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Mathematical Problem Solving through the Lens of Ethics and Aristotelian Attitude: A Case Study

José M. Marbán [1,*] and Joaquín Fernández-Gago [2]

[1] Department of Didactics of Experimental Sciences, Social Sciences and Mathematics, University of Valladolid, 47011 Valladolid, Spain
[2] Department of Didactics of Social Sciences, Experimental Sciences and Mathematics, University of Malaga, 29071 Malaga, Spain; jfernandezg@uma.es
* Correspondence: josemaria.marban@uva.es

Abstract: Attitudes are central to the affective domain in mathematics. However, we still have a long way to go to reach a clear consensus on the characterization of such construct and its operationalization, as there are a wide variety of intertwined approaches depending on the theoretical framework that accompanies each of them. In this article, we explore the idea of incorporating ethics as part of a new characterization of attitude in problem solving contexts by means of an instrumental case study consisting of a key informant 10th grade student. Data were collected via classroom observations, questionnaires, semi-structured interviews and written productions. Results were analyzed using the lens of Aristotelian Ethics and the Theory of Creative Intelligence and show how happiness, responsibility and strength are key concepts that help to describe and understand the way the student behaves when solving mathematical problems. We conclude that besides the traditional attention paid to attitudes towards math, it is worth directing our attention towards attitudes influenced by ethics and personal affect as a whole.

Keywords: attitude; ethics; case study; mathematics; problem solving

MSC: 97C20

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1. Introduction

There is a general consensus that the mathematical competencies of citizens in certain societies, including Spain, are disturbingly low. This perception, which had previously been brought to light by different local diagnostic tests at the end of the 20th century, has resurged in the last two decades after reports were published by international programs such as TIMSS and, fundamentally, PISA, causing the social and political eye to focus on the search for reasons behind this deficiency in mathematics education.

A significant amount of attention has been paid to researching the latent factors behind academic achievement in mathematics education together with research into several possible interrelationships that might influence it. Some authors have focused their attention on analyzing the relationship between attitudes and other affective elements, such as anxiety within mathematics learning and, more specifically, in contexts of mathematical problem solving [1–7]. Despite major advances in our understanding of the interaction between the affective and cognitive domains, conclusive data on causal issues have not been found yet.

The present article focuses particularly on the component of the affective domain that has received the most attention in studies related to this field: attitude. This construct is critical to the understanding and interpretation of situations and behavior in different contexts of mathematics education. Attitude is a construct closely related to emotion and belief systems and although the main objective is to analyze attitudes, this article also aims to highlight the relevance of ethics in attitude when solving mathematics problems, shining light on this question and leading the way towards a new characterization of attitude.

Attitude as a construct has been a focus of research in mathematics education in the last decades, and is generally associated with the affective domain, as defined by [1]: a construct of an amalgamation of attitudes, beliefs and emotions. Later, Ref. [8] would include a new component encompassing values, moral judgement, and ethical judgement all as one intervening factor in the decision-making process before problem solving. More specifically, Ref. [8] (p. 212) explain: "But what has this to do with mathematical learning and problem solving? The importance of the values/morals/ethics component of affect pertains to the individual's feelings, tacit or overt, about learning, problem solving success, mathematic behavior, etc., as (morally desirable) virtues or values".

Using a structure in which the attitude construct does not explicitly appear, Ref. [9] (p. 137) relate the above-mentioned factor with meta-affect when affirming "Beliefs, values, and ethics also play a role in meta-affect, as they influence how an emotion functions ecologically in the individual's personality—i.e., the purpose served by the emotional feeling". In this structure, there is a component called *mathematical integrity*, which refers to the postures related to when a mathematical problem is considered "right," when the learner's understanding suffices, when a problem solution is considered satisfactory or when solving a mathematics problem is considered to be worth it. These studies do not mention how different ways of conceiving ethics can influence behavior of the problem solver.

Within the affective domain, attitudes play a large part in mathematics education and have generated a considerable amount of research. Ref. [10] point out that, in the world of mathematics, attitudes have been less clearly defined than in the field of psychology, where attitudes are thought of as a predisposition with a high emotional load that influences conduct; in this way, the influence attitudes have on the teaching–learning process and mathematics academic achievement is well-known in Refs. [11,12], along with the influence attitudes have on anxiety in mathematics [13–17]. Studies of this have found that students with better attitudes towards mathematics tend to have better perceptions of the usefulness of math, higher intrinsic motivation to study, better mathematic self-concept [18], higher confidence in mathematics learning [19] and, above all, they show more willingness to approach the subject [20,21].

According to [3,22,23], one can see the lack of consensus on attitude among researchers, as there are many theoretical frameworks to work from. At the same time, it is important to note the disparity between instruments used in the mediation and study of this area. In any case, most works generally observe certain aspects, tendencies and ideas that merit mentioning. One of them is the search for relationships and associations between attitude and academic achievement. The second aspect is relative to the use of dichotomic classifications of positive and negative attitudes towards mathematics. Finally, one aspect integral to the studies mentioned is the establishment of relationships and interrelationships between the constructs of attitudes, beliefs and emotions, constructs that have yet to be made "universal" not only in the context of mathematics education but also in many others, for instance, science education [24]. In this last case, the TMA model by [22,25], a three-part, three-dimensional model composed of emotional disposition towards math (emotion), vision of math (beliefs) and perceived competence in math (self-perception), has become an important reference to look at.

As such, research on attitudes in mathematics could be divided into positivists, works that focus on looking for the relationship between attitudes and achievements, and interprets, in other words, those studies focused on the understanding of the phenomenon more than on quantitative links between attitudes and achievements.

In the second group, it is important to highlight the work by [1], who reconceptualized the attitude construct by proposing a structure in which beliefs are elements that cause emotion to come up. Moreover, in this structure, attitudes are built from repeated emotional reactions, which establishes that both social context and individual experience contribute to the origin of beliefs. McLeod's work has inspired subsequential studies on beliefs, emotions, and attitudes to have two components: one that refers to cognition and the other to stability.

