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# Modulus of elasticity prediction through transversal vibration in cantilever beams and ultrasound technique of different wood species

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ABSTRACT

The prediction of the modulus of elasticity (MOE) of five species of different spectrum density woods, namely, *Populus* × *euramericana* I-214 (Poplar), *Fagus sylvatica* L. (Beech), *Quercus pyrenaica* L. (Oak), *Paulownia elongata* S.Y.Hu (Paulownia) and *Pinus sylvestris* L. (Scots pine) were examined through the natural frequency of vibration on cantilevered beams (transverse direction) and ultrasound (longitudinal direction). Cantilever beams are commonly used for other materials but limited information is available for wood materials tested in this manner. A total of 60 specimens with nominal dimensions of 40 × 60 × 1200 mm$^3$ were tested, which were visually graded according to UNE 56544:2022 and UNE 56546:2022 as first class, and finally the global bending stiffness was obtained from a four-point bending test. Utilising this data, a regression model was presented to predict the MOE.

Also, *Picea sitchensis* Trautv. & G.Mey (Sitka spruce) has been chosen as a blind species in order to validate the regression model of prediction of the MOE as a function of the dynamic MOE by ultrasound. Bending strength, modulus of elasticity and density were obtained according to the EN 408. In the prediction model using the dynamic MOE with vibrations, an r$^2$ of 95.9% was achieved for the induced vibration technique which was found to be slightly higher than the model for the ultrasound prediction which had an r$^2$ of 93.7%.

## 1. Introduction

In general, non-destructive testing techniques (NDT) applied to wood or wood-derived components are tools that can estimate the physical–mechanical properties and determine and ensure their structural integrity during construction. Furthermore, such techniques can identify damaged or deteriorated components without compromising the functionality of the structural element. To this end, a great number of studies have been performed in the last few decades using different methods such as stress wave techniques, transverse vibration, ultrasounds, X-ray or thermography, among other techniques [1–28].

One NDT that has gained importance in the last few years is the induced vibration technique which measures acoustic properties to predict and evaluate the physical–mechanical properties of wood and establish classification ranges based on these properties [7,14,29]. In this regard, studies have focused on estimating the behaviour of wood or wood-based components based on vibration frequencies (longitudinal and transverse), estimating their dynamic elastic modulus, and analysing the relationships between the dynamic parameters and the static variables of elasticity and resistance, namely the static modulus of elasticity and bending strength [30–32]. Similarly, several research projects have focused on studying the relationship between dynamic parameters and physical variables and between dynamic parameters and quality parameters, such as the presence of defects in the wood. In most of these studies, vibration analyses have been performed on simply supported beams and beams with both ends free [33,34]. Their conclusions are all similar: dynamic properties correlate sufficiently well with static variables, namely the modulus of elasticity and to a lesser degree with the bending strength, which can suitably predict the physical characteristics and quality structural parameters of wood [7,16,41,20,29,35–40].

On the other hand, ultrasonic techniques are one of the most widely

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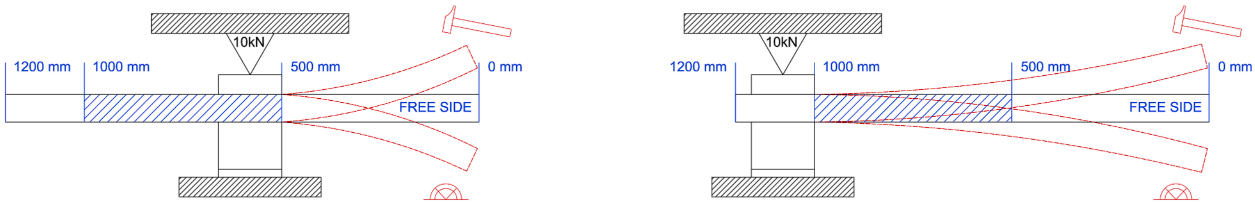


**Fig. 1.** Configuration of the cantilever beam. a) initial length, b) final length.

used non-destructive testing methods for timber, due to their sufficient accuracy and relative simplicity of testing. Thus, many studies have been carried out for timber grading and for non-destructive evaluation of in-use timber structures [42–47].

Although in order to grade determining timber properties by different methods that use this research approach (induced vibrational) [48], there is essentially no study or grading machine development based on the application of cantilever beams to evaluate the behaviour of wooden beams. However, there are a large number of projects that have utilised this approach but they are focused on different isotropic materials [16,49–58]. By definition, a cantilever has a rigid support on one end, which prevents any type of displacement or rotation, and the other end is free to vibrate [58]. Therefore, the behaviour of wood, which is an anisotropic and heterogeneous material when fixed in a cantilever may be an appropriate alternative to determine its vibrational behaviour and estimate its elastic behaviour. Furthermore, this type of support is easily reproducible in the laboratory.

The aim of this study was to determine the elastic behaviour of different timber species (with low, medium and high density) by determining the dynamic modulus of elasticity based on transversal vibration frequencies in cantilever beams, and compare them with ultrasonic and static bending tests.

## 2. Materials and methods

### 2.1. Testing materials

The testing was conducted using five different wood species, some of them commonly used in construction and structures such as *Populus × euramericana* I-214 (Poplar), *Fagus sylvatica* L. (Beech), *Quercus pyrenaica* L. (Oak), *Paulownia elongate S.Y. Hu* (Paulownia) and *Pinus sylvestris* L. (Scots pine). The test material consisted of 60 wooden joists (12 pieces per species) of 40 mm × 60 mm and 1200 mm (structural proportion dimensions, EN 408:2011 [59]) that were visual graded according to UNE 56544:2022 [60] and UNE 56546:2022 [61] and achieved a first class. The test material was conditioned in Wood Laboratory of Valladolid University (Spain) at 65 ± 5 % HR and 20 ± 2 °C until reaching constant mass. All pieces were measured and weighed to calculate their density at 12 % moisture content (*MC*), according to standard EN 384:2016 [62].

