Managing perceptions on the linguistic terms of qualitative scales

José Luis García-Lapresta*

IMUVA, PRESAD Research Group, Departamento de Economía Aplicada, Universidad de Valladolid, Spain

Rodion Iurev

ITMO University, Saint Petersburg, Russia
Departamento de Economía Aplicada, Universidad de Valladolid, Spain

David Pérez-Román

PRESAD Research Group, Departamento de Organización de Empresas y Comercialización e Investigación de Mercados, Universidad de Valladolid, Spain

Abstract

In this paper, we consider the problem of how individuals perceive the ordinal proximities between the linguistic terms of an ordered qualitative scale. We address the challenge of accurately measuring these ordinal proximities in ordered qualitative scales used in surveys. To tackle this issue, we have devised a visual procedure, managed through sliders, and an appropriate software that generates the metrizable ordinal measure which represents the ordinal arrangement of the proximities between the linguistic terms of an ordered qualitative scale. The procedure has been applied to real-world survey data to validate its effectiveness. The results indicate that the new proposal significantly outperforms traditional techniques in terms of accuracy and reliability, providing a robust tool for researchers in fields requiring precise ordinal data analysis.

Keywords: ordered qualitative scales; ordinal proximity measures; distance; surveys.

*Corresponding author.
Email addresses: lapresta@uva.es (José Luis García-Lapresta), rodion.iurev@itmo.ru,rodion.iurev@uva.es (Rodion Iurev), david.perez.roman@uva.es (David Pérez-Román)

1. Introduction

Numerous decision-making problems take as inputs the opinions that agents provide on a set of alternatives through ordered qualitative scales composed of linguistic terms.

The *European Union Statistics on Income and Living Conditions* (*EU-SILC*) use different questionnaires to know the opinions of European citizens about a large number of issues (see European Union, 2022). Some surveys include questions that should be answered in an ordered qualitative scale. For instance, {a heavy burden, somewhat a burden, not burden at all} to know the financial burden of the total housing cost; {with great difficulty, with difficulty, with some difficulty, fairly easily, easily, very easily} to know the ability to make ends meet; {yes, strongly limited; yes, limited; no, not limited} to know the limitation in activities because of health problems; and {with great difficulty, with some difficulty, easily, very easily} to know the accessibility of public transport, among many others.

We consider that an ordered qualitative scale is non-uniform whenever the psychological proximities between the pairs of consecutive terms of the scale are not perceived as identical. For instance, the last mentioned scale is non-uniform if 'easily' is perceived closer to 'very easily' than to 'with some difficulty', or if 'with some difficulty' is perceived closer to 'with great difficulty' than to 'easily'.

In order to handle non-uniform ordered qualitative scales in an ordinal way, ordinal proximity measures were introduced by García-Lapresta & Pérez-Román (2015). They describe the proximities between the linguistic terms of ordered qualitative scales through non-numerical degrees of proximity.

García-Lapresta et al. (2018) introduce the notion of metrizable ordinal proximity measure and provide a method for generating metrizable ordinal proximity measures through suitable sequences of questions for the case of ordered qualitative scales with four linguistic terms. They also introduce an aggregation procedure of metrizable ordinal proximity measures based on weighted metrics.

After García-Lapresta & Pérez-Román (2015) and García-Lapresta et al. (2018), several papers propose and analyze different decision-making procedures in the framework of non-uniform ordered qualitative scales through ordinal proximity measures (see García-Lapresta & González del Pozo (2023) for references).

When an ordered qualitative scale contains 3 or 4 linguistic terms, it is possible to generate a metrizable ordinal proximity measure from questions about how is perceived the closeness between the terms of the scale. However, when the scale has more than 4 linguistic terms it is difficult, almost impossible or even impossible, to generate the metrizable ordinal proximity measure from questions.

The main contribution of this paper consists of the design of a visual procedure for obtaining the proximities between the terms of an ordered qualitative scale by means of sliders. After the development of appropriate software, it is possible to generate metrizable ordinal proximity measures from the visual perceptions of the proximities between the terms of the scales.

The proposed procedure has been implemented in two real case studies, where respondents from three different countries (Italy, Russia and Spain) showed their perceptions on two scales used by public organizations. The outcomes are clear: individuals do not perceive those scales as uniform.

The rest of the paper is organized as follows. Section 2 is devoted to present metrizable ordinal proximity measures, their elicitation and how to generate them from numerical distances. Section 3 describes the visual procedure and contains the two real case studies. Finally, Section 4 includes concluding remarks.

2. Metrizable ordinal proximity measures

Consider a group of individuals that is required to show their opinions about some issues through the linguistic terms of an *ordered qualitative scale* (**OQS**) $\mathcal{L} = \{l_1, \dots, l_g\}$, whose terms are ordered from worst to best, $l_1 \prec \cdots \prec l_g$, with $g \geqslant 3$.

An ordinal proximity measure on an ordered qualitative scale $\mathcal{L}$ assigns an ordinal degree of proximity to each pair of linguistic terms of $\mathcal{L}$, satisfying several conditions. The ordinal degrees of proximity are arranged into a linear order $\Delta = \{\delta_1, \dots, \delta_h\}$, with $\delta_1 \succ \cdots \succ \delta_h$. Note that the elements of Δ are not numbers, and they only represent different degrees of proximity in an ordinal fashion: δ_1 is the maximum, δ_2 is the second, etc.

We now formally present the notion ordinal proximity measure.

