Environmental regulation and tax evasion when the regulator has incomplete information*

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Abstract

This paper analyzes the dynamic interaction between an environmental regulator and a polluting firm in a stock pollution Stackelberg game, where the regulator acts as the leader and the firm as the follower. The firm must determine the emissions required for production and pay a tax based on its reported emissions. The regulator chooses this tax on emissions to induce more environmentally respectful behavior of the firm. Evasion, defined as the gap between real and reported emissions can be discouraged using a fine. A central assumption in our analysis is that the regulator has incomplete information regarding the firm's objective function. The regulator does not know, but conjectures, how afraid the firm is of the fine for fraud. Based on this conjecture, the regulator estimates the firm's best-response functions and determines the tax. We compare the results when the regulator is accurate or misguided. Interestingly we find that when the regulator overestimates the firm's fear of the fine for fraud, social welfare can be greater than when he accurately estimates it.

Keywords: Dynamic regulation, Evasion, Incomplete information, Stackelbeg differential games

1 Introduction

The environment is affected both by consumption habits and the production options of firms. This paper is concerned with the production side. The emission of pollutants is a classic economic externality, where market solutions no longer internalize the environmental

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impact of production decisions. Therefore, governments in most countries try to influence economic agents' behavior by implementing various environmental protection regulations.[1] It is also common to introduce inspections and a penalty regime to ensure that firms comply with the rules.

In mainstream environmental economics, one common hypothesis is to assume that polluters perfectly comply with the optimal policy fixed by the environmental regulators. Alternatively, monitoring and enforcement issues become central in the literature when this assumption is dropped. Telle (2013) empirically analyzes the environmental audit method in many countries. Typically, the regulator does not have mechanisms to correctly predict the level of emissions by companies, but requires them to report their emissions and then inspects some of them. This system allows companies to make two decisions: how much to emit and how much to report. Differing reports are considered fraud and sanctioned if detected. However, even taking into account the possibility of being caught, underreporting of emissions can benefit the company, and indeed, this is usually the case (see Telle, 2013, and references therein).

We analyze the strategic interaction between a representative firm, whose productive activity generates emissions, and an environmental regulator. This latter is identified with a government that seeks to maximize social welfare. It implements an emission tax in order to control emissions to reduce negative effects on the environment and social welfare. The firm optimally determines the real emissions and the amount to be reported to the regulator, in order to maximize its profit. Net profits are defined by the production profits minus the taxes from reported emissions and the expected fine from tax evasion, given the possibility of being caught reporting less than the real emissions. The regulator does not have completely knowledge of the firm's objective function and he conjectures it. With this conjecture, he estimates the best-response functions of the firm and determines the optimal tax for reported emissions. The objective of the regulator is composed of three parts: as a government which seeks to maximize social welfare, it is concerned with the firm's production profits and also internalizes an environmental externality, defined through a pollutant stock stemming from the accumulation of emissions over time. Additionally, although the regulator has no zeal for tax and fine collection, we assume he disapproves of tax evasion. This represents a regulator behaving as a government, which internalizes the damage that fraud imposes on society. Thus, fraud represents a second externality of the firm on society. With two externalities a single policy instrument does not attain the first best.

[1] A revision of the policy options on environmental and natural resource management can be found in Sterner and Coria (2013).

The game is played *à la* Stackelberg, with the regulator taking the role of the leader and the firm acting as the follower. The standard hypothesis in the Stackelberg literature assumes that the regulator has perfect knowledge of the firm's reaction functions. This hypothesis immediately follows under the assumption of complete information. In our formulation, of incomplete information, the follower can hide information because the regulator does not completely know the firm's objective function and he has to conjecture it. In particular, the regulator can undervalue or overvalue the firm's fear of the fine for fraud, acting in an underconfident or overconfident manner. He can mismeasure the probability that the firm assigns to being caught and/or the magnitude of the fine for fraud expected by an evading firm. To the extent that the regulator errs in his conjecture, the resulting best-response functions could also be mistaken. The regulator makes decisions considering these estimated best-responses of real and reported emissions, which are functions of the tax and the pollutant stock.

The main objective of the paper is to measure the impact of the hypothesis of incomplete information. In particular, how the regulator's mismeasure of the firm's fear of the fine for fraud affects the stringency of the regulatory instrument, and as a result, social welfare.

We focus first on the social optimum, characterizing the optimal emissions that would be chosen by a firm that internalizes the environmental damage, or equivalently by a social planner who maximizes social welfare. The particular case, where by chance the regulator gets the firm's objective function right, will be denoted as the benchmark scenario. This accidental scenario would be equivalent to the game if played under complete information. In this case, the regulator happens to know the firm's true best-response functions (real and reported emissions). By contrast, in the general case the leader fails in his conjecture. Even in this case he succeeds in estimating the firm's best response regarding real emissions, but he over/under-conjectures the reaction of reported emissions to the tax. The equilibrium strategies of the real and reported emissions and the tax and social welfare, are compared against their counterparts in the benchmark scenario. We also characterize the situations where the firm is better off/worse off in the general scenario rather than in the benchmark scenario.

Because the government maximizes social welfare, one would expect society to be worse off when it fails to accurately conjecture the firm's objective function. Interestingly, our main result shows that if the regulator errs in his conjecture, society may be better off than if he succeeds in his estimation. Thus, under certain conditions, this model can lead to higher social welfare under incomplete information rather than under complete information. This occurs if the regulator is overconfident and believes the firm is very afraid of the fine for fraud. Then, he will feel more inclined to tax, so the firm will value the environment more strongly than in the benchmark case. The tax will typically be fixed above the benchmark case. As a result, there will be more fraud than in the case of a regulator who conjectures accurately. However, achieving lower emissions (closer to the social optimum) can also lead to higher social welfare than in the benchmark case. This occurs while the positive effect of lower emissions, and hence lower pollution (closer to the social optimum), remains greater than the negative effect of a higher evasion rate. The positive effect decreases with respect to the negative effect as the regulator's level of overvaluation of the firm's fear of the fine for fraud becomes larger.

This result cannot take place in a static setting. It is specific of the dynamic framework, where the higher propensity to tax triggers a self-regulation mechanism on the firm, seeking to reduce future pollution and taxes.

Given this result, one can conclude that the tax policy is more efficient from a social welfare perspective when the government slightly overvalues the firm's fear of the fine for fraud than in the case of an accurate regulator. Conversely, it will be less efficient when the government undervalues or strongly overvalues this fear.

A sensitivity analysis is carried out for the society's aversion to evasion, the intensity of the fine, and the environmental damage.[2] If the society is more averse to evasion, the regulator will fix a lower tax, but the tax will grow more strongly with the overvaluation of the firm's fear of the fine for fraud. Hence the region where a misguided regulator (and hence society) is better than an accurate regulator narrows. Conversely, a higher intensity of the fine for fraud implies a smoother increment of the tax with the overvaluation of the firm's fear of the fine. Moreover, a more punishing fine directly reduces evasion. Thus, the region where incomplete information improves social welfare widens. Finally, this region narrows with greater environmental damage.

An example that could fit our setting is as follows. In a given country or region a political green party wins the elections. This party wants to apply ambitious environmental policies, although it is concerned on the firm's fraud because it damages its credibility. However, this party shows an overoptimistic belief regarding firm's evasion. Knowing the government's strong determination to apply green policies, the firm is highly afraid of an increment in pollution which will strongly raise future taxes. Hence, this rise in the firm's environmental valuation leads to a self-regulation in terms of a reduction in current emissions, seeking to slow down the pollution stock growth. Given this self-regulation the finally adopted policy does not need to be so ambitious, opening up the possibility of social welfare improvements.

[2]This analysis is carried out under the assumption that the initial aversion is not too high with respect to the conjectured intensity of the fine.

The paper is organized as follows. Section 2 gives a short overview of the literature. Section 3 presents the model and the methodology used throughout this study. Section 4 characterizes the social optimum and the equilibria of the game in the benchmark and the general cases. These two scenarios are compared in Section 5. This section analyzes the effect of overvaluation/undervaluation of the true firm's fear of the fine for fraud on the equilibrium strategies and social welfare. A sensitivity analysis of the main results is carried out. Section 6 presents some conclusions and future research.

2 Literature review

From a dynamic game perspective, this paper contributes to the literature on environmental regulation when firms can commit fraud when reporting their emissions.

Fraudulent or criminal behavior and punishment in mainstream economics are theoretically analyzed in the pioneer work by Becker (1968). He proposes a theory on crime based on a cost-benefit analysis. The cost of enforcement for society should be confronted against the benefits from deterring criminal acts. The expected utility of committing fraudulent acts is inversely related to the probability of being discovered and the severity of the punishment. Tsebelis (1989) argues that using cost-benefit analysis to explain the relationship between fraudsters and inspectors is not correct, as both agents are rational. Therefore, applying game theory would be more effective for this type of interaction. The game theoretic approach in this literature on crime deterrence is generally based on a two-player simultaneous-move game, known as an inspection game. Contrary to Becker, Tsebelis concludes that the severity of punishment does not affect the agents' decision on evasion. The Tsebelis model was revisited by Pradiptyo (2007), who concludes that under certain conditions, attempts to increase the severity of the punishment would increase the probability of infraction.

The relationship between regulatory rigor and compliance in the environmental economic literature is analyzed in Downing and Watson (1974), Harford (1978), Jones and Scotchmer (1990), Keeler (1995), Arguedas (2008, 2013), and Lappi (2016). In this context, Macho-Stadler and Pérez-Castrillo (2006) assume that producers can decide real emissions but also the level of emissions to be reported. Within this setting, firms can choose to comply or not to comply with the environmental policy, but environmental taxes are always evaded by underreporting emissions. They prove that an increase in the monitoring budget will not necessarily promote an increment in compliance with environmental taxes. A similar result is obtained in Häckner and Herzing (2017) by imposing similar conditions. Oestreich (2017) examines a model similar to Macho-Stadler and Pérez-Castrillo (2006) and also puts the emphasis on the optimal definition (satisfying social efficiency) of an audit mechanism. This mechanism depends on the gap between reported emissions and a reference level. In a complementary way, Macho-Stadler (2008) proves that environmental taxes are preferred to other instruments when compliance is an issue. She shows that the optimal inspection policy can lead to a high emission flow associated with an elevated level of fraud. All these works consider a static framework, neglecting that pollution may accumulate over time and that agents' decisions have lasting effects on the environment. The present paper addresses these aspects, analyzing a dynamic game. Thus, current decisions will affect not only on current emissions but also future pollution and players' actions.

