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Grado en Ingeniería eléctrica

Título del TFG

**CONSTRUCTAL OPTIMIZATION OF
PHOTOVOLTAIC STRUCTURES**

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Resumen y palabras clave (Abstract y Keywords)

Este proyecto de investigación consiste en encontrar que geometría es la óptima (cuando hablamos de óptima nos referimos a menores pérdidas eléctricas posibles) en una célula fotovoltaica elemental.

Las simulaciones consisten en dos diferentes tipos de grosores de la banda conductora, con tres conductividades diferentes, simulando distintos materiales conductores.

Después de encontrar que geometría es la más eficiente, partiremos de esa geometría para estudiar en este caso, el par de células elementales óptimo (este caso es el primer orden, girando 90 grados la célula y aplicando una simetría debajo de ella).

Partiremos del par de células elementales óptimo, haciendo el mismo procedimiento descrito anteriormente hasta llegar a cuarto orden, que será la última simulación.

Paneles fotovoltaicos, optimización, elementos finitos, simulación, generación eléctrica.

This research project aims to determine the optimal geometry (referring to the geometry that minimizes electrical losses) for a basic photovoltaic cell.

The simulations involve two different thicknesses of the conducting layer, with three different conductivities, simulating various conducting materials.

After identifying the most efficient geometry, we will proceed to study the optimal pair of elementary cells in this case, which involves rotating the cell 90 degrees and applying symmetry below it.

Starting from the optimal pair of elementary cells, we will follow the same procedure described above, progressing up to the fourth order, which will be the final simulation.

Photovoltaic panels, optimization, finite elements, simulation, electricity generation.



CONSTRUCTAL OPTIMIZATION OF PHOTOVOLTAIC STRUCTURES

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1. Introduction

Photovoltaic cells, also known as solar cells, are electronic devices that convert sunlight into usable electricity. They are made of materials called semiconductors, which have the ability to absorb photons of light and release electrons.

The engineering behind photovoltaic cells involves understanding the principles of semiconductor physics, electronic circuit design, and solar energy conversion.

Photovoltaic cells generate both voltage and current when exposed to sunlight. The voltage is a measure of the electrical potential difference between the positive and negative terminals of the cell, while the current is a measure of the flow of electrons through the circuit.

When the photovoltaic cell is exposed to sunlight, the energy from photons (particles of light) is absorbed by the semiconductor material, which causes electrons to be released. These electrons move through the material under the influence of an electric field, resulting in the generation of an electrical current. The voltage of the cell is determined by the type of semiconductor material used and the design of the cell.

The voltage and current generated by photovoltaic cells are directly related to the amount of sunlight received. In other words, the more intense the sunlight, the greater the amount of voltage and current produced by the cell.

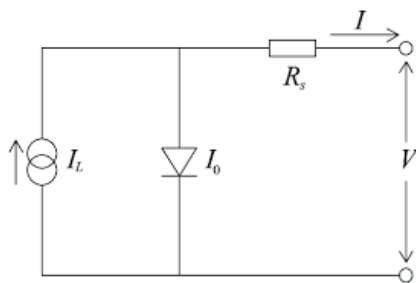


Fig 1 Equivalent Scheme

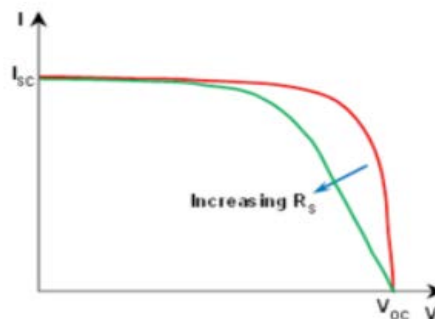


Fig 2 changing \$R_S\$ from ideality

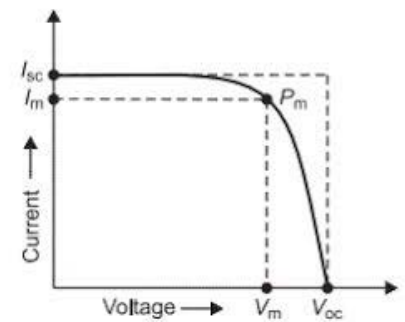


Fig 3 Maximum power point

Figures from <https://www.avisolar.com>

The main goal of this study is get the lowest series resistance for the maximum power point will be the highest possible.

For it, we have to play with different factor as the band conductivity or the geometry of the PV panel.



2. The constructal optimization of the collector

The constructal optimization of PV (photovoltaic) collectors involves designing and optimizing the configuration of solar panels to maximize the collection of solar energy and improve overall efficiency.

The surface area of the PV collector determines the amount of solar energy that can be captured. Constructal optimization involves determining the optimal size and shape of the PV panels to achieve the highest possible energy collection.

The electrical connections between PV panels impact the overall system efficiency. Constructal optimization considers the routing and arrangement of electrical wiring to minimize losses and ensure optimal power transmission within the collector system.

The main goal of this study is find the properly aspect ratio on the elementary cell and then rotate 90 degrees and mirrored to test different pairs of this elementary cell (first order) until reach the lowest Rseries.

After that, it will test until fourth order where it will depart of the first order optimal solution, rotating 90 degrees and mirrored as the procedure before.

It will compare the results with two different band height: 0,01 and 0,02. All the constructal process is the same.

3. Elementary cell

The amount of the material of the collector and the total area and the “productive” area of the PV cell will be constant.

3.1 Mathematical model

We assume that the elemental PV cell operates under DC conditions, so the potential is always constant.

The mathematical model for the electric field is then made of the following equations:

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{w'''}{\sigma_o} = 0 \text{ emitter} \quad (1)$$

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} = 0 \text{ collector} \quad (2)$$

The optimal solution of the problem is:

$$V(x, y) = \frac{w'''}{2\sigma_o(H_o y - y^2)} + V_o(x, y) \quad (3)$$

Where:

V = electrical potential

w''' = PV current source

σ_o = electrical conductivity

H_o = Height of elementary cell

All the parameters used in these simulations are non-dimensional.

3.2 The optimization principle

It will be explained how to draw a model for an elementary cell with the aspect ratio “ $f = H/L = 1$ ”. Every aspect ratio will be draw to the same way, the only difference is reediting the “ f ” constant value, the others constants will change automatically.

In this simulation of the elementary cell, it will find the optimal geometry of this cell (the lowest Rseries).

In addition, it will be redraw the new aspect ratio for each “ f ”.

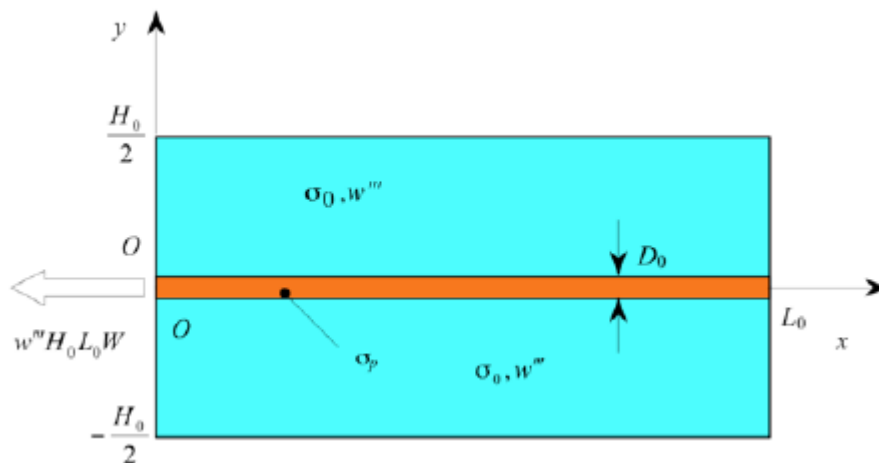


Fig 4 Elemental cell

The following figure shows how the simulation would be drawn in the software

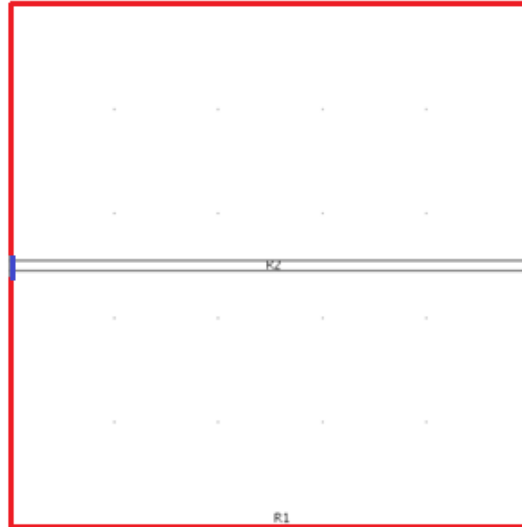


Fig 5 An example of an elementary cell drawn.

The previous figure is builded with aspect ratio factor “f” equal at 1.

Here:

$$f = \frac{H}{L} \text{ aspect ratio}$$

$$L = \frac{1}{\sqrt{f}} \text{ width}$$

$$H = \sqrt{f} \text{ height}$$

$$D = 0,01\sqrt{f} / 0,02\sqrt{f} \text{ strip height}$$

As above will be define two different boundaries.

The external boundary, marking in red (the limit of the collector with the air) as an electric insulation (current through the boundary = 0).

The limit of the aluminium band with the air in left side will be define as a ground (Voltage = 0) marking in blue.

It will be fill the current source and electric conductivity. It will be integrated to get the value of the current. This value will not be the same for every simulation since the number of cells are going to change.

For PV silicon cell, $Q = 1$ (It produce the current) and $\sigma_0=1$

For the Aluminium band, $Q = 0$ (It does not produce current, only flow) is going to simulate with the following conductivities:

$$\sigma_p=100, \sigma_p=1000 \text{ and } \sigma_p=10000$$

Otherwise, if the geometry is keeping the same aspect ratio (as in the elementary cell), the current will keep constant.

3.3 Solve the simulation

With the steps described in the chapter 3.2, it can mesh and calculate the solution in the simulator software as it can be seen in Fig. 6.

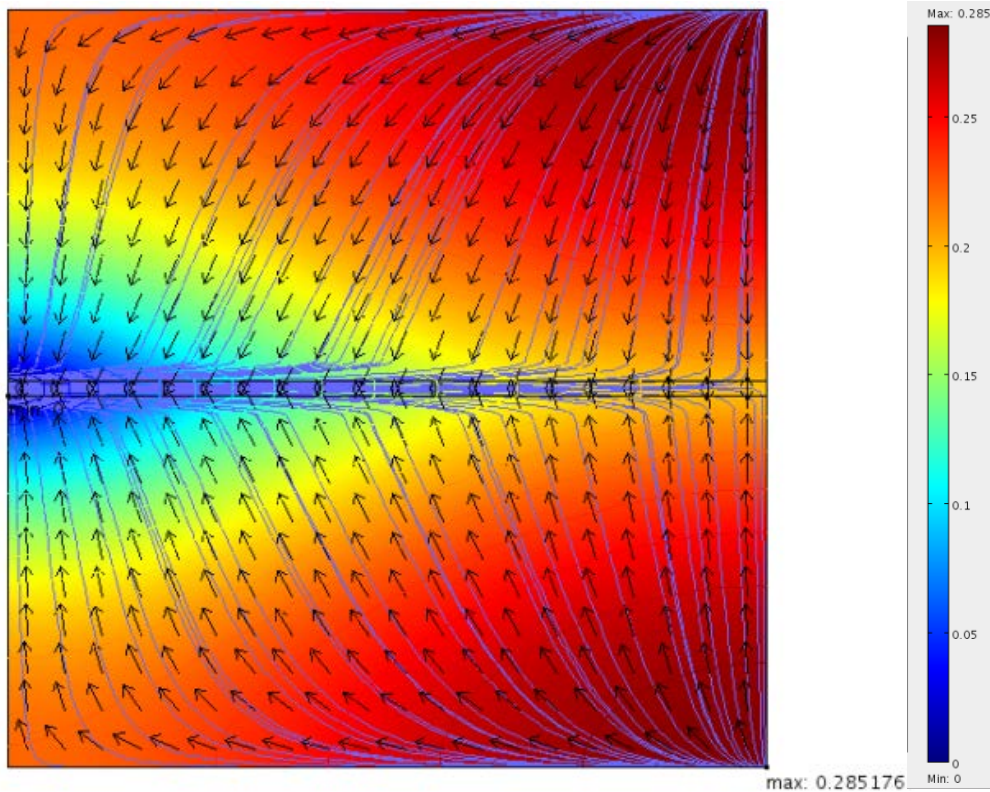


Fig 6. An example of a simulation using aspect ratio equal one ($f = 1$) and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

Simulations are conducted from $f = 0,2$ to $f = 2,5$ to determinate the lowest Rseries.

3.4 Results and conclusions

After solve every aspect ratio, it will be build a table with the different Vmax value, for calculate which is the best geometric aspect ratio to has the lowest losses as it is the goal of this report.

The current for all the aspect ratio will be the same, $I = 0,98$ for $Do = 0,02$ and $I = 0,99$ for $Do = 0,01$.

We will calculate the Rseries using the following equation:



$$R_{series} = \frac{V_{max}}{I} \quad (4)$$

In the following tables, will be show different simulations with different conductivities for each aspect ratio and each Band height (Do). The lowest Rseries shall be searched for each conductivity marked in green.

Do = 0,02														
σp = 100														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	0,873	0,607	0,404	0,327	0,293	0,285	0,281	0,280	0,285	0,296	0,310	0,326	0,343	0,362
I	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980
Rserie	0,890	0,619	0,412	0,333	0,299	0,291	0,286	0,285	0,291	0,302	0,316	0,332	0,350	0,369
σp = 1000														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	0,140	0,113	0,105	0,114	0,131	0,140	0,150	0,170	0,192	0,214	0,237	0,260	0,283	0,307
I	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980
Rserie	0,143	0,115	0,107	0,117	0,133	0,143	0,153	0,174	0,196	0,219	0,242	0,265	0,289	0,313
σp = 10000														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	0,036	0,044	0,065	0,087	0,110	0,122	0,134	0,158	0,181	0,205	0,229	0,253	0,277	0,301
I	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980	0,980
Rserie	0,037	0,045	0,066	0,089	0,113	0,124	0,137	0,161	0,185	0,209	0,234	0,258	0,282	0,307

Do = 0,01														
σp = 100														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	1,346	0,938	0,618	0,489	0,425	0,406	0,392	0,377	0,372	0,374	0,381	0,392	0,405	0,420
I	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990
Rserie	1,360	0,947	0,625	0,494	0,429	0,410	0,396	0,381	0,376	0,378	0,385	0,396	0,409	0,424
σp = 1000														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	0,250	0,186	0,148	0,145	0,154	0,161	0,169	0,187	0,207	0,228	0,250	0,273	0,296	0,319
I	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990
Rserie	0,252	0,188	0,149	0,146	0,156	0,163	0,171	0,189	0,209	0,230	0,253	0,276	0,299	0,322
σp = 10000														
f	0,2	0,3	0,5	0,7	0,9	1	1,1	1,3	1,5	1,7	1,9	2,1	2,3	2,5
Vmax	0,049	0,053	0,070	0,092	0,114	0,126	0,138	0,162	0,186	0,210	0,234	0,258	0,283	0,307
I	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990	0,990
Rserie	0,049	0,053	0,071	0,093	0,115	0,127	0,139	0,164	0,188	0,212	0,236	0,261	0,286	0,310

Table 1. Simulations data with different aspect ratio

Marked in green are the lowest Rseries.

Here:

D_o = width of the conductive band of the elementary cell

σ_p = conductivity factor

f = aspect ratio

I = current

R_{serie} = series resistance

The table 1 shows the R_{serie} for each aspect ratio, so it will be chosen the lowest R_{serie} for each conductivity. It can be seen clearly in the figure 7 to 12.

