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Robust H_∞ Control of Takagi-Sugeno Systems with Actuator Saturation

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Abstract The robust static output feedback (SOF) control for continuous-time Takagi-Sugeno systems subject to actuator saturation is solved here, including H_∞ performance guarantees. Based on a polytopic model of the saturation, sufficient conditions are proposed for designing these controllers in terms of Linear Matrix Inequalities (LMIs). With the aid of some special derivations, bilinear matrix inequalities are converted into a set of linear matrix inequalities which can be solved easily without requiring iterative algorithms or equality constraints, moreover, the output matrix of the considered system does not require to be full row rank. Finally, some examples are presented to show the validity of the proposed methodology.

Keywords Takagi-Sugeno (T-S) fuzzy systems · Static Output Feedback (SOF) · Actuator saturation · Linear matrix inequalities (LMIs).

1 INTRODUCTION

In recent years, a large number of researchers are studying Takagi-Sugeno (T-S) systems because, although most physical and systems are nonlinear, they can be adequately represented by this class of fuzzy systems. In fact, it has been demonstrated [1] that T-S fuzzy models can represent exactly some non-linearities of systems, by using fuzzy IF-THEN rules. Thus, these T-S fuzzy models provide a simple and effective method to complement other nonlinear control strategies (such as Mamdani

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fuzzy logic control [2]) by using mathematical nonlinear models. In fact, some significant results have already been proposed using this formalism, most of them using the linear matrix inequality (LMI) formalism. For example, the stability and stabilization problems of nonlinear systems described by T-S fuzzy systems can be found in [3–7], where results are mostly based on the quadratic Lyapunov function (LF) approach. Since uncertainty often leads to instability, robust stability of uncertain T-S fuzzy systems is regarded as an important issue. Robust control of T-S fuzzy systems with uncertainties was studied in [9–12]. Sufficient LMI conditions guaranteeing the existence of H_∞ controllers for T–S fuzzy dynamic systems was presented in [8]. These results were found to be conservative, so effort concentrated on deriving less conservative results, for example, in [13–15]. It must be pointed out that most of these references focused on state feedback. However, in most real-life applications, state variables are not always completely measurable. Hence, researchers are now paying attention to output feedback. More precisely, as dynamic output feedback increases the complexity of the controller, efforts concentrate on static output feedback (SOF): See [20] and references therein. SOF control of T-S fuzzy systems is already attracting a lot of attention [16–19]. Design of SOF controllers is challenging, as it is linked with solving Bilinear Matrix Inequalities (BMI), which are numerically complex. Many results concerning SOF design were developed. In [21] and [22], the SOF controllers have been proposed by using two-step method, where a state feedback controller design was obtained from the first step, whereas the second one is the solution of the LMI problem. Thus, the design depends on the state feedback controller gain obtained in the first step. For T-S fuzzy models, there are some results for SOF design: based on a quadratic Lyapunov function and some matrix transformations, a procedure to calculate SOF controller was given in [23]. In [24], a cone complementary algorithm was studied. [25] proposed an iterative algorithm based on the common quadratic Lyapunov function. In these last works, the conditions are bi-linear in the decision variables, so iterative algorithms were used to numerically solve the stabilization problem, which represents a weakness of the approaches. In other studies [10, 26–28] the BMI problem for SOF controls has been solved by inserting an equality constraint condition for the Lyapunov matrix, which makes the numerical solution more complex. To overcome this, a sufficient condition which does not require equality constraints was proposed in [40] and [41], for robust static output feedback H_∞ controller problem for linear systems with polytopic uncertainties and uncertain fuzzy systems respectively.

None of the previous cited works deals with saturation in the control signal, which is inherent to most real-life applications. If these saturations are not taken into account, stability tests and performance measures are not valid. Several methodologies have been suggested for dealing with systems with actuator saturation. For instance, stability of saturated linear systems was investigated in [29]. Saturated output feedback was considered in [30–32]. With time-varying delay, the problems of descriptor systems and nonlinear systems have been investigated, in [34] and [35], respectively. [36] has studied the stabilization of discrete-time systems with actuator saturation based on multiple Lyapunov function and slack variable matrices. It is worth noting that there are still few contributions addressing SOF H_∞ control subject to actuator saturation: one may refer to [30, 37, 33]. In particular, the singular value decomposition approach was proposed in [30,37] to design SOF H_∞ controller with actuator saturation in terms of eigenvalue problem for both, linear discrete-time and linear continuous-time systems, respectively. In [31] the robust observer-based output feedback controller design of uncertain time delay systems subject to actuator saturation was obtained, although based on an iterative LMI optimization.

Other results related to the current proposal are the following. In [47,50], a state feedback controller was designed to guarantee asymptotic stability of systems of neutral type, but the H_∞ norm performance is not considered. More recently, in [48] and [49], a state feedback controller has been designed to stabilize the system and guarantee an H_∞ norm disturbance attenuation performance. All this previous results for saturated systems assumed the plant to be linear. For nonlinear systems that can be described as T-S fuzzy systems, the adaptive sliding mode control was investigated in [45] by using the singular value decomposition of system input matrix. [46] proposed a descriptor formulation, which avoids the coupling terms between the feedback gains and the Lyapunov matrices, in order to obtain more relaxed conditions. Note that both [45] and [46] did not investigate the performance. To overcome this, in the present contribution, guarantees of H_∞ performance are explicitly considered in the design of a robust SOF controller. It must be pointed out that although [33] investigated the robust H_∞ SOF of T-S fuzzy systems under actuator saturation, this was done by inserting equality constraint conditions, and the output matrix of the considered system is required to be of full row rank. With the later constraint, a particular class of matrices is imposed. In this paper, with the aid of some special derivations, all complex couplings between Lyapunov matrices and feedback gain matrices are avoided. Hence, the proposed method leads to less conservative results than previous works in the literature, but it can also be applied to a larger class of systems.

Thus, in this paper, we propose a new approach for the robust SOF H_∞ control for continuous-time T-S fuzzy systems subject to actuator saturation. Based on the use of the concept of decay rate in the quadratic Lyapunov function, sufficient condition for this problem is formulated in the form of LMIs. More precisely, the design of robust SOF H_∞ control subject to actuator saturation is solved, where the value of the decay-rate is imposed. In comparison with the above mentioned LMI design methods, the proposed method requires neither equality constraints as in [33] nor transformation matrices which are hard to be satisfied. Moreover, this method can handle T-S fuzzy systems with multiple output matrices of subsystems and these matrices are not explicitly required to be of full row rank as in most of existing works [33,27], which overcomes the drawback induced by the previous approaches. Hence, the proposed method not only guarantees the asymptotic stability of the closed-loop system in presence of actuator saturation, but also can be applied to a larger class of T-S fuzzy systems. This paper is organized as follows: All the preliminaries and notations used throughout this document are introduced in Section 2. The SOF H_∞ design methodology is described in Section 3, while in Section 4 the proposed methodology is demonstrated by numerical and practical examples. Some conclusions are given in Section 5.

2 Problem formulations

Consider the following T-S fuzzy model that represents continuous-time systems with actuator saturation and parametric uncertainties [33]:

Plant Rule i: IF $\varsigma_1(t)$ is M_{i1} AND ... AND $\varsigma_s(t)$ is M_{is} THEN

$$\begin{cases} \dot{x}(t) = \bar{A}_i(t)x(t) + \bar{B}_{2i}(t)\sigma(t) + \bar{B}_{1i}(t)\omega(t) \\ z(t) = \bar{C}_{1i}(t)x(t) + \bar{D}_{2i}(t)\sigma(t) + \bar{D}_{1i}(t)\omega(t) \\ y(t) = \bar{C}_{2i}x(t) \end{cases} \tag{1}$$

where $i = 1,2,\dots,r$, and r represent the rule number, $\varsigma(t) = [\varsigma_1(t)\varsigma_2(t)\dots\varsigma_s(t)]$ are known premise variables, M_{ij} are fuzzy sets, $x(t) \in \mathbb{R}^n$ denotes the state vector, $\sigma(t) \in \mathbb{R}^m$ is the saturated control input, $y(t) \in \mathbb{R}^{m_2}$ is the measurement output, $z(t) \in \mathbb{R}^{m_3}$ is the controlled output, $\omega(t) \in \mathbb{R}^{m_4}$ is the disturbance, that belongs to $\mathscr{L}_2[0,\infty) := \{\omega(t) : ||\omega(t)||_{\mathscr{L}_2} := \sqrt{\int_0^\infty ||\omega(t)||^2 dt} < \infty\}$. The uncertain matrices $\bar{A}_i(t)$, $\bar{B}_{2i}(t)$, $\bar{B}_{1i}(t)$, $\bar{C}_{1i}(t)$, $\bar{D}_{2i}(t)$, $\bar{D}_{1i}(t)$ and $\bar{C}_{2i}$ are decomposed as follows:

