Perpetuating regional asymmetries through income transfers

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December 20, 2019

Abstract

This paper studies the effect of income transfers on the distribution of economic activity through a modified footloose entrepreneurs model. The model incorporates some key features of the Dutch Disease literature: sectorial mobility and non-tradable goods. If foreign competition is high (high trade openness), transfers could cause a Dutch Disease in the short and long run. For intermediate levels of foreign competition, Dutch Disease appears only in the short run. And, for low levels, the recipient region always benefits from the income transfers. Additionally, when economies of scale are large, the transfers could perpetuate a core-periphery structure.

JEL classification: H30, R12

Key Words: new economic geography, Dutch disease, income transfers

*The author have been partially supported by Ministerio de Economía y Competitividad, Gobierno de España (ECO2017-82227-P). The author acknowledge the support by COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy evaluation"

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1 Introduction

Since Krugman’s seminal work (Krugman, 1991), there has been a better understanding of the forces that shape the geographical distribution of economic activity. New Economic Geography (NEG) states that reductions in tariff and transport costs lead to a core-periphery structure of the economy. In this regard, international and interregional income transfers are recognized instruments to compensate spatial economic disparities; e.g. the European Union spends almost one third of its budget on these kinds of programs (Baldwin et al. 2005).

According to the NEG literature, income transfers enlarge the market size of the recipient region. Thus, the smaller, or peripheral, region becomes more attractive for firms to settle in, and regional disparities are reduced. The market size effect is the key element behind this mechanism. For example, Rowthorn (2010) explains that fiscal transfers in Great Britain can help to reduce the North-South regional disparities. Bickenbach et al. (2013) point out that public transfers toward East Germany increase the market potential of this region and nourish the dispersion of the economic activity. For the Chilean economy, Modrego et al. (2014) simulate a positive shock in the market potential of Santiago, which reassembles the public transfers program applied in the country. Their results suggest that the number of firms will increase, especially in the beneficiary region and the surrounding area.

However, a negative relation between income transfers and industrial employment is sometimes observed. As an example, Figure 1 depicts the negative correlation between growth rates of the industrial labor (industry plus manufacturing, divided by total labor) and the disposable income (divided by primary income) for the European regions (NUTS2) between 2005 and 2014. Although, it is a simple correlation, it raises the question of the effectiveness of transfers; which is no new concern for economists. Moreover, Yanno and Nugent (1999) show that aid flows are associated with contractions of the
tradable sectors for the cases of Burkina Faso, Congo, Lesotho, Liberia, Senegal and Yemen. Bulir and Lane (2002) also present some evidence of the decline of tradable sectors for a sample of aid-dependent countries. Subramanian and Rajan (2005) find that aid flows deteriorate the competitiveness of the tradable sectors in developing countries. For Uganda, Adam (2005) also finds some evidence of a reduction in the tradable sectors for the short run only, pointing out that in the long run this reduction is exceeded by the positive effects. Choueiri et al. (2008) study the effects of the EU transfers on the new state members. They detect that transfers to household’s income tend to deteriorate the balance of trade and decrease the ratio of tradable to non-tradable prices. Baskaran et al. (2017) find that intergovernmental transfers do not encourage economic growth in West German states over the period 1975-2005.

Figure 1: Industry and Income Transfers (2005-2014). Source: Eurostat

Most of these studies rely on the Dutch Disease (DD) literature for a possible explanation of the negative relation observed between transfers (aid flows) and the tradable sector. This literature first appeared to explain the de-industrialization process faced by the Netherlands as a consequence of the discovery of important gas reserves in the North Sea in the 1960s. Since then, the DD literature has spread to the study of other kinds of booms, like foreign aid (White, 1992; Nkusu, 2004; Selaya et al., 2010; Taguchi, 2017), income and fiscal transfers (Gabrisch, 1997; Breau et al., 2016), remittance flows (Bourdet et al, 2006; Chowdhury et al., 2014; Uddin et al., 2017), public expenditure (Adam et al., 2003), or capital inflows (Athukorala et al., 2003; Lartey, 2007; Moosa, 2017).

The basic models of the DD are the Salter-Swan model (Swan, 1960, 1963) or Salter-Swan-Corden-Dornbusch model (Corden and Neary 1982; Dornbusch, 1991; Corden, 1994) which consist of a small open economy with two tradable sectors (booming or
resource sector, and a lagging or manufacturing sector) and a non-tradable sector (service sector), with perfect competition in all of them. According to this literature, a technical improvement in the booming sector has two effects: the resource movement and the spending effects. The marginal product of labor increases in the booming sector, attracting labor at the expense of the other two sectors (resource movement effect). On the other hand, the extra income from the booming sector is spent partially in the non-tradable sector, which increases the price of non-tradable goods, and wages of the economy (spending effect). Thus, an appreciation of the real exchange rate takes place, and the country becomes less competitive in the international markets, so harming the tradable sectors (Corden and Neary, 1982; Corden, 1984; Van Wijnbergen, 1986; Krugman, 1987; Yano and Nugent, 1999). Noticeably, if instead of a technical improvement, the boom is a large windfall of economic resources, such as fiscal or income transfers, aid flows, remittances, public expenditure, or capital flows, only the spending effect is present.

The conclusions of the NEG and the DD literatures clash, at least in their theoretical developments. The aim of this paper is to study the effects of income transfers on the spatial distribution of the economic activity, by using key elements from these two literatures. A modified Footloose Entrepreneur Model (Forslid and Ottaviano, 2003) is developed, that considers some key features: income transfers, a non-tradable sector, sectorial mobility of labor, and a slightly differentiated agricultural good. One of the main results is that transfers are not always beneficial for the recipient region. Under some conditions they can exacerbate or even create regional disparities, rather than mitigate them.

Although the study of income transfers in NEG models is not widespread- probably because the results seems straightforward because of the market potential - there are some interesting works in the field. Baldwin et al. (2005) developed a footloose capital (FC) model that considers income transfer between regions. They find that transfers
tend to boost industrial activity in the recipient region unless there are some differences between the endowments of labor and capital within each region. Additionally, there are some related works that study unproductive public expenditure in a NEG framework. In these models, the public expenditure is devoted to consumption goods in order to analyze the market potential (see Commendatore et al., 2018 for a survey on productive and unproductive public expenditures in NEG models). When these public expenditures are “liberalized” the public sector of one region can purchase goods produced in the other region, which can be seen as a transfer of incomes between the regions. Trionfetti (1997) proposed a Core-Periphery model (Krugman, 1991) with unproductive public expenditure. When he considers the case of transfers between regions, the demand for industrial goods produced in the recipient region increases due to the higher income in that region, but it also reduces due to the fall in the foreign demand. If the net result is an increase (resp. decrease) in the demand, the number of firms in the recipient region would increase (resp. decrease) because of what he called the pull effect, which is the result of a higher market potential. Trionfetti (2001) extends the NEG model proposed by Krugman and Venables (1995) by incorporating unproductive public procurements. He finds that liberalized public expenditure is irrelevant in determining the industrialization pattern between the regions. Brülhart et al. (2004) show that an economy with a large home-biased public expenditure on its own industry, which can be seen as a net recipient of transfers, tends to have a larger number of firms (pull effect), and that it also reduces the likelihood of industrial agglomeration in the other economy (spread effect). All these results are in line with the standard predictions of the NEG models: a higher income or demand increases the market potential and attracts more industrial firms to the net recipient region. However, none of these works tries to explain how the effects of transfers described by the new economic geography and the Dutch Disease literatures interact.

One of the reasons why NEG models do not usually incorporate the effects described in the Dutch Disease literature could be that some of the main assumptions need to be
relaxed. For example, in the original Core-Periphery (CP) model (Krugman, 1991) sectorial mobility should be included. This simple change leads to many difficulties. Because of sectorial mobility, in the long run the whole population could move to the other region, and the periphery would be completely unpopulated. Additionally, the competition effect becomes weaker as the fixed market disappears. Furthermore, if after incorporating sectorial mobility, the agricultural sector remains unchanged, the agriculture price equalization would imply wage equalization within and between regions, which overrides the spending effect. The Footloose Capital (FC) and Footloose Entrepreneur (FE) models (Martin and Rogers, 1995; Forslid and Ottaviano, 2003) already incorporate labor mobility between sectors. Nevertheless, to obtain results similar to a Dutch Disease, the agricultural sector needs to be modified (or discarded) in order to avoid wage equalization. However, the equalization of wages across sectors and regions is what makes them tractable.

In this paper, a modified FE model (Forslid and Ottaviano, 2003) that considers some of the assumptions made by the DD literature is presented. First, a non-tradable sector with constant returns to scale is incorporated, which is a key element of the DD mechanism (Corden et al., 1982). Second, following Fujita et al. (1999), it is assumed that the agricultural goods are homogenous within each region, but slightly differentiated between regions. This assumption is made to avoid inter-regional wage equalization, which allows differences between non-tradable prices of the regions. And third, inter-regional transfers are allowed. A particular feature of the setup of the model is that each sector has a different "transport cost": agricultural goods are freely tradable, industrial goods face an iceberg transport cost, and services are non-tradable.

A similar model to the one proposed in this paper can be found in Moncarz, et al. (2017). The authors study how intergovernmental transfers affect manufacturing production in an FC model. However, there are same important differences between our model and theirs. First, in addition to the non-tradable sector, they incorporate a public sector whose only mission is to hire workers, so competing with the private sector for the
labor force. Second, they remove the agricultural sector. And third, all their results rely on numerical simulations. The model developed in this paper presents a more general structure by maintaining the agricultural sector. This difference becomes important in the short-run analysis. Additionally, analytical results are obtained which give a better understanding of the links between the two literatures, and the effects of transfers in the short and long run. Finally, an FE model is used instead of an FC model. This allows the study of transfers in the case of either stable or unstable solutions.

Another work that brings the DD effect into a NEG model is Takatsuka et al. (2015), who study the impact of a resource boom in the distribution of the economic activity by introducing a different natural resource in each region (so avoiding wage equalization) that is used as an input in the industrial production and as a final consumption good. The DD in Takatsuka et al. (2015) appears due to a shock in the demand (final or intermediate) for resource goods of one of the regions, which draws labor from the industrial sector and increases the wage in that region. On the other hand, as long as the resource good is also used as an industrial input, the firms in the region that experience the boom have an advantage because they are closer to the source of their main input. In the model proposed in this paper, the shock comes in the form of an increase in the disposable income without changing the preferences of households and without imposing any assumption about a preference for one good over the others. Additionally, the model presented here is a modified FE model, as opposed to the static model of Takatsuka et al. (2015), allowing the differentiation between the short run, when firms can adjust their level of production, and the long run, when migration of entrepreneurs is allowed. As pointed out, the FE model permits the study of stable and unstable solutions, which is not possible with a static model.

It is found that income transfers play a double role in the model. On the one hand, the increase in the disposable income of the recipient region attracts firms, due to the market size effect, in accordance with the NEG literature predictions. On the other, the
expenditure shock increases the wages of workers, making industrial production more expensive, which shrinks the industrial activity, as explained by the DD literature.