In order to validate and ensure that the statistical model worked properly, a blind control sample comprising 10 *Picea sitchensis* Trautv. & G.Mey (Sitka spruce), of Irish provenance, with dimensions of 40 mm × 100 mm × 4500 mm in length and 12 % MC, was randomly selected (from a batch > 200 units) and was tested. This additional wood species was chosen because it has a wide range of elasticity values and is widely used in the European market. These tests were realised at the Timber Engineering Laboratory of the College of Science and Engineering at the University of Galway in Ireland.

### 2.2. Non-destructive testing by transversal free vibration

Each piece was placed in a cantilever configuration (one end fixed and the other end free) and clamped under a load of 10 kN to provide the optimum rigidity conditions. The test beams were attached to the main support using wood components and clamps to ensure uniform pressure. The initial free span length was 500 mm, which was later increased by a length, *d*, of 25 mm for each new position until a free span length of 1000 mm was achieved (Fig. 1).

The tests were carried out using 21 different free span lengths (500 mm/25 mm each step) for five species (Paulownia, Poplar, Scotland Pine, Beech and Oak) and, for each one, four tests were conducted for the two orientations of the section: vertical, *V*, and horizontal, *H*, where a total of 1008 tests per species and orientation (21 span lengths/orientation × 4 repetitions/span length × 12 beams/species) were completed. Seven free span lengths were tested for Spruce, where a total of 280 tests per orientation (7 span lengths/orientation × 4 repetitions/span length × 10 beams) were completed.

The vibration frequency in each cantilever beam was generated by applying a hammer blow perpendicularly onto the top part of the transverse face of the piece followed by measuring and recording the vibration using a signal receiver (microphone) and wave analyser (FFT analyser).

The natural frequency of transversal vibrations of a cantilever beam, according to the Euler-Bernoulli theory [63], can be calculated by Equation (1):

$$f_n = C_n\sqrt{\frac{(E_{dyn}{}^*I)}{\rho^*S^*L^4}} \tag{1}$$

where $f_n$ is the natural frequency (Hz), $C_n$ is a constant that depends on the vibration mode ($n$), e.g. $n_1$ = fundamental mode, $n_2$ = first harmonic, etc., taking the values: mode 1 = 0.5602, mode 2 = 5.5014, mode 3 = 9.8198…; $E_{dyn}$ is the dynamic modulus of elasticity (Pa); $I$ is the area moment of inertia of the beam cross-section ($m^4$); $\rho$ is the bulk density ($kg/m^3$); $S$ is the area of the cross-section ($m^2$) and $L$ is the free span length of the cantilever beam (m).

Equation (1) can also be rewritten as per Equation (2):

$$f_n{}^2 = C_n{}^{2*}\frac{(E_{dyn}{}^*I)}{\rho^*S}{}^*\frac{1}{L^4} \tag{2}$$

Equation (2) is similar to the general equation for a linear model, $y = mx + b$, in the form slope ($m$) –intercept ($b$) if it is assumed $y = f_n^2$ and $\times = (1/L^4)$, $m = C_n^2{}^*(E_{dyn}{}^*I)^*(\rho^*S)^{-1}$ and $b = 0$. In this way, for each free length ($L$), it will be possible to obtain a frequency value. Thus, plotting (x,y) points, the slope can be determined and the dynamic modulus of elasticity will be calculated as is indicated in Equation (3):

$$E_{dyn} = m^*\frac{\rho^*S}{C_n{}^{2*}I} \tag{3}$$

where $m$ is the slope of the regression line ($x = 1/L^4$, $y = f^2$).

### 2.3. Non-destructive testing by ultrasonic technique

In addition to the NDT by transversal free vibration, all pieces were tested by the Sylvatest DUO ultrasound equipment (CBS-CBT, Lausanne, Switzerland) with conical 22 kHz sensors. The transducers were placed at the ends of the samples (direct method).

The dynamic modulus of elasticity ($E_{din\ ultra}$, MPa) was obtained according to Equation (4).

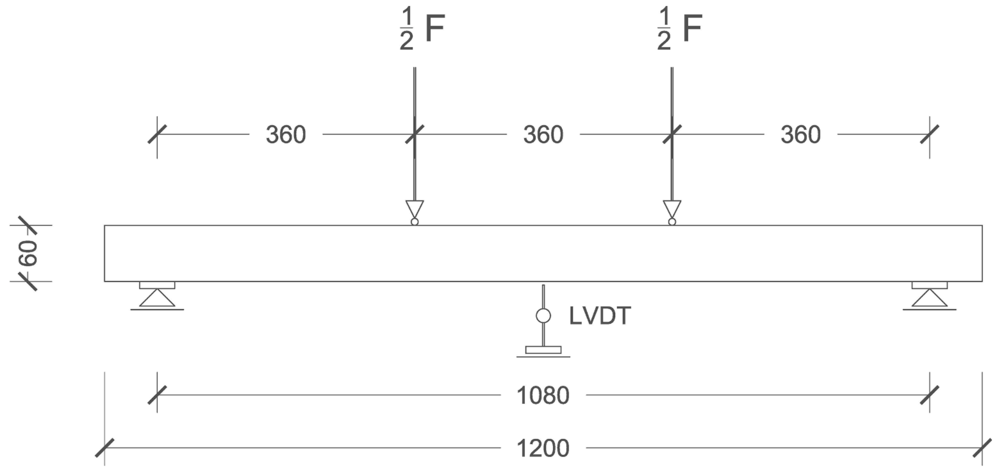


**Fig. 2.** Static bending testing according to EN 408:2011 [59] (dimensions in mm).

$$E_{din\ ultra} = V^2 \cdot \rho \tag{4}$$

Where V is the ultrasonic velocity (m/s) and ρ is the density (kg/m$^3$) of each piece.