Definition 1. *(García-Lapresta & Pérez-Román, 2015) An* ordinal proximity measure (**OPM**) on $\mathcal{L}$ with values in Δ *is a mapping* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$*, where* $\pi(l_r, l_s) = \pi_{rs}$ *represents the degree of proximity between* l_r *and* l_s*, satisfying the following conditions:*

1. Exhaustiveness*: For every* $\delta \in \Delta$*, there exist* $l_r, l_s \in \mathcal{L}$ *such that* $\delta = \pi_{rs}$*.*
2. Symmetry*:* $\pi_{sr} = \pi_{rs}$*, for all* $r, s \in \{1, \dots, g\}$*.*

3. Maximum proximity: $\pi_{rs} = \delta_1 \;\Leftrightarrow\; r = s$, *for all* $r, s \in \{1, \dots, g\}$.
4. Monotonicity: $\pi_{rs} \succ \pi_{rt}$ *and* $\pi_{st} \succ \pi_{rt}$, *if* $1 \leqslant r < s < t \leqslant g$.

An OPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is called *uniform* if $\pi_{r(r+1)} = \pi_{s(s+1)}$ for all $r, s \in \{1, \dots, g-1\}$, i.e., $\pi_{12} = \pi_{23} = \cdots = \pi_{(g-1)\,g}$.

Every OPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ is represented by a $g \times g$ symmetric matrix with coefficients in Δ: (π_{rs}). Note that all the elements in the main diagonal are δ_1.

This matrix will be called *proximity matrix associated with* π.

Taking into account the conditions of Definition 1, we would only need to show the upper half proximity matrix.

Note that $\pi_{rs} = \delta_h$ if and only if $(r, s) = (1, g)$ or $(r, s) = (g, 1)$ (see García-Lapresta & Pérez-Román, 2015, Prop. 2).

Remark 1. Every OPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ generates a weak order on the set

$$X = \left\{(r, s) \in \{1, \dots, g\}^2 \mid r < s\right\} \setminus \{(1, g)\}$$

allocating at the first tier those $(r, s) \in X$ such that $\pi_{rs} = \delta_2$, in the second tier those $(r, s) \in X$ such that $\pi_{rs} = \delta_3$, and so on, and in the last tier those $(r, s) \in X$ such that $\pi_{rs} = \delta_{h-1}$.

These tiers only contain the essential information about the ordinal degrees of proximity between the linguistic terms of the OQS $\mathcal{L}$. They do not include the pairs $(r, s) \in \{1, \dots, g\}^2$ that correspond to trivial ordinal degrees of proximity: (r, r), because $\pi_{rr} = \delta_1$ for every $r \in \{1, \dots, g\}$; and $(1, g)$, because $\pi_{1g} = \delta_h$.

A relevant family of OPMs is the one of metrizable OPMs (**MOPM**). They were introduced by García-Lapresta et al. (2018) and they use linear metrics on OQSs.

Definition 2. *(García-Lapresta et al., 2018). A* linear metric *on* $\mathcal{L}$ *is a mapping* $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ *satisfying the following conditions for all* $r, s, t \in \{1, \dots, g\}$:

1. $d(l_r, l_s) \geqslant 0$.
2. $d(l_r, l_s) = 0 \;\Leftrightarrow\; l_r = l_s$.
3. $d(l_s, l_r) = d(l_r, l_s)$.
4. $d(l_r, l_t) = d(l_r, l_s) + d(l_s, l_t)$, *if* $r < s < t$.

It is worth noting that every linear metric on an OQS is univocally determined from the distances between consecutive linguistic terms of the OQS (see García-Lapresta et al., 2018, Remark 1).

Definition 3. *(García-Lapresta et al., 2018). An OPM* $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta$ *is* metrizable *if there exists a linear metric* $d : \mathcal{L} \times \mathcal{L} \longrightarrow \mathbb{R}$ *such that* $\pi_{rs} \succ \pi_{tu} \Leftrightarrow d(l_r, l_s) < d(l_t, l_u)$*, for all* $r, s, t, u \in \{1, \ldots, g\}$*. We say that* π *is generated by* d*.*

An individual whose perceptions about the ordinal proximities between the linguistic terms of an OQS are represented by a MOPM behaves *as if he or she had in mind* a linear metric on the OQS[1]. It means that the ordinal degrees of proximity are generated by the numerical distances between the pairs of linguistic terms. For instance, if $\mathcal{L} = \{l_1, l_2, l_3, l_4\}$, $d(l_1, l_2) = 1.3$, $d(l_2, l_3) = 2.2$ and $d(l_3, l_4) = 1$, we have

$$\begin{aligned} & d(l_3, l_4) = 1 < d(l_1, l_2) = 1.3 < d(l_2, l_3) = 2.2 < \\ & d(l_2, l_4) = 3.2 < d(l_1, l_3) = 3.5 < d(l_1, l_4) = 4.5. \end{aligned}$$

These distances generate the MOPM $\pi : \mathcal{L} \times \mathcal{L} \longrightarrow \Delta = \{\delta_1, \ldots, \delta_7\}$ defined as $\pi_{rr} = \delta_1$ (for every $r \in \{1, 2, 3, 4\}$), $\pi_{34} = \delta_2$, $\pi_{12} = \delta_3$, $\pi_{23} = \delta_4$, $\pi_{24} = \delta_5$, $\pi_{13} = \delta_6$ and $\pi_{14} = \delta_7$, with the following associated proximity matrix

$$\begin{pmatrix} \delta_1 & \delta_3 & \delta_6 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_5 \\ & & \delta_1 & \delta_2 \\ & & & \delta_1 \end{pmatrix}.$$

2.1. Eliciting metrizable OPMs

We now focus on the proximity matrices associated with the MOPMs formed by 3 or 4 terms.