Assuming dynamic settings, most theoretical papers on environmental regulation issues analyze the interaction between a representative firm and an inspection agency. Although the models are simplistic, inspection and penalty mechanisms tend to be described very carefully. Some of these papers assume perfect compliance and study the optimal environmental policies, for example, Beavis and Dobb (1986), Hartl (1992), Conrad (1992), Falk and Mendelsohn (1993) or Benford (1998). Another set of papers allows for non-compliance and analyzes optimal dynamic enforcement. Among these papers, some authors consider an exogenous standard, for example, Harrington (1988), Harford and Harrington (1991), Raymond (1999), Friesen (2003), or Zhang and Xu (2016). Closer to our work, Arguedas et al. (2017, 2020) allow the policy instrument to be endogenously chosen. In these two papers, the regulator has perfect knowledge (complete information) of the level of noncompliance of the emission limits. Thus, tax evasion aspects are neglected. In the present paper, we consider a dynamic game where the regulatory instrument is endogenously chosen, and further consider the existence of fraud or tax evasion.[3] To give entrance to fraud, we introduce a second fundamental aspect, assuming a regulator who shows incomplete information.

Jiang and Liu (2017) identify several sources of incomplete information in an attack-defense game: noisy signals, secrecy policy and/or "false" targets, lack of transparency or rationality, the valuation of targets and/or the number of interactions. Among them, we align ourselves with secrecy policy when the firm can evade emission taxes by reporting emissions below their actual value. Considering that the firm can deceive the regulator, the main novelty of the present paper is to assume that the regulator holds incomplete infor-

[3] In the literature, tax evasion is widely acknowledged as a contributor to negative externalities. In Grinols and Mustard (2001), fraudulent activity generates social costs, which involve real resources used for apprehension, trial, incarceration and rehabilitation of criminals, or costs arising from increased police presence or auditing in areas with higher fraud rates. According to Çule and Fulton (2009), such corruption hampers successful revenue collection, subsequently impacting the provision of public goods to society. Additionally, Wilks and Zimbelman (2004) emphasize that the prospect of fraud increases market volatility and disrupts its smooth functioning.

mation regarding the firm's objective function. In particular, since the existence of fraud leads the regulator to sanction evading firms, we assume that the regulator mismeasures the effect of the fine for fraud on the firm's net profits. Maybe the regulator believes that the firm overestimates/underestimates the probability of being caught.

This assumption of incomplete information of the regulator aligns with authors like Tasic (2009, 2011), Carlsson and Johansson-Stenman (2012), or Dudley (2019 and references therein), who have a critical view on the standard hypothesis in the economic literature, which neglects the possibility of cognitive limitations and bias for the regulator, although not for other economic agents.[4] Thus, we contribute to the literature which questions the assumption of a perfectly rational regulator. Specifically, in this paper, the regulator might either underestimate or overestimate the firm's fear of fraud penalties, displaying either under-confidence or overconfidence.

We also contribute to the dynamic game theory literature by considering a Stackelberg leader who does not have complete knowledge of the follower's objective function and hence, does not accurately know his best-response functions (see, for example, Dockner *et al.* (2000) or Long (2010).

3 The model

A representative company produces a consumer good with emissions as the only input. Consider $Y(t)$ and $e(t)$ as production and production emissions at time t. For mathematical convenience and in line with the literature on pollution dynamic games (see, for example, Jørgensen *et al.* 2010), we assume that the output is a quadratic function on emissions:

$$Y(t) = \frac{e(t)(A - e(t))}{2}, \quad A > 0.$$

The output is a strictly concave function on emissions and reaches its maximum when $e(t) = A$.

Emissions accumulate as a pollution stock, P, that generates environmental damage. The time evolution of the stock is described by the following differential equation:

$$\dot{P}(t) = e(t) - \delta P(t), \quad P(0) = P_0,$$

[4] Alternatively, our formulation could be applicable for a non-biased regulator who is facing an over/under confident firm. See the literature on corporate bias and executive overconfidence (Schrand and Zechman 2012 or Cao *et al.* 2024 among others).

where $\delta > 0$ is the environment assimilation capacity (or recovery rate) and $P_0 > 0$ is the initial pollution stock.

We assume that the government acts as a regulator seeking to hold the company accountable for the environmental externality that it generates. He imposes an emission tax rate, $\tau(t)$, to internalize this environmental damage. Not knowing the exact amount of emissions, this tax is not applied to real emissions but to the emissions that the company reports, $R(t)$. The company has two options: i) to declare authentic emissions or ii) to commit fraud, i.e. to declare less than the authentic emissions, $e(t) - R(t) > 0$, in order to pay a lower fee for the generated emissions[5].

A cheating firm has to be aware that it will be penalized if fraud is detected. The firm is audited with probability p_A and, if fraud is detected, a penalty is imposed. Under the assumption of a growing marginal penalty with the size of evasion, the expected fine is given by the product of the probability of inspection, p_A, times the fine rate, f, times the square of the size of evasion. For shortness, the product $2p_A f$ is denoted by letter β, which represents the intensity of the fine. In consequence, the expected fine can be written as:[6]

$$\frac{\beta}{2}(e(t) - R(t))^2. \tag{1}$$

We consider a second-best situation in which the regulator can determine the emission tax. However he cannot control the fine system. On the one hand, changing the probability of inspection, p_A, can be linked to large adjustment costs (hiring/firing costs of auditors) and hence economically unaffordable. On the other hand, we also consider an exogenously fixed fine rate, f, assumption which can be supported under two alternative scenarios. The regulator can avoid changing the fine rate, because it is legally and/or politically costly to implement. Alternatively, it can represent the case where taxation and auditing are carried out by two different regulatory bodies. A central government would determine taxes, considering a fixed fine rate, since inspection activities are carried out by a local government/agency.

At each time t, the firm decides on real emissions and how much of these emissions to report in order to maximize profits, defined by the income from production minus the amount of the fine for fraud and the tax paid on reported emissions:[7]

$$F_F\left(e(t), R(t), \tau(t)\right) = e\left(t\right)\left(A - \frac{e(t)}{2}\right) - \frac{\beta}{2}(e(t) - R(t))^2 - \tau\left(t\right)R\left(t\right).$$

[5]Note that we are assuming a positive tax. If conversely the regulator fixed an emission subsidy, then the firm would commit fraud by reporting more than the real emissions, $e(t) - R(t) < 0$.

[6]Alternatively, one might consider that the probability of being caught is not only determined by the probability of inspection but it is also proportional to the size of fraud, $p_A(e(t) - R(t))$. Hence, the fine in (1) is also applicable under the assumption of a constant marginal fine.

[7]The subscript F stands for the firm.

The interaction between the regulator and the company is described as a differential game, *à la* Stackelberg, in which the regulator is the leader, and the company is the follower. This paper focuses on feedback stage-wise Stackelberg equilibrium in a linear-quadratic differential game. To have a time-consistent equilibrium, we assume that the regulatory policy and the company's decisions depend on the pollution stock, P. At each instant, the regulator fixes and announces the fee on reported emissions depending on the stock of pollution. The company must decide the real emissions and the reported emissions as functions of the pollution stock.[8]

Next, we explain the resolution of the Stackelberg game. First, we solve the follower's problem computing the firm's best-response functions. Second, we present the maximization problem of the leader. In the standard formulation, with complete information, the regulator knows these best-response functions and determines the equilibrium tax. However, in our formulation with incomplete information, the regulator has to estimate the best-response functions by conjecturing the firm's objective function.

3.1 The firm's maximization problem

The firm decides the real and reported emissions which maximize the net profits' present value over an infinite time horizon, as functions of the policy announced by the regulator. For simplicity of the exposition, from now on, the time argument will be removed, so that the dynamic maximization problem for the company is given by:

$$\max_{e,R} \int_0^\infty \left[e\left(A - \frac{e}{2}\right) - \frac{\beta}{2}(e-R)^2 - \tau R \right] e^{-\rho t} dt \tag{2}$$

$$\text{s.t.: } \dot{P} = e - \delta P, \quad P(0) = P_0, \tag{3}$$

where $\rho > 0$ is the time discount factor.

The regulator suffers environmental damage from the pollution; hence, the optimal tax is a function of the pollutant stock. Hence, the tax adjusts according to the severity of the environmental problem. Therefore, since the regulator announces a stock-dependent tax, the firm is concerned about the level of the pollutant stock when making its optimal decisions.

[8] We analyze a game where the regulator has a stagewise first-mover advantage, an instantaneous advantage at each time. This solution satisfies sub-game perfection, which is the most credible concept (see, Dockner *et al.* 2000, Haurie *et al.* 2012). As usual in this type of differential game with an infinite time horizon, we assume that the agents (the company and the regulator) use stationary strategies, and consequently their strategies and value functions do not depend explicitly on time but exclusively on the pollution stock.

The company takes the role of the follower in the Stackelberg game. To characterize its best-response functions, we solve problem (2)-(3) using the following Hamilton-Jacobi-Bellman equation (HJB):

$$\rho V_F(P) = \max_{e,R}\{F_F(e,R,\tau) + V_F'(P)(e-\delta P)\}, \tag{4}$$

where $V_F(P)$ represents the firm's value function.

The first-order conditions of the RHS of the HJB equation above characterize the best-response functions. The interior real and reported emissions are functions of the state variable, P, and the regulatory variable, τ, and are given by:[9]

$$e^{br}(P;\tau) = A - \tau + V_F'(P), \tag{5}$$

$$R^{br}(P;\tau) = A - \tau\left(1+\frac{1}{\beta}\right) + V_F'(P). \tag{6}$$

A higher emission tax induces a one-to-one reduction in real emissions and a stronger reduction in reported emissions. The smaller the β, and hence the expected fine for evasion, the stronger the reduction in reported emissions with a rise in the emission fee.