$D_o = 0,02$

It will be plot for each conductivity the R_{serie} values for each aspect ratio

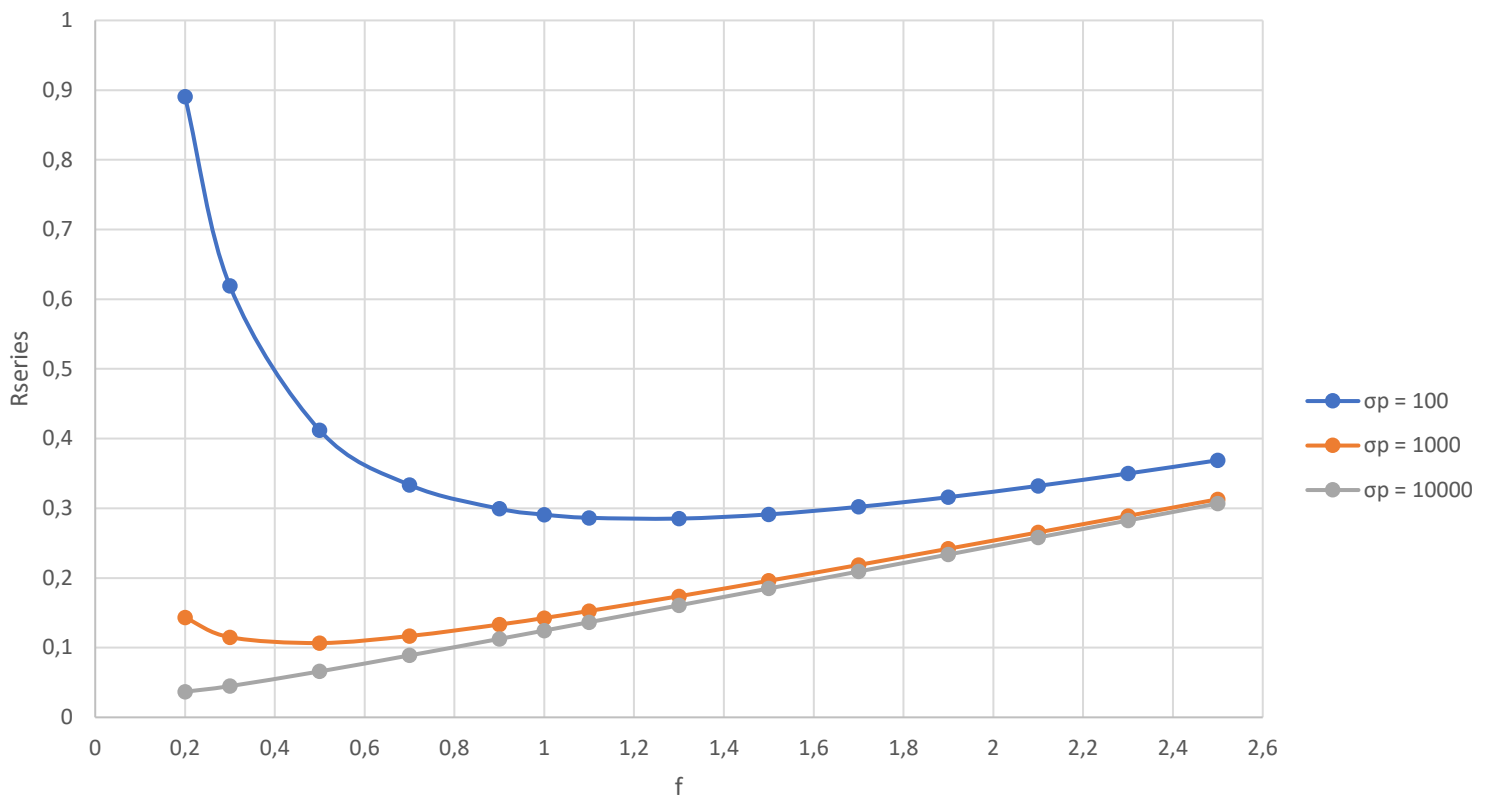


Fig 7. Graph of R_{serie} for each aspect ratio

- For $\sigma_p = 100$, the trend is convex, being the minimum value of resistance for the aspect ratio equal at 1,3.
- for $\sigma_p = 1000$, the trend is similar as the sigma before, but is not so marked, being the minimum value of resistance for the aspect ratio equal at 0,5.
- for $\sigma_p = 10000$, the trend is linear in whole "f" interval, being the minimum value of resistance for the aspect ratio equal at 0,2. It can be determinate $R1_{serie} < R2_{serie} < \dots < R_{serie}$, so $R1$ is the lowest R_{serie} .

Let's do a zoom in to see clearer the minimum:

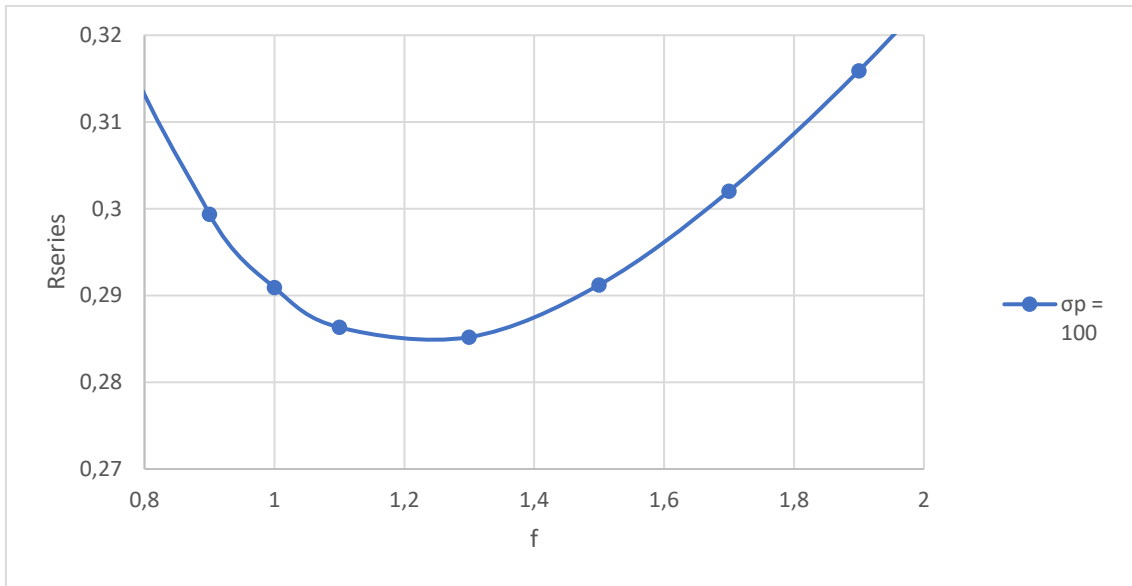


Fig 8. Graph of Rseries zoomed for $\sigma_p = 100$

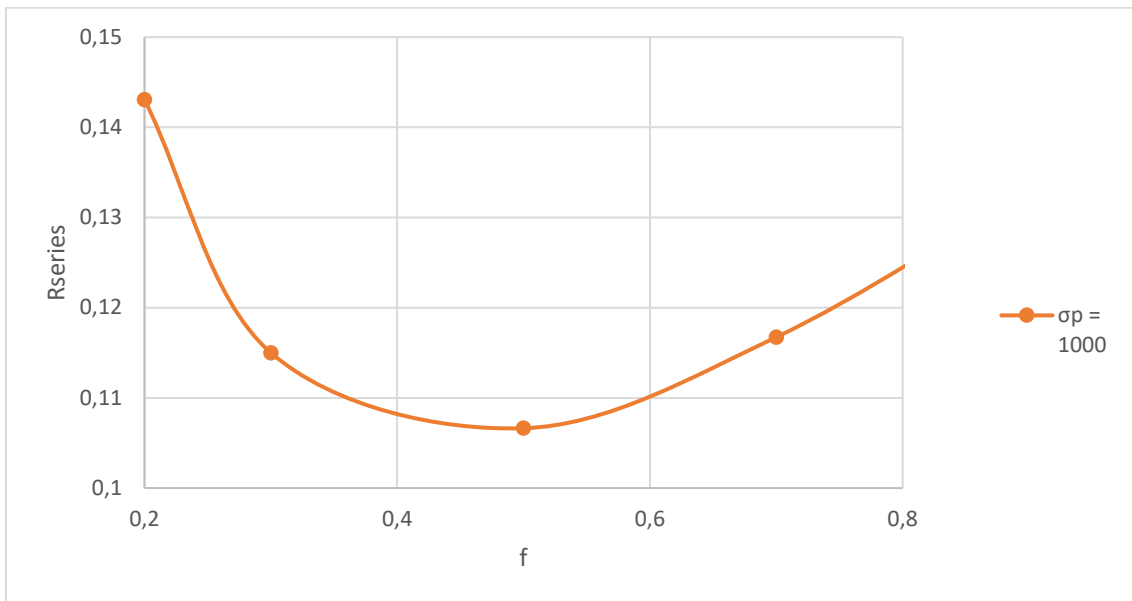


Fig 9. Graph of Rseries zoomed for $\sigma_p = 1000$

Do = 0,01

It will be plot for each conductivity the Rseries values for each aspect ratio.

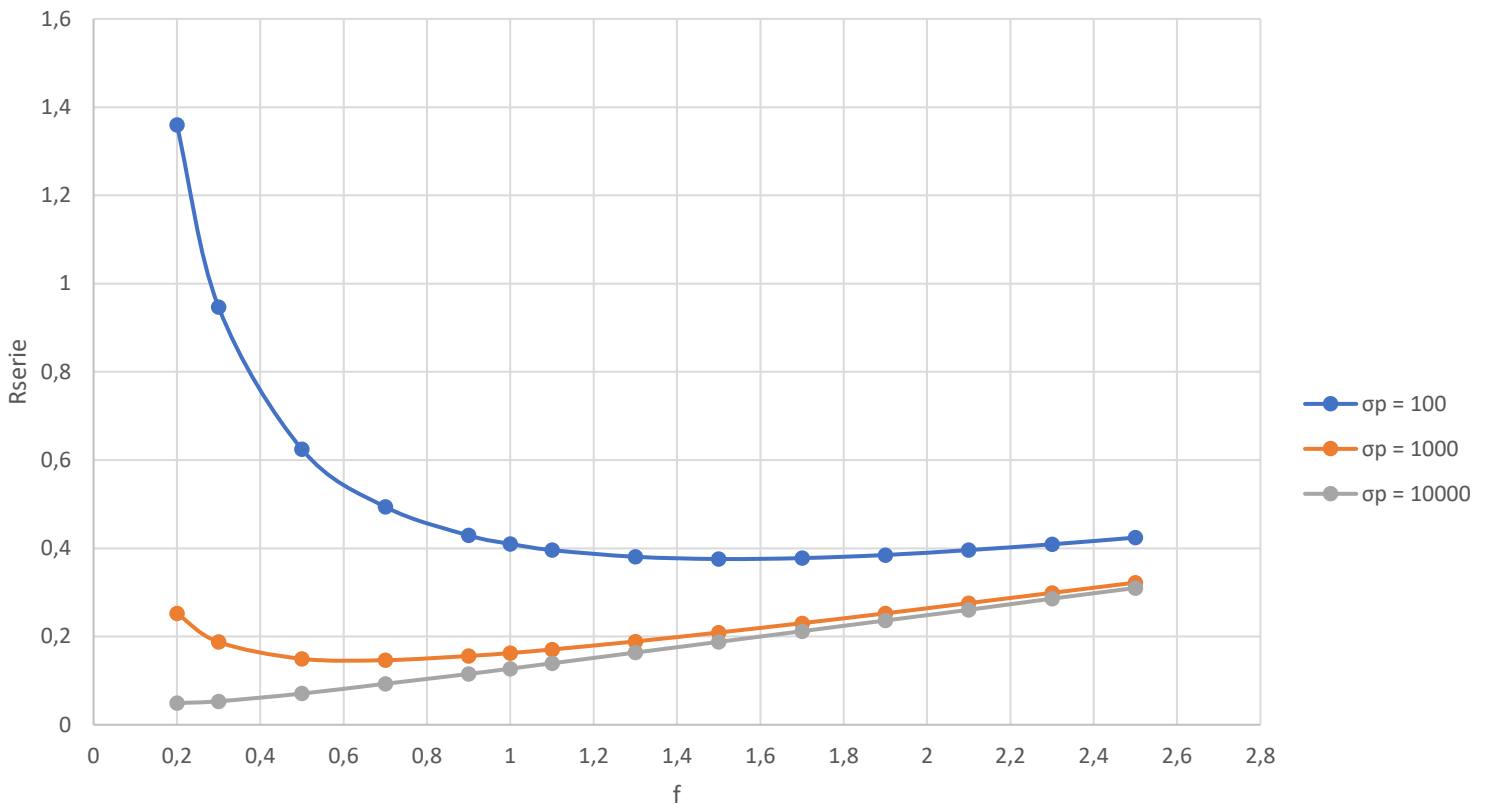


Fig 10. Graf of Rseries for each aspect ratio

- For $\sigma_p = 100$, the trend of this resistance is convex, being the minimum value of resistance for the aspect ratio equal at 1,5.
- for $\sigma_p = 1000$, the trend of this resistance is similar as the sigma before, but is not so marked, being the minimum value of resistance for the aspect ratio equal at 0,7.
- for $\sigma_p = 10000$, the trend is linear in whole "f" interval, being the minimum value of resistance for the aspect ratio equal at 0,2. It can be determinate $R1_{series} < R2_{series} < \dots < R_{series}$, so R1 is the lowest Rseries.

Let's do a zoom in to see clearer the minimum:

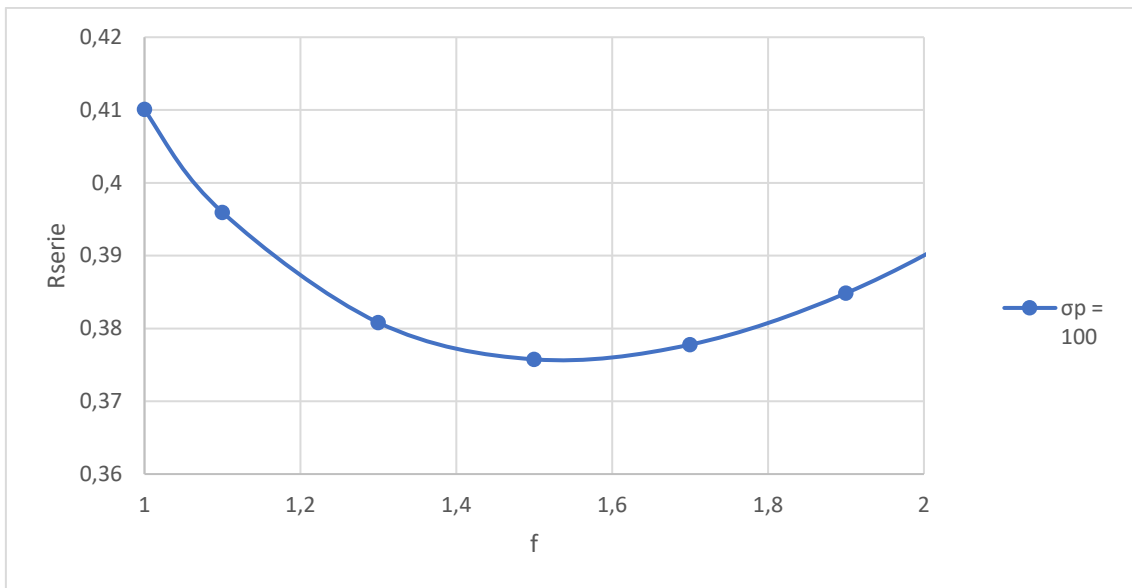


Fig 11. Graph of Rseries zoomed for $\sigma_p = 100$

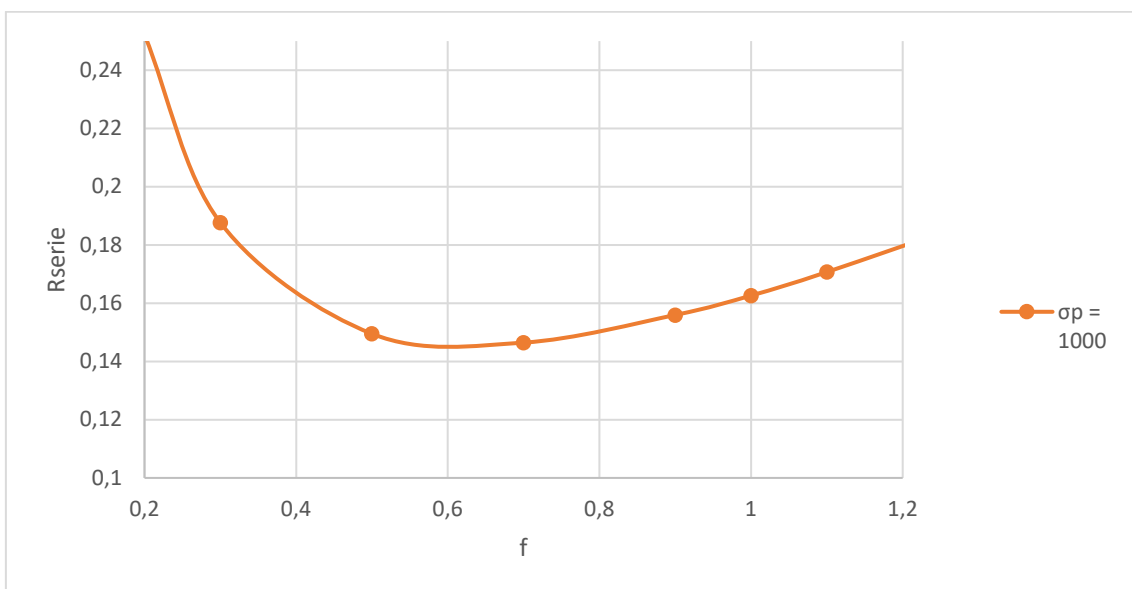


Fig 12. Graph of Rseries zoomed for $\sigma_p = 1000$

4. The first order ensemble

The goal of this chapter is find the optimal elementary pair cells for each conductivity. It is important since we are going to depart from this optimal result.

