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Multiclass AQM on a TCP/IP router: A control theory approach

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Summary

Active queue management (AQM) is a well-known technique to improve routing performance under congested traffic conditions. It is often deployed to regulate queue sizes, thus aiming for constant transmission delay. This work addresses AQM using an approach based on control theory ideas. Compared with previous results in the literature, the novelty is the consideration of heterogeneous traffic, ie, multiclass traffic. Thus, each traffic class may have different discarding policies, queue sizes, and bandwidth share. This feature brings the proposal nearer to real network management demands than previous approaches in the literature. The proposed technique assumes that each class already has a simple controller, designed a priori, and focuses on designing a static state-feedback controller for the multiclass system, where the design is based on using LMIs for the calculations. For this, optimization problems with LMI constraints are proposed to compute the state-feedback gains that ensure stability for a large set of admissible initial conditions. These conditions ensure not only closed-loop stability but also some level of performance. As far as we know, this is the first control theory based approach for the AQM problem on TCP/IP routers that allows a multiclass AQM while also considering time-varying delays and input saturation. This is an important step to frame AQM in a more formal, yet realistic context, enabling it to address important service level agreement (SLA) directives. The proposal is tested on a simulated system at the end of this paper, showing the feasibility and performance of the approach in the presence of multiclass traffic.

KEYWORDS

AQM, congested traffic conditions, control theory, heterogeneous traffic, time-varying delay, input saturation

1 | INTRODUCTION

The transport control protocol (TCP) has already more than 30 years of usage in internet applications. The growing performance requirements are demanding improved techniques that are based on TCP to satisfy new applications within existing network resources. In this sense, active queue management (AQM) arose as an effective way to smoothly regulate the traffic rate and delay variation, the so-called jitter, which threaten multimedia and on demand network services[1]: for this, AQM adapts TCP by selectively discarding packets. Thus, these AQM mechanisms assist the TCP/IP congestion control reducing internet traffic congestion and improving the Quality of Service (QoS) demands of users and applications.[2,3] According to Bigdeli and Haeri,[4] these techniques can be categorized in various groups such as heuristic methods, that are mainly developed by computer scientists; mathematical schemes, such as game theory-based algorithms, and control theory-based approaches (see the work of Domańska et al[5] for details). This paper focuses on control theory-based techniques, that are based on the model proposed in the work of Misra et al[6]: the most resounding so far is the proportional integral (PI) controller proposed in the work of Hollot et al.[7] Numerous techniques have been proposed in the literature (see, for example, other works[4,8-11] and references therein). Among them, we emphasize the work of Aoul et al,[8] in which fuzzy control is used for the TCP/IP AQM. However, the inherent time delays and saturation of the probability discard levels were not sufficiently considered in the router queue dynamics. As the degrees of freedom are chosen a priori, we believe that a wider set of criteria should be considered on the control synthesis. A model predictive control approach for TCP/IP AQM was developed in the work of Marami et al,[11] although there is a trade off between the computational burden of considering nonlinear TCP/IP dynamics, and the accuracy of the result under real network conditions. As an effort to reduce the computational burden, while still not considering time delays or saturations in the model, Bigdeli and Haeri[4] proposed an ARMarkov-based model predictive control for AQM of the TCP/IP router.

Note that the works above do not consider sufficiently the saturation of the discarding probability, which is a nonnegative number smaller than one. Since the control actions in AQM are the discarding probabilities, AQM approaches that ignore the input saturation are not adequate because saturations are known to deteriorate performance and might lead to unacceptable losses of stability. In order to mitigate the saturation effect on the stability of such systems, local/global stabilization in a specified domain of attraction has been studied previously,[12-22] and antiwindup techniques have been proposed in other works[23-28] where the emphasis was on the transient performance caused by the saturation. In this sense, the aforementioned works[23-26] proposed delay dependent antiwindup techniques to ensure the closed-loop stability of TCP/IP AQM routers. Note that these approaches have been based on an a priori given dynamic feedback law, which disregards input saturation. An antiwindup compensator is then synthesized over it to be active when the saturation is reached. The drawback of this approach is the computing resources required to implement them in a real network, which may make it unfeasible. An alternative to this approach is direct synthesis, where a feedback law is synthesized considering the saturation bounds in the design. This is addressed in the present work, where a static feedback law is chosen, as its implementation is far less resource consuming than a dynamic one.

It must be noted that these previous works consider the traffic homogeneous, ie, they do not differentiate one TCP flow from another. However, data connections must be secured for some applications (for example, financial and e-banking transfers), whereas others perform better under less varying delays (like VoIP), and some have no special specifications (like regular e-mail traffic). However, multiclass traffic implies considering multiple delays in the TCP/IP traffic connections. Hence, using a unique, single delay AQM policy for the whole traffic is not likely to address the specifications that telecom carriers and internet providers hold with high demanding customers.[29,30] In this sense, the present work contributes to the AQM problem by explicitly allowing different time delays for multiple traffic classes: the available link bandwidth is modeled as a time-variant disturbance while formally ensuring closed-loop stability and a performance level of the TCP/IP router. From a formal point of view, this paper proposes a static state feedback for time-varying delay systems with saturating inputs that ensure its closed-loop stability and a level of performance. Moreover, an optimization problem is formulated where its objective consists in finding a control law that maximizes (an estimate of) the domain of attraction in order to ensure the stability for a wide set of initial conditions.

Notation. $\mathcal{C}$ is the set of continuous functions of $[-\tau(t), 0]$ in $\Re^n$, ϕ is a function belonging to $\mathcal{C}$, and $\delta = \max_{[-\tau(t),0]}\|\phi\| = \max_{[-\tau(t),0]}\|\dot{\phi}\|$; $\bar{\lambda}$ is the maximal eigenvalue.

2 | PROBLEM FORMULATION

A static state feedback is proposed here to ensure a closed-loop stability and a certain level of performance of the system under study, which is modeled as a time-varying delay systems with saturating inputs. This section first presents the detailed modeling of an AQM router, showing that it is a nonlinear dynamic system. Then, based on some reasonable assumptions, a simpler model is derived, which would be used for the design of the proposed controller. This controller provides less performance degradation of the overall system when the saturation is active. To design this controller, the problem is reformulated into a state tracking problem, since the output is the combination of internal states.

