Commitment vs. noncommitment behaviors in natural resource conflicts: A case study of groundwater resources

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Abstract

We examine the problem of natural resource exploitation when an exceptional extraction of a resource (groundwater) is needed and devoted to a different use than its regular use. The study applies a two-stage Stackelberg game to examine the strategic behavior of players who compete for water. The leader, with varying weights assigned to the different uses and environmental concern, is the manager of the new (nonregular) resource use, who only intervenes in the second stage of the game. The follower is a regular (agricultural) resource user. We examine the crucial resource of groundwater, introducing two types of Stackelberg equilibria (open-loop and feedback) that can arise depending on agents' commitment behavior. We compare the extraction behaviors of the leader and the follower for the two equilibria and the effects on the final state of the resource and agents' profits. Unexpectedly, we demonstrate that situations can occur in which noncommitment strategies could be more favorable than commitment strategies in terms of the final aquifer stock and the regular user's profits. To avoid that noncommitment strategies are implemented in these circumstances, the weights assigned by the leader to the different uses will play an important role.

Keywords: Groundwater resource; Multiple uses; Asymmetric players; Commitment and noncommitment behaviors; Stackelberg dynamic game
JEL: Q25, C72

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1 Introduction

Natural resource management is an important issue and the sustainability of these resources is a concern shared by political agents and citizens. Avoiding the depletion of natural resources has led decision makers to implement different exploitation strategies with the common objective of achieving sustainable resource exploitation under management. When natural resource management is shared by different agents that compete for its use, strategic behaviors of the agents arise. Although this situation can occur in the management of different natural resources, we focus on water, one of the most valuable resources on the planet, and in particular, groundwater resources.

Groundwater depletion is a major challenge in which groundwater resources have an increasingly important role, not only for irrigation (a regular use in the majority of aquifers), but also for domestic, industrial, and recreational purposes. One circumstance that occurs and illustrates the increased competition for groundwater resources is the need for exceptional extraction of groundwater for nonregular use such as the construction of a water reserve, transfer between basins, and special needs for domestic or recreational water (e.g., irrigation of golf courses, swimming pools, water sport complexes, and other uses). In this study, we examine a problem of groundwater resource exploitation, mainly used for a unique purpose (e.g., irrigation), that exceptionally faces the entrance of a new use in the system, which causes a problem of water scarcity for the regular user.

In this context, a benevolent water authority (e.g., a water agency) is needed to manage how much groundwater could be extracted for the new (nonregular) use so that different resource users can simultaneously exploit and profit from the resource. Because of the exceptional groundwater needed for this new nonregular use, we consider that a benevolent water authority that manages this new use over the regular use acts as the leader in a Stackelberg game. Hence, we do not focus on the implementation of public policies that facilitate socially optimal resource allocation since we consider the extraction of water for nonagricultural use to be exceptional; therefore, the specific problem we examine does not require public policy implementation. We concentrate on investigating and characterizing a second-best equilibrium, through the resolution of a Stackelberg game between the water agency (the leader) and the agricultural user (the follower). Specifically, we focus on the comparison of two Stackelberg equilibrium concepts that correspond to different agents' commitment behavior, and analyze how the agents' differing behaviors when making decisions regarding extraction concerning new (nonregular) use (the leader) and regular use (the follower) affect the sustainability of the resource and the decision makers' profits.

Indeed, our study contrasts with previous literature that focuses on the centralized management of aquifers and consequently seeks to identify the efficient or Pareto solution for the groundwater management problem. Groundwater quantity issues are primarily addressed in the literature using optimal control theory (Koundouri et al. (2017) for a review), and most studies examine different types of uncertainties (Tsur and Zemel (2014) for a review), particularly the problem of water scarcity (de Frutos Cachorro et al. (2014)). In de Frutos Cachorro et al. (2014), water scarcity is considered through modeling an exogenous shock to the groundwater resource (namely, a decrease in the recharge rate of the aquifer) to analyze optimal extraction paths and the social costs of optimal adaptation to the shock. In contrast to previous research, our approach importantly introduces the strategic aspect in the behavior of the different decision-makers and treats water scarcity as an endogenous shock to the groundwater resource.

The extraction of any natural resource has three characteristics. First, the extraction problem is intrinsically dynamic; second, strategic interdependencies exist when the extraction decisions of one agent affect their profits as well as the profits of the other agents; and third, strategic and forward-looking behavior, in the sense that the decision makers consider the present and future consequences of their own actions and those of the other agents. These three problem characteristics make it extremely suitable for modeling and exploration as a dynamic game (Jørgensen et al. (2010)). In this context, the use of dynamic games has been largely justified in the literature on water management to examine problems in which the dynamic and strategic interactions that occur between decision makers (i.e., the water agency and the regular user) are considered (Madani (2010) for a review). Two main externalities could arise in this type of problem under decentralized management, and consequently, noncooperative equilibria are generally inefficient (in terms of stock and/or welfare) with respect to cooperative solutions. First, a strategic (or stock) externality emerges because of competition between the different users for the limited resource. Next, extraction by one user lowers the resource stock, resulting in increased extraction cost for other users, which is a cost externality. Indeed, game theory literature focuses on cooperative and/or noncooperative (Nash equilibrium) solutions for irrigation users (Negri (1989) and Rubio and Casino (2001)) under water scarcity (de Frutos Cachorro et al. (2019)) or more complex problems due to farmers' heterogeneity (Saleh et al. (2011)), competition between uses (de Frutos Cachorro et al. (2021)), and spatial characteristics such as multicell aquifers (e.g., Saak and Peterson (2007)), among others. However, as explained in Kicsiny (2017), when water conflicts enter the problem, Stackelberg (or leader–follower) equilibria offer a more realistic representation of the problem in practical cases in terms of previous classic Nash equilibria. As noted previously, in this study, since the exceptional need of water for a nonregular use leads to a water conflict between users, it is appropriate and interesting to consider that the different agents compete à la Stackelberg and play hierarchically. To investigate this specific situation, we formulate our problem as a two-period discrete time game. The water agency is the leader and as such makes extraction decisions regarding the new (nonregular) use in the first place and exclusively in the second period of the game. Subsequently, the agent who makes decisions about regular use (the follower in the Stackelberg game) makes resource extraction decisions in the two periods depending on the leader's actions.

General Stackelberg dynamic games could be classified according to their relevance at theoretical and/or empirical levels. The books by Dockner et al. (2000) and Başar and Olsder (1999) are well-known references for Stackelberg dynamic games in continuous or discrete time, respectively. Some studies offer interesting findings from a theoretical perspective, although their application to real cases is restricted (Nie (2005), Erdlenbruch et al. (2014)). To the best of our knowledge, only a few previous articles focus on comparing different types of Stackelberg equilibria that correspond to agents' different commitment behaviors, particularly open-loop and feedback Stackelberg equilibria. This is extremely important in practice since each equilibrium concept can be considered more realistic than the other, depending on the information that is available to each player. Implementing feedback strategies requires that the current stock of the aquifer can be observed by decision agents. Therefore, in some settings open-loop strategies are more realistic if the stock of water is unobservable or only observable with a delay. Furthermore, in the open-loop equilibrium, the leader makes a commitment regarding extraction behavior and the follower believes this commitment and chooses the resource extraction based on this belief. The problem with Stackelberg open-loop equilibria is that they are generally inconsistent[1] over time and therefore less realistic. The feedback Stackelberg equilibrium does not have this disadvantage. This equilibrium is consistent over time, and the players do not commit to their extraction behavior over time, but make decisions depending on the stock of the resource at the beginning of each period. In particular, Nie (2005) analyzes and compares open-loop (commitment) and feedback (noncommitment) Stackelberg equilibria for a general setting, concluding that feedback Stackelberg strategies are more efficient

[1]The problem of temporal inconsistency is due to the fact that if the leader optimally decides to perform a number of actions over several periods, and if other economic agents (in our case, the agent who decides the regular use) believe in this commitment and choose their actions under this belief, then, at some period in the future, the leader would want to deviate from this commitment (Kydland and Prescott (1977)).

for the leader's objective than open-loop strategies. Most empirical studies use general algorithms to determine approximate solutions to complex problems, primarily focusing on a specific equilibrium type (Kicsiny et al. (2014) and Xu et al. (2019)).

In the literature combining Stackelberg problems, water scarcity, and competition between groundwater uses, Kicsiny et al. (2014) address the problem of a local authority (the leader) who would manage the use of a water reserve for different types of uses in a given time period by first reserving a minimally guaranteed quantity for both uses, and by subsequently assigning a proportion of the water reserve available for domestic use. The follower (a representative agent of the farmers) then decides the proportion of the available reserve left for irrigation after the leader's decision. However, Kicsiny et al. (2014), only analyze the feedback Stackelberg solution, and in contrast to our study, the water conflict is continuous over time; therefore, the local authority fixes minimum and maximum quantities for both uses at each period before making extraction decisions for nonagricultural use. Moreover, a crucial difference between our study and Kicsiny et al. (2014) is that the dynamics in our formulation are in the stock of the aquifer (renewable resource), whereas in Kicsiny et al. (2014), the dynamics are in the available water; that is, the water remaining from the total reserve (nonrenewable resource).

In this study, we examine whether noncommitment (feedback) and commitment (open-loop) Stackelberg strategies could be more profitable for the sustainability of the resource (in terms of final aquifer stock level) and/or for the agents' profits. To the best of our knowledge, this study is novel in the sense that it proposes and compares different Stackelberg equilibria for resource management in a context of competition between different groundwater uses.

Our paper also differs from previous literature in several ways. First, and most importantly, we are interested in modeling a specific situation that requires a water authority (e.g., a water agency) to manage an exceptional extraction of water for nonregular use. To do so, we construct a two-stage discrete problem in which the leader only intervenes when necessary (in this case, in the second stage of game in which the nonregular water use takes place), while in Nie (2005) and Kicsiny et al. (2014), a leader–follower approach is applied at each step of the game in which both the leader and follower are active players at all stages. Our specific game depicts three external effects, the cost and strategic effects in the second period, where both agents compete for the limited resource, and a third intertemporal effect that arises because the follower's extraction in the first period lowers the available stock for the second period, resulting in increased extraction cost for the leader in the second period. Second, we assume that the leader (the water agency) is a benevolent entity that decides how much groundwater can be extracted for the new (nonregular) use. For this purpose, the leader considers the profits from the different uses (regular and nonregular uses) in the objective to avoid a water conflict while also allowing for the possibility of assigning different weights to different uses. Indeed, this kind of objective that considers both users' profits has already been introduced in the literature regarding water conflicts between different water uses (Maas et al. (2017)), considering equal weights for the different uses in the objective. Furthermore, the leader is environmentally concerned, specifically about the sustainability of the natural resource; therefore, the leader values the final stock of the aquifer in the objective function. In contrast, the decision-agent who determines the regular use of the resource (the follower) assumes the role of a profit maximizing firm. Since the two agents (the leader and the follower) determine extraction strategies to maximize different objectives and compete for water, our problem can be understood as a mixed duopoly (De Fraja and Delbono (1990))[2]; however, in contrast to the majority of the literature on mixed duopolies in which firms compete in quantities through the price–demand function (e.g., recent studies such as Lee and Park (2021), De Chiara and Manna (2022), Zhu et al. (2022), and Delbono et al. (2023)), in this study, firms compete for water through the cost function, more specifically, through the pumping costs that depend on the stock of the aquifer and the extracted quantity.

As described above, we are particularly interested in analyzing and comparing the agents' extraction behaviors for different Stackelberg equilibria depending on the type of existing commitment between decision makers. With this aim, we analytically solve and compare the commitment (open-loop) and noncommitment (feedback) equilibria. Our theoretical results demonstrate that committed strategies lead to higher stock levels than uncommitted strategies when the leader assigns a weight to the profits from regular use that equals the one assigned to the profits from the new (nonregular) use. Finally, we perform numerical simulations to analyze decision makers' profits and determine whether the main results concerning final stock levels remain valid when the leader assigns different weights to different uses. We demonstrate that the introduction of this aspect into the model is extremely relevant, representing one of the main drivers of final results. Notably, the numerical results suggest that situations can emerge in which uncommitted strategies could be more favorable than committed strategies, not only in terms of the final resource stock, but also in terms of profit of the agent deciding on regular use. However, the water agency (the leader) always obtains higher profits under committed strategies; therefore,

[2]We thank an anonymous reviewer for bringing this literature to our attention.

the decision makers' interests (the water agency and the agent deciding on the regular use) cannot be aligned. As a consequence, since the agent deciding on the regular use is the only active player in the first period, this agent could induce the water agency to employ uncommitted strategies in the second period by playing uncommitted strategies in the first period. In this case, since the objective of the water agency could be seen as a measure of social welfare (it considers total surplus from groundwater exploitation and a term that measures its environmental concern about final stock levels), the implemented strategies would not coincide with strategies that procure a higher value of the water agency's objective to the detriment of social welfare.