Beliefs are considered to be more stable and cognitive while emotions are less stable and less cognitive.

Refs. [19,26] for support, arguing that beliefs are enormously influential on the surge of emotions and that repeated emotional reactions are the origins of attitudes. Lastly, it is important to note that for McLeod, the social (cultural) context and individual experiences are a large contributing factor to the creation of beliefs. This model, despite having a large theoretical base, has received some criticism stating that the concept of attitude is a blind spot, seen as a mix of cognitive (beliefs), affective (emotions) and conative (behavior) aspects. In response, attitude is beginning to be linked to the observer and not the participant.

In social psychology, on the other hand, recent theories present consensus on the multidimensionality of the attitude construct understood as a tripartite model with three components: affective, cognitive and behavioral. If this model is applied to mathematics education, attitude can, therefore, be characterized into three components: emotional disposition towards maths, beliefs about maths and behavior related to maths. The previous model tends to fit attitudes into only one general emotional disposition. Nonetheless, one limitation to the threefold notion of attitude as a combination of beliefs, emotions and behavior, is the circular reasoning pointed out by [27] (p. 346) when affirming that "we accept that beliefs influence thoughts and actions, and sometimes hidden beliefs, which operate subconsciously, are detected through how people act or think". There are also limitations to the notion of attitude as emotional disposition, despite its advantages, as it does not include cognitive aspects. Positive attitude can be defined in terms of positive emotions, for example. Being able to define attitudes as positive or negative is critical to the development of this construct and the framework of this study.

A third definition that avoids circular reasoning has been proposed by [28]. According to them, attitudes towards maths can be described as a model of beliefs and emotions associated with maths. Other authors such as [29] differentiate between attitude towards mathematics, with a clear affective component and mathematical attitudes, with a clear cognitive component. Ref. [30] establishes various types of attitudes, such as appropriate initial attitude and the enjoyment of a challenge, and two purely mathematical attitudes: the inductive attitude and the precision and rigor attitude.

Lastly, some authors such as [22] contemplate the idea of attitude in association with the attitude of some teachers, such as complaining in response to the uncontrollable errors of their students.

As one can see, there is a clear lack of consensus among authors on the construct of attitude, which must not be interpreted as a weakness or limitation but rather a natural manifestation of the complexity of the affective domain itself in mathematics. This situation calls for a clear conceptualization of the components and constructs referenced by every author in their works to avoid possible adverse effects for the lack of consensus. Consequently, the present authors wish to precisely describe the starting point of this study, based on the synthesis given insofar in which the main focal points influencing the theoretical framework of this paper have been stated along with those that have influenced the configuration of the affective domain components are taken into consideration here. Thus, concerning the relationship in the first place that the attitude construct has with emotions and beliefs, we maintain that there are self-perception beliefs and beliefs about mathematics, as proposed by [22,25], that influence the behavior of students. Because the tripartite model is in use, one must implicitly accept that when the student's attitude is being studied, one must also look into their emotional disposition and beliefs. Moreover, in the present study, one must consider that students' own beliefs with respect to values/morals/ethics can explain their actions. In the second place, this study steps away from qualifying attitudes as positive or negative, because, as [31] proports, one can have a positive or negative attitude relative to the point of view from which the task is evaluated, and both are driven by the person's values. In the third place, in line with [30], the present study marks two different aspects of attitude. The first is psychological, which is related to the enjoyment of a challenge, and

the other is related to mathematics in practice. In the fourth place, this characterization is based on the voluntary disposition of the individual, which sometimes goes unnoticed in other characterizations. Lastly, comparable to [32], this paper extends the characterization of attitude to attitude towards mathematics problems. Finally, we think it is worth mentioning [33] although it is not concerned with attitudes as it has somehow influenced the authors when facing this study.

In sum, this study takes into account all the critical aspects as mentioned and commented on in this section. Using a particular theoretical framework which includes the attitude construct, the behavior of one student playing the role of an instrumental case study is analyzed through interpretative lens, with comments and a complimentary analysis supported by the previously mentioned theoretical framework and taking into account the behavior of students in other studies, such as those of [32,33], when faced with mathematics problems. As such, the case is thought as a contribution to deepen the topic of attitude and to provide useful information to refine already existing theories by means of conclusions that might lead to starting points for the statement of certain hypothesis or to the design of new case studies. Hence, the case plays a secondary and supporting role in arriving at the formulation of statements about the object of study and, in particular, to provide preliminary tentative answers to the following research questions: is it feasible to create/extend existing characterizations of attitude by including Aristotelian Ethics; and, subsequently, is it relevant to use this characterization to analyze how some students perform in mathematical problem solving?

Next, the theoretical framework of this study is presented, followed by the methodological design and the presentation and discussion of the results derived mainly from interviews and writings associated with mathematical problem solving. The paper ends with conclusions and includes some comments on the limitations of the study as well as possible future lines of research.

2. Theoretical Framework

Aristotelian Ethics and the Theory of Creative Intelligence [34] together make up the bases of the theoretical framework of the present study. Hence, now we proceed to first explaining Aristotelian Ethics for later evaluating said ethics through Marina's work.

It is important to begin with the definition of ethics and morals used in this article. Here, morals are understood as customs or norms that a society accepts and follows and that are associated with certain values. On the other hand, the word "ethics" is understood in the way that [35] (p. 1058) explained " . . . ethical historians have limited their studies to those ideas of moral character with a philosophical basis, that is to say, that instead of being supposed, ethics are examined to their core; in other words, they are philosophically justified". Therefore, ethics includes the principles we base ourselves on and that determine our actions within what is moral. As the participating student rationally reflected on if her actions to resolve the problems were linked to being happier or not, the present study is on the level of ethics.