2.1 | Nonlinear TCP/AQM dynamics

Numerous models have been provided in the literature that represent adequately the dynamical responses of the system at hand (see, for example, the work of Misra et al,[6] Ariba et al,[9] and Liu et al[10] and references therein). In this paper, following the work of Misra et al,[6] we consider that the network consists of senders and receivers that use internet TCP protocol to transfer data, and a AIMD algorithm to control the congestion; moreover, traffic is here differentiated in classes, following the modus operandi of the telecom industry when Quality of Service (QoS) and service level agreements (SLA) are concerned. Then, a dynamic model of TCP behavior is described by the following coupled differential equations[6]:

$$\begin{aligned} \dot{W}_1(t) &= \frac{1}{t_{r1}(t)} - \frac{W_1(t)W_1(t-t_{r1}(t))}{2t_{r1}(t-t_{r1}(t))}p_1(t-t_{r1}(t)) \\ \dot{q}_1(t) &= -C_1(t) + \frac{N_1}{t_{r1}}W_1(t), \end{aligned} \tag{1}$$

where $W_1(t)$ is the average TCP window size (packets), $q_1(t)$ is the average queue length (packets), $t_{r1}(t) = \frac{q_1(t)}{C_{1_0}} + T_{p_1}$ is the round trip time (s) in which T_{p_1} is the propagation delay (s), $C_1(t)$ is the link capacity (packets/s), N_1 is the loading factor (number of TCP sessions), and $p_1 \in [0,1]$ is the probability of packet marking/dropping. The window size and queue length are positive and bounded, that is, $W_1 \in [0, W_{1_m}]$ and $q_1 \in [0, q_{1_m}]$, where W_{1_m} and q_{1_m} denote maximum window size and buffer capacity, respectively.

The window size equation in (1) describes the growing of the window in the phase of bandwidth probing (the first term on the right hand side), whereas the second term on the right hand side corresponds to its multiplicative decrease following the packet marking probability p_1. This second term comes from the convention in AIMD that the window size is halved once a congestion loss is detected, and the marking/dropping is implemented to distribute the losses in proportion to a flow's bandwidth share, which is $p_1(t)\frac{W_1(t)}{t_{r1}(t)}$.

In the linearization of Equation (1), it is important to configure the equilibrium point as this depends on the probability of packet marks, especially. The probability p_{1_0} at an equilibrium point satisfies the condition $p_{1_0} = \frac{2N_1^2}{(q_{1_0}+T_{p_1}C_{1_0})^2}$. This condition can be derived from Equation (1) at the equilibrium point $(W_{1_0}, q_{1_0}, p_{1_0})$. In the TCP/AQM network, each value of $(W_{1_0}, q_{1_0}, p_{1_0})$ is positive, and the probability p_0 is less than or equal to 1. Therefore for a given triplet of network parameters (N_1, C_{1_0}, T_{p_1}), let $(W_{1_0} = \frac{t_{r1}C_{1_0}}{N_1}, q_{1_0} = C_{1_0}(t_{r1} - T_{p_1}), p_{1_0} = \frac{2}{W_{1_0}^2})$ be a possible operating point. Now, define $\delta\mathfrak{C}_1 = \mathfrak{C}_1 - \mathfrak{C}_{1_0}$ with $\mathfrak{C} = W, q, p, C$. Then, we can obtain the linearized version of (1) that will be used to derive the controller as follows:

$$\begin{aligned} \delta\dot{W}_1(t) =& -\frac{N_1}{t_{r1}^2 C_{1_0}}(\delta W_1(t) + \delta W_1(t-t_{r1}(t))) - \frac{1}{t_{r1}^2 C_{1_0}}(\delta q_1(t) + \delta q_1(t-t_{r1}(t))) \\ & - \frac{t_{r1}C_{1_0}^2}{2N_1^2}\delta p_1(t-t_{r1}(t)) + \frac{t_{r1}-T_{p_1}}{t_{r1}^2 C_{1_0}}(\delta C_1(t) + \delta C_1(t-t_{r1}(t))) \\ \delta\dot{q}_1(t) =& \frac{N_1}{t_{r1}}\delta W_1(t) - \frac{1}{t_{r1}}\delta q_1(t) - \frac{T_{p_1}}{t_{r1}}\delta C_1(t) \\ t_{r1}(t) =& \frac{\delta q_1(t)}{C_{1_0}} + t_{r1}. \end{aligned} \tag{2}$$

2.2 | Modeling of the TCP/AQM system

Rewriting (2) in a state space form yields

$$\begin{aligned} \dot{x}_1(t) &= A_1x_1(t) + A_{d_1}x_1(t-\tau_1(t)) + B_1u_1(t-\tau_1(t)) + B_{w_1}w_1(t) \\ y_1(t) &= C_{y_1}x_1(t) \\ z_1(t) &= C_{z_1}x_1(t), \end{aligned} \tag{3}$$

in which

$$x_1(t) = \begin{bmatrix} \delta W_1(t) \\ \delta q_1(t) \end{bmatrix}, A_1 = \begin{bmatrix} \frac{-N_1}{t_{r1}^2 C_{1_0}} & \frac{-1}{t_{r1}^2 C_{1_0}} \\ \frac{N_1}{t_{r1}} & \frac{-1}{t_{r1}} \end{bmatrix}, A_{d_1} = \begin{bmatrix} \frac{-N_1}{t_{r1}^2 C_{1_0}} & \frac{-1}{t_{r1}^2 C_{1_0}} \\ 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \frac{-t_{r1} C_{1_0}^2}{2N_1^2} \\ 0 \end{bmatrix}, B_{w_1} = \begin{bmatrix} \frac{t_{r1}-T_{p1}}{t_{r1}^2 C_{1_0}} & \frac{t_{r1}-T_{p1}}{t_{r1}^2 C_{1_0}} \\ \frac{-T_{p1}}{t_{r1}} & 0 \end{bmatrix}, w_1(t) = \begin{bmatrix} \delta C_1(t) \\ \delta C_1(t - t_{r1}(t)) \end{bmatrix},$$

$$u_1(t) = \delta p_1(t), C_{y_1} = [\,0\ \ 1\,], y_1(t) = \delta q_1(t), C_{z_1} = \begin{bmatrix} 0 & \frac{1}{C_{1_0}} \end{bmatrix}, z_1(t) = t_{r1}(t) - t_{r1}.$$

In Equation (3), the states variables represent the congestion window and the queue size, respectively, and the input represents the marking probability. On the other hand, as the available link bandwidth is time varying, as it cannot be measured accurately, and as bandwidth variations caused by short-term sudden flows (unresponsive flow and heterogeneous flow) are unavoidable, it is more practical to take $w_1(t)$ as a disturbance. Furthermore, the delay $\tau_1(t)$ is a positive integer that is assumed to be time dependent and to satisfy $0 \le \tau_1(t) \le h_{\tau_1}$ and $0 \le \dot{\tau}_1(t) \le d_1 < 1$, where h_{τ_1} and d_1 are known positive and finite integers.

