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RESEARCH ARTICLE

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Observer-based controller for positive polynomial systems with time delay

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Summary

This paper deals with some synthesis problems for a class of positive polynomial systems with time delay, for which the state vector takes nonnegative values whenever the initial conditions are nonnegative. First, the synthesis of state feedback controllers is solved using a Lyapunov-Krasovskii approach, including the requirement of positiveness. In a second part, observers are included. In both cases, the design conditions are presented in terms of sum of squares, which can be numerically and symbolically solved via SOSTOOLS and a semidefinite program solver. Finally, four numerical examples are given to illustrate the proposed approach.

KEYWORDS

polynomial observer, positive polynomial systems, SOSTOOLS, sum of squares

1 | INTRODUCTION

In real world, many dynamical systems involve variables that are always confined to the positive orthant. These systems, whose all state trajectories remain forever nonnegative whenever the initial conditions are nonnegative, are called positive systems. They have been considered appropriate to describe many biological, ecological, chemical, social science, and economic processes and communication networks (see other works[1-7] and the references therein).

Seeking new approaches to study the problems of stability for positive systems is being paid much attention. In the context of positive systems, there are many fundamental results concerned with the reachability/controllability, the realizability the eigenvalue regions of positive systems, etc.

On the other hand, time delay often occurs in many practical systems such as chemical processes, telecommunication, mechanical systems, etc. As a consequence, LMI approaches have been proposed to tackle analysis and design of linear positive systems with time delay.[8,9] In the work of Cui et al,[10] the exponential stability for positive singular systems with distributed delays has been developed. H_∞ control via dynamic output feedback for positive systems with multiple delays has been investigated in the work of Zhang et al.[11] Significant efforts have been made to develop stability analysis methods and various techniques have been obtained. The standard methods which use common Lyapunov-Krasovskii (L-K) functional, often lead to conservative results (see other works[7,12-17] and the references therein).

Recent developments on polynomial theory, especially the sum of squares (SOS) theory,[18] have provided a feasible solution to ensure the global nonnegativity for polynomial functions. Sufficient conditions for positivity of polynomial systems are formulated as a sum of squared polynomials. The existence of SOS decomposition has been shown to be equivalent to a semidefinite programming feasible problem. Many toolboxes such as SOSTOOLS[19] have been developed to solve this problem. With the development of SOS techniques, a new avenue is open to analyze and synthesize controllers for nonlinear systems with polynomial dynamics. The problems of stability analysis and H_∞ control for 2-D polynomial Roesser model are considered in the work of Li et al.[20] The SOS approach was proposed in the work of Zhu et al[21] to deal with the problem of H_∞ optimal control of polynomial nonlinear systems. In the work of Iben-Ammar et al,[22] an L-K functional approach for observer-based control for a class of polynomial systems with time delay, where the Lyapunov matrices are depending in the measurable states, is investigated. However, research rarely focuses on positive nonlinear systems; by contrast, they concentrate on general nonlinear systems. Up to now, we can only find two references[23,24] studying the problems of stability and positivity of polynomial fuzzy model–based control systems with time delay, using a linear programming (LP) approach. Polynomial L-K functions have proved useful for designing stability criteria for complex polynomial systems. It has been noticed in the literature that some of these criteria are less conservative than others, and entire hierarchies have been proposed, with better performance of the criterion when increasing the order of the Lyapunov matrix. We point out that with an LP approach,[23,24] we cannot increase the order of the Lyapunov function as λ should be constant. This restriction is lifted in the current work by using polynomial Lyapunov matrices in the L-K functional selected.

Thus, this work deals with positive polynomial systems with time-varying delay. Our main objective is to develop a new method and techniques for the analysis and the synthesis of the positive polynomial systems with time delay. In a first part, the stabilization problem is considered, enforcing the stability and the positivity of the closed loop system under state feedback controls. A polynomial model is employed to represent the nonlinear system with time delay. A polynomial controller is used to stabilize the nonlinear system and make the closed-loop system positive. Moreover, SOS-based positive and stability conditions are obtained to determine the system positivity and stability. To the best knowledge of the authors, most works have focused on the positive T-S fuzzy model–based control systems, whereas very limited work has been done on the positive polynomial model–based control systems with time delay. In a second part, the observer based polynomial controller design problem is considered to ensure the stability of the resulting closed loop system, the convergence of estimation errors, and the positivity of both the states and their estimated values.

The remainder of the paper is organized as follows. In Section 2, preliminaries of polynomial systems and controllers are presented. The analysis of the positive polynomial systems with time delay is investigated in Section 3. Section 4 treats and solves the positive polynomial observation problem. In Section 5, numerical examples are given to demonstrate the effectiveness of the proposed methodology. Finally, Section 6 provides the conclusions.

Notation. $\mathbb{R}^n$ denotes the set of n-dimensional real vectors. $S > 0$ (respectively, $S \geq 0$) means that S is symmetric positive definite (respectively, symmetric positive semidefinite) matrix. $()^T$ represents the transposed matrix; A matrix is called a Metzler matrix if its off-diagonal elements are always nonnegative.

2 | PRELIMINARIES

The following preliminaries will be used in the derivation of the main results. Furthermore, we provide a brief review of the SOS decomposition that will be useful in the following sections.

Definition 1. A polynomial $Y(x)$ is SOS if there exists a set of polynomials $q_i(x), i = 1, \dots, s$, such that

$$Y(x) = \sum_{i=1}^{s} q_i^2(x). \tag{1}$$

Moreover, if $Y(x)$ is SOS, then $Y(x) > 0, \forall x$.

Proposition 1 (See the work of Tanaka et al[25]). *Let $D(x(t))$ be an $n \times n$ symmetric polynomial matrix of degree $2d$ in $x(t)$. Furthermore, let $\check{x}(x(t))$ be a column vector whose entries are all monomials in $x(t)$ with degree no greater than d and consider the following:*

(1) $\eta^T(t)D(x(t))\eta(t)$ *is SOS, where* $\eta(t) \in \mathbb{R}^n$.
(2) *There exists a positive semidefinite matrix Q such that* $\eta^T(t)D(x(t))\eta(t) = (\eta(t) \otimes \check{x}(x(t)))^T Q(\eta(t) \otimes \check{x}(x(t)))$, *where* $\otimes$ *denotes the Kronecker product.*
(3) $D(x(t)) \geq 0$ *for all* $x(t) \in \mathbb{R}^n$.