One question that comes up at this point is why one must consider values/morals/ethics when understanding the actions taken to resolve a problem. One possible response has been provided earlier through our reference to [8] and will also be detailed here. Other authors who have considered ethics in mathematics education are [36–38]. We could also include here [39], even though he does not speak explicitly about ethics in his decision-making theory, as the principles exposed are part of the directions that a teacher can follow, which are deeply related to ethics. So, there are two approaches to dealing with ethics in mathematics education: on one side, emphasis on the teacher's decisions and, on the other side, emphasis on the decisions that students make. Our work is of the second type.

Ref. [9] establishes relationships between the set conformed by beliefs, values and ethics, on the one side, and individual meta-affect, on the other. Unlike [9], the authors of the present work consider that taking ethics/moral/values into account does not constitute by itself having meta-affective awareness. Moreover, certain convictions about this

ethics/morals/values "mixed bag," some of which are potentially beliefs, might model and influence in behavior when solving problems and thus involve meta-affectivity as well as how they can be a directing force behind meta-affectivity. Under these premises, since ethics is the most inclusive construct of the above-mentioned "mixed bag", from now on we will refer only to ethics.

Aristotle begins *Nichomachean Ethics* [40] writing about happiness and rationalizing the existence of a supreme being, a being that is *eudaimonia* or happiness. The problem is in what one considers to be happiness. For Aristotle, there are three different types of lives: the voluptuous life, the political life and the theoretical life. So, those who choose the first orbit identify happiness with pleasure. For those who choose, as Aristotle puts it, the voluptuous life, any other type of satisfaction would be ephemeral. The second type of life as categorized by Aristotle is one that is political, with happiness rooted in honor. Aristotle treats this one with more respect because recognition of another man's honor and deserving honor was morally relevant yet had degraded to confusion with fame. While fame comes from a mistaken evaluation or ridicule, the essence of honor comes from a moral evaluation. Even so, Aristotle did not consider honor but rather the power to reason as the maximum level of life to be reached.

Although Aristotelian notions included in the present study are described below, it is important to add that the present authors have not carried out exhaustive research into all the notions contemplated by Aristotle. In Book II, Aristotle speaks of moral virtue as a state of rational choice when deciding the best actions to take that balance the relationship between pleasure and pain. Aristotle affirms: "Hence the importance, as Plato points out, of having been definitely trained from childhood to like and dislike the proper things; this is what good education means" (p. 93). In this way, if the student manages to confront a problem despite the discomfort of not understanding it or not knowing how to start, then he/she either rejects the temptation to give up or the student accepts that learning can be beneficial, all despite negative emotions that can come up in the process, presenting coherency with Aristotle's ethics.

In Book III, the definitions of voluntarily and involuntarily appear. The involuntary act is due to a first principle extrinsic to man, such as force or ignorance, leaving the rational choice more restricted than the voluntary act in the sense that the act is within our power and is fruit of deliberation. The idea of attitude when confronted with a mathematics problem is within rational choice and, as we will see, is a choice of events before a challenge.

Book III focuses on courage as the term between fear and confidence and it affirms, as well, that the brave are action-takers. This could be the same in the case of some students who, despite feeling fear when they cannot resolve a problem, decide to act.

Apart from the authors who have studied the construct of attitude in Mathematics Education, some of whom have already been mentioned previously, the present study is fundamentally based on the works of Marina [34,41]. This author affirms that there is an intelligence responsible for receiving information, elaborating upon it, and producing responses, and it is called *computational intelligence*. There is another intelligence, *executive intelligence*, in charge of creating ideas based on the information produced by computational intelligence. By Marina's definition, a person can access, control, and direct their mental activity. Moreover, any intellectual task requires the same mental activities according for this author, from writing a poem to solving a mathematics problem. The way of doing so is what will determine if the person is more intelligent or less intelligent. Thus, for the author, intelligence does not exist as an independent ability. One cannot perceive, remember, imagine, compare, conceptualize, decide and, apart from that, be intelligent. One must use an adjective to indicate that intelligence has to do with the way of using one's faculties, for example: there is an intelligent way of watching, intelligent memory, an intelligent way of imagining, intelligent motivation, intelligent language and so on with other aspects. To use intelligence, one must invent projects. The present authors will focus on the project, invented or not, by our student to solve mathematics problems and investigate whether that project serves a greater project.

Ref. [34] (p. 184) considers that the voluntary construct is not in opposition to motivation, and affirms "Because of this, it is important to highlight that drive is the ability to give oneself orders and obeying meditated values and not only feelings.". How the student in this study talks to herself in her project will be present in the analysis. Ref. [34] (p. 140) establishes failures in intelligence and how they can be tied to beliefs, specifically beliefs related to emotions or feelings, affirming that "Under apparently spontaneous and original feelings there are fundamentally implicit beliefs taking place.".

For [41] (p. 163) attitudes are more mediated by intelligence than emotions. In the tension that exists between ideas/occurrences being produced by computational intelligence and then becoming actions, one can try to control the intelligence where such ideas/occurrences are produced. Thus, Marina characterizes attitude in the following way:

> It has to do with the voluntary disposition of intelligence, which can access, control, and manage mental operations to achieve a determined style of occurrences. This differs from character, which is also a fixed style of occurrences, because it is voluntary. This differs from habits because it is not automatic. The importance of attitude within vital economy is derived from the fact that it is the point where intelligence negotiates with feelings.

If a person begins to feel emotion when presented with a problem, tension or conflict between the desire to feel good and the desire to resolve the problem is created, which can create contradictory ideas such as either to abandon the situation, try to manage emotions and resolve the problem or even justify whether carrying out the problem, such as a challenge, makes sense or not. From the TMA point of view, the tripartite system used by the present authors would describe the actions taken, the attitude component, in terms of beliefs and emotions, while, for Marina, part of this description would also include the voluntary decision to manage these tensions and contradictions. Furthermore, passive attitudes could be present when there is certain stimulation and we decide that executive functions do not go beyond setting a goal of non-action, and active attitude when one decides to act after being confronted with certain information. An attitude of negotiation could also be present when faced with contradictory occurrences and taking sides. A flexible attitude could be present if one manages different plans and a defensive attitude when one tries to support their argument. An Aristotelian attitude is one in which a person, when confronted with difficulties, adopts an attitude based on happiness, moral virtue and strength in the Aristotelian sense. Attitude can be derived, or not, from mathematics attitudes (see Figure 1) such as inductive attitude or attitude of precision and rigor, as [30] proposes.