Considering that there are several classes of TCP flows passing through the router, each of them with specific delays, we can rewrite (3) in the following new, general state-space model of a multiclass AQM+TCP system:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_m(t) \end{bmatrix} &= \begin{bmatrix} A_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & A_m \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} + \begin{bmatrix} A_{d_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & A_{d_m} \end{bmatrix} \begin{bmatrix} x_1(t-\tau_1(t)) \\ \vdots \\ x_m(t-\tau_m(t)) \end{bmatrix} \\ &+ \begin{bmatrix} B_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_m \end{bmatrix} \begin{bmatrix} u_1(t-\tau_1(t)) \\ \vdots \\ u_m(t-\tau_m(t)) \end{bmatrix} + \begin{bmatrix} B_{w_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_{w_m} \end{bmatrix} \begin{bmatrix} w_1(t) \\ \vdots \\ w_m(t) \end{bmatrix} \\ \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} &= \begin{bmatrix} C_{y_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_{y_m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} \\ \begin{bmatrix} z_1(t) \\ \vdots \\ z_m(t) \end{bmatrix} &= \begin{bmatrix} C_{z_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_{z_m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix}. \end{aligned} \tag{4}$$

The simplification of system (4) gives us the opportunity to obtain the following system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} \left(A_{d_{\tau_i}} x(t-\tau_i(t)) + B_{\tau_i} u(t-\tau_i(t)) \right) + B_w w(t) \\ y(t) &= C_y x(t) \\ z(t) &= C_z x(t), \end{aligned} \tag{5}$$

where the plant model to obtain (5) is given explicitly as $\psi(t) = [\psi_1^T(t) \dots \psi_m^T(t)]^T$, $\psi = x, u, w, y, z$, $F = \text{diag}\{F_1, \dots, F_m\}$, $F = A, B_w, C_y, C_z$, $H_{\tau_1} = \text{diag}\{H_1, \dots, 0\}, \dots, H_{\tau_m} = \text{diag}\{0, \dots, H_m\}$, and $H = A_d, B$.

It is more practical to take $w(t)$ as the link capacity disturbance, where $w(t)$ is assumed to be bounded with finite energy, that is, $w(t) \in \mathcal{L}_2$. Hence, for a scalar ω, the disturbance is $\|w(t)\|_2^2 = \int_0^\infty w^T(t) w(t) dt \le \omega^{-1} < \infty$.

Assuming that our AQM uses a state-feedback controller $u(t) = Kx(t)$ of the following type:

$$u(t) = \begin{bmatrix} K_{11}\delta W_1(t) + K_{12}\delta q_1(t) \\ \vdots \\ K_{m1}\delta W_m(t) + K_{m2}\delta q_m(t) \end{bmatrix}. \tag{6}$$

Each component of the input vector $u(t)$ is subject to amplitude limitations defined by $|u_i(t)| \le u_{0_i}$, $u_{0_i} > 0$, where $i = 1, \dots, m$. Due to the control bounds, the effective control signal to be applied to (5) is given by $u(t) = \text{sat}(Kx(t), u_0)$, where $\text{sat}(Kx(t), u_0)$ is the saturation function described by $\text{sat}(Kx(t), u_0) = [\text{sat}(K_1 x(t)) \dots \text{sat}(K_m x(t))]^T$, where $\text{sat}(K_i x(t)) = \text{sign}(K_i x(t)) \min\{u_{0_i}, |K_i x(t)|\}$.

Then, from $u(t-\tau_i(t)) = \text{sat}(K_i x(t-\tau_i(t)), u_{i_0})$, system (5) reads

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} \left(A_{d_{\tau_i}} x(t-\tau_i(t)) + B_{\tau_i} \text{sat}(K_i x(t-\tau_i(t)), u_{i_0}) \right) + B_w w(t) \\ y(t) &= C_y x(t) \\ z(t) &= C_z x(t). \end{aligned} \tag{7}$$

To establish the results, the following preliminaries are required.

Lemma 1 (See the work of Zhang et al[31]).
Let x be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any symmetric positive definite matrix R, appropriately sized matrices N_1, N_2, and delay τ

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(s) R \dot{x}(s) ds \le \begin{bmatrix} x(t) \\ x(t-\tau(t)) \end{bmatrix}^T \left(2 \begin{bmatrix} N_1 & -N_1 \\ N_2 & -N_2 \end{bmatrix} + \tau(t) \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} R^{-1} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}^T \right) \begin{bmatrix} x(t) \\ x(t-\tau(t)) \end{bmatrix}. \tag{8}$$

Lemma 2 (See the work of Cao et al[32]).
Let Λ be the set of all $m \times m$ diagonal matrices with elements that are either one or zero. Then, there are 2^m elements D_j in Λ and denote $D_j^- = I_m - D_j$, which are also elements of Λ.

The controller design goal will be mathematically transformed to embed $\text{sat}(Kx(t), u_0)$ within a convex hull of a group of linear feedbacks (to avoid saturation). Given two gain matrices K and H, the matrix set $\{D_j K + D_j^- H\}$ is formed by choosing some rows of K and the rest from H. For this, the set $\text{sat}(Kx(t), u_0) \in Co\{D_j K + D_j^- H\} x(t)$ is defined for two given gain matrices K and H and all $x(t)$ that satisfy $|H_i x(t)| \le u_{0_i}$.

Remark 1. It should be noted that condition (8) is an extended Jensen's inequality and is beneficial in formulating convex criteria (see the work of Briat[32]).

Using Lemma 2, the closed-loop system (7) becomes

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \sum_{j=1}^{2^m} \sum_{i=1}^{m} \lambda_j A_{dK_{ij}} x(t-\tau_i(t)) + B_w w(t) \\ y(t) &= C_y x(t) \\ z(t) &= C_z x(t), \end{aligned} \tag{9}$$

where $\sum_{j=1}^{2^m} \lambda_j = 1$, $\lambda_j \ge 0$, and $A_{dK_{ij}} = A_{d_{\tau_i}} + B_{\tau_i}(D_{i_j} K_i + D_{i_j}^- H_i)$.

Definition 1. The domain of attraction Φ of the origin for the studied system is defined as the set of all points x_0 of the state space for which $x(0) = x_0$ leads to a trajectory $x(t)$ that converges asymptotically to the origin for $w(t) = 0$. In other words,

$$\Phi = \left\{ x(t) \in \Re^n;\ \lim_{t\to\infty} x(t) = 0 \right\}.$$

Remark 2. There are initial conditions x_0 such that the trajectories converge to the origin, ie, $x(t) \to 0$ as $t \to \infty$, but also initial conditions leading to divergent trajectories, ie, $x(t) \to \infty$ as $t \to \infty$. For this reason, a natural objective is then to obtain a 'good' estimate of the domain of attraction by means of regions of asymptotic stability. In this case, we are interested in finding a region (or a union of regions) of stability, which 'best' fits in Φ. This is, in general, not an easy task, since the form of the domain of attraction is generally complex, and unknown. In practice, we aim to find a region of stability with a simple form (an ellipsoid or polyhedra), that can be 'maximized' considering some simple geometric criteria.

Definition 2.

* A region R_S is said to be a region of stability for the studied system if $R_S \subset \Phi$ and $0 \in R_S$.