### 2.4. Density and static bending testing

Finally, every wood sample was tested in a universal testing machine following a four-point bending test configuration in accordance with standard EN 408: 2011 [59]. The test configuration is shown in Fig. 2.

The global bending modulus of elasticity (MOEGTO) was obtained by recording the deformations measured during the test with a linear variable differential transformer (LVDT) located at the center of the span. The tests were performed with a constant displacement speed of 10 mm/min. The section of the stress–strain curve for which the beam's stiffness was calculated corresponded to between 10 % and 40 % of the estimated maximum load for all beams. When 40 % of the estimated maximum load was reached the test was finished and the sample was removed without any damage. The MOEGTO was determined according to Equation (5) (EN 408:2011 [59]).

$$MOEGTO = \frac{3al^2 - 4a^3}{2bh^3\left(2\frac{w2-w1}{F2-F1} - \frac{6a}{5Gbh}\right)} \tag{5}$$

Where $a$ (mm) is the distance between a loading point and the nearest support in a bending test, $l$ (mm) is the distance between supports, $b$ (mm) is the width of the cross-section, $h$ (mm) is the height of cross-section, $w$ (mm) is the deformation, $F$ (N) is the force, $G$ (MPa) is the transverse modulus of elasticity. In this case, G was taken as infinite according to EN 408:2011 [59].

### 2.5. Statistical treatment of experimental data

All statistical analyses were performed using R software (v. 4.1.1:2021) [64]. The assumptions of independence, normality, and homoscedasticity were checked for the variables under study ($E_{din\ vibra}$, $E_{din\ ultra}$, MOEGTO, and density). The normality of the data was tested for all populations using the Shapiro–Wilk normality test and Q–Q normal probability plots. The homoscedasticity requirement was contrasted by Bartlett's test. Since it was not met on numerous occasions, the usual comparative analysis by ANOVA could not be used. Robust comparison methods Welch's heteroscedastic F-test with trimmed means and Winsorized variances were used instead. This robust procedure tests for equality of means by replacing the usual means and variances with trimmed means and Winsorized variances [65,66], together with bootstrapping and comparison using robust homogeneous groups.

For the validation of the models, a 10-fold cross-validation method was used. 10-fold cross validation is a method used to evaluate the accuracy and performance of a machine learning model. The objective is to determine the reliability of a model's predictions when it is trained and tested on different data sets. In this method, the data set is divided into 10 equal parts or folds. 9 of the folds are used to train the model while the remaining fold is used to test it. This process is repeated 10 times, with each fold serving as the test set once. The 10-fold cross validation helps to reduce the risk of overfitting, which is when a model is too closely tailored to the training data and does not perform well on new data. By testing the model on different sets of data, the 10-fold cross validation provides a better estimate of its performance on new data. The results from the 10 tests are then averaged to obtain a final score, which provides a more accurate representation of the model's performance. The objective of 10-fold cross validation is to create a more

**Table 1**
Descriptive analyses: density.

| Specie | Density (kg/m$^3$) | | | Normality test | | Homoscedasticity test | Robust homog. Groups |
|---|---|---|---|---|---|---|---|
| | | | | Shapiro-Wilk | | Bartlett test | |
| | Mean ± IC** | CV (%) | range | W | p-value | p-value | |
| *Paulownia elongata* | 243.35 ± 14.23 | 9.2 | 68.0 | 0.913 | 0.234 | 0.026 * | e |
| *Populus × euramericana* | 389.37 ± 28.56 | 11.5 | 151.5 | 0.974 | 0.954 | | d |
| *Picea sitchensis* | 409.97 ± 27.37 | 9.3 | 109.6 | 0.903 | 0.2339 | | d |
| *Pinus sylvestris* | 569.43 ± 19.10 | 5.3 | 94.2 | 0.945 | 0.574 | | c |
| *Fagus sylvatica* | 672.75 ± 36.24 | 8.5 | 133.0 | 0.793 | 0.008 * | | b |
| *Quercus pyrenaica* | 766.16 ± 37.90 | 7.8 | 194.0 | 0.864 | 0.055 | | a |

* p-value < 0.05 implies that the assumption of homoscedasticity is not met.
** IC 95% Interval of Confidence.

