International Journal of Ecological Economics and Statistics;
Volume. 39, Issue No. 1; Year 2018; Int. J. Ecol. Econ. Stat.;
ISSN 0973-1385 (Print), ISSN 0973-7537 (Online)

Congestion control of data network by using anti-windup approach

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ABSTRACT

An Active Queue Management (AQM) scheme is design to control congestion in data networks, which includes anti-windup to deal with control signal saturation. More precisely, a methodology is proposed to design advanced AQM systems capable of regulating queue size even in the presence of significant disturbances. Hence, we first provide sufficient conditions for stabilization for the equivalent class of systems, which are derived in terms of LMI: this makes possible to derive optimization solutions that ensure performance and stability for a large domain of initial conditions. This approach is validated with a numerical example that illustrates the methodology, and the improvements with respect to previous congestion control solutions.

Keywords: AQM, congestion control, anti-windup, signal saturation, stabilization.

Mathematics Subject Classification: 90B18, 34H05, 41A40, 93D15.
Journal of Economic literature (JEL) classification: C02, C60, C62, C63.

1 Introduction

Novel AQM algorithms are already playing a key role in data networks to cope with the increasing user demands: Voice IP (VoIP) and video streaming are already pushing current data networks to the congestion limits, as packet size and session duration vary significantly.

A significant research is being devoted to develop more efficient AQM techniques: The simplest is Drop Tail, that drops packets arriving at a router when its buffer is full, which leads to performance degradation due to excessive time-outs and restarts (Hollot, Misra, Towsley and Gong, 2002). As an improvement, the Random Early Detection (RED) (Floyd and Jacobson, 1993; Misra, Gong and Towsley, 2000) randomly drops/marks packets arriving at the router before it is full to avoid busty traffics on the feedback signal (Hollot et al., 2002). However, the low-pass filter used, which limits the closed-loop bandwidth (Hollot et al., 2002). Consequently, variants of RED have been proposed for better congestion control: for example, (Ott, Lakshman and Wong, 1999; Wang, Li, Hou, Sohraby and Lin, 2004). Random Exponential Marking (REM) (Athuraliya, Low, Li and Yin, 2001) uses both the queue length and the input rate as a congestion indication. In parallel, a fluid model of TCP dynamical behavior was derived in (Misra et al., 2000), which makes possible to design controllers using traditional Control Theory approaches, such as PI (Hollot, Misra, Towsley and Gong, 2001), PD (Sun, Chen, Ko, Chan and Zukerman, 2003) or PID (Yanfie, Fengyuan and Chuang, 2003): this approach is augmented here do deal with saturations in the control signals. Most of these techniques do not take into account input saturation, but in AQM the control action is the discarding probability, frequently saturates, as it is a nonnegative number smaller than one. Accordingly, any practical AQM should take into account this, as input saturations deteriorate the performance and create instabilities.

Thus, this paper concentrates on the augmentation of congestion controllers with anti-windup schemes, to deal with saturation due to inherent variations in traffic. Some anti-windup techniques have been proposed in (Bender, 2013; El Fezazi, El Haoussi, Tissir and Tadeo, 2015; El Fezazi, El Haoussi, Tissir, Husain and Zakaria, 2016; El Fezazi, Lamrabet, El Haoussi, Tissir, Alvarez and Tadeo, 2016; Tarbouriech, Da Silva and Garcia, 2004; Tissir, 2014), where the emphasis is on the transient performance caused by the saturation. The approach here is inspired by these previous results, but system discretization due to periodic sampling is explicitly taken into account. Moreover, as stability during saturation is a central issue, the anti-windup compensation is designed to enlarge the domain of initial conditions that mathematically ensure that the closed-loop system trajectories remain bounded. The mathematical objective is then to design a controller capable of regulating the queue size at the router around a desired value guaranteeing the stability, explicitly taking into account link capacity disturbances and time-varying delay, using the discrete-time equivalent of the linearized TCP congestion window model, and incorporating an anti-windup compensator. The proposed synthesis methodology is based on Lyapunov functionals and LMI conditions that guarantee closed-loop stability of the TCP/AQM system and minimization of the $\mathcal{L}_2-$gain of the disturbance to the system output. Then, the design of anti-windup controller is performed using LMIs.

It must be pointed out that the results are developed starting from a discrete-time state space model of TCP/AQM based on the dynamic models developed by (Misra et al., 2000) of the Transmission Control Protocol (TCP), which is derived using the assumptions that the data traffic is equivalently represented by a fluid flow and the packet losses can be described by a Poisson process. These assumptions have been shown to be valid in practice, as TCP has been designed to be fair, and the packet losses are multiple.

At the end of the paper, some simulation results will be presented to show the effectiveness of the proposed method, and a comparison is made with other recent methods.

2 Problem Formulation

This section presents the problem and discusses some AQM models.

2.1 Dynamic Model of an AQM Router

In this paper, the network in Figure 1 is considered, with multiple server machines connect to multiple client machines in a computer network. The network consists of n senders, n receivers, and 1 bottleneck routers, which transports packets from senders to receivers. Large-scale networks can be simplified as in Figure 1 in case of designing congestion controllers, where one router is bottleneck in the network, running TCP flows.

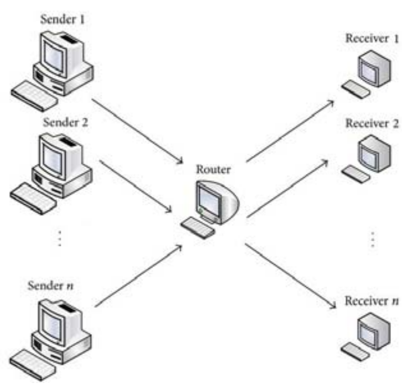

Figure 1: Network topology used.

A model for this network was developed using fluid-flow and stochastic differential equation analysis in (Misra et al., 2000)). Assuming that the AQM scheme implemented at the router marks packets using Explicit Congestion Notification (ECN) (Yan, Gao and Ozbay, 2005) to inform the TCP sources of impending congestion, and ignoring the TCP timeout mechanism, the model that relates average values of the network variables is described by the following coupled, nonlinear differential equations:

$$\begin{aligned} \dot{W}(t) &= \frac{1}{RTT(t)} - \frac{W(t)W(t-RTT(t))}{2RTT(t-RTT(t))}p(t-RTT(t)) \\ \dot{q}(t) &= -C(t) + \frac{N(t)}{RTT(t)}W(t) \end{aligned} \tag{2.1}$$

where

$W(t)$ is the average TCP window size (packets);
$q(t)$ is the average queue length (packets);
$RTT(t)$ is the round trip time $= \frac{q(t)}{C(t)} + T_p$ (secs);
C is the link capacity (packets/secs);

T_p is the propagation delay (secs);
N is the number of sessions;
$p \in [0 \ \ 1]$ is the probability of packet marking/dropping.