Although the proposed model has large open economies and monopolistic competition in the industrial sector, unlike to standard DD models, due to the difference in the transport costs of the sectors, a DD can occur. In the terminology of the DD literature, the increase in the disposable income of the recipient region causes a spending effect in the service sector, as expected, but also in the industrial sector. Income transfers increase the market potential of the recipient region where, at the same time, transport costs make the local industrial goods more attractive, leading to a rise in their demands. The wage, and the industrial and service prices rise, so lowering the competitiveness and shrinking all the sectors of the recipient region. A DD emerges if the effect of the higher prices offsets the spending effect in the industrial sector (market size effect).

In particular, for the short run the model suggests that the agricultural and the non-tradable sectors shrink and expand, respectively, while de-industrialization takes place if the trade openness is high. In this case, because of the high competition from foreign firms, the benefits of the transfers to the local industry are limited. In the long run, however, the net changes in wages and in cost of living favor the recipient region. Thus, if the transport costs are high, the recipient region can end up attracting industrial firms, even if in the short run some de-industrialization has taken place. But, if the foreign competition is strong (high trade openness), the Dutch Disease, which took place in the short run, can overcome all other positive effects derived from the transfers, leading to a long-run DD.

Additionally the DD outcomes in the long run can take two different forms. The first is the more "standard" de-industrialization, where some firms decide to abandon the recipient region due to the higher production costs. The second emerges as the result of the interaction of NEG forces with the DD. When economies of scale are large, the transfers could perpetuate a core-periphery structure, where the recipient region is a
rural periphery. Thus, income transfers can create or even exacerbate regional disparities rather than mitigate them.

The remainder of the paper is organized as follows: section 2 introduces the model; section 3 studies the short run and the effect of transfers on the regional economic structure; section 4 analyses the long run and the effect of transfers on the location of industry; section 5 concludes.

2 The Model

Following Forslid et al. (2003) an FE model is used, with two regions \((j = 1, 2)\) and three sectors: industrial, agricultural and services. As in the original FE model, there are two types of population. Entrepreneurs \((H_j)\), which are mobile between regions; and workers \((L_j)\) which can not move between regions, but can move freely between sectors within the same region (this is also a feature of the original model).

The industrial sector has increasing returns to scale, with a fixed cost in entrepreneurs and variable cost in workers.\(^4\) There is monopolistic competition (Dixit and Stiglitz, 1977); and goods are tradable between regions with iceberg transport costs. The agricultural sector has constant returns to scale, and only employs workers. Following Fujita et al. (1999), there is perfect competition within the region, but products are slightly differentiated between regions, i.e., the regions produce different types of agricultural goods. The reason for this assumption is to avoid wage equalization of workers.\(^5\) While the CP model assumes no mobility between sectors, and the FE model allows sectorial mobility, but with equalized wages, the proposed model incorporates the mobility and avoids equalization of wages. Additionally, agricultural goods are freely tradable between the regions. The service sector, or non-tradable sector, also has constant returns to scale, employing only workers. There is perfect competition, and services are non-tradable between regions.\(^6\)
Finally, there is a supra-regional authority whose only function is to collect taxes and assign transfers between the regions. This authority maintains a balanced budget.

2.1 Households

Households seek to maximize their utility, which has the form of a nested Cobb-Douglas (across sectors) and CES (over the varieties) used in the original Krugman model (1991). Thus, a representative household in region 1 solves the following consumption problem,

$$\max_{c_1,c_2,c_{A1},c_{A2}} U_1 = C_{M1}^{\mu_1}C_{s1}^{1-\mu_1-\mu_2}C_{A1}^{\mu_2}$$

s.t. $$y_1^d = \int_0^{n_1} c_{1i}p_1 idi + \int_0^{n_2} c_{2i}p_2i \tau di + \frac{1}{2} c_{A1}p_{A1} + \frac{1}{2} c_{A2}p_{A2} + C_{s1}p_{s1}$$

with

$$C_{M1} = \left( \int_0^{n_1} c_{1i}^{\frac{\sigma-1}{\sigma}} d\frac{1}{\sigma} + \int_0^{n_2} c_{2i}^{\frac{\sigma-1}{\sigma}} d\frac{1}{\sigma} \right)^{\frac{\sigma}{\sigma-1}}$$

$$C_{A1} = \left( \frac{1}{2} c_{A1}^{\frac{\sigma-1}{\sigma}} + \frac{1}{2} c_{A2}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where $$C_{M1}$$ and $$C_{A1}$$ are consumption indexes of industrial and agricultural goods respectively; $$C_{s1}$$ is the consumption of services; $$c_{ji}$$ is the consumption of variety $$i$$ produced in region $$j$$; $$n_j$$ are the number of varieties in region $$j$$; $$c_{A_j}$$ is the consumption of agricultural good produced in region $$j$$; $$y^d_j$$ is the disposable income per household; $$p_{ji}$$ is the (fob) price of each industrial good; $$p_{s1}$$ is the price of the services in region 1; $$p_{A_j}$$ is the price of the agricultural good produced in region $$j$$; $$\mu_1 \in (0, 1), \mu_2 \in (0, 1)$$ are the proportions of the disposable income devoted to expenditure in industrial goods and services; $$\sigma > 1$$ is the elasticity of substitution of the industrial and agricultural goods (which are assumed to be equal for analytical purposes); and $$\tau > 1$$ is the iceberg transport cost. The same problem is solved by households in region 2.
From the first order conditions of the maximization problem (1)-(4), the following demand functions are obtained:

\[ C_M = \mu_1 \frac{y_j}{P_1}, \quad C_s = \mu_2 \frac{y_j}{p_{s1}}, \quad C_A = (1 - \mu_1 - \mu_2) \frac{y_j}{P_A} \] \hspace{1cm} (5)

\[ c_{1i} = C_M \left( \frac{p_{1i}}{P_1} \right)^{-\sigma}, \quad c_{2i} = C_M \left( \frac{p_{2i}}{P_1} \right)^{-\sigma} \] \hspace{1cm} (6)

\[ c_{A1} = C_A \left( \frac{p_{A1}}{P_A} \right)^{-\sigma}, \quad c_{A2} = C_A \left( \frac{p_{A2}}{P_A} \right)^{-\sigma} \] \hspace{1cm} (7)

where \( P_1 \) and \( P_A \) are the price indexes for region 1, that is,

\[ P_1 = \left( \int_0^{n_1} p_{1i}^{1-\sigma} di + \int_0^{n_2} (p_{2i})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \] \hspace{1cm} (8)

\[ P_A = \left( \frac{1}{2} p_{A1}^{1-\sigma} + \frac{1}{2} p_{A2}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \] \hspace{1cm} (9)

Mirror-image formulas hold for consumers in region 2. Additionally, because agricultural goods are assumed to be freely tradable between regions, the agricultural price index is the same for both regions.

### 2.2 Agricultural Sector

The agricultural good is produced with constant returns to scale in perfect competition. It is assumed that one unit of labor is required to produce one unit of agricultural good. Due to the free entry condition profits of a firm \( i \) in country \( j \), \( \pi_{ij} \), must equal zero, with

\[ \pi_{Aji} = p_{A_j} A_{ji} - w_j l_{A_j} \]

then,

\[ p_{A_j} = w_j \]

where \( w_j \) is the nominal wage paid to workers in region \( j \); \( A_{ij} \) is the production of each firm in region \( j \); and \( l_{A_j} \) is the labor employed by each agricultural firm. Because we assume
that the number of agricultural firms in each region is equal to 1/2, total agricultural employment and production in each region is $L_{A_j} = l_{A_j}/2 = A_j = A_{ij}/2$. Note that the mobility of workers between regions equalizes nominal wages within each region.

2.3 Industrial Sector

A firm $i$ in the industrial sector of region $j$ employs workers ($l_{xji} = \beta x_{ji}$) and a fixed amount of entrepreneurs ($f$) to produce industrial goods, $x_{ji}$. The resulting cost function involves a constant marginal cost and a fixed cost, giving rise to increasing returns to scale.

$$\text{Cost} = fw_{H_j} + (\beta x_{ji})w_j$$

where $w_{H_j}$ is the wage of the entrepreneurs in region $j$ to produce variety $i$, and $x_{ji}$ is the output.

It is assumed that there is a large number of manufacturing firms, each producing a single product in monopolistic competition. Given the definition of the manufacturing aggregate (3), the elasticity of demand facing any individual firm is $-\sigma$. Then, the profit-maximizing price behavior of a representative firm in region $j$ is

$$p_{ji} = \beta \frac{\sigma}{\sigma - 1}w_j$$

Since firms are identical and they face the same wage, manufactured good prices are equal for all varieties in each region and the superscript $i$ can be dropped. Similar equations apply in region 2. Comparing the prices of representative products, yields

$$\frac{p_1}{p_2} = \frac{w_1}{w_2}$$

Because there is free entry in the sector, a firm’s profits must equal zero. Using this
condition, the price rule (11) and $l_{x_{ji}} = \beta x_{ji}$, it is obtained that

$$x^*_{ji} = x^*_j = \frac{(\sigma - 1) f}{\beta} \left( \frac{w_{H_j}}{w_j} \right) = \frac{f w_{H_j}}{p_j / \sigma}$$

(13)

$$l^*_{x_{ji}} = l^*_j = (\sigma - 1) f \left( \frac{w_{H_j}}{w_j} \right)$$

(14)

The output and labor employed per firm is the same in each region, so the subscript $i$ can be dropped. Note that the number of firms multiplied by the entrepreneurs per firm must equal the total number of entrepreneurs available in the region. Also, the number of firms multiplied by the workers per firm must equal the labor employed in the industrial sector. So, the number of firms must be

$$n_j = \frac{H_j}{f} = \frac{L_{E_j}}{(\sigma - 1) f} \left( \frac{w_j}{w_{H_j}} \right)$$

(15)

and

$$w_{H_j} = \frac{L_{E_j}}{H_j} \frac{w_j}{\sigma - 1}$$

(16)

where $L_{E_j} = \int_0^{n_j} l_{x_{ji}} di$ is the aggregate labor employed in the industrial sector of region $j$. The last expression (16) is the operating profit of each entrepreneur of a representative firm.

### 2.4 Service Sector

As in the agricultural sector, services are produced with constant returns to scale in perfect competition. One unit of labor is required to produce one unit of services. Because of the free entry condition the price of the services is

$$p_{s_j} = w_j$$

(17)
3 Short-Run Equilibrium

In equilibrium, households maximize their utility, firms maximize their profits, there is free entry in all sectors, the supra-regional authority maintains a balanced budget, and market clearing conditions hold for the four markets: labor, agricultural goods, industrial goods and services.

The balanced budget of the supra-regional authority can be written as

\[ BB = t_1 Y_1 + t_2 Y_2 - T_1 - T_2 = 0 \]

where \( t_j \) is the tax rate imposed on households of region \( j \), and \( T_j \) is the income transfer received by households of region \( j \).

For simplification, it is assumed that only region 1 pays taxes, and only region 2 receives transfers, such that

\[ t_1 = t \in (0, 1) \text{ and } t_2 = 0 \]  \hspace{1cm} (18)
\[ T_1 = 0 \text{ and } T_2 = tY_1 \]  \hspace{1cm} (19)

Thus, \( t \) is the tax rate paid by region 1, or the rate of transfers received by region 2.