* For a positive scalar β and a symmetric positive definite matrix P, an ellipsoid D_e is defined by

$$D_e = \left\{x(t) \in \Re^n;\ \ x^T(t)Px(t) \le \beta^{-1}\right\}.$$

* A polyhedral set Θ is constructed as

$$\Theta = \left\{x(t) \in \Re^n;\ \ |H_i x(t)| \le u_{0_i}\right\}.$$

The objective of this paper is to use a system performance index as the basis for the problem of AQM-based congestion control in data networks based on the dynamical model (2). Thus, the following auxiliary function is defined:

$$J(t) = \dot{V}(t) + \frac{1}{\gamma} z^T(t)z(t) - w^T(t)w(t).$$

Then, we work to find the controller (6) such that the closed-loop system satisfies the system performance index

$$\int_0^{\infty} \left(\frac{1}{\gamma} z^T(t)z(t) - w^T(t)w(t)\right) dt < 0, \tag{10}$$

whereas the prescribed scalar γ should be as minimal as possible. For this, let the Lyapunov functional candidate be

$$V(t) = x^T(t)Px(t) + \sum_{i=1}^{m} \left(\int_{t-\tau_i(t)}^{t} x^T(s)Q_i x(s)ds + \int_{-\tau_i(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_i\dot{x}(s)dsd\theta\right), \tag{11}$$

where $P = \text{diag}\{P_1, \ldots, P_m\} = P^T > 0$, $Q_1 = Q_1^T > 0, \ldots, Q_m = Q_m^T > 0$, and $R_1 = R_1^T > 0, \ldots, R_m = R_m^T > 0$ need to be determined by optimization.

Problem 1. Based on the analysis above, the problem to be studied in this paper can be illustrated as follows. Considering the TCP/IP router model in (5), we aim to put forward a feasible method for designing a reliable H_∞ controller (6) and seeking the corresponding controller gains $K_1, \ldots, K_m$ such that the closed-loop system is asymptotically stable, and for a given performance index $\gamma > 0$, the H_∞ norm of the closed-loop transfer function $G(s)$ satisfies[33,34]

$$||G(s)||_\infty^2 = \frac{||z(t)||_2^2}{||w(t)||_2^2} < \gamma \tag{12}$$

or equivalently the performance index (10) holds.

3 | MAIN RESULTS

In this paper, we adopt the H_∞ performance approaches in order to ensure closed-loop stability and a certain level of performance of time-varying delay systems with saturating inputs of the TCP/AQM system. The sufficient conditions for stability are given in terms of the existence of solutions of some LMIs, based on Lyapunov functionals.

3.1 | Multiclass traffic

Let us now establish sufficient conditions that guarantee the stability and stabilization of the TCP/IP router control system for multiclass traffic in order to solve Problem 1 stated in Section 2.

Theorem 1. *Consider TCP/IP router control system (9) with reliable H_∞ controller (6), if there exist symmetric positive definite matrices $\overline{P} = \text{diag}\ \{\overline{P}_1, \ldots, \overline{P}_m\}$, $\overline{Q}_1, \ldots, \overline{Q}_m$, $\overline{R}_1, \ldots, \overline{R}_m$, appropriately sized matrices X, $\overline{N}_{11}, \ldots, \overline{N}_{1m}$,*

$\overline{N}_{21}, \ldots, \overline{N}_{2m}, U_1, \ldots, U_m, G_1, \ldots, G_m$, *and a real scalar* α *satisfying the following conditions:*

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \widehat{\Sigma}_{11} & \ldots & \widehat{\Sigma}_{1m} & B_w & h_{\tau_1}\overline{N}_{11} & \ldots & h_{\tau_m}\overline{N}_{1m} & XC_z^T \\ * & \Sigma_{22} & \widehat{\Sigma}_{21} & \ldots & \widehat{\Sigma}_{2m} & \alpha B_w & 0 & \ldots & 0 & 0 \\ * & * & \widetilde{\Sigma}_{11} & \ldots & 0 & 0 & h_{\tau_1}\overline{N}_{21} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \ldots & \widetilde{\Sigma}_{mm} & 0 & 0 & \ldots & h_{\tau_m}\overline{N}_{2m} & 0 \\ * & * & * & \ldots & * & -I & 0 & \ldots & 0 & 0 \\ * & * & * & \ldots & * & * & -h_{\tau_1}\overline{R}_1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \ldots & * & * & * & \ldots & -h_{\tau_m}\overline{R}_m & 0 \\ * & * & * & \ldots & * & * & * & \ldots & * & -\gamma I \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} \overline{P} & G_{1_i}^T & \ldots & G_{m_i}^T \\ * & \beta u_{1_{0i}}^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \ldots & \beta u_{m_{0i}}^2 \end{bmatrix} \geq 0 \tag{14}$$

$$\beta - \omega \leq 0, \tag{15}$$

where G_i *is the ith row of* G *and*

$$\Sigma_{11} = AX^T + XA^T + \sum_{i=1}^{m}\left(\overline{N}_{1i} + \overline{N}_{1i}^T + \overline{Q}_i\right), \Sigma_{12} = \overline{P} - X^T + \alpha XA^T$$

$$\Sigma_{22} = \sum_{i=1}^{m} h_{\tau_i}\overline{R}_i - \alpha(X + X^T), \widehat{\Sigma}_{1i} = A_{d_{\tau_i}}X^T + B_{\tau_i}\left(D_{i_j}U_i + D_{i_j}^- G_i\right) - \overline{N}_{1i} + \overline{N}_{2i}^T$$

$$\widehat{\Sigma}_{2i} = \alpha A_{d_{\tau_i}}X^T + \alpha B_{\tau_i}\left(D_{i_j}U_i + D_{i_j}^- G_i\right), \widetilde{\Sigma}_{ii} = -\overline{N}_{2i} - \overline{N}_{2i}^T - (1 - d_i)\overline{Q}_i.$$

Then, system (9) is stabilized by the controller gains $K_1 = U_1X^{-T}, \ldots, K_m = U_mX^{-T}$ *and the trajectories are bounded for every initial condition in the region*

$$\mathcal{R}_1 = \left\{\phi \in C[-\overline{h}_\tau, 0], \max_{[-\overline{h}_\tau, 0]} \|\phi\| \leq \frac{\varpi_1}{\iota_1}\right\}, \overline{h}_\tau = \max\{h_{\tau_1}, \ldots, h_{\tau_m}\} \tag{16}$$

with any ϖ_1 *satisfying* $\varpi_1 \leq \beta^{-1} - \omega^{-1}$ *and*

$$\iota_1 = \sum_{i=1}^{m}\left\{\overline{\lambda}(X^{-1}\overline{P}X^{-T}) + h_{\tau_i}\overline{\lambda}\left(X^{-1}\overline{Q}_iX^{-T}\right) + \frac{h_{\tau_i}^2}{2}\overline{\lambda}\left(X^{-1}\overline{R}_iX^{-T}\right)\right\}.$$