As explained by (Misra et al., 2000), the first differential equation in (2.1) describes the TCP window control dynamic and the second equation models the bottleneck queue length, from an accumulated difference between packet arrival rate and link capacity. The congestion window size $W(t)$ increases by one every RTT when no congestion is detected, and is halved when congestion is detected.

To derive the anti-windup mechanism (2.1) is linearized around the equilibrium point (that depends on the nominal probability of packet marks). This nominal probability p_0 fulfills $p_0 = \frac{2N^2}{(q_0+T_pC_0)^2}$, and at the equilibrium point $\mathfrak{I} = (W_0 = \frac{RTTC_0}{N}, q_0 = C_0(RTT - T_p), p_0 = \frac{2}{W_0^2})$. In the TCP/AQM network, each value of $\mathfrak{I}$ is positive and the probability p_0 is less than or equal to 1. If we define $\delta\mathfrak{C} = \mathfrak{C} - \mathfrak{C}_0$ with $\mathfrak{C} = W, q, p, C$, then, we can expressed the linearized version of (2.1) as follows

$$\begin{aligned}
\delta\dot{W}(t) &= \frac{-N}{RTT^2C_0}\Big(\delta W(t) + \delta W(t - RTT(t))\Big) - \frac{1}{RTT^2C_0}\Big(\delta q(t) + \delta q(t - RTT(t))\Big) \\
&\quad - \frac{RTTC_0^2}{2N^2}\delta p(t - RTT(t)) + \frac{RTT - T_p}{RTT^2C_0}\Big(\delta C(t) + \delta C(t - RTT(t))\Big) \\
\delta\dot{q}(t) &= \frac{N}{RTT}\delta W(t) - \frac{1}{RTT}\delta q(t) - \frac{T_p}{RTT}\delta C(t) \\
RTT(t) &= \frac{\delta q(t)}{C_0} + RTT
\end{aligned} \tag{2.2}$$

2.2 TCP/AQM System Modelling in state space

Rewriting (2.2) in state space form yields

$$\begin{aligned}
\dot{x}(t) &= A_0x(t) + A_1x(t - \tau(t)) + B_0u(t - \tau(t)) + B_1w(t) \\
y(t) &= C_{y_c}x(t) \\
z(t) &= C_{z_c}x(t)
\end{aligned} \tag{2.3}$$

in which

$$x(t) = \begin{bmatrix} \delta W(t) \\ \delta q(t) \end{bmatrix},\ A_0 = \begin{bmatrix} \frac{-N}{RTT^2C_0} & \frac{-1}{RTT^2C_0} \\ \frac{N}{RTT} & \frac{-1}{RTT} \end{bmatrix},\ A_1 = \begin{bmatrix} \frac{-N}{RTT^2C_0} & \frac{-1}{RTT^2C_0} \\ 0 & 0 \end{bmatrix},$$
$$B_0 = \begin{bmatrix} \frac{-RTTC_0^2}{2N^2} \\ 0 \end{bmatrix},\ B_1 = \begin{bmatrix} \frac{RTT-T_p}{RTT^2C_0} & \frac{RTT-T_p}{RTT^2C_0} \\ \frac{-T_p}{RTT} & 0 \end{bmatrix},\ w(t) = \begin{bmatrix} \delta C(t) \\ \delta C(t - RTT(t)) \end{bmatrix},$$
$$u(t) = \delta p(t),\ C_{y_c} = \begin{bmatrix} 0 & 1 \end{bmatrix},\ y(t) = \delta q(t),\ C_{z_c} = \begin{bmatrix} 0 & \frac{1}{C_0} \end{bmatrix},\ z(t) = RTT(t) - RTT$$

where the states variables are then the congestion window and queue sizes, and the input represents the marking probability.

As the AQM controller is by nature a discrete-time system, a discretized model of (2.3) assuming periodic sampling is

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-d(k)) + Bu(k-d(k)) + B_w w(k) \\ y(k) &= C_y x(k) \\ z(k) &= C_z x(k) \end{aligned} \tag{2.4}$$

where $A = e^{A_0 T}$, $A_d = \int_0^T e^{A_0 s} A_1 ds$, $B = \int_0^T e^{A_0 s} B_0 ds$, $B_w = \int_0^T e^{A_0 s} B_1 ds$, $C_y = C_{y_c}$, and $C_z = C_{z_c}$ and $d(k)$ is a positive integer representing the time delay of the system (that by nature is time-varying) and satisfies $d_m \leq d(k) \leq d_M$ where d_m and d_M are known positive finite integers.

The disturbance vector $w(k)$ is assumed to be limited in energy, that is, $w(k) \in \mathcal{L}_2$. Hence for some scalar δ, the bounds on the disturbance $w(k)$ are the following:

$$\|w(k)\|_2^2 = \sum_{k=0}^{\infty} w^T(k) w(k) \leq \delta^{-1} < \infty \tag{2.5}$$

An anti-windup compensator is going to be proposed that shows graceful performance degradation of the overall system in the presence of saturation. To this end, we reformulate the problem into a state tracking problem, since the output is the combination of internal states.

In order to regulate (2.4) around a desired working point, we assume an controller, stabilizing in absence of control bounds of the form:

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c y(k) \\ y_c(k) &= C_c x_c(k) + D_c y(k) \end{aligned} \tag{2.6}$$

As a consequence of the saturation, the interconnection with (2.4) is given by $u(k) = sat(y_c(k))$, where $sat(y_c(k)) = sign(y_c(k)) min\{|y_c(k)|, u_0\}$.