4.1 Mathematical model

The optimization problem is now the determination of the optimal aspect ratio H1 L1, or how many elementary PV cells must be connected so that the maximum

voltage on the assembly is minimized. Analytically, the optimal number of constituent PV elementary cells is then:

$$n_{1,opt} = \frac{(\sigma D_1)^{1/2}}{2^{1/4}(\sigma \phi_o)^{1/8}} \quad (5)$$

The total amount of good conductive material (D1)

$$\left(\frac{D_1}{D_o}\right)_{opt} = \frac{(\sigma D_1)^{1/2}}{2} \quad (6)$$

4.2 The optimization principle

The main purpose of this simulation is determinate the optimal aspect ratio of H1 and L1, how many elementary PV cells must be connected for resistance of the assembly is minimized.

Remember from the chapter before:

$$R_{series} = \frac{V_{max}}{I_{total}} \quad (7)$$

Where I_{total} is the current produced by the cell.

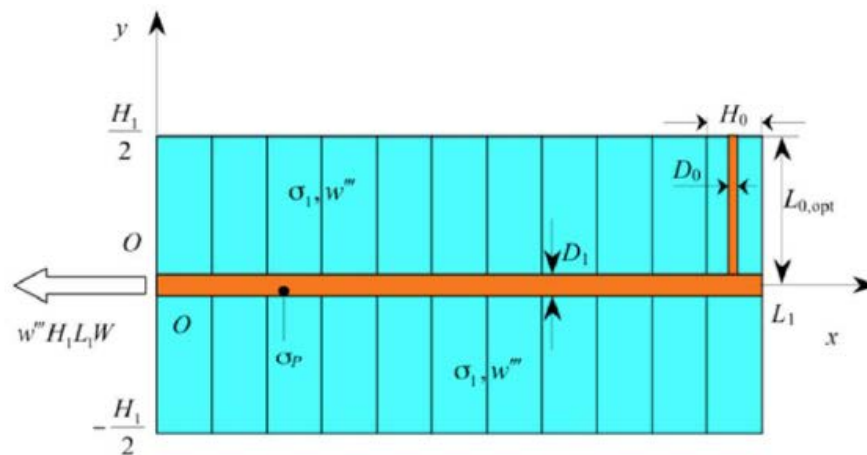


Fig 13 The essemble of the first order

Where $D_1 = 2D_o$

The elementary cell geometry used for this assembly is the optimal obtained in the chapter before for each conductivity.

The following figure shows how the simulation would be drawn in the software



Fig 14 An example of a pair of elementary cells

Do = 0,02

- For $\sigma_p = 100$, It will leave from “f = 1,3”.
- For $\sigma_p = 1000$, It will leave from “f = 0,5”.
- For $\sigma_p = 10000$, It will leave from “f = 0,2”.

Do = 0,01

- For $\sigma_p = 100$, It will leave from “f = 1,5”.
- For $\sigma_p = 1000$, It will leave from “f = 0,7”.
- For $\sigma_p = 10000$, It will leave from “f = 0,2”.

Two types of boundary conditions will be used.

The external boundary, marked in red (the limit of the PV cell with the air) as an electric insulation (current through the boundary = 0).

The limit of the aluminium band with the air in left side will be define as a ground (Voltage = 0) marked in blue.

Now the current values are not constant, since it has to draw with diferent pairs of elemetary cells.

4.3 Solve the simulation

With the steps described in the section 3.2, it can mesh and calculate the solution in the simulator software As it can be seen in the Fig. 15.

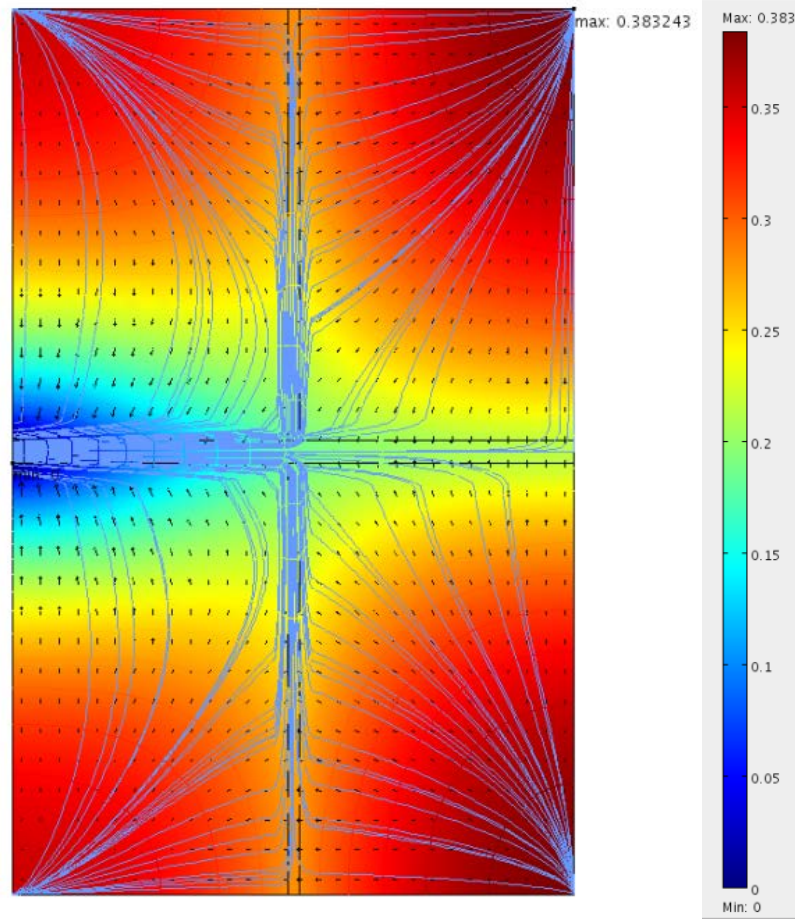


Fig 15 An example of a pair of elementary cells, $\sigma p = 100$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen above, the simulation shows the value and position of the maximum Voltage point.

For two pairs of elementary cells:

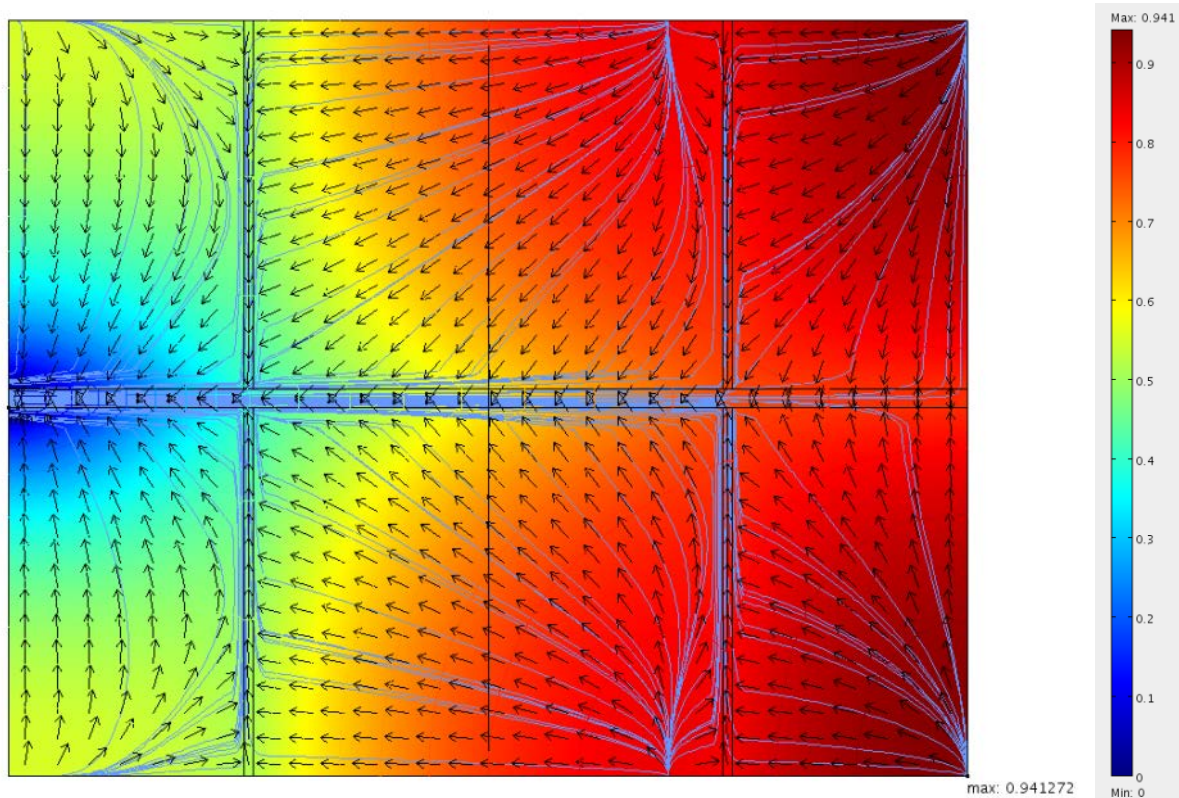


Fig 16 An example of two pair of elementary cells, $\sigma_p = 1000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

The V_{max} and I_{total} is increasing as it is adding pairs of elementary cells.

It will be repeat adding pairs elementary cells with the same process until find which is the lowest R_{series} .

4.4 Results and conclusions

After of solve with different pair of elementary cells, it will be build a table with the different V_{max} value, to calculate which is the best number of pairs to has the lowest losses as it is the goal of this report.



Do=0,02					
σp = 100					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,3832	0,9414	1,8602	3,1306	4,7520
I (A/m)	1,9600	3,9200	5,8800	7,8400	9,8000
Rserie (Ω)	0,1955	0,2402	0,3164	0,3993	0,48490
σp = 1000					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,1246	0,1901	0,3026	0,4598	0,6613
I (A/m)	1,9600	3,9200	5,8800	7,8400	9,8000
Rserie (Ω)	0,0636	0,0485	0,0515	0,0586	0,0675
σp = 10000					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,0384	0,0453	0,0572	0,0740	0,0954
I (A/m)	1,9600	3,9200	5,8800	7,8400	9,8000
Rserie (Ω)	0,0196	0,0116	0,0097	0,0094	0,0097
Do=0,01					
σp = 100					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,5521	1,5278	3,1065	5,2768	8,0382
I (A/m)	1,9800	3,9600	5,9400	7,9200	9,9000
Rserie (Ω)	0,2788	0,3858	0,5230	0,6663	0,81194
σp = 1000					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,1802	0,3101	0,5309	0,8385	1,2326
I (A/m)	1,9800	3,9600	5,9400	7,9200	9,9000
Rserie (Ω)	0,0910	0,0783	0,0894	0,1059	0,1245
σp = 10000					
Pair of elementary Cells	1	2	3	4	5
Vmax (V)	0,0534	0,0668	0,0904	0,1236	0,1662
I (A/m)	1,9800	3,9600	5,9400	7,9200	9,9000
Rserie (Ω)	0,0270	0,0169	0,0152	0,0156	0,0168

Table 2. Simulations data with different elementary pair cells

Marked in green the lowest Rseries

The table 2 shows the Rseries for each pair of elementary cells, so it will be chosen the lowest Rseries for each conductivity. It can be seen clearly in the figures 17 to 22.

Do = 0,02

Plotting it can be see:

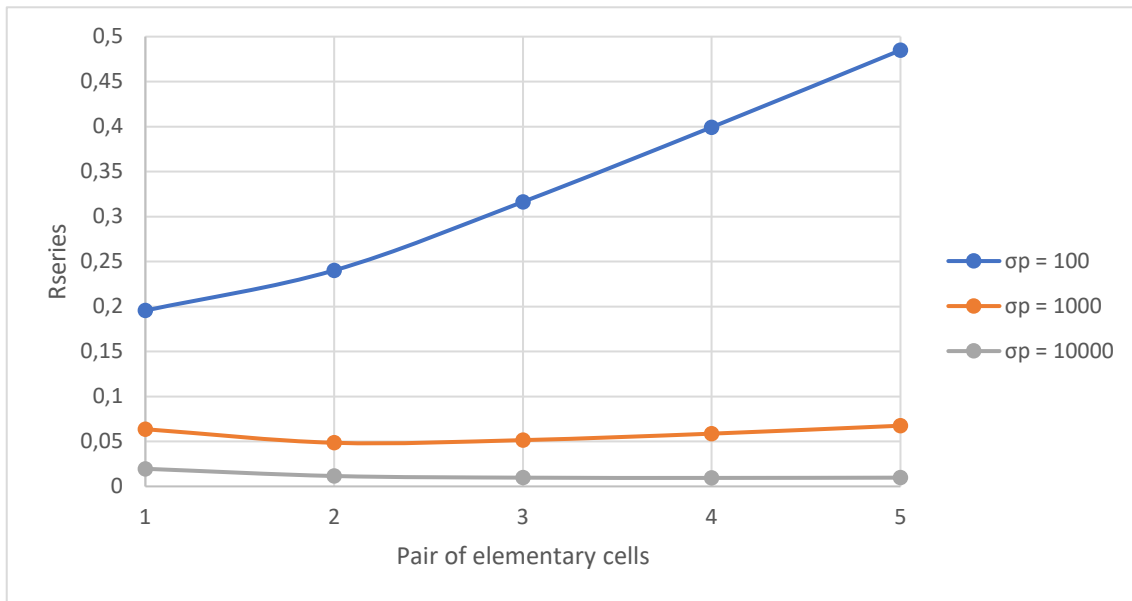


Fig 17. Graph of Rseries for each elementary pair cells

- for $\sigma_p = 100$, the trend of this resistance is increasing almost linearly in whole “f” interval, being the minimum value of resistance one pair of elementary cells.

It can be determinate $R1_{serie} < R2_{serie} < \dots < R_{serie}$ so R1 (one pair of elementary cells) is the lowest Rseries.

- for $\sigma_p = 1000$, the trend of this resistance is convex, being the minimum value of resistance two pairs of elementary cells.
- for $\sigma_p = 10000$, the trend of this resistance is convex, being the minimum value of resistance four pairs of elementary cells.

Let’s do a zoom in to see clearer the minimum:

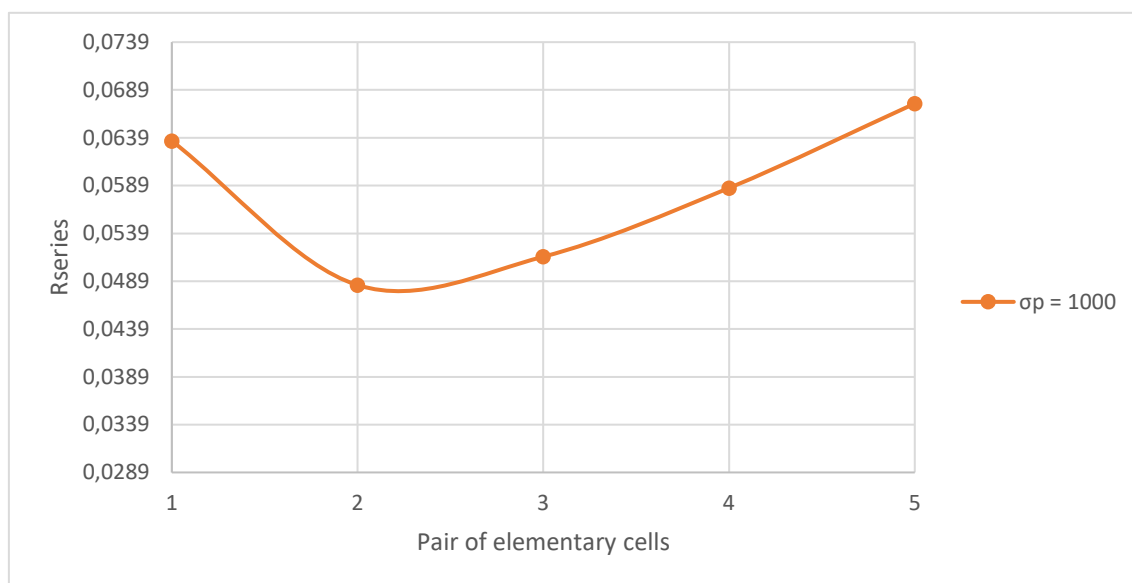


Fig 18. Graph of Rseries for each elementary pair cells zoomed for $\sigma_p = 1000$