Proof. Taking the derivative of the proposed Lyapunov function (11), we have

$$\dot{V}(t) = 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{m}\left(x^T(t)Q_ix(t) - x^T(t - \tau_i(t))Q_ix(t - \tau_i(t)) + \tau_i(t)\dot{x}^T(t)R_i\dot{x}(t) - \int_{t-\tau_i(t)}^{t}\dot{x}^T(s)R_i\dot{x}(s)ds\right). \tag{17}$$

From Equation (9), for any matrices Y_1 and Y_2, the following relation is true:

$$2\left[x^T(t)Y_1 + \dot{x}^T(t)Y_2\right]\left[-\dot{x}(t) + Ax(t) + \sum_{j=1}^{2^m}\sum_{i=1}^{m}\lambda_j A_{dK_{ij}}x(t - \tau_i(t)) + B_w w(t)\right] = 0. \tag{18}$$

Then, applying Lemma 1 to the latter term of (17) and taking into account (18), we can write

$$J(t) \leq \sum_{j=1}^{2^m}\lambda_j\eta^T(t)\Pi\eta(t), \tag{19}$$

where

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \widehat{\Pi}_{11} & \dots & \widehat{\Pi}_{1m} & Y_1 B_w \\ * & \Pi_{22} & \widehat{\Pi}_{21} & \dots & \widehat{\Pi}_{2m} & Y_2 B_w \\ * & * & \widetilde{\Pi}_{11} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \dots & \widetilde{\Pi}_{mm} & 0 \\ * & * & * & \dots & * & -I \end{bmatrix}, \eta(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ x(t-\tau_1(t)) \\ \vdots \\ x(t-\tau_m(t)) \\ w(t) \end{bmatrix},$$

and

$$\Pi_{11} = Y_1 A + A^T Y_1^T + \sum_{i=1}^{m} \left(N_{1i} + N_{1i}^T + h_{\tau_i} N_{1i} R_i^{-1} N_{1i}^T + Q_i \right) + \frac{1}{\gamma} C_z^T C_z$$
$$\Pi_{12} = P - Y_1 + A^T Y_2^T, \Pi_{22} = \sum_{i=1}^{m} h_{\tau_i} R_i - Y_2 - Y_2^T$$
$$\widehat{\Pi}_{1i} = Y_1 A_{dK_{ij}} - N_{1i} + N_{2i}^T + h_{\tau_i} N_{1i} R_i^{-1} N_{2i}^T, \widehat{\Pi}_{2i} = Y_2 A_{dK_{ij}}$$
$$\widetilde{\Pi}_{ii} = -N_{2i} - N_{2i}^T - (1-d_i) Q_i + h_{\tau_i} N_{2i} R_i^{-1} N_{2i}^T.$$

It is clear that if $\Pi < 0$, then

$$J(t) = \dot{V}(t) + \frac{1}{\gamma} z^T(t) z(t) - w^T(t) w(t) < 0. \tag{20}$$

From Π, it is easy to see that Y_2 is nonsingular and, consequently, Y_1 is invertible. Then, setting $Y_2 = \alpha Y_1$, we apply the Schur complement to (19) and multiply it by $\mathrm{diag}\{Y_1^{-1}, \dots, Y_1^{-1}, I, Y_1^{-1}, \dots, Y_1^{-1}, I\}$ and its transpose, on the left and on the right, respectively, after which we introduce the changes of variables $Y_1^{-1} = X$ and $X \Omega X^T = \overline{\Omega}$ with $\Omega = P$, $Q_1, \dots, Q_m, R_1, \dots, R_m, N_{11}, \dots, N_{1m}, N_{21}, \dots, N_{2m}$.

Now, let us replace $A_{dK_{1j}}, \dots, A_{dK_{mj}}$ by $A_{d_{\tau_1}} + B_{\tau_1}(D_{1_j} K_1 + D_{1_j}^- H_1), \dots, A_{d_{\tau_m}} + B_{\tau_m}(D_{m_j} K_m + D_{m_j}^- H_m)$, respectively, and put $U_1 = K_1 X^T, \dots, U_m = K_m X^T, G_1 = H_1 X^T, \dots, G_m = H_m X^T$ to get (13).

Since the matrix (13) holds, integrating both sides of the inequality (20) from 0 to ∞, we have

$$V(\infty) - V(0) + \int_0^{\infty} \left(\frac{1}{\gamma} z^T(t) z(t) - w^T(t) w(t) \right) dt < 0. \tag{21}$$

Then, since the system is stable ($V(\infty) = 0$), we can conclude that condition (12) is verified where $V(0) = 0$ (zero initial condition). On the other hand, from (21), it can be concluded that the trajectories of $x(t)$ never leave the ellipsoid D_e, provided that $x(t) \in \Theta$. In this sense, the inclusion of D_e in Θ ensures that $x(t) \in \Theta$, ie, it enforces the validity of Lemma 2. This is ensured through the verification of the following condition:

$$\begin{bmatrix} P & H_{1_i}^T & \dots & H_{m_i}^T \\ * & \beta u_{1_{0i}}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & \beta u_{m_{0i}}^2 \end{bmatrix} \geq 0. \tag{22}$$

For this reason, the relation (14) is obtained by premultiplying and postmultiplying matrix (22) by $\mathrm{diag}\{X, I\}$ and $\mathrm{diag}\{X^T, I\}$, respectively, where $G_1 = H_1 X^T, \dots, G_m = H_m X^T$.

Inequality (15) ensures that $\beta^{-1} - \omega^{-1} \geq 0$ and, consequently, the existence of a set of initial conditions ϕ such that

$$V(0) = x^T(0) P x(0) + \sum_{i=1}^{m} \left(\int_{-h_{\tau_i}}^{0} x^T(s) Q_i x(s) ds + \int_{-h_{\tau_i}}^{0} \int_{\theta}^{0} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta \right)$$
$$\leq \sum_{i=1}^{m} \left\{ \left(\overline{\lambda}(P) + h_{\tau_i} \overline{\lambda}(Q_i) \right) \|\phi\|^2 + \frac{h_{\tau_i}^2}{2} \overline{\lambda}(R_i) \|\dot{\phi}\|^2 \right\} \leq \varpi_1.$$

The simultaneous verification of (13)- (15) ensures that (21), $\forall \phi \in \mathcal{R}_1$. This concludes the proof of (16) and thus the proof of Theorem 1. □

Next, we focus on guaranteeing the stability of the following input saturated delay systems without saturated actuator:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_{d_\tau}x(t-\tau(t)) + B_\tau \text{sat}(Kx(t), u_0) + B_w w(t) \\ z(t) &= C_z x(t). \end{aligned} \tag{23}$$

This system can be seen as a particular case of system (7). Then, the following corollary gives a condition to stabilize system (23).

