts where the retailer's first-period advertising negatively affects second-period sales, the increase in sales in the first period due to manufacturer advertising is highly critical to compensate for the loss of sales in the second period and to ensure the profitability of channel activities over both periods. This explains why only a handful of companies, such as Tesla, choose not to advertise their products (Light, 2020).

From the retailer's perspective, we analytically obtain the following results.

Proposition 4. *The retailer prefers Scenario 1, where the manufacturer undertakes advertising and offers a cooperative advertising support, to Scenario 2 where only cooperative advertising support is provided.*

Proposition 5. *The retailer prefers Scenario 3, in which the manufacturer undertakes her own advertising in addition to the retailer advertising, to Scenario 4 where the manufacturer takes no advertising responsibility.*

These two propositions show the retailer's preference for Scenarios 1 and 3. They do not, however, give a complete picture of the comparisons between the four scenarios, as shown in Fig. 3.

Fig. 3 suggests that, similar to the case for the manufacturer, Scenarios 2 and 4 wherein the manufacturer does not invest in national advertising are not desirable for the retailer. Instead, the retailer prefers either Scenario 1 or Scenario 3, depending on the parameter values. Interestingly, regardless of the values of the other parameters, the retailer always welcomes Scenario 3 to the detriment of Scenario 1 when his first-period advertising negatively impacts second-period sales. This is to avoid boosting retailer advertising that can harm or marginally increase sales in period 2. Scenario 1 is mainly preferred when retailer advertising has a relatively small impact on sales in the first period. Otherwise, Scenario 1 is preferred when either the first-period effect of retailer advertising is relatively high and/or the effect of manufacturer advertising is very high. This means that in most areas in the parameter space, the retailer obtains his largest profit when the manufacturer exclusively undertakes advertising and offers no cooperative advertising support (Scenario 3). In this case, Scenario 1, in which the manufacturer directly undertakes advertising and offers a cooperative advertising program, comes in second place despite the fact that it increases demand and leads to a higher first-period retail price compared to Scenario 3. A possible explanation of this result is the free-riding phenomenon that allows the retailer to substantially reduce his advertising expenditures in Scenario 3 compared to Scenario 1. The manufacturer's cooperative advertising in Scenario 1 mitigates this phenomenon, but the resulting increase in sales cannot compensate for the erosion of the retailer's margin, due to the accrued investments in local advertising. Another perspective is that when the retailer's advertising has a negative impact on second-period sales, the manufacturer has no choice but to try to invest more in Scenario 3 by adjusting both her own advertising and pricing decisions. These adjustments benefit the retailer who takes this opportunity to set a higher second-period retail price in Scenario 3.

From Propositions 2 to 5 and Figs. 2 and 3, we can see that channel members' preferences for the different advertising scenarios may diverge. In particular, while they both agree on the fact that advertising responsibility should not be left to the retailer with or without any support (Scenarios 2 and 4), each of them holds a different view about how the manufacturer should allocate her advertising resources. Whenever possible, the manufacturer prefers to spend her advertising efforts on both own advertising and a cooperative advertising program that aims at stimulating retailer advertising. This involves, among other things, providing cooperative advertising support to the retailer even when the retailer's advertising has a moderately negative impact on second-period sales. All other things being equal, it is only when the negative impact of retail advertising on second-period sales is significant that cooperative advertising should not be offered. From the retailer's perspective, for example, the manufacturer should not offer cooperative advertising in addition to own advertising in the context where local advertising negatively impacts second-period sales. These divergent views and many others previously discussed suggest that in some areas of the parameter space, the manufacturer prefers Scenario 1, while the retailer is better off implementing Scenario 3.

In the context of a conventional marketing channel where participation in cooperative advertising programs is voluntary, once the manufacturer has announced a positive support rate and her national advertising effort in Scenario 1, it could be in the retailer's best interest to stay out of the proposed cooperative advertising program and to deliberately advertise less (as he would do in Scenario 3). Such an opportunistic approach aims to take advantage of the manufacturer's national advertising and leads to suboptimal strategies and profits. This being said, even in such a context, the manufacturer can still convince the retailer to participate in a cooperative advertising program in Scenario 1 by sharing with him any surplus that is made over Scenario 3 as a side payment or through the provision of additional services as part of the cooperative advertising program. For instance, in addition to sharing part of the retailer's advertising costs, manufacturers often supply pre-prepared layouts and creative copies to retailers to further reduce their advertising expenses (Martín-Herrán and Sigué, 2017a). Alternatively, knowing the preferences of the retailer (Fig. 3), the manufacturer could rule out Scenario 1 when it does not serve the best interests of the retailer and consider instead Scenario 3 as the second best solution.

6. Conclusion

This paper investigates manufacturers' optimal allocation of resources to national and cooperative advertising programs in a bilateral monopolistic marketing channel. Manufacturers' cooperative advertising supports the retailers' efforts, which stimulate immediate sales but may harm future demand, whereas national advertising aims at building brand equity and stimulates immediate and future sales. These effects then influence channel members' prices, margins and profitability over time. To study this problem, we developed a two-period game-theoretic model considering a manufacturer and retailer advertising decisions as well as pricing strategies over a two-period horizon. Our model represents the differences between manufacturer and retailer advertising with regards to how they affect sales in the short (immediate) and long (post advertising) terms. We characterized equilibrium solutions for four advertising scenarios for the manufacturer and endogenously identified the conditions under which each scenario is possible at equilibrium. We find that these conditions critically depend, among others, on the nature (positive or negative) and size of the long-term effects of retailer advertising, which are often overlooked in the literature. By comparing channel members' profits across these four scenarios, we obtained the following findings.