The subindices appearing in the matrices A's correspond to those δ's placed just over the main diagonal, $\pi_{12}, \pi_{23}, \ldots, \pi_{(g-1)\,g}$, i.e., those ordinal degrees of proximity between the pairs of consecutive terms of the OQS.

For $g = 3$, there are three OPMs and all of them are metrizable. If an agent declares $\pi_{12} = \pi_{23}$, $\pi_{12} \succ \pi_{23}$ or $\pi_{12} \prec \pi_{23}$, then the corresponding MOPMs are associated with the proximity matrices A_{22} (see Fig. 1), A_{23} (see Fig. 2) or A_{32} (see Fig. 3), respectively:

[1] It is not realistic to think that individuals may directly assign numerical distances between pairs of linguistic terms (for instance, between 'very good' and 'excellent'). It is similar to the case of rational preferences; they are representable by utility functions, although individuals do not use them in their pairwise comparisons (see, for instance, Bridges & Mehta, 1995).

$$A_{22} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ & \delta_1 & \delta_2 \\ & & \delta_1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_4 \\ & \delta_1 & \delta_3 \\ & & \delta_1 \end{pmatrix}$$

$$A_{32} = \begin{pmatrix} \delta_1 & \delta_3 & \delta_4 \\ & \delta_1 & \delta_2 \\ & & \delta_1 \end{pmatrix}.$$

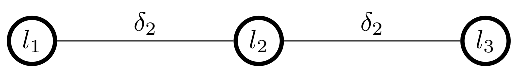


Figure 1: OPM with associated proximity matrix A_{22}.

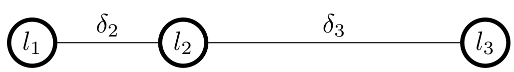


Figure 2: OPM with associated proximity matrix A_{23}.

Figure 3: OPM with associated proximity matrix A_{32}.

Thus, if we know the answer to a single question, then we obtain the corresponding MOPM.

For $g \geqslant 4$, individuals may have difficulties to directly eliciting the MOPMs that reflect their perceptions about the ordinal proximities between the terms of the OQS.

For $g = 4$, this can be done through appropriate sequences of 2-4 questions, whose answers allow us to obtain one out of the 25 MOPMs (see García-Lapresta et al., 2018, Subsect. 2.3). We note that the mentioned questions are devised in such a way that they prevent individuals from incurring inconsistencies.

The procedure starts by asking the individual about the proximities between l_1, l_2 and l_3. Three answers are possible: l_2 is closer to l_1 than to l_3 ($\pi_{21} \succ \pi_{23}$), l_2 is closer to l_3 than to l_1 ($\pi_{23} \succ \pi_{21}$), or the proximity between l_2 and l_1 is the same that the proximity between l_2 and l_3 ($\pi_{21} = \pi_{23}$). The next question depends on the previous answer. The sequential procedure continues with similar questions that compare the closeness between the remaining pairs of terms of the OQS until we obtain one of the 25 MOPMs.

For $g = 5$, to prevent individuals' inconsistencies, again some sequences of appropriate questions about the proximities between the linguistic terms of the OQS could be addressed to agents for obtaining the corresponding MOPMs. In this case, between 4 and 10 questions are needed for obtaining one out of the 473 MOPMs. However, the interdependency between those questions can produce mental exhaustion in the agents.

Clearly, for $g > 5$ it is almost impossible to follow the same procedure for eliciting MOPMs. For instance, there are 18,262 MOPMs for $g = 6$.

Consequently, a different procedure is needed for eliciting MOPMs for OQS that have more than 4 terms.

2.2. Generating metrizable OPMs from distances

Since a linear metric on an OQS is univocally determined from the distances between consecutive linguistic terms (see García-Lapresta et al., 2018, Remark 1), if an agent could provide these distances, then it is possible to generate the only MOPM compatible with the provided distances. In such a case, the `MATLAB` program presented in García-Lapresta & Pérez-Román (2020) generates the corresponding MOPM.

Remark 2. If agents perceive an OQS in different ways, it could be useful to merge their perceptions in order to generate a global MOPM on the OQS. García-Lapresta et al. (2018, Sect. 4) propose a procedure to aggregate MOPMs. In this paper, we follow a different approach that is developed in Section 3.

3. Visual procedure

In some scenarios, individuals may have difficulties to assign exact numerical values to vague notions (see, for instance, Zimmer, 1983). In an experiment about some transitivity conditions, Świtalski (2001) asked a group of 44 students about the intensity of their preferences between five destinations for holidays. Instead of using numbers to declare such intensities of preference in the comparison of each pair of destinations, the author considered a visual perspective: students had to fix a sign X in an interval of length 10 cm.

In our case, we are interested in how agents perceive the closeness between the linguistic terms of an OQS in order to generate the MOPM that best represents the closeness between the terms of the OQS.

We note that the linguistic terms of an OQS $\mathcal{L} = \{l_1, \ldots, l_g\}$ are linearly ordered in one dimension, $l_1 \prec \cdots \prec l_g$. To facilitate individuals eliciting their perceptions on the proximities between the linguistic terms of the OQS, in this paper we have devised a visual procedure through a slider where the positions of the linguistic terms of the OQS can be placed moving the handles. Fig. 5 (p. 12) and Fig. 8 (p. 15) show the sliders used in Study 1 and Study 2, respectively.

The slider interface is intuitive, aligning the visual procedure with cognitive psychology, where visual methods are known to reduce the cognitive load on respondents, allowing them to express their perceptions more naturally without the need to provide explicit numerical assignments.

The choice of visual methods, particularly the slider, offers significant advantages over traditional methods such as direct questioning or pairwise comparisons. Traditional methods often require respondents to make explicit numerical judgments, which can be cognitively more demanding and less appropriate in vagueness environments. In contrast, the visual procedure allows respondents to express their perceptions in a more intuitive and less mentally taxing manner.