The remainder of this paper is organized as follows. Section 2 describes the Stackelberg game and the model resolution for the two types of commitment behavior. We conduct comparisons between theoretical results for open-loop and feedback Stackelberg equilibria in Section 3 for the case in which the water agency (the leader) assigns an equal weight to the two uses (regular and nonregular) in its objective function. In Section 4, we complete our analysis through numerical simulations, relaxing the previous hypothesis and allowing the leader to weigh the two uses in its profits function differently. A further analysis is presented in Section 5, examining how the previous results could change if the follower applies a discount rate to the second-period profits. We detail our conclusions in Section 6. All the proofs are presented in the appendices. The supplementary material includes detailed tables showing the results of our numerical simulations and the Maple program used to generate these results.

2 The game

We investigate a problem in which a regular user of a groundwater resource (e.g., an agent in charge of making extraction decisions concerning agricultural use) faces the announcement that another exceptional extraction for a nonagricultural use will occur in the second period. This leads to a competition problem between the two uses for the limited stock of the aquifer over the two periods. To avoid groundwater overexploitation, a water authority is needed to manage how much water can be extracted for the new use in such a way that both users can simultaneously and profitably exploit the common resource.

We formulate our problem as a discrete two-stage Stackelberg model with two decision makers who have different objectives. First, there is the agent in charge of the extraction of the resource for regular use (i.e., irrigation in the majority of the aquifers), who is the follower aiming to maximize profits. Second, there is a water authority (e.g., a water agency), who is the leader of the Stackelberg game aiming to maximize the sum of total surplus derived from groundwater exploitation, which is defined as a weighted sum of profits from both uses, and an environmental term that considers the final stock value of the resource in the objective function and measures the leader's environmental concern.

Both agents maximize their objectives, considering the aquifer dynamics over the two periods. The stock of the aquifer is the state variable and, at the initial time, this stock is denoted by G_0. Agents' extraction decisions over the two periods (i.e., extraction of the leader for the new (nonagricultural) use in the second period and extractions of the follower for the agricultural use in the first and second periods) are the decision variables of the problem.

2.1 Game formulation

2.1.1 Aquifer dynamics

In the first period, the aquifer is exclusively exploited by the follower, and we denote the amount of water extracted by the follower in this period by g_{1f}. Hence, the stock of the aquifer at the end of the first period, G_1, is as follows:

$$G_1 = G_0 - g_{1f} + r, \tag{1}$$

where r represents the constant recharge of the aquifer over the first period.

In the second period, the leader and the follower make extraction decisions, with g_{2l} denoting the leader's extraction for the new (nonagricultural) use and g_{2f} denoting the follower's extraction for irrigation purposes. For simplicity, assuming that the recharge of the aquifer over the second period is identical to that over the first period, the stock of the aquifer at the end of the second period, G_2, is as follows:

$$G_2 = G_1 - g_{2f} - g_{2l} + r. \tag{2}$$

2.1.2 Revenue and cost functions of the different users

Concerning the regular use (irrigation), we consider the agent deciding this use faces a (per period) linear demand for irrigation $g_{tf} = a - bp_{tf}^{w}$ $(a, b > 0)$, where g_{tf} represents the water extraction and p_{tf}^{w} the price of water in period t. Taking the differential game of groundwater exploitation of Rubio and Casino (2001) as a reference, we assume that the decision-agent is a price taker in the output market, the agricultural production function exhibits constant returns to scale, and production factors other than water and land are optimized, conditioned on the water extraction. As a result, the price of water equals the value of the marginal product of water and the (per period) follower's revenue function can be obtained by integrating the inverse demand function as follows:

$$R_{tf}(g_{tf}) = \int p^w_{tf}(g_{tf})\, dg_{tf} = \int \frac{a - g_{tf}}{b}\, dg_{tf} = \frac{a}{b} g_{tf} - \frac{1}{2b} g^2_{tf}, \quad t = 1, 2. \tag{3}$$

In what follows, we denote $a_f = \frac{a}{b}$ and $b_f = \frac{1}{b}$ to simplify the notation.

Unlike the profits from the regular use defined in (3), for analytical tractability, we assume that the new (nonagricultural) use faces perfectly elastic demand; therefore, the marginal revenue for water is constant for this use. Examples of this exceptional extraction for nonagricultural use could be the construction of a water reserve, transfer between basins, or a special need for domestic or recreational water (e.g., irrigating golf courses, swimming pools, or water sport complexes). The revenue function of the new aquifer user in the second period is therefore linear with respect to the water extracted for this new use, g_{2l}, and can be defined as follows:

$$R_{2l}(g_{2l}) = a_l g_{2l} \quad \text{with} \quad a_l > 0. \tag{4}$$

As in the previous literature regarding exploitation of groundwater resources (Negri (1989), Rubio and Casino (2001) and de Frutos Cachorro et al. (2019)), individual (per period) pumping costs depend on the stock of the aquifer at the end of the period, G_t, and the extracted quantity of water in this period by agent i, g_{ti}, as follows:

$$C_{ti}(G_t, g_{ti}) = (z - cG_t) g_{ti}, \quad t = 1, 2, \quad i = f, l, \tag{5}$$

where z and c are positive parameters. More specifically, z corresponds to the maximum unit (or marginal) cost, indicating the marginal cost when $G = 0$.[3]

2.1.3 Leader and follower objectives

The players (the leader and the follower) make extraction decisions to maximize different objectives. This can be seen as a mixed duopoly following the definition of classic studies such as De Fraja and Delbono (1990), in the sense that one firm (the agent deciding on the agricultural use) maximizes profits, and another environmentally and socially concerned (or public) firm (the water agency) maximizes the sum of total surplus derived

[3] We assume that the marginal pumping costs are positive and check a posteriori in all the numerical simulations that this hypothesis is satisfied. In particular, $G_0 < z/c$ by assumption.

from groundwater exploitation and an environmental term that considers the final stock value of the resource while competing for groundwater. However, in contrast to the majority of previous literature examining mixed duopolies in which firms compete in quantities through the price–demand function (e.g., recent studies such as Lee and Park (2021), De Chiara and Manna (2022), Zhu et al. (2022), and Delbono et al. (2023)), in this study, firms compete for water through the cost function (see Equation (5)).

The follower's objective: The decision-agent of the agricultural use (the follower) aims to maximize total profits over the two periods, where the profits at period t are determined by the following expression:

$$\Pi_{tf}(g_{tf}, G_t) = R_{tf}(g_{tf}) - C_{tf}(G_t, g_{tf}) = a_f g_{tf} - \frac{b_f}{2} g_{tf}^2 - (z - cG_t)g_{tf}, \quad t = 1, 2, \tag{6}$$

with functions R_{tf} and C_{tf} given by (3) and (5), respectively.

The leader's objective: We assume that the leader is an environmentally and socially concerned (or public) water authority with the objective of maximizing the sum of the total surplus[4] from groundwater exploitation, defined as a weighted sum of profits derived from agricultural, Π_f, and nonagricultural, Π_l, uses, and an environmental term that adds the possibility that the leader values the final resource stock in the objective function, as follows:

$$\theta \left(\sum_{t=1}^{2} \Pi_{tf} \right) + (1-\theta)\Pi_{2l} + AG_2, \tag{7}$$

with θ, $0 \leq \theta < 1$, $A \geq 0$ and,

$$\Pi_{2l}(g_{2l}, G_2) = R_{2l}(g_{2l}) - C_{2l}(G_2, g_{2l}) = a_l g_{2l} - (z - cG_2)g_{2l}, \tag{8}$$

with functions R_{2l} and C_{2l} given by (4) and (5), respectively.

Parameter θ represents the weight that the water agency assigns to the agricultural profits. The greater the weight θ, the more significant the agricultural use of the aquifer for the leader; and vice versa, the lower θ, the more important the new use of the aquifer for the leader.

[4]In the literature on mixed duopolies, a different objective function is commonly used by socially concerned firms, i.e., to maximize social welfare defined as the sum of consumers' and producers' surplus (see De Fraja and Delbono (1990) for a review). Furthermore, some studies include the possibility of assigning different weights to the consumer surplus in the objective function (De Chiara and Manna (2022), Zhu et al. (2022)) or incorporate an environmental damage function (Lee and Park (2021)) in the case of environmentally concerned firms.

2.2 Game resolution under different commitment behaviors

In a general Stackelberg game, or leader–follower game, the leader makes decisions first and the follower subsequently makes decisions depending on the actions of the leader. Different types of Stackelberg equilibria can be computed depending on the commitment behavior between the two agents over the two-period game. We next analytically solve the game for open-loop (commitment solution) and feedback (noncommitment) Stackelberg equilibria. While the former involves time-inconsistent solutions, the latter procures time-consistent solutions (see proofs in Appendix A).

In this study, we assume that agriculture requires water continuously to grow crops. Moreover, we are interested in examining the problem in which a new use needs water to procure a new activity. Consequently, we are interested in positive extractions of water both for agricultural (the follower's extraction) and nonagricultural (the leader's extraction) use and the positive stock of the aquifer at the end of the two periods. In summary, we focus on interior and strictly positive solutions; hence, corner solutions are not analyzed.

2.2.1 Open-loop Stackelberg equilibrium

In an open-loop Stackelberg equilibrium, the leader commits at $t = 0$ to the path of extraction for the new use in the second period. The follower then believes the leader's commitment and subsequently chooses extraction paths over the two-period game under this belief.

Accordingly, the follower first determines their extraction behavior in the two periods, g_{1f} and g_{2f}, assuming the leader's extraction policy in the second period, g_{2l}. Since the follower's objective is to maximize profits over the two periods, the follower faces the following problem:

$$\begin{aligned} \max_{g_{1f}\geq 0,\ g_{2f}\geq 0} \quad & \sum_{t=1}^{2} \Pi_{tf}(g_{tf}, G_t), \\ & \text{s.t: } (1), (2) \\ & G_1, G_2 \geq 0 \end{aligned} \tag{9}$$

where function Π_{tf} is given by (6). We then obtain the follower's best-reaction functions, $\tilde{g}_{1f}(g_{2l})$ and $\tilde{g}_{2f}(g_{2l})$, representing the follower's extractions over the two periods as functions of the leader's extraction in the second period. Next, the leader determines the water extraction in the second period, g_{2l}, to maximize a weighted sum of the two-period profits of the follower (considering the follower's best-reaction functions, $\tilde{g}_{1f}(g_{2l})$ and $\tilde{g}_{2f}(g_{2l})$) and the profits derived from extraction for the new use in the second period, as well as the term measuring the leader's environmental concern as follows:

$$\begin{aligned} \max_{g_{2l}\geq 0} \quad & \left\{\theta\left(\sum_{t=1}^{2}\Pi_{tf}(\tilde{g}_{tf}(g_{2l}),G_t)\right)+(1-\theta)\Pi_{2l}(g_{2l},G_2)+AG_2\right\}, \\ & \text{s.t: } (1),\ (2) \\ & G_1,G_2\geq 0. \end{aligned} \tag{10}$$

Functions Π_{tf} and Π_{2l} are given by (6) and (8), and θ, with $0\leq\theta<1$ is the weight assigned to the follower's profits. The equilibrium strategies of this model formulation are examined using numerical examples in Section 4. Here, to ease the presentation, we restrict our attention to the case $\theta=1/2$ (i.e., the case in which the leader equally weighs both users' profits in the objective function). Solving the previous problem with $\theta=1/2$ (see Appendix B.1 for details), we obtain g_{2l}^{OL}, where the superscript OL indicates open-loop equilibrium. Finally, substituting g_{2l}^{OL} in the follower's best-reaction functions, we obtain g_{1f}^{OL} and g_{2f}^{OL}:

$$g_{1f}^{OL}=\frac{2(G_0c+cr+a_f-z)b_f^2+c(6(G_0c-z)+5(cr+a_f)-2A+a_l)b_f+c^2(2(G_0c+z)-4A+3a_l-a_f)}{(b_f+2c)(b_f+3c)(2b_f+c)}, \tag{11}$$

$$g_{2f}^{OL}=\frac{(G_0c+2cr+2A-a_l+2a_f-z)b_f+c(G_0c+3cr+4A-3a_l+4a_f-z)}{(b_f+3c)(2b_f+c)}, \tag{12}$$

$$g_{2l}^{OL}=\frac{(G_0c+2cr-2A+a_l-z)b_f^2+c(G_0c+3cr-6A+4a_l-3a_f-z)b_f+(3a_l-3a_f-4A)c^2}{c(b_f+2c)(2b_f+c)}. \tag{13}$$

Once the optimal extraction strategies are characterized, we can obtain the states of the aquifer at the end of the two periods for previous extraction behavior as follows:

$$\begin{aligned} G_1^{OL}=&\frac{1}{(2b_f+c)(b_f+2c)(b_f+3c)}\left\{2(G_0+r)b_f^3+(9(G_0c+cr)-2a_f+2z)b_f^2\right.\\ &\left.+c(11G_0c+12cr+2A-a_l-5a_f+6z)b_f+c^2(4G_0c+6cr+4A-3a_l+a_f+2z)\right\}, \end{aligned} \tag{14}$$

$$\begin{aligned} G_2^{OL}=&\frac{1}{c(2b_f+c)(b_f+3c)}\left\{(G_0c+2cr+2A-a_l+z)b_f^2+c(2G_0c+5cr+6A-4a_l-a_f+5z)b_f\right.\\ &\left.+c^2(G_0c+3cr+4A-3a_l+a_f+2z)\right\}. \end{aligned} \tag{15}$$

The mathematical expressions concerning the players' profits from optimal extraction strategies and the states of the aquifer are extremely long and are omitted for brevity.