Then, (1) $\Longrightarrow$ *(3) and (1)*$\Longleftrightarrow$*(2). The property (1)* $\Longrightarrow$ *(3) will be used in the proof of Theorems 1 and 2 and Corollary 1.*

2.1 | Polynomial systems

A polynomial model for a nonlinear plant is defined as follows:

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + A_\tau(x(t))x(t-\tau(t)) + B(x(t))u(t) \\ x(t) = \psi(t), t \in [-\overline{\tau}, 0], \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}_+^n$, $x(t-\tau(t)) \in \mathbb{R}_+^n$, and $u(t) \in \mathbb{R}^p$ are, respectively, the state, the delay state, and the control input vectors. $A(x(t)) \in \mathbb{R}^{n\times n}$ is a Metzler matrix, $A_\tau(x(t)) \in \mathbb{R}^{n\times n}$ is a nonnegative polynomial matrix, and $B(x(t)) \in \mathbb{R}^{n\times p}$ is a polynomial matrix. $\tau(t) \in \mathbb{R}_+$ is the time-varying delay that a continuous function bounded as follows:

$$\begin{cases} 0 \leq \tau(t) \leq \overline{\tau} \\ \dot{\tau}(t) \leq d \leq 1. \end{cases} \tag{3}$$

Definition 2 (See the work of Liu et al[2]). A system (2) is positive if the initial condition $\psi(.) \geq 0$ holds and the system state vector $x(t) \geq 0$ for all $t \geq 0$.

Lemma 1 (See the work of Li et al[23]). *A system (2) is guaranteed to be positive, when* $u(t) = 0$, *if* $A(x(t))$ *is a Metzler matrix and* $A_\tau(x(t))$ *is positive matrix.*

3 | STATE FEEDBACK POLYNOMIAL CONTROLLER DESIGN

A polynomial controller is proposed for the nonlinear system represented by the polynomial model (2) in such way that the resulting governed polynomial system is positive and asymptotically stable. In other words, we look for the following state-feedback polynomial control law

$$u(t) = K(x(t))x(t), \tag{4}$$

where polynomial matrix $K(x(t)) \in \mathbb{R}^{p\times n}$.

The polynomial model (2) and polynomial controller (4) constitute a positive polynomial model–based control system. From (2) and (4), we have the closed-loop system as follows:

$$\begin{cases} \dot{x}(t) = [A(x(t)) + B(x(t))K(x(t))]x(t) + A_\tau(x(t))x(t-\tau(t)) \\ x(t) = \psi(t), t \in \left[-\overline{\tau}, 0\right], \end{cases} \tag{5}$$

where polynomial matrix $K(x(t)) \in \mathbb{R}^{p\times n}$ has to be determined such that system (5) is asymptotically stable and satisfy

$$A(x(t)) + B(x(t))K(x(t)) \quad \text{is} \quad \text{Metzler} \tag{6}$$

$$A_\tau(x(t)) \geq 0. \tag{7}$$

In the sequel, for briefness we use x, x_α, x_τ, and $\dot{x}$ to denote $x(t)$, $x(\alpha)$, $x(t-\tau(t))$, and $\dot{x}(t)$, respectively.

In the following section, we study the stabilization and positiveness of the governed polynomial system (5).

Theorem 1. *Closed-loop system (5) is asymptotically stable and positive if there exist symmetric polynomial matrices* $X(\bar{x}) \in \mathbb{R}^{N\times N}$ *and* $S(x) \in \mathbb{R}^{N\times N}$ *and polynomial matrix* $M(x) \in \mathbb{R}^{p\times N}$ *such that the following SOS-based conditions are satisfied:*

$$v_1^T(X(\bar{x}) - \epsilon_1(\bar{x})I)v_1 \quad \text{is} \quad \text{SOS} \tag{8}$$

$$v_2^T(S(x) - \epsilon_2(x)I)v_2 \quad \text{is} \quad \text{SOS} \tag{9}$$

$$-v_3^T(\Psi(x) + \epsilon_3(x)I)v_3 \quad \text{is} \quad \text{SOS} \tag{10}$$

$$v_4^T\left([A(x)X(\bar{x}) + B(x)M(x)]_{ij} - \epsilon_4(x)I\right)v_4 \quad \text{is} \quad \text{SOS} \quad \text{for} \quad i \neq j \tag{11}$$

$$v_5^T\left(A_\tau(x)X(\bar{x}) - \epsilon_5(x)I\right)v_5 \quad \text{is} \quad \text{SOS}, \tag{12}$$

where v_i denotes vectors that are independent of x. $\epsilon_i(x)$ is a slack variable (a radially unbounded positive definite polynomial) to keep the positivity of the SOS conditions, and

$$\Psi(x) = \begin{pmatrix} \psi^{11}(x) & \psi^{12}(x) \\ * & \psi^{22}(x) \end{pmatrix}, \tag{13}$$

in which

$$\psi^{11}(x) = A(x)X(\bar{x}) + B(x)M(x) + X(\bar{x})A^T(x) + M^T(x)B^T(x) - \sum_{l\in L}\frac{\partial X(\bar{x})}{\partial x_l}\left(A^l(x)x + A_\tau^l(x)x_\tau\right) + S(x) \tag{14}$$

$$\psi^{12}(x) = A_\tau(x)X(\bar{x}) \tag{15}$$

$$\psi^{22}(x) = -(1-d)S(x). \tag{16}$$

Moreover, a stabilizing feedback gain $K(x)$ can be obtained from $X(\bar{x})$ and $M(x)$ as follows:

$$K(x) = M(x)X^{-1}(\bar{x}). \tag{17}$$

Proof. Here, SOS-based conditions (11) and (12) ensure that the matrix $A(x)X(\bar{x})+B(x)M(x)$ is Metzler and $A_\tau(x)X(\bar{x}) \geq 0$, respectively.

Considering the polynomial L-K functional as

$$V(x) = x^TX^{-1}(\bar{x})x + \int_{t-\tau(t)}^{t} x_\alpha^T\tilde{S}(x)x_\alpha \mathrm{d}\alpha, \tag{18}$$

with $X(\bar{x}) \in \mathbb{R}^{n\times n}$ and $\tilde{S}(x)$ being symmetric polynomial matrices in $\bar{x}$ and x, respectively.

Let $A^l(x)$ and $A_\tau^l(x)$ signify the lth row of $A(x)$ and $A_\tau(x)$, respectively, $L = [l_1, l_2, \dots, l_n]$ signify the row indices of $B(x)$ whose analogous row is equal to zero, and define $\bar{x} = [x_{l_1}, x_{l_2}, \dots, x_{l_n}]$.