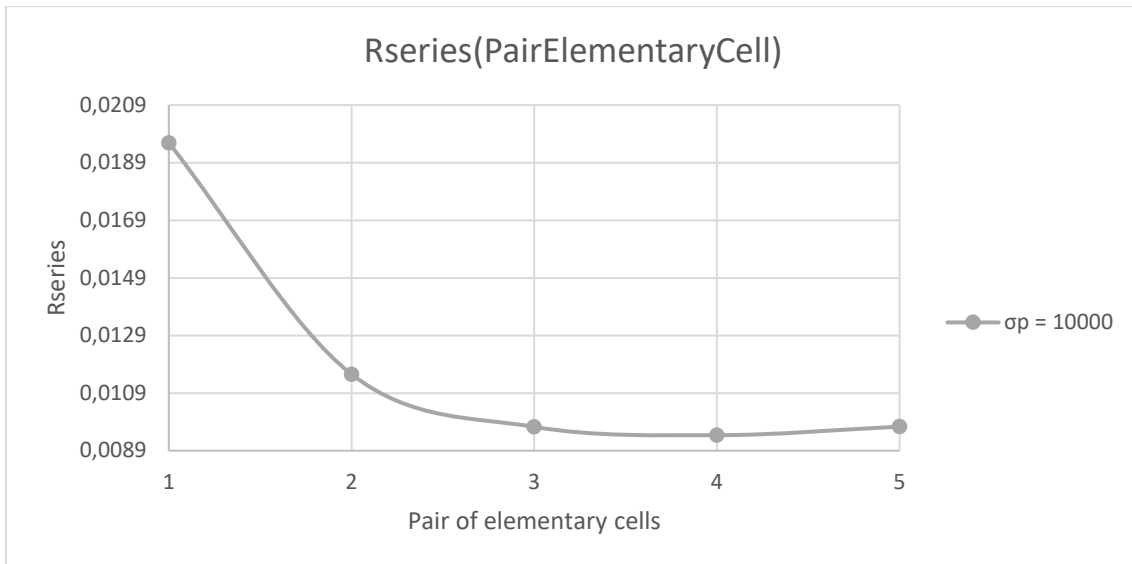


Fig 19. Graph of Rseries for each elementary pair cell zoomed for $\sigma_p = 10000$

Do = 0,01

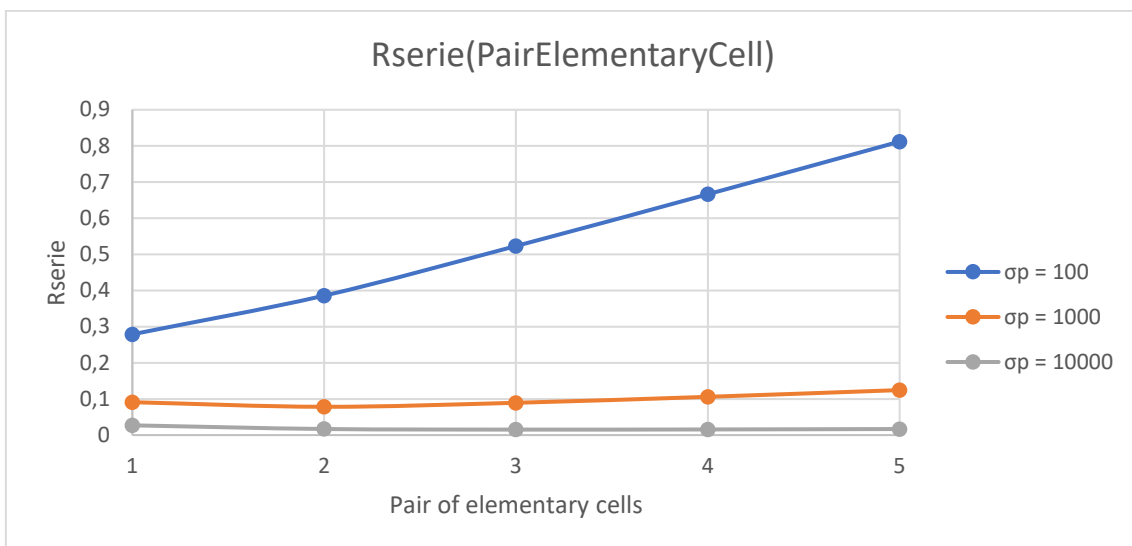


Fig 20. Graph of Rseries for each elementary pair cells

- for $\sigma_p = 100$, the trend of this resistance is increasing linearly in whole “f” interval, being the minimum value of resistance one pair of elementary cells.

It can be determinate $R1serie < R2serie < \dots < Riserie$ so $R1$ (one pair of elementary cells) is the lowest Rseries.

- for $\sigma_p = 1000$, the trend of this conductivity is convex, being the minimum value of resistance two pairs of elementary cells.
- for $\sigma_p = 10000$, the trend of this conductivity is convex, being the minimum value of resistance three pairs of elementary cells.

Let's do a zoom in to see clearer the minimum:

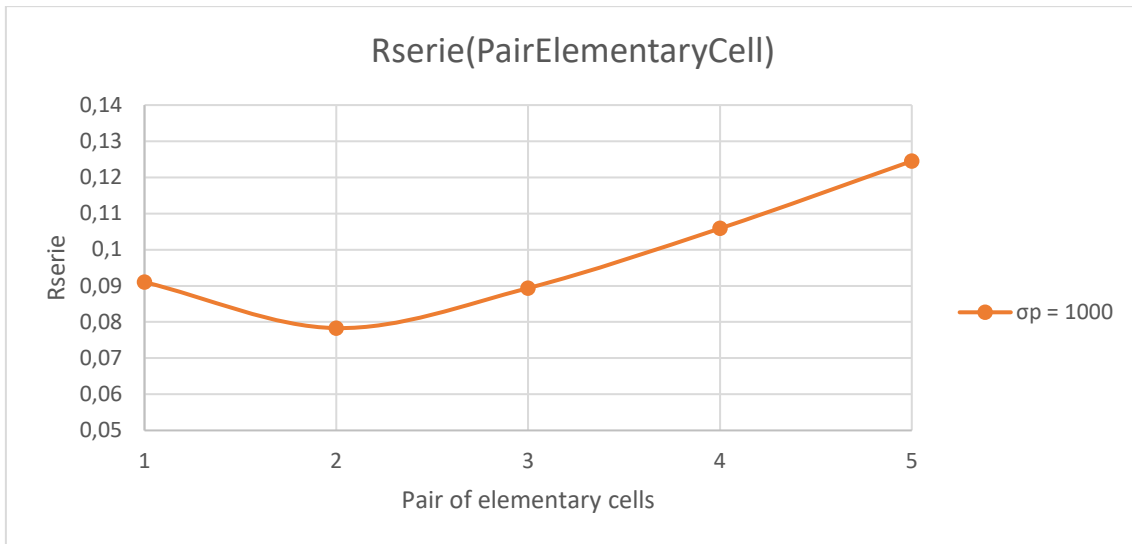


Fig 21. Graph of Rseries for each elementary pair cells zoomed for $\sigma_p = 1000$

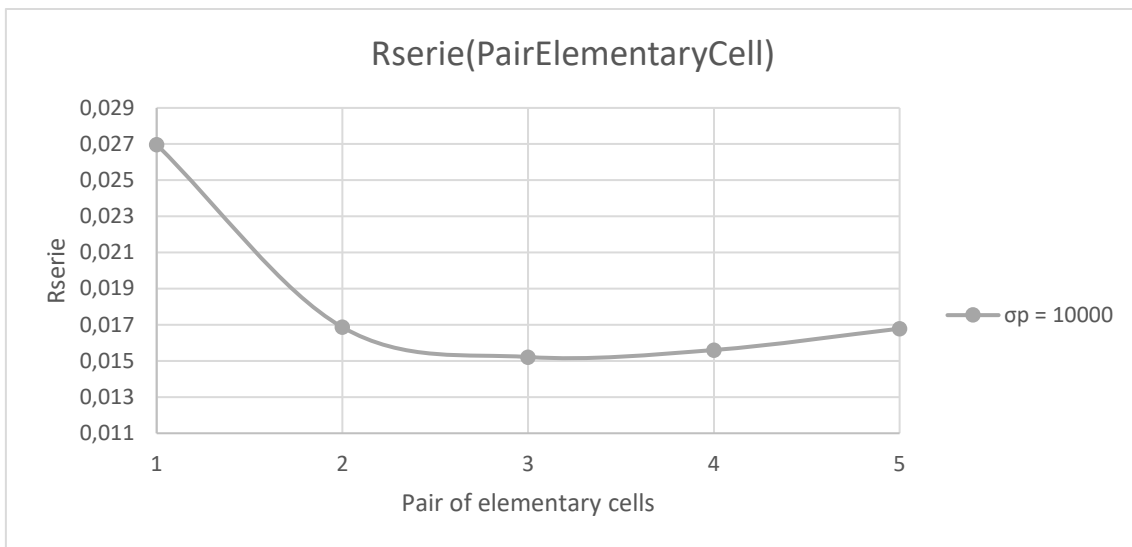


Fig 22. Graph of Rseries for each elementary pair cells zoomed for $\sigma_p = 10000$

5. The second order ensemble

The best second order assembly is made of two optimized first order assemblies where $H2 = H1 = 2L1_{opt} = 2L2$ and $D2 = 2D1$.

5.1 Mathematical model

To determine the minimum voltage and width $D2$ optimal (Fig.23):

$$\Delta V_{a_2, \min} = \frac{3}{8} \left(1 + \frac{1}{3^{1/2}} \right) \frac{w''' H_0^2}{\sigma_0} \quad (8)$$

$$\frac{D_{2, \text{opt}}}{H_0} = \frac{4}{3^{1/2}} \left(\frac{\sigma_p}{\sigma_0} \right)^{1/2} \left(\frac{D_0}{H_0} \right)^{3/2} \quad (9)$$

It can apply the same before equations for higher orders than two, but the second one can be written generally as:

$$D_{i, \text{opt}} = \frac{2^i}{3^{1/2}} D_0 \left(\frac{D_0 \sigma_p}{H_0 \sigma_0} \right)^{1/2} \text{ for } i \geq 2 \quad (10)$$

5.2 The optimization sequence

The main purpose of this simulation is determinate the minimum Rseries as the chapter before.

The Fig.23 is a simplified geometry:

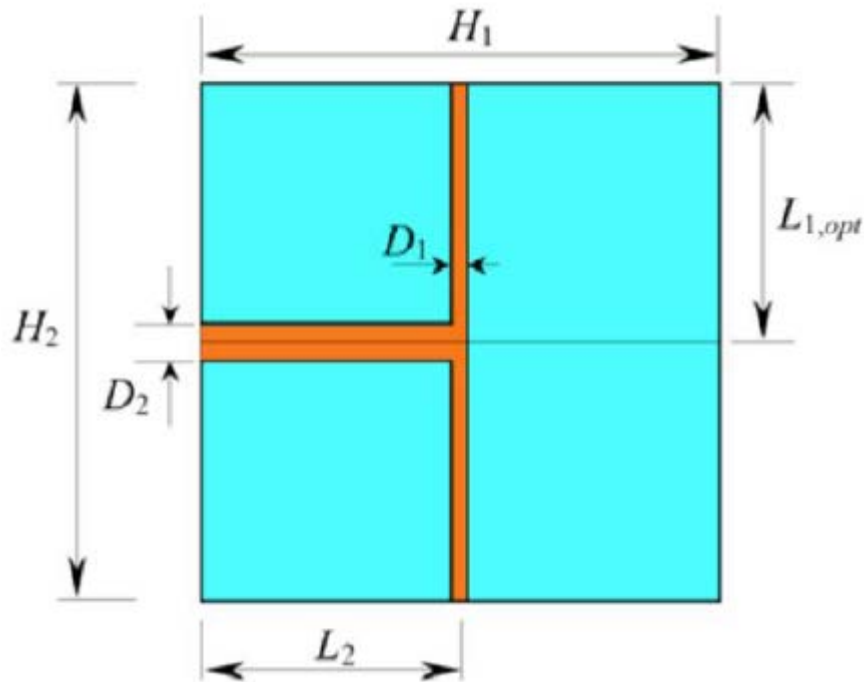


Fig 23 Ensemble of the second order

Where the conductive band height is $D_2 = 2D_1 = 4D_0$

- For $\sigma_p = 100$, It will leave from the first order with the optimal pairs, is this case is 1 pair of elementary cells.
- For $\sigma_p = 1000$, It will leave from the first order with the optimal pairs, is this case is 2 pair of elementary cells.
- For $\sigma_p = 10000$, It will leave from the first order with the optimal pairs, is this case is 4 pair of elementary cells.

Do = 0,01

- For $\sigma_p = 100$, It will leave from the first order with the optimal pairs, is this case is 2 pair of elementary cells.
- For $\sigma_p = 1000$, It will leave from the first order with the optimal pairs, is this case is 2 pair of elementary cells.
- For $\sigma_p = 10000$, It will leave from the first order with the optimal pairs, is this case is 3 pair of elementary cells.

In the simulator is drawn like:



Fig 24 Second construct: two optimized first order constructs with Do = 0,02.

It will be drawn every conductivity case.

It will be define two different boundaries.

The external boundary, marked in red (the limit of the PV cell with the air) as an electric insulation (current through the boundary = 0).

The limit of the aluminium band with the air in left side will be define as a ground (Voltage = 0) marked in blue.

Do = 0,02

- For $\sigma_p = 100$, the current is 4
- For $\sigma_p = 1000$, the current is 7,92
- For $\sigma_p = 10000$, the current is 15,76

Do = 0,01

- For $\sigma_p = 100$, the current is 4
- For $\sigma_p = 1000$, the current is 7,96
- For $\sigma_p = 10000$, the current is 11,92

5.3 Solve the simulation

With the steps described in the section 3.2, it can mesh and calculate the solution in the simulator software as it can be seen in the figure 25 to 30.

Do = 0,02

For $\sigma_p = 100$

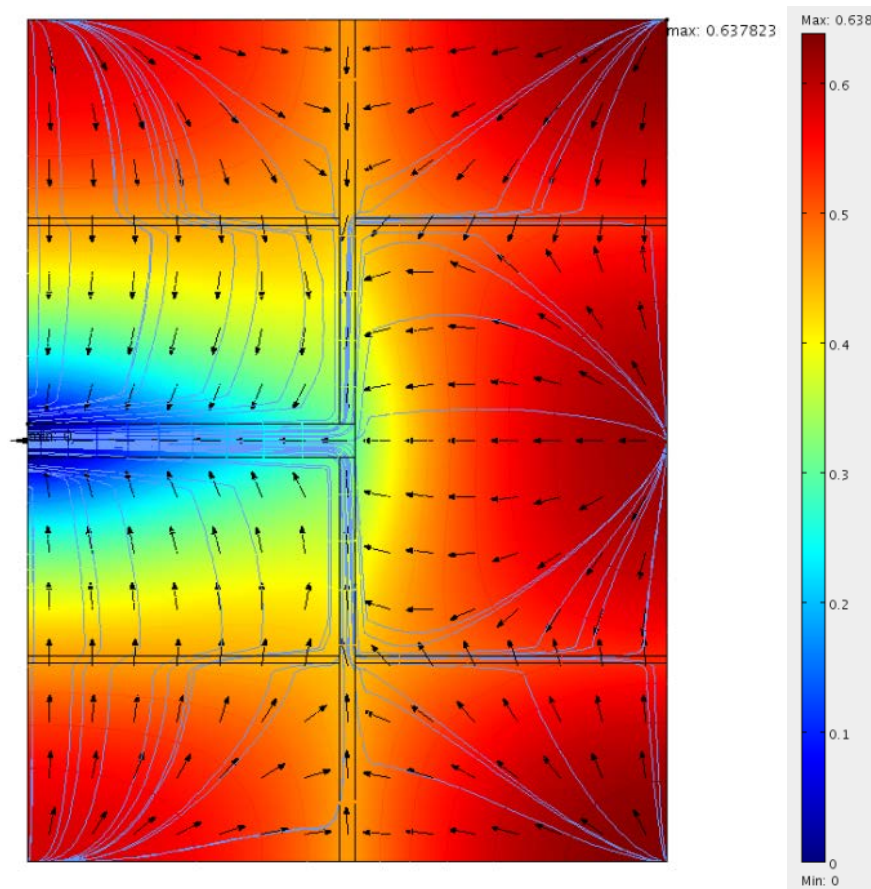


Fig 25 Optimal design of the second order ensemble for $\sigma_p = 100$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,6378$

$I_{total} = 4$

For $\sigma_p = 1000$

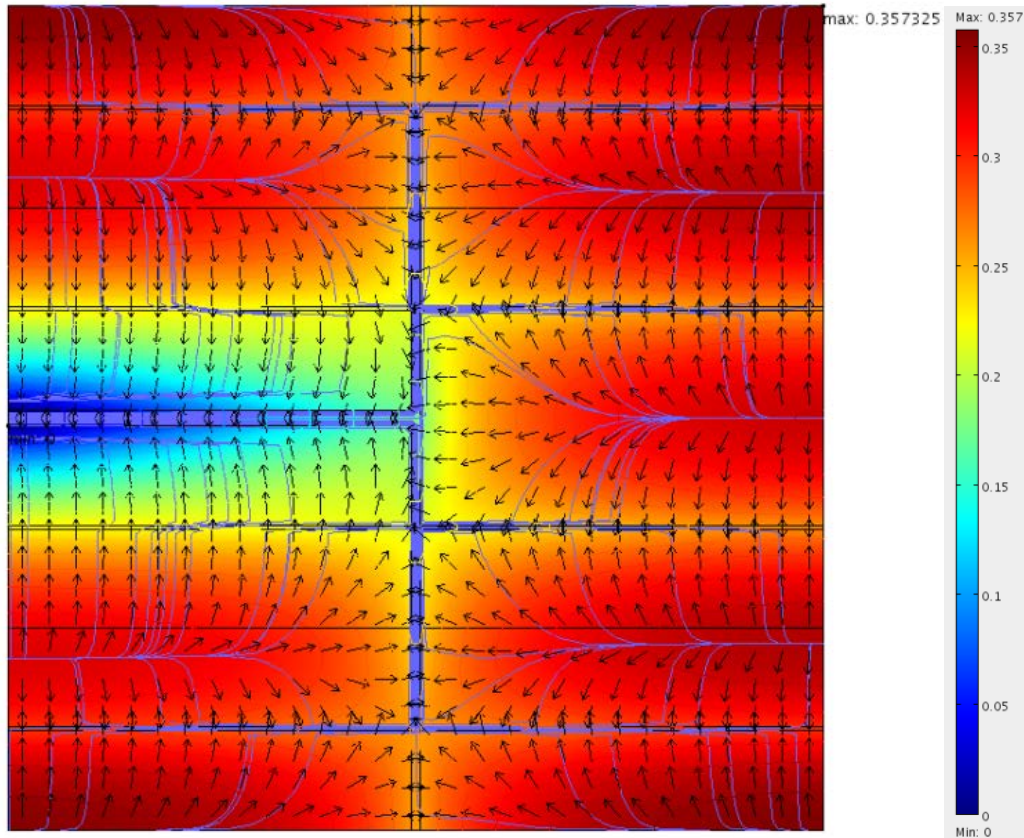


Fig 26 Optimal design of the second order ensemble for $\sigma_p = 1000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,3573$