**Agricultural market:** total supply must equal total demand, aggregating the demand functions (7) and using the total output of the agricultural sector \( A_j = L_{A_j} \),

\[ L_{A_j} = \frac{1 - \mu_1 - \mu_2}{2} \left( \frac{P_{A_j}}{P_A} \right)^{1-\sigma} \frac{(Y_1^d + Y_2^d)}{w_j} \]  \hspace{1cm} (20)

**Service market:** total supply must equal total demand. Using \( C_{s_1} \) from expression (5),

\[ L_{s_j} = \mu_2 \frac{Y_j^d}{w_j} \]  \hspace{1cm} (21)
where $L_{sj}$ is the total labor employed in the non-tradable sector of region $j$.

**Labor market:** as a result of the free labor mobility assumption between the three sectors, and by using equations (20) and (21), the labor market clearing condition states that

$$L_j = L_{Ej} + L_{Aj} + L_{sj}$$  \hspace{1cm} (22)

$$L_{Ej} = L_j - \frac{(1 - \mu_1 - \mu_2)}{2} \left( \frac{p_{A1}}{P_A} \right)^{1-\sigma} \left( \frac{Y_1^d + Y_2^d}{w_j} \right) - \mu_2 \frac{Y_j^d}{w_j}$$  \hspace{1cm} (23)

**Industrial market:** Using the demand equations (6), the industrial price index (8), the equilibria for both industrial sectors are

$$x_1^* = \mu_1 p_1^{-\sigma} \left( \frac{Y_1^d}{P_1^{1-\sigma}} + \frac{Y_2^d}{P_2^{1-\sigma} \tau^{1-\sigma}} \right)$$  \hspace{1cm} (24)

$$x_2^* = \mu_2 p_2^{-\sigma} \left( \frac{Y_1^d}{P_1^{1-\sigma} \tau^{1-\sigma}} + \frac{Y_2^d}{P_2^{1-\sigma}} \right)$$  \hspace{1cm} (25)

Using equations (13)-(16), (18)-(19), and (20)-(23) the previous two equations can be reduced to the single one:

$$CA_2 \equiv \mu_1 \left[ \frac{\phi n_2 p_2^{1-\sigma} Y_1^d}{P_1^{1-\sigma}} - \frac{\phi n_1 p_1^{1-\sigma} Y_2^d}{P_2^{1-\sigma}} \right] + \frac{(1 - \mu_1 - \mu_2)}{2P_A} \left[ \frac{Y_1^d}{p_{A2}^{\sigma-1}} - \frac{Y_2^d}{p_{A1}^{\sigma-1}} \right] + \left( tY_1 \right) = 0$$  \hspace{1cm} (26)

where $\phi \equiv \tau^{1-\sigma} \in (0, 1)$ is an index of openness. Equation (26) guarantees that the current account of region 2 is balanced. In the first square brackets industrial exports and imports of region 2 are exhibited. Within the second square brackets are the agricultural exports and imports respectively of region 2. And the value of the net transfers received by region 2 are in the last brackets. Note that the current account for region 1 is $CA_1 = -CA_2$.

Regarding the incomes, on normalizing $L_1 + L_2 = 1$ and $H_1 + H_2 = 1$, total regional
incomes are

\[ Y_1 = H w_H + L w_L \]  \hspace{1cm} (27)  
\[ Y_2 = (1 - H) w_H + (1 - L) w_L \]  \hspace{1cm} (28)

where, to simplify notation, the subscript 1 is dropped, such that, \( H_1 = H \) and \( L_1 = L \), while

\[ Y_{1d} = (1 - t) Y_1 \quad \text{and} \quad Y_{2d} = Y_2 + t Y_1 \]  \hspace{1cm} (29)

Equation (26), together with expressions (27)-(29), implicitly defines the ratio of nominal wages \((w \equiv w_1/w_2)\) as a function of \( H \) (see Proof of Proposition 1 at the Appendix).

**Proposition 1** The current account equation (26) defines a positive relation between the proportion of entrepreneurs, \( H \), and the ratio of nominal wages, \( w \).

**Proof.** See the Appendix. \( \blacksquare \)

As the number of firms increases in one of the regions, labor demand rises, and the competition among the firms for labor causes a rise in nominal wages.

### 3.1 Income transfers in the short-run

The aim of this section is to understand what happens to the regions in the short-run, when income transfers increase. In particular, it seeks to understand how the productive structures of the regions change if \( t \) increases. The effect of the rate of transfers over nominal wages is established in the following proposition.

**Proposition 2** An increase in the tax rate of region 1, that is, an increase in the rate of income transfers, \( t \), tends to diminish the ratio of nominal wages, \( w \), for each value of \( H \), and increase the total transfer received by region 2, \( T_2 \).
Proof. See the Appendix. ■

The change in the ratio of nominal wages comes through two channels or effects: a spending effect on the service sector (as in the DD literature) and a spending effect on the industrial sector (the market size effect in the NEG literature). First, looking at expressions (27)-(29), an increase in the rate of transfers tends to raise the disposable income of region 2, and reduce the disposable income of region 1. This causes a trade imbalance (commercial deficit) in the first two terms of current account equation (26), while transfers increase in the third term. However, part of the expenditure is devoted to non-tradable goods, which implies that, in region 1, the trade surplus is not enough to pay the transfers. The current account deficit in region 1 causes a downward pressure on prices and wages. This is the spending effect (in the service sector) from the DD literature, and depends on the existence of a non-tradable sector ($\mu_2 > 0$).\(^8\)

Second, because of transport costs, households have a preference for locally produced goods. Then, when an income transfer takes place, the industrial sector of region 2 gains more from the higher disposable income of region 2 than it loses from the lower disposable income of region 1. The opposite happens in region 1. As a consequence, the industrial trade imbalance is higher than in the case of costless trade. This additional spending effect on the industrial sector is the market size effect from the NEG literature, and depends on the existence of transport costs ($\phi < 1$).\(^9\)

In addition, the second part of Proposition 2 has important implications. On the one hand, total transfers ($T_2 = tY_1$) increase in spite of the reduction in the wage ratio that diminishes income of region 1. On the other hand, disposable income of region 2 increases because of the change in the wage ratio and also because of the increase in transfers.

Now the short-run effect of income transfers can be addressed. The focus is put on region 2, the recipient region, while all changes in region 1 are equal, but of the opposite sign. This can be analyzed in two steps.
In the first step, when $t$ increases, the transfers received by region 2 rise, which implies a positive shock on $Y^d_2$. In the industrial goods markets, the changes in the disposable incomes cause a decrease in the demand of the industrial goods produced in region 1 and an increase in the ones produced in region 2. This is due to the existence of transport costs. In the non-tradable sector, demand increases in region 2 and decreases in region 1. No change is observed in the agricultural sector. Because agricultural goods are not subject to transport costs, the increase in the households’ demand of region 2 is completely compensated by a decrease in households’ demand of region 1.

However, these new equilibria in each of the good markets imply an excess of labor demand in region 2. Thus, in the second step, in order equilibrate the labor market, wages of region 2 rise (a decrease of $w$). As region 2 wages go up, supply in each good market contracts, until all markets are again at equilibrium. Thus, at the new equilibrium the nominal wage ratio ($w$) is lower, and income transfers ($T_2$) are higher.

Additionally, as anticipated, the productive structure of the regions also changes. Clearly, the agricultural sector of region 2 shrinks. The case of the non-tradable sector is less evident. However, because the employment on the non-tradable sector depends only on the disposable income, this sector expands (see the proof of Proposition 2 in the Appendix). Non-tradable goods do not face any competition from foreign goods, thus they benefit from the large demand expansion.

Finally, the industrial sector is the most difficult to explore analytically. The following proposition states the effect of transfers on the industrial sector in the symmetric equilibrium, which is defined as

$$L = 1 - L = 1/2, \quad H = 1 - H = 1/2, \quad t = 0 \text{ and } w = 1 \quad (30)$$

Note that the symmetric equilibrium is a solution of equation (26).
Proposition 3  At the symmetric equilibrium, if the rate of income transfers, \( t \), increases marginally from zero, then there exists a value \( \phi^{sr} \) such that the industrial sector will:

i) shrink in region 1 (decrease of \( L_{E_1} \)) and expand in region 2 (increase of \( L_{E_2} \)) for low trade openness (\( \phi < \phi^{sr} \));

ii) expand in region 1 (increase of \( L_{E_1} \)) and shrink in region 2 (decrease of \( L_{E_2} \)) for high trade openness (\( \phi > \phi^{sr} \)).

Proof. See the Appendix.

These results are in line with the arguments made before. The lower the trade openness is, the lower is the trade between regions and the foreign competition. As a consequence, the reduction in the disposable income of region 1, \( Y_{d1} \), has a minor negative effect on the demand of industrial goods produced in region 2. Thus, the majority of the transfer received by region 2 is spent locally. The result is a large expansion of the demand for industrial goods produced in region 2. The subsequent contraction of the supply in this region does not manage to reverse this initial expansion. In the extreme case of a closed economy (\( \phi = 0 \)), the industrial sector behaves like the non-tradable sector.

The opposite happens when the trade openness is high. The demand expansion is limited because of the large foreign competition. Meanwhile, the excess of labor demand puts pressure on wages to rise (in region 2), causing a contraction in the supply, and as a consequence of this, the industrial sector shrinks. In the extreme case of free trade (\( \phi = 1 \)), the industrial sector, as a whole, behaves like the agricultural sector.

The other parameters of the model also give interesting insights on the effects of transfers in the short run. The following proposition states the relation between the shrink/expansion of the industrial sector, with the size of the non-tradable sector and the elasticity of substitution.
Proposition 4 \textit{The }\phi^{sr}\textit{ threshold diminishes,}

\begin{itemize}
  \item[i)] as the proportion of disposable income devoted to the consumption of non-tradable goods, \( \mu_2 \), increases;
  \item[ii)] and, as the elasticity of substitution, \( \sigma \), increases.
\end{itemize}

\textbf{Proof.} See the Appendix. \( \blacksquare \)

The larger \( \mu_2 \), the greater the impact of the demand shock on the non-tradable sector. In region 2, the positive demand shock benefits the non-tradable sector more in the first place, while the increase in the demand for industrial goods is going to be smaller. Then, in order to avoid deindustrialization in this region, lower competition is needed to ensure that the supply contraction does not reverse the weak demand expansion. The opposite happens in region 1.\(^ {10} \)

On the other hand, if there is no service sector (\( \mu_2 = 0 \rightarrow \phi_{(\mu_2=0)}^{sr} = 1 \)), the results are straightforward. In region 2, the positive demand shock affects only the industrial sector, which generates an upward pressure on the wages of the region. As wages in region 2 increase, the supply of agricultural goods and industrial goods contracts (prices go up) until a new equilibrium is reached. The agricultural sector must shrink, so the industrial sector must expand (this is true at all the equilibria, not only at the symmetric one). In region 1, the agricultural sector always expands, so the industrial sector always shrinks.

Additionally, a high elasticity of substitution, \( \sigma \), reduces the threshold \( \phi^{sr} \) and hence increases the range of values of the trade openness for which a short-run DD takes place. A higher elasticity implies that consumers are more willing to substitute the consumption of one variety for another. In this scenario, the rise in the price of the recipient region due to the transfers shifts the consumption in favor of the industrial production of region 1 more than if \( \sigma \) were low. Thus, to avoid this shift overcoming the positive demand
shock of transfer, higher barriers to foreign competition are needed, that is, lower trade openness.