$$\begin{aligned} \bar{A}_i(t) &= A_i + \Delta A_i(t), & \bar{B}_{1i}(t) &= B_{1i} + \Delta B_{1i}(t) \\ \bar{B}_{2i}(t) &= B_{2i} + \Delta B_{2i}(t), & \bar{C}_{1i}(t) &= C_{1i} + \Delta C_{1i}(t) \\ \bar{D}_{1i}(t) &= D_{1i} + \Delta D_{1i}(t), & \bar{D}_{2i}(t) &= D_{2i} + \Delta D_{2i}(t) \\ \bar{C}_{2i} &= C_{2i} \end{aligned} \tag{2}$$

in order to avoid complexity in designing the output feedback control law, we assume that there is no uncertainties on the measured output. where A_i, B_{1i}, B_{2i}, C_{1i}, D_{1i}, D_{2i} and C_{2i} are constant matrices of compatible dimensions, $\Delta A_i(t)$, $\Delta B_{1i}(t)$, $\Delta B_{2i}(t)$, $\Delta C_{1i}(t)$, $\Delta D_{1i}(t)$, and $\Delta D_{2i}(t)$ are time-varying parametric uncertainties described by:

$$\begin{aligned} &\Delta A_i(t) = M_i\Theta_i(t)N_{Ai}, \;\; \Delta B_{1i}(t) = M_i\Theta_i(t)N_{B1i}, \\ &\Delta B_{2i}(t) = M_i\Theta_i(t)N_{B2i}, \;\; \Delta C_{1i}(t) = M_{zi}\Theta_i(t)N_{C1i}, \\ &\Delta D_{1i}(t) = M_{zi}\Theta_i(t)N_{D1i}, \;\; \Delta D_{2i}(t) = M_{zi}\Theta_i(t)N_{D2i}, \end{aligned} \tag{3}$$

where M_i, M_{zi}, N_{Ai}, N_{B1i}, N_{B2i}, N_{C1i}, N_{D1i} and N_{D2i}, $i = 1,2,...r$, are known real constant matrices, and $\Theta_i(t)$ are unknown time-varying matrix functions satisfying: $\Theta_i(t)^T\Theta_i(t) \le I$.

The defuzzification process of the T-S system (1) provides the equivalent representation [33]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \eta_i(\varsigma(t))[\bar{A}_i(t)x(t) + \bar{B}_{2i}(t)\sigma(t) + \bar{B}_{1i}(t)\omega(t)] \\ z(t) = \sum_{i=1}^{r} \eta_i(\varsigma(t))[\bar{C}_{1i}(t)x(t) + \bar{D}_{2i}(t)\sigma(t) + \bar{D}_{1i}(t)\omega(t)] \\ y(t) = \sum_{i=1}^{r} \eta_i(\varsigma(t))[\bar{C}_{2i}x(t)] \end{cases} \tag{4}$$

in which

$$\eta_i(\varsigma(t)) = \frac{w_i(\varsigma(t))}{\sum_{i=1}^{r} w_i(\varsigma(t))}, \; w_i(\varsigma(t)) = \prod_{j=1}^{s} M_{ij}(\varsigma_j(t))$$

where $M_{ij}(\varsigma_j(t))$ is the grade of membership of $\varsigma_j(t)$ in M_{ij}, and $w_i(\varsigma(t))$ represents the weight of the i^{th} rule. In this paper, we assume that $w_i(\varsigma(t)) \geq 0$, for $i = 1,2,\ldots,r$, and $\sum_{i=1}^{r} w_i(\varsigma(t)) > 0$ for all t. Therefore, we get $\eta_i(\varsigma(t)) \geq 0$, for $i = 1,2,\ldots,r$ and $\sum_{i=1}^{r} \eta_i(\varsigma(t)) = 1$ for all t.

A difference from [17,18,25,27], is that this paper considers explicitly the effect of actuators saturation, in addition to uncertainties and H_∞ performance. The saturation function $sat[.]$ is defined as follows:

$$\begin{aligned} \sigma(t) &= sat(u(t),\bar{u}) \\ &= [sat(u_1(t),\bar{u}_1), sat(u_2(t),\bar{u}_2), \ldots, sat(u_m(t),\bar{u}_m)]^T \end{aligned} \tag{5}$$

$$sat(u_l(t),\bar{u}_l) = \begin{cases} \bar{u}_l \;\; if \;\; u_l(t) > \bar{u}_l \\ u_l(t) \;\; if \;\; -\bar{u}_l \leq u_l(t) \leq \bar{u}_l \\ -\bar{u}_l \;\; if \;\; u_l(t) < \bar{u}_l \end{cases} \tag{6}$$

where $\bar{u} \in \mathbb{R}^m$ denotes the saturation level, $u(t) \in \mathbb{R}^m$ is the control input, $\bar{u}_m$ and u_m denote the i^{th} element of $\bar{u}$ and u, respectively.

The SOF controller for the system (4) used here is based on the concept of parallel distributed compensation (PDC) [27]. Thus, the controller is composed by r rules of the following form:

Controller Rule i : IF $\varsigma_1(t)$ is M_{i1} AND ... AND $\varsigma_s(t)$ is M_{is} THEN

$$u(t) = F_i y(t) \tag{7}$$

where $i = 1,2\ldots,r$ and F_i are the feedback gain matrices that will be calculated to provide robust performance. The overall SOF controller is then inferred as follows:

$$u(t) = \sum_{i=1}^{r} \eta_i(\varsigma(t)) F_i y(t) \tag{8}$$

In order to facilitate the design of the controller, the saturations are modelled using the equivalent polytopic representation in [38,42]. For this, the following definitions and lemmas are needed:

Let E be the set of $m \times m$ diagonal matrices whose diagonal elements are 1 or 0. There are 2^m elements in E and we denote its elements as E_s, $s = 1,...2^m$, and denote $E_s^- = I - E_s$. It is easy to see that $E_s^- \in E$ if $E_s \in E$.

Lemma 1 *[43] Let $u, \mu \in \mathbb{R}^m$ with $u = [u_1 \; u_2,...,u_m]^T$ and $\mu = [\mu_1 \; \mu_2,...,\mu_m]^T$. Suppose that $|\mu_i| \leq \bar{u}_i$ for $i = 1,2,...,m$. If $x \in \bigcap_{j=1}^{r} \Im(H_j)$ for $x \in \mathbb{R}^n$, then:*

$$\Im(H_j) = \{x \in \mathbb{R}^n : |h_i^j x| \leq \bar{u}_i\} \tag{9}$$

$$sat(u,\bar{u}) \in co\{E_s u + E_s^- \mu : s \in [1,2^m]\} \tag{10}$$

where co denotes the convex hull.
where H_j is $m \times n$ matrix and h_i^j is the i^{th} row of H_j. $\Im(H_j)$ is a polyhedral set. Consequently, $sat(u,\bar{u})$ can be rewritten as

$$\begin{cases} sat(u,\bar{u}) = \sum_{s=1}^{2^m} \zeta_s (E_s u + E_s^- \mu), \\ \sum_{s=1}^{2^m} \zeta_s = 1, \qquad 0 \le \zeta_s \le 1 \end{cases}$$

For $x(t_0) = x_0 \in \mathbb{R}^n$, denote the state trajectory of system (4) as $\psi(t,x_0)$. The domain of attraction of the origin is given as

$$\mathbb{S} := \{x_0 \in \mathbb{R}^n : \lim_{t\to\infty} \psi(t,x_0) = 0\} \tag{11}$$

where $\mathbb{S}$ is an invariant set, which means that all the trajectories starting from within it will remain in it forever. Furthermore, we are interested in getting an estimate $\Xi_\delta \subset \mathbb{S}$ of the domain of attraction, where

$$\Xi_\delta = \{x_0 \in \mathbb{R}^n : \max |x_0| \le \delta\}$$

where $\delta > 0$ denote a scalar to be maximized.

Definition 1 *[42] Let ε be a compact set, and let $W(x(t))$ be a positive scalar function, defined by:*

$$\varepsilon(P,\rho) = \{x \in \mathbb{R}^n : \ x(t)^T P x(t) \le \rho\}.$$

and

$$W(x(t)) = x(t)^T P x(t)$$

where $P \in \mathbb{R}^{n\times n}$ denotes a symmetric positive definite matrix. The ellipsoid $\varepsilon(P,\rho)$ is said to be contractively invariant set if $\dot{W}(x(t)) < 0$, $\forall x \in \varepsilon(P,\rho) \setminus \{0\}$. Thus, if an ellipsoid ε is contractively invariant for a system, it is inside its domain of attraction.

Lemma 2 *[39,42,43] An ellipsoid $\varepsilon(P,\rho)$ is inside $\bigcap_{j=1}^{r} \Im(H_j)$ if and only if*

$$h_i^j \left(\frac{P}{\rho}\right)^{-1} (h_i^j)^T \le \bar{u}_i^2 \qquad i = 1,\ldots,m \tag{12}$$

where h_i^j is the i^{th} row of the matrix H_j.