To guarantee the positivity of extraction decisions and aquifer state variables, we assume that the sufficient conditions summarized in Condition 1 in Appendix D.1 are fulfilled.

Since both players' goal is to achieve maximum value for their objectives (problems (9) and (10)), the concavity of the objective functions of the leader and the follower with respect to their decision variables must be guaranteed. In Appendix B.2, we derive these concavity conditions and demonstrate that condition $\theta = 1/2$ ensures this property.

2.2.2 Feedback Stackelberg equilibrium

In a feedback Stackelberg equilibrium, the follower chooses extraction behavior at each step after the leader has decided and announced the strategy. The problem must be solved using backward induction. As the leader does not extract water in the first period, there is only one decision maker (the follower) in this first period. Because the leader is not an active player in the first period, the feedback equilibrium of the problem can be seen as a "degenerated Stackelberg", for which a solution can be obtained following a three-step procedure (see Appendix C.1 for details).

Since the game is solved via backward induction, in the first step, the follower determines extraction behavior in period two, g_{2f}, assuming the leader's extraction policy in the second period, g_{2l}, and the extraction in the first period, g_{1f}. The follower must then solve the following problem in the second period:

$$\begin{aligned} \max_{g_{2f} \geq 0} \quad & \Pi_{2f}(g_{2f}, G_2), \\ & \text{s.t: } (1), (2) \\ & G_1, G_2 \geq 0, \end{aligned} \tag{16}$$

with function Π_{2f} given by (6). The solution to this problem gives g_{2f} as a function of g_{2l} and g_{1f}, $\hat{g}_{2f}(g_{2l}, g_{1f})$. In the second step, after substituting $\hat{g}_{2f}(g_{2l}, g_{1f})$ in the leader's problem, the leader determines extraction behavior in period two, g_{2l}. The leader's problem is as follows:

$$\max_{g_{2l}\geq 0} \quad \theta\Pi_{2f}\left(\hat{g}_{2f}(g_{2l},g_{1f}),G_2\right)+(1-\theta)\Pi_{2l}(g_{2l},G_2)+AG_2, \tag{17}$$
$$\text{s.t: (1), (2),}$$
$$G_1,G_2\geq 0,$$

with functions Π_{2f} and Π_{2l} given by (6) and (8), and $0\leq\theta<1$. The solution to this problem establishes g_{2l} as a function of g_{1f}, i.e., $\hat{g}_{2l}(g_{1f})$. Finally, substituting the leader's reaction function in the second stage of the game in the follower's problem, the follower's problem to solve in the first period is as follows:

$$\max_{g_{1f}\geq 0} \quad \Pi_{1f}(g_{1f},G_1)+\Pi_{2f}\left(\hat{g}_{2f}(\hat{g}_{2l}(g_{1f}),g_{1f}),G_2\right), \tag{18}$$
$$\text{s.t: (1), (2),}$$
$$G_1,G_2\geq 0,$$

with function Π_{tf} given by (6). From the solution to this problem, we obtain the follower's extraction strategy in the first period, g_{1f}^{FB}, where the superscript FB denotes feedback equilibrium. Subsequently, we replace the latter value in the leader's and follower's reaction functions in period two, obtaining g_{2l}^{FB}, g_{2f}^{FB}, i.e., the leader's and follower's optimal strategies in the second period. We next present all optimal strategies and aquifer states derived from these strategies for the particular and important case of $\theta=1/2$:

$$\begin{aligned}
g_{1f}^{FB} &= \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{4(G_0c+cr+a_f-z)b_f^3\right.\\
&+c(11(G_0c-z)+10cr-2A+a_l+10a_f)b_f^2+c^2(7(G_0c-z)+5cr-4A+3a_l+4a_f)b_f\\
&\left.+c^3(G_0c-2A+2a_l-a_f-z))\right\}, \qquad (19)\\
g_{2f}^{FB} &= \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{2(G_0c+2cr+2A-a_l+2a_f-z)b_f^3\right.\\
&+c(5(G_0c-z)+10cr+14A-9a_l+14a_f)b_f^2+c^2(4(G_0c-z)+11cr+14A-12a_l+16a_f)b_f\\
&\left.+c^3(G_0c+3cr+4A-4a_l+5a_f-z))\right\}, \qquad (20)\\
g_{2l}^{FB} &= \frac{1}{c(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{2(G_0c+2cr-2A+a_l-z)b_f^4\right.\\
&+c(7(G_0c-z)+16cr-22A+13a_l-6a_f)b_f^3+c^2(7(G_0c-z)+19cr-40A+29a_l-22a_f)b_f^2\\
&\left.+c^3(2(G_0c-z)+6cr-28A+25a_l-23a_f)b_f+6c^4(a_l-a_f-A)\right\}, \qquad (21)
\end{aligned}$$

$$G_1^{FB} = \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{4(G_0+r)b_f^4+4(4G_0c+4cr-a_f+z)b_f^3\right.$$
$$+c(21G_0c+22cr+2A-a_l-10a_f+11z)b_f^2+c^2(11G_0c+13cr+4A-3a_l-4a_f+7z)b_f$$
$$\left.+c^3(2G_0c+3cr+2A-2a_l+a_f+z)\right\}, \tag{22}$$

$$G_2^{FB} = \frac{1}{c(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{2(G_0c+2cr+2A-a_l+z)b_f^4\right.$$
$$+c(7G_0c+16cr+18A-11a_l-2a_f+13z)b_f^3+c^2(9G_0c+23cr+28A-21a_l-2a_f+23z)b_f^2$$
$$\left.+c^3(5G_0c+14cr+18A-16a_l+3a_f+13z)b_f+c^4(G_0c+3cr+4A-4a_l+2a_f+2z)\right\}. \tag{23}$$

Once again, the mathematical expressions concerning optimal profits are extremely long; hence, are omitted for brevity.

All the conditions collected under Condition 1 in Appendix D.1 are sufficient conditions ensuring positive resource extractions and stocks in the feedback case.

As in the open-loop equilibrium, the concavity of the leader's and follower's objective functions with respect to the corresponding decision variables in the three steps of the game resolution (i.e., in problems (16), (17), and (18)) must be guaranteed. In Appendix C.2, we derive the conditions that ensure the concavity of the different objective functions and prove that these conditions are satisfied for $\theta = 1/2$.

We next compare the extraction strategies, aquifer states, and agents' profits for different commitment behaviors. In Section 3, we focus on theoretical results for the case in which the weight assigned to agricultural use equals the weight of nonagricultural use, $\theta = 1/2$. A numerical analysis of different values that the leader assigns to θ and a robustness analysis for other parameters are performed in Section 4.

3 Theoretical results: Open-loop vs. Feedback Stackelberg equilibria

In this section, we compare both agents' extraction behavior, aquifer states, and players' profits for the different types of equilibria (see equations (11) to (15) for the open-loop case and (19) to (23) for the feedback case). The results for both equilibria can be compared if Condition 1 in Appendix D.1 is satisfied; therefore, we assume this condition in what follows. Please recall that our attention is restricted to the case in which the leader's weight for the new use equals the weight for agricultural use, $\theta = 1/2$.

We first compare the follower's and the leader's extraction for each period depending on the type of commitment behavior.

Proposition 1 *The follower's extraction in the first period, g_{1f}, and the leader's extraction in the second period, g_{2l}, are greater in the feedback case than in the open-loop case. The opposite is obtained for the follower's extraction behavior in the second period, g_{2f}.*

Proof: See Appendix D.2.

One of the results indicating that the leader's extraction behavior in the second period is more aggressive in the feedback case than in the open-loop case can be explained by the fact that the leader has extra information about stock levels at the beginning of each period in the feedback case than in the open-loop case. Consequently, the leader is able to better adapt to only using the resource in the second period by increasing extraction. In addition, as explained in the introduction, we can interpret the entrance of a new use in the second period, and therefore the leader's extraction, as an endogenous shock to the groundwater resource, implying a water scarcity problem for the follower. In the literature regarding shocks in optimal groundwater management, de Frutos Cachorro et al. (2014) treats water scarcity as an exogenous shock to the groundwater resource and shows that the higher the intensity of the shock (which could be equivalent to higher extraction from the leader in our case), the higher the impatience effect and therefore, the higher the extraction before the shock occurrence (which could be equivalent to the follower's extraction in the first period in our case). The same reasoning can be applied in this work to explain that $g_{2l}^{FB} > g_{2l}^{OL}$ implies $g_{1f}^{FB} > g_{1f}^{OL}$. Furthermore, the follower in the feedback case adapts earlier (in the first period) to anticipate extraction losses in the second period (i.e., $g_{2f}^{FB} < g_{2f}^{OL}$) due to competition with the leader, by increasing extraction in the first period in comparison to the open-loop case (i.e., $g_{1f}^{FB} > g_{1f}^{OL}$).

We next compare both players' total extractions over the two periods under the two scenarios concerning the players' behavior (open-loop and feedback). Using the notation $\text{Total} = g_{1f} + g_{2f} + g_{2l}$, we obtain the following proposition.

Proposition 2 *The total amount of water extracted by the follower and the leader over the two periods is higher in the feedback case, $Total^{FB}$, than in the open-loop case, $Total^{OL}$.*

Proof: See Appendix D.3.

Focusing on the impact of extraction decisions on resource stocks in the two periods, the following corollary is evident from Propositions 1 and 2.

Corollary 1 *The resource state after the first and second periods, i.e., G_1 and G_2, respectively, is higher in the open-loop case than in the feedback case.*

This means that the players' commitment regarding extraction behavior over the two periods is positive for the resource state with respect to the noncommitment case. This result aligns with those obtained in previous research characterizing Nash equilibria (Negri (1989), Rubio and Casino (2001), de Frutos Cachorro et al. (2019)). Noncommitment strategies exacerbate the competition between the different users for the available stock (strategic externality); therefore, exacerbating resource exploitation.

As the previous results should not necessarily be maintained for other values of θ, we next run numerical simulations to analyze our previous results for any θ. We also assess the robustness of the previous results by performing a sensitivity analysis for other important parameters of the model.

4 Numerical results

Parameter	Description	Value
a_f	Coefficient of revenue agricultural use (linear term)	4.5
z	Marginal pumping cost intercept	4
c	Marginal pumping cost slope	0.281
G_0	Initial stock level	10
r	Natural recharge rate	5
θ	Weight assigned by the leader to agricultural use	$\theta \in \{0.4, 0.5, 0.581, 0.59, 0.655\}$
b_f	Coefficient of revenue agricultural use (nonlinear term)	$b_f \in \{0.01, 0.1, 0.5, 1\}$
a_l	Marginal revenue from alternative use	$a_l \in \{4.8, 6, 6.2\}$
A	Coefficient of the leader's valuation of the final stock	$A \in \{0, 0.07\}$

Table 1: Parameter values of the model.

We next perform numerical simulations to analyze whether the main results obtained in the previous sections concerning the agents' extraction behavior and final resource stock remain unchanged after relaxing the assumption $\theta = 1/2$. We complete the analysis by examining the agents' profits for these numerical scenarios. For this purpose, we use values of the parameters that are listed in Table 1. More specifically, we fix values corresponding to model parameters above the horizontal line and run several simulations with respect to parameters below the line. In particular, we consider different weights that the leader can assign to agricultural use (θ) and other important parameters of the model as the coefficient (nonlinear term) of the follower's revenue function (b_f), the marginal revenue of alternative use (a_l), and the coefficient of the leader's valuation of final stock levels (A).

As shown in Appendices B.1 and C.1, for any value of θ in [0,1], we obtain analytical solutions for these general cases, and in this section, we run numerical simulations for different parameter values with Maple (2022).

We next examine whether the results of the extraction decisions and the final resource stock (Propositions 1 and 2 and Corollary 1) remain valid when the leader does not equally weigh the two different resource uses; that is, θ differs from 1/2, and we analyze profits per period and per agent. The key results of the numerical simulations are summarized in Table 2 (see also Table 3 and Tables in the Appendix E for detailed results). The Maple program and values for all numerical simulations can be found in the supplementary material.

Focusing first on the description of the key results concerning the final stock levels described in Table 2, the simulated results suggest that when the leader assigns a higher weight to the profits from agricultural use than to those from the alternative use ($\theta > \frac{1}{2}$), noncommitment strategies could be more favorable than commitment strategies in terms of final stock levels (see first row Table 2). This result is the opposite to what we showed for the case $\theta = 1/2$. In fact, parameter b_f of the follower's revenue function, or similarly, parameter a_l of the leader's revenue function[5], also has an important role concerning key results on final stock levels, and previous results are obtained when the value of b_f is low enough, i.e., when the follower's productivity is high enough to compete with the new use[6]

[5]Results of the robustness analysis with respect to a_l are summarized in Tables 7 and 8 in Appendix E.2.