The software supporting this visual procedure is robust and user-friendly. It is built on the Django 4.1 framework (Django Software Foundation), with a SQLite3 database for fast and reliable data storage.

The visual part is developed using the modified `Javascript` library `noUiSlider` (noUiSlider). Thanks to this combination, it becomes possible to conduct surveys at any point, collecting data from users for further processing in any format.

After sending this data, information about each term and its position on the selected scale is stored in the database.

Once we know the numerical distances between the consecutive linguistic terms of an OQS, we generate the only MOPM that is compatible with these distances (see Subsection 2.2).

It worths mentioning that individuals do not realize that the positions they allocate the terms of the OQS with the slider generate numerical distances between the linguistic terms of the OQS. They only have to show their perceptions in a visual way.

In order to the slider can generate all possible MOPMs on the OQS, it is necessary to select an appropriate granularity of the slider for each value of g. We have considered a total granularity of $G = g!$, with a granularity between consecutive linguistic terms of $\frac{G}{g-1} = g \cdot (g-2)!$ Table 1 shows these granularities for $g = 3, 4, 5, 6, 7$.

Taking into account the obtained numerical distances between the linguistic terms of the OQS $\mathcal{L} = \{l_1, \ldots, l_g\}$, for each individual $i \in \{1, \ldots, m\}$: $d_i(l_r, l_s)$, for all $r, s \in \{1, \ldots, g\}$ such that $r < s$, these distances can be aggregated in

g	$g \cdot (g-2)!$	$G = g!$
3	3	6
4	8	24
5	30	120
6	144	720
7	840	5,040

Table 1: Granularities.

order to obtain a collective linear metric on the OQS, hence a MOPM on $\mathcal{L}$ (see García-Lapresta et al., 2018, Remark 1).

This task can be made through an aggregation function (see Beliakov et al., 2007). By simplicity, we have used the arithmetic mean:

$$\bar{d}(l_r, l_s) = \frac{1}{m} \cdot \sum_{i=1}^{m} d_i(l_r, l_s),$$

for all $r, s \in \{1, \ldots, g\}$ such that $r < s$.

Although the visual devised procedure and the corresponding developed software can be implemented in any OQS, it is important to note that the number of linguistic terms in OQSs is usually between 3 and 7 (see Miller, 1956; Lozano et al., 2008).

We have considered two real case studies where the participants showed their perceptions about two OQSs frequently used by different organizations. Study 1 deals with an OQS of 4 linguistic terms, while Study 2 do the same with an OQS of 5 linguistic terms.

Since the perception of an OQS may depend on the problem who is analyzed[2], in both studies we included a brief introduction to the online survey with a description on the scenario where the OQS is used.

3.1. Study 1

A total of 35 students (11 men, 22 women and 2 undefined) from the degree on Commerce of the University of Valladolid (Spain) completed a survey online. It started with a brief introduction:

[2]For instance, the psychological distance between the linguistic terms 'never' and 'rarely' can be different if the question is "How often do you go to the cinema?" or "How often do you use cocaine?".

On the question "Do you often have difficulty doing any of these activities on your own?" (preparing meals, shopping, using the telephone, managing medication, light housework, occasional heavy housework, taking care of finances and everyday administrative tasks) EUROSTAT uses the scale {I can't make it on my own / Yes, with great difficulty / Yes, with some difficulty / No difficulty}.

Indicate your perception of the proximity between the elements of the scale.

Once the respondents selected their gender (male, female, I prefer not to specify), they had to show their perceptions on the OQS of 4 linguistic terms included in Table 2 in two different ways:

- Through a questionnaire of a sequence of 2-4 questions (Fig. 4 shows the first question according to the algorithm presented in García-Lapresta et al. (2018, Fig. 4)).
- By means of a slider (Fig. 5 shows the English version of the screen with the slider).

l_1	l_2	l_3	l_4
I can't make it on my own	Yes, with great difficulty	Yes, with some difficulty	No difficulty

Table 2: Meaning of the linguistic terms in Study 1.

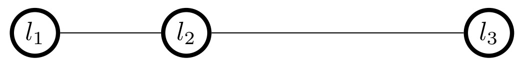
Half of the respondents started with the questionnaire and the other half with the slider (the system assigned it randomly).

The survey ended with a final question: Which of the survey options (slider or questionnaire) seemed more convenient? A 51.4% of the respondents preferred the slider, a 28.6% of them preferred the questionnaire, and a 20% of them were indifferent between the two procedures. Table 3 shows the results by gender.

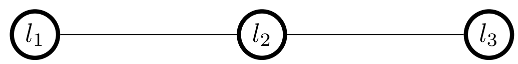
The received data were stored in the format of a relational database with access to it in the `SQL` language. `SQLite2` was chosen as the DBMS (Data Base Management System), which is characterized by high speed and reliability. Some limitations of this database were not critical in this task.

Select the correct statement:

◯ "Yes, with great difficulty" (l_2) is closer to "I can't make it on my own" (l_1) than to "Yes, with some difficulty" (l_3)



◯ "Yes, with great difficulty" (l_2) is on equal distance to "Yes, with some difficulty" (l_3) than to "I can't make it on my own" (l_1)



◯ "Yes, with great difficulty" (l_2) is closer to "Yes, with some difficulty" (l_3) than to "I can't make it on my own" (l_1)

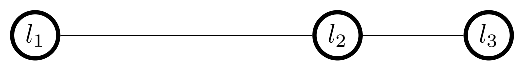


Figure 4: Questionnaire used in Study 1.

more convenient	male	female	undefined
slider	45.6%	59.1%	0.0%
questionnaire	54.4%	18.2%	0.0%
indifferent	0.0%	22.7%	100.0%

Table 3: Results by gender.