Corollary 1. *If there exist symmetric positive definite matrices $\overline{P}$, $\overline{Q}$, $\overline{R}$, appropriately sized matrices X, $\overline{N}_{14}$, $\overline{N}_{24}$, U, G, and a real scalar α satisfying the following conditions:*

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & B_w & h_\tau \overline{N}_{14} & XC_z^T \\ * & \Upsilon_{22} & \Upsilon_{23} & \alpha B_w & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & h_\tau \overline{N}_{24} & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -h_\tau \overline{R} & 0 \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0, \begin{bmatrix} \overline{P} & G_i^T \\ * & \beta u_{0i}^2 \end{bmatrix} \geq 0, \beta - \omega \leq 0, \tag{24}$$

where

$$\begin{aligned} \Upsilon_{11} &= AX^T + B_\tau(D_jU + D_j^- G) + \overline{N}_{14} + \overline{Q} + XA^T + (D_jU + D_j^- G)^T B_\tau^T + \overline{N}_{14}^T \\ \Upsilon_{12} &= \overline{P} - X^T + \alpha XA^T + \alpha(D_jU + D_j^- G)^T B_\tau^T, \Upsilon_{22} = h_\tau \overline{R} - \alpha(X + X^T) \\ \Upsilon_{13} &= A_{d_\tau}X^T - \overline{N}_{14} + \overline{N}_{24}^T, \Upsilon_{23} = \alpha A_{d_\tau}X^T, \Upsilon_{33} = -\overline{N}_{24} - \overline{N}_{24}^T - (1-d)\overline{Q}. \end{aligned}$$

Then, system (23) is stabilized by the controller gain $K = UX^{-T}$, and the trajectories are bounded for every initial condition in the region

$$\mathcal{R}_{1_0} = \left\{ \phi \in C[-h_\tau, 0], \max_{[-h_\tau,0]} \|\phi\| \leq \frac{\varpi_{1_0}}{\iota_{1_0}} \right\}$$

with any ϖ_{1_0} satisfying $\varpi_{1_0} \leq \beta^{-1} - \omega^{-1}$ and

$$\iota_{1_0} = \overline{\lambda}(X^{-1}\overline{P}X^{-T}) + h_\tau \overline{\lambda}(X^{-1}\overline{Q}X^{-T}) + \frac{h_\tau^2}{2}\overline{\lambda}(X^{-1}\overline{R}X^{-T}).$$

Proof. In order to obtain (24), it suffices to follow the same steps as in the Proof of Theorem 1, considering $P_1 = \cdots = P_m = P$, $Q_1 = \cdots = Q_m = Q$, $R_1 = \cdots = R_m = R$, $N_{11} = \cdots = N_{1m} = N_{14}$, and $N_{21} = \cdots = N_{2m} = N_{24}$. □

Remark 3. In deriving Theorem 1, the slack variables Y_1 and Y_2 are introduced by Equation (18) in order to reduce the conservatism of the asymptotic stability conditions. It can be seen from the proof that $\dot{V}(t)$ remains unaffected by the slack variables Y_1 and Y_2. Hence, these matrices lead to a more flexible LMI condition in (13) and, consequently, reduce the conservatism of Theorem 1. This advantage can be appreciated in the numerical examples at the end of this paper.

Remark 4. Some new stability conditions for time-delay systems are obtained by using the extended Jensen's inequality[31] to bound the quadratic term $\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds$ instead of cross terms, which is proved to be less conservative than those reported in the literature. In particular, the extended Jensen inequality contains more free matrices and covers Jensen inequality as a special case, which leads to less conservative results (see the work of Ji and Su[35]).

3.2 | Implementation constraints

Since δW_1 is not available at routers in real networks, we use an approximation as follows[36-38]:

$$\delta W_1 = \frac{t_{r1}}{N_1}\left(\frac{N_1}{t_{r1}}W_1 - C_1\right) = \frac{t_{r1}}{N_1} \cdot \dot{q}_1.$$

Hence, the control signal (6) for the network becomes as follows:

$$\delta p = \begin{bmatrix} \begin{bmatrix} K_{11}\frac{t_{r1}}{N_1} & K_{12} \end{bmatrix} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \begin{bmatrix} K_{m1}\frac{t_{rm}}{N_m} & K_{m2} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \dot{q}_1 \\ q_1 - q_{1_0} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \dot{q}_m \\ q_m - q_{m_0} \end{bmatrix} \end{bmatrix}.$$

3.3 | Optimization problems

The proposed conditions in Theorem 1 are in LMI form, so they can be easily considered in convex optimization problems. Two problems of interest are now presented.

3.3.1 | Minimization of γ

As in the work of Bender,[23] in order to minimize γ where $\beta^{-1} = \omega^{-1}$, a solution is provided by the following problem, with zero initial conditions:

$$\begin{aligned} &\min \quad \gamma \\ &\text{subject to } (13)-(15). \end{aligned} \tag{25}$$

3.3.2 | Maximization of δ

Now, we consider the free-disturbance case ($w(t) = 0$). In this stage, we develop a methodology to estimate the largest possible domain of initial conditions for which it can be ensured that the closed-loop system trajectories remain bounded. Then, we introduce the new matrix variables $\overline{S}^{-1} = \widetilde{S}$ with $S = P, Q_1, \dots, Q_m, R_1, \dots, R_m$, and $X^{-1} = \widetilde{X}$, and we impose the following conditions:

$$\begin{aligned} &\begin{bmatrix} \sigma_{\widetilde{P}} I & \widetilde{X} \\ \widetilde{X}^T & \widetilde{P} \end{bmatrix} \geq 0, \begin{bmatrix} \sigma_{\widetilde{Q}_1} I & \widetilde{X} \\ \widetilde{X}^T & \widetilde{Q}_1 \end{bmatrix} \geq 0, \dots, \begin{bmatrix} \sigma_{\widetilde{Q}_m} I & \widetilde{X} \\ \widetilde{X}^T & \widetilde{Q}_m \end{bmatrix} \geq 0, \\ &\begin{bmatrix} \sigma_{\widetilde{R}_1} I & \widetilde{X} \\ \widetilde{X}^T & \widetilde{R}_1 \end{bmatrix} \geq 0, \dots, \begin{bmatrix} \sigma_{\widetilde{R}_m} I & \widetilde{X} \\ \widetilde{X}^T & \widetilde{R}_m \end{bmatrix} \geq 0. \end{aligned} \tag{26}$$

It follows that condition (16) is satisfied if the following LMI holds:

$$\sum_{i=1}^{m} \delta^2 \left\{ \sigma_{\widetilde{P}} + h_{\tau_i} \sigma_{\widetilde{Q}_i} + \frac{h_{\tau_i}^2}{2} \sigma_{\widetilde{R}_i} \right\} \leq \beta^{-1}. \tag{27}$$