[6]Note that the follower's inverse demand function $p^w_{tf} = \frac{a - g_{tf}}{b}$ can be rewritten as $p^w_{tf} = a_f - b_f g_{tf}$ because $a = a_f/b_f$ and $b = 1/b_f$ (see subsection 2.1.2 for details). Therefore, a decrease in b_f means that a higher price is paid for the same quantity of water extracted, leading to higher productivity of the output product.

or when the value of a_l is low enough, i.e., for a less competitive new use.

Key results	**Mathematical assumption**	**Significance to literature**	**Policy recommendation**
Final stock levels higher in FB	$\theta > \frac{1}{2}$ and b_f low enough	Contradicts the literature of Nash equilibria (e.g. de Frutos Cachorro et al. (2019))	**Noncommitment strategies** could be more **favorable** than commitment strategies in terms of the **sustainability of the resource**.
Follower's total profit higher in FB	$\theta >> \frac{1}{2}$ and $A > 0$	Contradicts the literature of Nash equilibria (e.g. de Frutos Cachorro et al. (2019))	**Noncommitment strategies** could be more **profitable** for the **follower** than commitment strategies.
Leader's total profit higher in FB	Never	Contradicts the literature of Stackelberg equilibria (e.g. Nie (2005)	The leader will always prefer **commitment strategies**.

Table 2: Key results of numerical simulations for different coefficients (nonlinear term) of the revenue function (b_f), different weights that the leader assigns to the agricultural use (θ), and different valuations of the final stock level by the leader (A), where $>>$ means "much higher", $>$ means "slightly higher".

The key results regarding the final stock levels are primarily attributable to the follower's extraction behavior in the first and second period (see rows 1 and 2 in Table 3 for a summary of the main results or columns 1 and 2 in Table 5 in Appendix E for detailed results). When the leader assigns a lower or equal weight to the agricultural use profits than to those from the alternative use ($\theta \leq \frac{1}{2}$), the follower seems to adapt by increasing extraction in the first period in the feedback case compared with the open-loop case (see row 1 in Table 3), to anticipate extraction losses in the second period due to competition with the new use. Indeed, the opposite is observed in the second period (see row 2 in Table 3), where the follower's extraction is lower in the feedback case than in the open-loop case when $\theta \leq \frac{1}{2}$. In fact, the leader's extraction is consistently higher in the feedback case than in the open-loop case in these scenarios (see row 3 in Table 3 or detailed results in Appendix Table 5 column 3). When θ increases (i.e., agricultural use becomes increasingly important for the leader compared with the new use ($\theta > \frac{1}{2}$)), this "anticipation" or fear of a water shortage is reduced in the feedback case compared with the open-loop case. Consequently, the follower focuses on extracting more in the second period (see row 2 in Table 3) and less in the first period (see row 1 in Table 3) in the feedback case than in the open-loop case, leading to a reduction in the leader's extraction in the second period of the feedback case due to competition (see row 3 in Table 3). Therefore, the total extractions over the two periods become less significant in the feedback case than in the open-loop case (see row 4 in Table 3). In other words, a higher final stock level is obtained in the feedback case than in the open-loop case.

Summary results	**Mathematical assumption**
1. Follower's first-period extraction higher in FB	$\theta \leq \frac{1}{2}$ OR ($\theta > \frac{1}{2}$ and b_f high enough)
2. Follower's second-period extraction lower in FB	$\theta \leq \frac{1}{2}$ OR ($\theta > \frac{1}{2}$ and b_f high enough)
3. Leader's second-period extraction higher in FB	$\theta \leq \frac{1}{2}$ OR ($\theta > \frac{1}{2}$ and b_f high enough)
4. Two-period total extractions lower in FB	$\theta > \frac{1}{2}$ and b_f low enough
5. Follower's first-period profit lower in FB	$\theta > \frac{1}{2}$ and b_f low enough
6. Follower's second-period profit higher in FB	$\theta >> \frac{1}{2}$ and b_f low enough
7. Follower's total profit higher in FB	$A > 0$, $\theta >> \frac{1}{2}$ and b_f low enough
8. Leader's total profit higher in FB	Never

Table 3: Summary of main results of the numerical simulations for different coefficients (nonlinear term) of the revenue function (b_f), different weights assigned to agricultural use by the leader (θ), and different valuations of the final stock level by the leader (A), where $>>$ means "much higher", $>$ means "slightly higher".

The key results concerning the agents' profits also suggest that noncommitment strategies could be more profitable for the follower than the commitment case (see second row in Table 2). This result could occur when the leader assigns a much higher weight to the agricultural use profits than the alternative use profits ($\theta >> \frac{1}{2}$) and considers the final stock value in the objective function ($A > 0$). Indeed, when the leader values the sustainability of the resource, the leader's second-period extraction for the new use is reduced to preserve stock levels, and this reduction is more significant in the feedback case than in the open-loop case.[7] This entails less competition for water in the second period when playing noncommitment strategies.[8] In other words, this conservative extraction behavior for the resource opens more opportunities for the follower to extract and accumulate profits in

[7] For example, for the specific values $\theta = 0.655$ and $b_f = 0.1$, when $A > 0$ (more specifically, when $A = 0.07$), the leader's second-period extraction for the new use decreases by 1.21 (1.69) units in the open-loop (feedback) case with respect to scenario A=0.

[8] Again, for the same parameter values (i.e., $\theta = 0.655$ and $b_f = 0.1$), when $A = 0.07 > 0$, total

the second period[9] in the noncommitment case (see row 6 Table 3), achieving higher total profits in the noncommitment case than in the commitment case (see row 7 Table 3).

Finally, as shown in the third row of Table 2 (and row 8 Table 3), the leader will always prefer commitment strategies. This result apparently contradicts the initial intuition gained from the literature of Standard Stackelberg games (Nie (2005)) in which noncommitment strategies ensure higher profits for the leader than commitment strategies. Indeed, our problem can be seen as a "degenerated Stackelberg game" in the sense that the leader does not extract for the new use in the first period, and the objective function of the leader considers a weighted sum of profits of regular and new uses and a term measuring the leader's environmental concern. Our results are then closer to those found when Nash equilibria are characterized (de Frutos Cachorro et al. (2019)). However, as the follower (i.e. the decision-agent of the agricultural use) is the only resource user in the first period, if situations arise in which noncommitment behavior produces higher agricultural profits than commitment behavior, the follower could force the leader to play noncommitment strategies in the second period by playing noncommitment strategies in the first period. Therefore, the policy implications derived from this study indicate that the leader should avoid assigning a high weight to the regular user's profits (i.e., a high θ) to ensure that commitment strategies are played out.

5 Further analysis

In this section, we extend the analysis performed in the previous section by introducing the possibility that the follower applies a discount to second-period profits in the objective function. Moreover, we conduct a sensitivity analysis regarding the weight assigned by the leader to agricultural use (θ) and characterize the intervals of θ for which the key results are obtained.

We consider a short, two-period planning horizon, focusing on the case in which the follower equally values the profits of both periods in the objective function, and therefore, the follower does not apply a discount to the second-period profits. This could be justified, for example, when the two periods refer to two consecutive irrigation seasons of spring and summer in the same year. However, it could indeed be interesting to analyze how our

extractions in the second period decrease by 0.59 (0.66) units in the open-loop (feedback) case with respect to scenario A=0.

[9] Please note that the opposite result is observed concerning the follower's profits in the first period, which are lower in the feedback case than in the open-loop case when $\theta >> \frac{1}{2}$ (see row 5 Table 3).

main results might change when the follower applies a discount factor to the second-period profits in the commitment and noncommitment scenarios. To this goal, we add parameter β, the discount factor, in the follower's objective function, which now reads $\Pi_{1f} + \beta\Pi_{2f}$, and perform a sensitivity analysis with respect to θ, for fixed value model parameters (i.e., parameter values in Table 1 with $b_f = 1$, $a_l = 6$, $A = 0.07$). The key results for the discount factors ($\beta = 0.1, 0.5$ and 1) are presented in Table 4. Detailed results for a fixed value of $\theta = 1/2$ are summarized in Tables 9 and 10 in Appendix F.

	$\beta = 0.1$ ($\theta \in [0.366, 0.678]$)	$\beta = 0.5$ ($\theta \in [0.13, 0.667]$)	$\beta = 1$ ($\theta \in [0.094, 0.656]$)
Final stock levels higher in FB	$\theta \in [0.366, 0.678]$	$\theta \in [0.438, 0.667]$	$\theta \in [0.637, 0.656]$
Follower's total profit higher in FB	$\theta \in [0.366, 0.678]$	$\theta \in [0.413, 0.667]$	$\theta \in [0.654, 0.656]$
Leader's total profit higher in FB	Never	Never	Never

Table 4: Key results of numerical simulations concerning the weight assigned to agricultural use (θ) by the leader and for different discount factors ($\beta = 0.1, 0.5$ and 1). Feasible sets are presented in parentheses.

Being the case $\beta = 1$ the scenario used in the previous sections (the no-discount case), a lower value of β ($0 < \beta < 1$) represents a positive discount case applied to the follower's second-period profits – that is, when the new user enters in the game – and therefore to a less competitive scenario for the available stock in the second period. More specifically, when β decreases, i.e., when the follower becomes more impatient, the follower's fear of a water shortage in the second period is reduced in the noncommitment case with respect to the commitment case, leading to higher extraction.[10] As a result, higher profits are obtained in the feedback case than in the open-loop scenario. In fact, the second row of Table 4 reveals that the interval in which the follower obtains higher profits in the feedback case than in the open-loop case expands as β decreases. A very impatient follower (i.e., a low β) will always prefer not to commit with the leader as the follower is the only user of the resource in the first period. In contrast, the leader will always prefer committed strategies (see the third row of the Table). Therefore, as explained in the previous section, some circumstances in which the leader's and follower's interests are not aligned can arise. As a consequence, the follower could force the leader to play uncommitted strategies in the second period by playing uncommitted strategies in the first period. In this case, since the

[10]For the numerical example in Appendix F, when $\beta = 1$, the follower's total extraction is higher by 0.3 units in the open-loop case than in the feedback case, while the opposite arises when $\beta = 0.1$, with the total extractions lower by 0.63 units in the open-loop case than in the feedback case.

leader's profits could be seen as a measure of social welfare, the strategies implemented would not coincide with the strategies that allow to attain a higher social welfare.

The findings have important policy implications regarding the strategies that would ultimately be implemented. When $\beta = 1$ (the case without discount), the follower is more patient about extraction behavior over the two periods; therefore, the leader's and the follower's preferences are generally aligned for any value of θ in the feasible set (i.e., for $\theta \in [0.094, 0.656]$). Committed strategies would be implemented except in the case of an extremely high θ value (i.e., for $\theta \in [0.654, 0.656]$). In the case of an intermediate discount factor, $\beta = 0.5$, the follower is more impatient and the strategies implemented are highly dependent on the weight assigned by the leader to agricultural use. For lower values of θ (i.e., $\theta \in [0.13, 0.413]$) the leader's and follower's interests are aligned, and both achieve higher profits in the open-loop case. For higher values of θ (i.e., $\theta \in [0.413, 0.667]$, the leader's and follower's interests diverge and the follower would compel the leader to play uncommitted strategies. To avoid this conflict and ensure that committed strategies are implemented, the leader should not assign a high weight to the agricultural user's profits in the objective function. Finally, in the case of a very impatient follower (a low value of β, $\beta = 0.1$), the leader's and follower's interests never coincide. Uncommitted strategies would then be implemented to the detriment of social welfare.

6 Conclusions and extensions

In this study, we investigate the challenge of groundwater resource exploitation that is primarily used for a unique purpose (irrigation) and faces the exceptional entrance of a new use in the system, indicating a potential problem of water scarcity for the agricultural resource user. Therefore, a water agency is required to manage how much groundwater can be extracted for the new (nonagricultural) user. To model this circumstance, we construct a two-stage discrete Stackelberg game in which the leader (the water agency) only intervenes when the new use occurs (in the second stage), and the follower is an agent who makes decisions regarding agricultural use. This study analyzes and compares the extraction behaviors of the different agents (the water agency and the decision-agent for the agricultural use) for different Stackelberg equilibria that represent various commitment behaviors and the consequences of these extraction policies for the final state of the resource and agents' profits. We compute and compare open-loop (commitment solution) and feedback (noncommitment) equilibria.