The following anti-windup compensator is proposed to mitigate the performance degradation induced by the saturation, ensuring asymptotic stability of the closed-loop system:

$$\begin{aligned} x_a(k+1) &= A_a x_a(k) + B_a \psi(y_c(k)) \\ y_a(k) &= C_a x_a(k) + D_a \psi(y_c(k)) \end{aligned} \tag{2.7}$$

Note that, $\psi(y_c(k))$ corresponds to a decentralized dead-zone nonlinearity:

$$\psi(y_c(k)) = y_c(k) - sat(y_c(k)) \tag{2.8}$$

Considering the anti-windup compensator, the controller (2.6) is rewritten as follows

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c u_c(k) + y_a(k) \\ y_c(k) &= C_c x_c(k) + D_c u_c(k) \end{aligned} \tag{2.9}$$

It follows that the augmented system corresponding to the closed-loop system (2.4)-(2.6)-(2.7) can be represented by the following equation

$$\begin{aligned} \xi(k+1) &= \mathbb{A}\xi(k) + \mathbb{A}_d \xi(k-d(k)) - \mathbb{B}\psi(\mathbb{K}\xi(k-d(k))) + \mathbb{B}_d \psi(\mathbb{K}\xi(k)) + \mathbb{B}_w w(k) \\ z(k) &= \mathbb{C}_z \xi(k) \end{aligned} \tag{2.10}$$

where we define the following matrices:

$$\xi(k)=\begin{bmatrix} x(k)\\ x_c(k)\\ x_a(k)\end{bmatrix},\ \mathbb{A}=\begin{bmatrix} A & 0 & 0\\ B_cC_y & A_c & C_a\\ 0 & 0 & A_a\end{bmatrix},\ \mathbb{A}_d=\begin{bmatrix} A_d+BD_cC_y & BC_c & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\mathbb{B}=\begin{bmatrix} B\\ 0\\ 0\end{bmatrix},\ \mathbb{B}_d=\begin{bmatrix} 0\\ D_a\\ B_a\end{bmatrix},\ \mathbb{B}_w=\begin{bmatrix} B_w\\ 0\\ 0\end{bmatrix},\ \mathbb{K}=\begin{bmatrix} D_cC_y & C_c & 0\end{bmatrix},\ \mathbb{C}_z=\begin{bmatrix} C_z & 0\end{bmatrix}$$

As mentioned before, we aim to ensure a large set of initial states. For this, an estimate of domain attraction will be used:

$$\Xi=\Big\{\phi_i(k),\ -d_M\le k\le 0:\max\|\phi_i(k)\|\le\kappa\Big\},\ i=1,\dots,m$$

In mathematical terms, we are interested in the synthesis of the anti-windup compensator (2.7) (i.e. in computing matrices A_a, B_a, C_a, and D_a), which ensures that the closed-loop trajectories of the system remain bounded for any disturbance satisfying (2.5). Moreover, it should ensure an upper bound for the $\mathcal{L}_2-$gain between the disturbance $w(k)$ and the regulated output $z(k)$ defined as follows (Chaibi, Tissir, Hmamed and Idrissi, 2013; El Fezazi, Tissir, El Haoussi, Alvarez and Tadeo, 2017; El Haoussi and Tissir, 2007; Tissir, 2009)

$$\|\mathfrak{F}_{zw}\|_\infty^2=\frac{\|z(k)\|_2^2}{\|w(k)\|_2^2}=\frac{\sum_{k=0}^{\infty}z^T(k)z(k)}{\sum_{k=0}^{\infty}w^T(k)w(k)}<\gamma \tag{2.11}$$

where $\mathfrak{F}_{zw}$ is the closed-loop transfer function from $w(k)$ to $z(k)$. This ratio would be minimized for a given set of expected network parameters.

3 Main Results

We first derive some delay-dependent conditions for stability of the TCP/AQM system with the proposed anti-windup compensators. These conditions are given in terms of the existence of solutions of some LMIs, based on Lyapunov functionals. This method provides a computable criteria to check the stability for time-varying delays in the general case of dynamic system. For the developments below the following is required:
For a matrix G we define the polyhedral set

$$\mathcal{S}=\Big\{\xi(k)\in\Re^n;|(\mathbb{K}_{(i)}-G_{(i)})\xi(k)|\le u_{0_{(i)}}\Big\}$$

The following lemma will be used later in this paper

Lemma 3.1. *(Tarbouriech et al., 2004) If $\xi(k)\in\mathcal{S}$, then the following relation is verified for any diagonal positive matrix T*

$$\psi^T(\mathbb{K}\xi(k))T\Big[\psi(\mathbb{K}\xi(k))-G\xi(k)\Big]\le 0$$

Remark 3.1. Lemma 3.1 allows a direct formulation of conditions in LMI form; moreover, the obtained anti-windup synthesis conditions can be applied to stable or unstable systems, being less conservative when the open-loop system is unstable.

Finally, for a positive scalar μ the trajectories of the system must not leave the set

$$\varepsilon(P,\mu)=\Big\{\xi(k)\in\Re^n;\ \xi^T(k)P\xi(k)\le\mu^{-1}\Big\}$$

3.1 Stability Results

Theorem 3.2. *If there exists positive definite symmetric matrices $\widehat{P}$, $\widehat{Q}$, $\widehat{R}$, and appropriately sized matrices $\widehat{T}_1$, $\widehat{T}_2$, $\widehat{G}_1$, $\widehat{G}_2$, $\widehat{Y}_1$, $\widehat{Y}_2$, $\widehat{Y}_3$, $\widehat{Y}_4$, $\widehat{Y}_5$, $\widehat{Y}_6$ such that the LMIs (3.1)-(3.3) are verified*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} & \Pi_{26} & \Pi_{27} & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & \Pi_{36} & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & \Pi_{47} & 0 \\ * & * & * & * & \Pi_{55} & 0 & \Pi_{57} & 0 \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix} < 0, \ \widehat{Q} < \widehat{R}, \tag{3.1}$$

$$\begin{bmatrix} \widehat{P} & \widehat{P}\mathbb{K}_{(i)}^T - \widehat{G}_{1_{(i)}}^T \\ * & \mu u_{0_{(i)}}^2 \end{bmatrix} \geq 0, \begin{bmatrix} \widehat{P} & \widehat{P}\mathbb{K}_{(i)}^T - \widehat{G}_{2_{(i)}}^T \\ * & \mu u_{0_{(i)}}^2 \end{bmatrix} \geq 0, \tag{3.2}$$