From the best-response functions (5) and (6), it follows that when the firm chooses real and reported emissions, it evades till the point at which the marginal fine equates to the emission tax:

$$\beta[e^{br}(P;\tau) - R^{br}(P;\tau)] = \tau. \tag{7}$$

As a result, and regardless of the pollution stock and the regulator's chosen tax, the evasion is always proportional to the emission tax, at rate $1/\beta$.

3.2 The regulator's maximization problem

The standard hypothesis in a hierarchical game is to consider a perfectly informed regulator who knows the best-response functions of the follower. However, we assume that the regulator in this game only has partial information regarding the objective of the firm and he has to estimate the best-response functions of the firm from his possibly mistaken conjecture about the firm's objective function. We assume that the leader conjectures the company's objective function as follows:[10]

$$F_F^C(e,R,\tau) = e(t)\left(A - \frac{e(t)}{2}\right) - \frac{\beta\alpha}{2}(e(t)-R(t))^2 - \tau(t)R(t). \tag{8}$$

[9]The superscript br stands for "best response".

[10]Here, superscript C stands for "conjectured".

This equation presents the same three terms of the firm's objective function. While the regulator is correct about output and taxes, he might be wrong about the repercussion of the fine on the firm's profits. For example, the regulator might overestimate or underestimate the likelihood that the firm assigns to being discovered. Parameter α measures the mistake made by the regulator when conjecturing how the fine for fraud affects the firm. If $\alpha > 1$ (< 1), the regulator over (under) conjectures this repercussion. Henceforth, we refer to α as the regulator's belief of the firm's fear of the fine for fraud, or regulator's belief, for shortness.

Thus, the regulator solves the dynamic maximization problem of the firm, but taking into account the conjectured firm's objective function, $F_F^C(e, R, \tau)$, given in (8). As a result, the estimated firm's interior best-response real and reported emissions are given by:[11]

$$e^{ebr}(P;\tau) = A - \tau + V_F'(P), \tag{9}$$

$$R^{ebr}(P;\tau) = A - \tau\left(1 + \frac{1}{\beta\alpha}\right) + V_F'(P). \tag{10}$$

Note that the regulator is correct in the estimated best response for real emissions ($e^{br} = e^{ebr}$), but he fails to estimate reported emissions correctly ($R^{br} \neq R^{ebr}$). If $\alpha > 1$, the regulator believes that the firm overestimates the probability of being caught and/or the fine rate. Consequently, the regulator believes that the firm will not reduce reported emissions so sharply with increments in the emission tax. Thus, the estimated reported emissions function reacts less sharply than the actual reported ones. The opposite occurs for $\alpha < 1$.

The regulator is concerned about the company's profits and the environmental damage caused by the pollution stock. This is because we understand the regulator to be a benevolent government. We believe that the government acts in the best interest of society, which includes the firm's profit. Moreover, in the same line, the taxes and fines collected are returned to society (in the form of a lump-sum transfer). The gains in government revenues (and hence public spending) is exactly offset by the losses in the firm's profits, making the fines and the taxes welfare-neutral. For that reason, taxes and fines do not enter the regulator's objective function. However, we believe that the regulator is negatively affected by the existence and the amount of deception, in the sense that he does not like to be cheated. Consequently, the regulator's objective function is defined as[12]:

$$F_L(e, R, \tau, P) = e\left(A - \frac{e}{2}\right) - \frac{d}{2}P^2 - \frac{\phi}{2}(e - R)^2,$$

[11] The superscript *ebr* stands for "estimated best response".

[12] The subscript L stands for "leader" (regulator).

with $d > 0$ the environmental damage. The third term in the regulator's objective function represents the cost for society associated with fraud. Thus, when the government maximizes social welfare, he faces two externalities on society generated by the firm: the first from the stock of pollution and the second linked to fraud. A single policy instrument does not allow to attain the first best.[13] Parameter ϕ represents the damage from fraud on society. It measures the cost borne by the society linked to the tax imposition, since the tax induces evasion. Society does not like the firm reporting emissions above or below real emissions. The cost of being cheated is given by a quadratic function in the magnitude of the evasion.

In this leader-follower configuration, the regulator decides on the optimal regulatory instrument, here a tax on emissions, τ, considering the estimated best-response functions in (9) and (10). Thus, the regulator faces the following dynamic optimization problem:

$$\max_{\tau} \int_0^{\infty} \left[e^{ebr}\left(P;\tau\right) \left(A - \frac{e^{ebr}\left(P;\tau\right)}{2}\right) - \frac{d}{2}P^2 - \frac{\phi}{2}(e^{ebr}\left(P;\tau\right) - R^{ebr}\left(P;\tau\right))^2 \right] e^{-\rho t} dt, \tag{11}$$

$$\text{s.t.:} \dot{P} = e^{ebr}\left(P;\tau\right) - \delta P;\ P(0) = P_0. \tag{12}$$

Focusing on feedback Stackelberg equilibrium, the regulator sets the strategies in terms of the pollution stock, P. His optimal decision is determined from the RHS of the HJB equation:

$$\rho V_L(P) = \max_{\tau} \{F_L(e^{ebr}\left(P;\tau\right), R^{ebr}(P;\tau), \tau, P) + V_L'(P)(e^{ebr}\left(P;\tau\right) - \delta P)\}, \tag{13}$$

where $V_L(P)$ is the regulator's value function. Once the optimal regulatory instrument is determined, the company's optimal real and reported emissions are calculated, as a function of the pollution stock, taking into account the true (not the estimated) best-response functions in (5) and (6).

Next, we characterize the optimal strategies of the regulator and the company when the leader conjectures the firm's objective function either accurately or mistakingly.

As a first step, prior to the analysis of the regulatory game with strategic interactions, we analyze the case of the social optimum with a unique decision maker in the next section. Second, we analyze the benchmark scenario with accurate estimation. In the subsequent section, we discuss the general scenario with a misguided regulator.

[13]When fraud has no effect on the society, $\phi = 0$, the second externality disappears. Therefore, the first best can be attained when deception is not an issue.

4 Equilibrium strategies

This section characterizes the equilibrium strategies when the regulator is accurate or misguided. It initially presents the social optimum for comparison purposes.

4.1 Social optimum

At the social optimum, emissions are chosen by a firm which internalizes the environmental damage, and hence with no need for environmental regulation. This is equivalent to a social planner who maximizes social welfare:

$$\max_{e} \int_0^\infty \left[e \left(A - \frac{e}{2} \right) - \frac{d}{2} P^2 \right] e^{-\rho t} dt, \tag{14}$$

subject to (3). The problem is solved using dynamic programming to characterize the optimal solution in feedback form. Thus, it can be easily compared against the optimal strategies in the subsequent scenarios with strategic interaction. From the first-order condition, the optimal emissions for this problem read (see Appendix A.1 for details):[14]

$$\begin{aligned} e_{SO}(P) &= A + V'_{SO}(P) \\ &= A + \left[\rho + 2\delta - \sqrt{(\rho + 2\delta)^2 + 4d} \right] \left\{ \frac{P}{2} + \frac{A}{\rho + \sqrt{(\rho + 2\delta)^2 + 4d}} \right\}, \end{aligned} \tag{15}$$

where $V_{SO}(P)$ denotes the value function of the social planner. This standard result states that the optimal emissions equal the amount that maximizes instantaneous income, A, minus the marginal damage from an additional unit of pollution (increasing in P). Notice that, since pollution is a bad, $V'_{SO}(P)$ takes a negative value, as (15) straightforwardly shows.

4.2 Benchmark case

In this section, we focus on the case where the regulator accurately conjectures the firm's objective function. Hence his estimated best-response functions in (9) and (10) match the true best-response functions in (5) and (6). Thus, he solves the optimization problem in (11)-(12), but replacing the estimated by the true best-response functions. This exercise will serve as a benchmark against which to compare the general scenario where the regulator errs in his conjecture.

[14] The subscript SO stands for "social optimum".

From the optimality condition for the regulator's problem, the interior equilibrium tax can be written as a constant fraction of the gap in marginal valuations of the environment between the regulator and the firm.[15]

$$\tau_B(P) = \frac{\beta^2}{\beta^2 + \phi}[(V^F)'(P) - (V^L)'(P)]. \tag{16}$$

The gap between the players' marginal valuations represents a greater overvaluation of the environmental problem by the regulator than that made by the firm. Given that the regulator internalizes the damage from pollution, he will value the environment more than the firm, implying a positive gap.

The constant of proportionality can be interpreted as the propensity to tax by the regulator per unit of discrepancy in the environmental valuation between the regulator and the firm. Henceforth, this will be denoted PTT. The PTT rises with the intensity of the fine, β, and decreases with the social damage associated with evasion, ϕ.

Plugging (16) into the true best-response functions (5)-(6), the equilibrium real and reported emissions follow:

$$e_B(P) = A + \frac{\beta^2}{\beta^2 + \phi}(V^L)'(P) + \frac{\phi}{\beta^2 + \phi}(V^F)'(P), \tag{17}$$

$$R_B(P) = A + (V^F)'(P) - \frac{\beta(1+\beta)}{\beta^2 + \phi}[(V^F)'(P) - (V^L)'(P)]. \tag{18}$$

From these expressions, it follows that the evasion is also proportional to the gap between the two players' environmental valuation:

$$e_B(P) - R_B(P) = \frac{\beta}{\beta^2 + \phi}\left[(V^F)'(P) - (V^L)'(P)\right].$$

4.2.1 The regulator disregards evasion, $\phi = 0$

Here we analyze the particular case where the size of the tax evasion is irrelevant to the society, and hence the regulator's objective function is defined by the final output minus the damage from pollution. By replacing $\phi = 0$ in (16), (17), and (18), the equilibrium

[15] Subscript B stands for "benchmark case". Note that the marginal valuation of a bad, such as the pollution stock, is negative. Thus, we define the marginal valuation of the environment by one agent as the opposite of its marginal valuation of pollution. As a result, the gap in marginal valuations of pollution between the follower and leader is equivalent to the gap in marginal valuations of the environment between the leader and follower.

strategies now read:

$$\begin{aligned}
\tau_{B0}(P) &= (V^F)'(P) - (V^L)'(P), \\
e_{B0}(P) &= A + (V^L)'(P), \\
R_{B0}(P) &= A + \frac{(1+\beta)(V^L)'(P) - (V^F)'(P)}{\beta}.
\end{aligned} \tag{19}$$

In this scenario, the regulator settles on a tax not only proportional but equal to the gap between the players' valuation of pollution. Consequently, the emissions only depend on the value the regulator gives to the environment. Indeed, the emissions strategy coincides with that in the social optimum. Interestingly, in this scenario there is no externality associated with evasion, which corresponds to a case with a single externality and one policy instrument. Then, the optimal social welfare is attained.