First, theoretical results are provided for the case in which the leader weighs the profits from agricultural use and the alternative/new use equally. We analytically demonstrate that the leader's extraction behavior in the second period is more aggressive in the feedback case than in the open-loop case. Hence, the follower in the feedback case adapts earlier to anticipate extraction losses of the second period due to competition with the leader by augmenting extraction in the first period in comparison with the open-loop case. In other words, the follower's extraction behavior is more (less) aggressive in the feedback case than in the open-loop case in the first (second) period. This results in lower total extractions, or equivalently, higher final stock levels in the commitment case than in the non-commitment case. These theoretical results align with the existing literature that considers simultaneous play and characterizes Nash equilibria (Negri (1989), Rubio and Casino (2001), de Frutos Cachorro et al. (2019)).

We then perform numerical simulations to examine the robustness of the previous results when relaxing the previous assumption, $\theta = \frac{1}{2}$, conducting robustness analyses for different weights assigned by the leader to the agricultural use. We also carry out sensitivity analyses for other important parameters of the model related to the agents' revenue and the leader's environmental concern. First, we find that the theoretical results regarding final stock levels remain qualitatively unchanged when the weight assigned by the leader to the profits from agricultural use is lower than that assigned to the new use profits ($\theta < \frac{1}{2}$). In contrast, the simulated results demonstrate that when the leader assigns a higher weight to the profits from agricultural use than those from the alternative use ($\theta > \frac{1}{2}$), noncommitment strategies could result in higher final stock levels than commitment strategies. This is primarily due to reduction in the fear of water shortage from the decision-agent for agricultural use in the noncommitment case and a consequent decrease in total extraction over the two periods with respect to the commitment case. This result is obtained when the follower's productivity is high enough to compete with the new use or when the marginal revenue of the alternative use is low enough (i.e., for a less competitive new use).

The simulated results also show that noncommitment strategies could be more favorable for the sustainability of the resource (i.e., higher stock levels at the end of the second period) and could also be more profitable for the follower in comparison to commitment strategies. The rationale for this interesting result could be that the leader allows for more conservative extraction behavior for the new use by assigning a higher weight to agricultural use profits than alternative use profits and valuing the sustainability of the resource. This opens more possibilities for the decision-agent for agricultural use to extract and accumulate profits in the noncommitment case in comparison to the commitment case.

Nie (2005) obtains a similar result for the leader's profits. However, since our problem can be seen as a "degenerated Stackelberg game", in the sense that the leader only extracts in the second period, in contrast to Nie (2005), the leader always obtains higher profits in the commitment case than in the noncommitment case. The same result is achieved for the profits of the decision-agent of the agricultural use, when the weight assigned by the leader to the profits from agricultural use is lower or equal to that assigned to the new use profits. Consequently, in this sense, these results are closer to those that are often found for Nash equilibria (de Frutos Cachorro et al. (2019)).

Finally, we extend our analysis by introducing the possibility that the follower applies a discount to the second-period profits in the objective function. The numerical results demonstrate that when the follower has a low or middle level of impatience and the leader assigns a high weight to the agricultural profits, uncommitted strategies are more favorable than committed strategies in terms of final resource stock and the regular user's profits. The same result is obtained for a very impatient follower, and is independent of the weight assigned to the agricultural use in the leader's objective. In any case, the leader will always prefer committed strategies.

In conclusion, committed strategies are continuously more profitable for the leader; however, some circumstances can occur in which uncommitted strategies produce higher profits for the follower than committed strategies. Depending on the discount factor applied by the follower (i.e., level of impatience) and/or the weight assigned to the agricultural profits by the leader, the leader's and follower's interests may not be aligned; therefore, as the only user of the resource in the first period, the follower could compel the leader to play uncommitted strategies in the second period by playing uncommitted strategies in the first period. Consequently, one of the primary policy implications of our study is that to ensure that committed strategies are implemented, the leader must avoid assigning too much weight to the agricultural profits in the objective function when the follower has a low or middle level of impatience.

Several possible research extensions emerge from this study. First, as we consider the follower to be the unique decision-agent for agricultural use, it could be interesting to introduce different followers such as multiple heterogeneous farmers, playing simultaneously à la Nash between them, and à la Stackelberg with the leader. We could also investigate the corresponding efficient solution to our problem in which the leader could be a social planner who makes all the extraction decisions considering the same objective function, to compute and analyze potential policy implications. Finally, we could apply our theoretical model to a real case using available data.

A Study of time-consistency

To verify the time-inconsistency of the open-loop equilibrium and the time-consistency of the feedback equilibrium in our model, we use the pure definition of time-consistency described in Kydland and Prescott (1977) and adapted to our setup. A policy plan g_{2l} is consistent if, for $t = 2$, g_{2l} maximizes the leader's objective function, taking as given previous decisions, and the strategy selected coincides with the optimal decision rule.

In the open-loop case, the problem the leader faces at $t = 2$ is described in (10), where the follower's strategies can be expressed as functions of g_{2l} as follows:

$$g_{1f} = \tilde{g}_{1f}(g_{2l}), \tag{24}$$

$$g_{2f} = \tilde{g}_{2f}(g_{2l}). \tag{25}$$

Denoting the leader's objective function as $\bar{\Pi}_l^{OL}(g_{1f}, g_{2f}, g_{2l})$, the leader aims to find g_{2l} that maximizes the objective, subject to restrictions (24) and (25).

The necessary condition for an interior solution is as follows:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0 \iff \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \frac{\partial \tilde{g}_{1f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2f}} \frac{\partial \tilde{g}_{2f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0. \tag{26}$$

If past decision (g_{1f}) is given, the previous necessary condition is the following:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0 \iff \cancel{\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \frac{\partial \tilde{g}_{1f}}{\partial g_{2l}}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2f}} \frac{\partial \tilde{g}_{2f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0. \tag{27}$$

The leader's strategy is time-consistent if the conditions in (26) and (27) coincide, i.e., if the following occurs:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \underbrace{\frac{\partial \tilde{g}_{1f}}{\partial g_{2l}}}_{\neq 0} = 0 \iff \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} = 0.$$

From equations (6), (8), and (10), we can easily demonstrate the following:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} = -\theta c(g_{1f} + g_{2f}) - (1 - \theta) c g_{2l} < 0.$$

This demonstrates that the open-loop equilibrium described in this study cannot be time-consistent.

In the feedback case, the problem that the leader must solve at $t = 2$ is described in (17), where the follower's strategy at $t = 2$ can be expressed as a function of g_{1f} and g_{2l}

as follows:

$$g_{2f} = \hat{g}_{2f}(g_{1f}, g_{2l}). \tag{28}$$

Denoting the leader's objective function as $\hat{\pi}_l^{FB}(g_{1f}, g_{2f}, g_{2l})$, the leader aims to find g_{2l} to maximize the objective, subject to restriction (28).

Then, the necessary condition for an interior solution is the following:

$$\frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2l}} = 0 \iff \frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2f}} \frac{\partial \hat{g}_{2f}}{\partial g_{2l}} + \frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2l}} = 0. \tag{29}$$

This expression clearly coincides with the necessary condition when g_{1f} is given; thus, the leader's plan is time-consistent in the feedback information case.

B Open-loop Stackelberg equilibrium

B.1 Derivation of the open-loop Stackelberg equilibrium

The follower's objective function in (9), representing the sum of profits over the two periods once G_1 and G_2 have been replaced by their expressions in (1) and (2), are as follows:

$$\bar{\Pi}_f(g_{1f}, g_{2f}, g_{2l}) = g_{2f}(a_f + c(G_0 + 2r - g_{1f} - g_{2f} - g_{2l}) - z) + (a_f - (z - c(G_0 - g_{1f} + r)))g_{1f}.$$

Assuming an interior solution, maximizing $\bar{\Pi}_f(g_{1f}, g_{2f}, g_{2l})$ with respect to g_{1f} and g_{2f} gives the follower's best-reaction function as follows:

$$\tilde{g}_{1f}(g_{2l}) = \frac{(b_f + c)(a_f + cG_0 - z) + c(b_f r + c g_{2l})}{(b_f + c)(b_f + 3c)}, \tag{30}$$

$$\tilde{g}_{2f}(g_{2l}) = \frac{(b_f + c)(a_f + cG_0 - z) + c(b_f(2r - g_{2l}) + c(3r - 2g_{2l}))}{(b_f + c)(b_f + 3c)}. \tag{31}$$

The follower's optimal two-period profits as a function of g_{2l} are as follows:

$$\begin{aligned}\bar{\Pi}_f(\tilde{g}_{1f}(g_{2l}), \tilde{g}_{2f}(g_{2l}), g_{2l}) = \frac{1}{2(b_f + c)(b_f + 3c)} &\{2(a_f - z)(b_f + c)(a_f + c(2G_0 - g_{2l} + 3r) - z) \\ &+ c^2(b_f + c)\left(2G_0^2 - 2G_0 g_{2l} + 6G_0 r + g_{2l}^2 - 4g_{2l} r + 5r^2\right) + c^3(g_{2l} - r)^2\}.\end{aligned}$$

Then, the leader's objective in (10) becomes the following:

$$\theta \bar{\Pi}_f(\tilde{g}_{1f}(g_{2l}), \tilde{g}_{2f}(g_{2l}), g_{2l}) + (1 - \theta)\Pi_{2l}(g_{2l}, G_2) + AG_2, \tag{32}$$

where once the follower's best-reaction functions have been substituted, $\Pi_{2l}(g_{2l}, G_2)$ and AG_2 are as follows:

$$\begin{aligned}
\Pi_{2l}(g_{2l}, G_2) &= \frac{g_{2l}}{b_f + 3c}(c(-2a_f+(b_f+c)(G_0-g_{2l}+2r)+c(r-g_{2l}))+a_l(b_f+3c)-z(b_f+c)),\\
AG_2 &= \frac{A}{b_f + 3c}(-2a_f + (b_f + c)(G_0 - g_{2l} + 2r) + c(r - g_{2l}) + 2z).
\end{aligned}$$

Assuming an interior solution, the maximization of (32) with respect to g_{2l}, gives the leader's optimal strategy, which is expressed as follows:

$$\begin{aligned}
g_{2l}^{OL} &= \frac{1}{c(b_f + 2c)(2b_f(1-\theta) + c(2-3\theta))}\{(b_f + c)\left[cr(2b_f + 3c) + a_l(1-\theta)(b_f + 3c)\right.\\
&\quad -A(b_f + 2c) - 2a_f c - (b_f + c)(z - cG_0) + \theta((b_f + 2c)(z - cG_0) + a_f c)]\\
&\quad -c\theta r(b_f + 2c)(2b_f + 3c)\}\,. \qquad (33)
\end{aligned}$$

The expression in (13) corresponds to the above expression, which is particularized at $\theta = 1/2$.

The follower's optimal strategies are obtained by replacing g_{2l} with expression (33) in the follower's best-reaction functions in (30) and (31), which are expressed as follows:

$$\begin{aligned}
g_{1f}^{OL} &= \frac{1}{(b_f + 2c)(b_f + 3c)(2b_f(1-\theta) + c(2-3\theta))}\{c(a_l(b_f + 3c) - A(b_f + 2c))\\
&\quad +2a_f\left(b_f^2 + 3b_f c + c^2\right) - a_f\theta(b_f + c)(2b_f + 5c) - a_l c\theta(b_f + 3c)\\
&\quad -\left(2b_f^2 + 7b_f c + 5c^2\right)(z - cG_0) + cr\left(2b_f^2 + 6b_f c + 3c^2\right)\\
&\quad -(b_f + 2c)(2b_f c(G_0 + r) - 2b_f z + c(4cG_0 + 3cr - 4z))\}\,,\\
g_{2f}^{OL} &= \frac{(1-\theta)(2a_f(b_f+2c)-a_l(b_f+3c)+(b_f+c)(cG_0-z)+cr(2b_f+3c))-A(b_f+2c)}{(b_f + 3c)(2b_f(1-\theta) + c(2-3\theta))}.
\end{aligned}$$

We obtain the follower's optimal strategies in (11) and (12) by replacing θ by 1/2 in the expressions above.

The optimal profits of the leader and the follower can be obtained by replacing the optimal extraction strategies in the agents' profit functions.

B.2 Concavity conditions

The concavity of the follower's objective function in (9) with respect to the decision variables g_{1f} and g_{2f} is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix are $h_{11} = -b_f - 2c, h_{12} = -c, h_{21} = -c$, and

$h_{22} = -b_f - 2c$; therefore, $h_{11} < 0$ and $h_{11}h_{22} - h_{12}h_{21} = b_f^2 + 4b_f c + 3c^2 > 0$, indicating that the quadratic form is negative definite and the follower's objective function is strictly concave.

The follower's best-responses to g_{2l} are given by (30) and (31), provided that these expressions are positive. (30) is always positive under condition $a_f > z$ (one of the conditions that we impose to ensure the positivity of extraction decisions and state variables; see Condition 1 in Appendix D.1), and (31) is positive if $g_{2l} < \frac{(b_f+c)(a_f+cG_0-z)+cr(2b_f+3r)}{c(b_f+2c)}$. The leader is interested in the follower's positive extractions and maximizes (32) under the last condition. The concavity of the leader's objective function with respect to decision variable g_{2l} is ensured if the second derivative of this function is negative with respect to g_{2l}. The sign of this derivative is given by the sign of $2b_f(-1+\theta)+c(-2+3\theta)$. Therefore, the concavity of the leader's objective function requires $\theta < (2b_f + 2c)/(2b_f + 3c)$ and $g_{2l}^{OL} < \frac{(b_f+c)(a_f+cG_0-z)+cr(2b_f+3r)}{c(b_f+2c)}$.