Conditions (8)-(9) imply that $X(\bar{x})$ and $S(x)$ are positive definite matrices. Deriving L-K function (18) along the trajectory of system (2), we obtain

$$\dot{V}(x) = \dot{x}^TX^{-1}(\bar{x})x + x^TX^{-1}(\bar{x})\dot{x} + x^T\dot{X}^{-1}(\bar{x})x + x^T\tilde{S}(x)x - (1-\dot{\tau}(t))x_\tau^T\tilde{S}(x)x_\tau. \tag{19}$$

Furthermore, we have

$$\dot{X}^{-1}(\bar{x}) = \sum_{l=1}^{n}\frac{\partial X^{-1}(\bar{x})}{\partial x_l}\dot{x}_l. \tag{20}$$

x_l denotes the lth row of $\bar{x}$. Then, $\dot{X}^{-1}(\bar{x})$ can be rewritten as

$$\dot{X}^{-1}(\bar{x}) = \sum_{l=1}^{n}\frac{\partial X^{-1}(\bar{x})}{\partial x_l}(A^l(x)x + A_\tau^l(x)x_\tau). \tag{21}$$

Combining (3), (19), and (21), yields

$$\begin{aligned}\dot{V}(x) \leqslant{} & \left(x^T[A(x)+B(x)K(x)]^T + x_\tau^TA_\tau^T(x)\right)X^{-1}(\bar{x})x + x^TX^{-1}(\bar{x})\left([A(x)+B(x)K(x)]x\right. \\ & \left.+A_\tau x_\tau\right) + x^T\left(\sum_{l\in L}\frac{\partial X^{-1}(\bar{x})}{\partial x_l}\left(A^l(x)x + A_\tau^l(x)x_\tau\right)\right)x + x^T\tilde{S}(x)x - (1-d)x_\tau^T\tilde{S}(x)x_\tau.\end{aligned} \tag{22}$$

We obtain

$$\dot{V}(x) \leqslant \vartheta^T(\Pi(x))\vartheta, \tag{23}$$

where $\begin{cases} \vartheta^T = [x^T, x_\tau^T] \\ \bar{A}(x) = A(x) + B(x)K(x) \end{cases}$

$$\Pi(x) = \begin{pmatrix} \pi^{11}(x) & \pi^{12}(x) \\ * & \pi^{22}(x) \end{pmatrix} \tag{24}$$

$$\pi^{11}(x) = X^{-1}(\bar{x})\bar{A}(x) + \bar{A}^T(x)X^{-1}(\bar{x}) + \sum_{l\in L} \frac{\partial X(\bar{x})^{-1}}{\partial x_l}(A^l(x)x + A_\tau^l(x)x_\tau) + \tilde{S}(x) \tag{25}$$

$$\pi^{12}(x) = X^{-1}(\bar{x})A_\tau(x) \tag{26}$$

$$\pi^{22}(x) = -(1-d)\tilde{S}(x). \tag{27}$$

Based on (10), we have

$$\Psi(x) < 0. \tag{28}$$

Multiplying (28) from the both sides by diag$[X^{-1}(\bar{x}), X^{-1}(\bar{x})]$, we obtain

$$\bar{\Pi}^1(x) < 0, \tag{29}$$

where

$$\bar{\Pi}^1(x) = \begin{pmatrix} \sigma^1(x) & X^{-1}(\bar{x})A_\tau(x) \\ * & -(1-d)X^{-1}(\bar{x})S(x)X^{-1}(\bar{x}) \end{pmatrix} < 0 \tag{30}$$

$$\begin{aligned}\sigma^1(x) = {} & X^{-1}(\bar{x})A(x) + X^{-1}(\bar{x})B(x)K(x) + A^T(x)X^{-1}(\bar{x}) + K^T(x)B^T(x)X^{-1}(\bar{x}) + \sum_{l\in L} \frac{\partial X^{-1}(\bar{x})}{\partial x_l}(A^l(x)x + A_\tau^l(x)x_\tau) \\ & + X^{-1}(\bar{x})S(x)X^{-1}(\bar{x}).\end{aligned} \tag{31}$$

Accordingly, if (29) holds, then $\dot{V}(x) < 0$ from (23). □

4 | OBSERVER-BASED POLYNOMIAL CONTROLLER DESIGN PROBLEM

Before proceeding, knowing that the dual system of system (2) is asymptotically stable if and only if system (2) is asymptotically stable, then we simply use the stability of the dual system.

In this section, we consider the following positive polynomial system with time delay:

$$\begin{cases} \dot{x}(t) = A(x)x + A_\tau(x)x_\tau + B(x)u(t) \\ y(t) = Cx(t) \\ x(t) = \psi(t), t \in [-\overline{\tau}, 0], \end{cases} \tag{32}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control vector, and $y(t) \in \mathbb{R}^{n_y}$ is the output vector.
$A(x)$, $A_\tau(x)$, and $B(x)$ are polynomial matrices, and C is constant matrix, where $A(x) \in \mathbb{R}^{n_x \times n_x}$ is Metzler, $A_\tau(x) \geq 0$, $B(x) \geq 0$, and $C \geq 0$.
$\tau(t) \in \mathbb{R}$ is the time-varying delay, a differentiable continuous function, satisfying the conditions given in (3).
Let us consider the following polynomial observer–based controller:

$$\begin{cases} \dot{\hat{x}} = A(\hat{x})\hat{x} + A_\tau(\hat{x})\hat{x}_\tau + B(\hat{x})u + L(\hat{x})(y - \hat{y}) \\ \hat{y} = C\hat{x} \\ u = K(\hat{x})\hat{x}, \end{cases} \tag{33}$$

where $\widehat{x} \in \mathbb{R}^{n_x}$ is the estimated state, $\widehat{y} \in \mathbb{R}^{n_y}$ is the estimated output vector, and $L(\widehat{x})$ and $K(\widehat{x})$ are the polynomial observer gains and polynomial controller gains to be designed, respectively.
The augmented positive polynomial model based observer control system is written as

$$\begin{cases} \dot{\widetilde{x}} = G(x, \widehat{x})\widetilde{x} + M(x, \widehat{x})\widetilde{x}_\tau \\ \widetilde{x} = \widetilde{\psi}, t \in [-\bar{\tau}, 0], \end{cases} \tag{34}$$

where

$$\widetilde{x} = \begin{bmatrix} \widehat{x} \\ x - \widehat{x} \end{bmatrix}, \widetilde{x}_\tau = \begin{bmatrix} \widehat{x}_\tau \\ x_\tau - \widehat{x}_\tau \end{bmatrix}, \widetilde{\psi} = \begin{bmatrix} \psi \\ \widehat{\psi} \end{bmatrix} \tag{35}$$

$$G(x,\widehat{x}) = \begin{bmatrix} G^{11}(\widehat{x}) & G^{12}(\widehat{x}) \\ G^{21}(x,\widehat{x}) & G^{22}(x,\widehat{x} \end{bmatrix}$$

$$\begin{aligned} G^{11}(\widehat{x}) &= A(\widehat{x}) + B(\widehat{x})K(\widehat{x}) \\ G^{12}(\widehat{x}) &= L(\widehat{x})C \\ G^{21}(x,\widehat{x}) &= (A(x) - A(\widehat{x})) - (B(x) - B(\widehat{x}))K(\widehat{x}) \\ G^{22}(x,\widehat{x}) &= A(x) - L(\widehat{x})C \end{aligned}$$

$$M(x,\widehat{x}) = \begin{bmatrix} A_\tau(\widehat{x}) & 0 \\ A_\tau(x) - A_\tau(\widehat{x}) & A_\tau(x) \end{bmatrix}.$$

Lemma 2. *There exist two orthogonal matrices $U \in \mathbb{R}^{n_y \times n_y}$ and $Z \in \mathbb{R}^{n_x \times n_x}$ such that*

$$U^T C Z = \begin{bmatrix} \widetilde{C} & 0 \end{bmatrix}, \tag{36}$$

where $\widetilde{C} = \text{diag}\{c_1, c_2, \dots, c_{n_y}\}$ and $c_i (i = 1, \dots, n_y)$ are nonzero singular values of C.