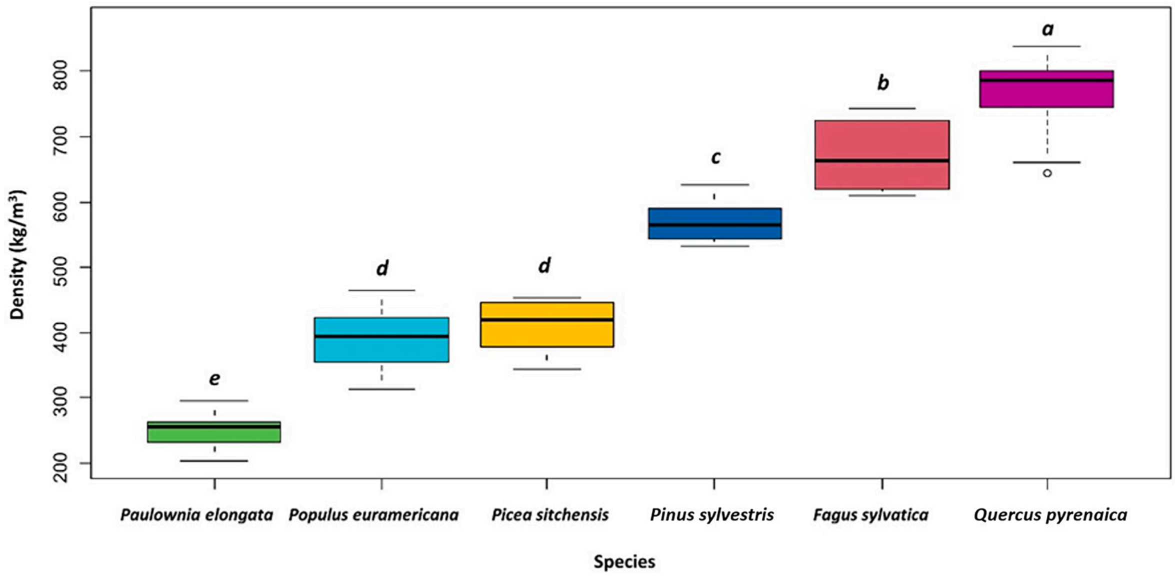


**Fig. 3.** Density box-plots of each species tested.

**Table 2**
Descriptive analyses: MOEGTO.

| Specie | MOEGTO (MPa) | | | Normality test | | Homoscedasticity test | Robust homog. Groups |
|---|---|---|---|---|---|---|---|
| | | | | Shapiro-Wilk | | Bartlett test | |
| | Mean ± IC** | CV (%) | range | W | p-value | p-value | |
| *Paulownia elongata* | 4933.1 ± 186.9 | 6.0 | 849.1 | 0.925 | 0.335 | 2.87e-04 * | d |
| *Populus* × *euramericana* | 9173.9 ± 1009.3 | 17.3 | 5081.3 | 0.959 | 0.767 | | c |
| *Picea sitchensis* | 6649.3 ± 1237.1 | 26.0 | 4620.3 | 0.904 | 0.242 | | c d |
| *Pinus sylvestris* | 13462.6 ± 365.4 | 4.3 | 2012 | 0.978 | 0.976 | | a |
| *Fagus sylvatica* | 12139.0 ± 542.2 | 7.0 | 2429 | 0.844 | 0.031 | | b |
| *Quercus pyrenaica* | 12437.3 ± 934.3 | 11.8 | 4908.7 | 0.945 | 0.568 | | a b |

* p-value < 0.05 implies that the assumption of homoscedasticity is not met.
** IC 95% Interval of Confidence.

reliable and robust model by validating its accuracy on multiple data sets [67]. In this case, each cross-validated (10 fold) was repeated 5 times and the average statistical values were obtained. To obtain more robust estimators, the operation was repeated 200 times and the mean values were obtained. Finally, another model check was performed, involving inputting values from a spruce species with distinct geometric and anatomical characteristics from those used to create the model (as was described in 2.1. Testing materials) and verifying that they fall

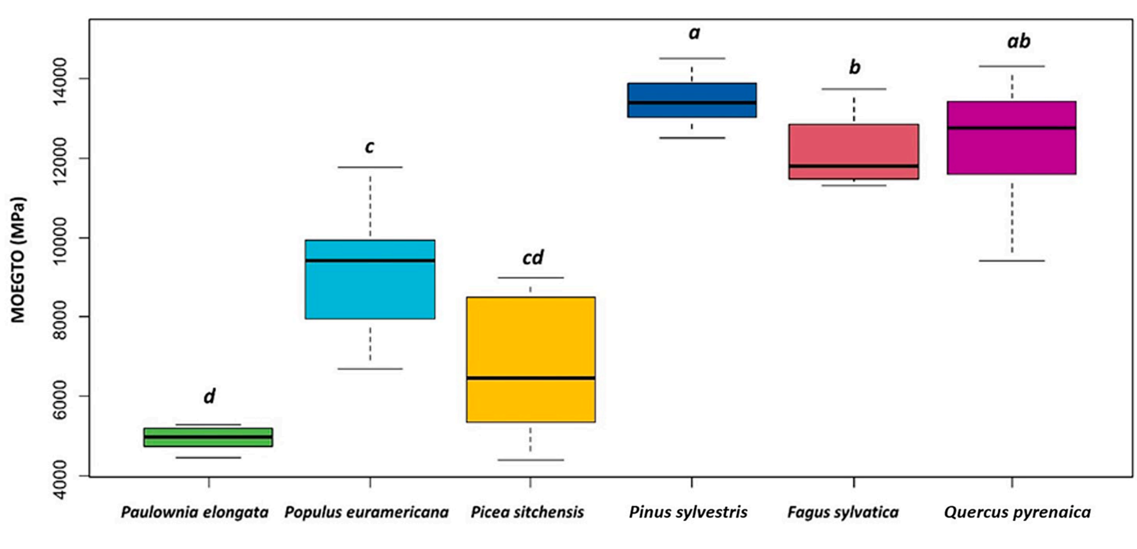


**Fig. 4.** MOEGTO box plot of each species tested.

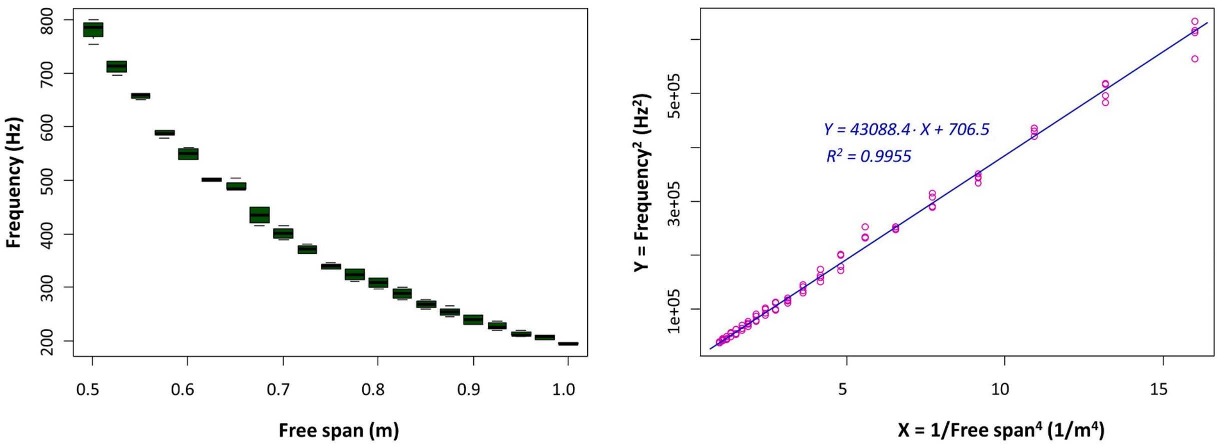


**Fig. 5.** Paulownia's example of the relationship between cantilever beam free span to vibration and its first natural frequency of vibration (left), and its transformation to linearise this relationship (right).