Lemma 3 *[41] For matrices T, Q, V, and W and a non-zero scalar ξ, the inequality*

$$T + W^T Q^T + QW < 0, \tag{13}$$

is fulfilled if the following condition holds:

$$\begin{bmatrix} T & * \\ \xi Q^T + VW & -\xi V - \xi V^T \end{bmatrix} < 0.$$

Lemma 4 *[10] Given constant matrices X, Y and an unknown constant matrix Δ satisfying the constraint $\Delta^T\Delta < I$, for any scalar $\varepsilon > 0$ the following inequality holds:*

$$X\Delta Y + Y^T\Delta^T X^T \leq \varepsilon XX^T + \varepsilon^{-1}Y^T Y$$

Remark 1 *The robust SOF H_∞ stabilization of T-S fuzzy systems under actuator saturation addressed in [33] was obtained by using $\mu = \sum_{j=1}^{r}\eta_j H_j x(t)$ and the polyhedral set defined by* (9). *However, in this paper we use*

$$\begin{aligned}\mu &= \sum_{j=1}^{r}\eta_j H_j y(t)\\ &= \sum_{j=1}^{r}\sum_{l=1}^{r}\eta_j\eta_l H_j[C_{2l}x(t)]\\ \Im y(H_j,\bar{u}) &= \{x\in\mathbb{R}^n | h_\lambda^j y(t)| \leq \bar{u}_\lambda\}\end{aligned} \tag{14}$$

By some routine manipulations, $\varepsilon(P,\rho)$ can be proven to be inside $\bigcap_{j=1}^{r}\Im y(H_j,\bar{u})$ if and only if

$$h_\lambda^j C_{2l}\left(\frac{P}{\rho}\right)^{-1} C_{2l}^T (h_\lambda^j)^T \leq \bar{u}_\lambda^2 \qquad \lambda = 1,\ldots,m,\ \ j,l = 1,...,r \tag{15}$$

so the proposed sets are really appropriate for controller design. Note that compared with method based on (12) *used in [33], the equation* (15) *involves more inequalities which results in a greater number of LMIs. Moreover, our results will be expressed directly in terms of LMIs, avoiding the equality constraints or the use of transformation matrices, unlike the SOF in [33]. This simplifies the design procedure, and widens the set of systems that can be stabilized by SOF.*

By applying Lemma 1 and Remark 1, if $x \in \bigcap_{j=1}^{r}\Im(H_j,\bar{u})$ then:

$$sat(u(t),\bar{u}) = \sum_{s=1}^{2^m}\zeta_s\sum_{j=1}^{r}\eta_j(\varsigma(t))\sum_{l=1}^{r}\eta_l(\varsigma(t))[E_sF_j + E_s^- H_j]y(t) \tag{16}$$

From (4) and (16), the closed-loop fuzzy system can equivalently be expressed as follows:

$$\begin{cases}\dot{x}(t) = \sum_{s=1}^{2^m}\zeta_s\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\eta_i\eta_j\eta_l[\bar{A}_{sijl}x(t) + \bar{B}_{1i}(t)\omega(t)]\\ z(t) = \sum_{s=1}^{2^m}\zeta_s\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\eta_i\eta_j\eta_l[\bar{C}_{sijl}x(t) + \bar{D}_{1i}(t)\omega(t)]\end{cases} \tag{17}$$

where

$$\begin{aligned}&\bar{A}_{sijl} = \bar{A}_i(t) + \bar{B}_{2i}(t)[E_sF_jC_{2l} + E_s^- H_jC_{2l}]\\ &\bar{C}_{sijl} = \bar{C}_{1i}(t) + \bar{D}_{2i}(t)[E_sF_jC_{2l} + E_s^- H_jC_{2l}]\\ &\eta_i = \eta_i(\varsigma(t)),\ \ \eta_j = \eta_j(\varsigma(t)),\ \ \eta_l = \eta_l(\varsigma(t)).\end{aligned}$$

The objective of the proposed controller design is then to find gains F_i, $i = 1,\ldots,r$ of the control law (8), such that the closed-loop system (17) satisfies the following two conditions in the presence of actuators saturation:
(1) SOF stability: The T-S fuzzy system (17) is asymptotically stable when $\omega(t) = 0$.
(2) H_∞ Performance: Subject to zero initial conditions, the controlled output $z(t)$ satisfies, for square integrable disturbance input $\omega(t)$,

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt. \tag{18}$$

3 Main results

In this section, the objective is to design a SOF H_∞ controller subject to actuator saturation, that stabilizes the T-S fuzzy system (17), and yields a maximum size of the estimated domain of attraction.

Theorem 1 *The closed-loop system* (17) *is asymptotically stable with H_∞ performance γ, such that, $\varepsilon(P,\rho)$ is inside $\bigcap_{j=1}^{r} \Im(H_j,\bar{u})$, if, given a scalar $\xi > 0$, there exist a symmetric positive definite matrix X, matrices Y_i, Z_i, V, and positive scalars ε_1, ε_2 and β, for $i, j = 1,2,\ldots,r$ and $s = 1,\ldots,2^m$ satisfying the following conditions:*

$$\begin{gathered} \Pi_{\lambda jj} \leq 0 \\ \lambda = 1,2,\ldots,m;\;\; j = 1,2,...,r \end{gathered} \tag{19}$$

$$\begin{gathered} \Pi_{\lambda jl} + \Pi_{\lambda lj} \leq 0 \\ \lambda = 1,2,\ldots,m;\;\; j = 1,2,...,r-1;\;\; l = j+1,...,r \end{gathered} \tag{20}$$

$$\phi_{siii} < 0,\;\; i = 1,2,\ldots,r \tag{21}$$

$$\begin{gathered} \phi_{siij} + \phi_{siji} + \phi_{sjii} < 0 \\ 1 \leq i \neq j \leq r \end{gathered} \tag{22}$$

$$\begin{gathered} \phi_{sijl} + \phi_{silj} + \phi_{sjli} + \phi_{sjil} + \phi_{slij} + \phi_{slji} < 0 \\ 1 \leq i \neq j \neq l \leq r \end{gathered} \tag{23}$$

where

$$\Pi_{\lambda jl} = \begin{bmatrix} -\bar{u}_\lambda^2 & * & * \\ C_{2l}^T(z_\lambda^j)^T & -X & * \\ \xi(z_\lambda^j)^T & C_{2l}X - VC_{2l} & -\xi V - \xi V^T \end{bmatrix} \tag{24}$$

z_λ^j is the λ^{th} row of the matrices Z_j.

$$\phi_{sijl} = \begin{bmatrix} \Upsilon_{sijl} & * \\ \Gamma_{sijl} & -\xi V - \xi V^T \end{bmatrix} \tag{25}$$

$$\Upsilon_{sijl} = \begin{bmatrix} \Psi_{sijl} & * & * & * & * \\ B_{1i}^T & -\gamma^2 I & * & * & * \\ \Upsilon_{sijl}^{31} & D_{1i} & -I+\varepsilon_2 M_{zi} M_{zi}^T & * & * \\ \Upsilon_{sijl}^{41} & N_{B1i} & 0 & -\varepsilon_1 I & * \\ \Upsilon_{sijl}^{51} & N_{D1i} & 0 & 0 & -\varepsilon_2 I \end{bmatrix} \quad (26)$$

$$\begin{aligned}
\Upsilon_{sijl}^{31} &= C_{1i}X + D_{2i}E_s Y_j C_{2l} + D_{2i}E_s^- Z_j C_{2l} \\
\Upsilon_{sijl}^{41} &= N_{Ai}X + N_{B2i}E_s Y_j C_{2l} + N_{B2i}E_s^- Z_j C_{2l} \\
\Upsilon_{sijl}^{51} &= N_{C1i}X + N_{D2i}E_s Y_j C_{2l} + N_{D2i}E_s^- Z_j C_{2l} \\
\Psi_{sijl} &= XA_i^T + A_i X + B_{2i}[E_s Y_j C_{2l} + E_s^- Z_j C_{2l}] + [C_{2l}^T Y_j^T E_s + C_{2l}^T Z_j^T E_s^{-T}]B_{2i}^T \\
&\quad + \beta X + \varepsilon_1 M_i M_i^T
\end{aligned}$$

and

$$\Gamma_{sijl} = \left[\, \Gamma_{sijl}^{11} \; 0 \; \Gamma_{sij}^{13} \; \Gamma_{sij}^{14} \; \Gamma_{sij}^{15} \,\right] \quad (27)$$

$$\begin{aligned}
\Gamma_{sijl}^{11} &= \xi\left(Y_j^T E_s^T + Z_j^T E_s^{-T}\right) B_{2i}^T + (C_{2l}X - VC_{2l}) \\
\Gamma_{sij}^{13} &= \xi\left(Y_j^T E_s^T + Z_j^T E_s^{-T}\right) D_{2i}^T \\
\Gamma_{sij}^{14} &= \xi\left(Y_j^T E_s^T + Z_j^T E_s^{-T}\right) N_{B2i}^T \\
\Gamma_{sij}^{15} &= \xi\left(Y_j^T E_s^T + Z_j^T E_s^{-T}\right) N_{D2i}^T
\end{aligned}$$

Moreover, the controller gains F_i, for the SOF controller (8) can be obtained from $F_i = Y_i V^{-1}$. Matrices H_i can be obtained from $H_i = Z_i V^{-1}$, $i = 1, \dots, r$.