$$\mu - \delta \leq 0, \tag{3.3}$$

where

$$\begin{aligned} \Pi_{11} &= -\widehat{P} + \widehat{Q} + (d_M - d_m)\widehat{R} - \widehat{Y}_1 - \widehat{Y}_1^T, \ \Pi_{12} = \widehat{Y}_1 - \widehat{Y}_2^T, \ \Pi_{22} = -\widehat{Q} + \widehat{Y}_2 + \widehat{Y}_2^T, \\ \Pi_{13} &= \widehat{Y}_1 - \widehat{Y}_3^T, \ \Pi_{23} = \widehat{Y}_2 + \widehat{Y}_3^T, \ \Pi_{33} = \widehat{Y}_3 + \widehat{Y}_3^T, \ \Pi_{14} = -\widehat{Y}_4^T, \Pi_{24} = \widehat{G}_1^T + \widehat{Y}_4^T, \\ \Pi_{34} &= \widehat{Y}_4^T, \ \Pi_{44} = -2\widehat{T}_1, \ \Pi_{15} = \widehat{G}_2^T - \widehat{Y}_5^T, \ \Pi_{25} = \widehat{Y}_5^T, \ \Pi_{35} = \widehat{Y}_5^T, \ \Pi_{55} = -2\widehat{T}_2, \\ \Pi_{16} &= -\widehat{Y}_6^T, \ \Pi_{26} = \widehat{Y}_6^T, \ \Pi_{36} = \widehat{Y}_6^T, \ \Pi_{66} = -I, \ \Pi_{17} = \widehat{P}\mathbb{A}^T, \ \Pi_{27} = \widehat{P}\mathbb{A}_d^T, \\ \Pi_{47} &= -\widehat{T}_1\mathbb{B}^T, \ \Pi_{57} = \widehat{T}_2\mathbb{B}_d^T, \ \Pi_{67} = \mathbb{B}_w^T, \ \Pi_{77} = -\widehat{P}, \ \Pi_{18} = \widehat{P}\mathbb{C}_z^T, \ \Pi_{88} = -\gamma I \end{aligned}$$

Then, there exists an anti-windup compensator (2.7) which ensures that the trajectories of the system (2.10) converge asymptotically to the origin and are bounded for all initial conditions in the ball

$$\begin{aligned} \kappa^2 \ \leq \ & \left(\mu^{-1} - \delta^{-1}\right) \Big/ \Big\{ \overline{\lambda}(\widehat{P}^{-1}) + (d_M + d_m)\overline{\lambda}(\widehat{P}^{-1}\widehat{Q}\widehat{P}^{-1}) \\ & + \frac{(d_M - d_m + 1)(d_M + d_m)}{2}\overline{\lambda}(\widehat{P}^{-1}\widehat{R}\widehat{P}^{-1}) \Big\} \end{aligned} \tag{3.4}$$

with $\overline{\lambda}$ the maximal eigenvalue and $\kappa = \max \|\phi\|$.

Proof. Consider the following Lyapunov functional

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) \\ &= \xi^T(k)P\xi(k) + \sum_{l=k-d(k)}^{k-1} \xi^T(l)Q\xi(l) + \sum_{l=-d_M+2}^{-d_m+1} \sum_{m=k+l-1}^{k-1} \xi^T(m)R\xi(m) \end{aligned} \tag{3.5}$$

Computing the difference of the Lyapunov functional gives

$$\begin{aligned}
\Delta V_1(k) &= \xi^T(k+1)P\xi(k+1) - \xi^T(k)P\xi(k), && (3.6)\\
\Delta V_2(k) &= \sum_{l=k+1-d(k+1)}^{k} \xi^T(l)Q\xi(l) - \sum_{l=k-d(k)}^{k-1} \xi^T(l)Q\xi(l)\\
&= \xi^T(k)Q\xi(k) - \xi^T(k-d(k))Q\xi(k-d(k)) + \sum_{l=k+1-d_m}^{k-1} \xi^T(l)Q\xi(l)\\
&\quad - \sum_{l=k+1-d(k)}^{k-1} \xi^T(l)Q\xi(l) + \sum_{l=k+1-d(k+1)}^{k-d_m} \xi^T(l)Q\xi(l), && (3.7)\\
\Delta V_3(k) &= \sum_{l=-d_M+2}^{-d_m+1} \Big[\sum_{m=k+l}^{k} \xi^T(m)R\xi(m) - \sum_{m=k+l-1}^{k-1} \xi^T(m)R\xi(m) \Big]\\
&= (d_M - d_m)\xi^T(k)R\xi(k) - \sum_{l=k+1-d_M}^{k-d_m} \xi^T(l)R\xi(l) && (3.8)
\end{aligned}$$

As $\forall Q < R$, one can easily see that

$$-\sum_{l=k+1-d(k)}^{k-1} \xi^T(l)Q\xi(l) \le -\sum_{l=k+1-d_m}^{k-1} \xi^T(l)Q\xi(l), \tag{3.9}$$

$$\sum_{l=k+1-d(k+1)}^{k-d_m} \xi^T(l)Q\xi(l) \le \sum_{l=k+1-d_M}^{k-d_m} \xi^T(l)R\xi(l) \tag{3.10}$$

Then, from (2.10) and (3.6)-(3.10), it follows that

$$\begin{aligned}
\Delta V(k) \le\ & \Big[\mathbb{A}\xi(k) + \mathbb{A}_d\xi(k-d(k)) + \mathbb{B}_d\psi(\mathbb{K}\xi(k)) - \mathbb{B}\psi(\mathbb{K}\xi(k-d(k))) + \mathbb{B}_w w(k)\Big]^T P\Big[\mathbb{A}\xi(k)\\
&+\mathbb{A}_d\xi(k-d(k)) + \mathbb{B}_d\psi(\mathbb{K}\xi(k)) + \mathbb{B}_w w(k) - \mathbb{B}\psi(\mathbb{K}\xi(k-d(k)))\Big]\\
&+\xi^T(k)(-P + (d_M - d_m)R + Q)\xi(k) - \xi^T(k-d(k))Q\xi(k-d(k)) && (3.11)
\end{aligned}$$