Moreover, even when the industry expands, an income transfer policy makes the economy of the recipient region more dependent. The region becomes more industrialized at the expense of agriculture. Nevertheless, the overall base economic activities (agriculture plus industry) face a contraction. The income generated by the inter-regional trade diminishes (total net export falls) as a consequence of the shifts operating in the economic structure of the region. The trade deficit needs the transfers to maintain the equilibrium of the current account.

To summarize the results of this subsection, because each sector has different levels of trade openness, when an income transfer policy is applied, independently of the name given to each of the sectors: i) the one with highest competition (high trade openness) faces a contraction; ii) the one with lowest competition (low trade openness) experiences an expansion; and iii) the sector with intermediate competition (intermediate trade openness) can shrink or expand depending on the strength of the spending effect on the non-tradable sector and the market size effect.

4 Dynamics and Long Run

Entrepreneurs are mobile between regions and they choose to migrate if they gain in terms of real profits from doing so. The entrepreneurs reallocation is driven by the following dynamics:

\[ H = H (1 - H) \left( \frac{V_1}{V_2} - 1 \right) \]

(31)

where,

\[ V_j = \frac{w_{H_j}}{P_j^{\mu_1} P_j^{\mu_2} P_A^{1-\mu_1-\mu_2}} \]

(32)

is the indirect utility (real profits) of an entrepreneur in region \( j \).
Transfers (and taxes) are deliberately excluded from the real profits in (32). The reason behind this is that only the effect of income transfers is under study. If they were included in the real profits there would be an additional effect to consider: a tax/subsidy policy directly apply to industrial firms.

Using equations (8)-(9), (15), (16), and \( w_1 = w_2 = p_1/p_2 \), the entrepreneurs dynamics can be restated as

\[
\dot{H} = H (1 - H) \left\{ \frac{L_{E_1}}{L_{E_2}} \frac{1 - H}{H} w^{1-\mu_2} \left[ \frac{H w^{1-\sigma} + (1 - H) \phi}{H \phi w^{1-\sigma} + (1 - H)} \right]^{\mu_1} - 1 \right\}
\]  

(33)

All interior solutions of equation (33) must satisfy: \( 0 < H < 1 \) and \( \dot{H} = 0 \). That is, the ratio \( V \equiv V_1/V_2 \) is

\[
V(H, w) = \frac{L_{E_1}(w)}{L_{E_2}(w)} \frac{1 - H}{H} w^{1-\mu_2} \left[ \frac{H w^{1-\sigma} + (1 - H) \phi}{H \phi w^{1-\sigma} + (1 - H)} \right]^{\mu_1} = 1.
\]  

(34)

where \( w \) satisfies equation (26). Note that the symmetric equilibrium (30) is also a solution of equation (34). Additionally, since the number of entrepreneurs per firm, \( f \), is equal for both regions, the ratio (34) is also the ratio of the real operating profits of firms.

4.1 Equilibria and Stability

A long-run equilibrium is a stationary point of the dynamic equation (33), where entrepreneurs do not have incentives to move from one region to the other. For analytical simplicity the stability properties are studied around the symmetric equilibrium (30), recalling that the conclusions are valid in a close neighborhood of this equilibrium.

The black-hole-condition (BHC: \( d \equiv \frac{\mu_1}{\sigma - 1} \geq 1 \)) and the no-black-hole-condition (NBHC: \( d \equiv \frac{\mu_1}{\sigma - 1} < 1 \)) defined for the original FE model (Forslid and Ottaviano, 2003), are used only for classification purposes. However, contrary to the original model, in this modified
FE model there always exists a range of values for \( \phi \) such that the dispersion equilibrium is stable, even when the BHC holds. The following proposition states the stability properties of the symmetric equilibrium (30).

**Proposition 5** The symmetric equilibrium presents the following stability properties:

i) when the black hole condition holds (\( d \geq 1 \)), there exists a threshold for the trade openness (\( \phi^r \)), such that, the symmetric equilibrium is unstable for \( \phi \in (0, \phi^r) \), and stable for \( \phi \in (\phi^r, 1) \);

ii) when the no black hole condition holds, and \( \bar{d} < d < 1 \), there exist two thresholds for the trade openness (\( \phi^b \) and \( \phi^r \)), such that, the symmetric equilibrium is stable for \( \phi \in (0, \phi^b) \), is unstable for \( \phi \in (\phi^b, \phi^r) \), and is stable for \( \phi \in (\phi^r, 1) \);

iii) when the no black hole condition holds and \( d \leq \bar{d} \), the symmetric equilibrium is stable for all \( \phi \in (0, 1) \).

**Proof.** See the Appendix.

If the NBHC holds with intermediate economies of scale (\( \bar{d} < d < 1 \)), for high transport costs the market crowding effect dominates and industrial activity is disperse. For intermediate levels of transport costs, agglomeration takes place due to the market size and the cost of living effects. And for low levels of transport costs, a new dispersion phase arises. This last dispersion phase takes place due to the competition for the limited labor supply, also known as the factor market competition (Ottaviano and Puga, 1998). The peripheral region has lower labor costs due to the absence of industrial firms. Then, if an individual firm decides to move to this region, it can take advantage of the lower costs. Nevertheless, this firm will have to face transport costs in order to reach the larger market. When the transport costs are low enough, the decision to move will result in a positive profit, and more firms will be willing to abandon the larger market in order to
benefit from the combination of low transport and labor costs, so generating a dispersion of the industrial activity.

If the BHC holds \( (d \geq 1) \), for high and intermediate values of transport costs, agglomeration forces dominate (black hole, in the original FE model). However, for low transport cost the factor market competition exceeds the agglomeration forces and dispersion becomes stable. At the other extreme, when the NBHC holds and economies of scale are very low \( (d \leq \bar{d} < 1) \) agglomeration forces are too weak compared, first, to the market crowding effect, and second, to the factor market competition, after. As a result, the symmetric equilibrium is stable for all values of the trade openness \( (\phi) \).

These three cases are depicted in Figure 2, where the solid lines indicate the stable equilibria, and the dashed lines indicate the unstable equilibria. The corresponding vector fields are also plotted in the same figure for further illustration of the stability properties. Additionally, Figure 2 also shows that when the symmetric equilibrium is unstable two other stable agglomeration equilibria appear, while the basins of attraction are delimited by the unstable equilibria.

Figure 2: Bifurcation Diagrams in the space \((\phi, H)\)

4.2 Income transfers in the long run

This section addresses the effect of income transfers when entrepreneurs are allowed to migrate. Instead of asking how workers move from one sector to the other, the question here is: how do firms relocate when transfers are applied? For the sake of simplicity a marginal increase in \( t \) from zero around the symmetric equilibrium is analyzed. Again, the properties derived for the symmetric equilibrium hold in a close neighborhood of this equilibrium.
Four possible cases may arise as $t$ increases marginally from zero. When the equilibrium is stable the number of entrepreneurs, $H$, can increase ($SH^+$) or decrease ($SH^-$); and when the equilibrium is unstable, $H$ can also increase ($UH^+$) or decrease ($UH^-$).

In the context of the model the aim of the income transfers is to favor the recipient region (region 2) by increasing its number of industrial firms. This is what happens in case $SH^-$, while the case $SH^+$ can be clearly defined as a DD outcome in the long run, since the transfers are pushing out firms from region 2 to region 1. The other two cases, $UH^+$ and $UH^-$ take place when the equilibrium is unstable. The change in $H$ when the symmetric equilibrium is unstable can be seen as a movement of the boundary that defines the basin of attraction of the agglomeration equilibria. In the case $UH^+$ the basin of attraction of the agglomeration equilibrium in region 2 widens, so the recipient region benefits from the income transfers. On the other hand, in case $UH^-$, region 1 benefits from an increase in the basin of attraction of its agglomeration equilibrium, thus, this case can also be defined as a long-run DD outcome.

**Proposition 6** At the symmetric equilibrium, when $t$ increases marginally from zero, there exists a value $\phi_{lr} \in (0, 1)$ such that:

i) When $\phi < \phi_{lr}$, region 2 benefits from the income transfer ($SH^-$ or $UH^+$)

ii) When $\phi > \phi_{lr}$, region 2 experiences a long-run DD outcome ($SH^+$ or $UH^-$).

where $\phi_{lr}$ is defined by expressions (77)-(79) and (82) in the Appendix.

**Proof.** See the Appendix. ■

Proposition 6 states that transfers can initially be beneficial for the recipient economy but, as the trade opens, the increase in the wages of the region can deteriorate the competitiveness of its tradable sectors and harm the industrial development. And, as long as the agricultural sector is not too small, the higher the elasticity of substitution,
the higher the range of $\phi$ for which region 2 would experience a long-run DD, as in the short run (see proof of Proposition 6 in the Appendix).

The mechanism behind the results of Proposition 6 can be disentangled by studying the effects of transfers in the ratio of real operating profits through expression (35). In a close neighborhood of the symmetric equilibrium (30), if $\phi < \phi^{lr}$ transfers tend to reduce this ratio, which favors the recipient region, and if $\phi > \phi^{lr}$ transfers tend to increase it, leading to a DD outcome for the recipient region.

$$
\frac{dV}{dt} = \left[ \frac{dL_{E_1}}{dt} - \frac{dL_{E_2}}{dt} \right] + \left[ \frac{dw}{w} \right]_{CA_2=0} - \left[ \frac{\mu_2}{w} + \mu_1 \frac{\partial(P_1/P_2)}{P_1/P_2} \right] \frac{dw}{dt} \bigg|_{CA_2=0} 
$$

The overall effect of expression (35) can be decomposed into three elements. The expression in the first square brackets is the volume effect derived from a change in employment, that is, changes in the operating profits due to changes in the volume of production. The expression in the second square brackets is the effect of the price on the operating profits. And the expression in the third square brackets is the change in the cost of living: non-tradable price and industrial price index. Moreover, the changes in the labor force are the results of the changes already studied in the short run (Proposition 3). Thus, expression (35) not only determines the effect on the indirect utilities, but also summarizes the links between the short and the long run.

Additionally, as it has been pointed out before, the recipient region can benefit from a transfer policy or can experience a DD outcome in different ways depending on the stability of the symmetric equilibrium. The following corollary of Proposition 6 states when each case can be observed.

**Corollary 7** Due to the stability properties of the symmetric equilibrium,

i) Case $SH^-$ takes place when $\phi < \min \left[ \phi^{lr}, \phi^b \right]$ or $\phi^r < \phi < \phi^{lr}$.
ii) Case $SH^+$ takes place when $\phi > \max[\phi^{lr}, \phi^r]$.

iii) Case $UH^-$ takes place when $\max[\phi^{lr}, \phi^b] < \phi < \phi^r$.

iv) Case $SH^+$ takes place when $\phi^b < \phi < \min[\phi^{lr}, \phi^r]$.

Figures 3 (a) - (e) illustrate Corollary 7 in the space $(\mu_2, \phi)$, for different values of $d$ (see the Appendix for the analytical derivation of the figures). These figures show the stability and instability regions of the symmetric equilibrium, and they also depict the DD regions for the short and for the long run. The four cases of Corollary 7 emerge from the intersection of these regions.