First, the manufacturer should implement Scenario 1 by investing in national advertising and providing cooperative advertising to the retailer when retailer advertising positively impacts future sales. Alternatively, when retailer advertising harms future sales, depending on the values of parameters, the manufacturer should exclusively either provide cooperative advertising (Scenario 2) or undertake national advertising (Scenario 3).

On the other hand, the retailer is better off when the manufacturer exclusively invests in national advertising and retail advertising harms future sales. Otherwise, depending on the advertising effects, the retailer can accept either Scenario 1 or Scenario 3. Interestingly, the preferences of the two channel members do not always converge. In fact, under certain market conditions, the manufacturer prefers Scenario 1, while the retailer prefers Scenario 3. This leads to suboptimal strategies and profits, as the retailer does not participate in the cooperative advertising program offered by the manufacturer if Scenario 1 is chosen.

Finally, our results indicate that Scenarios 2 and 4 where the manufacturer does not invest in national advertising are not desirable for both the manufacturer and retailer.

These findings have three major theoretical and managerial implications. First, with respect to works that study the allocation of advertising responsibilities in channels, our research contributes new insights. In particular, when manufacturer and retailer advertising are differentiated, leaving the responsibility of channel advertising to the retailer is the worst possible scenario for both the manufacturer and the retailer. This is different from previous findings in the literature which consider alternative channel structures (e.g., Sigué and Chintagunta, 2009). The involvement of the manufacturer in channel advertising both through own advertising campaigns and push activities such as cooperative advertising helps improve channel profits. However, under certain conditions, cooperative advertising is not optimal and channel members are each better off when the manufacturer invests in national advertising alone. These conditions depend heavily on how damaging the first-period retailer advertising is on the second-period sales. Therefore, the manufacturer should avoid supporting retailer advertising, which significantly harms second-period sales. A company such as Toyota follows this rule and discourages retail promotional activities that favor short-term sales to the detriment of long-term sales (Martín-Herrán and Sigué, 2017a).

Second, unless the manufacturer decides to enhance the attractiveness of Scenario 1 where both national and cooperative advertising activities are undertaken for the retailer, in areas where the two channel members' advertising preferences diverge, this strategy can lead to suboptimal outcomes. This may explain why auto makers such as Ford, Honda, and Toyota supply their dealers with various customizable marketing resources such as display ads, TV and radio spots, direct mail, and print ads, which can be considered as additional side payments (Shreve, 2018), in addition to undertaking national advertising and offering cooperative advertising programs to support dealers' local advertising. As well, in this industry, manufacturers give priority to national advertising when they must focus exclusively on one advertising instrument. In fact, empirical evidence supports the view that manufacturer advertising activities stimulate more current sales than does retailer advertising (see Herrington and Dempsey, 2005).

Third, compared to the current literature on cooperative advertising that focuses on identifying the conditions for which these arrangements improve channel profits, we have shown that a broader perspective should be taken into account. In particular, even when cooperative advertising is profitable for both the manufacturer and the retailer, a profit-maximizing retailer may not participate in a cooperative advertising arrangement in the context where the manufacturer simultaneously undertakes own advertising and offers a cooperative advertising program. Because participation in cooperative advertising is not mandatory, the retailer would prefer to free ride on manufacturer advertising and increase his profits at the expense of the manufacturer. While the obstacles to participation in cooperative advertising programs are numerous (see Borrell Associates, 2015; Galloway, 2016), this research supports the view that an explanation for the existence of unused cooperative funds may be that the proposed programs are not attractive enough for the retailers who act opportunistically and prefer situations where manufacturers carry out only their own advertising programs. A direct implication of this finding is that manufacturers may have to beef up their programs by offering side payments or by providing additional support services to enhance retailers' participation in cooperative advertising programs.

Finally, this work relies on some simplifying assumptions that limit its generalization. Some of them could be relaxed in future research. For instance, we have considered cooperative advertising as a special case of a non-price push promotional activity. The trade-offs that manufacturers have to make between their own advertising and other non-price push promotional activities can also be explored. This will require building alternative models that take into account the specificity of each promotional tool considered. Another relevant extension will be to consider advertising resource allocation in a context where there is competition at the level of the manufacturer or/and the retailer. Advertising decisions in such a context may be different from those where the focus is only on vertical interactions between channel members. Further, alternative demand function formulations can also be considered, as it is well-known that advertising decisions depend on advertising response curves.

Declaration of competing interest

None.

Appendix A. Proof of Proposition 1 and optimal total manufacturer's and retailer's profits

In this appendix, the superscript (i) stands for Scenario *i*.