Using the Newton-Leibniz formula, for any appropriately dimensioned matrices $Y_{1,\dots,6}$ the following holds:

$$\begin{aligned}
&\Big[\xi^T(k)Y_1 + \xi^T(k-d(k))Y_2 + \sum_{j=k-d(k)}^{k-1} y^T(j)Y_3 + \psi^T(\mathbb{K}\xi(k-d(k)))Y_4\\
&+\psi^T(\mathbb{K}\xi(k))Y_5 + w^T(k)Y_6\Big]\Big[-\xi(k) + \xi(k-d(k)) + \sum_{j=k-d(k)}^{k-1} y(j)\Big] = 0 && (3.12)
\end{aligned}$$

where $y(j) = \xi(j+1) - \xi(j)$.
Then, applying Lemma 3.1 and taking into account (3.11)-(3.12), the following inequality holds

$$\Delta V(k) - w^T(k)w(k) + \frac{1}{\gamma}z^T(k)z(k) \le \eta^T(k)(\Upsilon + L^T PL)\eta(k) \tag{3.13}$$

where

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} & \Upsilon_{15} & \Upsilon_{16} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \Upsilon_{25} & \Upsilon_{26} \\ * & * & \Upsilon_{33} & \Upsilon_{34} & \Upsilon_{35} & \Upsilon_{36} \\ * & * & * & \Upsilon_{44} & 0 & 0 \\ * & * & * & * & \Upsilon_{55} & 0 \\ * & * & * & * & * & \Upsilon_{66} \end{bmatrix}, \ L = \begin{bmatrix} \mathbb{A}^T \\ \mathbb{A}_d^T \\ 0 \\ -\mathbb{B}^T \\ \mathbb{B}_d^T \\ \mathbb{B}_w^T \end{bmatrix}^T, \ \eta(k) = \begin{bmatrix} \xi(k) \\ \xi(k-d(k)) \\ \sum_{j=k-d(k)}^{k-1} y(j) \\ \psi(\mathbb{K}\xi(k-d(k))) \\ \psi(\mathbb{K}\xi(k)) \\ w(k) \end{bmatrix},$$

and

$$\begin{aligned} \Upsilon_{11} &= -P + Q + (d_M - d_m)R - Y_1 - Y_1^T + \frac{1}{\gamma}\mathbb{C}_z^T\mathbb{C}_z, \ \Upsilon_{12} = Y_1 - Y_2^T, \\ \Upsilon_{22} &= -Q + Y_2 + Y_2^T, \ \Upsilon_{13} = Y_1 - Y_3^T, \ \Upsilon_{23} = Y_2 + Y_3^T, \ \Upsilon_{33} = Y_3 + Y_3^T, \\ \Upsilon_{14} &= -Y_4^T, \ \Upsilon_{24} = G_1^T T_1^T + Y_4^T, \ \Upsilon_{34} = Y_4^T, \ \Upsilon_{44} = -2T_1, \ \Upsilon_{15} = G_2^T T_2^T - Y_5^T, \\ \Upsilon_{25} &= \Upsilon_{35} = Y_5^T, \ \Upsilon_{55} = -2T_2, \ \Upsilon_{16} = -Y_6^T, \ \Upsilon_{26} = \Upsilon_{36} = Y_6^T, \ \Upsilon_{66} = -I \end{aligned}$$

Then, it is clear that if

$$\Upsilon + L^T P L < 0 \tag{3.14}$$

then

$$\Delta V(k) - w^T(k)w(k) + \frac{1}{\gamma} z^T(k)z(k) < 0 \tag{3.15}$$

Accordingly, the following condition is obtained by applying the Schur complement to (3.14)

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} & \Upsilon_{15} & \Upsilon_{16} & \Upsilon_{17} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \Upsilon_{25} & \Upsilon_{26} & \Upsilon_{27} \\ * & * & \Upsilon_{33} & \Upsilon_{34} & \Upsilon_{35} & \Upsilon_{36} & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 & \Upsilon_{47} \\ * & * & * & * & \Upsilon_{55} & 0 & \Upsilon_{57} \\ * & * & * & * & * & \Upsilon_{66} & \Upsilon_{67} \\ * & * & * & * & * & * & \Upsilon_{77} \end{bmatrix} < 0, \tag{3.16}$$

where

$$\Upsilon_{17} = \mathbb{A}^T P, \ \Upsilon_{27} = \mathbb{A}_d^T P, \ \Upsilon_{47} = -\mathbb{B}^T P, \ \Upsilon_{57} = \mathbb{B}_d^T P, \ \Upsilon_{67} = \mathbb{B}_w^T P, \ \Upsilon_{77} = -P$$

Pre- and post-multiplying (3.16) by $\Delta = diag\{P^{-1}, P^{-1}, P^{-1}, T_1^{-1}, T_2^{-1}, I, P^{-1}\}$, applying the Schur complement and taking the following changes of variables

$$\begin{aligned} &\widehat{P} = P^{-1}, \ \widehat{\Omega} = \widehat{P}\Omega\widehat{P}, \ \Omega = Q, R, Y_1, Y_2, Y_3, \ \widehat{\Lambda} = \Lambda\widehat{P}, \ \Lambda = G_1, G_2, Y_6, \\ &\widehat{T}_1 = T_1^{-1}, \ \widehat{T}_2 = T_2^{-1}, \ \widehat{Y}_4 = \widehat{T}_1 Y_4 \widehat{P}, \ \widehat{Y}_5 = \widehat{T}_2 Y_5 \widehat{P}. \end{aligned}$$

we obtain the inequality (3.1) of Theorem 3.2.

Since (3.1) holds, the condition (3.15) is satisfied. Now, summing up (3.15) from 0 to ∞ with respect to k yields

$$V(\infty) < V(0) + \sum_{k=0}^{\infty}\Big(w^T(k)w(k) - \frac{1}{\gamma}z^T(k)z(k)\Big) \tag{3.17}$$

Under the zero initial condition $V(0) = 0$ and by noting that $V(\infty) \geq 0$, we have (2.11) which implies that system (2.10) has its restricted $\mathcal{L}_2-$gain from $w(k)$ to $z(k)$ less than γ.
The LMIs in (3.1) and (3.2) ensure that the trajectories are contained inside the ellipsoid $\varepsilon(P, \mu)$, $\forall k$, once $\varepsilon(P, \mu) \subset \mathcal{S}$. This is verified by the following conditions

$$\begin{bmatrix} P & \mathbb{K}_{(i)}^T - G_{1_{(i)}}^T \\ * & \mu u_{0_{(i)}}^2 \end{bmatrix} \geq 0, \begin{bmatrix} P & \mathbb{K}_{(i)}^T - G_{2_{(i)}}^T \\ * & \mu u_{0_{(i)}}^2 \end{bmatrix} \geq 0$$

These matrices give the ellipsoidal inclusion LMIs (3.2) by pre- and post-multiplied by $\Delta^{'} = diag\{\widehat{P}, I\}$. Moreover, from the Lyapunov functional (3.5), it follows that

$$V(0) \leq \Big\{\overline{\lambda}(P) + (d_M + d_m)\overline{\lambda}(Q) + (d_M - d_m + 1)\frac{d_M + d_m}{2}\overline{\lambda}(R)\Big\}\|\phi\|^2 = \beta \tag{3.18}$$

Then, we have

$$\xi^T(k)P\xi(k) \leq V(k) \leq V(0) + \|w(k)\|_2^2 \leq \beta + \delta^{-1} \leq \mu^{-1}$$

Hence, for all k the trajectories of the system do not leave the set $\varepsilon(P, \mu)$, concluding the proof. □

We focus now on guaranteing the stability of the following system, which can be seen as a particular case of system (2.10).