**Table 3**
Regression lineal ($x = 1/L^4$, $y = f^2$) Residual Standard Error (RSE) and multiple R-squared per specie.

| Specie | *Vertical (V)* | | | | *Horizontal (H)* | | | |
|---|---|---|---|---|---|---|---|---|
| | RSE | | Multiple R-squared | | RSE | | Multiple R-squared | |
| | Min | Max | Min | Max | Min | Max | Min | Max |
| *Paulownia elongata* | 15,750 | 31,380 | 0.993 | 0.998 | 7519 | 12,090 | 0.994 | 0.997 |
| *Populus × euramericana* | 44,230 | 65,960 | 0.963 | 0.984 | 22,940 | 37,210 | 0.951 | 0.976 |
| *Pinus sylvestris* | 36,860 | 72,190 | 0.957 | 0.983 | 2981 | 11,490 | 0.995 | 0.999 |
| *Fagus sylvatica* | 148,700 | 418,500 | 0.974 | 0.998 | 60,120 | 88,020 | 0.993 | 0.998 |
| *Quercus pyrenaica* | 124,800 | 216,800 | 0.995 | 0.998 | 68,930 | 110,200 | 0.993 | 0.997 |

within the model's prediction range. This allows for assessing the accuracy of the model's performance in the other five species.

## 3. Results and discussion

### 3.1. Density and static bending testing

Descriptive analyses of the density values are shown in Table 1:

Where it can be seen that the mean value of density per species ranged between 243 kg/m$^3$ for Paulownia and 766 kg/m$^3$ for Oak. The robust homogeneous group analysis establishes that there are statistically significant differences between all species except Poplar and Spruce (Fig. 3).

The density values were similar to those published for these species by other authors in *Populus × euramericana* I-214 [8,12,68] in *Pinus sylvestris*, [30] in *Pinus nigra*, [33] in *Quercus pyrenaica* and *Fagus sylvatica*, [69] in *Paulownia elongata*, and [34] in *Picea sitchensis.*

Descriptive analyses of MOEGTO values are shown in Table 2.

It can be seen that the different species cover a wide range of mean values from approximately 4900 MPa for Paulownia to 13500 MPa for Scots pine. The high value of Scots pine, which forms a homogeneous group with oak, is noteworthy. It is also worth noting Spruce's low value and high variability, mainly because the pieces present a greater number of singularities as they come from an unclassified batch (Fig. 4).

The MOEGTO values in *Populus × euramericana, Pinus sylvestris, Quercus pyrenaica* and *Fagus sylvatica* were higher than those published for the same species by other authors [8,12] in poplar, [30,68], in pinus, and [33] in oak. This is due to the fact that the pieces tested were free of singularities or defects that affect the mechanical properties of the wood. However, in the case of *Picea sitchensis*, the values were lower than those published by other authors [34]. This could be because they were pieces with knots and juvenile wood in which the bending properties are lower [70].

### 3.2. Non-destructive testing by transversal free vibration

First, $E_{din\ vibra}$ slope ($m$) was calculated using the values obtained for each of the 4 trials per free span. Fig. 5 shows an example of the Paulownia relationship between free beam length to vibration and its first natural frequency of vibration, and its transformation to linearise this relationship.

The results obtained for the $E_{din\ vibra}$ slope ($m$) of all the species tested in the two directions are shown in Table 3.

It can be seen that the $R^2$ values in all cases are higher than 95 % ($R^2 \geq 0.95$). This makes it possible to determine the value of the slope with great accuracy. The values for spruce are not reflected as they are only

**Table 4**
Transversal vibration dynamic modulus of elasticity $E_{din\ vibra}$ (*H* and *V*) per species.

| Specie | $E_{din\ vibra}$ *V* (MPa) | | $E_{din\ vibra}$ *H* (MPa) | | Paired *t*-test |
|---|---|---|---|---|---|
| | Mean* (CV%) | Min – Max* (p value SW test) | Mean* (CV%) | Min – Max* (p value SW test) | $E_{din}V$ vs $E_{din}H$ p-value |
| *Paulownia elongata* | 5239.8 (12.0) | 4251.1 – 6636.6 (0.791) | 5143.5 (11.3) | 4280.7 – 6497.6 (0.618) | 0.0690 |
| *Populus × euramericana* | 9271.2 (16.54) | 6996.2 – 11860.5 (0.636) | 9354.9 (16.9) | 6733.0–11691.2 (0.356) | 0.3211 |
| *Pinus sylvestris* | 13105.4 (5.0) | 11692.7 – 14047.1 (0.648) | 13155.14 (4.3) | 12140.0 – 13804.7 (0.262) | 0.5603 |
| *Fagus sylvatica* | 13855.4 (8.6) | 12155.2 – 15412.4 (0.730) | 13918.5 (7.4) | 12205.3 – 15353.8 (0.562) | 0.2881 |
| *Quercus pyrenaica* | 14493.5 (8.6) | 12178.6 – 16038.0 (0.131) | 14566.8 (10.1) | 11987.1 – 16863.4 (0.899) | 0.9957 |