Equilibrium strategies and optimal manufacturer's and retailer's profits for Scenario 1:

The equilibrium strategies for Scenario 1 are:

$$w_1^{(1)} = \frac{g\left(16c_m\left(5\beta\delta+64c_r-12\left(\beta^2+\delta^2\right)\right)+(1-\alpha)\alpha^2\left(\alpha\delta^2+20\beta\delta+128\alpha c_r-24\alpha\beta^2\right)\right)}{2\left(32c_m\left(32c_r-9\beta^2-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right)\right)}, \tag{A.1}$$

$$p_1^{(1)} = \frac{g\left(16c_m\left(-24\beta^2+25\beta\delta+192c_r-36\delta^2\right)-(\alpha-1)\alpha^2\left(-48\alpha\beta^2+3\alpha\delta^2+40\beta\delta+384\alpha c_r\right)\right)}{4\left(32c_m\left(-9\beta^2+32c_r-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right)\right)}, \tag{A.2}$$

$$a_m^{(1)} = \frac{\alpha g\left(\alpha\left(-36\beta^2+30\beta\delta+\delta^2\right)+6\delta(5\beta-4\delta)+128(\alpha+1)c_r\right)}{32c_m\left(-9\beta^2+32c_r-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right)}, \tag{A.3}$$

$$t^{(1)} = \frac{8c_m\left(-6\beta^2\delta+6\beta\delta^2+32\beta c_r+48c_r\delta+\delta^3\right)-(\alpha-1)\alpha^2\left(\alpha\beta\delta^2+32\alpha\beta c_r-48c_r\delta-\delta^3\right)}{16c_r\left(8c_m(6\beta+5\delta)-(\alpha-1)\alpha^2(6\alpha\beta-5\delta)\right)}, \tag{A.4}$$

$$a_r^{(1)} = \frac{4g\left(8c_m(6\beta+5\delta)+(1-\alpha)\alpha^2(6\alpha\beta-5\delta)\right)}{32c_m\left(-9\beta^2+32c_r-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right)}, \tag{A.5}$$

$$w_2^{(1)} = \frac{g\left(16c_m\left(32c_r-(\delta-3\beta)^2\right)+(\alpha-1)\alpha^2(\delta(3\alpha\beta-2\delta)+64c_r)\right)}{32c_m\left(-9\beta^2+32c_r-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right)}, \tag{A.6}$$

$$p_2^{(1)} = \frac{3g\left(16c_m\left(32c_r-(\delta-3\beta)^2\right)+(\alpha-1)\alpha^2(\delta(3\alpha\beta-2\delta)+64c_r)\right)}{2(32c_m\left(-9\beta^2+32c_r-6\delta^2\right)-\alpha^2\left(\alpha^2\left(\delta^2-36\beta^2\right)+60\alpha\beta\delta+128\left(\alpha^2+1\right)c_r-24\delta^2\right))}. \tag{A.7}$$

The optimal manufacturer's and retailer's profits in Scenario 1 read:

$$\Pi_M^{(1)} = \frac{g^2\left(8c_m\left(-36\beta^2+60\beta\delta+256c_r-23\delta^2\right)-(\alpha-1)^2\alpha^2\left(128c_r+\delta^2\right)\right)}{8\left[32c_m(32c_r-9\beta^2-6\delta^2)-\alpha^2(\alpha^2(\delta^2-36\beta^2)+60\alpha\beta\delta+128(\alpha^2+1)c_r-24\delta^2)\right]}, \tag{A.8}$$

$$\Pi_R^{(1)} = \frac{g^2 Num\Pi_R^{(1)}}{16\left[32c_m(32c_r-9\beta^2-6\delta^2)-\alpha^2(\alpha^2(\delta^2-36\beta^2)+60\alpha\beta\delta+128(\alpha^2+1)c_r-24\delta^2)\right]^2}, \tag{A.9}$$

where

$$\begin{aligned} Num\Pi_R^{(1)} =\ & 256c_m^2(3(108\beta^4-192\beta^3\delta+155\beta^2\delta^2-88\beta\delta^3+56\delta^4)+8192c_r^2-128c_r(30\beta^2-11\beta\delta+19\delta^2)) \\ & -32(\alpha-1)\alpha^2c_m\left(\delta\left(\alpha\left(96\beta^2\delta+71\beta\delta^2-36\beta^3-12\delta^3\right)-48\delta\left(2\beta\delta-\beta^2+\delta^2\right)\right)+8192(\alpha-1)c_r^2\right. \\ & \left.-64c_r\left(12(4\alpha-3)\beta^2+8(\alpha-1)\beta\delta+(23\alpha-28)\delta^2\right)\right)+(\alpha-1)^2\alpha^4\left(\delta^2\left(\alpha^2\left(132\beta^2+\delta^2\right)\right.\right. \\ & \left.\left.-224\alpha\beta\delta+96\delta^2\right)+16\,384\left(\alpha^2+1\right)c_r^2-256c_r\left(\alpha^2\left(24\beta^2-\delta^2\right)-38\alpha\beta\delta+14\delta^2\right)\right). \end{aligned}$$

Equilibrium strategies and optimal manufacturer's and retailer's profits for Scenario 2:

The equilibrium strategies for Scenario 2 are:

$$w_1^{(2)} = \frac{1}{4}g\left(2+\frac{\beta(6\beta+5\delta)}{32c_r-9\beta^2-6\delta^2}\right), \tag{A.10}$$

$$p_1^{(2)} = \frac{1}{8}g\left(6+\frac{5\beta(6\beta+5\delta)}{32c_r-9\beta^2-6\delta^2}\right), \tag{A.11}$$

$$a_m^{(2)} = 0, \tag{A.12}$$

$$t^{(2)} = \frac{6\beta\delta(\delta-\beta)+16c_r(2\beta c_r+3\delta)+\delta^3}{16c_r(6\beta+5\delta)}, \tag{A.13}$$

$$a_r^{(2)} = \frac{g(6\beta+5\delta)}{32c_r-9\beta^2-6\delta^2}, \tag{A.14}$$

$$w_2^{(2)} = \frac{1}{2}g\left(1+\frac{\delta(6\beta+5\delta)}{32c_r-9\beta^2-6\delta^2}\right), \tag{A.15}$$

$$p_2^{(2)} = \frac{3}{4}g\left(1+\frac{\delta(6\beta+5\delta)}{32c_r-9\beta^2-6\delta^2}\right). \tag{A.16}$$