Notably, for $\theta = 1/2$, the expression $2b_f(-1+\theta)+c(-2+3\theta)$ is always negative.

C Feedback Stackelberg equilibrium

C.1 Derivation of the feedback Stackelberg equilibrium

We determine the feedback Stackelberg equilibrium using backward induction.

In the first stage, the follower decides the amount of extraction in period two and solves the problem in (16). Once G_1 and G_2 are replaced by their expressions in (1) and (2), the follower's objective function in the second period is as follows:

$$\widetilde{\Pi}_{2f}(g_{1f}, g_{2f}, g_{2l}) = g_{2f}(a_f + c(G_0 - g_{1f} - g_{2f} - g_{2l} + 2r) - z).$$

Assuming an interior solution, maximizing $\widetilde{\Pi}_{2f}(g_{1f}, g_{2f}, g_{2l})$ with respect to g_{2f} gives the follower's second-period best-reaction function as follows:

$$\hat{g}_{2f}(g_{2l}, g_{1f}) = \frac{a_f + c(G_0 - g_{1f} - g_{2l} + 2r) - z}{b_f + 2c}. \tag{34}$$

The follower's optimal second-period profits are as follows:

$$\widetilde{\Pi}_{2f}(g_{1f}, \hat{g}_{2f}(g_{2l}, g_{1f}), g_{2l}) = \frac{(a_f + c(G_0 - g_{1f} - g_{2l} + 2r) - z)^2}{2(b_f + 2c)}.$$

In the second step, the leader determines the extraction in period two, considering the follower's extraction in this period that is given in (34). Therefore, the leader's objective in (17) is the following:

$$\theta\widetilde{\Pi}_{2f}(g_{1f}, \hat{g}_{2f}(g_{2l}, g_{1f}), g_{2l}) + (1-\theta)\Pi_{2l}(g_{2l}, G_2) + AG_2, \tag{35}$$

where $\Pi_{2l}(g_{2l}, G_2)$ and AG_2 once G_1, G_2, and g_{2f} are replaced by their expression in (1), (2), and (34), respectively, are as follows:

$$\begin{aligned}
\widetilde{\Pi}_{2l}(g_{2l}, g_{1f}) &= \frac{g_{2l}(-a_f c + a_l(b_f + 2c) + (b_f + c)(c(G_0 - g_{1f} - g_{2l} + 2r) - z))}{b_f + 2c}, \\
AG_2 &= \frac{A(-a_f + b_f(G_0 - g_{1f} - g_{2l} + 2r) + cG_0 - cg_{1f} - cg_{2l} + 2cr + z)}{b_f + 2c}.
\end{aligned}$$

Assuming an interior solution, maximizing (35) with respect to g_{2l} gives the leader's extraction in the second period as a function of the follower's extraction in the first period, as follows:

$$\hat{g}_{2l}(g_{1f}) = \frac{A(b_f+c)+a_f c+a_l(\theta-1)(b_f+2c)+(b_f(\theta-1)+c(2\theta-1))(c(G_0-g_{1f}+2r)-z)}{c(\theta(2b_f+3c)-2(b_f+c))}. \tag{36}$$

In the third and final step, the follower decides the extraction in period one considering the leader's reaction function in the second period given in (36). The follower's objective function in the first period is the following:

$$\begin{aligned}
\widetilde{\Pi}_{1f}(g_{1f}) &= \frac{(A(b_f+c)-(\theta-1)(a_f(2b_f+3c)-a_l(b_f+2c)+(b_f+c)(c(G_0-g_{1f}+2r)-z)))^2}{2(b_f+2c)(2b_f(1-\theta)+c(2-3\theta))^2} \\
&+ a_f g_{1f} - \frac{1}{2} g_{1f}(b_f g_{1f} - 2c(G_0 - g_{1f} + r) + 2z).
\end{aligned}$$

Assuming an interior solution, the maximization of $\widetilde{\Pi}_{1f}(g_{1f})$ with respect to g_{1f}, gives the optimal strategy, g_{1f}^{FB}, as follows:

$$g_{1f}^{FB} = \frac{M_3 b_f^3 + M_2 b_f^2 + M_1 b_f + M_0}{\left(2b_f^2(\theta-1)+b_f c(6\theta-5)+c^2(5\theta-3)\right)\left(2b_f^2(\theta-1)+b_f c(8\theta-7)+c^2(7\theta-5)\right)}, \tag{37}$$

where

$$\begin{aligned}
M_3 &= 4(\theta-1)^2(a_f + c(G_0 + r) - z), \\
M_2 &= c(\theta-1)(A + 2a_f(9\theta-7) + \theta(a_l + 19(cG_0 - z) + 18cr) - a_l - 15(cG_0 - z) - 14cr), \\
M_1 &= c^2(2A(\theta-1) + \theta(\theta(28a_f+3a_l+31(cG_0-z)+29cr)-42a_f-6a_l-48(cG_0-z)-44cr) \\
&+ 15a_f + 3a_l + 2(9(cG_0 - z) + 8cr)), \\
M_0 &= c^3\left(A(\theta-1)+\theta^2(15a_f+2a_l+17(cG_0-z)+16cr)-2\theta(9a_f+2a_l+11(cG_0-z)+10cr)\right. \\
&+ \left. 5a_f + 2a_l + 7(cG_0 - z) + 6cr\right).
\end{aligned}$$

(19) corresponds to (37) for the particular case $\theta = 1/2$.

The optimal strategy for the leader's extraction g_{2l}^{FB} is obtained by replacing g_{1f} with the expression in (37) in the leader's second-period best-reaction function in (36) as follows:

$$g_{2l}^{FB} = \frac{N_4 b_f^4 + N_3 b_f^3 + N_2 b_f^2 + N_1 b_f + N_0}{c\left(2b_f^2(\theta-1)+b_f c(6\theta-5)+c^2(5\theta-3)\right)\left(2b_f^2(\theta-1)+b_f c(8\theta-7)+c^2(7\theta-5)\right)}, \quad (38)$$

where

$$\begin{aligned}
N_4 &= 2(\theta-1)(A+(\theta-1)(a_l+cG_0+2cr-z)),\\
N_3 &= c(A(13\theta-12)+(\theta-1)(-2a_f(\theta-2)+\theta(15a_l+13(cG_0-z)+28cr)\\
&- 2(7a_l+5(cG_0-z)+11cr))),\\
N_2 &= c^2(5A(6\theta-5)+\theta(\theta(-10a_f+41a_l+31(cG_0-z)+73cr)+28a_f-76a_l\\
&- 48(cG_0-z)-115cr)-17a_f+35a_l+18(cG_0-z)+44cr),\\
N_1 &= c^3(A(30\theta-22)+\theta(\theta(-17a_f+49a_l+32(cG_0-z)+84cr)+43a_f-86a_l\\
&- 43(cG_0-z)-115cr)-23a_f+37a_l+14(cG_0-z)+38cr),\\
N_0 &= c^4(A(11\theta-7)-2a_f(\theta(5\theta-11)+5)+2a_l(\theta-1)(11\theta-7)\\
&+ 2(\theta(6\theta-7)+2)(c(G_0+3r)-z)).
\end{aligned}$$

(21) is (38), which is particularized at $\theta = 1/2$.

Finally, the optimal strategy for the follower's extraction in the second period, g_{2f}^{FB} is obtained by replacing g_{1f} and g_{2l} with the expressions in (37) and (38), respectively, into the follower's second-period best-reaction function in (34) as follows:

$$g_{2f}^{FB} = \frac{P_4 b_f^4 + P_3 b_f^3 + P_2 b_f^2 + P_1 b_f + P_0}{\left(2b_f^2(\theta-1)+b_f c(6\theta-5)+c^2(5\theta-3)\right)\left(2b_f^2(\theta-1)+b_f c(8\theta-7)+c^2(7\theta-5)\right)}, \quad (39)$$

where

$$\begin{aligned}
P_3 &= 2(\theta-1)((\theta-1)(2a_f-a_l+cG_0+2cr-z)-A),\\
P_2 &= c(A(8-9\theta)+(\theta-1)(\theta(18a_f-11a_l+7(cG_0-z)+16cr)-2(8a_f-5a_l+3(cG_0-z)+7cr))),\\
P_1 &= c^2(A(10-13\theta)+(\theta-1)(\theta(28a_f-20a_l+8(cG_0-z)+21cr)-2(11a_f-8a_l+3(cG_0-z)+8cr))),\\
P_0 &= c^3(3\theta-2)((\theta-1)(5a_f-4a_l+cG_0+3cr-z)-2A).
\end{aligned}$$

(20) is obtained replacing $\theta = 1/2$ in (39).

The optimal profits of the leader and the follower are obtained by replacing the optimal extraction strategies in the agents' profit functions.

C.2 Concavity conditions

In the second period, the follower's objective function in (16) is strictly concave with respect to the decision variable g_{2f}, because $\frac{\partial^2 \Pi_{2f}}{\partial g_{2f}^2} = -b_f - 2c < 0$. The follower's best-response is given by (34), provided it is positive. Since we ask for positive solutions (in extractions and aquifer levels), $G_2 = G_0 - g_{1f} - g_{2l} - g_{2f} + 2r > 0$, that is, $G_0 - g_{1f} - g_{2l} + 2r > g_{2f}$. As $g_{2f} > 0$ and $a_f > z$ (see positivity Condition 1.A in Appendix D.1), we determine that (34) is positive.

In the second period, the concavity of the leader's objective function in (17) for the decision variable g_{2l} requires $\frac{\partial^2 \Pi_{2l}}{\partial g_{2l}^2} = \frac{c(2b_f(\theta-1)+c(3\theta-2))}{b_f+2c} < 0$. Therefore, this concavity condition reduces to $2b_f(\theta - 1) + c(3\theta - 2) < 0$. The leader's best response is (36), provided that this expression is positive. The denominator of (36) is negative, then (36) is positive if the numerator $A(b_f+c)+a_f c+a_l(\theta-1)(b_f+2c)+(b_f(\theta-1)+c(2\theta-1))(c(G_0-g_{1f}+2r)-z)$ is negative.

In the first period, the follower's objective function in (18) is strictly concave with respect to the decision variable g_{1f}, iff $c^2(\theta-1)^2(b_f+c)^2 - (b_f+2c)^2(2b_f(\theta-1)+c(3\theta-2))^2 < 0$.

It can be easily checked that the two conditions ensuring the concavity of the leader's and follower's objective functions are always satisfied for the particular case $\theta = 1/2$.

D Open-loop vs. Feedback Stackelberg equilibria

D.1 Positivity conditions

We next determine the conditions ensuring the positivity of agents' optimal strategies and aquifer states over the two periods for open-loop and feedback equilibria. We characterize these conditions under the assumptions that $\theta = 1/2$ and $a_l > a_f > z$ are satisfied.

The sufficient conditions for the positivity of the players' optimal strategies and aquifer states over the two periods are as follows:

Condition 1: **A**: $a_l > a_f > z$, **B**: $G_0c - 2A > 0$, **C**: $3a_l - 3a_f - 4A > 0$, **D**: $G_0c + 3cr + 4A - 4a_l + 2a_f + 2z > 0$, **E**: $G_0c + 3cr - 14A > 0$, **F**: $5G_0c + 14cr + 18A - 16a_l + 3a_f + 13z > 0$ and **G**: $9G_0c + 23cr + 28A - 21a_l - 2a_f + 23z > 0$.

In a first step, we demonstrate that Conditions 1.A, 1.B, 1.C, 1.D, and 1.E ensure that the agents' optimal strategies are positive over the two periods and for the two types of equilibria. Notably, if $A = 0$, which represents the extreme case where the regulator is not concerned about the aquifer stock at the end of the second period, conditions 1.A to 1.E reduce to Condition 1.A and $G_0c + 3cr - 4a_l + 5a_f - z > 0$.

- From (11),

$$g_{1f}^{OL} = \frac{2(G_0c+cr+a_f-z)b_f^2+c(6(G_0c-z)+5(cr+a_f)-2A+a_l)b_f+c^2(2(G_0c+z)-4A+3a_l-a_f)}{(b_f+2c)(b_f+3c)(2b_f+c)},$$

  and $G_0c-2A>0$ (Condition 1.B) implies $g_{1f}^{OL}>0$.

- From (12),

$$g_{2f}^{OL} = \frac{(G_0c+2cr+2A-a_l+2a_f-z)b_f+c(G_0c+3cr+4A-3a_l+4a_f-z)}{(b_f+3c)(2b_f+c)},$$

  and

$$\min(G_0c+2cr+2A-a_l+2a_f-z, G_0c+3cr+4A-3a_l+4a_f-z)>0, \quad (40)$$

  implies $g_{2f}^{OL}>0$.