Proof Convex sum property and the SOF conditions (19) and (20) imply that

$$\sum_{\lambda=1}^{m} \eta_\lambda \left[\sum_{j=1}^{r} \eta_j^2 \Pi_{\lambda jj} + \sum_{j=1}^{r-1} \sum_{l=j+1}^{r} \eta_j \eta_l (\Pi_{\lambda jl} + \Pi_{\lambda lj}) \right] = \sum_{\lambda=1}^{m} \eta_\lambda \sum_{j=1}^{r} \sum_{l=1}^{r} \eta_j \eta_l \Pi_{\lambda jl}$$

and, from (19) and (20), one concludes that

$$\Pi_{\lambda jl} = \begin{bmatrix} -\bar{u}_\lambda^2 & * & * \\ C_{2l}^T (z_\lambda^j)^T & -X & * \\ \xi (z_\lambda^j)^T & C_{2l}X - VC_{2l} & -\xi V - \xi V^T \end{bmatrix} < 0 \quad (28)$$

If the LMI conditions in (28) hold, then the feasible solution of these conditions satisfies $-\xi V - \xi V^T < 0$, which implies that matrix V is non-singular.

Using Lemma 3 with

$$T_{\lambda jl} = \begin{bmatrix} -\bar{u}_\lambda^2 & * \\ C_{2l}^T (z_\lambda^j)^T & -X \end{bmatrix}, \; Q_{\lambda j} = \begin{bmatrix} z_\lambda^j \\ 0 \end{bmatrix}, \; W_l = V^{-1} \left[\, 0 \; C_{2l}X - VC_{2l} \,\right], \quad (29)$$

the LMI constraint (28) is equivalent to

$$T_{\lambda jl} + W_l^T Q_{\lambda j}^T + Q_{\lambda j} W_l < 0 \tag{30}$$

defining $Z_j = H_j V$, the inequality (30) can be rewrite as

$$T_{\lambda jl} + \begin{bmatrix} 0 & * \\ (XC_{2l}^T - C_{2l}^T V^T)(h_\lambda^j)^T & 0 \end{bmatrix} \leq 0. \tag{31}$$

Substituting $T_{\lambda jl}$ in (29) into (31), we get

$$\begin{bmatrix} -\bar{u}_\lambda^2 & * \\ XC_{2l}^T (h_\lambda^j)^T & -X \end{bmatrix} \leq 0. \tag{32}$$

Defining $X = (\frac{P}{\rho})^{-1}$, constraint (32) can be written by Schur complement as the inequality (15).

Since (21)-(23) hold, we can write that

$$\begin{aligned}
&\sum_{s=1}^{2^m} \zeta_s \Big[\sum_{i=1}^{r} \eta_i^3 \phi_{siii} + \sum_{i=1}^{r} \sum_{j=1, i\neq j}^{r} \eta_i^2 \eta_j \Big(\phi_{siij} + \phi_{siji} + \phi_{sjii} \Big) \\
&+ \sum_{i=1}^{r-2} \sum_{j=i+1}^{r-1} \sum_{l=j+1}^{r} \eta_i \eta_j \eta_l \Big(\phi_{sijl} + \phi_{silj} + \phi_{sjil} + \phi_{sjli} + \phi_{slij} + \phi_{slji} \Big) \Big] \\
&= \sum_{s=1}^{2^m} \zeta_s \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} \eta_i \eta_j \eta_l \phi_{sijl} < 0
\end{aligned}$$

which is satisfied if

$$\phi_{sijl} = \begin{bmatrix} \Upsilon_{sijl} & * \\ \Gamma_{sijl} & -\xi V - \xi V^T \end{bmatrix} < 0. \tag{33}$$

where Υ_{sijl} and Γ_{sijl} are defined in (26)
By Lemma 3 with

$$Q_{sij} = \begin{bmatrix} B_{2i}(E_s Y_j + E_s^- Z_j) \\ 0 \\ D_{2i}(E_s Y_j + E_s^- Z_j) \\ N_{B2i}(E_s Y_j + E_s^- Z_j) \\ N_{D2i}(E_s Y_j + E_s^- Z_j) \end{bmatrix}, \; W_l = V^{-1} \begin{bmatrix} C_{2l} X - V C_{2l} \; 0 \; 0 \; 0 \; 0 \end{bmatrix},$$

the inequality (33) implies that,

$$\varphi_{sijl} = \Upsilon_{sijl} + Q_{sij} W_l + W_l^T Q_{sij}^T < 0. \tag{34}$$

(34) can be rewritten as:

$$\varphi_{sijl} = \Upsilon_{sijl} + \begin{bmatrix} \bar{\phi}^{11}_{sijl} & * & * & * & * \\ 0 & 0 & * & * & * \\ \bar{\phi}^{31}_{sijl} & 0 & 0 & * & * \\ \bar{\phi}^{41}_{sijl} & 0 & 0 & 0 & * \\ \bar{\phi}^{51}_{sijl} & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{35}$$

where

$\bar{\phi}^{11}_{sijl} = B_{2i}(E_sY_j + E_s^- Z_j)V^{-1}(C_{2l}X - VC_{2l}) + (XC_{2l}^T - C_{2l}^T V^T)V^{-T}(Y_j^T E_s^T + Z_j^T E_s^{-T})B_{2i}^T$

$\bar{\phi}^{31}_{sijl} = D_{2i}(E_sY_j + E_s^- Z_j)V^{-1}(C_{2l}X - VC_{2l})$

$\bar{\phi}^{41}_{sijl} = N_{B2i}(E_sY_j + E_s^- Z_j)V^{-1}(C_{2l}X - VC_{2l})$

$\bar{\phi}^{51}_{sijl} = N_{D2i}(E_sY_j + E_s^- Z_j)V^{-1}(C_{2l}X - VC_{2l})$

Using the change of variable $F_j = Y_jV^{-1}$, $H_j = Z_jV^{-1}$ and substituting (26) into (35) we get:

$$\varphi_{sijl} = \begin{bmatrix} \Psi_{sijl} & * & * & * & * \\ B_{1i}^T & -\gamma^2 I & * & * & * \\ \varphi^{31}_{sijl} & D_{1i} & -I + \varepsilon_2 M_{zi} M_{zi}^T & * & * \\ \varphi^{41}_{sijl} & N_{B1i} & 0 & -\varepsilon_1 I & * \\ \varphi^{51}_{sijl} & N_{D1i} & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{36}$$

where

$\Psi_{sijl} = XA_i^T + A_iX + B_{2i}[E_sF_jC_{2l}X + E_s^- H_jC_{2l}X] + [XC_{2l}^T F_j^T E_s + XC_{2l}^T H_j^T E_s^{-T}]B_{2i}^T + \beta X + \varepsilon_1 M_i M_i^T,$

$\varphi^{31}_{sijl} = C_{1i}X + D_{2i}E_sF_jC_{2l}X + D_{2i}E_s^- H_jC_{2l}X,$

$\varphi^{41}_{sijl} = N_{Ai}X + N_{B2i}E_sF_jC_{2l}X + N_{B2i}E_s^- H_jC_{2l}X,$

$\varphi^{51}_{sijl} = N_{C1i}X + N_{D2i}E_sF_jC_{2l}X + N_{D2i}E_s^- H_jC_{2l}X,$

Pre-and post-multiplying the inequality (36) by $diag(X^{-1}, I, I, I, I)$ gives

$$\bar{\varphi}_{sijl} = \begin{bmatrix} \bar{\Psi}_{sijl} & * & * & * & * \\ B_{1i}^T P & -\gamma^2 I & * & * & * \\ \bar{\varphi}^{31}_{sijl} & D_{1i} & -I + \varepsilon_2 M_{zi} M_{zi}^T & * & * \\ \bar{\varphi}^{41}_{sijl} & N_{B1i} & 0 & -\varepsilon_1 I & * \\ \bar{\varphi}^{51}_{sijl} & N_{D1i} & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{37}$$

where

$\bar{\Psi}_{sijl} = A_i^T P + PA_i + PB_{2i}[E_sF_jC_{2l} + E_s^- H_jC_{2l}] + [C_{2l}^T F_j^T E_s + C_{2l}^T H_j^T E_s^{-T}]B_{2i}^T P + \beta P + \varepsilon_1 PM_i M_i^T P$

$\bar{\varphi}^{31}_{sijl} = C_{1i} + D_{2i}E_sF_jC_{2l} + D_{2i}E_s^- H_jC_{2l}$