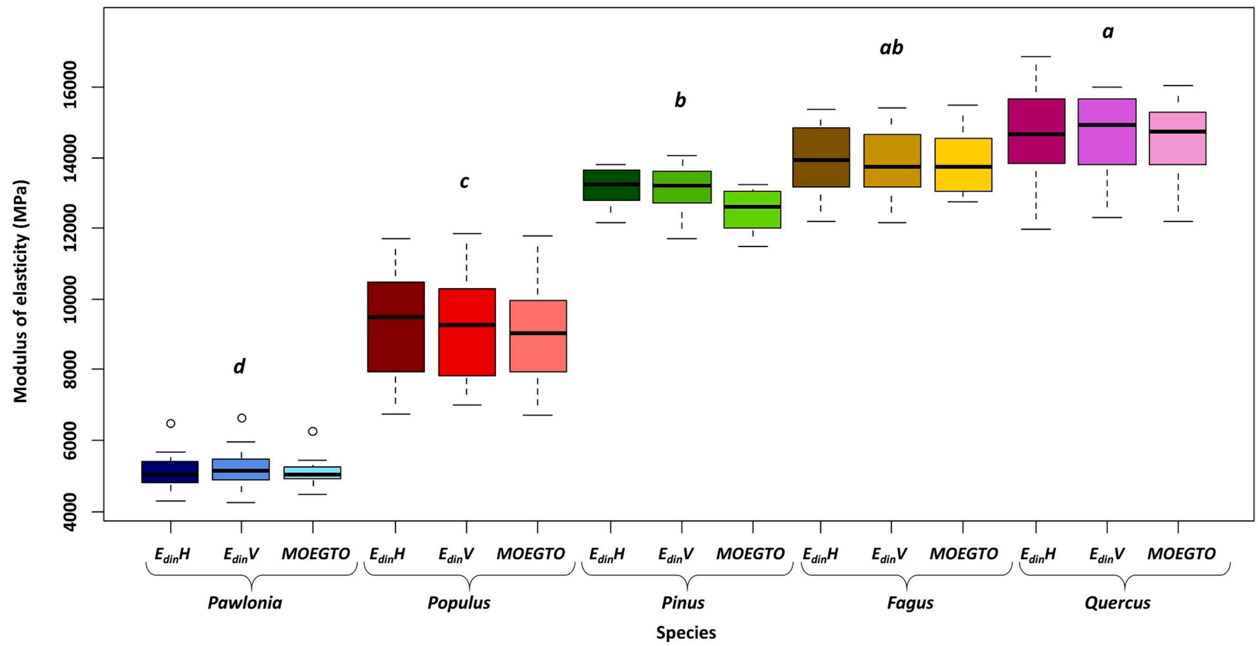


**Fig. 6.** Box plot of horizontal $E_{din\ vibra}$, vertical $E_{din\ vibra}$, and MOEGTO per species.

**Table 5**
Ultrasound dynamic modulus of elasticity.

| Specie | $E_{din\ ultra}$ (MPa) | | | Normality test | | Homoscedasticity test | Robust homog. Groups |
|---|---|---|---|---|---|---|---|
| | | | | Shapiro-Wilk | | Barlett test | |
| | Mean ± IC | CV | range | W | p-value | p-value | |
| *Paulownia elongata* | 7060.8 ± 1009.0 | 22.5 | 68.0 | 0.903 | 0.176 | 0.561 | d |
| *Populus × euramericana* | 12570.5 ± 1304.9 | 16.3 | 6598.1 | 0.926 | 0.335 | | b |
| *Picea sitchensis* | 9405.0 ± 1539.0 | 22.9 | 14617.5 | 0.976 | 0.937 | | c |
| *Pinus sylvestris* | 16780.3 ± 652.3 | 6.1 | 3198.9 | 0.930 | 0.377 | | a |
| *Fagus sylvatica* | 15894.6 ± 937.4 | 9.3 | 4465.9 | 0.942 | 0.528 | | a |
| *Quercus pyrenaica* | 15917.1 ± 1133.9 | 11.2 | 194.0 | 0.964 | 0.841 | | a |

tested in one orientation as a model check.

Table 4 shows the mean values, dispersion values, and normality tests for the transverse vibration dynamic modulus of elasticity ($E_{din\ vibra}$) for vertical (V) and horizontal (H) orientation.

There were no statistically significant differences between the values obtained in the horizontal or vertical direction (p-value > 0.05) as shown by the pairwise comparisons for each species, which coincides with other authors [7,68,71] in different pine species.

The mean $E_{din}$ values were higher than those published by other authors in simply supported vibration tests because the pieces analysed in this study were free of defects compared to the lower values in structural wood of the same species [7,8,12,33,68].

The comparisons made between the values of horizontal $E_{din\ vibra}$, vertical $E_{din\ vibra}$, and MOEGTO by species were similar in all cases. Homogeneous groups identified alphabetically were established as shown in Fig. 6.

### 3.3. Non-destructive testing by ultrasonic technique

Table 5 shows the mean values, dispersion values, and normality tests for the ultrasound time of flight dynamic MOE ($E_{din\ ultra}$). A robust homogeneous groups test is also presented for each species.