Proof. Since $rank(C) = n_y$. □

Theorem 2. *Consider polynomial model (32) with observer-based polynomial control (33). If there exist symmetric matrices $\widetilde{P}_1$, $\widetilde{P}_2^{11} \in \mathbb{R}^{n_y \times n_y}$, $\widetilde{P}_2^{22} \in \mathbb{R}^{(n_x - n_y)\times(n_x - n_y)}$, S_1 and S_2, polynomial matrix $W^K(\widehat{x})$, and positive polynomial matrix $W^L(\widehat{x})$ such that the following SOS-based conditions are satisfied:*

$$v_1^T(\widetilde{P}_1 - \epsilon_1 I)v_1 \text{ is SOS} \tag{37}$$

$$v_2^T(P_2 - \epsilon_2 I)v_2 \text{ is SOS} \tag{38}$$

$$v_3^T(S_1 - \epsilon_3 I)v_3 \text{ is SOS} \tag{39}$$

$$v_3^T(S_2 - \epsilon_3 I)v_3 \text{ is SOS} \tag{40}$$

$$\begin{gathered} v_4^T\left(\left[A(\widehat{x})\widetilde{P}_1 + B(\widehat{x})W^K(\widehat{x})\right]_{ij} - \epsilon_4(\widehat{x})I\right)v_4 \text{ is SOS for } i \neq j \\ v_5^T\left(\left[A(x)P_2 - W^L(\widehat{x})C\right]_{ij} - \epsilon_5(x,\widehat{x})I\right)v_5 \text{ is SOS for } i \neq j \\ v_6^T\left(\left(A(x) - A(\widehat{x})\right)\widetilde{P}_1 + \left(B(x) - B(\widehat{x})\right)W^K(\widehat{x}) - \epsilon_6(x,\widehat{x})I\right)v_6 \text{ is SOS} \\ v_7^T\left(\left(A_\tau(x) - A_\tau(\widehat{x})\right)\widetilde{P}_1 - \epsilon_7(x,\widehat{x})I\right)v_7 \text{ is SOS} \end{gathered} \tag{41}$$

$$-\widetilde{\eta}^T\left(\Upsilon(x,\widehat{x}) + \epsilon_8(x,\widehat{x}I)\widetilde{\eta} \text{ is SOS,} \tag{42}$$

where

$$P_2 = Z\widetilde{P}_2Z^T \quad \text{and} \quad \widetilde{P}_2 = \begin{bmatrix} \widetilde{P}_2^{11} & 0 \\ 0 & \widetilde{P}_2^{22} \end{bmatrix}, \tag{43}$$

$U \in \mathbb{R}^{n_y \times n_y}$, $Z \in \mathbb{R}^{n_x \times n_x}$, and $\widetilde{C}$ are given by applying Lemma 2

$$\Upsilon(x,\widehat{x}) = \begin{bmatrix} \Upsilon^1(x,\widehat{x}) & \Upsilon^2(x,\widehat{x}) \\ * & \Upsilon^3 \end{bmatrix}, \tag{44}$$

where

$$\Upsilon^1(x,\hat{x}) = \begin{bmatrix} \Upsilon^{11}(\hat{x}) & \Upsilon^{12}(x,\hat{x}) \\ * & \Upsilon^{22}(x,\hat{x}) \end{bmatrix}$$

$$\Upsilon^2(x,\hat{x}) = \begin{bmatrix} \Upsilon^{13}(\hat{x}) & \Upsilon^{14}(x,\hat{x}) \\ * & \Upsilon^{24}(x,\hat{x}) \end{bmatrix}$$

$$\Upsilon^3 = \begin{bmatrix} -(1-d)S_1 & 0 \\ * & -(1-d)S_2 \end{bmatrix}$$

$$\begin{aligned}
\Upsilon^{11}(\hat{x}) &= A(\hat{x})\widetilde{P}_1 + B(\hat{x})W^K(\hat{x}) + \widetilde{P}_1 A(\hat{x})^T + \left(W^K(\hat{x})\right)^T B(\hat{x})^T + S_1 \\
\Upsilon^{12}(x,\hat{x}) &= W^L(\hat{x})C + \widetilde{P}_1(\hat{x})(A(x) - A(\hat{x}))^T + (W^K(\hat{x}))^T (B(x) - B(\hat{x}))^T \\
\Upsilon^{13}(\hat{x}) &= \widetilde{P}_1 A_\tau(\hat{x})^T \\
\Upsilon^{14}(x,\hat{x}) &= \widetilde{P}_1 (A_\tau(x) - A_\tau(\hat{x}))^T \\
\Upsilon^{22}(x,\hat{x}) &= A(x)P_2 - W^L(\hat{x})C + P_2 A(x)^T - C^T (W^L(\hat{x}))^T + S_2 \\
\Upsilon^{24}(x,\hat{x}) &= P_2 A_\tau(x)^T.
\end{aligned}$$

v_i denotes vectors that are independent of x and $\hat{x}$, and $\widetilde{\eta}$. $\epsilon_i(x)$ is a slack variable (a radially unbounded positive definite polynomial) to keep the positivity of the SOS condition.

Then, closed-loop polynomial system (34) is asymptotically stable, and polynomial controller and observer gains are given by

$$K(\hat{x}) = W^K(\hat{x})\widetilde{P}_1^{-1} \tag{45}$$

$$L(\hat{x}) = W^L(\hat{x})U\widetilde{C}(\widetilde{P}_2^{11})^{-1}\widetilde{C}^{-1}U^T. \tag{46}$$

Proof. Since the stability of the system implies the stabilization of its dual, we will be interested in the stability of the dual system (see the works of El-Hajjaji et al[1,26] for more details). Thus, we can deal with the stability of the dual system.

Considering the polynomial L-K functional as

$$V(\widetilde{x}) = \hat{x}^T \widetilde{P}_1 \hat{x} + e_x^T P_2 e_x + \int_{t-\tau(t)}^{t} \widetilde{x}_\alpha^T S \widetilde{x}_\alpha \mathrm{d}\alpha, \tag{47}$$

where $e_x = x - \hat{x}$ is the estimation error via the observer and $\widetilde{P}_1$, P_2, and S are symmetric positive definite matrices to be determined.