By contrast, when evasion represents a cost for the society, $\phi > 0$, the PTT, $\beta^2/(\beta^2 + \phi)$, is less than one and the emissions are no longer independent of how the firm values pollution. Equilibrium emissions in (17) are given by A plus a convex combination between the marginal valuation of pollution by the firm and the regulator. Because the regulator internalizes pollution $(V^L)'(P) < (V^F)'(P) < 0$, the convex combination in (17) is larger than the leader's valuation in (19). Therefore, emissions are greater than when the society disregards evasion. As a result, the social optimum cannot be reached in this general case.

4.2.2 Numerical illustration

This section computes the equilibrium strategies in the general case when the firm's tax evasion generates an externality on society. These strategies in (16)-(18) are expressed in terms of the derivatives of the firm's and regulator's value functions. In order to have a full characterization of the strategies, these value functions need to be determined. Given the linear-quadratic structure of the different games, we conjecture quadratic value functions:

$$V_B^F(P) = a_B^{2F}\frac{P^2}{2} + a_B^{1F}P + a_B^{0F}, \quad V_B^L(P) = a_B^{2L}\frac{P^2}{2} + a_B^{1L}P + a_B^{0L}. \tag{20}$$

To determine the values of the coefficients of these quadratic functions, one needs to solve the Riccati equations stemming from the system of equations:

$$\rho V_B^F(P) = F_F(e_B(P), R_B(P), \tau_B(P)) + (V_B^F)'(P)(e_B(P) - \delta P), \tag{21}$$

$$\rho V_B^L(P) = F_L(e_B(P), R_B(P), \tau_B(P), P) + (V_B^L)'(P)(e_B(P) - \delta P). \tag{22}$$

Four triples of solutions for these coefficients are obtained. Here we are assuming that the equilibrium that is effectively implemented is the one which gives higher payoff to the leader in the hierarchical game. We also look for solutions with bounded optimal time paths.

The four solutions can be found analytically, and two of them can be dropped due to the non-converge of the optimal pollution path to a finite steady-state pollution stock. However, it is not possible to analytically determine which of the remaining two solutions gives a higher value function for the regulator, except in the simplest case with $\phi = 0$.[16] To distinguish between these two solutions, we perform a numerical analysis considering the following parameter values:

$$\delta = 0.1,\ \rho = 0.03,\ d = 0.001,\ \beta = 0.3,\ A = 1,\ \phi = 0.1. \tag{23}$$

For the set of parameter values in (23), the equilibrium strategies and the steady-state pollution stock are:[17]

$$\begin{aligned} &e_B(P) = 0.98 - 0.0020P, \quad R_B(P) = 0.95 - 0.0089P, \\ &\tau_B(P) = 0.0084 + 0.0021P, \quad P_B^{SS} = 9.58. \end{aligned}$$

An increase in the pollution stock positively impacts the emission tax fixed by the regulator. As the environmental problem worsens, the regulation gets stricter. Consequently, real and reported emissions decrease, the latter at a much faster rate (more than four times larger).

4.3 General case

Here we analyze the problem (11)-(12) when the regulator misjudges the effect of the fine for fraud on the firm revenues. The mistake made by the regulator when conjecturing how the fine for fraud affects the firm is measured by parameter α. If $\alpha > 1$ (< 1) the regulator over (under) values this repercussion. And for $\alpha = 1$, he does not fail in his conjecture and the benchmark case is recovered. Since $\alpha = 1$ corresponds to the complete information scenario, the sign of $1-\alpha$ characterizes either undervaluation if positive, or overvaluation if negative. And the absolute value of $1 - \alpha$ measures the intensity of the over/under-conjecture of the regulator. In what follows, we analyze situations where α runs from 0 to 2. The former corresponds to a regulator who completely undervalues the firm's fear of the fine for fraud, and the latter a situation where he overvalues this fear twice its true value.

[16] In this case, the two sets of solutions achieve the social optimum. Interestingly, each solution gives rise to a different tax, reported emissions and evasion. As a result, each solution is linked to a different value function for the firm.

[17] The superscript SS stands for "steady state".

As shown in expression (10), if the regulator believes that the firm is very afraid of the fine for fraud, $\alpha > 1$, his overconfidence makes him believe that the firm will slightly reduce the reported emissions with increments in the emission tax (the opposite applies if the regulator is underconfident and believes that the firm is slightly afraid of the fine $\alpha < 1$).[18]

The first-order condition in the RHS of equation (13) gives the interior equilibrium tax:[19]

$$\tau_\alpha(P) = h(\alpha)[(V^F)'(P) - (V^L)'(P)], \tag{24}$$

where

$$h(\alpha) = \frac{\alpha^2\beta^2}{\alpha^2\beta^2 + \phi}.$$

The PTT is now dependent on the regulator's belief in the firm's fear of the fine for fraud, $h(\alpha)$. It also depends on the intensity of the fine, β, and the social damage associated with evasion and internalized by the regulator, ϕ, as in the benchmark case. Additionally, the regulator's belief, α, also indirectly affects the emission tax through its effect on the value functions, and therefore, on the gap between the players' valuation of the environment, $(V^F)'(P) - (V^L)'(P)$.

Plugging (24) into the true best-response functions (5)-(6), the firm's real and reported interior equilibrium emissions are:

$$e_\alpha(P) = A + \frac{\alpha^2\beta^2}{\alpha^2\beta^2 + \phi}(V^L)'(P) + \frac{\phi}{\alpha^2\beta^2 + \phi}(V^F)'(P), \tag{25}$$

$$R_\alpha(P) = A + (V^F)'(P) - \frac{\alpha^2\beta(1+\beta)}{\alpha^2\beta^2 + \phi}[(V^F)'(P) - (V^L)'(P)]. \tag{26}$$

Note that, even if the regulator fails to estimate the true best response for reported emissions, the linear relationship between the evasion and the optimal emission tax in (7) still remains.

The PTT, $h(\alpha)$, is an increasing S-shaped function that satisfies, $h(0) = 0$ and $\lim_{\alpha\to\infty} h(\alpha) = 1$. The greater the firm's fear of being fined from the regulator's perspective, α, the wider his underestimation of evasion and hence, the greater the PTT. The particular case $h(1) = \beta^2/(\beta^2 + \phi)$, corresponds to the PTT in the definition of the equilibrium tax in the benchmark case in (16).

[18] It is important to note that our formulation could also be applicable if, instead of the regulator displaying a behavioral bias, it was the firm exhibiting a cognitive limitation unknown to the regulator. In this case, the scenario where $\alpha > 1$ ($\alpha < 1$) would represent the firm acting with more (less) confidence and less (more) fear of the fine than the regulator had assumed.

[19] The subscript α stands for the "general scenario".

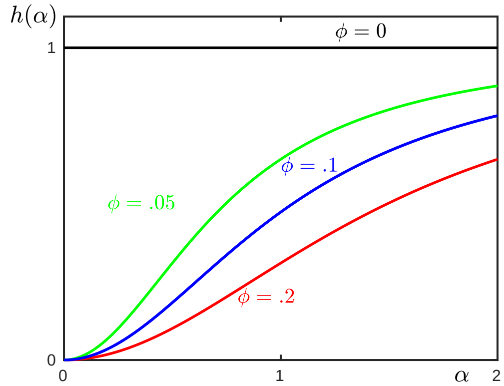

Figure 1: $h(\alpha)$ for $\phi = 0$, $\phi = 0.03$ and $\phi = 0.1$.

Figure 1 depicts the PTT function for $\beta = 0.3$ and four values of $\phi \in \{0, 0.05, 0.1, 0.2\}$. Assume that we are in the extreme case with $\phi = 0$, where the regulator bears no cost linked to tax imposition. Regardless of the conjecture made by the regulator, the PTT will be constant and equal to 1. This seeks to force the firm to fully internalize the environmental externality that firm's emissions impose on society. Although it seeks that the firm emits exactly the same as at the social optimum, this does not occur, because the firm deceives. In the general case with $\phi > 0$, $h(\alpha)$ is S-shaped. The smaller the value of ϕ ($\phi = 0.05$, the green line), the closer the $h(\alpha)$ curve to the straight line ($\phi = 0$). Opposite to this, the higher the value of ϕ ($\phi = 0.2$, the red line), the greater the externality from evasion, the lower the PTT, and the lower the proportion of the pollution externality that the regulator wants the firm to internalize.

4.3.1 Numerical illustration

Following similar reasoning as in the benchmark case, we conjecture quadratic value functions:

$$V_\alpha^F(P) = a_\alpha^{2F}\frac{P^2}{2} + a_\alpha^{1F}P + a_\alpha^{0F}, \quad V_\alpha^L(P) = a_\alpha^{2L}\frac{P^2}{2} + a_\alpha^{1L}P + a_\alpha^{0L}.$$

The coefficients for these value functions follow from the Riccati equations stemming from:

$$\rho V_\alpha^F(P) = F_F(e_\alpha(P), R_\alpha(P), \tau_\alpha(P)) + (V_\alpha^F)'(P)(e_\alpha(P) - \delta P),$$
$$\rho V_\alpha^L(P) = F_L(e_\alpha(P), R_\alpha(P), \tau_\alpha(P), P) + (V_\alpha^L)'(P)(e_\alpha(P) - \delta P).$$

Due to its complexity, the selection between the four solutions for the coefficients of the value functions is numerically implemented. We compute the four sets of solutions for these coefficients for a $[0, 2]$ interval in parameter α. As already commented, the interval $[0, 2]$ is chosen for α, because the error in the regulator's belief is measured by the distance of this parameter to 1. In this scenario, we discard one solution because it is unstable for every

$\alpha \in [0,2]$. We select the only remaining solution which is stable for every $\alpha \in [0,2]$, and moreover provides higher regulator value function than the other two when they are stable. This is true for any value[20] of $P \in [0, P_{\max}]$.