$I_{total} = 7,92$

For $\sigma_p = 10000$

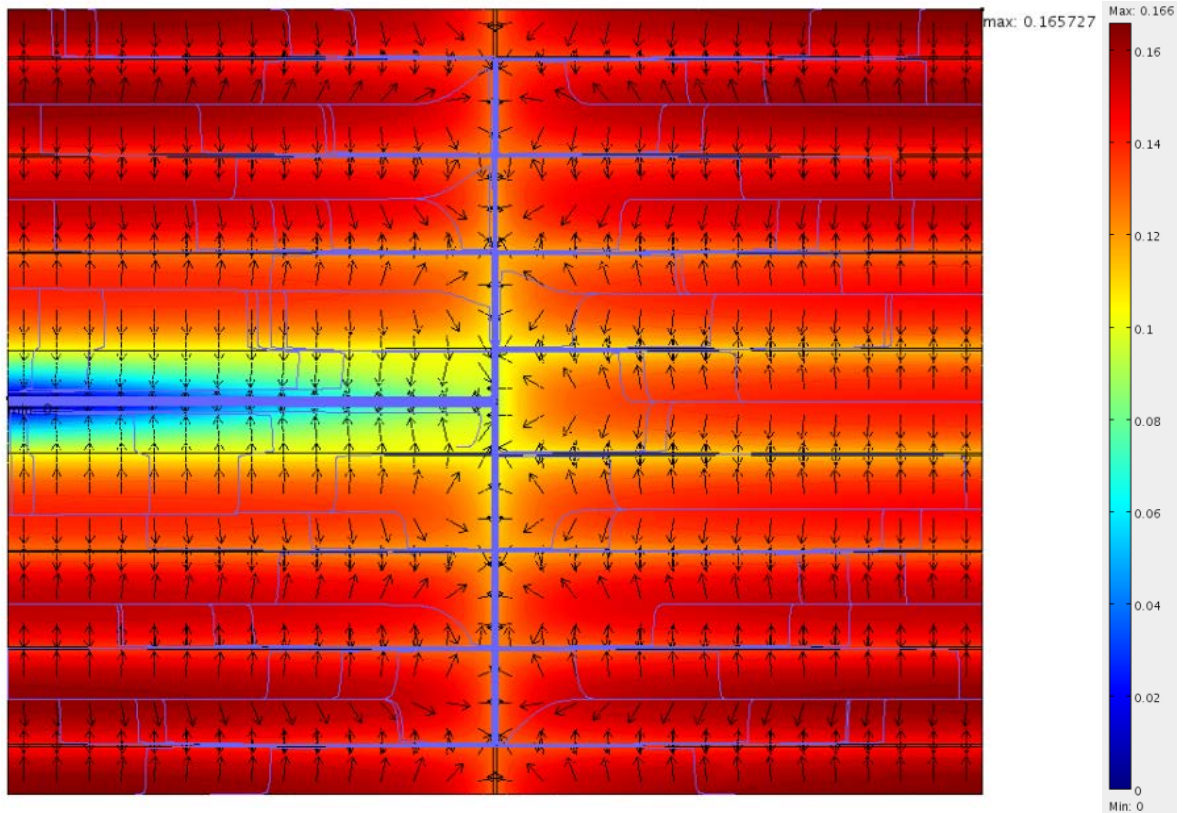


Fig 27 Optimal design of the second order ensemble for $\sigma_p = 10000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,1657$

$I_{total} = 15,76$

Do = 0,01

For $\sigma_p = 100$

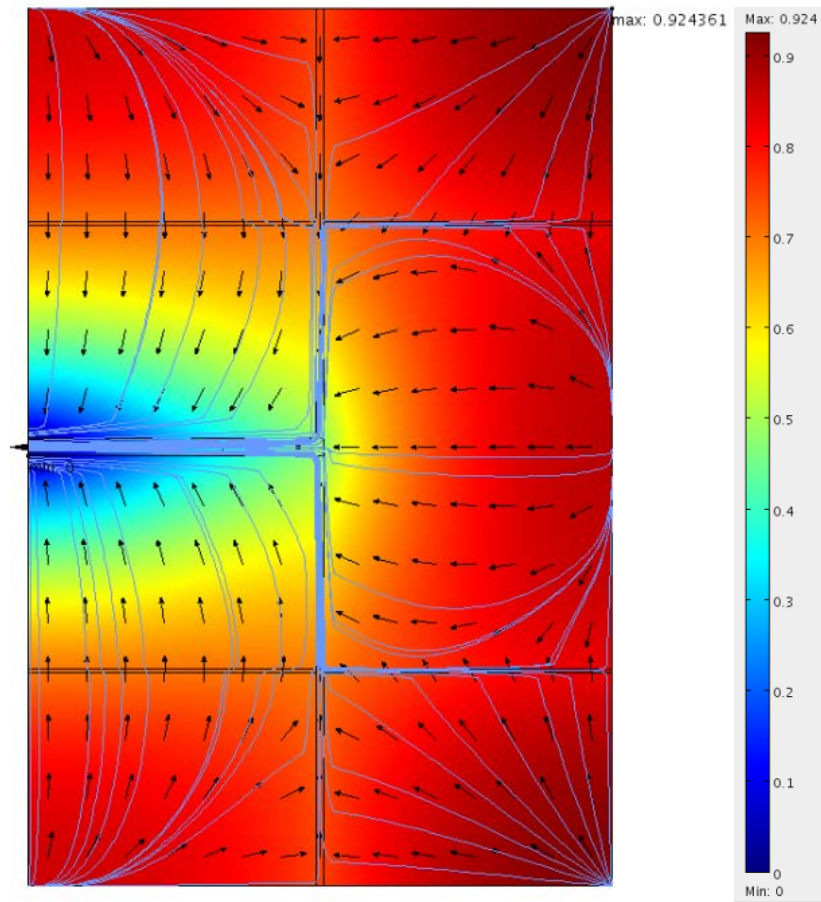


Fig 28 Optimal design of the second order ensemble for $\sigma_p = 100$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,9243$

$I_{total} = 4$

For $\sigma_p = 1000$

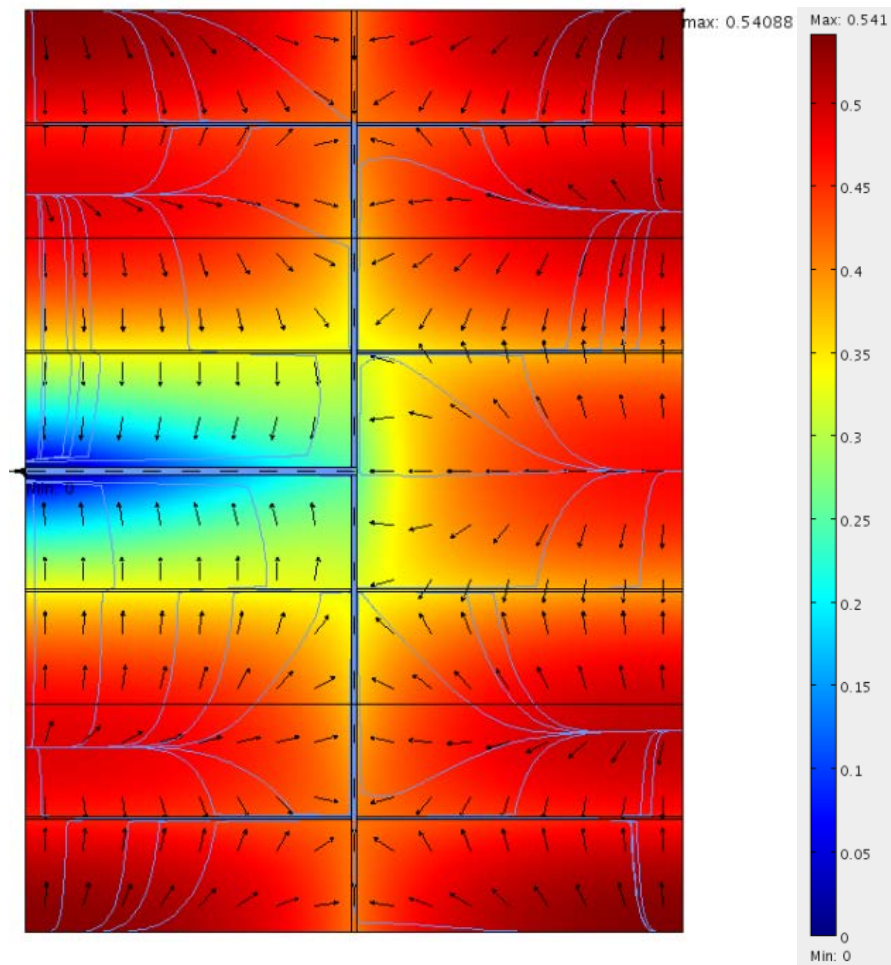


Fig 29 Optimal design of the second order ensemble for $\sigma_p = 1000$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,5408$

$I_{total} = 7,96$

For $\sigma_p = 10000$

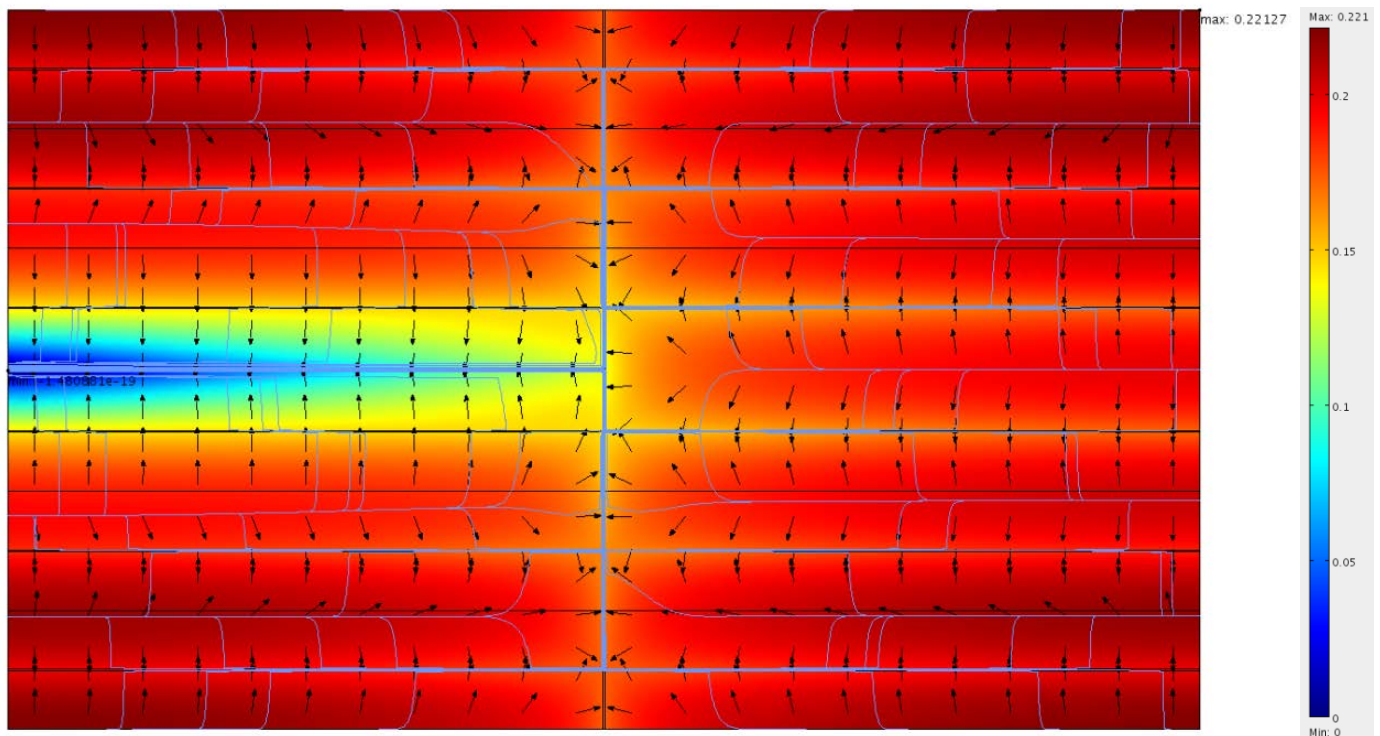


Fig 30 Optimal design of the second order ensemble for $\sigma_p = 10000$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,2212$

$I_{total} = 11,92$

5.4 Results and conclusions

Now it will be calculate the R_{series} of each sigma.

$$R_{series} = \frac{V_{max}}{I_{total}} \quad (11)$$

Do = 0,02

- For $\sigma_p = 100$, the R_{series} is 0,1594
- For $\sigma_p = 1000$, the R_{series} t is 0,0451
- For $\sigma_p = 10000$, the R_{series} is 0,0105

Do = 0,01

- For $\sigma_p = 100$, the Rseries is 0,231
- For $\sigma_p = 1000$, the Rseries t is 0,0679
- For $\sigma_p = 10000$, the Rseries is 0,0185

6. The third order ensemble

The best third order assembly is made of wo optimized first order assemblies where $H_3 = 2H_2 \cdot L_3 = H_2/2$ and $D_3 = 2D_2$.

6.1 Mathematical model

The mathematical model is described in the chapter before.

6.2 The optimization sequence

The main purpose of this simulation is determinate the minimum Rseries as the chapter before.

The Fig.31 is a simplified geometry:

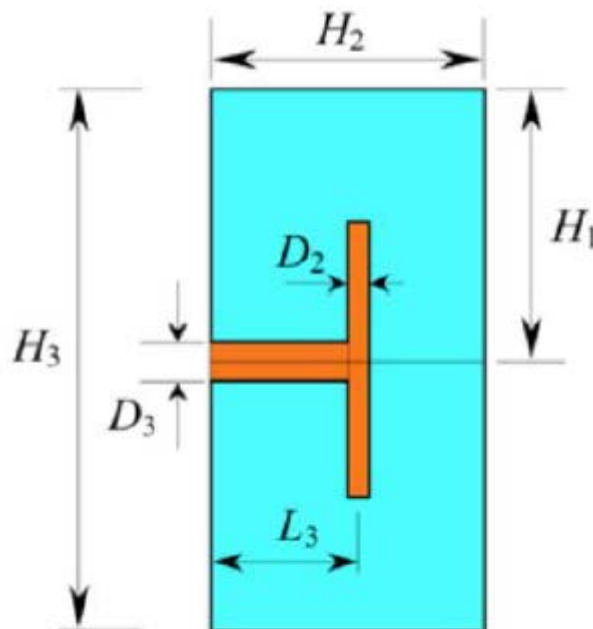


Fig 31 Ensemble of the second order

Where the conductive band height is $D_3 = 2D_2 = 4D_1 = 8D_0$

For each conductivity, it will leave from the optimal second order described in the chapter before.

In the simulator is drawn like:

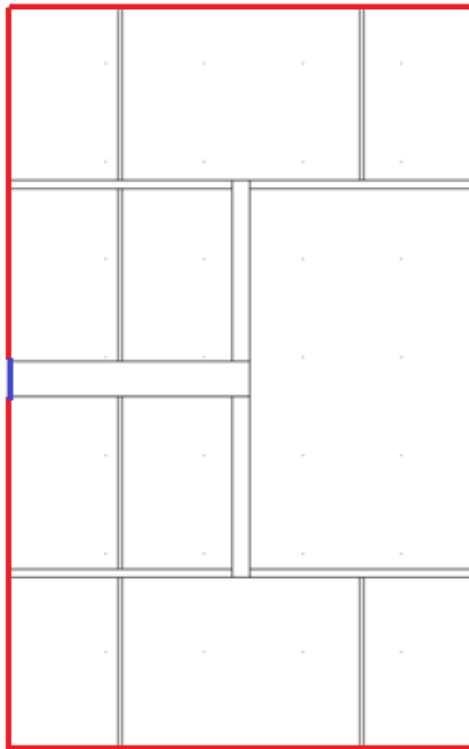


Fig 32 Third construct: two optimized second order constructs

It will be drawn every conductivity case.