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k - d(k)) + Bu(k) + B_w w(k) \\ y(k) &= C_y x(k) \\ z(k) &= C_z x(k) \end{aligned} \tag{3.19}$$

Then, the augmented system is given by

$$\begin{aligned} \xi(k+1) &= \mathbb{A}\xi(k) + \mathbb{A}_d\xi(k - d(k)) + (\mathbb{B}_d - \mathbb{B})\psi(\mathbb{K}\xi(k)) + \mathbb{B}_w w(k) \\ z(k) &= \mathbb{C}_z\xi(k) \end{aligned} \tag{3.20}$$

The following corollary gives a condition to stabilize system (3.19)

Corollary 3.3. *If there exists positive definite symmetric matrices $\widehat{P}$, $\widehat{Q}$, $\widehat{R}$, and appropriately sized matrices $\widehat{T}_2$, $\widehat{G}_2$, $\widehat{Y}_1$, $\widehat{Y}_2$, $\widehat{Y}_3$, $\widehat{Y}_5$, $\widehat{Y}_6$ such that the LMIs (3.21)-(3.22) are verified*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{25} & \Pi_{26} & \Pi_{27} & 0 \\ * & * & \Pi_{33} & \Pi_{35} & \Pi_{36} & 0 & 0 \\ * & * & * & \Pi_{55} & 0 & \Pi_{57} - \widehat{T}_2\mathbb{B}^T & 0 \\ * & * & * & * & \Pi_{66} & \Pi_{67} & 0 \\ * & * & * & * & * & \Pi_{77} & 0 \\ * & * & * & * & * & * & \Pi_{88} \end{bmatrix} < 0, \ \widehat{Q} < \widehat{R}, \tag{3.21}$$

$$\begin{bmatrix} \widehat{P} & \widehat{P}\mathbb{K}_{(i)}^T - \widehat{G}_{2_{(i)}}^T \\ * & \mu u_{0_{(i)}}^2 \end{bmatrix} \geq 0, \ \mu - \delta \leq 0 \tag{3.22}$$

Then, there exists an anti-windup compensator as defined in (2.7) which ensures that the trajectories of the system (3.20) converge asymptotically to the origin and are bounded for every initial condition in the following ball

$$\begin{aligned} \kappa^2 \ \leq \ & \Big(\mu^{-1} - \delta^{-1}\Big)\Big/\Big\{\overline{\lambda}(\widehat{P}^{-1}) + (d_M + d_m)\overline{\lambda}(\widehat{P}^{-1}\widehat{Q}\widehat{P}^{-1}) \\ & + \frac{(d_M - d_m + 1)(d_M + d_m)}{2}\overline{\lambda}(\widehat{P}^{-1}\widehat{R}\widehat{P}^{-1})\Big\} \end{aligned} \tag{3.23}$$

with $\kappa = \max\|\phi\|$.

Remark 3.2. In deriving Theorem 3.2, the slack variable $Y_{1,\ldots,6}$ were introduced in order to offer additional degrees of freedom for the optimization. It can be seen from the above Proof that $\Delta V(k)$ remains unaffected by equation (3.12), so these matrices lead to more flexible LMI conditions in (3.1), reducing possible conservatism in Theorem 3.2 and the subsequent results, as will be shown in the numerical examples.

3.2 Implementation Constraints

The control signal for the network is given by

$$u(k) = sat(\mathbb{K}\xi(k)) = sat\Big(\begin{bmatrix} \mathbb{K}_1 & \mathbb{K}_2 & \mathbb{K}_3 & \mathbb{K}_4 \end{bmatrix}\begin{bmatrix} \delta W(k) \\ \delta q(k) \\ x_c(k) \\ x_a(k) \end{bmatrix}\Big) \tag{3.24}$$

As the used AQM are based on state feedback, and having in mind that the state $\delta W(k) = W(k) - W_0$, is not directly available at routers in real networks, it is necessary to take this constraint into account. Some authors have proposed to use an observer to estimate this state (Chen, Hung, Liao and Yan, 2007; Manfredi, Di Bernardo and Garofalo, 2009) whereas others expressed $W(k) - W_0$ as a function of the rate mismatch at routers, which can be estimated (Zhang, Ye, Ma, Chen and Li, 2007). The approach in this paper uses the following approximation:

$$W(k) - W_0 = \frac{RTT}{N}\Big(\frac{NW(k)}{RTT} - C_0\Big) = \frac{RTT}{N}\Big(flow\, rate - C_0\Big) \tag{3.25}$$

Furthermore, as in (Athuraliya *et al.*, 2001), it must be pointed out that the rate of mismatch is the rate at which the queue length grows when the buffer is nonempty. Therefore, we can approximate it by $\frac{\delta q}{T}$ where $\frac{1}{T}$ is the sampling frequency. Hence, (3.24) becomes

$$\delta p(k) = sat\Big(\begin{bmatrix} 0 & \mathbb{K}_1\frac{RTT}{NT} + \mathbb{K}_2 & \mathbb{K}_3 & \mathbb{K}_4 \end{bmatrix}\xi(k)\Big)$$

Finally, it must be pointed out that to implement our AQM controller (3.1), we first discretize (2.3).