Deriving Lyapunov function (47), we obtain

$$\begin{aligned}
\dot{V}(\tilde{x}) &= \dot{\hat{x}}^T \widetilde{P}_1 \hat{x} + \hat{x}^T \widetilde{P}_1 \dot{\hat{x}} + \dot{e}_x^T P_2 e_x + e_x^T P_2 \dot{e}_x + \widetilde{x}^T S \widetilde{x} - (1 - \dot{\tau}(t))\widetilde{x}_\tau^T S \widetilde{x}_\tau \\
&= 2\widetilde{x}^T \widetilde{P} \dot{\widetilde{x}} + \widetilde{x}^T S \widetilde{x} - (1 - \dot{\tau}(t))\widetilde{x}_\tau^T S \widetilde{x}_\tau.
\end{aligned} \tag{48}$$

where

$$\widetilde{P} = \begin{bmatrix} \widetilde{P}_1 & 0 \\ 0 & P_2 \end{bmatrix} \text{ and } \widetilde{x} = \begin{bmatrix} \hat{x} \\ e_x \end{bmatrix}. \tag{49}$$

Changing $\dot{\widetilde{x}}$ by its expression of the dual system, function $\dot{V}(\widetilde{x})$ will be of the form

$$\dot{V}(\tilde{x}) = 2\tilde{x}^T \tilde{P}[G^T(x,\hat{x})\tilde{x} + M^T(x,\hat{x})\tilde{x}_\tau] + \tilde{x}^T S \tilde{x} - (1 - \dot{\tau}(t))\tilde{x}_\tau^T S \tilde{x}_\tau. \tag{50}$$

The time derivative of $V(\tilde{x})$ satisfies

$$\dot{V}(\widetilde{x}) \leq \widetilde{\eta}^T \Omega(x,\hat{x})\widetilde{\eta}, \tag{51}$$

where $\widetilde{\eta}^T = [\widetilde{x}^T, \widetilde{x}_\tau^T]$.

$$\Omega(x,\hat{x}) = \begin{bmatrix} \Omega^{11}(x,\hat{x}) & \Omega^{12}(x,\hat{x}) \\ * & -(1-d)S \end{bmatrix} \tag{52}$$

$$\Omega^{11}(x,\hat{x}) = \widetilde{P}G^T(x,\hat{x}) + G(x,\hat{x})\widetilde{P} + S$$
$$\Omega^{12}(x,\hat{x}) = \widetilde{P}M^T(x,\hat{x}).$$

If the following conditions hold, $\dot{V}(\widetilde{x}) < 0$ at $\widetilde{x} \neq 0$

$$\Omega(x,\hat{x}) \prec 0. \tag{53}$$

By applying Lemma 2 and taking into account that P_2 is in form of (43), we obtain

$$\begin{aligned} L(\hat{x})CP_2 &= L(\hat{x})U \begin{bmatrix} \tilde{C} & 0 \end{bmatrix} Z^T Z \begin{bmatrix} \tilde{P}_2^{11} & 0 \\ 0 & \tilde{P}_2^{22} \end{bmatrix} Z^T \\ &= L(\hat{x})U \begin{bmatrix} \tilde{C}\tilde{P}_2^{11} & 0 \end{bmatrix} Z^T \\ &= L(\hat{x})U\tilde{C}\tilde{P}_2^{11}\tilde{C}^{-1}U^T U \begin{bmatrix} \tilde{C} & 0 \end{bmatrix} Z^T. \end{aligned}$$

Let $W^L(\hat{x}) = L(\hat{x})U\widetilde{C}\widetilde{P}_2^{11}\widetilde{C}^{-1}U^T$, and we obtain

$$L(\hat{x})CP_2 = W^L(\hat{x})C. \tag{54}$$

Define

$$S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}, \quad W^K(\hat{x}) = K(\hat{x})\widetilde{P}_1. \tag{55}$$

Substituting (55) into (44), we obtain

$$\Upsilon(x,\hat{x}) \prec 0. \tag{56}$$

The SOS conditions in Theorem 2 imply (56); then, we have $\dot{V}(\widetilde{x}) < 0$.

Moreover, SOS-based conditions (41)-(41) ensure that the closed-loop system is positive. □

Remark 1. Because of the equivalence of stability between original polynomial observer–based control system(34) and its dual system, the polynomial observer–based control (34) is guaranteed to be asymptotically stable and positive if the stability and positive conditions in Theorem 2 are satisfied.

When we consider the polynomial model (32) without delay, we obtain the following corollary.

Corollary 1. *Consider polynomial model (32) and observer-based polynomial control (33) without delay. If there exist symmetric matrices $\widetilde{P}_1$, $\widetilde{P}_2^{11} \in \mathbb{R}^{n_y \times n_y}$ and $\widetilde{P}_2^{22} \in \mathbb{R}^{(n_x-n_y)\times(n_x-n_y)}$, polynomial matrix $W^K(\hat{x})$, and positive polynomial matrix $W^L(\hat{x})$ such that the following SOS-based conditions are satisfied:*

$$v_1^T(\widetilde{P}_1 - \epsilon_1 I)v_1 \text{ is SOS} \tag{57}$$

$$v_2^T(P_2 - \epsilon_2 I)v_2 \text{ is SOS} \tag{58}$$

$$\begin{gathered} v_3^T([A(\hat{x})\widetilde{P}_1 + B(\hat{x})W^K(\hat{x})]_{ij} - \epsilon_3(\hat{x})I)v_3 \text{ is SOS for } i \neq j \\ v_4^T([A(x)P_2 - W^L(\hat{x})C]_{ij} - \epsilon_4(x,\hat{x})I)v_4 \text{ is SOS for } i \neq j \\ v_5^T((A(x) - A(\hat{x}))\widetilde{P}_1 + (B(x) - B(\hat{x}))W^K(\hat{x}) - \epsilon_5(x,\hat{x})I)v_5 \text{ is SOS} \\ -\widetilde{x}^T(\Upsilon^1(x,\hat{x}) + \epsilon_6(x,\hat{x}I)\widetilde{x} \text{ is SOS,} \end{gathered} \tag{59}$$

where

$$\Upsilon^1(x,\hat{x}) = \begin{bmatrix} \Upsilon^{11}(\hat{x}) & \Upsilon^{12}(x,\hat{x}) \\ * & \Upsilon^{22}(x,\hat{x}) \end{bmatrix}$$

$$\Upsilon^{11}(\hat{x}) = A(\hat{x})\tilde{P}_1 + B(\hat{x})W^K(\hat{x}) + \tilde{P}_1 A(\hat{x})^T + (W^K(\hat{x}))^T B(\hat{x})^T$$
$$\Upsilon^{12}(x,\hat{x}) = W^L(\hat{x})C + \tilde{P}_1(\hat{x})(A(x) - A(\hat{x}))^T + (W^K(\hat{x}))^T(B(x) - B(\hat{x}))^T$$
$$\Upsilon^{22}(x,\hat{x}) = A(x)P_2 - W^L(\hat{x})C + P_2 A(x)^T - C^T(W^L(\hat{x}))^T.$$

Then, closed-loop polynomial system (34) without time delay is asymptotically stable, and polynomial controller and observer gains are given by

$$K(\hat{x}) = W^K(\hat{x})\tilde{P}_1^{-1} \tag{60}$$

$$L(\hat{x}) = W^L(\hat{x})U\tilde{C}(\tilde{P}_2^{11})^{-1}\tilde{C}^{-1}U^T. \tag{61}$$

Proof. By following the same procedure as the proof of Theorem 2, we can conclude that the closed-loop polynomial system (34) without delay is asymptotically stable. □

5 | ILLUSTRATIVE EXAMPLES

To demonstrate the validity and applicability of the obtained theoretical results, numerical and practical simulations are presented.