The optimal manufacturer's and retailer's profits in Scenario 2 read:

$$\Pi_M^{(2)} = \frac{g^2\left(-36\beta^2+60\beta\delta+256c_r-23\delta^2\right)}{32\left(32c_r-9\beta^2-6\delta^2\right)}, \tag{A.17}$$

$$\Pi_R^{(2)} = \frac{g^2(3(108\beta^4-192\beta^3\delta+155\beta^2\delta^2-88\beta\delta^3+56\delta^4)+8192c_r^2-128c_r(30\beta^2-11\beta\delta+19\delta^2))}{64\left(32c_r-9\beta^2-6\delta^2\right)^2}. \tag{A.18}$$

Equilibrium strategies and optimal manufacturer's and retailer's profits for Scenario 3:

The equilibrium strategies for Scenario 3 are:

$$w_1^{(3)} = \frac{4g(c_m(\delta^3(\beta-\delta)+4\delta^2(\beta^2+8c_r)+4(\beta^2-4c_r)(16c_r-\beta\delta))+8(\alpha-1)\alpha^2c_r(\beta(\delta-\alpha\beta)+4\alpha c_r))}{Den^{(3)}}, \tag{A.19}$$

$$p_1^{(3)}= \frac{2g\left(16(\alpha-1)\alpha^2 c_r(\beta(\delta-\alpha\beta)+6\alpha c_r)-c_m\left(\delta(\beta-\delta)\left(8\beta^2-3\delta^2\right)+768c_r^2-16c_r\left(8\beta^2-\beta\delta+6\delta^2\right)\right)\right)}{Den^{(3)}}, \tag{A.20}$$

$$a_m^{(3)}= \frac{\alpha g\left(-\delta^2(\beta-\delta)(\alpha\beta-\delta)-256(\alpha+1)c_r^2+16c_r\left(4\alpha\beta^2-3(\alpha+1)\beta\delta+2\delta^2\right)\right)}{Den^{(3)}}, \tag{A.21}$$

$$t^{(3)}= 0, \tag{A.22}$$

$$a_r^{(3)}= \frac{g\left((\alpha-1)\alpha^2\left(16c_r(2\alpha\beta-\delta)-\delta^2(\alpha\beta-\delta)\right)-8c_m\left(2\beta^2\delta+(2\beta+\delta)\left(16c_r-\delta^2\right)\right)\right)}{Den^{(3)}}, \tag{A.23}$$

$$w_2^{(3)}= \frac{8g\left((1-\alpha)\alpha^2 c_r(\delta(\alpha\beta-\delta)+16c_r)-c_m\left(\beta\delta^2(\beta-\delta)+128c_r^2-8c_r\left(4\beta^2-2\beta\delta+\delta^2\right)\right)\right)}{Den^{(3)}}, \tag{A.24}$$

$$p_2^{(3)}= \frac{12g\left((1-\alpha)\alpha^2 c_r(\delta(\alpha\beta-\delta)+16c_r)-c_m\left(\beta\delta^2(\beta-\delta)+128c_r^2-8c_r\left(4\beta^2-2\beta\delta+\delta^2\right)\right)\right)}{Den^{(3)}}, \tag{A.25}$$

where

$$Den^{(3)} = \alpha^2\left(\delta^2(\delta-\alpha\beta)^2+256\left(\alpha^2+1\right)c_r^2-32c_r(\alpha\beta-\delta)(2\alpha\beta-\delta)\right)-8c_m\left(256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4\right).$$

The optimal manufacturer's and retailer's profits in Scenario 3 read:

$$\Pi_M^{(3)} = \frac{g^2\left(32(\alpha-1)^2\alpha^2c_r^2-c_m\left(\delta^2(\beta-\delta)^2+512c_r^2-32c_r\left(2\beta^2-3\beta\delta+\delta^2\right)\right)\right)}{Den}, \tag{A.26}$$

$$\Pi_R^{(3)} = \frac{g^2 Num\Pi_R^{(3)}}{Den^2}. \tag{A.27}$$

where

$$\begin{aligned}Num\Pi_R^{(3)} = &\left(4c_m^2\left(\delta^4(\beta-\delta)^2\left(4\beta^2+\delta^2\right)+131\,072c_r^4-4096c_r^3\left(12\beta^2-6\beta\delta+7\delta^2\right)\right.\right.\\&+256c_r^2\left(16\beta^4-24\beta^3\delta+29\beta^2\delta^2-14\beta\delta^3+9\delta^4\right)-16c_r\delta^2\left(20\beta^4-32\beta^3\delta+18\beta^2\delta^2-10\beta\delta^3+5\delta^4\right)\big)\\&-16(\alpha-1)\alpha^2c_mc_r\left(\delta^5(\delta-\alpha\beta)+4096(\alpha-1)c_r^3-256c_r^2\left(4(\alpha-1)\beta^2+(2\alpha-3)\delta^2\right)\right.\\&\left.+16c_r\delta^2\left(4(\alpha-1)\beta^2+3\alpha\beta\delta+(\alpha-3)\delta^2\right)\right)+(\alpha-1)^2\alpha^4c_r\left(\delta^4\left(-(\delta-\alpha\beta)^2\right)+4096\left(\alpha^2+1\right)c_r^3\right.\\&\left.\left.-256c_r^2\left(4\alpha^2\beta^2-6\alpha\beta\delta+3\delta^2\right)+16c_r\delta^2\left(5\alpha^2\beta^2-8\alpha\beta\delta+3\delta^2\right)\right)\right).\end{aligned}$$