The effect of α on the equilibrium tax in (24) combines two factors. A direct effect on the PTT and an indirect effect on the gap in the marginal valuation of the environment between the leader and the follower. The regulator's belief in the firm's fear of the fine for fraud undoubtedly rises his PTT. This effect is partially counterbalanced because a higher PTT induces a stronger valuation of the environment in the firm (see Figure 2 up-right) and a softer valuation in the regulator. Typically, the positive effect of a higher PTT is stronger than the negative effect of a narrower gap, which implies a positive relationship between the regulator's belief and the equilibrium tax (see Figure 2 down).

The comparison among the curves for the four different values of the pollution stock (1, 10, 20, and 30) in Figure 2 shows that the discrepancy in the players' valuation of the environment grows with the pollution stock. Importantly, the relative effect of α on this gap is less intense the greater the pollution stock. Thus, if pollution is high, the reaction of the tax to changes in α is mainly determined by the PTT. In contrast, when the pollution stock is small, the offsetting effect of a narrower gap in the players' valuation plays a more relevant role.[21]

[20] We have chosen $P_{\max} = 30$, three times its steady-state value in the benchmark case. In this section, we will see that the steady-state pollution stock does not critically change with α.

[21] Typically, this offsetting effect partially counterbalances the rise induced by a higher PTT. However, if the pollution stock is very small (much lower than its steady-state value) this offsetting effect can be so strong as to lead the emission tax down.

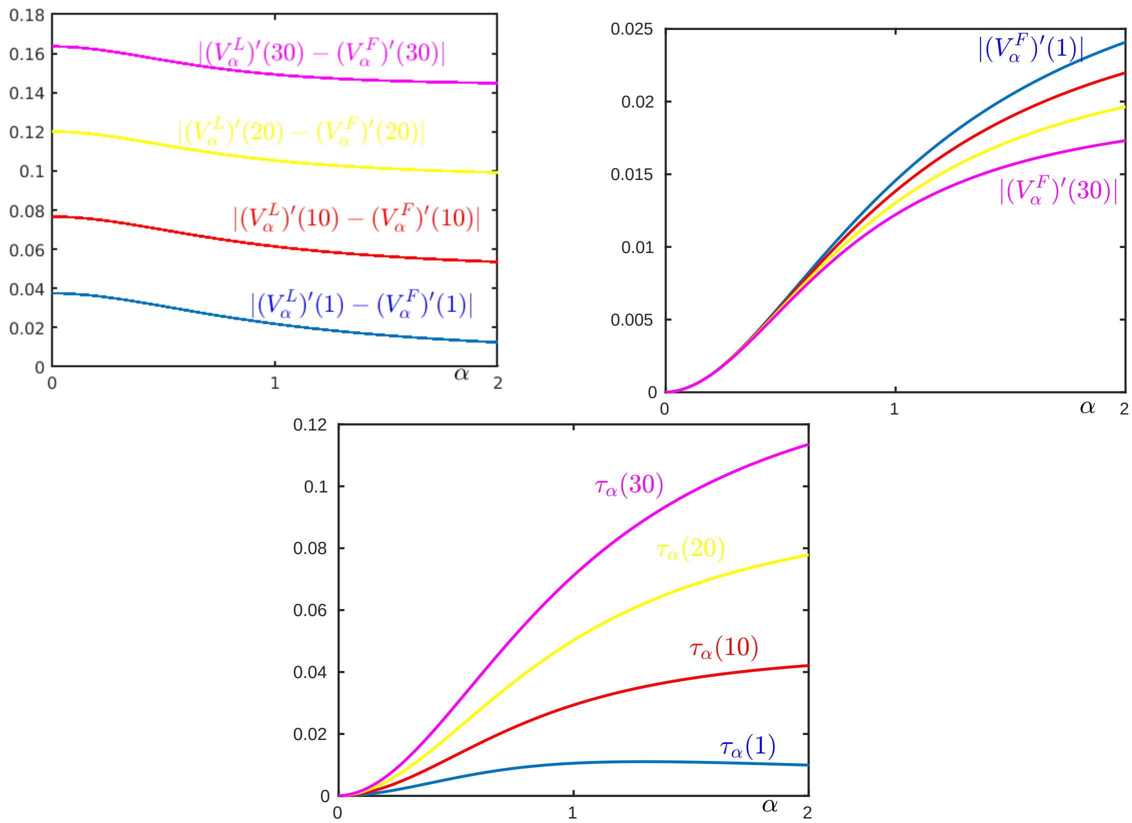

Figure 2: $|(V_\alpha^F)'(P_0) - (V_\alpha^L)'(P_0)|$ (up-left); $|(V_\alpha^F)'(P_0)|$(up-right); $\tau_\alpha(P_0)$ (down).

Figure 3 depicts the equilibrium real and reported emissions as a function of α again for the four values of the pollution stock previously considered. Typically (except for very low values of the pollution stock) real and reported emissions decrease with α as the emission tax becomes tighter. The reduction is stronger the greater the pollution stock. Note that a rise in the emission tax reduces reported emissions more strongly than real emissions.

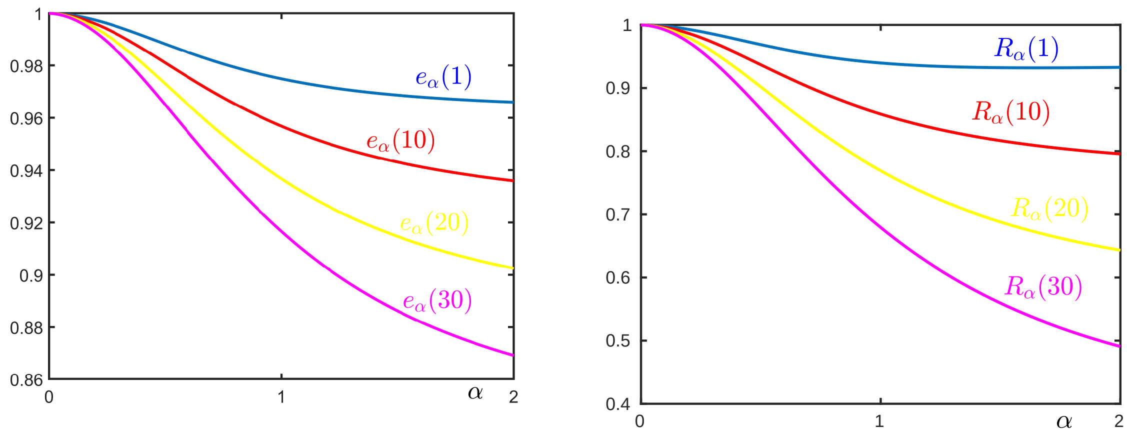

Figure 3: $e_\alpha(P)$(left); $R_\alpha(P)$ (right).

5 Comparison of the benchmark and the general cases

In what follows, we compare the scenarios with accurate and misguided conjecture to analyze the effect of the regulator's overvaluation/undervaluation of the exact firm's fear of being fined for fraud.

The comparison of the equilibrium strategies with accurate and misguided conjecture is summarized in Figures 4 and 5. These figures depict the level curves[22] of the gaps $\tau_\alpha^L(P)-\tau_B^L(P)$, $e_\alpha(P)-e_B(P)$ and $R_\alpha(P)-R_B(P)$ on the $\alpha-P$ plane. The more afraid of the fine for fraud the regulator believes the firm to be, the higher the PTT. Figure 4 shows that for α greater than one, the regulator raises the tax above the benchmark case. As a result, the firm's valuation of the environment also rises and emissions decrease, getting closer to their social optimum value (Figure 5 left), although there is an increase in fraud. The rise in the tax and the decrease in emissions is more acute the greater the pollution stock. Note that the reduction in reported emissions is stronger than the reduction in real emissions (Figure 5 right). Opposite reasoning applies for α lower than one.

[22]These level curves have been computed using Mathematica (version 12.3.1). Negative (positive) level curves represent combinations $\alpha-P$ where incomplete information implies a lower (greater) value of the corresponding decision variable.

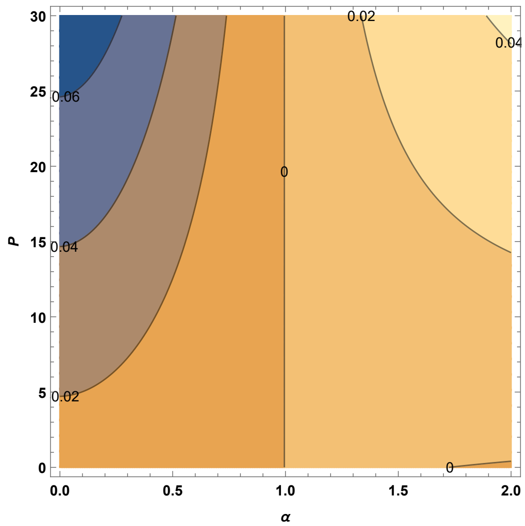

Figure 4: Level curves of the gap $\tau_\alpha(P) - \tau_B(P)$.