5.1 | Example 1

Consider the following continuous-time positive polynomial system with time delay:

$$\dot{x}(t) = A(x(t))x(t) + A_\tau(x(t))x(t - \tau(t)) + B(x(t))u(t), \tag{62}$$

where

$$A(x(t)) = \begin{bmatrix} -0.743 - x_1^2 - x_1 & -1 \\ 2 & -2 - x_1^2 \end{bmatrix}, A_\tau(x(t)) = \begin{bmatrix} 0.1x_1^2 & 0 \\ 0 & 0.2 \end{bmatrix} \quad \text{and} \quad B(x(t)) = \begin{bmatrix} 30 + 0.38x_1^2 \\ 0 \end{bmatrix}.$$

In this example, the following assumptions are considered, $\tau(t) = 0.5 + 0.1sin(t)$ and $d = 0.6$.
The following polynomial matrices are obtained by solving the SOS conditions in Theorem 1:

$$X(x_2) = \begin{bmatrix} 8.789e^{-16}x_2^2 + 1.815e^{-7} & -5.076e^{-16}x_2^2 + 5.107e^{-8} \\ * & -1.037e^{-16}x_2^2 + 1.157e^{-7} \end{bmatrix} \tag{63}$$

$$S(x_2) = \begin{bmatrix} 1.6287e^{-6}x_2^2 + 1.851e^{-7} & -1.726e^{-7}x_2^2 - 8.275e^{-8} \\ * & 2.856e^{-8}x_2^2 + 1.363e^{-7} \end{bmatrix} \tag{64}$$

$$M(x_2) = \begin{bmatrix} -6.05e^{-8}x_2^2 - 8.438e^{-9} & 1.186e^{8}x_2^2 + 6.968e^{-9} \end{bmatrix}. \tag{65}$$

The polynomial feedback gain $K(x_2)$ can be obtained from $X(x_2)$ and $M(x_2)$ as $K(x_2) = M(x_2)X^{-1}(x_2)$.
Figure 1A shows control results by the polynomial controller using Theorem 1. It can be seen that the controller guarantees not only the global asymptotic stability of the controlled system but also the positivity of the state variables. Figure 1B shows the responses of the control input.

5.2 | Example 2

To demonstrate the effectiveness of the proposed method in Theorem 2, we take the following polynomial system with time delay:

$$\begin{cases} \dot{x}(t) = A(x)x(t) + A_\tau(x)x(t - \tau(t)) + B(x)u(t) \\ y(t) = Cx(t), \end{cases} \tag{66}$$

where

$$A(x) = \begin{bmatrix} -4 & 0.4 \\ 0.1 & -2 - x_2^2 \end{bmatrix}, \; A_\tau(x) = \begin{bmatrix} 0.1x_2^2 & 0 \\ 0 & 0.2 \end{bmatrix}, \; B(x) = \begin{bmatrix} 2 \\ 0.1x_2^2 \end{bmatrix}, \; \text{and} \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

In this example, the following assumptions are considered, $\tau(t) = 0.21 + 0.29sin(t)$ and $d = 0.5$.

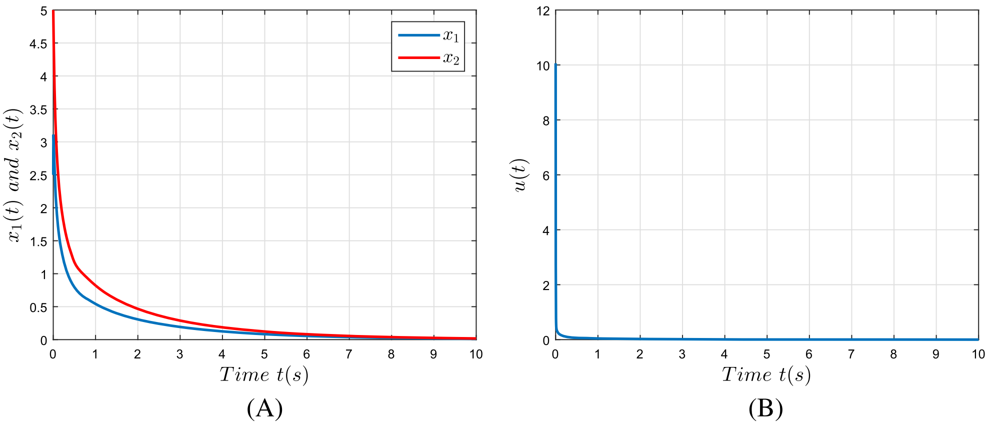

FIGURE 1 Simulation results for Example 1. A, States trajectories of $x(t)$; B, Control $u(t)$ [Colour figure can be viewed at wileyonlinelibrary.com]

The following polynomial matrices are obtained by solving the SOS conditions in Theorem 2:

$$\widetilde{P}_1 = \begin{bmatrix} 1.029e^{-7} & -7.156e^{-10} \\ * & 1.419e^{-8} \end{bmatrix} \tag{67}$$

$$P_2 = \begin{bmatrix} 1.845e^{-7} & 0 \\ * & 2.039e^{-5} \end{bmatrix} \tag{68}$$

$$S_1 = \begin{bmatrix} 2.794e^{-7} & -6.606e^{-8} \\ * & 3.678e^{-8} \end{bmatrix} \tag{69}$$

$$S_2 = \begin{bmatrix} 9.618e^{-7} & -3.584e^{-6} \\ * & 3.489e^{-5} \end{bmatrix} \tag{70}$$

$$K(\hat{x}_2) = \begin{bmatrix} -0.0146\hat{x}_2^2 + 0.2034 & 0.8888\hat{x}_2^2 + 3.0801 \end{bmatrix} \tag{71}$$

$$L(\hat{x}_2) = \begin{bmatrix} 0.7168\hat{x}_2^2 + 2.2486 \\ -0.0047\hat{x}_2^2 + 0.0428 \end{bmatrix}. \tag{72}$$

Figure 2A and Figure 2B show the evolution of state variables and their estimated values, for initial states $x(0) = \begin{bmatrix} 1.5 & 2 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T$. In fact, the controller guarantees the global asymptotic stability of the controlled system.

5.3 | Example 3

To demonstrate the effectiveness of the proposed method, we take the tunnel diode circuit system shown in Figure 3. The electronic circuit can be described by the following nonlinear model[27]:

$$\begin{cases} \dot{x}(t) = A(x)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{73}$$

where $x(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $y(t) = x_1 = V_{out}$ and $x_2 = i_L$.

$$A(x) = \begin{bmatrix} \frac{1}{R_L C_a} - \frac{1}{C_a}(0.002 + 0.01x_1^2) & \frac{1}{C_a} \\ \frac{1}{2L} & -\frac{R_E}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where $C_a = 25mF$ is the capacitor, $L = 20H$ is the inductance, $R_E = 200\Omega$, and $R_L = 2k\Omega$ are the resistors.