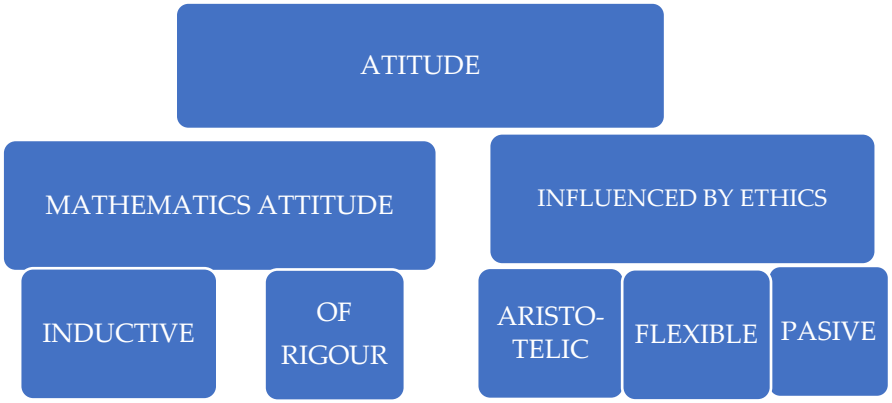

Figure 1. Attitude influenced by ethics and mathematics attitude; designed by the authors.

The theoretical framework of the present study, as presented in Ref. [42] includes computational and executive intelligence, as established by Marina (Figure 2). However, executive intelligence is defined here by the functions proposed by [43]: goal making, planning, carrying out a goal-oriented plan and the effective execution of such. For some

authors, attitude has three dimensions: beliefs, emotions and behavior, which the present model takes into account while not being characterized in its entirety by them.

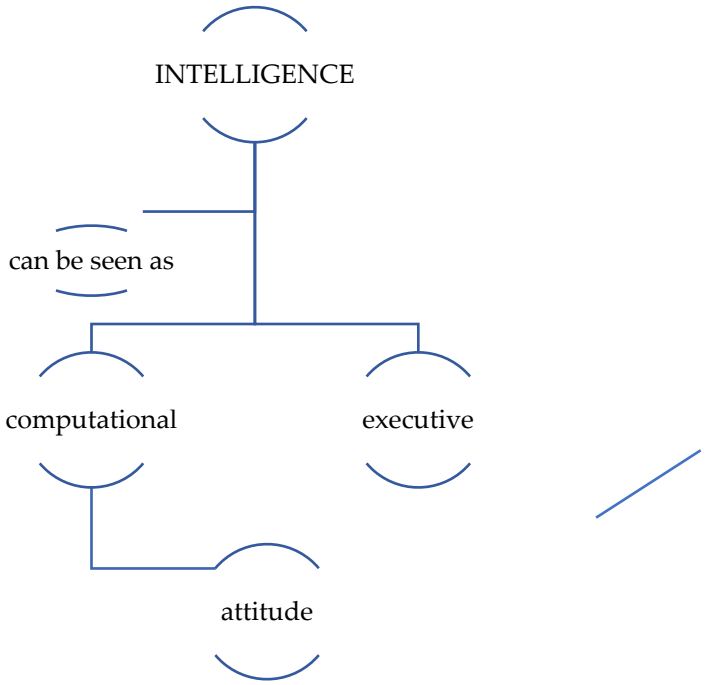

Figure 2. Attitude is a voluntary disposition of intelligence; designed by the authors.

The term "beliefs" refers to judgements, both conscious and unconscious, that we conform with because we are pledged to them. Beliefs act in groups or conglomerates and make up a system, as [44] establish. Beliefs that influence behavior when solving mathematics problems are, for example, beliefs about mathematics teaching, mathematics learning, about maths in general, and beliefs about the social context.

Feelings are integrated in blocks of information that include judgements by the subject involved in the situation and the predisposition of the subject to act. Emotions are transient feelings that appear abruptly with physical, conscious manifestations. As the balance of the situation is on a continuum, emotions can change quickly and tend to be connected in groups, showing shifts of emotions within the same group or across groups. For [45] (p. 47), one feeling can become another: "feelings can give way to other feelings".

In this study, the Aristotelian ideas of ethics has been filtered using Marina's ideas on ethics. In Ref. [34] (p. 17): "human intelligence is like an intelligence transfigured for freedom", and in that freedom a human being can "invent" two projects, one being happiness and the other perfection.

From the Aristotelian Ethics point of view, part of being human, and thus part of what goes into being happy, is reasoning. Ref. [34] (p. 62) says: "what is good for man is an active life of a being who can reason and, on one hand, obey that reasoning, and on the other had own it and think it". Here is where Marina expands upon the notion of happiness to include the notion of intelligence. Referring to the idea above, [34] (p. 162) reaffirms: "What this means is that, in reality, happiness consists of living intelligently. And as man has the intelligence to create, it all comes down to living creatively . . . ". This idea of happiness applies to the present study in the following questions: is solving these problems necessary to the progression of my academic life; will I be happier solving this problem or unhappy if I try and fail; and, if I am happier, what does that happiness mean? Mathematics education has indeed dealt somehow and sometimes with difficulties in a sense that somehow connects with the notion of happiness [32].

We try now to clarify how the group of notions appearing here make up the present study overall:

- Attitude, as defined by both Aristotle and Marina, involves voluntary acts. Marina, moreover, affirms that it is a choice between those presented by computational intelligence, and it is carried out by executive intelligence. When confronting a difficulty, different attitudes come into play such as defensive attitudes, passive attitudes, and Aristotelian attitudes.
- Ethics, both for Aristotle and Marina, are closely connected to happiness. The difference between them is that, for Aristotle, reasoning is an innate human ability and thus necessary for happiness to exist, while for Marina it is "living creatively".
- Attitudes can be affected by beliefs and emotions. The present study considers beliefs related to "living creatively".