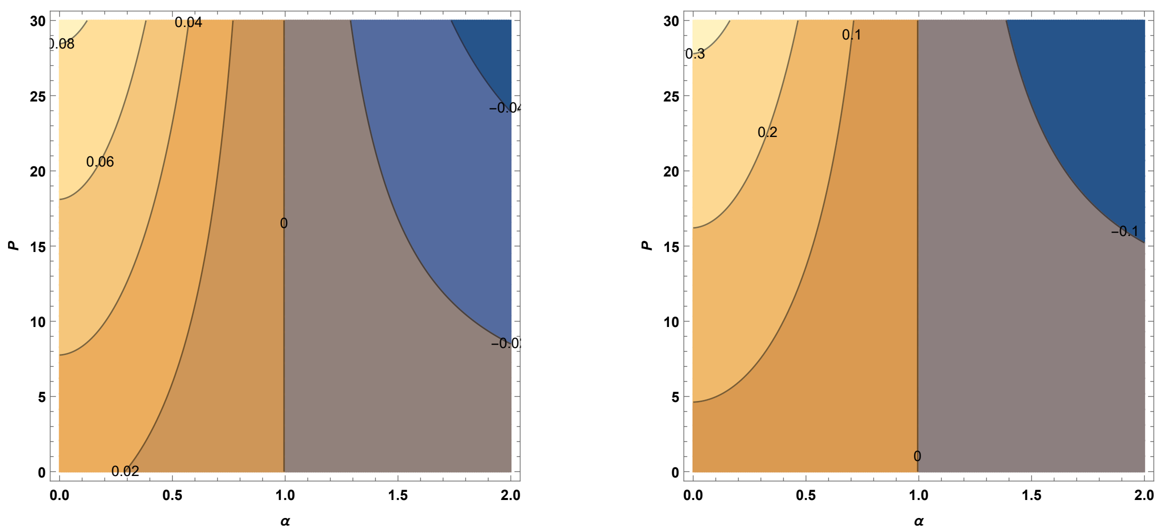

Figure 5: Level curves of the gaps $e_\alpha(P) - e_B(P)$ (left); $R_\alpha(P) - R_B(P)$ (right).

Next, we compare social welfare and firm's profit with accurate and misguided conjecture. Note that by identifying the regulator as the government, his value function represents social welfare. The main result of this paper concerns the comparison of social welfare.

Main result:
When the regulator overvalues the firm's fear of the fine for fraud, social welfare is higher with misguided rather than with accurate conjecture if overvaluation remains below a certain threshold. The threshold is lower the greater the pollution stock. Overvaluation implies less emissions and lower pollution stock closer to the social optimum. Although it also implies

more evasion and less production, this negative effect only partially counterbalances the positive effect as long as overvaluation does not surpass the threshold.

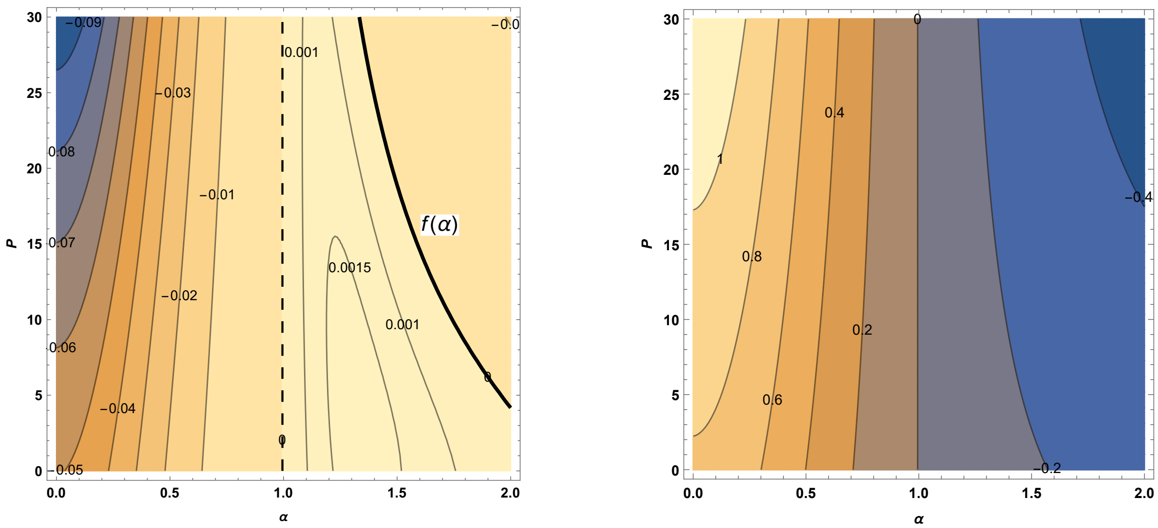

Figure 6: Level curves of the gaps $V_\alpha^L(P) - V_B^L(P)$ (left); $V_\alpha^F(P) - V_B^F(P)$ (right).

This main result is presented in Figure 6 (left), which depicts the level curves of the gap $V_\alpha^L(P) - V_B^L(P)$: the difference between the regulator's value function with accurate and misguided conjecture. Recall that the regulator is a benevolent central planner and hence his value function also represents social welfare. Negative level curves represent combinations of the regulator's conjecture, α, and the initial pollution stock, P, where incomplete information leads to a reduction in social welfare, $V_\alpha^L(P) < V_B^L(P)$. Conversely, positive level curves correspond to combinations where incomplete information is social welfare enhancing, $V_\alpha^L(P) > V_B^L(P)$. One would expect that the lack of information leads to social welfare losses. This presumption is confirmed when the regulator undervalues the firm's fear of the fine for fraud, to the left of $\alpha = 1$. However, this presumption is not necessarily true in the case of overvaluation, to the right of $\alpha = 1$, as the lightest region in Figure 6 (left) points out. This region is delimited by the vertical line $\alpha = 1$ where the regulator has complete information (or he conjectures accurately) and the downward-sloping curve $P = f(\alpha)$, where $V_\alpha^L(P) = V_B^L(P)$. This region (denoted henceforth as L-win), is characterized by positive level curves and hence, the leader attains greater welfare when misguided rather than when accurate. Figure 6 (left) also shows that the greater the pollution stock, the greater the losses (for $\alpha < 1$), or the lower the gains associated with the lack of information (for $\alpha > 1$).

In the L-win light region in Figure 6 (left), a misinformed regulator who mismeasures the firm's fear of the fine for fraud becomes better off than when he measures it accurately.

When $\alpha > 1$, the emission tax and the evasion are higher, and the real emissions are slightly lower than under the benchmark case, and closer to the social optimum. While this represents a gain for the regulator in terms of a lower pollution stock, it also implies a loss in terms of a lower final output plus higher evasion. For a slight overvaluation, the positive effect overcomes the negative one, but greater and greater overvaluation gives higher and higher relevance to the negative effect (relative to the positive effect) until eventually a saturation point is reached, where the reduction of emissions and the rise of evasion become welfare reducing. The drop in emissions and the rise of evasion becomes more pronounced the higher the pollution stock. As a result, the saturation point is reached sooner for larger rather than lower pollution stock values. Therefore, the lightest L-win area narrows with the pollution stock.

The comparison of the value functions of the firm when the regulator is misguided or accurate is presented in Figure 6 (right). If the regulator undervalues/overvalues the firm's fear of the fine for fraud, the firm is better/worse off. The lack of information for the leader is profitable for the firm if the regulator's belief is small and more so when the pollution stock is high. In that case, the tax is very lax, allowing for the firm to produce significantly more emissions. The region where the firm is better off when the leader is uninformed is denoted F-win. Note that L-win and F-win are disjoint regions, and no win-win situation ever happens.

5.1 Static versus dynamic

This section highlights that the main result does not hold in a static formulation of the game. In Appendix 6 we present and characterize the Stackelberg equilibrium for the equivalent game in a static formulation. The regulator gains with firms revenues and losses with evasion, just like in the dynamic setting. Moreover, he suffers damages from emissions, instead of from the accumulated pollution stock. We prove that incomplete information of the regulator always leads to social welfare losses. For example, if the regulator is overconfident ($\alpha > 1$), he would settle higher taxes, inducing an emissions reduction, but also higher evasion. The positive effect of a lower damage from emissions is overcome by the negative effects of lower production and higher evasion.

In the dynamic setting, the players' equilibrium strategies are also dependent on their marginal valuation of the environment. In consequence, under incomplete information, the assumption of an overconfident regulator ($\alpha > 1$), has associated a greater propensity to tax, just like in the static setting, plus an additional dynamic effect. Because the regulator believes that the firm is very afraid of the fine, this latter foresees a strong PTT. And a higher PTT implies a higher tax, especially for higher pollution stock. In consequence, the firm values pollution more negatively. As shown in (5) and (6), a stronger marginal valuation of the environment induces a reduction in real and reported emissions in the same amount. This dynamic self-regulation effect has no direct influence on evasion. Moreover, due to this self-regulation, the regulator does not need to raise tax that much, which hence implies a lower rise in evasion. And because evasion rises less, it is now possible that the positive effect of a lower damage from pollution surpasses the negative effects of lower production and (not so) higher evasion.

5.2 Sensitivity analysis and robustness

This section analyzes the sensitivity of the main result to changes in the main parameters: the damage that evasion represents for society, ϕ, the intensity of fine for fraud, β, and the environmental damage, d.

Result 1 *The L-win region narrows with the damage that evasion represents for society, ϕ.*

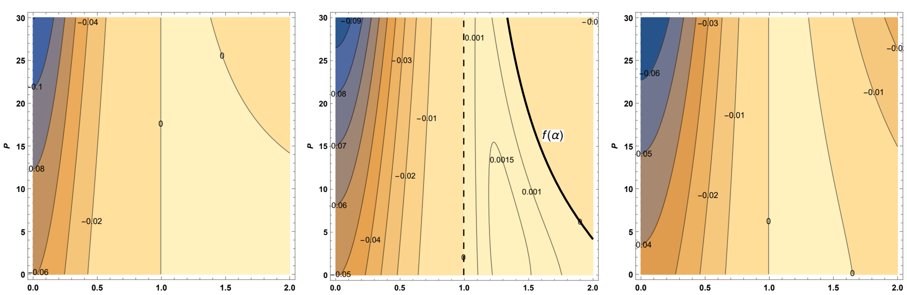

Figure 7: $V_\alpha^L(P) - V_B^L(P)$ for $\phi = 0.05$ (left), for $\phi = 0.1$ (center) and for $\phi = 0.2$ (right).