Equilibrium strategies and optimal manufacturer's and retailer's profits for Scenario 4:

The equilibrium strategies for Scenario 4 are:

$$w_1^{(4)} = \frac{1}{2}g\left(1-\frac{\beta\delta\left(-4\beta^2+4\beta\delta+16c_r+\delta^2\right)}{256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4}\right), \tag{A.28}$$

$$p_1^{(4)} = \frac{1}{4}g\left(\frac{\beta\left(\delta\left(8\beta(\beta-\delta)-3\delta^2\right)+16c_r(4\beta+\delta)\right)}{256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4}+3\right), \tag{A.29}$$

$$a_m^{(4)} = 0, \tag{A.30}$$

$$t^{(4)} = 0, \tag{A.31}$$

$$a_r^{(4)} = \frac{g\left(16c_r(2\beta+\delta)-\delta\left(-2\beta^2+2\beta\delta+\delta^2\right)\right)}{256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4}, \tag{A.32}$$

$$w_2^{(4)} = \frac{g\left[\delta\left(16c_r(2\beta+\delta)-\delta\left(-2\beta^2+2\beta\delta+\delta^2\right)\right)\right]}{2\left[256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4\right]}+\frac{g}{2}, \tag{A.33}$$

$$p_2^{(4)} = \frac{3}{4}g\left(\frac{\delta\left(16c_r(2\beta+\delta)-\delta\left(-2\beta^2+2\beta\delta+\delta^2\right)\right)}{256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4}+1\right), \tag{A.34}$$

The optimal manufacturer's and retailer's profits in Scenario 4 read:

$$\Pi_M^{(4)}= \frac{g^2\left(\delta^2(\beta-\delta)^2+512c_r^2-32c_r\left(2\beta^2-3\beta\delta+\delta^2\right)\right)}{8\left(256c_r^2-32c_r\left(2\beta^2+\delta^2\right)+\delta^4\right)}, \tag{A.35}$$

$$\Pi_R^{(4)} = \frac{g^2 Num\Pi_R^{(4)}}{16\left(256c_r^2 - 32c_r\left(2\beta^2 + \delta^2\right) + \delta^4\right)^2}, \quad (A.36)$$

where

$$\begin{aligned} Num\Pi_R^{(4)} = {} & \delta^4(\beta-\delta)^2\left(4\beta^2+\delta^2\right) + 131\,072c_r^4 - 4096c_r^3\left(12\beta^2 - 6\beta\delta + 7\delta^2\right) \\ & + 256c_r^2\left(16\beta^4 - 24\beta^3\delta + 29\beta^2\delta^2 - 14\beta\delta^3 + 9\delta^4\right) \\ & - 16c_r\delta^2\left(20\beta^4 - 32\beta^3\delta + 18\beta^2\delta^2 - 10\beta\delta^3 + 5\delta^4\right). \end{aligned}$$

We obtain the different subgame-perfect equilibrium solutions previously described using backwards induction for the game.

The game is played in four stages and is solved backwards. We next describe and solve the problem the players are facing at each stage of the game.

Stage 4: At this stage of the game, the retailer chooses the second-period price, p_2, in order to maximize his second-period profits. Therefore, the retailer's problem can be written as:

$$\max_{p_2} R_2, \quad (A.37)$$

where

$$R_2 = (p_2 - w_2)d_2, \quad (A.38)$$

denotes the retailer's second-period profits and d_2 is the second-period demand function given by

$$q_2 = g - p_2 + \delta a_r + \alpha^2 a_m. \quad (A.39)$$

The solution to problem (A.37) gives us the retailer's reaction function, that is, p_2 as a function of the wholesale price, w_2, and the retailer's and manufacturer's advertising in the first period, a_r and a_m.

For any $\alpha \in (0, 1)$, $\delta \in (-1, 1)$, and $\beta \in (0, 1)$, the retailer's second-period profit is a strictly concave function of his decision variable in this period, p_2. From the first-order optimality conditions for the problem in (A.37), the following expression can be derived:

$$p_2 = \frac{1}{2}\left(g + w_2 + \alpha^2 a_m + \delta a_r\right). \quad (A.40)$$

Stage 3: At this stage of the game, the manufacturer chooses the second-period wholesale price, w_2, in order to maximize her second-period profits. Therefore, the problem the manufacturer is facing can be written as:

$$\max_{w_2} M_2, \quad (A.41)$$

where

$$M_2 = w_2 q_2, \quad (A.42)$$

denotes the manufacturer's second-period profits and q_2 is the demand function in this period, defined in (A.39). At this stage of the game, the manufacturer knows the retailer's pricing reaction function derived in Stage 4, and therefore incorporates this information when deciding her optimal pricing strategies. Therefore, the reaction function in (A.40) has to be substituted in the manufacturer's objective function in (A.41).

The solution to this problem gives us the wholesale price, w_2, as a function of the retailer's and manufacturer's advertising in the first period, a_r and a_m.