Two types of boundary conditions will be used.

The external boundary, marked in red (the limit of the PV cell with the air) as an electric insulation (current through the boundary = 0).

The limit of the aluminium band with the air in left side will be define as a ground (Voltage = 0) marked in blue.

It will be integrated to get the value of the current.

Do = 0,02

- For $\sigma_p = 100$, the current is 8,20
- For $\sigma_p = 1000$, the current is 16
- For $\sigma_p = 10000$, the current is 31,64

Do = 0,01

- For $\sigma_p = 100$, the current is 8,12
- For $\sigma_p = 1000$, the current is 16,03
- For $\sigma_p = 10000$, the current is 23,88

6.3 Solve the simulation

With the steps described in the section 3.2, it can mesh and calculate the solution in the simulator software as it can be seen in the figure 33 to 38.

$$Do = 0,02$$

$$\sigma_p = 100$$

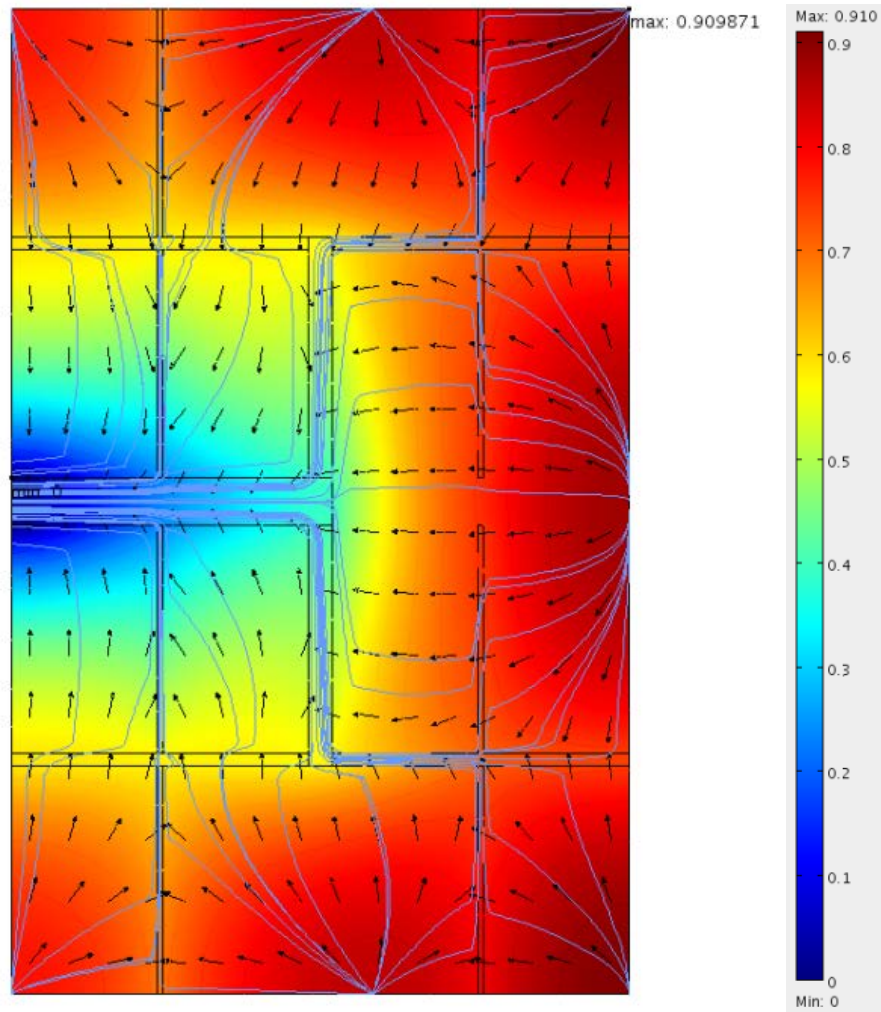


Fig 33 Optimal design of the third order ensemble for $\sigma_p = 100$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$$V_{max} = 0,9098$$

$$I_{total} = 8,20$$

$\sigma_p = 1000$

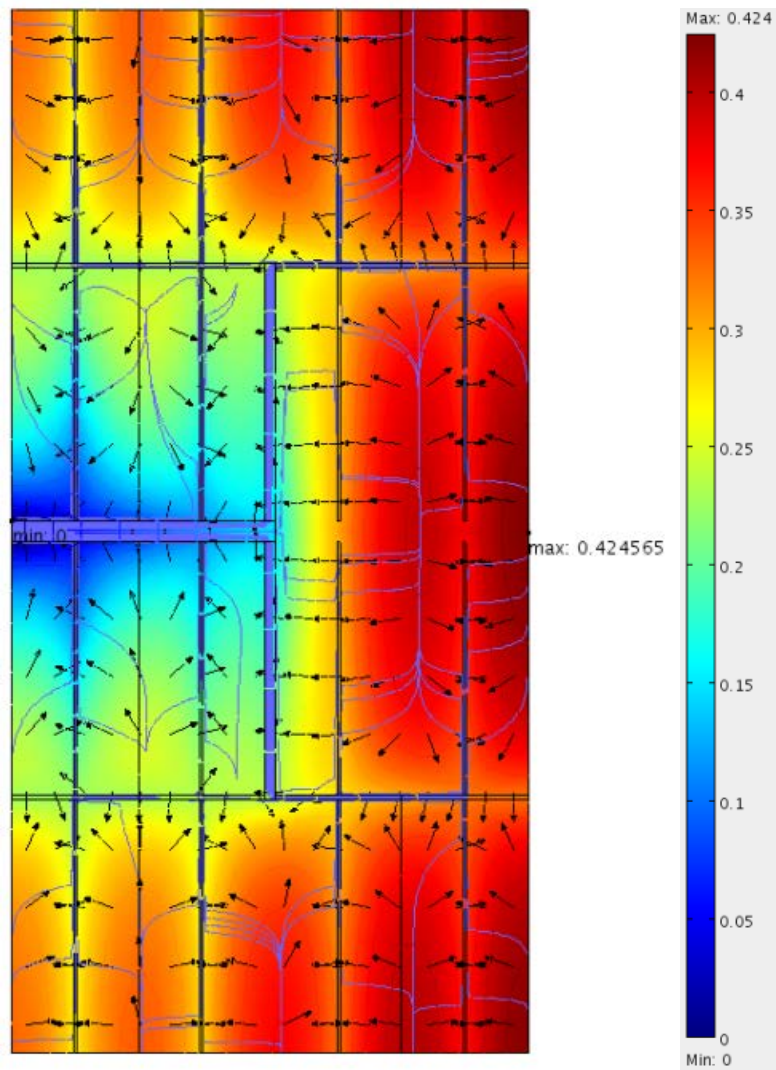


Fig 34 Optimal design of the third order ensemble for $\sigma_p = 1000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,4245$

$I_{total} = 16$

$\sigma_p = 10000$

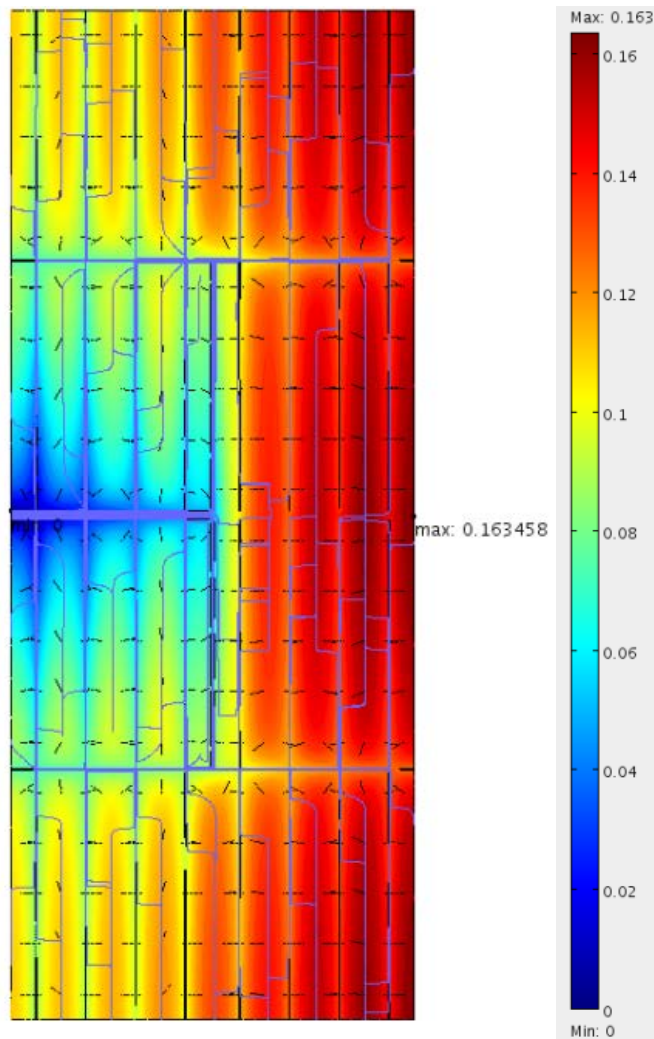


Fig 35 Optimal design of the third order ensemble for $\sigma_p = 100$ and $Do=0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,1634$

$I_{total} = 31,64$

Do = 0,01

For $\sigma_p = 100$

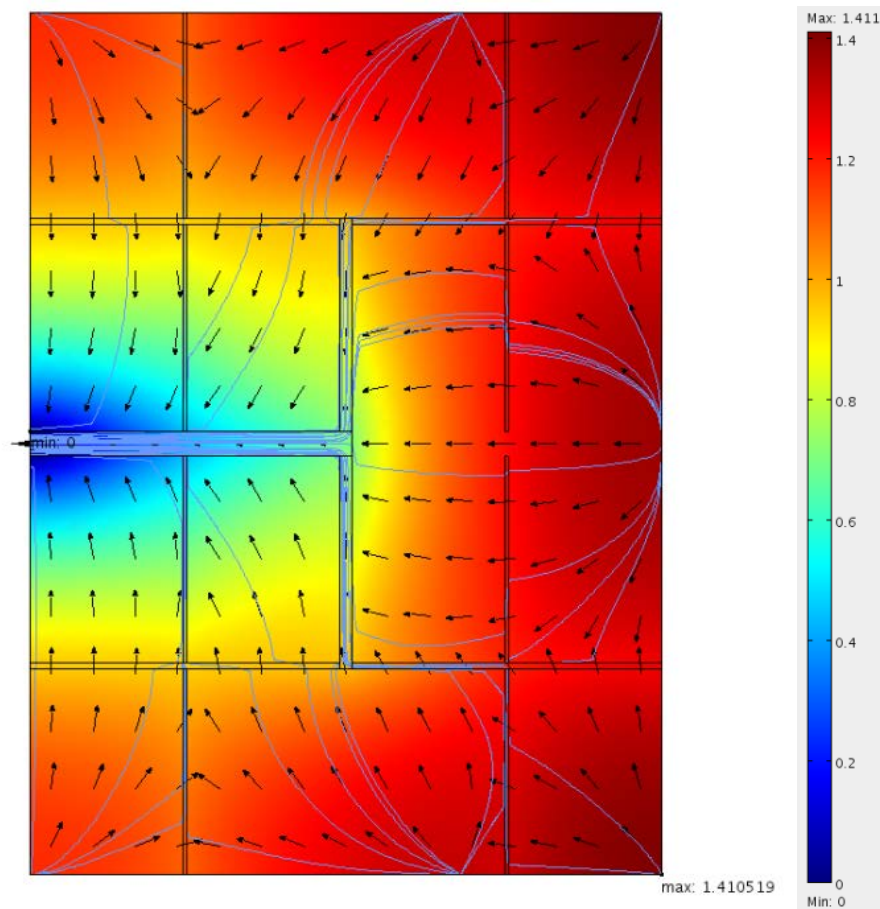


Fig 36 Optimal design of the third order ensemble for $\sigma_p = 100$ and $Do=0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 1,4105$

$I_{total} = 8,12$

For $\sigma_p = 1000$

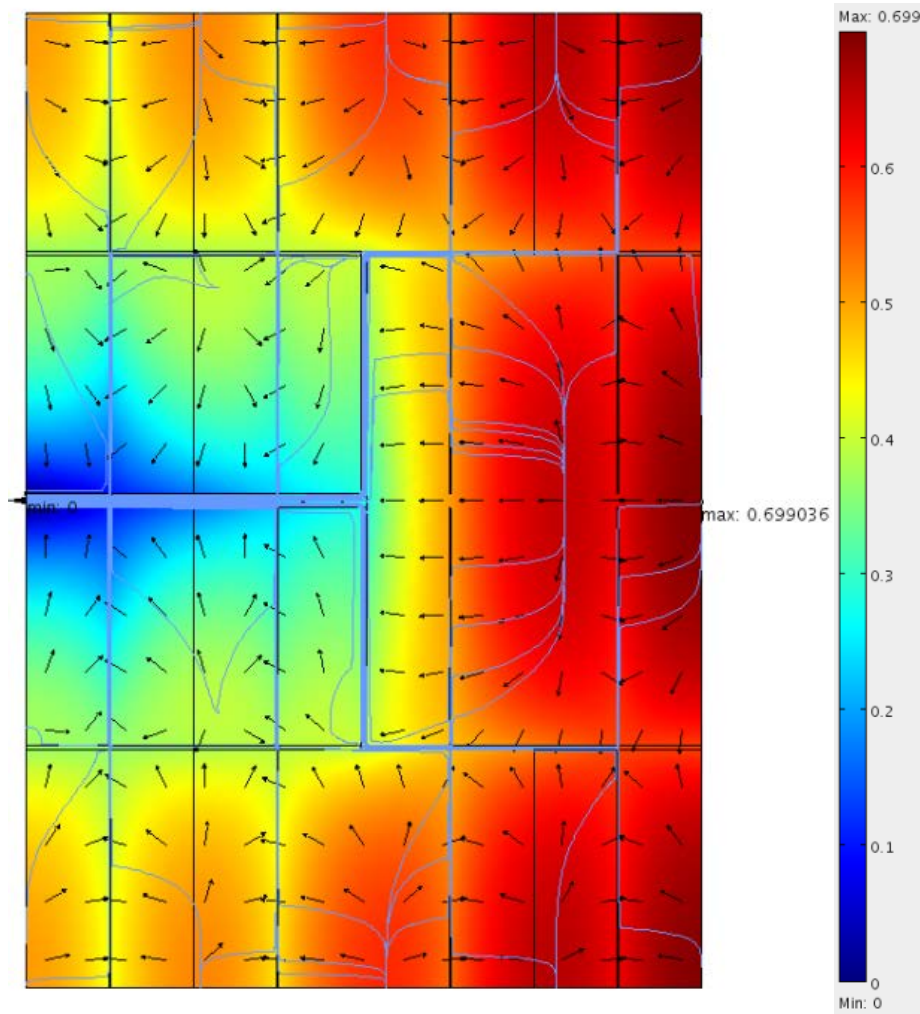


Fig 37 Optimal design of the third order ensemble for $\sigma_p = 1000$ and $Do=0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,699$

$I_{total} = 16,03$

For $\sigma_p = 10000$

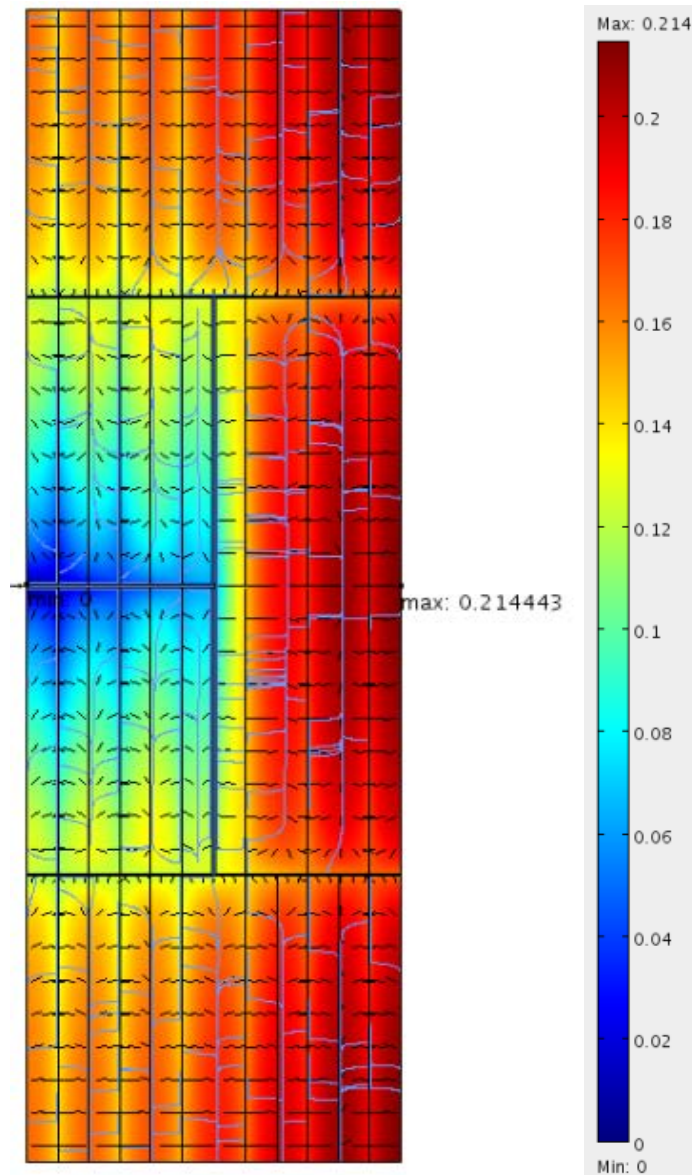


Fig 38 Optimal design of the third order ensemble for $\sigma_p = 10000$ and $Do=0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,2144$

$I_{total} = 23,88$

6.4 Results and conclusions

Now it will be calculate the Rseries of each sigma.

$$R_{series} = \frac{V_{max}}{I_{total}} \quad (12)$$

Do = 0,02

- For $\sigma_p = 100$, the Rseries is 0,1109
- For $\sigma_p = 1000$, the Rseries t is 0,0265
- For $\sigma_p = 10000$, the Rseries is 0,0051

Do = 0,01

- For $\sigma_p = 100$, the Rseries is 0,1737
- For $\sigma_p = 1000$, the Rseries t is 0,0436
- For $\sigma_p = 10000$, the Rseries is 0,00897

7 The fourth order ensemble

The best second order assembly is made of wo optimized first order assemblies where $H4 = H3 = 2H2 = 2L4_{opt}$ and $D4 = 2D3$.