As Figure 7 (center chart) shows, for the initial value $\phi = 0.1$, the L-win region fills the whole range $\alpha \in [1, 2]$ for low pollution stock, and this range decreases as the stock of pollution increases. For a lower cost of evasion $\phi = 0.05$ (left chart), the L-win region widens, allowing the regulator larger overvaluations of the true effect that the fine for fraud has on the firm, especially when the environmental problem is less serious (lower pollution stock). Conversely, for $\phi = 0.2$, the area of the L-win region narrows, and the reduction is more acute the lower the pollution stock.

The sensitivity to evasion has a twofold effect on the equilibrium policy imposed by the regulator. A higher sensitivity to evasion implies a smaller PTT, h(α). In addition, ϕ

also indirectly affects the environmental valuation gap between the regulator and the firm, $(V_\alpha^F)'(P_0) - (V_\alpha^L)'(P_0)$.

The slope of the PPT is affected by the harm the evasion imposes on the society:

$$\frac{dh'(\alpha)}{d\phi} = \frac{2\alpha\beta^2(\alpha^2\beta^2 - \phi)}{(\alpha^2\beta^2 + \phi)^3} \gtrless 0 \iff \alpha \gtrless \frac{\sqrt{\phi}}{\beta}.$$

For the parameter values in (23), $\sqrt{\phi}/\beta = 1.05$. Therefore, a higher ϕ implies a steeper PTT to the right of $\alpha > 1.05$. Moreover, its effect on how the gap $(V_\alpha^F)'(P_0) - (V_\alpha^L)'(P_0)$ narrows when α rises is negligible. Thus the effect of ϕ on how the equilibrium tax (typically) rises with α in the general case is mainly determined by its effect on $h'(\alpha)$. The more averse to evasion the society, the lower the tax, but also and more importantly the more responsive this tax to α (when α is large). As a result, if the regulator overvalues how the fine for fraud affects the firm ($\alpha > 1$), he will raise the tax more strongly. Therefore, the saturation point would be reached for a lower α, hence narrowing the L-win region.

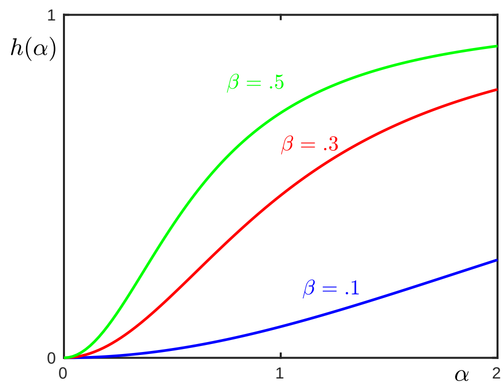

Figure 8: $h(\alpha)$ for $\beta = 0.1$, $\beta = 0.3$ and $\beta = 0.5$.

Result 2 *The L-win region widens with the intensity of fine for fraud, β.*

Figure 9 shows the welfare comparison for a misguided versus an accurate regulator for different values of the intensity of the fine. As one moves from lower to higher intensity of the fine (from left to right in Figure 9), the range where a misguided regulator is better off than an accurate regulator is enlarged, especially for low pollution stock.

Likewise for ϕ, the intensity of the fine for fraud, β, also generates two effects on τ, through the PTT and the gap $(V_\alpha^F)'(P_0) - (V_\alpha^L)'(P_0)$. On top of these two effects, β directly reduces evasion: τ/β.

The slope of the PPT is affected by the intensity of the fine for fraud in the exact opposite sign as the harm that evasion imposes on the society:

$$\frac{dh'(\alpha)}{d\beta} = \frac{4\alpha\beta\phi(\phi - \alpha^2\beta^2)}{(\alpha^2\beta^2 + \phi)^3} \gtrless 0 \iff \alpha \lessgtr \frac{\sqrt{\phi}}{\beta}.$$

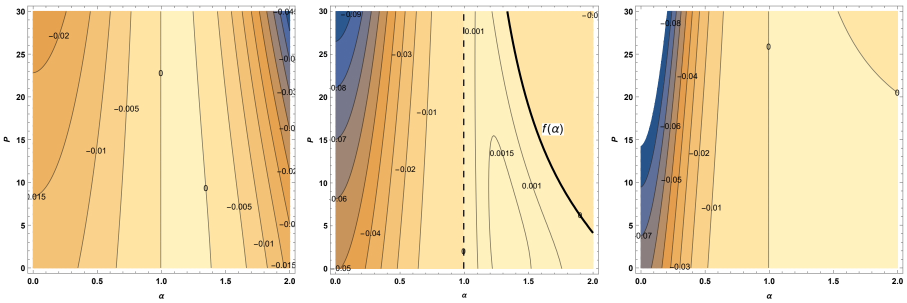

Figure 9: $V_\alpha^L(P) - V_B^L(P)$ for $\beta = 0.1$ (left), for $\beta = 0.3$ (center) and for $\beta = 0.5$ (right).

As Figure 8 shows, the PTT increases with the intensity of the fine for fraud, but also becomes smoother, provided that α is sufficiently high. Again, a change in β has a negligible effect on how the gap in the marginal valuations varies with α. Therefore, a more punishing fine implies a less intense increment in the tax. Moreover, a more intense fine also reduces evasion directly. As a result, if the regulator overvalues how the fine for fraud affects the firm, then he will raise the tax more gradually and, hence, evasion will also raise more gradually . The saturation point will be reached at a higher α, thus widening the L-win region.

Result 3 *The L-win region narrows with the environmental damage, d.*

As the environmental damage increases (moving from left to right in Figure 10), the L-win region narrows, especially when environmental pollution is less severe.

It is important to notice that the environmental damage parameter, d, does not modify the PTT. Thus, d only affects the equilibrium policy imposed by the regulator, through the gap between the regulator's and the firm's marginal valuation of the environment. Greater environmental damage implies a wider gap between the players' valuation and, more importantly, this gap becomes less responsive to changes in α, assuming that α is sufficiently large (certainly for $\alpha > 1$). Consequently, the variation of the tax with a rise in α is more strongly enhanced with the increasing PTT and less strongly offset by the decreasing gap between the marginal valuations of the players. Therefore, the tax grows faster with α and the L-win region becomes narrower, as shown in Figure 10.

The sensitivity analysis concerning all three parameters shows that a win-win case never occurs.

Remark 4 *The previous analysis is based on the interpretation of a regulator acting as a benevolent central planner who maximizes social welfare internalizing the pollution externality and the externality from fraud. Alternatively, one could assume that evasion does*

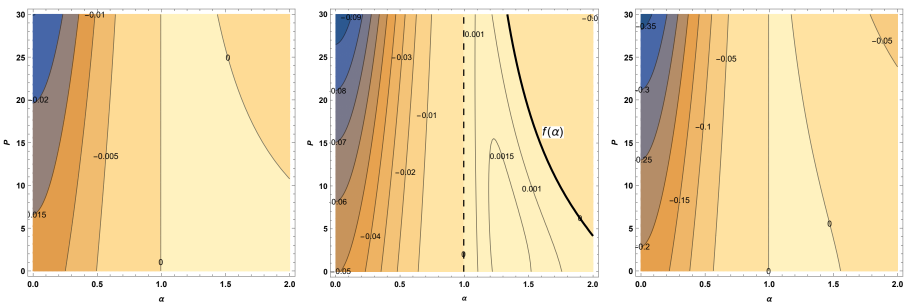

Figure 10: $V_{\alpha}^{L}(P) - V_{B}^{L}(P)$ for $d = 0.0005$ (left), for $d = 0.001$ (center) and for $d = 0.002$ (right).

not represent an externality on society, although it represents a (for example, reputational) cost for the regulator. Thus, social welfare is affected by pollution but unaffected by evasion. Under this interpretation, we have carried out the same analysis as in Section 5 and we have obtained that the overvaluation of the firm's fear of the fine unequivocally leads to higher social welfare.

6 Concluding remarks

The dynamic interaction between a regulator and a representative firm is analyzed as a Stackelberg differential game where pollution accumulates over time. The main aim of this paper is to study the effect of the regulator's incomplete information on environmental policy, the real and reported emissions, evasion, the pollution stock, and social welfare at the equilibrium. Under incomplete information, the regulator does not have full information regarding the firm's objective function and has to conjecture it. First we discuss the benchmark case where, by chance the regulator accurately conjectures the firm's objective function. In this benchmark case, it is easy to observe that the social optimum would be achieved provided that the evasion linked to the environmental policy had no associated cost for the regulator.

In the general case, the regulator fails to accurately conjecture the firm's objective function. In particular, the precise firm's fear of the fine for fraud. Given this conjecture, he computes what he believes to be the firm's best-response functions regarding real and reported emissions. To the extent that he fails in his conjecture, the equilibrium strategies of the players, the time path for the pollution stock, and social welfare differ from their expressions in the benchmark case.

The equilibrium emission tax in the general case can be above or below its value in the benchmark case. The interesting result occurs when the regulator overvalues the firm's fear of being fined for fraud. In that case, the propensity to tax by the regulator surpasses its benchmark value. As a reaction, the firm values the environment strongly, narrowing the gap between the leader's and the follower's valuations. Adding up both effects, the tax typically rises (except for very low values of the pollution stock). A higher tax leads emissions closer to their social optimum value, although also implying greater evasion. When overvaluation is small the positive effect of lower emissions surpasses the negative effect of higher evasion, attaining greater social welfare. Larger and larger overvaluation reduces the positive effect concerning the negative effect, until the latter eventually overcompensates the former.

It is important to notice that this result cannot be replicated in a static setting. The reason is that overvaluation by the regulator raises his propensity to tax. In the dynamic setting this makes the firm more afraid of pollution growth and hence of future large taxes. In consequence, the firm raises its valuation of the environment and self-regulates reducing current emissions. This allows for a more relaxed policy, which opens up the possibility of social welfare improvements. This self-regulation mechanism is absent in a static setting.