The mean $E_{din,ultra}$ values in *Populus × euramericana*, *Pinus sylvestris*, *Quercus pyrenaica* and *Fagus sylvatica* were higher than those published for the same species by other authors [8,12] in poplar, [7,30,68] in pinus, [33] in oak. This is likely due to the fact that the pieces tested

**Table 6**
MOEGTO prediction based on dynamic modulus of elasticity.

| Species | MOEGTO vs $E_{din\ ultra}$ | | | MOEGTO vs $E_{din\ vibra}$ | | |
|---|---|---|---|---|---|---|
| | p-value | RSE | Adjusted $R^2$ | (p-value) | RSE | Adjusted $R^2$ |
| *Paulownia elongata* | 1.710 e-03 | 184.4 | 0.607 | (4.0 e-04) | 160.3 | 0.703 |
| *Populus × euramericana* | 5.09 e-04 | 886 | 0.689 | (2.24 e-04) | 818 | 0.735 |
| *Picea sitchensis** | 8.1 e-04 | 919.5 | 0.714 | (1.4 e-03) | 903.8 | 0.719 |
| *Pinus sylvestris* | 7.29 e-03 | 413.7 | 0.582 | (2.08 e-05) | 234.5 | 0.836 |
| *Fagus sylvatica* | 9.41 e-04 | 505 | 0.649 | (9.7 e-05) | 404.9 | 0.775 |
| *Quercus pyrenaica* | 2.22 e-04 | 756.5 | 0.735 | (1.2 e-04) | 712.7 | 0.765 |
| Global | 2.2e-16 | 851.7 | 0.937 | (2.2e-16) | 685.3 | 0.959 |

*not included in the global model calculation.

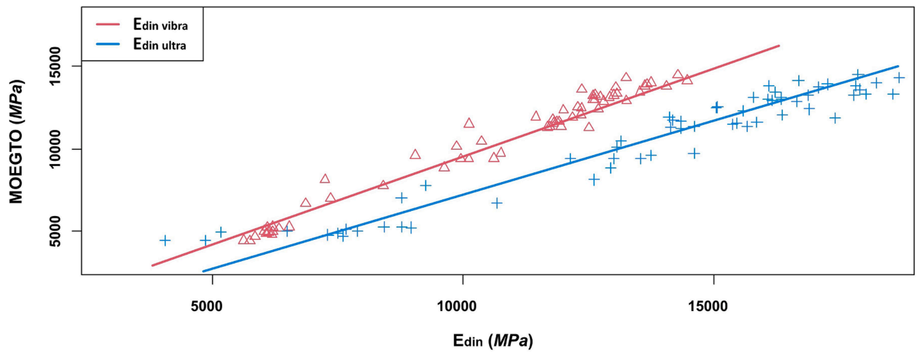


**Fig. 7.** Regression models $E_{din\ vibra}$ vs $E_{din\ ultra}$.

**Table 7**
Robust cross-validation tests indicators mean: RMSE, $R^2$, and MAE.

| | Vibration | SD | Ultrasound | SD |
|---|---|---|---|---|
| Intercept ($\sigma_{Int}$) | −1559.266 | 56.158 | −766.2172 | 171.732 |
| Slope ($\sigma_{Slope}$) | 1.125 | $6.26 \cdot 10^{-3}$ | 0.816 | $1.20 \cdot 10^{-2}$ |
| RMSE ($\sigma_{RMSE}$) | 664.528 | 297.519 | 853.939 | 182.247 |
| $R^2$ ($\sigma^2_R$) | 0.972 | $2.51 \cdot 10^{-2}$ | 0.958 | $2.38 \cdot 10^{-2}$ |
| MAE($\sigma_{MAE}$) | 525.8918 | 198.680 | 756.524 | 172.104 |

were free of singularities or defects that affect the mechanical properties of the wood.

### 3.4. MOEGTO prediction based on dynamic modulus of elasticity.

Table 6 shows the values of the main statistics of the regression models of the predictor variables ($E_{din\ vibra}$ and $E_{din\ ultra}$) concerning MOEGTO.

It can be seen that, for each species, the adjusted $R^2$ values were higher for vibration than for the ultrasound technique. The Residual Standard Error (RSE) value shows that, in all cases, the errors of the vibration model were lower than in the case of the ultrasound model.

The mean values of the dynamic modulus of elasticity obtained by ultrasound were higher than those obtained with the vibration method, as is shown in Fig. 7, an aspect that has already been reflected in previous works comparing both methods of non-destructive classification [7,12,33,68,72–74].

### 3.5. Models validation

The results of the robust cross-validation tests are shown in Table 7. The smaller the root mean square error (RMSE) and mean absolute