3. Research Methodology

As mentioned in the Introduction, the main purpose behind this article is to illustrate through a direct and particular approximation to the reality of a student facing mathematical problems how a new characterization of the attitude construct which includes an ethical component, a characterization that makes up our theoretical framework, can improve the understanding and interpretation of realities such as the one presented.

We are interested in the depths of the human being that can be unique and individual, not in those that represent a pattern that can be adjusted to an aesthetic of a stable reality of which we cannot even be sure of its existence. Therefore, this research focuses on a case study in the sense given by [46] for whom the wholistic, empirical, interpretive and empathetic character of a qualitative paradigm on the one hand, along with the uniqueness of the selected case study itself, allow for a deep understanding of an activity in concrete circumstances as well as a close relationship between researchers and the object of study, making the methodological procedure proposed here one of the most adequate for the purpose of this research. Thus, there is no need for any kind of statistical hypothesis.

At the same time, in line with the classification given by [47], the type of case study that best fits with the purposes of this research is an instrumental case study. In this sense, [48] says that the studied reality will therefore focus on one case, justifying the usefulness of such according to the following intentions: widen the knowledge base, recognize the unique character of the context, and make a contribution with it since similar studies have yet to be carried out.

3.1. Participants

The case under study consisted of a single participant, who we will now refer to as Claudia, a fictitious name to protect her anonymity. Claudia was a student from 4th ESO (10th grade in high school) and she was selected based on her qualities as a key informant, shown in previous years, in addition to the fact that the year before Claudia had a teacher who used problem solving as a teaching method and as part of the content as well and because in a previous questionnaire filled out by 60 students from 4th ESO (10th grade in high school), she showed her closeness to attitudes related to Aristotelian Ethics. An important point about Claudia is that in the beginning of 3rd ESO, she presented resistance to problem solving as a teaching model, that she managed to overcome, in such a way that she even obtained good marks in mathematics. In [42] the authors investigate a case study whose data could be labeled as attitude far from Aristotelian Ethics, being this work useful for our purpose in this study. The methodological focus of this study was carried out in the following way: at the start of a teaching unit, a problem was presented which could be solved using heuristics and without being familiar with the specific knowledge contained in that unit. Then such knowledge was taught to solve a similar problem to the first in the end. The 2nd ESO (8th grade) teacher was actually in favor of a much more traditional, fundamentally expositive, teaching method and homework that required low cognitive demand, which, all in all, provided interesting experiential baggage for the purposes of this research.

3.2. Data Collection Instruments

The instruments used for data collection were questionnaires, interviews and observation of the participant and a written production of three problems.

The questionnaire was designed first. In it there were mostly multiple-choice questions as well as questions on a Likert-type scale. To elaborate on the questionnaire that Claudia would fill out in the end, a pilot experiment with an initial design was first carried out on a group of students from 1st Bachillerato (11th grade) social science. After, the results of the experience were debated and analyzed by a discussion group made up of experts in mathematics education at the University of Huelva. These experts agreed that there was a need to include a category *about attitudes and values*, together with another two categories called *sense of the activities* and *about evaluation.* This group suggested that the following dimensions should be considered as part of the category *about attitudes and values:*

- Sense/feeling of challenge;
- Sense of mistake;
- Sense of management in the long term;
- Sense of doubt;
- Sense of ethics in actions.

Here, one can see three questions chosen from that questionnaire as to provide a better understanding of this instrument and its purpose:

> *B.10. In class the teacher has brought in an activity that seemed very interesting to me but when I had to respond to what it asked me, I had a lot of questions. Mark below what you think about this class:*

- I would rather not have the class because it causes stress and insecurity.
- It is worth it because it is more interesting and motivating.
- It is worth it because if I have questions that means I can learn.
- If I doubt then the teacher is doing something wrong.

In this question, participants are asked to write 1, 2, 3 or 4 next to each answer option with 1 meaning they slightly agree, 4 meaning that they strongly agree and 2 and 3 being intermediate states.

This question would provide information on the dimension *sense of challenge*, among others, within the category of the feeling of attitudes and values. If a 4 is put on the second or third option, then we would consider them closer to Aristotelian Ethics and manages difficulties and emotions such as doubt.

> *B.12. During the first trimester, the teacher has worked on curious, interesting problems with which he/she challenges the students. In those challenges, the student must make conjectures/hypotheses about what helps them solve the problems. What would you say to other classmates to convince them to work in this way?*

- That they should try the conjectures, so they could solve the problems little by little.
- That it is a better way, because you learn that solving problems happens in phases, steps, and that not everything is about simply knowing or not.
- I prefer to repeat something that the teacher has already completed.

This question would provide information on the *sense of challenge* dimension, among others, found within the category of feeling of attitudes and values. The first and second options are closer to Aristotelian Ethics.

> *C.1. If you happen to make a mistake on a problem, what goes through your mind? What do you do?*

- I feel frustrated, but I go back and look for another way and keep the parts that are correct.
- I feel frustrated, but I go back and look for another way.
- I get overwhelmed and go in circles looking for ideas but don't come up with anything.
- I skip it.

This question would provide information on the dimension of *sense of error*, within the category of feelings of attitudes and values. The first and second options were marked as closer to Aristotelian Ethics.

Once this pilot questionnaire was corrected, 60 students from 4th ESO (10th grade in high school) filled out the final version of the questionnaire. Then, and very much related to the above questionnaire, a semi-structured interview was designed to gather data on the attitude of the participant towards the dimensions represented in the questionnaire's categories and, as such, confirm or not confirm the interpretations of the present study.