Figure 3: Stability, Instability and Dutch Disease regions

When the economies of scale are low, Figures 3 (a) - (b), the only DD outcome is $SH^+$ where the number of firms decreases in the recipient region, creating regional asymmetries. Moreover, as long as the agricultural sector is not too small $(1 - \mu_1 - \mu_2$ not too small), lower economies of scale leads to a lower $\phi^{lr}$, which increase the DD regions in Figure 3 (see proof of Proposition 6 in the Appendix). This is somewhat a common DD scenario. However, as the economies of scale increase (low $\sigma$), Figures 3 (c) - (e), another DD scenario appears, $UH^-$. The unstable symmetric equilibrium moves downwards, increasing the size of the basin of attraction of the agglomeration equilibrium $H = 1$, helping to perpetuate a core-periphery structure.

Additionally, Figures 4 (a) and 4 (c) illustrate the DD outcomes in the space $(t, H)$, showing how, for a given $\phi$, an increase in the the rate of transfers could lead to regional disparities or to a wider basin of attraction of $H = 1$. The beneficial cases, $SH^-$ and $UH^+$, are also depicted in Figures 4 (b) and 4 (d) respectively.
Corollary 8 If $\bar{d} < d$, a marginally increase from zero of rate tax, $t$, leads to "broken" bifurcation diagrams.

Figure 5 illustrates the "broken" bifurcation diagrams that arise for $t = 0.05$ and the same parameters values used in Figure 5; $H = 1/2$ is also plotted (dotted line) as reference. The asymmetry introduced in the model by $t > 0$, eliminates the ambiguity of the core-periphery pattern, commonly interpreted as path dependency. While in the example of Figure 2 (b) which region becomes the core and which the periphery after $\phi^b$ is reached is unknown, this ambiguity disappears in Figure 5 (b). When the trade openness is low, the beneficial effect of transfers moves the stable interior equilibrium downwards, leading to an stable equilibrium where $H = 0$. However, for high trade openness, this equilibrium becomes unstable, and the interior equilibrium, with $H > 1/2$, is stable.

Moreover, the DD outcomes that appear in Figures 5 (a) - (c) are supported by the examples depicted in Figures 4 (a) and 4 (c). These last plots help to reject the idea that for high foreign competition (high trade openness), the recipient region needs higher level of transfers. In both DD examples depicted in Figure 4, a larger transfer rate only exacerbates the DD results.

Finally, Figures 6 (a) - (f) depict an example of initially asymmetric regions with $L = 0.6$. At the left of the figure there are no income transfers, and at the right a transfer policy is applied with $t = 0.05$. Because of the larger worker population, the interior stable equilibrium moves upwards in panels (a) - (c), leading to regional disparities or to a core-periphery structure, where, region 1 becomes the industrial core and region 2
the periphery unambiguously. These asymmetries could induce to the application of an income transfer policy such as the one depicted in panels (d) - (f). However, as pointed out before, only for high levels of transport costs are the transfers beneficial for region 2. As transport costs diminishes below some threshold, the recipient region experience a DD. Thus, the results and the economic intuition derived for the symmetric region case seem also valid for the case with asymmetric regions.

Figure 6: Initially Asymmetric Regions

5 Conclusions

This paper analyzes the effects of the implementation of transfers in a NEG model that incorporates some of the key features of the DD literature. A footloose entrepreneur model is modified by adding a non-tradable (service) sector. There is a supra regional authority that collects taxes and makes transfers from one region to the other exogenously. The main differences between the model proposed here and others from the NEG literature is the combination of the following elements: a non-tradable sector, labor mobility across sectors and slightly differentiated agricultural goods.

Under this setup, income transfers play a double role in the model. On the one hand, the increase in the disposable income of the recipient region causes a spending effect on the non-tradable sector, so increasing wages and making industrial production more expensive, as pointed out by the DD literature. On the other hand, the income transfers increase the market potential through the market size effect described by the NEG literature. A DD emerges if the disadvantages of the higher production costs (spending effect) offsets the advantages from a higher market potential (market size effect).
In the short-run, deindustrialization can take place for high values of trade openness. The high foreign competition limits the beneficial effects of the transfers, which are ultimately surpassed by the increase in the production costs. The last is a consequence of the competition between the industrial and the non-tradable sector for the labor force, which is fixed in each region.

In the long run, the net changes in the prices (wages) and in the cost of living favor the recipient region by increasing industrial nominal and real profits. If there is no de-industrialization in the short run, firms will move to the recipient region. If there is de-industrialization in the short run, the recipient region will only end up attracting firms if the positive effects of prices and the cost of living are stronger than the DD effect. To ensure this, low foreign competition is needed. But, if the competition is very high, the transfers will create or even exacerbate regional disparities instead of reducing them.

Additionally, for sufficiently high economies of scale, there is a range of trade openness for which the symmetric equilibrium becomes unstable. Then, a transfer policy could give rise to an expansion of the basin of attraction of the agglomeration equilibrium in the donor region, which can be seen as a different type of DD. In this case, the transfers tends to perpetuate a core-periphery structure.

It is also found that short-run results, which are associated with the Dutch Disease literature, condition the long-run results, which are associated with the New Economic Geography literature. The contribution of the DD literature to the NEG literature is that wage adjustments are important and that transfers have not always positive effects for the regional economies because of sectorial mobility. The contribution of the NEG literature to the DD literature is that, through the same wage adjustments, a previous short run de-industrialization could end up being a long-run industrialization because of the inter-regional mobility.
6 References


DOI: 10.1080/09668139708412461


DOI: 10.1016/0304-3878(87)90005-8


DOI: 10.1016/S0166-0462(00)00072-7


DOI: 10.1080/17421772.2010.516445


DOI: 10.5772/intechopen.68852.


7 Appendix

Derivation of Equation (26): Multiplying expression (24) by \( n_1p_1 \) and using \( \phi \equiv \tau^{1-\sigma} \)

\[
n_1p_1x_1^i = \mu_1 n_1p_1^{1-\sigma} \left( \frac{Y^d_1}{P_1^{1-\sigma}} + \frac{\phi Y^d_2}{P_2^{1-\sigma}} \right)
\]

replacing the left hand side for the corresponding expressions (11), (13) and (15)

\[
\frac{L_{E_1}w_1}{\sigma - 1} + L_{E_1}w_1 = \mu_1 n_1p_1^{1-\sigma} \left( \frac{Y^d_1}{P_1^{1-\sigma}} + \frac{\phi Y^d_2}{P_2^{1-\sigma}} \right)
\]

using expressions (16) and (23), and by adding and subtracting \((1 - \mu_1)Y^d_1\),

\[
H_1w_1 + L_1w_1 - Y^d_1 - \frac{1-\mu}{2} \left( \frac{p_{A_1}}{P_A} \right)^{1-\sigma} (Y^d_1 + Y^d_2) + (1 - \mu) Y^d_1 + \mu_1 Y^d_1 = \mu_1 n_1p_1^{1-\sigma} \left( \frac{Y^d_1}{P_1^{1-\sigma}} + \frac{\phi Y^d_2}{P_2^{1-\sigma}} \right)
\]

where \(\mu \equiv \mu_1 + \mu_2\). Considering the agricultural price index (9) and \(Y^d_1 = (1 - t)Y_1\), and after some manipulation, it yields

\[
tY_1 - \frac{1-\mu}{2P_A} \left( \frac{Y^d_1}{P_{A_2}} - \frac{Y^d_2}{P_{A_1}} \right) + \mu_1 Y^d_1 \left( 1 - \frac{n_1p_1^{1-\sigma}}{P_1^{1-\sigma}} \right) = \mu_1 n_1p_1^{1-\sigma} \frac{\phi Y^d_2}{P_2^{1-\sigma}}
\]

finally, by using the expression of industrial price index (8), equation (26) it is obtained.

Proof of Proposition 1: First, and hereinafter, labor in region 2 is taken as numerarie, then, \(w_2 = p_{s_2} = p_{A_2} = 1\) and \(p_2 = \beta \frac{\sigma}{\sigma - 1}\), and \(\mu \equiv \mu_1 + \mu_2\). Furthermore,

\[
w \equiv \frac{p_1}{p_2} = \frac{p_{A_1}}{p_{A_2}} = \frac{p_{s_1}}{p_{s_2}} = \frac{w_1}{w_2} = w_1
\]
Using expressions (16), (20)-(23), (29) and replacing these in (27) and (28) it is obtained,

\[ Y_1 = \frac{\sigma}{\sigma - \mu_1} \left[ Lw + \frac{t \mu_2 Lw}{\sigma - 1 + \mu_2 (1 - t)} + \frac{1 - \mu}{2 \mu_1} \frac{Lw - (1 - L) \mu_1}{\sigma - 1 + \mu_2 (1 - t)} \right] \] (37)

\[ Y_2 = \frac{\sigma}{\sigma - \mu_1} \left[ (1 - L) - \frac{t \mu_2 Lw}{\sigma - 1 + \mu_2 (1 - t)} + \frac{1 - \mu}{2 \mu_1} \frac{(1 - L) \mu_1 - Lw}{\sigma - 1 + \mu_2 (1 - t)} \right] \] (38)

\[ Y^w = Y_1 + Y_2 = Y_1^d + Y_2^d \quad \text{and} \quad Y^w = \frac{\sigma}{(\sigma - \mu_1)} \left[ Lw + (1 - L) \right] \] (39)

Using (8)-(9), (36), (15), and (37)-(39) the current account equation (26) can be rewritten as

\[ CA_2(H, w) \equiv s_y (1 - t) \left\{ \mu_1 \phi \left[ \frac{H w^{1-\sigma}}{H w^{1-\sigma} + 1 - H} + \frac{1 - H}{H w^{1-\sigma} + (1 - H) \phi} \right] + (1 - \mu) \right\} \\
+ ts_y - \left[ \mu_1 \phi \frac{H w^{1-\sigma}}{H w^{1-\sigma} + 1 - H} + (1 - \mu) \frac{w^{1-\sigma}}{1 + w^{1-\sigma}} \right] = 0 \] (40)

where \( s_y \equiv Y_1(w)/Y^w(w) \). Implicit differentiation of (40) leads to

\[ \left. \frac{dw}{dH} \right|_{CA_2=0} = -\frac{\partial CA_2}{\partial H} \frac{\partial CA_2}{\partial w} \] (41)

where,

\[ \frac{\partial CA_2}{\partial H} = -\frac{\mu_1 \phi w^{1-\sigma}}{(H w^{1-\sigma} \phi + 1 - H)^2} \left\{ 1 - s_y (1 - t) \frac{(1 - \phi^2) \left[ (H w^{1-\sigma})^2 - (1 - H)^2 \right]}{(H w^{1-\sigma} + (1 - H) \phi)^2} \right\} < 0 \] (42)

the second term in curly brackets could be negative or positive, but in the last case, it will always be lower than one, so the expression is always negative. Additionally,

\[ \frac{\partial CA_2}{\partial w} = \frac{\mu_1 \phi (\sigma - 1) H (1 - H)}{w \sigma (H w^{1-\sigma} \phi + 1 - H)^2} \left\{ 1 - s_y (1 - t) \frac{(1 - \phi^2) \left[ (H w^{1-\sigma})^2 - (1 - H)^2 \right]}{(H w^{1-\sigma} + (1 - H) \phi)^2} \right\} + \frac{(1 - \mu) (\sigma - 1) w^{-\sigma}}{(1 + w^{1-\sigma})^2} \\
+ \left. \frac{\partial s_y}{\partial w} \left\{ 1 - \mu (1 - t) + \mu_1 \phi (1 - t) \left[ \frac{H w^{1-\sigma}}{H w^{1-\sigma} \phi + 1 - H} + \frac{1 - H}{H w^{1-\sigma} + (1 - H) \phi} \right] \right\} \right|_{CA_2=0} \] (43)
Note that,
\[
\begin{align*}
\frac{\partial Y_1}{\partial w} &= \frac{\sigma}{\sigma - \mu_1} \left\{ L + \frac{\mu_4 L}{(\sigma - 1 + \mu_2)(1-t)} + \frac{1 - \mu}{(1+w^{1-s})^2} \frac{L(\sigma w^{1-s} + 1) + (1 - L)(\sigma - 1)w^{-\sigma}}{\sigma - 1 + \mu_2(1-t)} \right\} > 0 \\
\frac{\partial Y^w}{\partial w} &= \frac{\sigma}{\sigma - \mu_1} L
\end{align*}
\]