User responses were recorded in sessions (one session per survey for each user). Interview data in questionnaire format were stored as a matrix number (0 to 24).

The provided interface allows the researcher to download the distances between consecutive terms, and from this data to obtain the corresponding proximity matrix.

In Table 12, included in the Annex, we show the positions which the 35 respondents placed the four linguistic terms of Table 2 through the slider of Fig. 5.

For instance, respondent number 5 placed the linguistic terms in positions $(0, 8, 16, 24)$. Then, the distances between consecutive terms are $(8, 8, 8)$ and

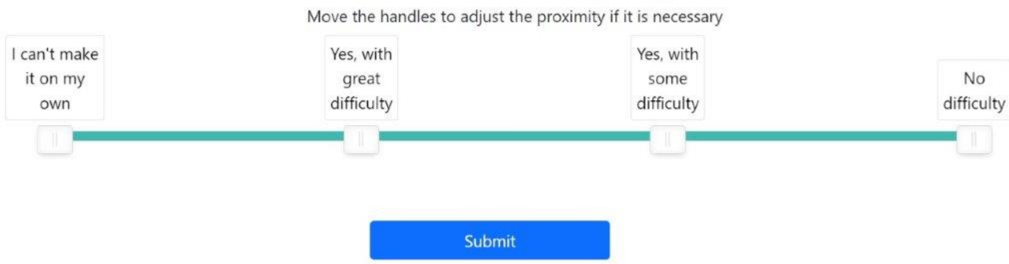


Figure 5: Slider used in Study 1.

the corresponding proximity matrix is

$$A_{222} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 \\ & \delta_1 & \delta_2 & \delta_3 \\ & & \delta_1 & \delta_2 \\ & & & \delta_1 \end{pmatrix}$$

Respondent number 7 slightly differs from respondent number 5: she placed the linguistic terms in positions $(0, 8, 19, 24)$. Now the distances between consecutive terms are $(8, 11, 5)$ and the corresponding proximity matrix is

$$A_{342} = \begin{pmatrix} \delta_1 & \delta_3 & \delta_6 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_5 \\ & & \delta_1 & \delta_2 \\ & & & \delta_1 \end{pmatrix}$$

In turn, respondent number 26 placed the linguistic terms in positions $(0, 8, 18, 24)$. Then, the distances between consecutive terms are $(8, 10, 6)$ and now the corresponding proximity matrix is

$$A_{243} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_5 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_6 \\ & & \delta_1 & \delta_3 \\ & & & \delta_1 \end{pmatrix}$$

Table 4 includes the average distances between consecutive linguistic terms and the dispersion (σ - standard deviation) for all the respondents. Note that, according to Table 1, for $g = 4$ the total granularity is 24, hence the sum of the average distances is 24.

	Average distance	Dispersion
l_1 *versus* l_2	$\bar{d}(l_1, l_2) = 7.8571$	2.929
l_2 *versus* l_3	$\bar{d}(l_2, l_3) = 8.5714$	3.507
l_3 *versus* l_4	$\bar{d}(l_3, l_4) = 7.5714$	2.522

Table 4: Average distances and dispersion in Study 1.

Table 5 contains the proximity matrices in the different groups.

Group	Proximity matrix
Total	A_{342}
Men	A_{243}
Women	A_{342}

Table 5: Proximity matrices in Study 1.

The obtained proximity matrices are shown below. Figures 6 and 7 include a visualization of the corresponding proximity matrices.

$$A_{243} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_5 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_6 \\ & & \delta_1 & \delta_3 \\ & & & \delta_1 \end{pmatrix} \qquad A_{342} = \begin{pmatrix} \delta_1 & \delta_3 & \delta_6 & \delta_7 \\ & \delta_1 & \delta_4 & \delta_5 \\ & & \delta_1 & \delta_2 \\ & & & \delta_1 \end{pmatrix}$$

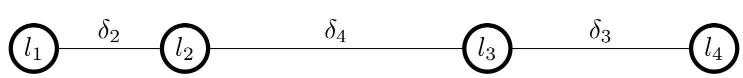


Figure 6: MOPM with associated proximity matrix A_{243}.

Taking into account Remark 1, in Table 6 the ordinal proximities between linguistic terms of the OQS are arranged, from the highest to the lowest. Note that in both cases we have obtained a linear order (every ordinal degree of proximity only appears once).

In order to know the correlation between the two linear orders of Table 6, we consider the Spearman's rank correlation coefficient (Spearman, 1904) (see also Daniel, 1990, pp. 358-365). The result is 0.8 and this means that the correlation is high.

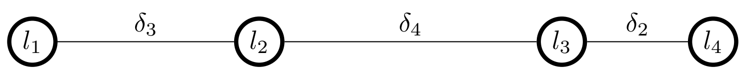


Figure 7: MOPM with associated proximity matrix A_{342}.

	A_{243}	A_{342}
δ_2	$(1, 2)$	$(3, 4)$
δ_3	$(3, 4)$	$(1, 2)$
δ_4	$(2, 3)$	$(2, 3)$
δ_5	$(1, 3)$	$(2, 4)$
δ_6	$(2, 4)$	$(1, 3)$

Table 6: Orders in Study 1.

3.2. Study 2

A total of 60 students from Italy (University of Palermo), Russia (ITMO University) and Spain (University of Valladolid), 20 for each country, participated in a survey online where they had to show their perceptions on the OQS of 5 linguistic terms included in Table 7. This OQS is regularly used by EUROFOUND (European Foundation for the Improvement of Living and Working Conditions).