For any $\alpha \in (0, 1)$, $\delta \in (-1, 1)$, and $\beta \in (0, 1)$, the manufacturer's second-period profit is a strictly concave function of her decision variable in this period, w_2. From the optimality first-order conditions for the problem expressed in (A.41) we get

$$w_2 = \frac{1}{2}\left(g + \alpha^2 a_m + \delta a_r\right). \quad (A.43)$$

Substituting this expression in the retailer's reaction function in (A.40) we obtain the second-period retail price as a function of the retailer's and manufacturer's advertising in the first period, a_r and a_m:

$$p_2 = \frac{3}{4}\left(g + \alpha^2 a_m + \delta a_r\right). \quad (A.44)$$

The second-period retailer's and manufacturer's optimal profits are obtained by substituting the expressions (A.43) and (A.44) in (A.38) and (A.42), and are given by

$$R_2 = \frac{1}{16}\left(g + \alpha^2 a_m + \delta a_r\right)^2, \quad (A.45)$$

$$M_2 = \frac{1}{8}\left(g + \alpha^2 a_m + \delta a_r\right)^2. \quad (A.46)$$

Before moving to the first period, it is worthwhile to highlight that the manufacturer's and retailer's strategies in the second period in the four scenarios described in Proposition 1 are given by (A.43) and (A.44). Similarly, the second-period retailer's and manufacturer's optimal profits in the four cases are given by (A.45) and (A.46), respectively.

Stage 2: Moving to the first period, the retailer chooses the retail price, p_1, and his advertising effort, a_r, in order to maximize his total profits during the two periods:

$$R = R_1 + R_2.$$

Taking into account the second-period retailer's profit (given by (A.45)), the retailer's total profit is as follows:

$$R = (p_1 - w_1)(\alpha a_m + a_r\beta + g - p_1) - a_r^2 c_r(1-t) + \frac{1}{16}(\alpha^2 a_m + a_r\delta + g)^2.$$

It can be easily proved that the retailer's total profit is a concave function in the retailer's first-period decision variables, p_1 and a_r, if and only if the following condition is satisfied:

$$\Delta = 4\beta^2 - 16c_r(1-t) + \delta^2 < 0. \tag{A.47}$$

Maximizing with respect to the first-period retail price and the retailer's advertising gives the following optimal reaction functions, in other words, the first-period retail price, p_1, and the retailer's advertising, a_r, as functions of first-period wholesale price, w_1, and the cooperative advertising support rate offered by the manufacturer to the retailer in the first period, t:

$$p_1 = \frac{\delta(\alpha a_m(\delta - \alpha\beta) + g(\delta - \beta)) + w_1\left(8\beta^2 + \delta^2\right) - 16c_r(1-t)(g + w_1 + \alpha a_m)}{2\left(4\beta^2 + \delta^2 - 16c_r(1-t)\right)}, \tag{A.48}$$

$$a_r = -\frac{\alpha a_m(\alpha\delta + 4\beta) + g(4\beta + \delta) - 4\beta w_1}{4\beta^2 + \delta^2 - 16c_r(1-t)}. \tag{A.49}$$

Stage 1: At this stage of the game, the manufacturer chooses the first-period wholesale price, w_1, and the cooperative advertising support rate offered to the retailer, t, in order to maximize her total profits:

$$M = M_1 + M_2.$$

Taking into account the second-period manufacturer's profits (given by (A.46)), the manufacturer's total profits are

$$M = \frac{1}{8}(\alpha^2 a_m + a_r\delta + g)^2 + w_1(\alpha a_m + a_r\beta + g - p_1) - a_m^2 c_m - a_r^2 c_r t.$$

Replacing the retailer's reaction functions given by (A.48) and (A.49), the manufacturer's total profits to be maximized read:

$$\begin{aligned} M = \; & \frac{1}{8}\left\{\frac{1}{\Delta}\left[64c_r(t-1)w_1(\alpha a_m + g - w_1) - 2\delta^2\left((\alpha-1)\alpha a_m(\alpha(\alpha+1)a_m + 2g) + (\alpha a_m + g - w_1)^2 + w_1^2\right)\right.\right. \\ & \left. - 4\beta\delta\left(\alpha^2 a_m + g\right)(2\alpha a_m + 2g - w_1)\right] + \frac{1}{\Delta^2}\left[\left(\delta^2 - 8c_r t\right)\left(\alpha a_m(\alpha\delta + 4\beta) + g(4\beta + \delta) - 4\beta w_1\right)^2\right] \\ & \left. - 8a_m^2 c_m + \left(\alpha^2 a_m + g\right)^2 \right\}, \end{aligned}$$

where Δ is defined in (A.47).

The maximization of M with respect to w_1, a_m and t gives two triples of solutions. One of the triple leads to a null retailer's advertising, and consequently, it is discarded because we are assuming that the retailer's advertising is strictly positive. The other triple of solutions are given by (A.1), (A.3) and (A.4).

For the triple given by (A.1), (A.3), and (A.4), the first, second and third minors of the Hessian matrix of function M with respect to the manufacturer's decisions variables in the first period, w_1, a_m and t, can be "a priori" positive or negative. In all the numerical simulations when the first scenario described in Proposition 1 is considered, we have checked that the quadratic form associated with the Hessian matrix is negative semidefinite, implying that M is a concave function, and that the interior solution given by (A.1), (A.3), and (A.4) is a maximum.

Substituting expressions (A.1), (A.3), and (A.4) in the retailer's reaction functions given in (A.48) and (A.49), we obtain the optimal first-period retail price and the optimal first-period rate of advertising carried out by the retailer in the first scenario of the statement Proposition 1, given by (A.2) and (A.5).