Remark 2. It is apparent that the above tunnel diode electronic circuit is modeled as a polynomial system, whereas in the work of Zhao et al,[27] the tunnel diode electronic circuit has been modeled as an uncertain system with polytopic uncertainty. In that paper, the state variable x_1 was restricted to be in the range of $[\bar{m}_1, \bar{m}_2]$, and an LMI approach was proposed to be robust with respect polytopic uncertainties. However, the electronic circuit in this paper is governed by the general model where the state matrix depends on the system state. Moreover, with the proposed approach, state

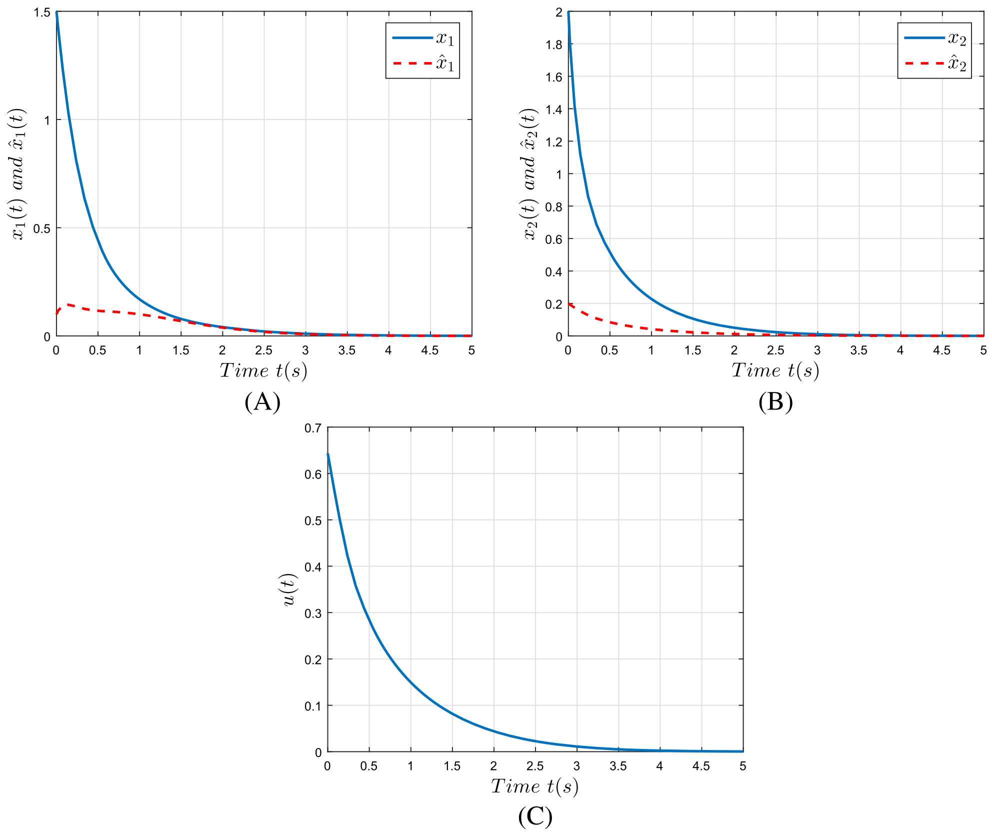

FIGURE 2 Simulation results for Example 2. A, Responses of state $x_1(t)$ and its estimation; B, Responses of state $x_2(t)$ and its estimation; C, Control $u(t)$ [Colour figure can be viewed at wileyonlinelibrary.com]

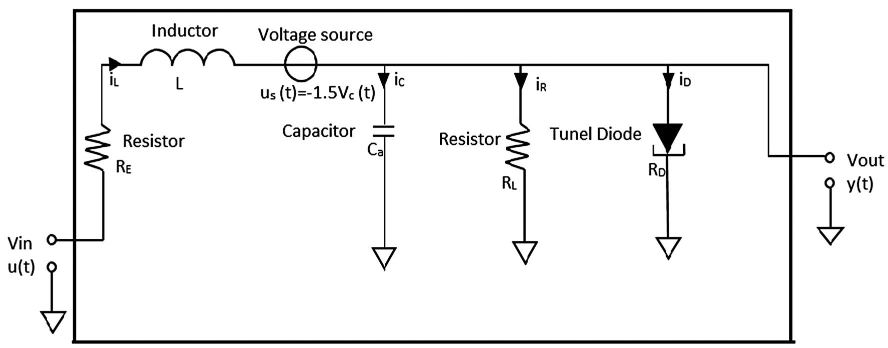

FIGURE 3 Tunnel diode circuit[27]

variable x_1 is in the range of $[0, +\infty[$. Therefore, our SOS-based approach offers more relaxed results. In addition, the SOS approach provides more relaxed results than the LMI approach.

The following solution is obtained by solving the SOS conditions in Corollary 1:

$$\widetilde{P}_1 = \begin{bmatrix} 2.262e^{-6} & -1.402e^{-6} \\ * & 3.322e^{-7} \end{bmatrix} \tag{74}$$

$$P_2 = \begin{bmatrix} 1.3637e^{-4} & 0 \\ * & 7.631e^{-8} \end{bmatrix} \tag{75}$$

$$K(\hat{x}_2) = \begin{bmatrix} -8.4762\hat{x}_2^2 - 1.0288 & -21.9630\hat{x}_2^2 - 3.5714 \end{bmatrix} \tag{76}$$

$$L(\hat{x}_2) = \begin{bmatrix} 0.01204\hat{x}_2^2 + 0.05816 \\ -0.00215\hat{x}_2^2 + 0.00105 \end{bmatrix}. \tag{77}$$