The survey started with a brief introduction:

> *The European Foundation for the Improvement of Living and Working Conditions (EUROFOUND) conducts numerous surveys to investigate social, labor and employment issues, covering a wide range of topics. In particular, when asking about how employees feel about their jobs, it asks respondents to indicate* ***how often*** *their job entails Working at very high speed / Working to tight deadlines / Learning new things.*
>
> *To answer these questions, the respondents have to use the scale* {*Never / Rarely / Sometimes / Often / Always*}.

The students elicited their perceptions about the OQS through the slider shown in Fig. 8.

The positions where the 60 respondents placed the five linguistic terms of Table 7 through the slider of Fig. 8 are included in the Annex: Table 13 for Italy, Table 14 for Russia and Table 15 for Spain.

l_1	l_2	l_3	l_4	l_5
Never	Rarely	Sometimes	Often	Always

Table 7: Meaning of the linguistic terms in Study 2.

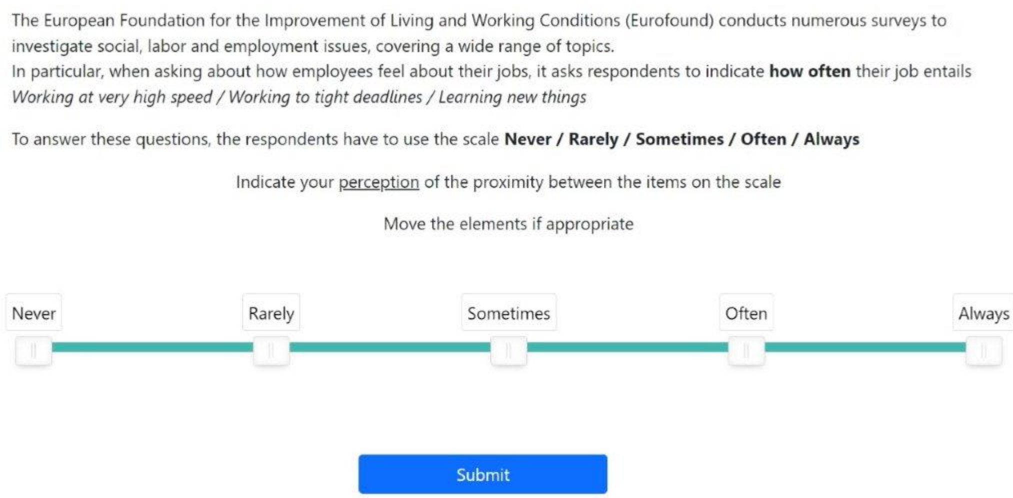


Figure 8: Slider used in Study 2.

Analogously to Study 1, Table 8 includes the average distances between consecutive linguistic terms and the dispersion (σ - standard deviation). Note that, according to Table 1, for $g = 5$ the total granularity is 120, hence the sum of the average distances is 120.

	Average distance	Dispersion
l_1 *versus* l_2	$\bar{d}(l_1, l_2) = 23.07$	7.998
l_2 *versus* l_3	$\bar{d}(l_2, l_3) = 33.52$	13.525
l_3 *versus* l_4	$\bar{d}(l_3, l_4) = 38.97$	15.254
l_4 *versus* l_5	$\bar{d}(l_4, l_5) = 24.45$	7.390

Table 8: Average distances and dispersion in Study 2.

Table 9 contains the proximity matrices in the different groups.

Note that the proximity matrix A_{2453} represents the collective perceptions of the OQS in 8 out the 12 groups: total, total men, total women, total Russia, total Spain, Russian men, Spanish men and Spanish women. In turn, A_{3542} corresponds to total Italy, Italian men and Italian women; and A_{2354} to Russian

Group	Proximity matrix
Total	A_{2453}
Total men	A_{2453}
Total women	A_{2453}
Total Italy	A_{3542}
Total Russia	A_{2453}
Total Spain	A_{2453}
Italian men	A_{3542}
Russian men	A_{2453}
Spanish men	A_{2453}
Italian women	A_{3542}
Russian women	A_{2354}
Spanish women	A_{2453}

Table 9: Proximity matrices in Study 2.

women.

The obtained proximity matrices are shown below. Figures 9, 10 and 11 include a visualization of the corresponding proximity matrices.

$$A_{2453} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_6 & \delta_9 & \delta_{11} \\ & \delta_1 & \delta_4 & \delta_8 & \delta_{10} \\ & & \delta_1 & \delta_5 & \delta_7 \\ & & & \delta_1 & \delta_3 \\ & & & & \delta_1 \end{pmatrix} \qquad A_{3542} = \begin{pmatrix} \delta_1 & \delta_3 & \delta_7 & \delta_{10} & \delta_{11} \\ & \delta_1 & \delta_5 & \delta_8 & \delta_9 \\ & & \delta_1 & \delta_4 & \delta_6 \\ & & & \delta_1 & \delta_2 \\ & & & & \delta_1 \end{pmatrix}$$

$$A_{2354} = \begin{pmatrix} \delta_1 & \delta_2 & \delta_6 & \delta_9 & \delta_{11} \\ & \delta_1 & \delta_3 & \delta_7 & \delta_{10} \\ & & \delta_1 & \delta_5 & \delta_8 \\ & & & \delta_1 & \delta_4 \\ & & & & \delta_1 \end{pmatrix}.$$

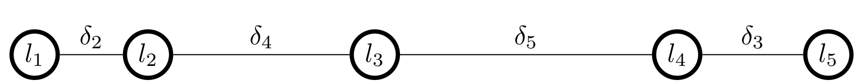


Figure 9: MOPM with associated proximity matrix A_{2453}.