$\bar{\varphi}^{41}_{sijl} = N_{Ai} + N_{B2i}E_sF_jC_{2l} + N_{B2i}E_s^- H_jC_{2l}$

$\bar{\varphi}^{51}_{sijl} = N_{C1i} + N_{D2i}E_sF_jC_{2l} + N_{D2i}E_s^- H_jC_{2l}$

we can write,
$\bar{\varphi}_{sijl} = \bar{\varphi}_{0_{sijl}} + \Delta\bar{\varphi}_{sijl}$
where

$$\bar{\varphi}_{0_{sijl}} = \begin{bmatrix} \bar{\varphi}^{11}_{0_{sijl}} & * & * \\ B^T_{1i}P & -\gamma^2 I & * \\ \bar{\varphi}^{31}_{0_{sijl}} & D_{1i} & -I \end{bmatrix} \tag{38}$$

$\bar{\varphi}^{11}_{0_{sijl}} = P[A_i + B_{2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})] + [A_i^T + (E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})^T B^T_{2i}]P + \beta P$
$\bar{\varphi}^{31}_{0_{sijl}} = C_{1i} + D_{2i}[E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}]$

and

$$\begin{aligned} \Delta\bar{\varphi}_{sijl} = {} & \varepsilon_1 \begin{bmatrix} PM_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PM_i \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_2 \begin{bmatrix} 0 \\ 0 \\ M_{zi} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_{zi} \end{bmatrix}^T \\ & + \varepsilon_1^{-1} \begin{bmatrix} (N_{Ai} + N_{B2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{B1i} \\ 0 \end{bmatrix} \begin{bmatrix} (N_{Ai} + N_{B2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{B1i} \\ 0 \end{bmatrix}^T \\ & + \varepsilon_2^{-1} \begin{bmatrix} (N_{C1i} + N_{D2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{D1i} \\ 0 \end{bmatrix} \begin{bmatrix} (N_{C1i} + N_{D2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{D1i} \\ 0 \end{bmatrix}^T \end{aligned} \tag{39}$$

using Lemma 4, we obtain
$\Delta\bar{\varphi}_{sijl} \geq \Delta\tilde{\varphi}_{sijl}$

$$\begin{aligned} \Delta\tilde{\varphi}_{sijl} = {} & \begin{bmatrix} PM_i \\ 0 \\ 0 \end{bmatrix} \Theta_i(t) \begin{bmatrix} (N_{Ai} + N_{B2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{B1i} \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} (N_{Ai} + N_{B2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{B1i} \\ 0 \end{bmatrix} \Theta_i^T(t) \begin{bmatrix} PM_i \\ 0 \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} 0 \\ 0 \\ M_{zi} \end{bmatrix} \Theta_i(t) \begin{bmatrix} (N_{C1i} + N_{D2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{D1i} \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} (N_{C1i} + N_{D2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l}))^T \\ N^T_{D1i} \\ 0 \end{bmatrix} \Theta_i^T(t) \begin{bmatrix} 0 \\ 0 \\ M_{zi} \end{bmatrix}^T \end{aligned} \tag{40}$$

$$\Delta\tilde{\varphi}_{sijl} = \begin{bmatrix} \Delta\tilde{\varphi}^{11}_{sijl} & * & * \\ N^T_{B1i}\Theta_i^T(t)M_i^T P & 0 & * \\ \Delta\tilde{\varphi}^{31}_{sijl} & M_{zi}\Theta_i(t)N_{D1i} & 0 \end{bmatrix} \tag{41}$$

$\Delta\tilde{\varphi}^{11}_{sijl} = PM_i\Theta_i(t)[N_{Ai} + N_{B2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})] + [N^T_{Ai} + (E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})^T N^T_{B2i}]\Theta_i^T(t)M_i^T P$
$\Delta\tilde{\varphi}^{31}_{sijl} = M_{zi}\Theta_i(t)[N_{C1i} + N_{D2i}(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})]$

So that

$\tilde{\varphi}_{sijl} = \bar{\varphi}_{0_{sijl}} + \Delta\tilde{\varphi}_{sijl} \leq \bar{\varphi}_{sijl} < 0.$
Let

$$\tilde{\varphi}_{sijl} = \begin{bmatrix} \tilde{\varphi}^{11}_{sijl} & * & * \\ \bar{B}^T_{1i}(t)P & -\gamma^2 I & * \\ \tilde{\varphi}^{31}_{sijl} & \bar{D}_{1i}(t) & -I \end{bmatrix} < 0 \tag{42}$$

$\tilde{\varphi}^{11}_{sijl} = P[\bar{A}_i(t) + \bar{B}_{2i}(t)(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})] + \beta P + [\bar{A}_i^T(t) + (E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})^T\bar{B}^T_{2i}(t)]P$
$\tilde{\varphi}^{31}_{sijl} = [\bar{C}_{1i}(t) + \bar{D}_{2i}(t)(E_sF_jC_{2l} + E_s^{-1}H_jC_{2l})]$

using the system matrices in (17),

$$\tilde{\varphi}_{sijl} = \begin{bmatrix} \bar{A}^T_{sijl}P + P\bar{A}_{sijl} + \beta P & * & * \\ \bar{B}^T_{1i}(t)P & -\gamma^2 I & * \\ \bar{C}_{sijl} & \bar{D}_{1i}(t) & -I \end{bmatrix} < 0. \tag{43}$$

Let us consider the Lyapunov function

$$V_1(t) = x^T(t)Px(t). \tag{44}$$

For the decay rate control design, the condition is defined as follows

$$\dot{V}_1(t) < -\beta V_1(t). \tag{45}$$

Taking the time derivative of LF (44) we get

$$\dot{V}_1(t) = \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t). \tag{46}$$

From (45) and (46) we have that

$$\dot{V}(t) = \dot{V}_1(t) + \beta V_1(t) < 0. \tag{47}$$

Thus if (43) holds, by Schur complement it can be easily verified that

$$\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0. \tag{48}$$

Integrating both sides of the inequality (48) from 0 to ∞ yields

$$\begin{aligned} &\int_0^\infty \dot{V}dt + \int_0^\infty (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t))dt \\ &= V(x(\infty)) - V(x(0)) + \int_0^\infty (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t))dt < 0 \end{aligned}$$

For zero initial condition, we obtain

$$\int_0^\infty (z^T(t)z(t)dt < \gamma^2 \int_0^\infty \omega^T(t)\omega(t))dt$$

This completes the proof.

Remark 2 *The SOF control of discrete-time T-S fuzzy systems with time-delays was investigated in [44], where the values of the disturbance attenuation and decay rate β were imposed. However, decay rates are used here as parameters: this makes possible to figure out the best attenuation possible, by finding the minimum value of γ that gives a feasible solution. Hence, the introduction of the scalar parameter β is not necessary to derive our results, but the selection of β provides extra free dimensions in the solution space for the design condition.*

Remark 3 *In [25] and [31], an iterative LMI approach has been developed to solve numerically the stabilization problem. However, in [25], the SOF control of continuous-time T-S fuzzy models is considered without the input saturation. In [31], the robust output feedback controller design of uncertain time-delay systems subject to actuator saturation was obtained. Moreover, the iterative LMI approach depends on the initial values. However, how to select the initial values is still an open problem. Thus, to avoid such drawback, an LMI design method is provided in this work without the need to any initial conditions, so our results are more suitable.*

Remark 4 *In the case where Lemma 3 is not used and the parameter $\beta = 0$, the inequality* (43) *is reduced to the inequality (16) in [33]. This explains theoretically, that our results are more general than that of [33] and can leads to less conservative results than the results of [33], and it can also be applied to a larger class of systems.*

We now study the estimation of the largest region inside $\varepsilon(P,\rho) = \{x \in \mathbb{R}^n | x(t)^T P x(t) \leq \rho\}$, with respect to the convex set $\mathscr{X}_N$.
For this, we define

$$\alpha_N(\varepsilon(P,\rho)) = \sup\{\alpha > 0 | (\alpha \mathscr{X}_N) \subset \varepsilon(P,\rho)\}.$$

where α is a scalar. Clearly if $\alpha_N(\varepsilon(P,\rho)) \geq 1$ then $\mathscr{X}_N \subset (\varepsilon(P,\rho))$.
The convex set $\mathscr{X}_N$ has two main forms [42]:

In the first case, $\mathscr{X}_N$ is an ellipsoid:

$$\mathscr{X}_N = \{x \in \mathbb{R}^n | x(t)^T N x(t) \leq 1\}. \tag{49}$$

where N is a positive definite matrix.