  Because the following inequalities apply:

$$4(G_0c+2cr+2A-a_l+2a_f-z)-(G_0c+3cr+4A-4a_l+2a_f+2z)>0,$$
$$G_0c+3cr+4A-3a_l+4a_f-z-(G_0c+3cr+4A-4a_l+2a_f+2z)>0,$$

  from (40), we obtain $G_0c+3cr+4A-4a_l+2a_f+2z>0$ (Condition 1.D), which implies $g_{2f}^{OL}>0$.

- From (13),

$$g_{2l}^{OL} = \frac{(G_0c+2cr-2A+a_l-z)b_f^2+c(G_0c+3cr-6A+4a_l-3a_f-z)b_f+(3a_l-3a_f-4A)c^2}{c(b_f+2c)(2b_f+c)};$$

  therefore, condition

$$\min(G_0c+2cr-2A, G_0c+3cr-6A)>0 \quad (41)$$

  and condition $3a_l-3a_f-4A>0$ (Condition 1.C) imply $g_{2l}^{OL}>0$.

  Because the two following inequalities apply:

$$G_0c+2cr-2A-(G_0c-2A)>0,$$
$$G_0c+3cr-6A-(G_0c+3cr-14A)>0,$$

  we have that $G_0c-2A>0$ (Condition 1.B) and $G_0c+3cr-14A>0$ (Condition 1.E) imply (41).

- From (19),

$$g_{1f}^{FB} = \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{4(G_0c+cr+a_f-z)b_f^3\right.$$
$$+c(11(G_0c-z)+10cr-2A+a_l+10a_f)b_f^2+c^2(7(G_0c-z)+5cr-4A+3a_l+4a_f)b_f$$
$$\left.+c^3(G_0c-2A+2a_l-a_f-z))\right\},$$

and

$$\min(7G_0c+5cr-4A, G_0c-2A) > 0$$

implies $g_{1f}^{FB} > 0$.

Because $7G_0c+5cr-4A-2(G_0c-2A) > 0$, then $G_0c-2A > 0$ (Condition 1.A) implies $g_{1f}^{FB} > 0$.

- From (20),

$$g_{2f}^{FB} = \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{2(G_0c+2cr+2A-a_l+2a_f-z)b_f^3\right.$$
$$+c(5(G_0c-z)+10cr+14A-9a_l+14a_f)b_f^2+c^2(4(G_0c-z)+11cr+14A-12a_l+16a_f)b_f$$
$$\left.+c^3(G_0c+3cr+4A-4a_l+5a_f-z))\right\},$$

then $G_0c+3cr+4A-4a_l+2a_f+2z > 0$ implies $g_{2f}^{FB} > 0$ because the following inequalities apply:

$$\begin{aligned}
5G_0c+10cr+14A-9a_l+14a_f-5z &> 3(G_0c+3cr+4A-4a_l+2a_f+2z),\\
4G_0c+11cr+14A-12a_l+16a_f-4z &> 3(G_0c+3cr+4A-4a_l+2a_f+2z),\\
2G_0c+4cr+4A-2a_l+4a_f-2z &> (G_0c+3cr+4A-4a_l+2a_f+2z).
\end{aligned}$$

- From (21),

$$g_{2l}^{FB} = \frac{1}{c(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{2(G_0c+2cr-2A+a_l-z)b_f^4\right.$$
$$+c(7(G_0c-z)+16cr-22A+13a_l-6a_f)b_f^3+c^2(7(G_0c-z)+19cr-40A+29a_l-22a_f)b_f^2$$
$$\left.+c^3(2(G_0c-z)+6cr-28A+25a_l-23a_f)b_f+6c^4(a_l-a_f-A)\right\},$$

and

$$\min(2G_0c+4cr-4A, 7G_0c+19cr-40A, 2G_0c+6cr-28A, -A+a_l-a_f) > 0 \quad (42)$$

implies $g_{2l}^{FB} > 0$.

Because the following inequalities apply:

$$
\begin{aligned}
&2G_0c + 4cr - 4A - 2(G_0c - 2A) > 0, \\
&7G_0c + 19cr - 40A - 6(G_0c + 3cr - 14A) > 0, \\
&-A + a_l - a_f - 1/4(3a_l - 3a_f - 4A) > 0,
\end{aligned}
$$

from (42), we obtain $G_0c - 2A > 0$ (Condition 1.B), $G_0c + 3cr - 14A > 0$ (Condition 1.E), and $3a_l - 3a_f - 4A > 0$ (Condition 1.C), which imply $g_{2l}^{FB} > 0$.

In a second step, we determine the sufficient conditions that guarantee that the aquifer stocks are positive over the two periods. We first obtain the sufficient conditions ensuring a positive aquifer stock at the end of the second period.

- From (15),

$$
\begin{aligned}
G_2^{OL} \;=\; & \frac{1}{c(2b_f + c)(b_f + 3c)} \left\{(G_0c{+}2cr{+}2A{-}a_l{+}z)b_f^2{+}c(2G_0c{+}5cr{+}6A{-}4a_l{-}a_f{+}5z)b_f \right. \\
& \left. +c^2(G_0c{+}3cr{+}4A{-}3a_l{+}a_f{+}2z)\right\}.
\end{aligned}
$$

  Because

$$
G_0c{+}2cr{+}2A{-}a_l{+}z > \frac{1}{3}(2G_0c{+}5cr{+}6A{-}4a_l{-}a_f{+}5z),
$$

  fulfilling the following two conditions:

$$
2G_0c + 5cr + 6A - 4a_l - a_f + 5z > 0, \tag{43}
$$

$$
G_0c + 3cr + 4A - 3a_l + a_f + 2z > 0, \tag{44}
$$

  guarantees $G_2^{OL} > 0$.

- From (23),

$$
\begin{aligned}
G_2^{FB} \;=\; & \frac{1}{c(2b_f^2{+}4b_fc{+}c^2)(2b_f^2{+}6b_fc{+}3c^2)} \left\{2(G_0c{+}2cr{+}2A{-}a_l{+}z)b_f^4 \right. \\
& +c(7G_0c{+}16cr{+}18A{-}11a_l{-}2a_f{+}13z)b_f^3{+}c^2(9G_0c{+}23cr{+}28A{-}21a_l{-}2a_f{+}23z)b_f^2 \\
& \left. +c^3(5G_0c{+}14cr{+}18A{-}16a_l{+}3a_f{+}13z)b_f{+}c^4(G_0c{+}3cr{+}4A{-}4a_l{+}2a_f{+}2z)\right\}.
\end{aligned}
$$

  Because the following two inequalities apply:

$$
\begin{aligned}
&(G_0c + 2cr + 2A - a_l + z) - \frac{1}{3}(2G_0c + 5cr + 6A - 4a_l - a_f + 5z) > 0, \\
&7G_0c + 16cr + 18A - 11a_l - 2a_f + 13z > 5G_0c + 14cr + 18A - 16a_l + 3a_f + 13z,
\end{aligned}
$$

then condition (43) and the following three conditions:

$$5G_0c + 14cr + 18A - 16a_l + 3a_f + 13z > 0 \quad \text{(Condition 1.F)}, \tag{45}$$
$$9G_0c + 23cr + 28A - 21a_l - 2a_f + 23z > 0 \quad \text{(Condition 1.G)}, \tag{46}$$
$$G_0c + 3cr + 4A - 4a_l + 2a_f + 2z > 0 \quad \text{(Condition 1.D)} \tag{47}$$

imply $G_2^{FB} > 0$.

Because $G_0c+3cr+4A-3a_l+a_f+2z > G_0c+3cr+4A-4a_l+2a_f+2z$, considering (44), we can conclude that Conditions 1.D, 1.F, 1.G, and (43) ensure that G_2^{OL} and G_2^{FB} are positive.

We next determine the sufficient conditions ensuring a positive aquifer stock at the end of the first period.

- From (48),

$$\begin{aligned} G_1^{OL} \;=\; & \frac{1}{(2b_f+c)(b_f+2c)(b_f+3c)}\left\{2(G_0+r)b_f^3+(9(G_0c+cr)-2a_f+2z)b_f^2\right.\\ & \left.+c(11G_0c+12cr+2A-a_l-5a_f+6z)b_f+c^2(4G_0c+6cr+4A-3a_l+a_f+2z)\right\}. \end{aligned}$$

  Because

$$\begin{aligned} & 9G_0c + 9cr - 2a_f + 2z - (4G_0c + 6cr + 4A - 3a_l + a_f + 2z) \\ = \; & 5G_0c + 3cr - 4A + 3a_l - 3a_f > 5G_0c + 3cr - 4A > 0, \end{aligned}$$

  where the last inequality stems from Condition 1.B, we obtain the following two conditions:

$$4G_0c + 6cr + 4A - 3a_l + a_f + 2z > 0, \tag{48}$$
$$11G_0c + 12cr + 2A - a_l - 5a_f + 6z > 0, \tag{49}$$

  which, combined with Condition 1.B, imply that $G_1^{OL} > 0$.

- From (22),

$$\begin{aligned} G_1^{FB} \;=\; & \frac{1}{(2b_f^2+4b_fc+c^2)(2b_f^2+6b_fc+3c^2)}\left\{4(G_0+r)b_f^4+4(4G_0c+4cr-a_f+z)b_f^3\right.\\ & +c(21G_0c+22cr+2A-a_l-10a_f+11z)b_f^2+c^2(11G_0c+13cr+4A-3a_l-4a_f+7z)b_f\\ & \left.+c^3(2G_0c+3cr+2A-2a_l+a_f+z)\right\}. \end{aligned}$$

Because

$$
\begin{aligned}
& 4G_0c + 4cr - a_f + z - \frac{1}{2}(4G_0c + 6cr + 4A - 3a_l + a_f + 2z) \\
= \;& 2cG_0 + cr - \frac{3}{2}a_f + \frac{3}{2}a_l - 2A > 2cG_0 + cr - 2A > 0, \text{ and} \qquad (50) \\
& 2G_0c + 3cr + 2A - 2a_l + a_f + z - (G_0c + 3cr + 4A - 4a_l + 2a_f + 2z) \\
= \;& G_0c - 2A + 2a_l - a_f - z > 0, \qquad (51)
\end{aligned}
$$

where the last inequality in (50) and the inequality in (51) stem from Conditions 1.A and 1.B, we obtain the following two conditions:

$$
\begin{aligned}
21G_0c + 22cr + 2A - a_l - 10a_f + 11z > 0, \qquad (52) \\
11G_0c + 13cr + 4A - 3a_l - 4a_f + 7z > 0, \qquad (53)
\end{aligned}
$$

which, combined with Conditions 1.A, 1.B, and 1.D, imply that $G_1^{FB} > 0$.

Considering (43), (49), and (52), and because

$$
\begin{aligned}
& 2(11G_0c + 12cr + 2A - a_l - 5a_f + 6z) - (21G_0c + 22cr + 2A - a_l - 10a_f + 11z) \\
= \;& G_0c + 2cr + 2A - a_l + z > \frac{1}{3}(2G_0c + 5cr + 6A - 4a_l - a_f + 5z) > 0,
\end{aligned}
$$

we can conclude that Conditions 1.A, 1.B, 1.D, (52), and (43) ensure the fulfillment of condition (49).

Considering conditions (43), (48), and (53), we obtain the following:

$$
\begin{aligned}
& (11G_0c + 13cr + 4A - 3a_l - 4a_f + 7z) - 2(2G_0c + 5cr + 6A - 4a_l - a_f + 5z) \\
= \;& 7G_0c + 3cr - 8A + 5a_l - 2a_f - 3z > 0, \\
& (4G_0c + 6cr + 4A - 3a_l + a_f + 2z) - (2G_0c + 5cr + 6A - 4a_l - a_f + 5z) \\
= \;& 2G_0c + cr - 2A + a_l + 2a_f - 3z > 0,
\end{aligned}
$$

where the inequalities stem from Conditions 1.A and 1.B; therefore, condition (43) implies conditions (48) and (53).

Furthermore, under Conditions 1.A and 1.B, the following inequality applies:

$$
\begin{aligned}
& 23(21G_0c + 22cr + 2A - a_l - 10a_f + 11z) - 19(9G_0c + 23cr + 28A - 21a_l - 2a_f + 23z) \\
= \;& 312G_0c + 69cr - 486A + 376a_l - 192a_f - 184z > 0,
\end{aligned}
$$

then Condition 1.G (46) implies condition (52).

Because under Conditions 1.A and 1.B,

$$
\begin{aligned}
& 46(2G_0c + 5cr + 6A - 4a_l - a_f + 5z) - 10(9G_0c + 23cr + 28A - 21a_l - 2a_f + 23z) \\
= \; & 2G_0c - 4A + 26a_l - 26a_f > 0,
\end{aligned}
$$

we determine that Condition 1.G (46) implies condition (43).

In summary, Conditions 1.A to 1.G ensure that the aquifer stock is positive at the end of the two periods.

In this section, all proofs were performed under Condition 1 to ensure the comparison between the different equilibria.