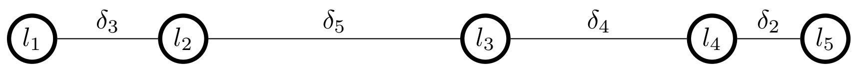


Figure 10: MOPM with associated proximity matrix A_{3542}.

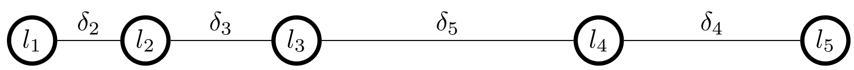


Figure 11: MOPM with associated proximity matrix A_{2354}.

Taking into account Remark 1, in Table 10 the ordinal proximities between linguistic terms of the OQS are arranged, from the highest to the lowest. Note that again in all cases are linear orders.

	A_{2453}	A_{3542}	A_{2354}
δ_2	$(1,2)$	$(4,5)$	$(1,2)$
δ_3	$(4,5)$	$(1,2)$	$(2,3)$
δ_4	$(2,3)$	$(3,4)$	$(4,5)$
δ_5	$(3,4)$	$(2,3)$	$(3,4)$
δ_6	$(1,3)$	$(3,5)$	$(1,3)$
δ_7	$(3,5)$	$(1,3)$	$(2,4)$
δ_8	$(2,4)$	$(2,4)$	$(3,5)$
δ_9	$(1,4)$	$(2,5)$	$(1,4)$
δ_{10}	$(2,5)$	$(1,4)$	$(2,5)$

Table 10: Orders in Study 2.

Table 11 shows the Spearman's rank correlation coefficients for all pairs of linear orders appearing in Table 10.

	A_{2453}	A_{3542}	A_{2354}
A_{2453}	1	0.93	0.97
A_{3542}	0.93	1	0.85
A_{2354}	0.97	0.85	1

Table 11: Spearman's rank correlation coefficients.

Note that the correlation is very high: from 0.85 to 0.97. The proximity matrices A_{2354} (Rusian women) and A_{3542} (total Italy, Italian men and Italian women) are the farthest from each other (with a correlation of 0.85). A higher correlation, 0.93, is between A_{3542} and A_{2453} (total, total men, total women, total Russia, total Spain, Russian men, Spanish men and Spanish women). The proximity matrices A_{2354} and A_{2453} have obtained the highest correlation, 0.97.

Summarizing, in both studies all groups perceive the corresponding OQSs as non-uniform. Some groups do not coincide in their perceptions, but they are very correlated.

4. Concluding remarks

In order to know the opinions of individuals about different issues, many public organizations and companies carry out a large number of surveys. Some of them are devised through questionnaires based on ordered qualitative scales. The reason is that human beings are more comfortable using words than numerical values.

It is important to note that not all the used ordered qualitative scales are uniform: sometimes individuals perceive different proximities between consecutive terms of the scales.

The notion of ordinal proximity measure allows managing the perceptions on the closeness between the linguistic terms of a scale by means of ordinal degrees of proximity, without numbers, in a pure ordinal fashion. The problem is how individuals can show their perceptions about those closenesses.

In this paper, we have designed a visual procedure through sliders that generates metrizable ordinal proximity measures by means of a software devised for this purpose. That procedure has been applied to two real case studies that involved respondents from three different countries (Italy, Russia and Spain). The obtained individual and collective metrizable ordinal proximity measures show that the considered ordered qualitative scales are not perceived as uniform. Consequently, it makes no sense to assign equidistant numerical values to the linguistic terms of those scales.

Individuals may have serious difficulties to directly provide the metrizable ordinal proximity measures that capture their perceptions about the psychological closeness between the linguistic terms of an ordered qualitative scale, and also to provide exact numerical distances between them. The proposed visual procedure gives a solution to this problem[3].

[3]On the relevance of visualization in different decision support systems and related refer-

In the two real case studies included in this paper, the participants had no difficulties in using the sliders. The devised software calculates the numerical distances between all pairs of linguistic terms of the ordered qualitative scale and also generates the metrizable ordinal proximity measures that represent the individual perceptions.

Once the metrizable ordinal proximity measures associated with the corresponding ordered qualitative scales are known, it is possible to apply them to different decision-making problems.

The visual procedure using sliders is versatile and can be applied in a variety of contexts other than those discussed in this paper. Potential applications include market research, where understanding consumer perceptions is crucial; political polling, where nuanced differences in opinion must be captured; and decision-making processes in fields such as healthcare, education, and public policy. The adaptability of this method makes it a powerful tool for researchers and practitioners across various disciplines.

Acknowledgments.

The authors are grateful to two anonymous reviewers for their useful comments and suggestions. The authors thank the respondents who participate in the real case studies and also Mariangela Sciandra for facilitating us to submit the survey to the Italian students. The financial support of the research project PID2021-122506NB-I00 funded by MICIU/AEI /10.13039/501100011033 and ERDF, EU, is acknowledged. Rodion Iurev thanks the Erasmus+ KA107 program managed by the International Relations Service of the University of Valladolid, Spain.

References

Basole, R. C., Huhtamäki, J., Still, K., & Russell, M. G. (2016). Visual decision support for business ecosystem analysis. *Expert Systems with Applications, 65*, 271–282.

Beliakov, G., Pradera, A., & Calvo, T. (2007). *Aggregation Functions: A Guide for Practitioners.* Springer, Heidelberg.

ences, see Basole et al. (2016).

Bridges, D. S., & Mehta, G. B. (1995). *Representations of Preference Orderings*. Springer, Berlin-Heidelberg.