3.3. Problems

Three problems annexed to the answer sheet were also designed as data collection instruments (Appendix A). In these problems, Claudia had to describe the corresponding problem-solving processes that she used. These processes were also accompanied by participant observation in the days before as well as an interview afterward to confirm if the written production of the problems were related to what the student actually did during the working session days that accompany the resolution of such problems.

Along with the previously mentioned problems, three new semi-structured interviews were also included. This time with their focus was on reflecting the student's attitude towards solving each of the problems mentioned above.

4. Results

In the interest of clarity, this section is divided into the presentation of the responses gathered through the interviews about the questionnaire, the presentation of some relevant dialogues that took place after one of most interesting problems for this study was solved and observations about Claudia in the before days to exam. The objective of the later was not to analyze the mathematical problem-solving process in and of itself but rather to create a classroom situation in which the attitude of the student and, more importantly, the problem-solving management skills that she would use with homework, could be detected. At least one week before the exam, the teacher assigned some problems for homework that could be solved independently or with other students. Students could get help or ask questions about them some days before the exam. These problems were not corrected in class.

4.1. Interviews about the Questionnaire

As we have already mentioned, Claudia and 59 other students filled out a questionnaire. The responses to such questionnaire are relevant for our aims as, for example, when Claudia scored 4th in the second and third answer options in question B.10, she was exhibiting closeness to Aristotelian Ethics, as well as she does in question B.12 when she marked 4 for the second answer option. For deepening her responses, we needed the semi-structured interview.

We now present in this section some extracts of the first interview with Claudia, and one extract of an annex to the first interview, that will be analyzed in the next section. The first interview focused on attitudes towards problem solving and started with asking if Claudia preferred homework that included challenges or exercises in which the role of the student was to look for a solution or if she preferred solving problems that required copying an algorithm. She was also asked if the role of the teacher should be, in her opinion, one that favors autonomy or one that transmits knowledge. Here is an extract from the interview:

> Interviewer: There are students who think that dealing with challenges and difficulties is a part of learning. What is your opinion?
>
> Claudia: Yeah, that's ok, because if not, we will never learn to do them. From my point of view, that is. Besides it was more difficult for me to pass and I passed . . . here, if we do them in class and have classmates to do them with, we help each other out, we can ask you while we are doing them, and that's great.

> Interviewer: What do you think about adding in adventure? It that motivating or not?
>
> Claudia: Well, at first you get stuck and are like: What now? But later, when you start getting it, it actually is cool because you can say you understand it and you feel good having figured it out.

Next, in the first interview too, Claudia was asked about the difficulties inherent to solving the problem:

> Claudia: First, really, I'm a bit down on myself . . . I say I don't know how to do it, I'm the first one to say I can't, then you read it again and twenty times more and, I don't know, you try to see if this problem is like another one or you start looking.
>
> Interviewer: What do you say to yourself?
>
> Claudia: It depends on the problem, depending on what it says, I do one thing or another or chose a strategy.
>
> Interviewer: And how do you stop that negativity?
>
> Claudia: After a while I look at it and say "Well, I have to start somewhere, do it sooner or later, and getting all worked up like that doesn't go anywhere." Well, a lot of times I start and make a mistake then get on the right path.

In the next part of the first interview, she is asked if the teacher should give mathematical challenges to the class:

> Interviewer: Some students have a hard time with math challenges. Despite that, would you keep class that way?
>
> Claudia: Well, the thing is, I like that way of giving class, but, yeah, not everyone works the same. So, I don't know. If you can tell it's more difficult for you and you're rushing, I don't know, I mean, I'm good at it but I see other people get more frustrated that way and, yeah, I've heard them say they'd like math to be the other way, but, really, you can get all the theory from you book you want, but if you don't understand there is no way.
>
> Interviewer: So even if the students have a hard time, you'd keep it the same.
>
> Claudia: I just think that they'd have a worse time in the end the other way (they might not think so) but I do. That giving class the other way for example, would make next year end up more difficult.

4.2. Participant Observation from Previous Problems

With the aim of understanding the participant observation included here in the instruments of data collection, one must consider both the existence of homework problems worked on independently and a few hours of question-and-answer session the day before the exam. The notes obtained from menus of this instrument consisted of noting down Claudia's questions about the problem descriptions, her ideas, her reasoning, her procedures, if she asked the teacher or her classmates, and how she used these resources to solve the problem. The teacher considered critical how to help students, and so agreed with "The teacher needs to hold knowledge of specific instructional strategies on how to help students to overcome or deal with being stuck" in [49] (p. 4), idea reaffirmed in [50]. From the observations in the days before the problem was faced, it is important to note that Claudia asked the teacher for help on the following tasks:

- Find out an angle knowing $\sin^2 x$;
- Apply a certain formula to solve a trigonometric problem;
- Solve a trigonometric problem with a specific formula (Claudia asks if her answer is correct; the teacher recommends she check it and, in fact, she does.);
- Calculate an angle in a rectangle triangle (Claudia shows her progress and asks if it is ok; a heuristic is suggested to her);

- Calculate a distance (Claudia does not understand the directions and asks for help on getting started; the heuristic of using a similar problem as a reference is suggested). The next day, Claudia shows the teacher a solution using scales, without using the specific knowledge in the didactic unit (the tangent), asking if this answer was ok. The teacher recommends she look at the heuristics used in the unit. In the end, the teacher shows her one solution using the tangent.

Among the problems Claudia could ask for help it is the one that asked her to find the relationship between the sine and cosine of a complimentary angle. Claudia did not ask for any help with this problem, which will be referenced as problem P in the rest of the paper.

4.3. Problems

As we have mentioned, we selected one of the problems that the student carried out. The selected problem is the following, that we will refer to as problem P', and that was the first problem in an exam for the whole classroom: "The two angels of a right triangle are called complimentary because their sum is 90 degrees. What is the compliment of 10 degrees? What relationship is there between the tangent of the angle and the tangent of the compliment"?