On comparing these expressions it can be observed that, \(\frac{\partial Y_1}{\partial w} > \frac{\partial Y^w}{\partial w}\), and \(Y^w > Y_1\), thus,
\[
\frac{\partial s_y}{\partial w} = \frac{\frac{\partial Y_1}{\partial w} Y^w - \frac{\partial Y^w}{\partial w} Y_1}{(Y^w)^2} > 0 \tag{44}
\]

Then,
\[
\frac{\partial CA_2}{\partial w} > 0 \tag{45}
\]

Considering the signs of (42) and (45), (41) must always be positive.

**Proof of Proposition 2:** From expressions (37) and (39) it can be obtained that
\[
\frac{\partial s_y}{\partial t} = \frac{\partial Y_1}{\partial t} = \frac{\mu_2}{\sigma - 1 + \mu_2(1-t)} s_y \tag{46}
\]

Then, deriving the current account equation (40) with respect to \(t\),
\[
\frac{\partial CA_2}{\partial t} = \frac{s_y}{(\sigma - 1 + \mu_2(1-t))} \left\{ (\sigma - 1 + \mu_2) - (\sigma - 1) \left[ \frac{\mu_1 H w^{1-s} - \sigma \phi}{H w^{1-s} + 1 - H} + \frac{\mu_2 (1-H) \phi}{H w^{1-s} + (1-H) \phi + (1-\mu)} \right] \right\} \tag{47}
\]

The sum in the square brackets is equal to or lower than 1, and \([(\sigma - 1 + \mu_2) - (\sigma - 1)(\mu_1 + 1 - \mu)] > 0\). Thus, the expression is always positive. Then:
\[
\frac{dw}{dt}_{CA_2=0} = -\frac{\partial CA_2}{\partial w} < 0
\]
For the second part of the proposition, equation (19) is divided by (39), such that

\[
\frac{T_2}{Y_w} = ts_y \quad \text{and} \quad \frac{Y^w - T_2}{Y_w} = (1 - ts_y)
\]

Deriving this expressions and expression (39) with respect to \( t \),

\[
\frac{d(ts_y)}{dt} = t \frac{\partial s_y}{\partial t} + s_y + t \frac{\partial s_y}{\partial w} \frac{dw}{dt} \bigg|_{CA_2=0} = \frac{\sigma - 1 + \mu_2}{\sigma - 1 + \mu_2(1-t)} s_y + t \frac{\partial s_y}{\partial w} \frac{dw}{dt} \bigg|_{CA_2=0} \quad > 0
\]

\[
\frac{d(1 - ts_y)}{dt} = - \frac{d(t s_y)}{dt} < 0
\]

\[
\frac{dY^w}{dt} = \frac{\sigma}{\sigma - \mu_1} L \frac{dw}{dt} \bigg|_{CA_2=0} < 0
\]

On looking at equations (43)-(47), it is clear that the first expression is always positive, while the last two are always negative. Thus, if \( T_2/Y^w \) increases and \( (Y^w - T_2)/Y^w \) decreases as \( t \) rises, \( dT_2/dt \) must be positive.

Proceeding in the same way for the disposable incomes,

\[
\frac{Y^d_1}{Y_w} = \frac{(1 - t) Y_1}{Y_w} = (1 - t) s_y \quad \text{and} \quad \frac{Y^d_2}{Y_w} = 1 - (1 - t) s_y
\]

by differentiating these expressions with respect to \( t \) it is obtained that

\[
\frac{d[(1 - t) s_y]}{dt} = - \frac{\sigma - 1}{\sigma - 1 + \mu_2 (1-t)} s_y + (1 - t) \frac{\partial s_y}{\partial w} \frac{dw}{dt} \bigg|_{CA_2=0} < 0
\]

\[
\frac{d[1 - (1 - t) s_y]}{dt} = - \frac{d[(1 - t) s_y]}{dt} \quad > 0
\]

Then, taking into account that \( dY^w/dt < 0 \), the last two expressions imply that

\[
\frac{dY^d_1}{dt} < 0 \quad \text{and} \quad \frac{dY^d_2}{dt} > 0
\]

**Proof of Proposition 3:** The change in the industrial sector as a proportion of the
labor force in the sector is:

\[
\frac{dL_{E_j}}{dt} = \left. \frac{\partial L_{E_j}/\partial t}{L_{E_j}} + \frac{\partial L_{E_j}/\partial w}{L_{E_j}} \frac{dw}{dt} \right|_{CA_2=0} \geq 0
\]

Using equations (23), (29), (37), (38), (43) and (47), the previous expression for region 1 at the symmetric equilibrium (30) is equal to

\[
\left. \frac{dL_{E_1}}{dt} \right|_{sym} = \frac{\sigma U(\phi)}{Z(\phi)} \geq 0
\]  

(48)

where \( U(\phi), \) and \( Z(\phi) > 0 \) for \( \phi \geq 0 \) \( (dZ(\phi)/d\phi > 0) \), are polynomials,

\[
U(\phi) = [2\mu_2 + \sigma (1 - \mu)] \phi^2 + 2\mu_2 (2\sigma - 1) \phi - \sigma (1 - \mu) \geq 0
\]  

(49)

\[
Z(\phi) = (\sigma - 1 + \mu_2) [4\mu_1 (\sigma - 1) \phi + (1 - \mu) (\sigma - 1) (1 + \phi)^2] + (\sigma - 1 + \mu_2) [1 + \frac{\sigma (1 - \mu)}{\sigma - 1 + \mu_2}] [(1 - \mu_2) (1 + \phi)^2 - \mu_1 (1 - \phi^2)] > 0
\]  

(50)

(51)

where \( Z(\phi) > 0 \) for all \( \phi \in [0,1] \). Then, the sign of expression (48) depends only on the numerator. The polynomial (49) has a unique positive root: \( P(\phi = \phi^{sr}) = 0 \) with \( \phi^{sr} \in (0,1) \), and

\[
\phi^{sr} = \frac{-\mu_2 (2\sigma - 1) + \sqrt{[\mu_2 (2\sigma - 1)]^2 + \sigma (1 - \mu) [2\mu_2 + \sigma (1 - \mu)]}}{[2\mu_2 + \sigma (1 - \mu)]}
\]  

(52)

Moreover, evaluating expression (48) for the extreme cases of \( \phi = 0 \) and \( \phi = 1 \) yields

\[
\left. \frac{dL_{E_1}}{dt} \right|_{sym} (\phi = 0) = -\frac{\sigma}{(\sigma - \mu_1)} < 0
\]

\[
\left. \frac{dL_{E_1}}{dt} \right|_{sym} (\phi = 1) = \frac{\mu_2 \sigma}{(1 - \mu_2) (\sigma - \mu_1)} > 0
\]

Then, expression (48) is negative for \( 0 \leq \phi < \phi^{sr} \) and positive for \( \phi^{sr} < \phi \leq 1 \). Proceeding
in the same way for region 2, (and by symmetry) it is obtained that
\[
\frac{dL_{E2}}{dt} \bigg|_{\text{sym}} = -\frac{dL_{E1}}{dt} \bigg|_{\text{sym}} = -\frac{\sigma U(\phi)}{Z(\phi)} \geq 0
\]

**Proof of Proposition 4:** From equation \(U(\phi) = 0\) (polynomial (49)) and the implicit differentiation, it is obtained that
\[
\frac{\partial \phi^{sr}}{\partial \mu_2} = -\frac{2\phi^{sr} [\phi^{sr} + (2\sigma - 1)] + \sigma [1 - (\phi^{sr})^2]}{2[2\mu_2 + \sigma (1 - \mu)]\phi^{sr} + 2\mu_2 (2\sigma - 1)} < 0 \quad (53)
\]
\[
\frac{\partial \phi^{sr}}{\partial \sigma} = -\frac{(1 - \mu) (\phi^{sr})^2 + 4\mu_2 \phi^{sr} - (1 - \mu)}{2[2\mu_2 + \sigma (1 - \mu)]\phi^{sr} + 2\mu_2 (2\sigma - 1)} < 0 \quad (54)
\]

While expression (53) is clearly negative, expression (54) is also negative since \(\mu_2 > 0\) and
\[
\frac{\partial \phi^{sr}}{\partial \sigma} < 0 \iff \phi^{sr} > \phi^* \quad \text{where } \phi^* \text{ is the unique positive root of the numerator of (54):}
\]
\[
\phi^* = \frac{-2\mu_2}{1 - \mu} + \sqrt{\left(\frac{2\mu_2}{1 - \mu}\right)^2 + 1} \quad (55)
\]

**Proof of Proposition 5:** The proof is divided in two parts. The first part proves the existence of the thresholds \(\phi^b\) and \(\phi^r\) that determine the stability/instability of the symmetric equilibrium. The second part derives the analytical expression for these thresholds.