Greater social welfare with a misguided rather than accurate conjecture is feasible when the regulator moderately overvalues the firm's fear of the fine for fraud. The robustness of this result was tested by running a sensitivity analysis with respect to three parameters. The area of the region where the misguided regulator is better off than an accurate regulator widens with the intensity of the fine, and it narrows with the society's aversion to evasion and the size of the environmental damage.

A first natural extension would be a regulator who mismeasures the relationship between firm's emissions and output. In this setting, the best-response functions estimated by the regulator can differ from the true ones and moreover, the regulator can introduce a wrong estimation of the firm's production function in his objective function. Thus, in this extension the lack of information directly enters his objective function, contrary to the case analyzed in this paper. A second extension, would be the study of the impact on the main result of introducing competition between several symmetric firms playing *à la* Nash.

Finally, as next steps and new developments, it would be interesting to introduce a variable measuring the firm's green reputation. This would be the "badwill" of the firm, defined as a stock variable, that increases with the company's repeated deceptions when discovered by the regulator. This idea can be linked with the literature on green consumption. In this setting, the firm could be allowed to carry out active actions in order to reduce its "badwill".

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Appendix

A.1 Social planner's problem (Section 4.1)

The solution to the social planner's problem in (14) is obtained from the following Bellman equation:

$$\rho V_{SO}(P) = \max_e \left\{ e\left(A - \frac{e}{2}\right) - \frac{d}{2}P^2 + V'_{SO}(P)(e - \delta P) \right\}, \tag{27}$$

where $V_{SO}(P)$ represents the social planner's value function.
From the first-order condition, the optimal emissions for this problem read:

$$e_{SO}(P) = A + V'_{SO}(P) \tag{28}$$

We conjecture a quadratic value function, $V_{SO}(P) = a_{2SO}P^2/2 + a_{1SO}P + a_{0SO}$. Plugging the optimal emissions into equation (27) and identifying coefficients in the left and the right hand sides one gets a system of 3 equations:

$$\rho \frac{a_{2SO}}{2} = \frac{a_{2SO}^2 - 2\delta a_{2SO} - d}{2},$$

$$\rho\, a_{1SO} = (A + a_{1SO})a_{2SO} - \delta a_{1SO},$$
$$\rho\, a_{0SO} = \frac{(A + a_{1SO})^2}{2}.$$

Two solutions for a_{2SO} are obtained from the first equation. From (3) and the optimal emissions in (28), the asymptotic stability of the steady-state of the pollution stock requires $a_{2SO} - \delta < 0$. The only solution satisfying this condition is:

$$a_{2SO} = \frac{\rho + 2\delta - \sqrt{(\rho + 2\delta)^2 + 4d}}{2}.$$

Plugging this solution into the second equation one get

$$a_{1SO} = \frac{A(\rho + 2\delta - \sqrt{(\rho + 2\delta)^2 + 4d})}{\rho + \sqrt{(\rho + 2\delta)^2 + 4d}}$$

Therefore, replacing a_{2SO} and a_{1SO} by their values in (28) the final expression of the optimal emission in (15) follows.

A.2 Benchmark case. Numerical illustration (Section 4.2.2)

Identifying coefficients in the LHS and the RHS of the system of equations (21) and (22) one gets a system of 6 Riccati equations. The two equations involving the quadratic coefficients of the value functions are:

$$\rho\,\frac{a_B^{2L}}{2} = \frac{\beta^2\left((a_B^{2L})^2 - 2a_B^{2L}\delta - d\right) - \phi\left((a_B^{2F})^2 - 2a_B^{2F}a_B^{2L} + 2a_B^{2L}\delta + d\right)}{2\left(\beta^2+\phi\right)},$$

$$\rho\,\frac{a_B^{2F}}{2} = \frac{(a_B^{2F})^2\left(\beta^3+\phi^2\right) - 2a_B^{2F}\left(a_B^{2L}\beta^2(\beta-\phi) + \delta\left(\beta^2+\phi\right)^2\right) + (a_B^{2L})^2(\beta+1)\beta^3}{2\left(\beta^2+\phi\right)^2}.$$

This system of two second-order polynomial equations can be analytically solved with the use of Mathematica (version 12.3.1), which returns the four pairs of solutions for a_B^{2F} and a_B^{2L}. We do not write here these expressions because they are highly cumbersome and give no insight.

From the first equation the coefficient a_B^{2L} can be written as two different functions of a_B^{2F}:

$$a_B^{2L}(a_B^{2F}) = \frac{\left(\beta^2+\phi\right)\left(2\delta+\rho+\sqrt{\frac{\phi(2\delta+\rho-2a_B^{2F})^2+\beta^2((2\delta+\rho)^2+4d)}{\beta^2+\phi}}\right) - 2a_B^{2F}\phi}{2\beta^2}, \quad (29)$$

$$a_B^{2L}(a_B^{2F}) = \frac{\left(\beta^2+\phi\right)\left(2\delta+\rho-\sqrt{\frac{\phi(2\delta+\rho-2a_B^{2F})^2+\beta^2((2\delta+\rho)^2+4d)}{\beta^2+\phi}}\right) - 2a_B^{2F}\phi}{2\beta^2}. \quad (30)$$

From (3) and the optimal emissions in (17), the asymptotic stability of the steady-state of the pollution stock requires:

$$\frac{a_B^{2F}\phi + a_B^{2L}\beta^2}{\beta^2+\phi} - \delta < 0.$$

It can be easily shown that replacing a_B^{2L} by its expression in (29) the stability condition does not hold. From this result we know that, at most two stable solutions exist. Once two of the four pairs of solutions are removed, we solve for the following equations involving the linear coefficients of the value functions:

$$\rho\,a_B^{1L} = \frac{Aa_B^{2L}\left(\beta^2+\phi\right) + a_B^{1F}\phi(a_B^{2L}-a_B^{2F}) + a_B^{1L}\phi(a_B^{2F}-\delta) + a_B^{1L}\beta^2(a_B^{2L}-\delta)}{\beta^2+\phi},$$

$$\begin{aligned}\rho\,a_B^{1F} = \frac{1}{\left(\beta^2+\phi\right)^2}&\left\{\beta^3(a_B^{2L}\beta(A+a_B^{1L}) + a_B^{1F}(a_B^{2F}-a_B^{2L}-\beta\delta) + a_B^{1L}(a_B^{2L}-a_B^{2F}))+\right.\\ &\left.\beta^2\phi(A(a_B^{2F}+a_B^{2L}) + a_B^{1F}(a_B^{2L}-2\delta) + a_B^{1L}a_B^{2F}) + \phi^2(a_B^{2F}(A+a_B^{1F}) - a_B^{1F}\delta)\right\},\end{aligned}$$

and the equations for the independent term coefficients:

$$\rho\,a_B^{0L} = \frac{\phi(A+a_B^{1F})(A-a_B^{1F}+2a_B^{1L}) + \beta^2(A+a_B^{1L})^2}{2\left(\beta^2+\phi\right)},$$

$$\rho\,a_B^{0F} = \frac{\beta^3\left(\beta(A+a_B^{1L})^2 + (a_B^{1F}-a_B^{1L})^2\right) + 2\beta^2\phi(A+a_B^{1F})(A+a_B^{1L}) + \phi^2(A+a_B^{1F})^2}{2\left(\beta^2+\phi\right)^2}.$$

In order to determine which equilibrium is indeed implemented, we compare the value function for the regulator for the two stable solutions, and choose the one which gives the highest value. We focus on the regulator as he is the leader in the Stackelberg game. This comparison cannot be carried out analytically and hence we rely on numerical analysis.

A.3 Static framework

This appendix shows that the static formulation of the model does not replicate the main result of the paper obtained in a dynamic setting, which states that incomplete information of the regulator can be welfare improving.

In this static setting the environmental damage is not given by the stock but by the flow of emissions. Hence there is no stock dynamics.

The true and the conjectured firm's objective functions match those in the dynamic case (only now decision variables are not time dependent). In consequence, the true and the conjectured best-response functions in (5), (6), (9) and (10), give their static counterparts, just canceling the term with $V_F'(P)$. Therefore, these functions do not depend on P.

The regulator's objective function now reads:

$$F_L(e, R, \tau) = e\left(A - \frac{e}{2}\right) - \frac{d}{2}e^2 - \frac{\phi}{2}(e - R)^2.$$

And the regulator's maximization problem is:

$$\max_{\tau} \left\{ e^{ebr}(\tau)\left(A - \frac{e^{ebr}(\tau)}{2}\right) - \frac{d}{2}(e^{ebr}(\tau))^2 - \frac{\phi}{2}(e^{ebr}(\tau) - R^{ebr}(\tau))^2 \right\}.$$

From the first order condition, the optimal tax reads:[23]

$$\tau_\alpha^s = \frac{\alpha^2 A \beta^2 d}{\alpha^2\beta^2(d+1) + \phi}.$$

Plugging this optimal tax on the true best-response functions, the optimal real and reported emissions follow:

$$e_\alpha^s = A\frac{\alpha^2\beta^2 + \phi}{\alpha^2\beta^2(d+1) + \phi}, \qquad R_\alpha^s = A\frac{\alpha^2\beta(\beta - d) + \phi}{\alpha^2\beta^2(d+1) + \phi}.$$

In the general case with incomplete information, the regulator's and hence the social welfare can be computed as $F_L(e_\alpha^s, R_\alpha^s, \tau_\alpha^s)$. Similarly, the social welfare with complete information, can be computed by replacing the strategies in the regulator's objective by their optimal

[23]Superscript s stands for static framework.

values when $\alpha = 1$: $F_L(e_1^s, R_1^s, \tau_1^s)$. The difference in social welfare with and without complete information then reads:

$$F_L(e_1^s, R_1^s, \tau_1^s) - F_L(e_\alpha^s, R_\alpha^s, \tau_\alpha^s) = \frac{\left(\alpha^2 - 1\right)^2 A^2 \beta^2 d^2 \phi^2}{2\left(\beta^2(d+1) + \phi\right)\left(\alpha^2\beta^2(d+1) + \phi\right)^2}.$$

This expression is strictly positive for any $\alpha \neq 1$. Therefore, incomplete information is never welfare improving in this static setting.