Therefore, we construct a feasibility problem as follows:

$$\begin{aligned} &\min \operatorname{tr}\left(\overline{P}\widetilde{P} + \overline{Q}_1\widetilde{Q}_1 + \dots + \overline{Q}_m\widetilde{Q}_m + \overline{R}_1\widetilde{R}_1 + \dots + \overline{R}_m\widetilde{R}_m + (X + X^T)(\widetilde{X} + \widetilde{X}^T) \right) \\ &\text{subject to } (13)-(15), (26), (27), \begin{bmatrix} \overline{S} & * \\ I & \widetilde{S} \end{bmatrix} \geq 0, \begin{bmatrix} X + X^T & * \\ I & \widetilde{X} + \widetilde{X}^T \end{bmatrix} \geq 0. \end{aligned} \tag{28}$$

On the other hand, from Theorem 1, taking α as a parameter of adjustment, as mentioned in Remark 5, we can see that the conditions are LMIs not only over the matrix variables but also over the objective scalar γ, which implies that γ can be included as an optimization variable to obtain a lower bound of the guaranteed H_∞ performance. That is, the controller design problem has been transformed into a set of LMI conditions. Based on these conditions, the multiobjective state-feedback controller design can be accomplished by using the following cone complementarity algorithm.

Step 1. Choose a small δ and a set $\left(\overline{S}, X, \widetilde{S}, \widetilde{X}, \sigma_{\widetilde{P}}, \sigma_{\widetilde{Q}_i}, \sigma_{\widetilde{R}_i}\right)_0 = \left(\overline{S}, X, \widetilde{S}, \widetilde{X}, \sigma_{\widetilde{P}}, \sigma_{\widetilde{Q}_i}, \sigma_{\widetilde{R}_i}\right)$ that satisfy the constrained minimization (28). Then, fix Δ, where $\delta = \delta + \Delta$.

TABLE 1 Comparison of stability radius for Example 1

Approach	δ	K
Da Silva et al[14]	83.55	$[-0.1950 \quad 0.0649]$
Chen et al[13]	84.61	$[-0.2223 \quad -0.0246]$
El Haoussi et al[20]	96.16	$[-10.2107 \quad 0.9563]$
Dey et al[15]	106.29	$[-0.6646 \quad -0.0239]$
El Fezazi et al[19]	115	$[-0.2426 \quad -0.0829]$
El Fezazi et al[16]	404	$[-0.1013 \quad -0.0495]$
Corollary 1	611	$[-0.1013 \quad -0.0492]$

Step 2. Solve the following LMI minimization problem in the matrix variables $\overline{S}$ and $\widetilde{S}$

$$\min \operatorname{tr}\Big(\overline{P}\widetilde{P}_0 + \overline{Q}_1\widetilde{Q}_{1_0} + \cdots + \overline{Q}_m\widetilde{Q}_{m_0} + \overline{R}_1\widetilde{R}_{1_0} + \cdots + \overline{R}_m\widetilde{R}_{m_0} + (X + X^T)\left(\widetilde{X}_0 + \widetilde{X}_0^T\right)$$
$$+ \overline{P}_0\widetilde{P} + \overline{Q}_{1_0}\widetilde{Q}_1 + \cdots + \overline{Q}_{m_0}\widetilde{Q}_m + \overline{R}_{1_0}\widetilde{R}_1 + \cdots + \overline{R}_{m_0}\widetilde{R}_m + \left(X_0 + X_0^T\right)(\widetilde{X} + \widetilde{X}^T)\Big)$$

subject to the LMIs in (28).

Step 3. Substitute the new matrix variables into (28). If the result is feasible, then set $\delta = \delta + \Delta$ and repeat Step 2; otherwise, $\delta = \delta - \Delta$, which is the desired estimate and finish.

Remark 5. For a given α, inequalities (13) and (24) are linear and the problem can easily be solved using YALMIP and SeDuMi software. Then, to find the optimal value of α, numerical search procedures can be used, for example, 'fminsearch program' of MATLAB, to maximize the estimate of the domain of attraction in addition to the cone complementarity algorithm.

4 | NUMERICAL EXAMPLES

The effectiveness of the proposed approach is illustrated by two examples. The first one is provided to check the validity of the results in the stability context, whereas the second one demonstrates the applicability of the technique to realistic problems.

Example 1. Consider the time-delay system (23) for which control values are saturated at ±15 and

$$A = \begin{bmatrix} 1 & 1.5 \\ 0.3 & -2 \end{bmatrix}, A_{d_\tau} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, B_\tau = \begin{bmatrix} 10 \\ 1 \end{bmatrix}.$$

Taking $\beta = 1, d = 0, h_\tau = 1$, and using the cone complementarity algorithm, the results obtained in Corollary 1 ensure the asymptotic stability of the system under consideration, where $\alpha = 94$. On the other hand, the estimated domain of attraction and the controller gain are shown in Table 1 in order to compare them with the results obtained in other works.[13-16,19,20]

It is clear that our approach is less conservative in stabilizing the closed-loop system with a larger estimate of the domain of attraction than those obtained in the aforementioned works.[13-16,19,20]

Example 2. Now, we illustrate our methodology using an example borrowed from the works of El Fezazi et al[18] and Bender[23] in which the TCP/IP router queue model is given in the form of (5). Three different TCP flows are considered, corresponding to the following numerical values of the maximum round trip time t_r, nominal link bandwidth C_0, number of connections N, and nominal queue size q_0

$$\begin{cases} t_{r1} = 0.30, t_{r2} = 0.25, t_{r3} = 0.20 \\ C_{1_0} = 3500, C_{2_0} = 3600, C_{3_0} = 3700 \\ N_1 = 70, N_2 = 75, N_3 = 80 \\ q_{1_0} = 175, q_{2_0} = 165, q_{3_0} = 150. \end{cases}$$

Thus, the nominal TCP window size $W_0 = \frac{t_r C_0}{N}$, the propagation delay $T_p = t_r - \frac{q_0}{C_0}$, and the probability of packet marking/dropping $u_0 = p_0 = \frac{2}{W_0^2}$ are estimated to be as follows:

$$\begin{cases} W_{1_0} = 15, W_{2_0} = 12, W_{3_0} = 9.25 \\ T_{p_1} = 0.25, T_{p_2} = 0.2042, T_{p_3} = 0.1595 \\ p_{1_0} = 0.0089, p_{2_0} = 0.0139, p_{3_0} = 0.0234. \end{cases}$$

Note that each class has its own average round trip time, and other specification parameters. The objective is the regulation of the router queue around a specific predetermined length, when its congested traffic is comprised by three different classes, coping with the saturation of the discharge probability (ie, input saturation) according to Theorem 1. Taking $\beta = 1$, for the specific time delays $h_{\tau_1} = 0.30$, $h_{\tau_2} = 0.25$, and $h_{\tau_3} = 0.20$ (and fixing $d_1 = d_2 = d_3 = 0.1$), it is possible to apply the stability results presented in Theorem 1, obtaining the following controller gains where $\alpha = 0.30$

$$K_1 = \begin{bmatrix} 2.3836 & -0.0020 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot 10^{-3}$$

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.7981 & -0.0089 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot 10^{-3}$$

$$K_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.5122 & -0.0478 \end{bmatrix} \cdot 10^{-3}.$$

Solving the algorithm (25), using MATLAB/YALMIP/SDPT3, the prescribed scalar is $\sqrt{\gamma} = 10^{-4}$.