**Part 1:** By differentiating \(V(H, w)\) from equation (34) with respect to \(H\),
\[
\frac{dV}{dH} = \frac{\partial V}{\partial H} - \frac{\partial V}{\partial w} \frac{\partial C A_2}{\partial H} \geq 0 \quad (56)
\]

If this expression is negative, the equilibrium is stable, and if it is positive the equilibrium is unstable. Evaluating expression (56) at the interior symmetric equilibrium (30) it is
obtained that

\[
\frac{dV}{dH}
\bigg|_{sym} = -4(1-d+\phi(1+d)) + \frac{4\mu_1 \phi (\sigma(1-\mu) + 1 - \mu_2)}{(1+\phi)^2 + (1-\mu)\phi} \frac{\mu_1 (1-\phi)}{(1+\phi)} \left[ \frac{\sigma^2 (1-\mu)}{\mu_2 (\sigma-1+\mu_2)} + 1 - \mu_2 - \mu_1 (\sigma - 1) \right] > 0
\]

where \( d \equiv \frac{\mu_1}{\sigma-1} \). Evaluating (57) at \( \phi = 1 \) yields,

\[
\frac{dV}{dH} \bigg|_{sym} (\phi = 1) = -4 \frac{\mu_1 (\sigma - 1 + \mu_2)^2}{\sigma (\sigma - \mu_1) (1 - \mu_2)} < 0
\]

Thus, when \( \phi = 1 \), the symmetric equilibrium is always stable. Additionally, evaluating expression (57) at \( \phi = 0 \) yields,

\[
\frac{dV}{dH} \bigg|_{sym} (\phi = 0) = 4 \left[ \frac{\mu_1}{\sigma - 1} - 1 \right]
\]

Which implies that, if the BHC holds, the symmetric equilibrium is unstable for \( \phi = 0 \), and stable otherwise. Furthermore, expression (57) can be rewritten as

\[
\frac{dV}{dH} \bigg|_{sym} = \frac{P(\phi)}{K(\phi)} = \frac{-A\phi^3 + B\phi^2 + C\phi + D}{K(\phi)} \geq 0
\]

where

\[
A \equiv (1 + d) \left[ 1 + \frac{\sigma (1-\mu)}{\sigma-1+\mu_2} \right] (1 - \mu_2 + \mu_1) + (1 - \mu) (\sigma - 1) > 0
\]

\[
B \equiv 4\mu_1 \left[ \frac{\sigma (1-\mu)}{\sigma-1+\mu_2} + 1 - \mu_2 + \mu_1 \right] - 2(1 + d) \left[ \frac{\sigma (1-\mu) (\sigma-\mu_1)}{\sigma-1+\mu_2} + \mu_1 (\sigma - 1) \right] - (1 - d) \left\{ \left[ \frac{\sigma (1-\mu)}{\sigma-1+\mu_1} + 1 \right] (1 - \mu_2 + \mu_1) + (1 - \mu) (\sigma - 1) \right\}
\]

\[
C \equiv 4\mu_1 \left[ \frac{\sigma (1-\mu)}{\mu_1 (\sigma-1+\mu_2)} + 1 - \mu \right] - 2(1 + d) \left[ \frac{\sigma (1-\mu) (\sigma-\mu_1)}{\sigma-1+\mu_2} + \mu_1 (\sigma - 1) \right]
\]

\[
D \equiv (d - 1) \frac{\sigma (1-\mu) (\sigma-\mu_1)}{\sigma-1+\mu_2}
\]

\[
K(\phi) \equiv \frac{4\mu_1 (\sigma-1)\phi + (1-\mu)(\sigma-1)(1+\phi)^2 + (1 + \frac{\sigma (1-\mu)}{\sigma-1+\mu_2}) (1 - \mu_2)(1+\phi) - \mu_1 (1-\phi^2)}{4(1+\phi)^{-1}} > 0
\]
Since expression (66) is positive for all values of $\phi \geq 0$, only $P(\phi)$ determines the sign of the expression (57). As $\phi \to \infty$, $P(\phi) \to -\infty$; and as $\phi \to -\infty$, $P(\phi) \to \infty$. Moreover, if $d \geq 1$, then $D \geq 0$. Also, when $d \geq 1$, $C > 0$, then there exists a threshold $\bar{\mu}_1(\sigma, \mu_2) \in (0, \min [1, \sigma - 1])$ for the parameter $\mu_1$, which can be expressed as

$d \equiv \frac{\bar{\mu}_1(\sigma, \mu_2)}{\sigma - 1}$, such that if $d < d < 1$, then $C > 0$, and there exist two real positive roots of the polynomial $P(\phi)$. And whenever $C < 0$, $B < 0$, according to expression (67), there are, therefore, no real positive roots.

$B - C = -2\mu_1 \frac{[(1+\mu_1)+2(1-\mu_2)]\sigma^2-[(1+\mu_1)+(1+3\mu_1)(1-\mu_2)]\sigma+(1-\mu_2)(1+2\mu_1)}{(\sigma-1)(\sigma-1-\mu_2)} < 0 \quad (67)$

**Part 2:** In order to obtain a closed form for the thresholds ($\phi^b$ and $\phi^r$) it is taken into account that $\phi^* = -1$ is always a solution of $P(\phi) = 0$. Then, this polynomial can be rewritten as

$P(\phi) = -(\phi + 1) [\phi^2 - (Tr) \phi + (Det)] \quad (68)$

where, $Tr \equiv \frac{B}{A} + 1$ and $Det \equiv -\frac{D}{A}$. Thus, the other two roots of $P(\phi)$ are

$\phi^b = \frac{Tr - \sqrt{(Tr)^2 - 4Det}}{2} \quad (69)$

$\phi^r = \frac{Tr + \sqrt{(Tr)^2 - 4Det}}{2} \quad (70)$

If $(Tr)^2 - 4Det > 0$, there are three cases: 1) if $Tr > 0$ and $Det > 0$, then $0 < \phi^b < \phi^r < 1$; 2) if $Tr \leq 0$ and $Det < 0$, then $\phi^b < 0 < \phi^r < 1$ and 3) if $Tr < 0$ and $Det > 0$, then $\phi^b < \phi^r < 0$. If $(Tr)^2 - 4Det = 0$, then $\phi^b = \phi^r \in [0, 1)$. If $(Tr)^2 - 4Det < 0$, then $\phi^b$ and $\phi^r$ are conjugated complexes.

Additionally, from these relations, $\bar{\mu}_1(\sigma, \mu_2) \in (0, \min [1, \sigma - 1])$ can be implicitly
defined as the value of $\mu_1$ that ensures that the following conditions are fulfilled:

$$Tr^2 - 4Det = 0 \text{ with } Tr > 0 \text{ and } Det > 0$$ (71)

$$\mu_1 - (\sigma - 1) < 0$$ (72)

[Figure 7: Regions of Bifurcation Points in the space $(\mu_1, \mu_2, \sigma)$]

The region above the plane in Figure 7 (a) corresponds to $d < 1$ (condition (72)). Only the parameter values below the dashed line of Figure 7 in the plane $(\mu_1, \mu_2)$ are feasible due to the parameter restriction: $\mu_1 + \mu_2 \equiv \mu \in (0, 1)$. The red surface in Figure 7 (b) depicts condition (71). Below this surface $Tr^2 - 4Det > 0$, and above $Tr^2 - 4Det < 0$. Thus, for each value of $\sigma$ and $\mu_2$, there exist a value $\mu_1 = \bar{\mu}_1(\sigma, \mu_2)$ such that $Tr^2 - 4Det = 0$. Moreover, Figure 7 (c) divides the space of parameters $(\mu_1, \mu_2, \sigma)$ in three regions: 1) below the gray plane, $d > 1$ and the symmetric equilibrium has only one bifurcation point, $\phi^r$; 2) above the gray plane and below the red surface, $\bar{d} < d < 1$ and the symmetric equilibrium has two bifurcation points, $\phi^b$ and $\phi^r$; and 3) above the red surface, $d < \bar{d} < 1$ and the symmetric equilibrium is stable for all values of $\phi$.

**Proof of Proposition 6:** By fully differentiating the system (40)-(33) with respect to $t$, it is obtained that

$$
\begin{pmatrix}
\frac{\partial C_A_2}{\partial w} & \frac{\partial C_A_2}{\partial H} \\
\frac{\partial V}{\partial w} & \frac{\partial V}{\partial H}
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{dt} \\
\frac{dH}{dt}
\end{pmatrix} =
\begin{pmatrix}
-\frac{\partial C_A_2}{\partial t} \\
-\frac{\partial V}{\partial t}
\end{pmatrix}
$$

Then, the change in the number of firms is

$$
\frac{dH}{dt} = \frac{(\frac{\partial C_A_2}{\partial H} \frac{\partial V}{\partial w} - \frac{\partial V}{\partial t} \frac{\partial C_A_2}{\partial w})}{(\frac{\partial C_A_2}{\partial w} \frac{\partial V}{\partial H} - \frac{\partial C_A_2}{\partial H} \frac{\partial V}{\partial w})}
$$
After some manipulation,
\[
\frac{dH}{dt} = -\frac{\partial V}{\partial t} + \frac{\partial V}{\partial w} \left( -\frac{\partial CA_2}{\partial w} \right)
= -\frac{\partial V}{\partial H} + \frac{\partial V}{\partial w} \left( -\frac{\partial CA_2}{\partial w} \right)
\]
(73)

The denominator is equal to the stability condition (57) in Proposition 5, while the numerator is the effect of a change in the rate of transfers \((t)\) over the ratio of indirect utilities \((V_1/V_2)\). Additionally, using (16) and (32), the numerator of (73) can be rewritten as
\[
\frac{dV}{dt} = \frac{V_1}{V_2} \left\{ \left[ \frac{dL_{E_1}}{dt} - \frac{dL_{E_2}}{dt} \right] + \left[ \frac{dw}{dt} \right]_0 \left[ \frac{CA_2}{w} \right] - \left[ \frac{\mu_2}{w} + \mu_1 \left( \frac{P_1/P_2}{\partial w} \right) \right] \frac{dw}{dt} \right\}_{CA_2=0}
\]
which is equal to expression (35). Evaluating at the symmetric equilibrium,
\[
\frac{dV}{dt}_{sym} = \frac{2}{Z(\phi)} \left[ \sigma U (\phi) - J (\phi) \right] \geq 0
\]
(74)

where
\[
J (\phi) \equiv (1 - \mu_2 + \mu_1) \left[ \mu_2 \sigma - \mu_1 \left( \sigma - 1 \right) \right] \phi^2
+ 2 \left[ \mu_2 (1 - \mu_2) \sigma + \mu_1^2 \left( \sigma - 1 \right) \right] \phi + (1 - \mu) \left[ \mu_2 \sigma + \mu_1 \left( \sigma - 1 \right) \right]
\]
(75)

\(U (\phi)\) and \(Z (\phi)\) are defined in (49) and (50), and \(J (\phi) > 0\). Thus, the sign is determined by the numerator. After some manipulations it is obtained that
\[
\sigma U (\phi) - J (\phi) = a\phi^2 + b\phi + c \geq 0
\]
(76)
where

\[ a \equiv 2\mu_2 \sigma - (1 - \mu_2 + \mu_1) [\mu_2 \sigma - \mu_1 (\sigma - 1)] + \sigma^2 (1 - \mu) > 0 \quad (77) \]

\[ b \equiv 2 [2\mu_2 \sigma (\sigma - 1) + \mu_2^2 \sigma - \mu_1^2 (\sigma - 1)] \geq 0 \quad (78) \]

\[ c \equiv -(1 - \mu) [\sigma^2 + \mu_2 \sigma + \mu_1 (\sigma - 1)] < 0 \quad (79) \]

Additionally, evaluating (74) at the extreme cases \( \phi = 0 \) and \( \phi = 1 \),

\[ \frac{dV}{dt} \bigg|_{\text{sym}} (\phi = 0) = -\frac{2\mu_1 (\sigma - 1) + \sigma (\sigma + \mu_2)}{\sigma (\sigma - 1)} < 0 \quad (80) \]

\[ \frac{dV}{dt} \bigg|_{\text{sym}} (\phi = 1) = 2\mu_2 \frac{\sigma - 1 + \mu_2}{\sigma - \mu_1 (1 - \mu_2)} > 0 \quad (81) \]

Thus, the polynomial (76) has only one positive root,

\[ \phi^{lr} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \in (0, 1) \quad (82) \]

Furthermore, because \( J(\phi) > 0 \) for all \( \phi \geq 0 \), then the following relation must hold:

\[ 0 < \phi^{sr} < \phi^{lr} < 1 \quad \text{when} \quad \mu_2 \in (0, 1 - \mu_1) \]

\[ \phi^{sr} = \phi^{lr} \quad \text{when} \quad \mu_2 = 0, 1 - \mu_1 \quad (84) \]

Combining these results with those from Proposition 5 properties i) and ii) of Proposition 6 are derived.