In the other case, $\mathscr{X}_N$ is a polyhedron:

$$\mathscr{X}_N = co\{x_0^1, x_0^2, \ldots, x_0^l\}. \tag{50}$$

where $x_0^1, x_0^2, \ldots, x_0^l$ are a priori given points in $\mathbb{R}^n$.
If $\mathscr{X}_N$ is given by (49), $\alpha_N(\varepsilon(P,\rho))$ is equivalent to $(\frac{N}{\alpha^2}) - X^{-1} \geq 0$; then, by Schur complement this last inequality can be written as:

$$\begin{bmatrix} \eta N & I \\ I & X \end{bmatrix} \geq 0 \tag{51}$$

with $\eta = \alpha^{-2}$, $(\frac{P}{\rho}) = X^{-1}$.
If $\mathscr{X}_N$ is given by (50), $\alpha_N(\varepsilon(P,\rho))$ is equivalent to $\alpha^2 (x_0^i)^T (\frac{P}{\rho}) x_0^i \leq 1$, $\forall i = 1, \ldots, l$, using Schur complement this last inequality can be equivalently written as:

$$\begin{bmatrix} \eta & (x_0^i)^T \\ x_0^i & X \end{bmatrix} \geq 0. \qquad (52)$$

If we aim to design a controller with H_∞ performance such that the domain of attraction of the closed-loop system is as large as possible, we need to establish a compromise between the maximization of the estimated size of the domain of attraction α and the minimization of the H_∞ attenuation level γ. We unify this by minimizing $\alpha^{-2}+\gamma^2$.

Theorem 2 *The closed-loop system* (17) *is asymptotically stable if for a given* $\xi > 0$ *there exist a symmetric positive definite matrix* X*, and matrices* Y_i*,* Z_i *and* V*, that satisfy the following optimization:*

$$\min(\alpha^{-2}+\gamma^2)\ s.t.$$

$$\begin{cases} LMI\ (19)-(23),\ and \\ \begin{bmatrix} \eta N & I \\ I & X \end{bmatrix} \geq 0 \\ or \\ \begin{bmatrix} \eta & (x_0^i)^T \\ x_0^i & X \end{bmatrix} \geq 0 \\ \quad i = 1,\dots,l \end{cases} \qquad (53)$$

Moreover, the attenuation level is guaranteed to be less than γ and the domain of attraction is maximized, with its size represented by α.

Remark 5 *There exist several convex methods for designing SOF controllers in the literature. For example, sufficient conditions with equality constraints were used in [27] and [28] for T-S fuzzy systems. Moreover, some methods for robust SOF* H_∞ *control that do not require constraints on system matrices have been proposed for polytopic linear systems and discrete-time T-S fuzzy systems, respectively, in [40] and [41]. Besides, based on a judicious use of Finsler's lemma in a T-S fuzzy Lyapunov control framework, SOF controllers subject to both control input and state constraints for discrete T-S fuzzy systems have been proposed in [32]. In comparison with the above-mentioned LMI design methods, this study proposes the robust SOF* H_∞ *control for continuous-time T-S fuzzy systems subject to actuator saturation, without any constraints on system matrices.*

4 Computer simulations

In this section, we provide numerical and a practical examples in order to highlight the effectiveness and the advantages of the proposed approach.

Example 1 *To show the merit of the proposed method we consider the following nonlinear system, borrowed from [33] and [27].*

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)+x_2(t)+\sin x_3(t)-0.1x_4(t)+(x_1^2(t)+1)u(t)+\omega(t)\\
\dot{x}_2(t) &= x_1(t)-2x_2(t)\\
\dot{x}_3(t) &= x_1(t)+x_1^2x_2(t)-0.3x_3(t)\\
\dot{x}_4(t) &= \sin x_3(t)-x_4(t)\\
z(t) &= x_1(t)+\sin(x_3(t))+0.3u(t)+0.1\omega(t)\\
y(t) &= x_1(t)+\sin(x_3(t))
\end{aligned} \tag{54}$$

Assuming that $x_1(t)\in$ [-a a], $x_3(t)\in$ [-b b] with $a>0$ and $b>0$, the nonlinear system (54) *can be equivalently represented by the following T-S fuzzy system:*

$$A_1=\begin{bmatrix}1&1&1&-0.1\\1&-2&0&0\\1&a^2&-0.3&0\\0&0&1&-1\end{bmatrix},\ A_2=\begin{bmatrix}1&1&\frac{\sin b}{b}&-0.1\\1&-2&0&0\\1&a^2&-0.3&0\\0&0&\frac{\sin b}{b}&-1\end{bmatrix},\ A_3=\begin{bmatrix}1&1&1&-0.1\\1&-2&0&0\\1&0&-0.3&0\\0&0&1&-1\end{bmatrix},$$

$$A_4=\begin{bmatrix}1&1&\frac{\sin b}{b}&-0.1\\1&-2&0&0\\1&0&-0.3&0\\0&0&\frac{\sin b}{b}&-1\end{bmatrix},\ B_{21}=\begin{bmatrix}1+a^2\\0\\0\\0\end{bmatrix},\ B_{23}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\ B_{22}=B_{21},\ B_{24}=B_{23}$$

$$B_{1i}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\ D_{1i}=0.1,\ D_{2i}=0.3,\ C_{11}=\begin{bmatrix}1&0&1&0\end{bmatrix},\ C_{12}=\begin{bmatrix}1&0&\frac{\sin b}{b}&0\end{bmatrix},$$

$$C_{13}=C_{11},\ C_{14}=C_{12},\ C_{21}=\begin{bmatrix}1&0&1&0\end{bmatrix},\ C_{22}=\begin{bmatrix}1&0&\frac{\sin b}{b}&0\end{bmatrix},\ C_{23}=C_{21},\ C_{24}=C_{22}$$

where the premise membership functions are taken the same as those used in [27]:

$$\left.\begin{aligned}
M_1^1(x_1) &= \frac{x_1^2}{a^2}\\
M_1^2(x_1) &= 1-M_1^1(x_1)\\
M_2^1(x_3) &= \begin{cases}\frac{b\sin x_3-x_3\sin b}{x_3(b-\sin b)} & x_3\neq 0\\ 1 & x_3=0\end{cases}\\
M_2^2(x_3) &= 1-M_2^1(x_1)
\end{aligned}\right\}$$

For comparison, we assume $M_i\in\mathbb{R}^{n\times n}$ and $M_{zi}\in\mathbb{R}^{n_z\times n_z}$ are identity matrices, $N_{B1i}=N_{D1i}=N_{D2i}=0$ and

$$N_{Ai}=\begin{bmatrix}0&0&0.1&0\\0&0&0&0\\0&0.1&0&0\\0&0&0.1&0\end{bmatrix},\ N_{B2i}=\begin{bmatrix}0.1\\0\\0\\0\end{bmatrix}^T,\ N_{C1i}=\begin{bmatrix}0\\0.1\\0\\0\end{bmatrix}^T$$

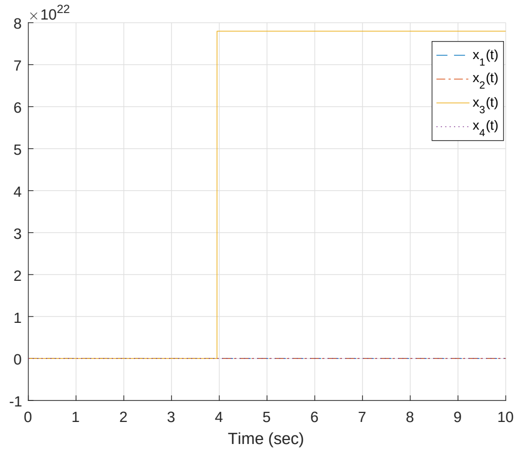

Fig. 1 Trajectories of the the open-loop system of (54).

Table 1 H_∞ attenuation bounds of Example (1), compared with [33]

Methods \ $\bar{u}$	1	2	3	4
[33], α_{max}	Infeasible	0.72	1.0576	1.3397
[33], γ	Infeasible	1.2965	0.8624	0.6517
Theorem 2, α_{max}	0.43	0.93	1.35	1.62
Theorem 2, γ	1.6131	0.8862	0.6599	0.4137

Fig. 1 shows the trajectories of the nonlinear system with $u(t) = 0$. The nonlinear system is unstable.

For $a = 1$, $b = 0.1$, and $\mathscr{X}_N = \{[1\ 0\ 0\ 0]^T\}$, the optimization problem of Theorem 2 will be solved for different saturation levels. The SOF H_∞ controllers is designed such that γ is minimized, and the domain of attraction is maximized. The obtained minimum values of γ and the maximum values of α are shown in Table 1. From Table 1, it is clear that increasing the value of saturation level decreases the level of guaranteed performance γ and the stability region expands. We can also see that the obtained attenuation level γ_{min} is smaller than those in [33], providing better performance. Moreover, in comparison with the results in [33], the SOF H_∞ control design is proposed without requiring equality constraints, which could make the numerical optimization more involved.