7.1 Mathematical model

The mathematical model is described in the chapter before.

7.2 The optimization sequence

The main purpose of this simulation is determinate the minimum Rseries as the chapter before.

The Fig.39 is a simplified geometry:

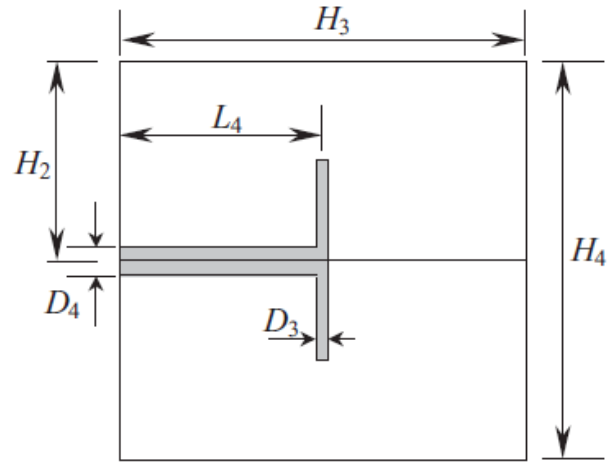


Fig 39 Ensemble of the fourth order

In the simulator is drawn like:

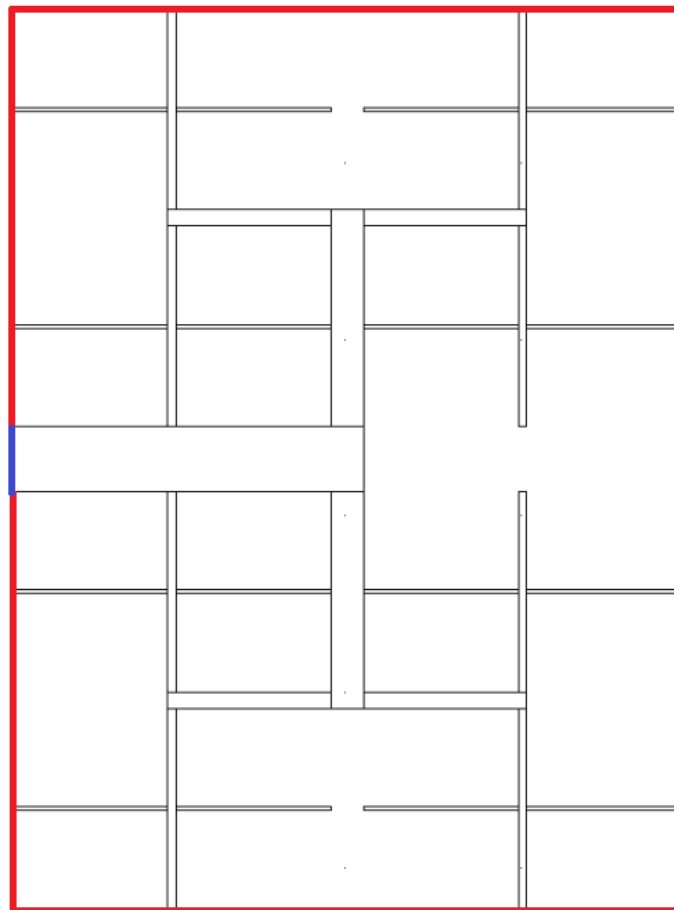


Fig 40 Fourth construct: two optimized third order constructs

Where the conductive band height is $D4 = 2D3 = 2D2 = 4D1 = 8D0$

It will be fill the current source and electric conductivity.

Two types of boundary conditions will be used.

The external boundary, marked in red (the limit of the PV cell with the air) as an electric insulation (current through the boundary = 0).

The limit of the aluminium band with the air in left side will be define as a ground (Voltage = 0) marked in blue.

It will be integrated to get the value of the current.

Do = 0,02

- For $\sigma_p = 100$, the current is 17,07
- For $\sigma_p = 1000$ the current is 32,64
- For $\sigma_p = 10000$ the current is 63,93

Do = 0,01

- For $\sigma_p = 100$, the current is 16,56
- For $\sigma_p = 1000$ the current is 32,38
- For $\sigma_p = 10000$ the current is 48,09

7.3 Solve the simulation

With the steps described in the section 3.2, it can mesh and calculate the solution in the simulator software as it can be seen in the figure 41 to 46.

Do = 0,02

For $\sigma_p = 100$

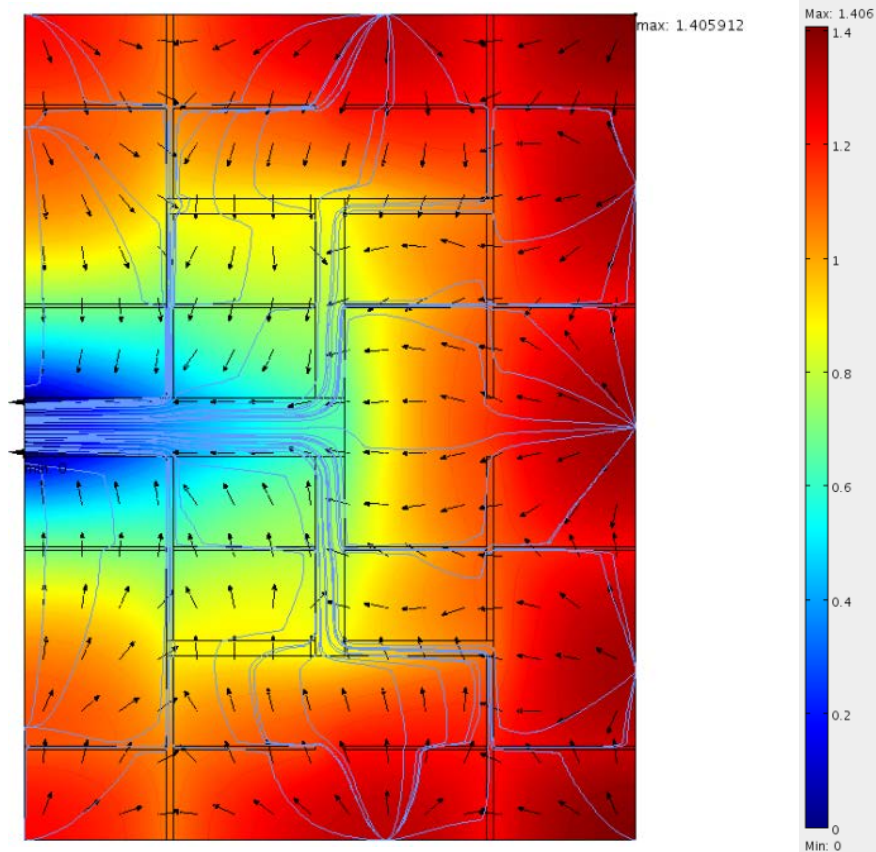


Fig 41 Optimal design of the third order ensemble for $\sigma_p = 100$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 1,4059$

$I_{total} = 17,07$

For $\sigma_p = 1000$

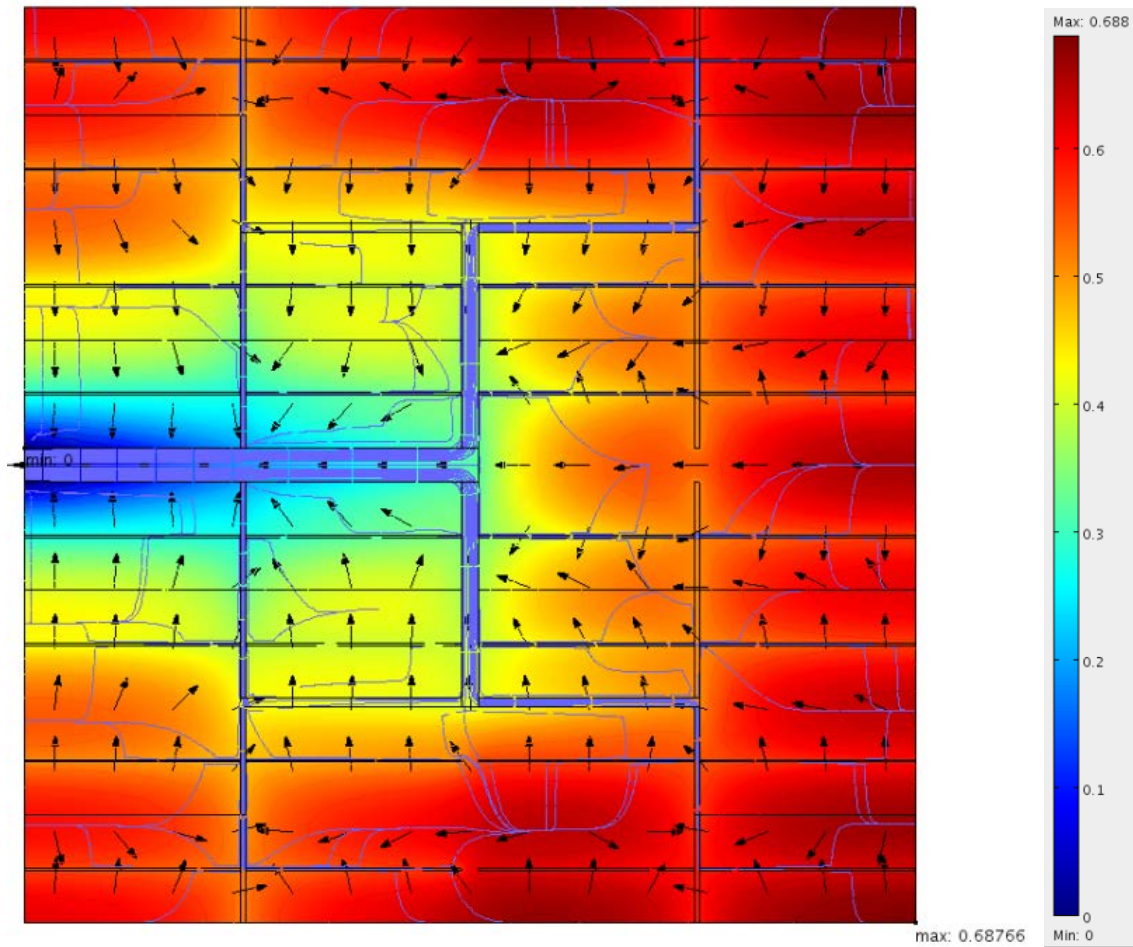


Fig 42 Optimal design of the third order ensemble for $\sigma_p = 1000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,6876$

$I_{total} = 32,64$

For $\sigma_p = 10000$

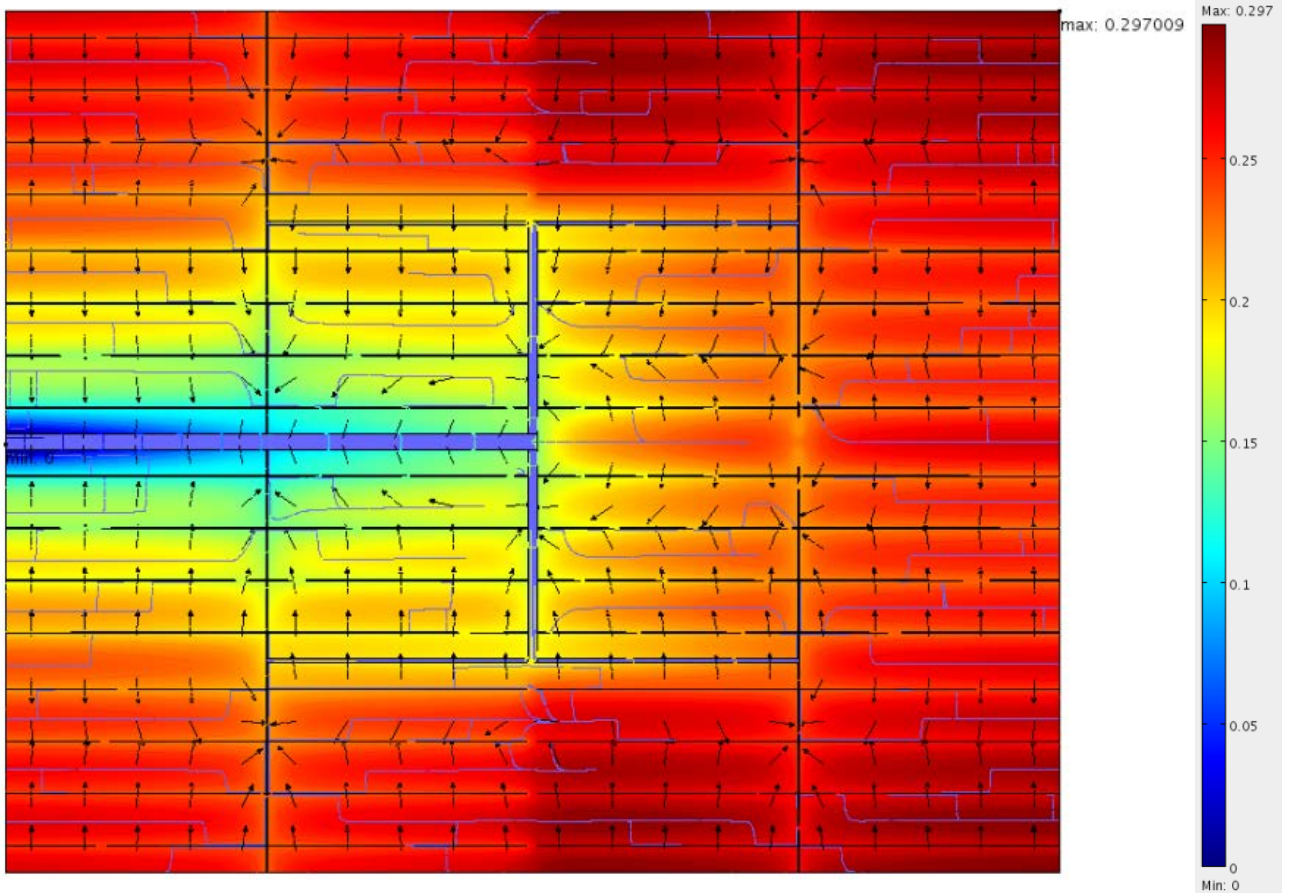


Fig 43 Optimal design of the third order ensemble for $\sigma_p = 10000$ and $Do = 0,02$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 0,2970$