3.3 Anti-windup Optimization

3.3.1 Minimization of γ

As (Bender, 2013), in order to minimize γ where $\mu^{-1} = \delta^{-1}$ a solution should be given for the following problem where the initial condition is null

$$\begin{aligned} &\min \gamma \\ &subject\ to\ (3.1) - (3.3) \end{aligned} \tag{3.26}$$

3.3.2 Maximization of κ

Now, we consider the free-disturbance case $(w(k) = 0)$. In practice the bounds on the time-varying delay d_M and d_m can be known. In order to ensure the stability of system (2.10) by using the Theorem 3.2, the admissible initial conditions must verify (3.4). Note that the smaller the maximal eigenvalues of $\widehat{P}^{-1}$, $\widehat{P}^{-1}\widehat{Q}\widehat{P}^{-1}$, and $\widehat{P}^{-1}\widehat{R}\widehat{P}^{-1}$, the larger κ for which (3.4) is verified. Hence, the problem of finding A_a, B_a, C_a, and D_a is transformed into the maximization of the region of stability that can be achieved by minimizing these maximal eigenvalues. With this aim, consider the following auxiliary LMIs as in (El Fezazi, El Haoussi, Tissir, Alvarez and Tadeo, 2017; El Haoussi, Tissir and Tadeo, 2014) where $\widetilde{P} = \widehat{P}^{-1}$, $\widetilde{Q} = \widehat{Q}^{-1}$, and $\widetilde{R} = \widehat{R}^{-1}$

$$\begin{bmatrix} \sigma_1 I & I \\ I & \widehat{P} \end{bmatrix} \geq 0, \begin{bmatrix} \sigma_2 I & \widetilde{P} \\ \widetilde{P} & \widetilde{Q} \end{bmatrix} \geq 0, \begin{bmatrix} \sigma_3 I & \widetilde{P} \\ \widetilde{P} & \widetilde{R} \end{bmatrix} \geq 0 \tag{3.27}$$

Consequently, the condition (3.4) implies that

$$\kappa^2 \Big\{ \sigma_1 + (d_M + d_m)\Big(\sigma_2 + \frac{d_M - d_m + 1}{2}\Big)\sigma_3 \Big\} \leq \mu^{-1} \tag{3.28}$$

Then, we construct a feasibility problem as follows

$$\begin{aligned} &\min \operatorname{tr}\big(\widehat{P}\widetilde{P} + \widehat{Q}\widetilde{Q} + \widehat{R}\widetilde{R}\big) \\ &\text{subject to (3.1)-(3.3),(3.27),(3.28),} \begin{bmatrix} \widehat{P} & * \\ I & \widetilde{P} \end{bmatrix} \geq 0, \begin{bmatrix} \widehat{Q} & * \\ I & \widetilde{Q} \end{bmatrix} \geq 0, \begin{bmatrix} \widehat{R} & * \\ I & \widetilde{R} \end{bmatrix} \geq 0 \end{aligned} \tag{3.29}$$

Based on the above conditions, the proposed controller can be designed for given d_M and d_m by using the following cone complementarity algorithm:

Step 3.1. Choose a small δ and set $\big(\widehat{P}, \widetilde{P}, \widehat{Q}, \widetilde{Q}, \widehat{R}, \widetilde{R}, \sigma_1, \sigma_2, \sigma_3\big)_0 = \big(\widehat{P}, \widetilde{P}, \widehat{Q}, \widetilde{Q}, \widehat{R}, \widetilde{R}, \sigma_1, \sigma_2, \sigma_3\big)$ that satisfies the constrained minimization (3.29). Then, fix Δ where $\delta = \delta + \Delta$.

Step 3.2. Solve the following LMI minimization problem in the matrix variables $\widehat{P}$, $\widetilde{P}$, $\widehat{Q}$, $\widetilde{Q}$, $\widehat{R}$, and $\widetilde{R}$

$$\begin{aligned} &\min \operatorname{tr}\big(\widehat{P}\widetilde{P}_0 + \widehat{Q}\widetilde{Q}_0 + \widehat{R}\widetilde{R}_0 + \widehat{P}_0\widetilde{P} + \widehat{Q}_0\widetilde{Q} + \widehat{R}_0\widetilde{R}\big) \\ &\text{subject to LMIs in (3.29)} \end{aligned}$$

Step 3.3. Substitute the new matrix variables into (3.29). If the result is feasible, then set $\delta = \delta + \Delta$ and repeat Step 2; otherwise, $\delta = \delta - \Delta$ is the desired estimate: Stop.

4 Simulations

In order to demonstrate the effectiveness and applicability of proposed design methodology, two examples are derived, with simulations provided to compare with existing works. The first one aims at studying the conservativeness. In the second example, some Matlab simulations are provided to compare the proposed controller with previous controllers.

Example 4.1. Consider the closed-loop system (3.20) with parameters for which control values are saturated at ± 10 where

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & -0.1 \\ 0 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$A_c = \begin{bmatrix} 0.0718 & 0.0389 \\ -0.0502 & -0.0012 \end{bmatrix}, B_c = \begin{bmatrix} -0.0213 & 0.0001 \\ 0.0621 & 0.0080 \end{bmatrix},$$
$$C_c = \begin{bmatrix} 0.0184 & 0.0213 \end{bmatrix}, D_c = \begin{bmatrix} -0.0228 & -0.0087 \end{bmatrix}$$

Using the cone complementarity algorithm, the results obtained in Corollary 3.3 ensures the asymptotic stability; the estimated domain of attraction for different delay ranges are shown in Table 1, which can be compared with the results in (Negi, Purwar and Kar, 2012).

Table 1: Values of κ obtained for several delay ranges.

Method	Reference (Negi et al., 2012)	This paper
$1 \le d(k) \le 3$	0.5151	2.3
$1 \le d(k) \le 4$	0.3852	1.5
$1 \le d(k) \le 5$	0.2918	1.1
$1 \le d(k) \le 6$	$Infeasible$	0.9

It is clear that the obtained stability radius κ is significantly larger than those obtained in (Negi et al., 2012). The corresponding matrices of (2.7) are $A_a = 0.3$, $B_a = 0.3$, $C_a = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}$, and $D_a = 0.5$.

Example 4.2. In this section we illustrate our methodology based on a TCP/IP router queue model borrowed from (Bender, 2013), given in the form of (2.3). The nominal parameters are $RTT = 0.2467$, $C_0 = 3750$, $q_0 = 175$, and $N = 60$. Then, we can deduce that $W_0 = \frac{RTTC_0}{N}$, $p_0 = \frac{2}{W_0^2}$, and $T_p = RTT - \frac{q_0}{C_0}$. Finally, the matrices of (2.6) are $A_c = 0$, $B_c = 1$, $C_c = 8.4969 \times 10^{-6}$, and $D_c = 1.6996 \times 10^{-5}$. We assume that control values are saturated at $u_0 = p_0$.