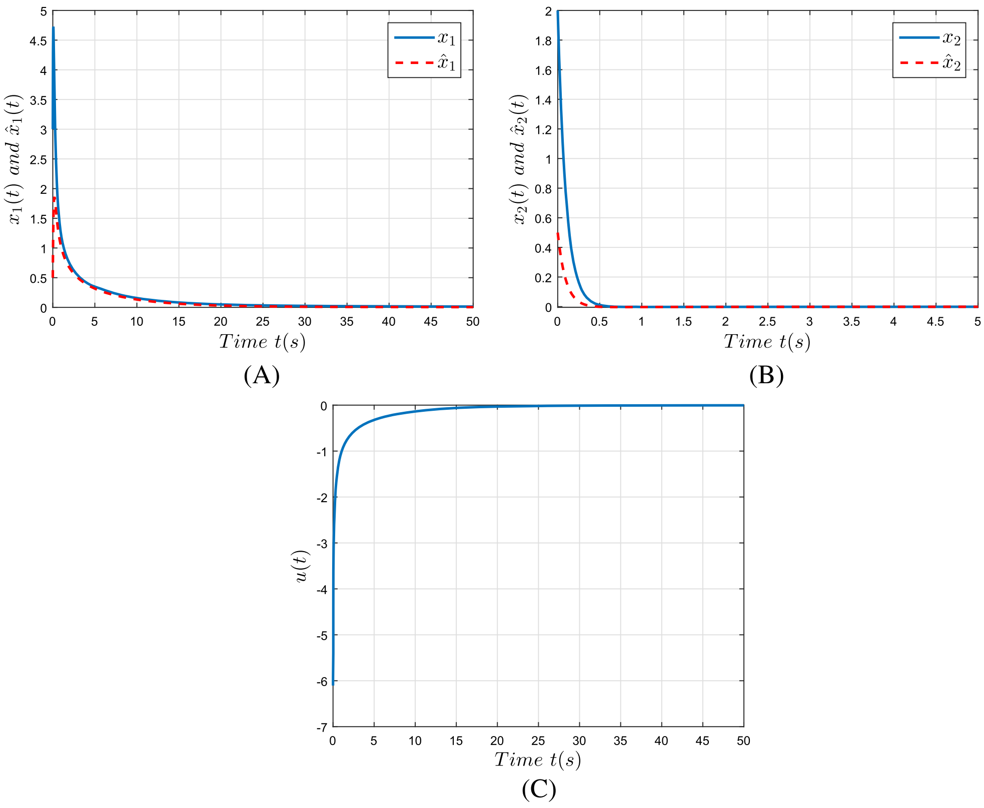

FIGURE 4 Simulation results for Example 3. A, Responses of state $x_1(t) = V_{\text{out}}$ and its estimation; B, Responses of state $x_2(t) = i_L$ and its estimation; C, Control $u(t)$ [Colour figure can be viewed at wileyonlinelibrary.com]

Figures 4A and 4B show control results, from initial states $x(0) = \begin{bmatrix} 3 & 2 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^T$, using the observer-based control law of Theorem 2. It can be seen that the controller guarantees the global asymptotic stability of the controlled system. Moreover, Figure 4C shows the response of the control input.

5.4 | Example 4

Consider the following continuous-time positive polynomial system:

$$\begin{cases} \dot{x}(t) = A(x)x(t) + B(x)u(t) \\ y(t) = Cx(t), \end{cases} \tag{78}$$

where

$$A(x(t)) = \begin{bmatrix} -5 - x_1^2 & -0.1 \\ 0.5 & -5 - x_2^2 \end{bmatrix}, \quad B(x(t)) = \begin{bmatrix} 0.2x_1^2 \\ 0.1x_2^2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The following polynomial matrices are obtained by solving the SOS conditions in Corollary 1:

$$\widetilde{P}_1 = \begin{bmatrix} 7.646e^{-9} & -4.022e^{-10} \\ * & 8.023e^{-10} \end{bmatrix} \tag{79}$$

$$P_2 = \begin{bmatrix} 2.786e^{-6} & 0 \\ * & 8.648e^{-9} \end{bmatrix} \tag{80}$$

$$K(\hat{x}_2) = \begin{bmatrix} 0.1687\hat{x}_2^2 + 4.8685 & 0.0569\hat{x}_2^2 + 2.2879 \end{bmatrix} \tag{81}$$

$$L(\hat{x}_2) = \begin{bmatrix} 0.0241\hat{x}_2^2 - 0.0134 \\ 0.0410\hat{x}_2^2 - 0.0119 \end{bmatrix}. \tag{82}$$

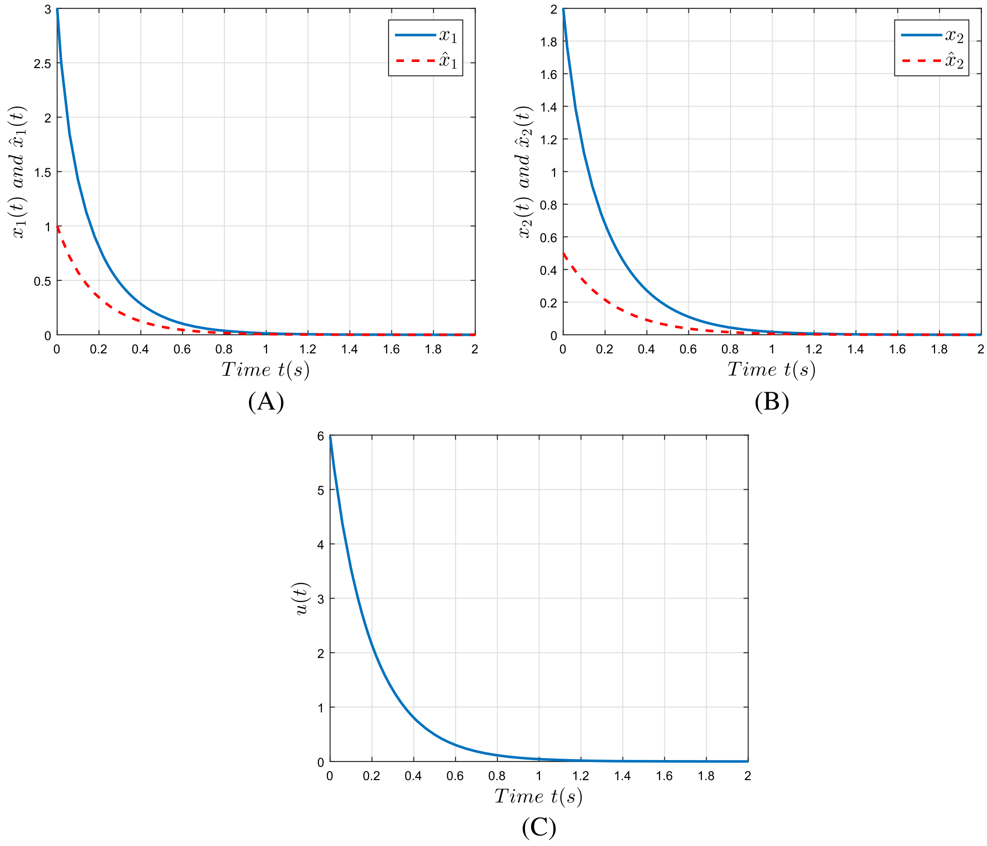

FIGURE 5 Simulation results for Example 4. A, Responses of state $x_1(t)$ and its estimation; B, Responses of state $x_2(t)$ and its estimation; C, Control $u(t)$ [Colour figure can be viewed at wileyonlinelibrary.com]

Figures 5A and 5B show control results by the polynomial observer using Corollary 1. The controller guarantees the asymptotic stability of the controlled system and the positivity of the state variables. Figure 5C shows the control input.

6 | CONCLUSION

In this paper, a novel approach to solve the design problem of positive observer-based polynomial control of positive polynomial systems with time delay has been proposed. A positive polynomial observer is developed to estimate the states of the positive polynomial system. Two results have been established in terms of SOS. In the first one, the stabilization problem have been considered, and sufficient conditions for its solvability have been proposed. Then, the observer based polynomial control design problem has been studied, and sufficient conditions for its solvability have been proposed. All the design conditions in the proposed approach can be solved via SOSTOOLS and SDP solver. To illustrate the validity of the design approaches, several illustrative examples have been provided. Our next subjects are to apply the advanced SOS robust stabilization conditions to more complex positive systems, eg, uncertain positive systems with interval delay in the presence of external disturbances, uncertain positive systems in the presence of sensor and/or actuator faults, etc.

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