Additionally, from polynomial (49) and the implicit differentiation, it is obtained that

\[ \frac{\partial \phi^{lr}}{\partial \sigma} = -\frac{\partial a}{\partial \sigma} (\phi^{lr})^2 + \frac{\partial b}{\partial \sigma} \phi^{lr} + \frac{\partial c}{\partial \sigma} \]

\[ \frac{\partial \phi^{lr}}{\partial \sigma} \cdot \frac{\partial \phi^{lr}}{\partial \sigma} + b \quad (85) \]

The denominator is positive since \( \phi^{lr} > -b/(2a) \). Hence, the sign of (85) depends on the numerator. Figure 8 depicts the region for which \( \partial \phi^{lr}/\partial \sigma > 0 \).
Figure 8 (a) shows that only for a very narrow range of values of the parameters \((\mu_1, \mu_2, \sigma)\) is the derivative (85) positive. Furthermore, Figure 8 (b) highlights that if the agricultural sector is not too small (approximately \(1 - \mu > 0.08\)), derivative (85) will be negative.

**Derivation of the Figures 3 (a) - (e):** First the focus is putted on \(\phi^{lr}\), which presents the same shape for all values of \(d\). Then, \(\phi^b\) and \(\phi^r\) are analyzed, by considering the different cases \((d < 1, d = 1 \text{ and } d > 1)\).

Differentiating of the polynomial (76) with respect to \(\mu_2\) yields

\[
\frac{\partial (\sigma U(\phi) - J(\phi))}{\partial \mu_2} = \sigma \left\{ 2\mu_2 \phi^2 + (\mu - \mu_1 \phi^2) + (\sigma - 1) \phi^4 - 4\mu_2 \phi + \sigma - 1 + \mu_2 \right\} + \mu_1 (\sigma - 1) > 0
\]

And the differential with respect to \(\phi\) is

\[
\frac{\partial (\sigma U(\phi) - J(\phi))}{\partial \phi} = 2a\phi + b > 0
\]

which is positive because \(\phi^{lr} > \frac{-b}{2a}\) (see expression (82)). Then, the implicit differentiation gives

\[
\frac{\partial \phi^{lr}}{\partial \mu_2} = -\frac{\partial (\sigma U(\phi) - J(\phi)) / \partial \mu_2}{\partial (\sigma U(\phi) - J(\phi)) / \partial \phi} < 0
\]

Additionally, evaluating the polynomial (76) at \(\mu_2 = 0\) and \(\mu_2 = 1 - \mu_1\),

\[\phi^{lr}(\mu_2 = 0) = 1 \text{ and } \phi^{lr}(\mu_2 = 1 - \mu_1) = 0\]

For \(\phi^b\) and \(\phi^r\) the simplest case, \(d = 1 \ (\sigma - 1 = \mu_1)\) is studied first. In this special case it is obtained that

\[\phi^b(\sigma - 1 = \mu_1) = 0 \text{ and } \phi^r(\sigma - 1 = \mu_1) = \frac{1 - \mu_2 - \mu_1 [4\mu_1^2 + \mu_1 (6\mu_2 - 1)] + 2\mu_2^2 + \mu_2 - 2}{1 - \mu_2 - \mu_1 [2\mu_1^2 + \mu_1 (2\mu_2 - 1)] + \mu_2 - 2}\]
Thus, only $\phi^r$ needs to be analyzed. Differentiating $\phi^r(\sigma - 1 = \mu_1)$ with respect to $\mu_2$,

$$\frac{\partial \phi^r(\sigma - 1 = \mu_1)}{\partial \mu_2} = \mu_2^2(5-2\mu_1^2) + \mu_2(2\mu_2-\mu_1^2) + 3\mu_1(1-\mu_2) + 2\mu_1[1+\mu_2(2-\mu_2)] + 2\mu_2^2(1-\mu_2) - (2\mu_1)^{-1}\{1-\mu_2-\mu_1[2\mu_1^2+\mu_1(2\mu_2-1)+\mu_2-2]\}^2 < 0$$

Additionally, note that the previous derivative tends to $-\infty$ when $\mu_2 = 1-\mu_1$. Evaluating $\phi^r(\sigma - 1 = \mu_1)$ at $\mu_2 = 0$ and $\mu_2 = 1 - \mu_1$:

$$\phi^r(\sigma - 1 = \mu_1, \mu_2 = 0) = \frac{1+2\mu_1+\mu_2^2-4\mu_1^3}{1+2\mu_1+\mu_2^2-2\mu_1^4}$$ and $\phi^r(\sigma - 1 = \mu_1, \mu_2 = 1 - \mu_1) = 0$

Bringing these results together, $d = 1$ yields $\phi^r(\mu_2 = 0) < \phi^{lr}(\mu_2 = 0)$ and $\phi^r(\mu_2 = 1 - \mu_1) = \phi^{lr}(\mu_2 = 1 - \mu_1) = 0$. Both thresholds diminish as $\mu_2$ increases, and they cross at least once within the interval $\mu_2 \in (0, 1 - \mu_1)$.

When $d > 1$ ($\sigma - 1 < \mu_1$), the BHC case, $\phi^b < 0$. Then, again, only $\phi^r$ needs to be studied. By differentiating expression (70) with respect to $\mu_2$,

$$\frac{\partial \phi^r}{\partial \mu_2} = \frac{1}{\sqrt{(Tr)^2 - 4Det}} \left[ \frac{\partial Tr}{\partial \mu_2} \phi^r - \frac{\partial Det}{\partial \mu_2} \right]$$

(86)

where

$$\frac{\partial Det}{\partial \mu_2} = \frac{-2\mu_1\sigma(\sigma-1-\mu_1)(\sigma-\mu_1)^2}{(\sigma+1+\mu_1)[\sigma^2-\sigma^2(1-\mu_2)+\mu_1(\sigma-1+\mu_2)]^2} > 0 \text{ if } \sigma - 1 < \mu_1$$

$$\frac{\partial Tr}{\partial \mu_2} - \frac{\partial Det}{\partial \mu_2} = \frac{-\{\sigma^2-\mu_1^2\sigma(1-\mu_2)+\sigma(1-\mu_2)(\sigma-1)\mid \sigma(2-\mu)-(1-\mu)\}}{[4\mu_1(\sigma-1)(\sigma-1+\mu_2)]^{-1}(\sigma+1+\mu_1)[\mu^2-\sigma^2(1-\mu_2)+\mu_1(\sigma-2)(\sigma-1+\mu_2)]^2} < 0$$

Then, expression (86) must be negative whenever $d > 1$. Now, evaluating $\phi^r$ at $\mu_2 = 0$ and $\mu_2 = 1 - \mu_1$ yields that $\phi^r \in (0, 1)$. Thus, when the BHC holds with inequality ($d > 1$), $\phi^r(\mu_2 = 0) < \phi^{lr}(\mu_2 = 0)$ and $\phi^r(\mu_2 = 1 - \mu_1) > \phi^{lr}(\mu_2 = 1 - \mu_1)$. As in the previous case, both thresholds diminish as $\mu_2$ increases, and they cross at least once within the interval $\mu_2 \in (0, 1 - \mu_1)$.

When $d < 1$ ($\sigma - 1 > \mu_1$), the analysis focuses on the case when the thresholds are
real numbers \((0 < \phi^b \leq \phi^r < 1)\), that is, when \(\bar{d} \leq d < 1\). From Proposition 5 a value \(\mu_2 = \mu_{2_0}\) (implicitly defined by \((Tr)^2 - 4Det = 0\)) can be defined, such that \(\phi_0 \equiv \phi^b = \phi^r\).

Then, by differentiating the polynomial \(O(\phi, \mu_2) \equiv \phi^2 - (Tr) \phi + Det = 0\) (see the polynomial (68)), and evaluating at \((\mu_{2_0}, \phi_0)\),

\[
\frac{\partial O}{\partial \phi}(\mu_{2_0}, \phi_0) = 2\phi - Tr|_{\phi_0} = 2\phi_0 - (\phi_0 + \phi_0) = 0 \\
\frac{\partial O}{\partial \mu_2}(\mu_{2_0}, \phi_0) > 0 \\
\frac{\partial^2 O}{\partial \phi^2}(\mu_{2_0}, \phi_0) = 2
\]

Thus, for the function \(\mu_2(\phi)\) implicitly defined by \(O(\phi, \mu_2) = 0\), it is obtained that

\[
\frac{d\mu_2}{d\phi}(\mu_{2_0}, \phi_0) = 0 \quad \text{and} \quad \frac{d^2\mu_2}{d\phi^2}(\mu_{2_0}, \phi_0) < 0
\]

which implies that the function \(\mu_2(\phi)\) (implicitly defined by \(O(\phi, \mu_2) = 0\)) has a maximum at \((\mu_{2_0}, \phi_0)\). In a close neighborhood of \(\mu_{2_0}\), \(\phi^b\) increases, and \(\phi^r\) diminishes as \(\mu_2\) increases until \(\mu_2 = \mu_{2_0}\). At this point, both thresholds converge to the value \(\phi_0\).
Notes

1 According to Eurostat definitions.

2 The first versions of the Footloose Entrepreneur Model were developed independently by Ottaviano (2001) and Forslid (1999).

3 The Dutch disease or de-industrialization in the short run could occur in our model as a result of the economic configuration of the regions. The bigger the non-tradable sector and the higher the competition, the higher the probability of ending up in a de-industrialization scenario. However, in Moncarz, et. al 2017, the Dutch disease always takes place in the short run, and is not a result of the economic configuration.

4 Alternatively, if the non-tradable services were also an input for industrial production all the results derived in this paper would remain unchanged.

5 Note that besides of avoiding wage equalization, the assumption of different regional agricultural good is more empirically accurate, as pointed out by Fujita et al. (1999).

6 Examples of non-tradable sectors are mainly: the construction sector, some finance and real state services, and public services. Piton (2017) estimates that, on average for the period 1995-2014, the weight of the non-tradable sectors in the total production for the European Union is larger than 40%.

7 See the derivation of equation (26) in the Appendix.

8 This channel can be shut down by setting $\mu_2 = 0$. In this case, all expenditure goes to tradable goods.

9 This channel can be shut down by making $\phi = 1$.

10 Another way of interpreting this is by focusing on the agricultural sector. Increasing
$\mu_2$ while holding constant $\mu_1$ is equivalent to a reduction of proportion of disposable income devoted to agricultural consumption $(1 - \mu_1 - \mu_2)$. Because this proportion is relatively small, the contraction of the supply will have a larger effect on the other sectors of the economy, which makes the industrial sector more likely to shrink.