Next, Claudia was interviewed about how she approached this problem P'. As neither P nor P' had been corrected, it was supposed that P' was more of a challenge for Claudia, as it was not an exercise from the book. Upon asking Claudia how she solved the problem, she replied:

> Claudia: Similar to what he did in class (referring to the question-and-answer days) with the sine and cosine.
>
> Interviewer: We could say the fundamental part of getting this problem was the "good effort" put into the previous one, the homework problem.
>
> Claudia: Yeah, not just the one that we already did but everything together . . . because in the previous problem like this it was cosine and sine, and in other problems, right, they talked about tangent, so it helped me relate it to this one. Wow, if I had only done the one before and not the other ones on the photocopies, I probably wouldn't have been able to get it.

Lastly, Claudia was asked about her questions and mental blocks, with the following response:

> Claudia: When I did the complementary to the tangent, I don't know, I hesitated about how to find the tangent . . . I understood sine and cosine better.
>
> Interviewer: Did you have any doubts about the complementary? How did you figure it out?
>
> Claudia: I turned the paper this way instead of that and I saw it clearer (she turned the paper 90 degrees, making the adjacent edge the opposite).

Next, we can see an image capture of Claudia's double check (Figure 3) after doing the proof, followed by its translation into English:

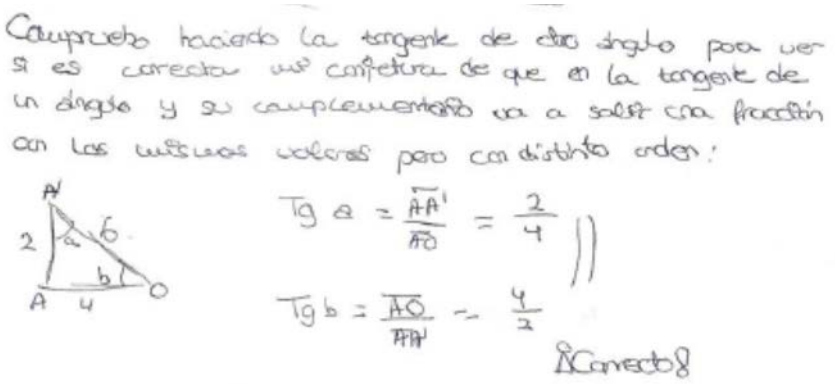

Figure 3. Claudia's double check after doing the proof. Source: Claudia's writing.

Translation of Claudia's writing: "I check doing the tangent of an angle for testing my guess is right about the tangent of an angle and its complementary will come out a fraction with the same values but in a different order: Right!"

5. Discussion

In this section, we interpret the data in the before section. We begin by the data on interviews, then we continue with the data on problems and observations, and we finish with an integration of the two analyses.

After the first extract of interview, we think that the fact that she approved of having challenges provides information about the category sense of attitudes and values and seems, so this indicates a certain strength when facing difficulties. She did not decide to give up and, in the ethical sense of her actions, it seemed important to her that she did it on her own. This student's problem-solving model consisted of a first phase of understanding with questions and heuristics from the Polya model [51] and then the next Attack Phase, by [52].

From the second extract, we infer that Claudia shows emotions such as fear as well as certain beliefs about herself. We can infer that Claudia is able to control her negative self-talk (which she clarified was the fear of not being able to complete the task) and that she has a certain strength in the Aristotelian sense, showing bravery when managing that fear and daring to go for it and not give up. She appreciates teacher help as a way to overcome difficulties, management that in [49] (p. 4) is considered desirable knowledge of teachers.

The above data can be expressed in a table (Table 1) as the following:

Table 1. Claudia's closeness to Aristotelian attitude. Source: Claudia's interviews.

Attitude	Non-Aristotelian	Intermediate States	Aristotelian
Strength facing difficulties			✓
Bravery facing fear			✓
Living creatively			✓

Lastly, the inference from the third extract can be included in the category related to sense of attitudes and values and, within that, the sense of ethics in one's actions. This is because dedication to resolution of the problem is involved and doing it on one's own despite difficulties (what Marina calls "living creatively") is considered here to be coherent with Aristotelian happiness. This attitude is confirmed with the observations. From these observations, one cannot affirm the presence of an attitude such as those in Aristotelian Ethics. If anything, the acceptance of the suggestions to use heuristics to help does not contradict the inferences made after the interviews about doing the work on her own or the strength to not give up.

For the problem and the later interview, Claudia reported on how her homework problems had influenced her work, one can infer something that merits including in the category sense of attitudes and values, which is perseverance, or her management of homework in the long run through asking for help. Additionally, the problem informs us that seems to be evidence of her strength to break through feelings of doubt and insecurities when blocked, which are included in the category of sense of attitudes and values. This strength and responsibility to do them for herself corroborates with the following dialogue when she tries to make conjectures and test them:

> Interviewer: You drew an example of a triangle.
>
> Claudia: Yeah, to help myself, and I made the tangent; I saw that the values were flipped over and I wrote another example to check it . . . and then I did the proof . . .

In sum, when emotions such as doubt, frustration and fear of not being able to solve the problem come up, an attitude similar to an Aristotelian one is put into action rooted in the beliefs of happiness and responsibility for learning to do it oneself and strength. This result connects with the idea established in [53] (p. 16) that "Thus the pupils learn that the effort pay off and brings desired, sweet profit in the end". Therefore, Claudia's Aristotelian-like attitude comes forth in inductive attitudes and precision and rigor (Figure 4).