Figure 1 presents the magnitude of the frequency response of the closed-loop transfer function: it can be seen that it fulfills condition (12) as the maximum is clearly below the prescribed bound.

The controller has been designed with the explicit objective of queue control, so some simulation results are presented in Figure 3 using the algorithm (25) to show the performance of the queue control. These simulations explore the regulation of the router queue around a specific predetermined length (given that a multitraffic approach is considered, we cannot make a fair comparison with single-traffic approaches). These simulations are based on using the following deviations on the initial values of the states: $x(0) = [10\ -10\ \ 10\ -10\ \ 10\ -10]^T$, and the Gaussian noise (for a limited time interval) as presented in Figure 2 in order to check the effect of random disturbances.

Figure 3 depicts the overall queue size and discards probability using the nominal values 175, 165, and 150, respectively, as references for the queues lengths and 0.0089, 0.0139, and 0.0234, respectively, as references for the probability of packet marking/dropping: it can be seen that all queue lengths are stabilized at the target value, showing that the designed controller drives all queue length rapidly to the desired value, with no significant oscillations or undershoots. On the other hand, as shown in Figure 3, the proposed method requires only a minor adaptation of the probability of packet drop to achieve this performance.

As a summary of these simulations, it can be concluded that a stable operating condition is reached and maintained by using the proposed controller, even when traffic is heterogeneous (ie, multiclass traffic). In addition to the ability to

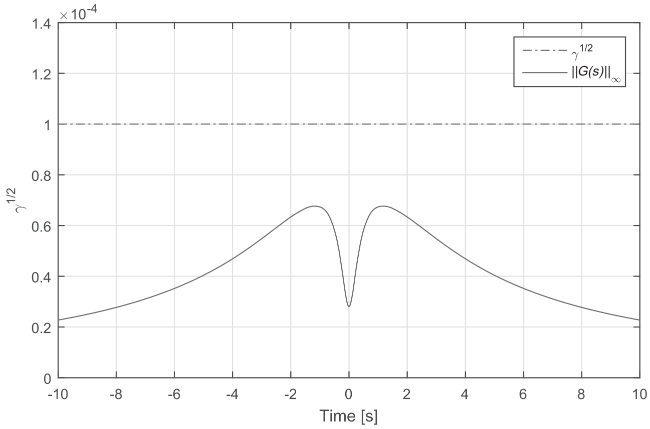

FIGURE 1 Verification of H_∞ performance

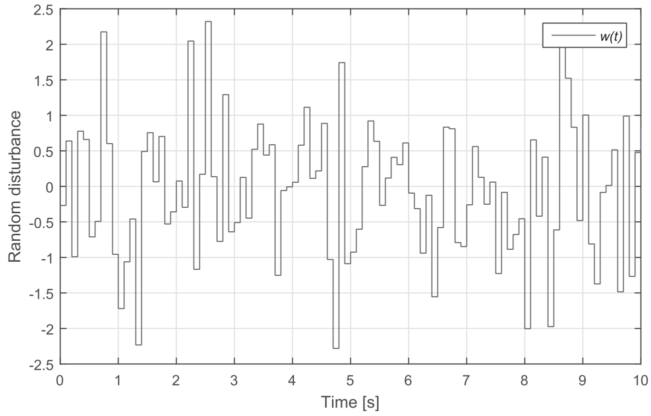

FIGURE 2 Random disturbance

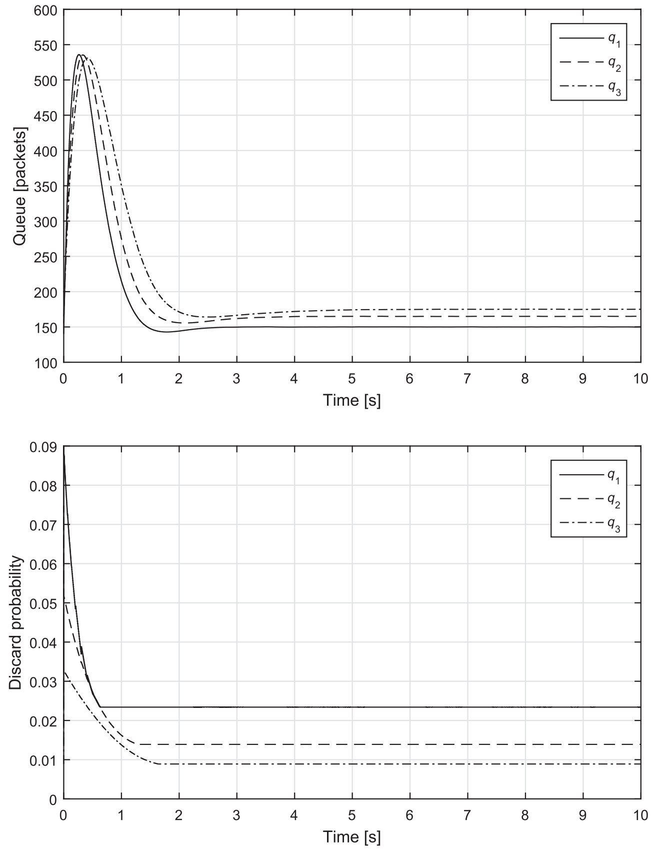

FIGURE 3 Variation of queue and discard probability over average value

choose the queue length set point, the proposed controller also reduces fluctuations in the instantaneous queue length: better queue control has been achieved. These good queue length regulation ensures that the delay in the packets is more controllable, which facilitates providing QoS guarantees when dealing with multiple Internet applications.

5 | CONCLUSION

A direct synthesis approach is used to address some AQM problems on TCP/IP routers under congested traffic using a control theory approach. Motivated by a multiclass AQM control problem, a static state feedback has been proposed to ensure stability and performance for a class of systems that represent these networks: time-varying delay systems with saturating inputs and link capacity disturbances. In mathematical terms, conditions are provided to guarantee the stability of the closed-loop system when the initial states are within a region of attraction. These conditions are given as LMIs and are more general than the previous results in the literature, as they take into account multiclass traffic. Moreover, its implementation is more simple than that of dynamic feedbacks, as static feedback is used. In order to illustrate the effectiveness of this methodology, a numerical example is presented consisting of three different classes of traffic through a congested router, where each class has its specificities. The results improve the router queue convergence and motivate the authors to continue their efforts toward achieving a simple solution to implement AQM for TCP/IP routers that is rigorous in terms of control theory.

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