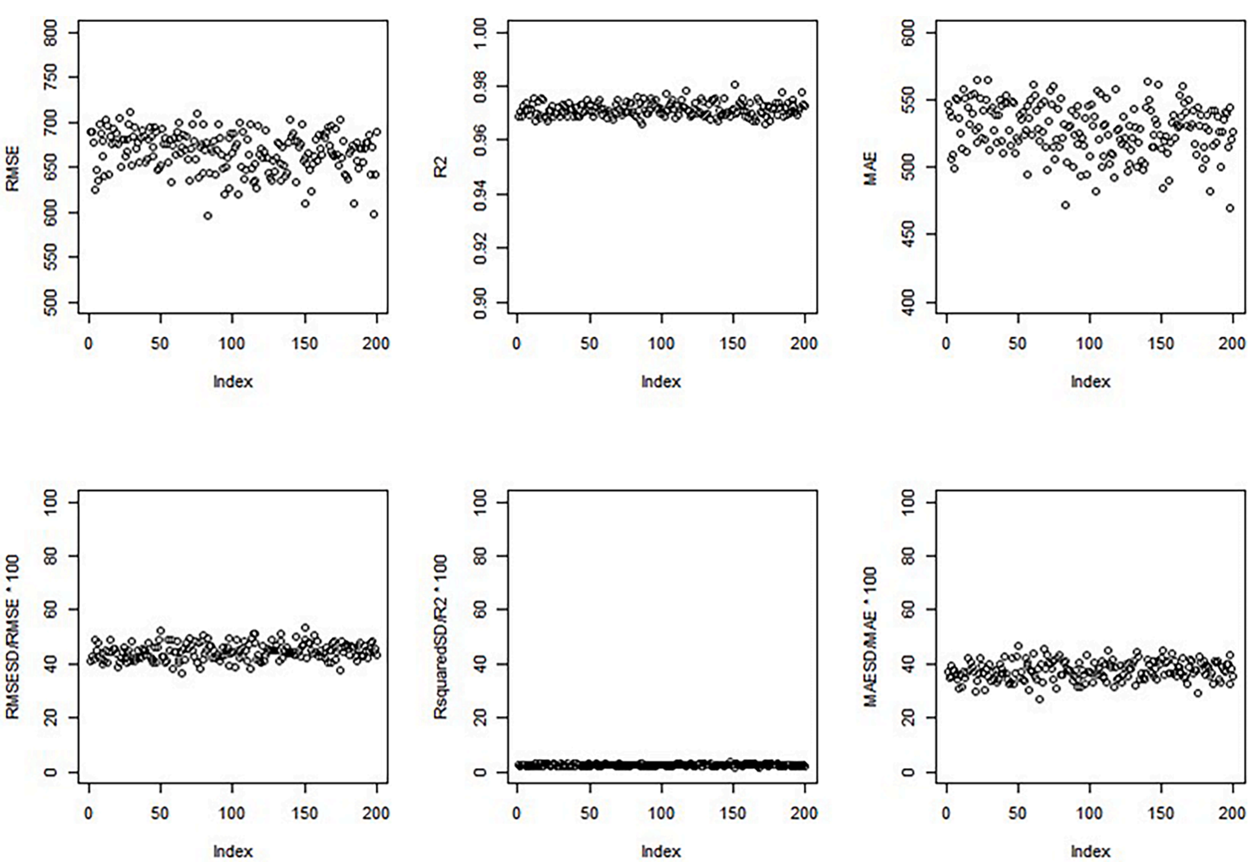


**Fig. 8.** Plot of the statistics for each of the 200 iterations of the cross-validation tests.

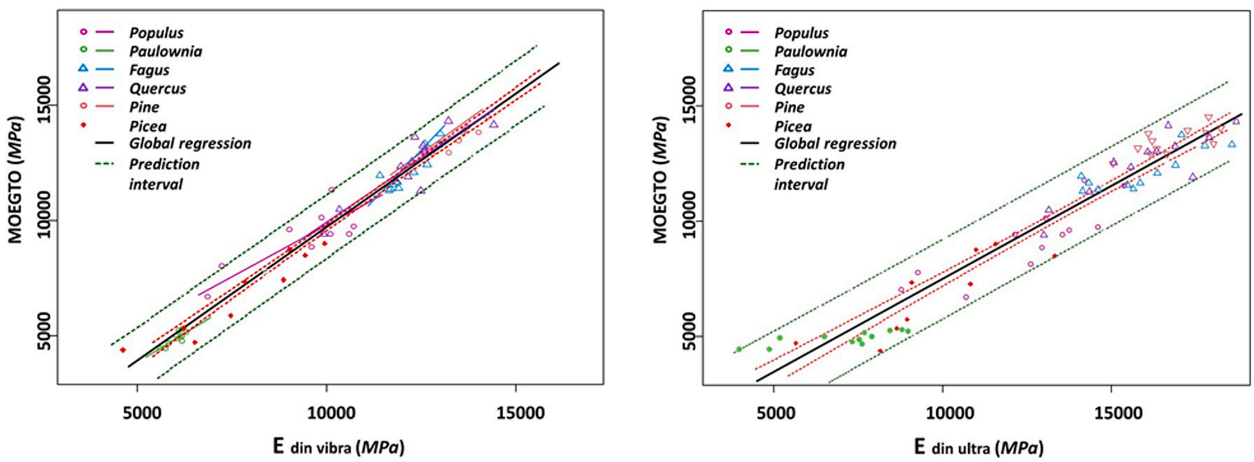


**Fig. 9.** General models MOEGTO vs $E_{din\ vibra}$, and MOEGTO vs $E_{din\ ultra}$.

error (MAE) value the better, and while the larger the $R^2$, too better. Consequently, the MOEGTO estimation model using transversal vibrations was the best one. Fig. 8 shows the values of the statistics for the 200 iterations of the cross-validation test. The robustness of the model is shown by the uniformity of the values of the statistics.

This is also observed in Fig. 9, where the dotted lines are further apart, i.e. to get the prediction right it needs more "width" concerning the regression line compared to vibrations, which in all statistics is better. These results coincide with other published articles in different pine species [7,68,71]. It can be seen that the prediction limits of the global regression contain all the points of the new species (*Picea sitchensis*).

## 4. Conclusions

The behaviour of the fundamental vibration frequency obtained for wood samples (*Populus* × *euramericana* I-214, *Fagus sylvatica* L., *Quercus pyrenaica* L., *Paulownia elongata* S.Y.Hu, *Pinus sylvestris* L., and *Picea sitchensis* Trautv. & G.Mey) in cantilever beams exhibited an inversely proportional relationship with the span length of the sample, which results in strong significant differences for the different species.

Taking into account the material used, which is heterogeneous in terms of its organic composition and anisotropic behaviour, the results obtained can be considered highly valuable. The final $R^2$ value obtained in the global modulus of elasticity of these different species for the prediction model as a function of density and vibration frequency was>95 % and higher than that obtained by ultrasound methods.

This study demonstrates that transversal vibrations in cantilever beams is an extremely precise tool for predicting the static modulus of elasticity of wood and for performing quality control and inspection of wood for structural use. This could be further developed into a useful tool for timber grading.

## CRediT authorship contribution statement

**Luis Acuña:** Conceptualization, Methodology, Investigation, Data curation, Writing – original draft. **Roberto Martínez:** Data curation, Writing – review & editing. **Eleana Spavento:** Methodology, Investigation, Writing – original draft. **Milagros Casado:** Methodology, Supervision, Investigation, Writing – original draft. **Javier Álvarez-Martínez:** Investigation. **Conan O'Ceallaigh:** Investigation, Writing – review & editing. **Annette M. Harte:** Writing – review & editing. **Jose-Antonio Balmori:** Methodology, Investigation, Writing – review & editing.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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