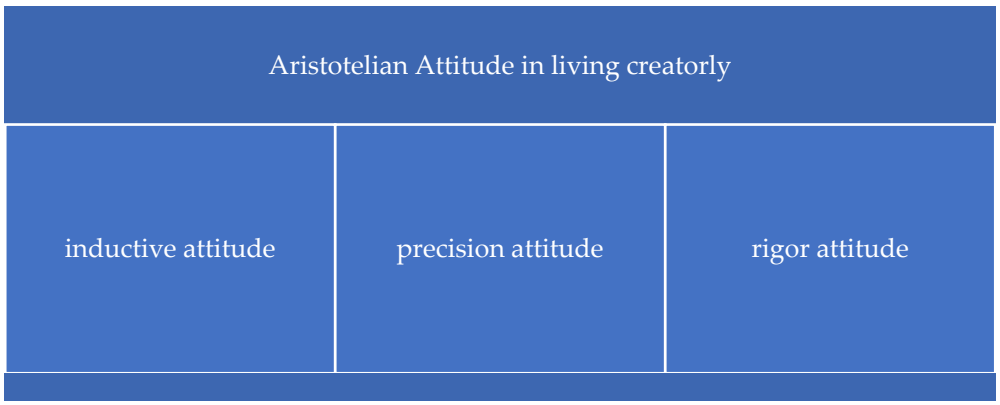

Figure 4. Claudia's attitude with living creatively comes forth in inductive attitudes and precision and rigor. Source: Claudia's interviews about problem.

A table or figure considering answers from the participant in [42] would have had all ✓in the non-Aristotelian column.

6. Conclusions

In the first place, returning to the main idea of this paper as described in the Introduction, this case study reinforces the idea that ethics can help to describe the way students behave. In [54] (p. 214), he states " . . . affective representation can increase or limit mathematic problem-solving potential", as he goes on to describe some emotional states that interact with this mathematic potential or not, when problem solving. Within those two styles, Claudia could identify puzzlement, enthusiasm and satisfaction. The present research, with a case study and Aristotelian Ethics, has intended to explain the process through which mathematical potential is reached. To do this, not only has it been important to analyze the problem being solved in the moment but also the problems solved beforehand and how the student manages the search for solutions in the long term. This result could help some teachers in their practices to manage homework in the long term, and thus improving student attitudes.

In the second place, the authors of this present study believe that, besides the traditional attention paid to attitudes toward maths, it can be also useful to direct our attention towards attitudes influenced by ethics and personal affect on the whole. More specifically, if students often show similar attitudes confronting academic difficulties, whether or not

it is within one subject, cases such as this study help to see when that attitude (with its specific qualities) is within the context of a mathematics task. Thus, the idea of attitude presented here connects to what [9] described regarding attitude in some teachers. Here, attitude within the binomial of ethics–mathematics can help to describe and understand student behavior in greater detail when problem solving.

Using the theoretical framework in the present study, it could be affirmed, for example, that Rita, the student in the study by [31], showed a defensive attitude in part, trying to justify her own goals and needs with a problem that she did not know how to solve by herself and her classmates had disregarded. Ref. [33] presented a student who could be categorized as having an inflexible attitude in part, yet the student participant became more flexible and accepted working on a team when she thought of it as "cheating." The above examples could have consequences in Educational Theory and in practices: in Educational Theory because our idea of attitude connects with the ideas of attitude teachers, ideas that are not considered by investigators such as [31], and in practices because the responsibility does not rest entirely on teachers, but it depends also on the ethics of students. Another example of our contribution is in [55], where the author explains that in the performance in problem solving five factors are involved, being one of them beliefs and affect. We believe that our characterization of attitude could be included in beliefs and affect. This result could have consequences in Educational Theory because it is possible that theoretical framework in [55] would explain the performances in a richer way, adding another dimension.

Finally, we must highlight once more that this case study was designed in order to open up questions about what the "best ethics" are for working with students and teachers in order to help them through major difficulties that traditionally come up in the planning process and the mathematic problem-solving process and help engender a more creative, active attitude in both processes and approach them in the most efficient way possible. Considering the results of this study, one possible hypothesis is that the best way is to use ethics that are compatible with happiness, a happiness that means using all different levels of intelligence, and, at the same time, contributes to a natural acceptance of the task of carrying out homework, whether depending on others or not. However, this must be confirmed with more research as evidently, all the affirmations and results of this study correspond to a unique case study and, therefore, cannot be applied beyond the potential interpretation given here, which does, in fact, help to understand the situations studied.

Inasmuch, this study supports the main hypothesis about the utility of including ethics as a component of the conceptual framework of attitudes that go into problem solving and the development of a better understanding of related phenomena, as in the case of Claudia. Nonetheless, despite these limitations, the results of the study point to continuing the underlying theme of the article though theoretical work on the construct of attitude from the perspective presented in this framework. To do so, we suggest that studies be carried out on a larger scale that transcend the particularities of one case and help generate a wider knowledge base through, for example, Grounded Theory based approaches

Finally, we consider that a very interesting element to be considered in future studies are those concerning the influence of ICT in attitudes of students to maths, following some of the ideas collected in [56–58], as well as the ones related to the influence of STEAM in attitudes of students as suggested in [50].

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Appendix A

THE MAGIC FLUTE

Gumersindo Pelaez has a flute measuring 26.8 cm, which he wants to store in a shoebox with dimensions 10, 15 and 20 cm. He says it will fit, while his friend Antoñito says it will not. Which of the two is right?

Use a formula to generalize the previous result.

SQUARES AND MATCHES II

How many matches are needed to build n^2 squares of side 1 forming another larger square, as in the following sequence?

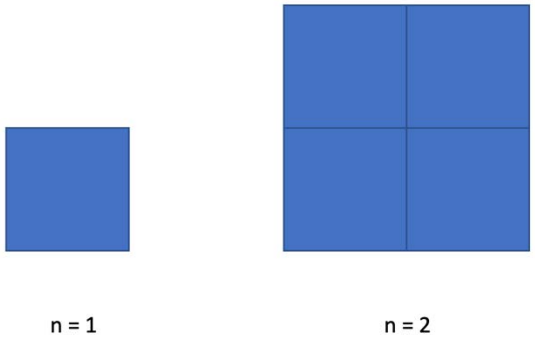

Figure A1. First two cases for the sequence of squares built with matches. Source: authors.

TANGENT OF THE COMPLEMENTARY

The two angels of a right triangle are called complementary because their sum is 90 degrees. What is the complementary of 10 degrees? What relationship is there between the tangent of the angle and the tangent of the complementary?

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