Some simulations are now presented: the solution of Theorem 2, with the saturation level $\bar{u} = 2$, and $\xi = 10^{-3}$, gives $\alpha_{max} = 0.93$, $\gamma_{min} = 0.8862$ and the following results:

$$F_1 = -1.9301,\ F_2 = -1.9309,\ F_3 = -4.3995,\ F_4 = -4.3745,$$
$$H_1 = -1.7193,\ H_2 = -1.7204,\ H_3 = -4.3995,\ H_4 = -4.3768.$$

Figures 2-5 show comparative evolution of the trajectories for the closed-loop nonlinear system (54) which are compared with those corresponding to the SOF with actuator saturation proposed in [33]. Initial conditions for the simulations are $x(0) =$

$[0.72\ \ 0\ \ 0\ \ 0]^T$ *and the external disturbance is* $\omega(t) = 0$*. The saturated control signal is displayed in Figure 6.*

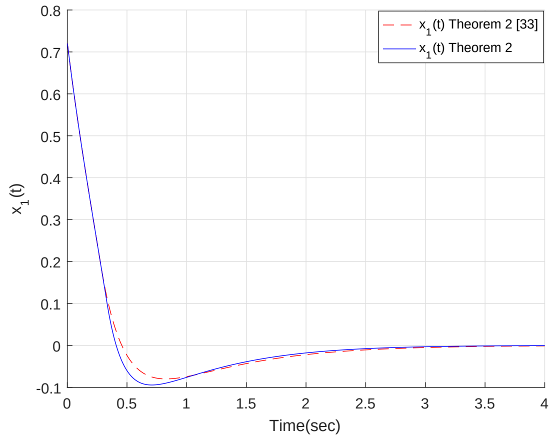

Fig. 2 Response of $x_1(t)$ for the nonlinear system (54) with the designed controller.

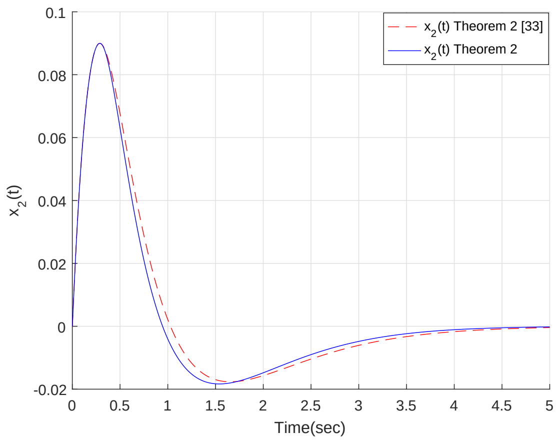

Fig. 3 Response of $x_2(t)$ for the nonlinear system (54) with the designed controller.

For $a = 5$ *and* $b = 4$*, there is no feasible solution using [33], but Theorem 2 provides a feasible solution: with* $\bar{u} = 2$, $\xi = 10^{-3}$*, we obtain* $\alpha_{max} = 0.83$, $\gamma_{min} = 1.1219$ *and the following gain matrices:*

$$F_1 = -1.8928,\ \ F_2 = -2.8531,\ \ F_3 = -4.3099,\ \ F_4 = -8.3440,$$
$$H_1 = -1.8913,\ \ H_2 = -2.8385,\ \ H_3 = -4.3102,\ \ H_4 = -8.3432.$$

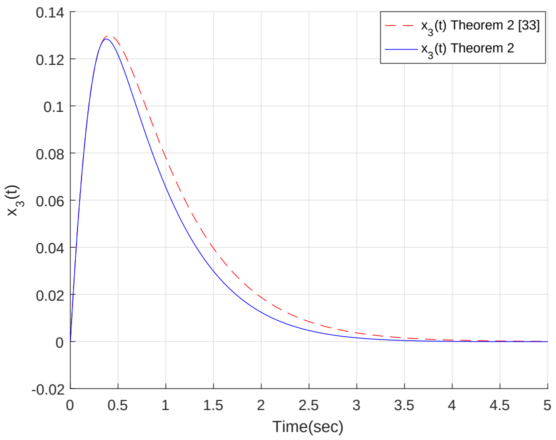

Fig. 4 Response of $x_3(t)$ for the nonlinear system (54) with the designed controller.

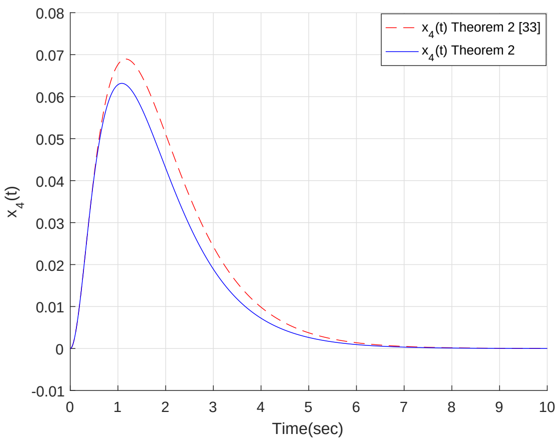

Fig. 5 Response of $x_4(t)$ for the nonlinear system (54) with the designed controller.

Some simulations are presented for the closed-loop system from the initial condition $x(0) = [0.5,\ -0.5,\ 0,\ 0.7]^T$ *and* $\omega(t) = 0$*: the states evolution are shown in Fig 7, whereas the saturated control inputs* $sat(u(t)))$ *and the (unsaturated) control inputs* $u(t)$ *are shown in Fig.8. The plot of ratio is shown in Fig. 9 under the initial condition* $x(0) = [0 \quad 0 \quad 0 \quad 0]^T$ *when* $\omega(t) = 0.25cos(10t)$*. It is clear that this ratio tends to a constant value, which is less than the prescribed value of* $\gamma = 1.1219$*.*
From these simulation results, we can see that the designed robust SOF H_∞ *presented in this paper is effective.*

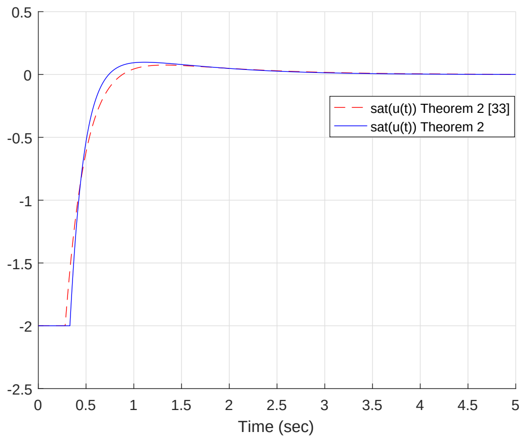

Fig. 6 The saturated input $sat(u(t))$ for the nonlinear system (54) with the designed controller.

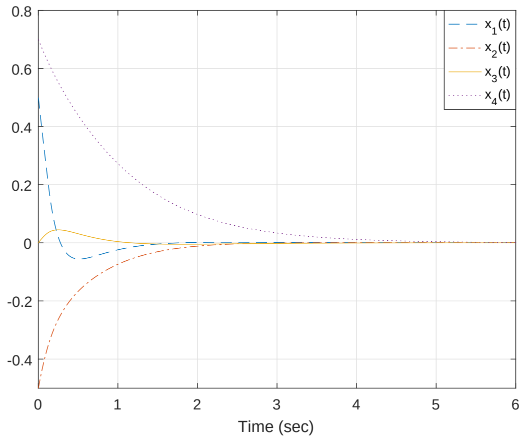

Fig. 7 Trajectories of the nonlinear system of (54).

Example 2 *In this example, we consider the control of a Permanent Magnet Synchronous Motor (PMSM) [33] with the following nonlinear model:*

$$\begin{cases} \dot{i}_d(t) = -\frac{R}{L} i_d(t) + n_p i_q(t) W(t) + \nu_d(t) \\ \dot{i}_q(t) = -\frac{R}{L} i_d(t) + n_p i_d(t) W(t) - \frac{\Psi}{L} W(t) + \nu_q(t) \\ \dot{W}(t) = \frac{\Psi}{L} i_q(t) - \frac{\tau}{J} W(t) + 2\omega(t) \end{cases} \tag{55}$$

where $i_q(t)$ and $i_d(t)$, $v_q(t)$ and $v_d(t)$ are the quadrature and direct input currents and voltages, respectively, $W(t)$ is the motor angular velocity, n_p is the number of pole-pairs, Ψ is the permanent-magnet flux, L is the direct quadrature-axis stator

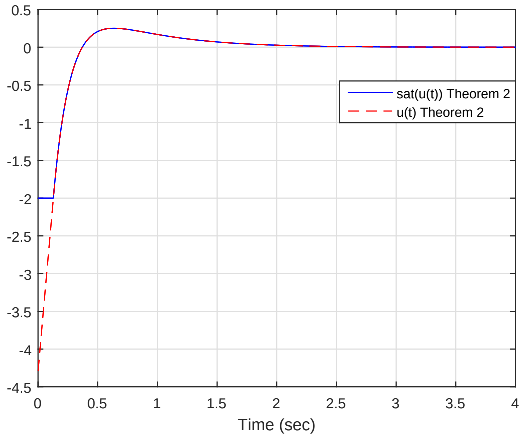

Fig. 8 The system control inputs (unsaturated and saturated) inputs for the nonlinear system of (54).