Then, applying the stability results presented in Theorem 3.2 and using the algorithm proposed in (3.26). For $0.1 \le d(k) \le 0.5$, $\mu = 1$, and $T = 0.1$, the corresponding matrices of (2.7) and prescribed scalar γ are $A_a = 0.2$, $B_a = 0.03$, $C_a = 0.05$, $D_a = 0.1$, and $\gamma = 0.01$, respectively.

The performance in reference queue tracking and disturbance rejection has been investigated through simulations. Two well-known AQM methods, RED and REM, were also simulated for comparison, with the parameters listed in Table 2. The transfer function for RED (Floyd and Jacobson, 1993) is $C_{RED}(s) = L_{RED}\frac{K_{RED}}{s+K_{RED}}$. For REM, the end-to-end marketing probability is $p \simeq (\log_e \omega_{REM}) \sum \rho(kT)$ where the update of the price $\rho(kT)$ in period T follows equation

(Athuraliya *et al.*, 2001)

$$\rho((k+1)T) = \max\left\{\rho(kT) + \eta\Big(b((k+1)T) - 0.99b(kT) - 1.75\Big), 0\right\}$$

Table 2: Controllers parameters.

Controllers	Parameters value
RED	$L_{RED} = 1.86 \times 10^{-4}$, $K_{RED} = 0.005$
REM	$\omega_{REM} = 1.001$, $\eta = 0.001$

Thus, using the algorithm proposed in (3.26) and the initial values $\xi_0 = [10 \ \ -10]^T$, the queue size regulation and deviations of drop probability are shown in Figure 3, when the Gaussian noise in Figure 2 is used as disturbance to check their effect.

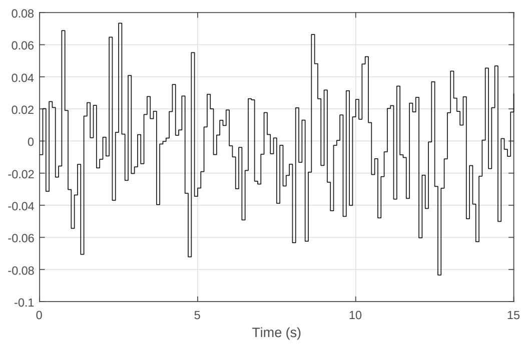

Figure 2: Random disturbance.

The simulation results show the performance of the proposed controller: it manages to maintain the queue length at the target value despite the inherent time-varying dynamics. On the contrary using a standard RED controller the queue length is far from the desired value, presenting a sluggish response, resulting in degraded utilization, losses, and high variance of queuing delay. Moreover, REM presents high variations, which in turn results in high and fluctuant values of the RTT, affecting the performance of the network and aggravating the quality of services. The drop rate (given by the rate probability) is smaller with the proposed controller than with standard RED and REM.

5 Conclusion

An AQM approach for congestion control has been discussed, that incorporates explicitly anti-windup to incorporate the effect of saturation of the control signal. The approach is based on a discrete-time TCP flow model with link capacity disturbance and time-varying delay: a sufficient condition for stability is proposed, which is then incorporated into an optimization algorithm to design anti-windup that ensure stability for the largest set of admissible initial states. Some simulation results are presented to demonstrate the effectiveness of the proposed approach.

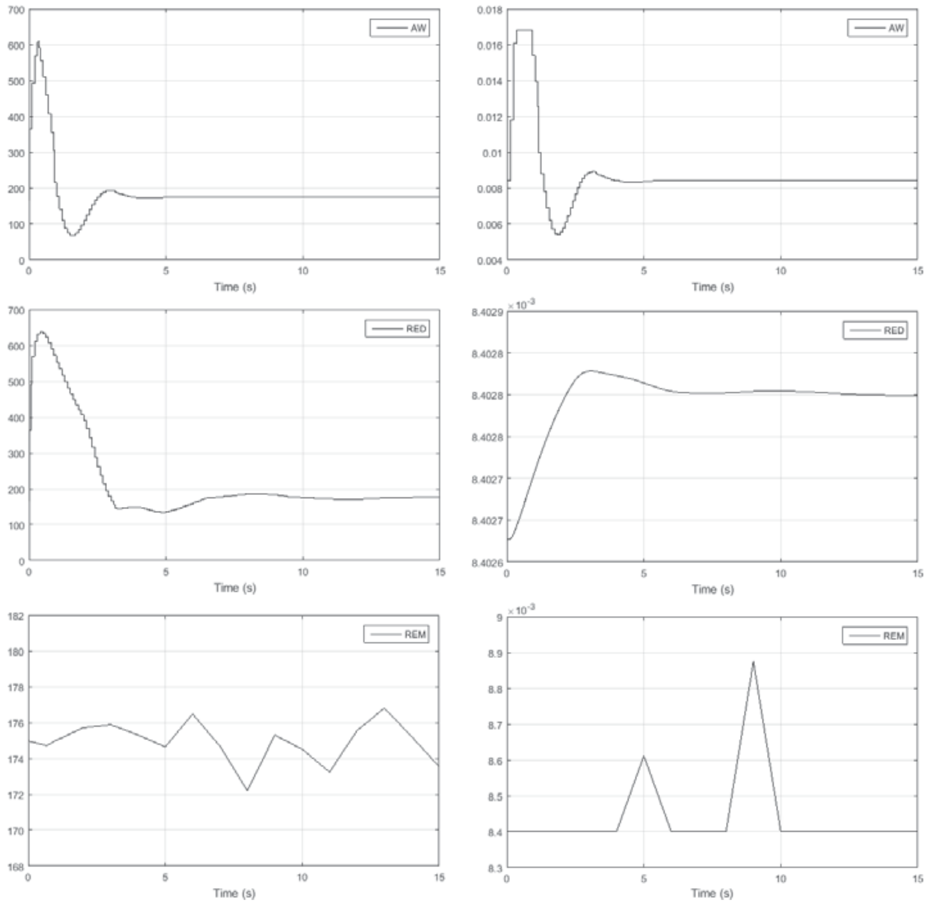

Figure 3: Variation over average value of queue (left) and discard probability (right).

Acknowledgment

We would like to thank Prof. Tadeo for many helpful discussions and Prof. Bender for providing assistance with the software. The last author has been funded by DPI2014-54530-R.

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