The final expressions of the optimal pricing strategies given by (A.6) and (A.7) in the first scenario of the statement of Proposition 1 can be obtained once w_1, p_1 as well as t_1, a_m, and a_r have been substituted by their expressions given by (A.1), (A.2), (A.4), (A.3), and (A.5).

Once all the equilibrium strategies in Scenario 1 are known, the optimal manufacturer's and retailer's profits can be easily computed; these profits are given by (A.8) and (A.9).

The strategies associated with the boundary solution $a_m = 0$ and $t > 0$, which constitutes the second scenario in Proposition 1 can be derived easily once the expressions of $w_1^{(2)}$ and $t^{(2)}$ are obtained from the optimality conditions derived from maximization of M with respect to w_1 and t, taking into account that $a_m = 0$. These optimality conditions read:

$$\begin{aligned} & 256c_r^2(t-1)^2(g - 2w_1) + 16c_r\left(\beta\delta g(2t-1) + 4\beta^2(g(2t-1) + (2-3t)w_1) + 2\delta^2(t-1)(g - 2w_1)\right) \\ & + \delta\left(g(\beta-\delta)(2\beta-\delta)(2\beta-\delta) - 2\delta^3 w_1\right) = 0, \\ & c_r(g(4\beta+\delta) - 4\beta w_1)\left(16c_r(g(4\beta(t+1) + \delta(5t-3)) + 4\beta(t-3)w_1) - g\left(16\beta^3 - 12\beta^2\delta + 20\beta\delta^2 + \delta^3\right)\right. \\ & \left. + 4\beta w_1\left(12\beta^2 + 7\delta^2\right)\right) = 0. \end{aligned}$$

Solving these equations we obtain two pairs of solutions. One of the pair of solutions leads to a null retailer's advertising, and is therefore discarded. The other pair of solutions is given by (A.10) and (A.13).

As in the previous case, replacing expressions (A.10) and (A.13) together with $a_m^{(2)} = 0$, in the retailer's reaction functions given in (A.48) and (A.49), we obtain the optimal first-period retail price and the optimal first-period rate of advertising carried out by the retailer in the second scenario given by (A.11) and (A.14).

Substituting all these first-period optimal strategies given by (A.10), (A.13), $a_m^{(2)} = 0$, (A.11) and (A.14), the final expressions of the optimal pricing strategies given by (A.15) and (A.16) in the second scenario of the statement of Proposition 1 can be obtained.

The optimal manufacturer's and retailer's profits in Scenario 2 are given by (A.17) and (A.18).

The optimal strategies in the third scenario in Proposition 1, corresponding to $a_m > 0$, $t = 0$, can be obtained following the same lines as in the previous scenario. These optimal strategies can be derived easily once the expressions of $w_1^{(3)}$ and $a_m^{(3)}$ are obtained from the optimality conditions derived from maximization of M with respect to w_1 and a_m, taking into account that $t = 0$. These optimality conditions read:

$$4\left(4cr-\beta^2\right)\left(16c_r(\alpha a_m+g-2w_1)-\beta\delta\left(\alpha^2 a_m+g\right)\right)+\delta^2\left(\delta^2-32c_r\right)(\alpha am+g-2w_1)$$
$$-\beta\delta^2\left(\alpha a_m(\alpha\delta+4\beta)+g(4\beta+\delta)\right)=0,$$
$$2\alpha\left(8(\beta(\delta-\alpha\beta)+4\alpha c_r)\left(\left(4c_r-\beta^2\right)\left(\alpha^2 a_m+g\right)+\beta\delta(\alpha a_m+g-w_1)\right)\right.$$
$$\left.+w_1\left(4\beta^2-16c_r+\delta^2\right)\left(\delta(\delta-\alpha\beta)-16c_r\right)\right)-8a_m c_m\left(4\beta^2-16c_r+\delta^2\right)^2=0.$$

Solving these equations we obtain $w_1^{(3)}$ and $a_m^{(3)}$, and replacing them into the first-period retailer's reaction functions given in (A.48) and (A.49) and the second-period pricing functions, the optimal strategies in Scenario 3 are given by given by (A.19) to (A.25).

The optimal manufacturer's and retailer's profits in Scenario 3 are (A.26) and (A.27).

Similarly, the optimal strategies in the fourth scenario in Proposition 1, corresponding to $a_m = t = 0$, can be obtained by following the same procedure as in the previous scenario. These optimal strategies can be derived easily once the expression of $w_1^{(4)}$ is obtained from the optimality condition derived from maximization of M with respect to w_1 taking into account that $a_m = t = 0$. This optimality condition reads:

$$4\left(4c_r-\beta^2\right)(16c_r(g-2w_1)-4\beta\delta g)+\delta^2\left(\left(\delta^2-32c_r\right)(g-2w_1)-\beta g(4\beta+\delta)\right)=0.$$

Solving this equation we obtain $w_1^{(4)}$ and replacing it together with $a_m^{(4)} = t^{(4)} = 0$ into the first-period retailer's reaction functions given in (A.48) and (A.49) and the second-period pricing functions, the optimal strategies for Scenario 4 are given by (A.28) to (A.34).

The optimal manufacturer's and retailer's profits in Scenario 4 are (A.35) and (A.36).

Appendix B. Proof of Propositions 2, 3, 4, and 5

In In this appendix, the superscript (i) stands for Scenario i. Under the assumptions $g = 1$, $c_m = c_r = 1$, we compare the manufacturer's and the retailer's profits across scenarios.