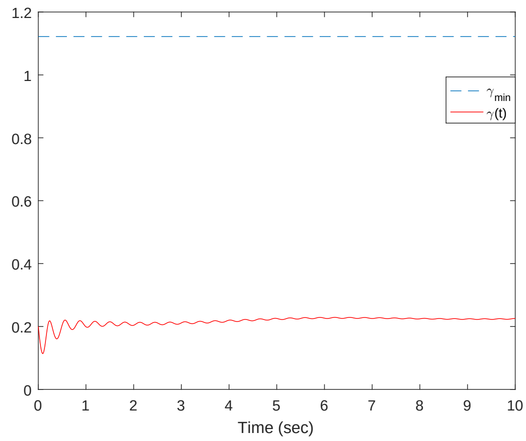

Fig. 9 The ratio $\gamma(t) = \sqrt{\frac{z^T(t)z(t)}{\omega^T(t)\omega(t)}}$ for the nonlinear system (54).

inductance, R is the stator winding resistance, J is the moment of inertia and τ is the viscous damping coefficient.

In the nominal case the controlled outputs are as follows:

$$\begin{cases} z_1(t) = 2(1+\rho)i_d(t) + 4i_q(t) + 4W(t) \\ \qquad + i_q(t)W(t) - W(t)^2 + 0.1\omega(t) \\ z_2(t) = W(t) \end{cases}$$

with ρ an uncertain parameter, in this paper we assume $\rho = 0$

Table 2 H_∞ attenuation bounds of Example (2), compared with [33]

	[33]	Theorem 2
α_{max}	1.5	1.85
γ	0.5	0.2760

As the nonlinear terms satisfy the conditions $W(t) \in [\theta_1,\ \theta_2]$, a TS fuzzy model has been proposed in [33] using a two-rule TS fuzzy model with the following parameters:

$$A_1 = \begin{bmatrix} -\frac{R}{L} & n_p\theta_1 & 0 \\ -n_p\theta_1 & -\frac{R}{L} & -\frac{\Psi}{L} \\ 0 & \frac{\Psi}{L} & -\frac{\tau}{J} \end{bmatrix},\ A_2 = \begin{bmatrix} -\frac{R}{L} & n_p\theta_2 & 0 \\ -n_p\theta_2 & -\frac{R}{L} & -\frac{\Psi}{L} \\ 0 & \frac{\Psi}{L} & -\frac{\tau}{J} \end{bmatrix},\ B_{21} = B_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},\ C_{11} = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}, C_{12} = \begin{bmatrix} 2 & 5 & 3 \\ 0 & 0 & 1 \end{bmatrix},\ D_{11} = D_{12} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$
$$D_{21} = D_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This result is obtained as M_{zi} is the identity matrix, M_i, N_{Ai}, N_{B1i}, N_{B2i}, N_{D1i} and N_{D2i} are zero matrices and

$$N_{C11} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The membership functions are the following:

$$\eta_1(W(t)) = \frac{\theta_2 - W(t)}{\theta_2 - \theta_1}, \qquad \eta_2(W(t)) = 1 - \eta_1(W(t)) \tag{56}$$

$x(t) = [i_d^T(t),\ i_q^T(t),\ W^T(t)]^T$ *and* $u(t) = [v_d^T(t),\ v_q^T(t)]^T$.
If $i_q(t)$ and $W(t)$ are the measurable variables, and the measurable outputs are the following:

$$y_1(t) = i_q(t)W(t), \qquad y_2(t) = W(t)$$

then

$$C_{21} = \begin{bmatrix} 0 & \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\ C_{22} = \begin{bmatrix} 0 & \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The following parameters have been used in the design and simulations: $n_p = 1$, $\Psi = 0.031 Nm/A$, $L = 0.01425 H$, $R = 0.9 \Omega$, $J = 4.5 \times 10^{-5} kgm^2$, $\tau = 0.0162 N/rad/s$. For comparison with previous results in [33], we choose the same data ($\theta_1 = -1$, $\theta_2 = 1$, and $N = I$) and the ellipsoid case. Solving the optimization problem of Theorem 2, Table 2 shows the results comparing with [33] when $\xi = 0.01$. From this Table 2, it can be seen that, by using the method in this paper, the values of γ and α can be greatly improved, thus providing better attenuation of disturbances.
The gain matrices corresponding to these results are:

$$F_1 = \begin{bmatrix} -5.3836 & -11.5986 \\ 6.4850 & -13.8292 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -3.9980 & -6.2608 \\ -32.4893 & -19.5988 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} -0.0029 & 0.0133 \\ -0.0066 & 0.0308 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} -0.2135 & -0.0281 \\ -0.4871 & -0.0638 \end{bmatrix}.$$

Some simulation results are shown in Figs. 10-12, from the initial state $x(0) = [1,\ -1,\ -0.5]^T$ *and* $\omega(t) = 0$. *Fig.10 presents the state responses of the closed-loop system* (17) *using the designed* H_∞ *controller, with saturations defined by* $sat(v_d(t)) = 1$, $sat(v_q(t)) = 2$. *Figs. 11-12 illustrate the evolution of the direct and quadrature input voltages* $\nu_d(t)$ *and* $\nu_q(t)$, *respectively. According to the simulation results, the proposed method stabilizes the permanent magnetic synchronous motor. This shows that the proposed method is effective to control practical systems with guaranteed attenuation levels.*

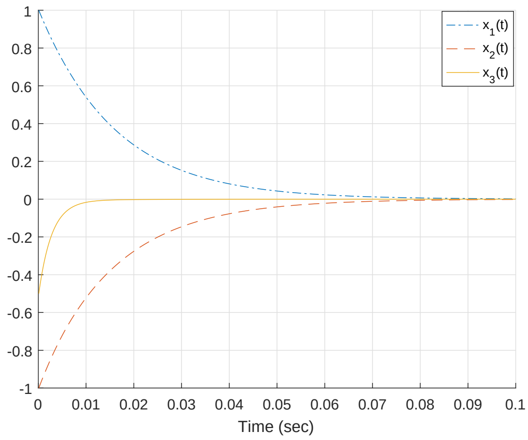

Fig. 10 Trajectories for the PMSM Example (55).

5 CONCLUSIONS

This paper has focused on designing Static Output Feedback controllers with guaranteed attenuation levels, for a class of nonlinear systems with input saturation. More precisely, a new method has been established for SOF H_∞ controller design where the controller gains can be obtain from LMI terms, and the resulting closed-loop system is asymptotically stable satisfying a prescribed level of H_∞ performance. The proposed approach is applicable for systems without disturbances in the measured output. On the other hand, the output matrices are not required to be full row rank.

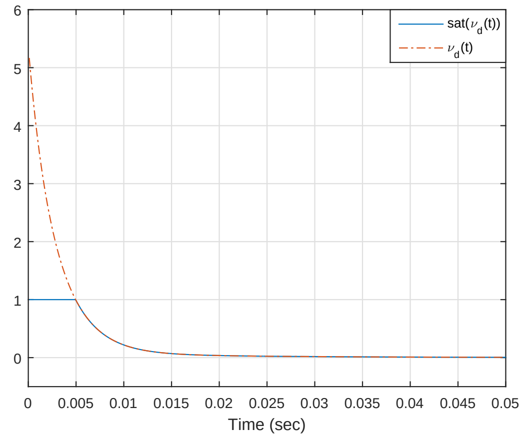

Fig. 11 Trajectory of direct input voltage $v_d(t)$ of Example (55).

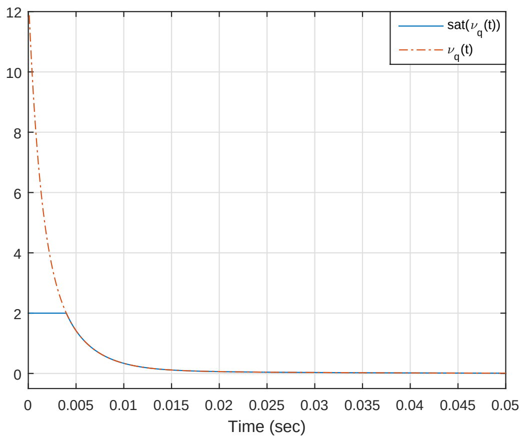

Fig. 12 Trajectory of quadrature input voltage $v_q(t)$ of Example (55).

In particular, it has been proved that the new proposed conditions are more relaxed than the existing ones with equality constraints between output matrix and Lyapunov matrix. It should be pointed out that compared with [33], Theorem 1 increases the number of LMI from $(m+r+2^m\ \frac{r^2}{2}(r+1))$ to $(\frac{r}{2}(r+1)m+2^m\ \frac{r^2}{2}(r+1))$ while increasing the size of each LMI. With the increases of IF$-$then rules the merit of the paper will be more apparent. Some examples are presented to demonstrate the validity and effectiveness of proposed approach. This result will be extended to finite frequency H_∞ control for a class of fuzzy delayed systems with input saturation systems in future research.

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