Daniel, W. W. (1990). *Applied Nonparametric Statistics* (2nd ed.). PWS-Kent, Boston.

Django Software Foundation, Django 4.1. `https://www.djangoproject.com/`.

European Union - Statistics on Income and Living Conditions. `https://doi.org/10.2907/EUSILC2004-2020V.2`.

García-Lapresta, J. L., & González del Pozo, R. (2023). Using scoring functions in a group decision-making procedure with heterogeneous experts and qualitative assessments. *International Journal of Approximate Reasoning, 161*, Article 109004.

García-Lapresta, J. L., González del Pozo, R., & Pérez-Román, D. (2018). Metrizable ordinal proximity measures and their aggregation. *Information Sciences, 448-449*, 149–163.

García-Lapresta, J. L., & Pérez-Román, D. (2015). Ordinal proximity measures in the context of unbalanced qualitative scales and some applications to consensus and clustering. *Applied Soft Computing, 35*, 864–872.

García-Lapresta, J. L., & Pérez-Román, D. (2020). Eliciting proximities between terms of ordered qualitative scales. *Proceedings of 2020 IEEE 10th International Conference on Intelligent Systems*, pp. 294–298.

Lozano, L. M., García-Cueto, E., & Muñiz, J. (2008). Effect of the number of response categories on the reliability and validity of rating scales. *European Journal of Research Methods for the Behavioral and Social Sciences, 4*, 73–79.

Miller, G. A. (1956). The magical number seven, plus or minus two: some limits on our capacity for processing information. *Psychological Review, 63*, 81–97.

noUiSlider. `https://refreshless.com/nouislider/`.

Spearman, C. (1904). The proof and measurement of association between two things. *The American Journal of Psychology, 15*, 72–101.

Świtalski, Z. (2001). Transitivity of fuzzy preference relations – an empirical study. *Fuzzy Sets and Systems, 118*, 503–508.

Zimmer, A. C. (1983). Verbal vs. numerical processing of subjective probabilities. In R. W. Scholz (Ed.), *Decision Making under Uncertainty*, North-Holland, Amsterdam, pp. 159–182.

Annex

Study 1											
Respondent	l_1	l_2	l_3	l_4	Gender	Respondent	l_1	l_2	l_3	l_4	Gender
1	0	5	19	24	female	19	0	15	20	24	female
2	0	15	18	24	undefined	20	0	13	16	24	female
3	0	5	13	24	female	21	0	5	8	24	male
4	0	8	17	24	female	22	0	7	16	24	female
5	0	8	16	24	female	23	0	8	16	24	male
6	0	12	18	24	male	24	0	8	16	24	female
7	0	8	19	24	female	25	0	8	16	24	female
8	0	8	19	24	female	26	0	5	18	24	male
9	0	8	17	24	male	27	0	6	18	24	female
10	0	8	16	24	undefined	28	0	6	17	24	female
11	0	2	13	24	female	29	0	10	18	24	female
12	0	10	16	24	male	30	0	6	12	24	female
13	0	6	19	24	male	31	0	3	19	24	female
14	0	11	15	24	male	32	0	3	21	24	male
15	0	8	16	24	female	33	0	7	15	24	female
16	0	9	12	24	male	34	0	8	16	24	female
17	0	10	18	24	female	35	0	8	16	24	female
18	0	8	16	24	male						

Table 12: Positions of linguistic terms in Study 1.

Study 2: Italy					
l_1	l_2	l_3	l_4	l_5	Gender
0	32	102	113	120	male
0	18	78	100	120	male
0	30	60	90	120	male
0	36	47	94	120	male
0	25	56	93	120	female
0	23	61	101	120	male
0	18	70	107	120	male
0	13	61	104	120	male
0	30	60	90	120	male
0	18	56	99	120	female
0	30	65	90	120	female
0	30	89	90	120	male
0	26	60	90	120	male
0	30	60	90	120	male
0	24	57	99	120	male
0	23	47	99	120	male
0	30	68	100	120	female
0	30	68	100	120	male
0	30	60	90	120	male
0	28	60	90	120	male

Table 13: Positions of linguistic terms in Study 2 (Italy).

Study 2: Russia					
l_1	l_2	l_3	l_4	l_5	Gender
0	18	74	112	120	male
0	30	60	93	120	female
0	1	67	98	120	male
0	23	49	90	120	male
0	11	42	75	120	female
0	13	41	96	120	female
0	22	60	101	120	male
0	40	71	106	120	male
0	27	54	91	120	male
0	12	23	110	120	male
0	30	60	90	120	female
0	30	60	90	120	female
0	15	81	99	120	male
0	21	33	90	120	male
0	32	46	82	120	female
0	18	51	97	120	undefined
0	10	19	114	120	male
0	20	28	90	120	female
0	41	68	97	120	male
0	4	18	94	120	male

Table 14: Positions of linguistic terms in Study 2 (Russia).

Study 2: Spain					
l_1	l_2	l_3	l_4	l_5	Gender
0	29	60	90	120	male
0	14	53	92	120	male
0	15	60	97	120	female
0	24	56	93	120	male
0	28	48	96	120	male
0	28	60	90	120	female
0	25	60	97	120	male
0	30	60	90	120	male
0	29	58	92	120	male
0	13	32	95	120	male
0	15	60	104	120	female
0	24	50	79	120	male
0	26	53	90	120	male
0	23	60	98	120	male
0	20	49	97	120	female
0	16	48	104	120	male
0	23	69	98	120	male
0	15	60	100	120	female
0	23	54	93	120	male
0	22	55	94	120	female

Table 15: Positions of linguistic terms in Study 2 (Spain).