Proof of Proposition 2

The difference of manufacturer's profits for Scenarios 1 and 2 gives:

$$\Pi_M^{(1)}-\Pi_M^{(2)}=\frac{\alpha^2\left(4\alpha\left(32-9\beta^2\right)+30(\alpha+1)\beta\delta+(\alpha-24)\delta^2+128\right)^2}{32\left(9\beta^2+6\delta^2-32\right)\left(-36(\alpha^4-8)\beta^2+60\alpha^3\beta\delta+(\alpha^4-24\alpha^2+192)\delta^2+128(\alpha^4+\alpha^2-8)\right)}.$$

From the expression above it is straightforward to conclude that:

$$\Pi_M^{(1)}-\Pi_M^{(2)}>0 \quad \text{for } \alpha\in(0,1),\ \beta\in(0,1),\ \delta\in(-1,1).$$

The comparison of manufacturer's profits for Scenarios 1 and 3 leads to:

$$\Pi_M^{(1)}-\Pi_M^{(3)}=-\frac{\left(\beta((1-\alpha)(-\alpha^3)(\delta^2+32)-16(3\delta^2+16))-(8-\alpha^2(1-\alpha))\delta(\delta^2+48)+48\beta^2\delta\right)^2}{8\left(36(\alpha^4-8)\beta^2-12\delta(5\alpha^3\beta+16\delta)+(24-\alpha^2)\alpha^2\delta^2-128(\alpha^2+1)\alpha^2+1024\right)\Delta_1},$$

with

$$\Delta_1=256\left(\alpha^2+1\right)\alpha^2-64\left(\alpha^4-8\right)\beta^2-32\left(\alpha^2-8\right)\delta^2+\delta\left(\alpha^4\beta^2\delta-2\alpha^3\beta\left(\delta^2-48\right)+\alpha^2\delta^3-8\delta^3\right)-2048.$$

The sign of $\Pi_M^{(1)} - \Pi_M^{(3)}$ is opposite to the sign of the denominator, and the latter is negative, because it is the product of a first positive factor and a second negative factor. Therefore, we can conclude that

$$\Pi_M^{(1)}-\Pi_M^{(3)}>0 \quad \text{for } \alpha\in(0,1),\ \beta\in(0,1),\ \delta\in(-1,1).$$

Proof of Proposition 3

The comparison of manufacturer's profits for Scenarios 3 and 4 leads to:

$$\Pi_M^{(3)}-\Pi_M^{(4)}=\frac{\alpha^2\left(\alpha\left(\beta^2(\delta^2-64)-\beta\delta(\delta^2-48)+256\right)-\beta\delta(\delta^2-48)+(\delta^2-16)^2\right)^2}{8\left(64\beta^2-\delta^4+32\delta^2-256\right)\Delta_1}.$$

Because both factors in the denominator are negative, we can conclude:

$$\Pi_M^{(3)}-\Pi_M^{(4)}>0 \quad \text{for } \alpha\in(0,1),\ \beta\in(0,1),\ \delta\in(-1,1).$$

Proof of Proposition 4

The comparison of retailer's profits for Scenarios 1 and 2 leads to:

$$\Pi_R^{(1)} - \Pi_R^{(2)} = \frac{Num(\Pi_R^{(1)} - \Pi_R^{(2)})}{64(9\beta^2+6\delta^2-32)^2(\Delta_2)^2},$$

where

$$\Delta_2 = 36\alpha^4\beta^2 - \alpha^4\delta^2 - 128\alpha^4 - 60\alpha^3\beta\delta + 24\alpha^2\delta^2 - 128\alpha^2 - 288\beta^2 - 192\delta^2 + 1024,$$

and $Num(\Pi_R^{(1)} - \Pi_R^{(2)})$ is a long polynomial expressions in terms of parameters α, δ, and β. We refrain from writing this expression. The denominator of the difference is positive, and hence the sign of the difference coincides with the sign of the numerator. Using the Reduce Command in Mathematica 11.1 that allows the analytical manipulation of the expression it can be easily shown that $Num(\Pi_R^{(1)} - \Pi_R^{(2)})$ is positive for $\beta \in \{0.25, 0.5, 0.75\}$ and any $\alpha \in (0, 1)$, $\delta \in (-1, 1)$.

Proof of Proposition 5

The comparison of retailer's profits for Scenarios 3 and 4 leads to:

$$\Pi_R^{(3)} - \Pi_R^{(4)} = \frac{Num(\Pi_R^{(3)} - \Pi_R^{(4)})}{16(64\beta^2-\delta^4+32\delta^2-256)^2(\Delta_3)^2},$$

where

$$\Delta_3 = \alpha^4\beta^2\delta^2 - 64\alpha^4\beta^2 + 256\alpha^4 - 2\alpha^3\beta\delta^3 + 96\alpha^3\beta\delta + \alpha^2\delta^4 - 32\alpha^2\delta^2 + 256\alpha^2 + 512\beta^2 - 8\delta^4 + 256\delta^2 - 2048,$$

and $Num(\Pi_R^{(3)} - \Pi_R^{(4)})$ is a long polynomial expressions in terms of parameters α, δ, and β. The denominator of the difference is positive, and hence the sign of the difference coincides with the sign of the numerator. Using the Reduce Command in Mathematica 11.1 it can be easily shown that $Num(\Pi_R^{(3)} - \Pi_R^{(4)})$ is positive for $\beta \in \{0.25, 0.5, 0.75\}$ and any $\alpha \in (0, 1)$, $\delta \in (-1, 1)$.

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