D.2 Proof of Proposition 1

$$
g_{1f}^{FB} - g_{1f}^{OL} = \frac{A_5 b_f^5 + A_4 b_f^4 + A_3 b_f^3 + A_2 b_f^2 + A_1 b_f + A_0}{(2b_f + c)(b_f + 2c)(b_f + 3c)\left(2b_f^2 + 4b_f c + c^2\right)\left(2b_f^2 + 6b_f c + 3c^2\right)}, \tag{54}
$$

where

$$
\begin{aligned}
A_5 &= 2c(G_0c + 2cr + 2A - a_l + 2a_f - z), \\
A_4 &= c^2(11G_0c + 24cr + 26A - 15a_l + 26a_f - 11z), \\
A_3 &= c^3(22G_0c + 53cr + 62A - 38a_l + 60a_f - 22z), \\
A_2 &= c^4(18G_0c + 49cr + 62A - 35a_l + 53a_f - 18z), \\
A_1 &= 5c^5(G_0c + 3cr + 4A - a_l + 2a_f - z), \\
A_0 &= 3c^6(a_l - a_f).
\end{aligned}
$$

We next demonstrate that under Condition 1, all the coefficients $A_i, i = 0, 1, \ldots, 5$ are positive; hence, $g_{1f}^{FB} - g_{1f}^{OL} > 0$.

A_5 is positive because the following inequalities apply:

$$
\begin{aligned}
& G_0c+2cr+2A-a_l+2a_f-z > G_0c+2cr+2A-a_l+a_f > G_0c+2cr+2A-a_l+z \\
& \quad > \frac{1}{3}(2G_0c + 5cr + 6A - 4a_l - a_f + 5z) > 0.
\end{aligned} \tag{55}
$$

The first two inequalities apply because $a_f > z$ (Condition 1.A), the third one applies because $a_l > a_f > z$ (Condition 1.A), and the last one applies because we already demonstrated that Condition 1.G (46) implies condition (43) ($2G_0c+5cr+6A-4a_l-a_f+5z > 0$) in section D.1.

A_4 is positive because the following inequality applies:

$$\begin{aligned} & 11G_0c + 24cr + 26A - 15a_l + 26a_f - 11z - 4(G_0c + 3cr + 4A - 4a_l + 2a_f + 2z) \\ = \; & 7G_0c + 12cr + 10A + a_l + 18a_f - 19z > 0, \end{aligned}$$

where the mathematical expression in Condition 1.D is used in the term in round brackets in the first line and the last inequality is implied by inequalities $a_l > a_f > z$ (Condition 1.A).

A_3 is positive because the following inequality applies:

$$\begin{aligned} & 22G_0c + 53cr + 62A - 38a_l + 60a_f - 22z - 10(G_0c + 3cr + 4A - 4a_l + 2a_f + 2z) \\ = \; & 12G_0c + 23cr + 22A + 2a_l + 40a_f - 42z > 0, \end{aligned} \tag{56}$$

where, once again, the mathematical expression in Condition 1.D is used in the term in round brackets in the first line and the last inequality is implied by inequalities $a_l > a_f > z$ (Condition 1.A).

A_2 is positive because the following inequality applies:

$$\begin{aligned} & 18G_0c+49cr+62A-35a_l+53a_f-18z-10(G_0c+3cr+4A-4a_l+2a_f+2z) \\ = \; & 8G_0c + 19cr + 22A + 5a_l + 33a_f - 38z > 0, \end{aligned}$$

where, for a third time, the mathematical expression in Condition 1.D is used in the term in round brackets in the first line and the last inequality is implied by inequalities $a_l > a_f > z$ (Condition 1.A).

A_1 is positive because the following inequality applies:

$$G_0c + 3cr + 4A - a_l + 2a_f - z > G_0c + 2cr + 2A - a_l + 2a_f - z > 0,$$

where last inequality stems from (55).

Therefore, we can conclude that the difference $g_{1f}^{FB} - g_{1f}^{OL}$ in (54) is positive.

$$g_{2f}^{OL} - g_{2f}^{FB} = \frac{B_3b_f^3 + B_2b_f^2 + B_1b_f + B_0}{(2b_f + c)(b_f + 3c)\left(2b_f^2 + 4b_fc + c^2\right)\left(2b_f^2 + 6b_fc + 3c^2\right)}, \tag{57}$$

where

$$\begin{aligned} B_3 &= c^2(3G_0c + 6cr + 6A + a_l + 2a_f - 3z), \\ B_2 &= c^3(5G_0c + 13cr + 16A + 5a_l - 5z), \\ B_1 &= c^4(2G_0c + 6cr + 8A + 7a_l - 5a_f - 2z), \\ B_0 &= 3c^5(a_l - a_f). \end{aligned}$$

Under condition 1.A ($a_l > a_f > z$), coefficients $B_i, i = 0, 1, \ldots, 3$ are positive; hence, $g_{2f}^{OL} - g_{2f}^{FB} > 0$.

$$g_{2l}^{OL} - g_{2l}^{FB} = -c\frac{C_4 b_f^4 + C_3 b_f^3 + C_2 b_f^2 + C_1 b_f + C_0}{(2b_f + c)(b_f + 2c)\left(2b_f^2 + 4b_f c + c^2\right)\left(2b_f^2 + 6b_f c + 3c^2\right)}, \tag{58}$$

where

$$\begin{aligned}
C_4 &= G_0 c + 2cr + 2A + 3a_l - 2a_f - z, \\
C_3 &= c(3G_0 c + 7cr + 8A + 15a_l - 12a_f - 3z), \\
C_2 &= c^2(3G_0 c + 8cr + 10A + 24a_l - 21a_f - 3z), \\
C_1 &= c^3(G_0 c + 3cr + 4A + 14a_l - 13a_f - z), \\
C_0 &= 3c^4(a_l - a_f).
\end{aligned}$$

Under condition 1.A ($a_l > a_f > z$), coefficients $C_i, i = 0, 1, \ldots, 4$ are positive; hence, $g_{2l}^{OL} - g_{2l}^{FB} < 0$.

D.3 Proof of Proposition 2

The difference of total extractions under the open-loop and feedback scenarios is as follows:

$$\text{Total}^{OL} - \text{Total}^{FB} = -\frac{c(b_f + c)(D_3 b_f^3 + D_2 b_f^2 + D_1 b_f + D_0)}{(2b_f + c)(b_f + 3c)\left(2b_f^2 + 4b_f c + c^2\right)\left(2b_f^2 + 6b_f c + 3c^2\right)}, \tag{59}$$

where

$$\begin{aligned}
D_3 &= 6A + 2a_f + a_l + 3cG_0 + 6cr - 3z > 0, \\
D_2 &= c(16A + 5a_l + 5cG_0 + 13cr - 5z) > 0, \\
D_1 &= c^2(8A - 5a_f + 7a_l + 2cG_0 + 6cr - 2z) > 0, \text{ and} \\
D_0 &= 3c^3(a_l - a_f) > 0.
\end{aligned}$$

The signs of the expressions above are from Condition 1.A; therefore, from (59), the sign of the difference $\text{Total}^{OL} - \text{Total}^{FB}$ is negative.

E Numerical results

E.1 Sensitivity analysis with respect to parameters b_f and A

Column		1	2	3	4	5	6	7	8	9	10	11	12
		$b_f = 0.01$				$b_f = 0.1$				$b_f = 1$			
		Follower		Leader		Follower		Leader		Follower		Leader	
Period		1	2	2	Total	1	2	2	Total	1	2	2	Total
$\theta = 0.4$	A=0	n.a	n.a	n.a	n.a	+	−	+	+	+	−	+	+
	A=0.07	n.a	n.a	n.a	n.a	+	−	+	+	+	−	+	+
$\theta = 1/2$	A=0	+	−	+	+	+	−	+	+	+	−	+	+
	A=0.07	+	−	+	+	+	−	+	+	+	−	+	+
$\theta = 0.581$	A=0	−	−	+	+	+	−	+	+	+	−	+	+
	A=0.07	−	−	+	+	+	−	+	+	+	−	+	+
$\theta = 0.59$	A=0	−	+	+	−	+	−	+	+	+	−	+	+
	A=0.07	−	+	−	−	+	−	+	+	+	−	+	+
$\theta = 0.655$	A=0	n.a.	n.a	n.a	n.a	−	+	−	−	+	−	+	+
	A=0.07	n.a	n.a	n.a	n.a	−	+	−	−	+	−	+	+

Table 5: Sign of differences between feedback and open-loop extraction results: + means $FB > OL$, – means $FB < OL$.

Column		1	2	3	4	5	6	7	8	9	10	11	12
		$b_f = 0.01$				$b_f = 0.1$				$b_f = 1$			
		Follower			Leader	Follower			Leader	Follower			Leader
Period		1	2	Total	Total	1	2	Total	Total	1	2	Total	Total
$\theta = 0.4$	A=0	n.a.	n.a.	n.a.	n.a.	+	−	−	−	+	−	−	−
	A=0.07	n.a.	n.a.	n.a.	n.a.	+	−	−	−	+	−	−	−
$\theta = 1/2$	A=0	+	−	−	−	+	−	−	−	+	−	−	−
	A=0.07	+	−	−	−	+	−	−	−	+	−	−	−
$\theta = 0.581$	A=0	−	−	−	−	+	−	−	−	+	−	−	−
	A=0.07	−	−	−	−	+	−	−	−	+	−	−	−
$\theta = 0.59$	A=0	−	+	−	−	+	−	−	−	+	−	−	−
	A=0.07	−	+	−	−	+	−	−	−	+	−	−	−
$\theta = 0.655$	A=0	n.a.	n.a.	n.a.	n.a.	−	+	−	−	+	−	−	−
	A=0.07	n.a.	n.a.	n.a.	n.a.	−	+	+	−	+	−	−	−

Table 6: Sign of differences between feedback and open-loop profit results: + means $FB > OL$, – means $FB < OL$.

E.2 Sensitivity analysis with respect to parameter a_l

We use the parameter values from Table 1 with $b_f = 0.1$ and $A = 0.07$ for the following numerical simulation, presenting a summary of results for different a_l and θ.

Column	1	2	3	4	5	6	7	8	9	10	11	12
	$a_l = 4.8$				$a_l = 6$				$a_l = 6.2$			
	Follower		Leader		Follower		Leader		Follower		Leader	
Period	1	2	2	Total	1	2	2	Total	1	2	2	Total
$\theta = 0.4$	+	–	+	+	+	–	+	+	+	–	+	+
$\theta = 1/2$	+	–	+	+	+	–	+	+	+	–	+	+
$\theta = 0.581$	–	–	+	+	+	–	+	+	+	–	+	+
$\theta = 0.59$	–	+	+	–	+	–	+	+	+	–	+	+
$\theta = 0.655$	n.a.	n.a	n.a	n.a	–	+	–	–	–	+	–	–

Table 7: Sign of differences between feedback and open-loop extraction results: + means $FB > OL$, – means $FB < OL$.

Column	1	2	3	4	5	6	7	8	9	10	11	12
	$a_l = 4.8$				$a_l = 6$				$a_l = 6.2$			
	Follower			Leader	Follower			Leader	Follower			Leader
Period	1	2	Total	Total	1	2	Total	Total	1	2	Total	Total
$\theta = 0.4$	+	–	–	–	+	–	–	–	+	–	–	–
$\theta = 1/2$	+	–	–	–	+	–	–	–	+	–	–	–
$\theta = 0.581$	–	–	–	–	+	–	–	–	+	–	–	–
$\theta = 0.59$	–	+	–	–	+	–	–	–	+	–	–	–
$\theta = 0.655$	n.a.	n.a.	n.a.	n.a.	–	+	+	–	–	+	–	–

Table 8: Sign of differences between feedback and open-loop profit results: + means $FB > OL$, – means $FB < OL$.

F Further analysis with respect to β, the discount factor

We conduct numerical simulation in this section using the parameter values from Table 1 with $\theta = 0.5$, $b_f = 0.1$ and $A = 0.07$, presenting a summary of results for different values of β.

Column	1	2	3	4	5	6	7	8	9	10	11	12
	$\beta = 0.1$				$\beta = 0.5$				$\beta = 1$			
	Follower		Leader		Follower		Leader		Follower		Leader	
Period	1	2	2	Total	1	2	2	Total	1	2	2	Total
$\theta = 1/2$	–	+	–	–	+	+	–	–	+	–	+	+

Table 9: Sign of differences between feedback and open-loop extraction results: + means $FB > OL$, – means $FB < OL$.

Column	1	2	3	4	5	6	7	8	9	10	11	12
	$\beta = 0.1$				$\beta = 0.5$				$\beta = 1$			
	Follower			Leader	Follower			Leader	Follower			Leader
Period	1	2	Total	Total	1	2	Total	Total	1	2	Total	Total
$\theta = 1/2$	–	+	+	–	+	+	+	–	+	–	–	–

Table 10: Sign of differences between feedback and open-loop profit results: + means $FB > OL$, – means $FB < OL$.

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