$I_{total} = 63,93$

Do = 0,01

For $\sigma_p = 100$

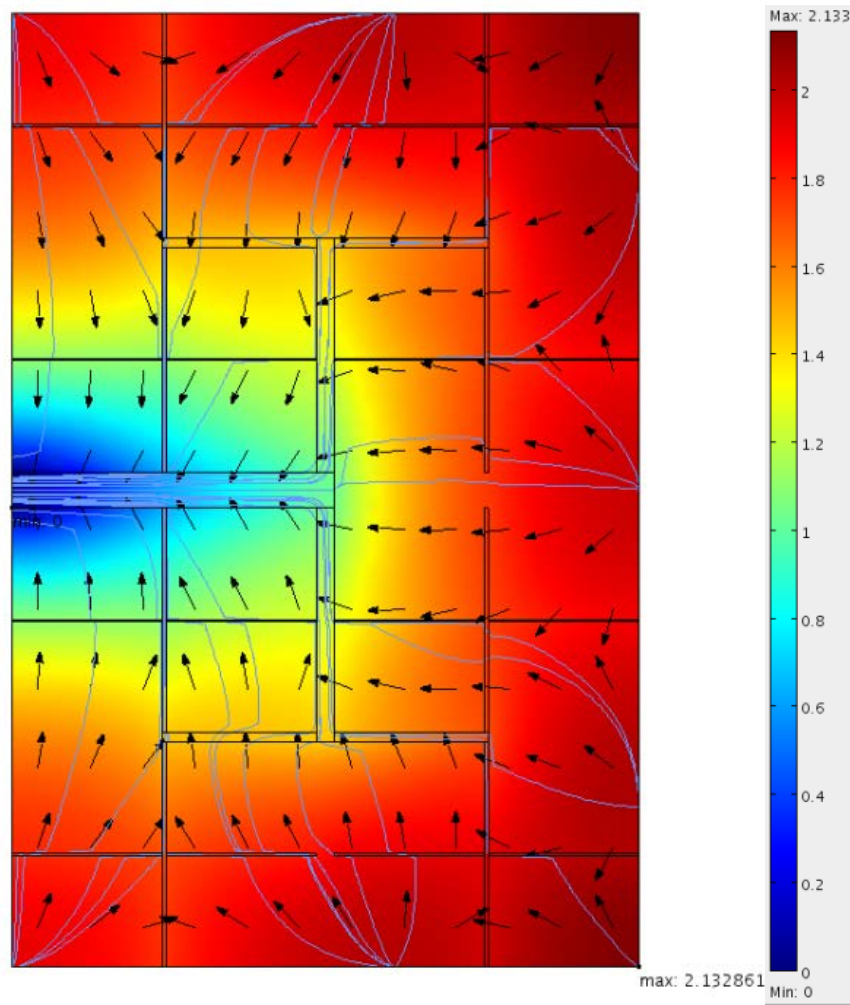


Fig 44 Optimal design of the third order ensemble for $\sigma_p = 100$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 2,1328$

$I_{total} = 16,56$

For $\sigma_p = 1000$

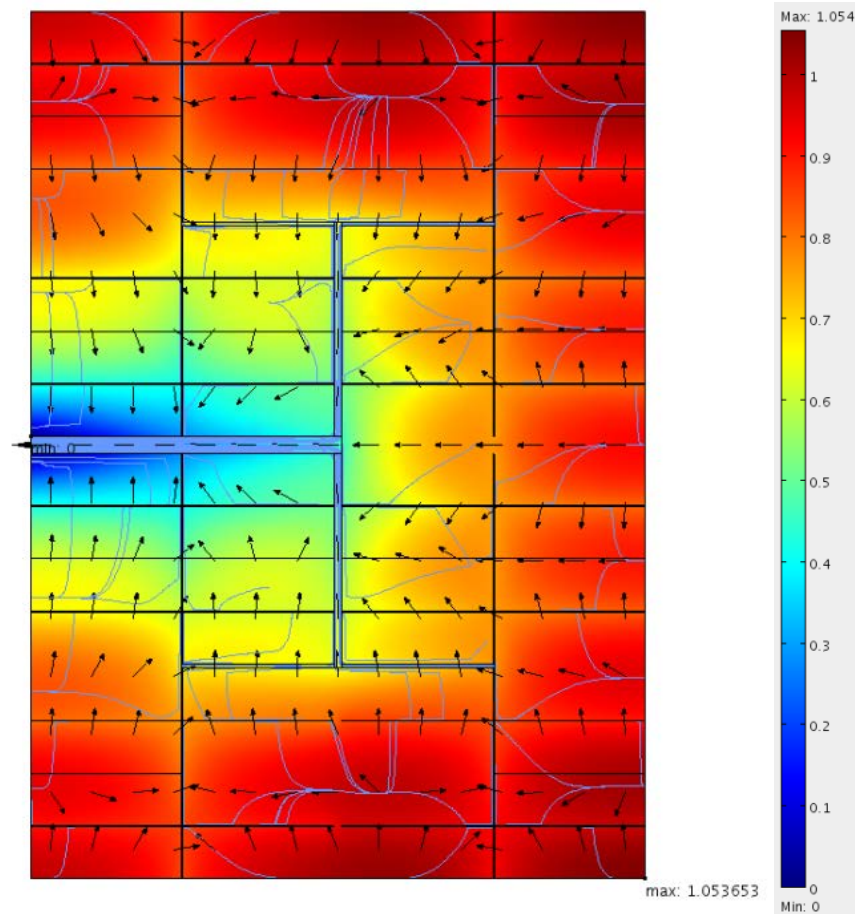


Fig 45 Optimal design of the third order ensemble for $\sigma_p = 1000$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$V_{max} = 1,0536$

$I_{total} = 32,38$

For $\sigma_p = 10000$

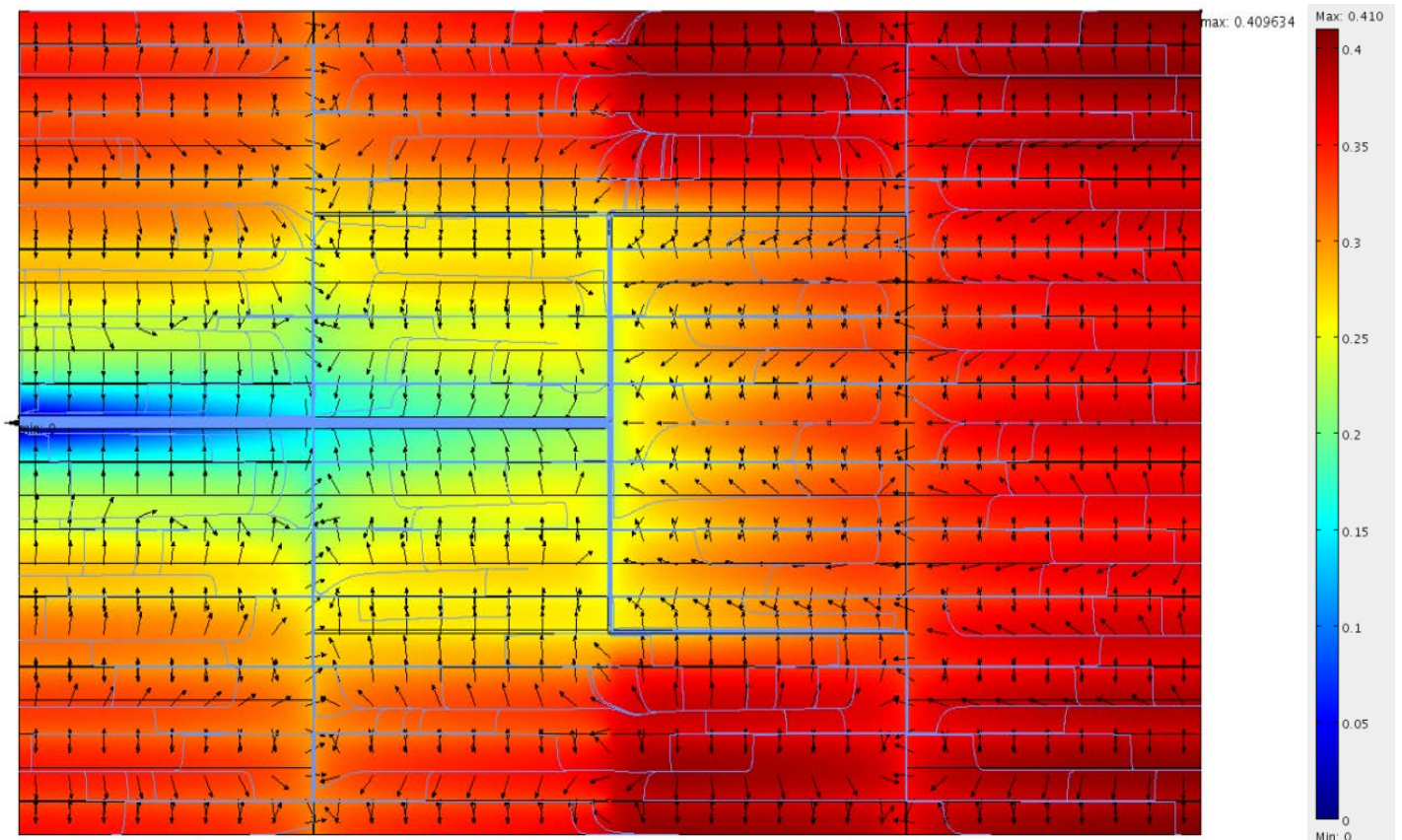


Fig 46 Optimal design of the third order ensemble for $\sigma_p = 10000$ and $Do = 0,01$

The arrows show the direction of the current and the lines the total current density.

As it can be seen, the simulation shows the value and position of the maximum Voltage point.

$$V_{max} = 0,4096$$

$$I_{total} = 48,09$$

7.4 Results and conclusions

Now it will be calculate the Rseries of each conductivity.

$$R_{series} = \frac{V_{max}}{I_{total}} \quad (13)$$

Do = 0,02

- For $\sigma_p = 100$, the Rseries is 0,0823
- For $\sigma_p = 1000$, the Rseries t is 0,0210



- For $\sigma_p = 10000$, the Rseries is 0,0046

Do = 0,01

- For $\sigma_p = 100$, the Rseries is 0,1287
- For $\sigma_p = 1000$, the Rseries t is 0,0325
- For $\sigma_p = 10000$, the Rseries is 0,0085

8 Final Conclusion

To better compare all the results, it will be arrange the final results in the next table.

Do=0,02				
$\sigma_p = 100$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,3832	0,6378	0,9098	1,4059
I (A/m)	1,9600	4,0000	8,2000	17,0100
Rserie (Ω)	0,1955	0,1595	0,1110	0,0827
$\sigma_p = 1000$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,1901	0,3575	0,4245	0,6876
I (A/m)	3,9200	7,9200	16,0000	32,6400
Rserie (Ω)	0,0485	0,0451	0,0265	0,0211
$\sigma_p = 10000$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,0740	0,1657	0,1634	0,2970
I (A/m)	7,8400	15,7600	31,6400	63,9300
Rserie (Ω)	0,0094	0,0105	0,0052	0,0046
Do=0,01				
$\sigma_p = 100$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,5521	0,9243	1,4105	2,1328
I (A/m)	1,9800	4,0000	8,1200	16,5600
Rserie (Ω)	0,2788	0,2311	0,1737	0,1288
$\sigma_p = 1000$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,3101	0,5408	0,6990	1,0536
I (A/m)	3,9600	7,9600	16,0300	32,3800
Rserie (Ω)	0,0783	0,0679	0,0436	0,0325
$\sigma_p = 10000$				
Order	1st	2nd	3rd	4th
Vmax (V)	0,0904	0,2212	0,2144	0,4096
I (A/m)	5,9400	11,9200	23,8800	48,0900
Rserie (Ω)	0,0152	0,0186	0,0090	0,0085

Table 3. Resume of all optimal data for each order

Graphing the previous results, it can be determinate the trend of them.

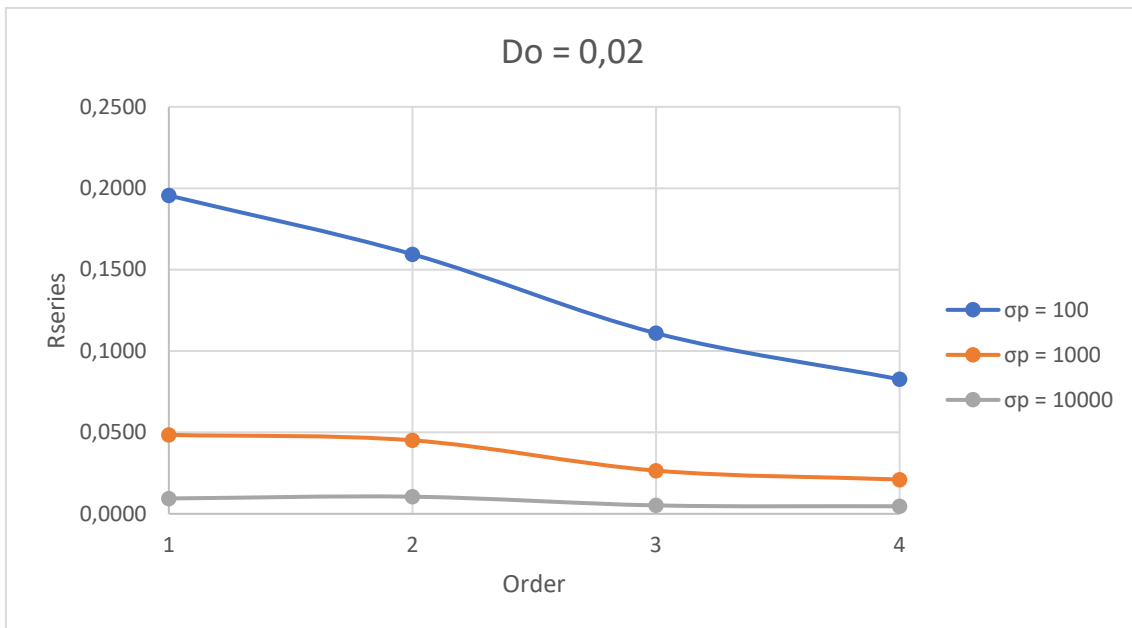


Fig 47. Graph of Rseries for each order (Do = 0,02)

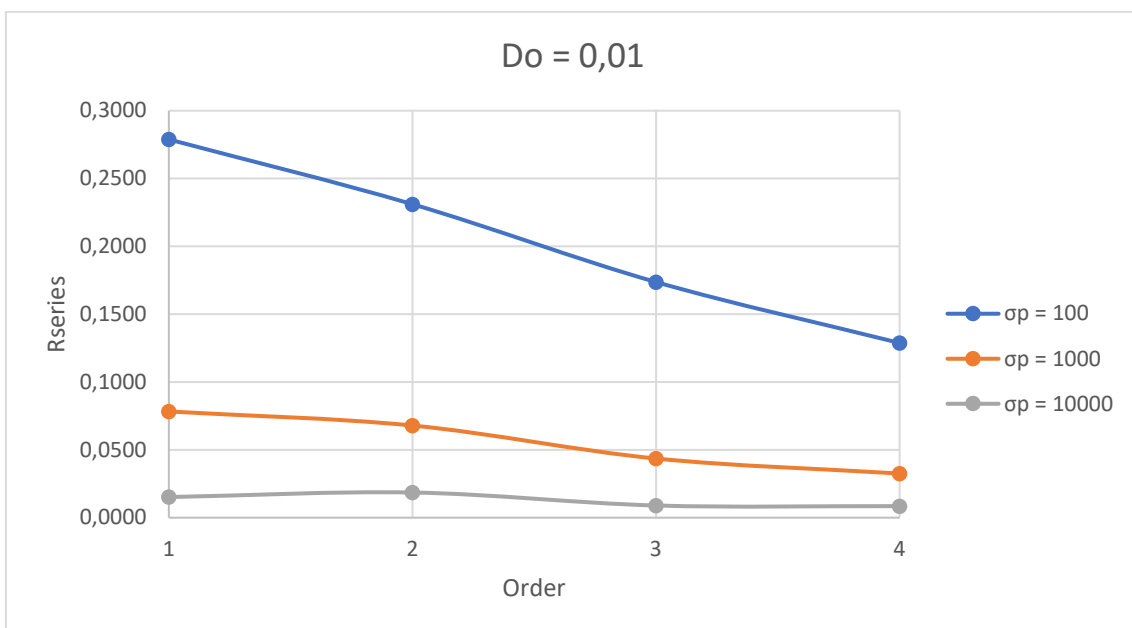


Fig 48. Graph of Rseries for each order (Do = 0,01)

- As it can be seen in the plots, the trend of each width conductive band is quite similar.
- Each order is built since the optimal elementary cell, being chosen the aspect ratio with the lowest series resistance tasted it with fourteen different aspect ratio as it can be seen in the chapter three.
- When it be increasing the order, the voltage and current is increasing since the size of the photovoltaic cells is bigger whereas the resistance (their ratio) is constant.



- It should be noted when it is going to increase the order, the Rseries does not change, it is invariant. That means less power loses (efficiency design), so if the order is higher, more optimal will be the design.
- Regarding the parsimony, that refers to the principle of simplicity and minimalism in design and operation. It will design and utilize solar panels in an efficient and cost-effective manner. It involves maximizing the energy output of the PV system while minimizing the use of resources, such as materials, space, and maintenance. There are some aspects affect directly as material efficiency, properly installation space, system optimization, maintenance and durability, Cost-effectiveness.
- Regarding the equipartition, it refers to the equal resistance path of the current through low and high conductivity areas - long path through high conductivity area and short path through low conductivity area (same voltage drop on these paths).

9. References

- A constructal approach to the optimal design of photovoltaic cells (*Alexandru M. Morega, Adrian Bejan*).
- National Renewable Energy Laboratory (NREL)(www.nrel.gov).
- SolarPower Europe: SolarPower Europe (www.solarpowereurope.org).
- Photovoltaics International (www.pv-tech.org)
- Avisolar (www.avisolar.com)