

Consensus Perspectives: Glimpses into Theoretical Advances and Applications

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Abstract The polysemic meanings of consensus are surveyed from several points of view, ranging from philosophical aspects and characterizations of several quantification measures within the Social Choice framework, paying also attention to aspects of judgment aggregation as well as fuzzy or linguistic approaches, to practical applications in Decision Making and Biomathematics, among others.

1 Introduction

Consensus [L., fr. consensus, pp. of *consentire*] **1**: group solidarity in sentiment and belief. **2 a**: general agreement, unanimity. **2 b**: the judgement arrived at by most of those concerned.

Consent 1: compliance in or approval of what is done or proposed by another, acquiescence. **2**: agreement as to action or opinion; *specif*: voluntary agreement by a people to organize a civil society and give authority to the government.

Webster's New Collegiate Dictionary

Consensus is a multi-faceted concept. Emerson [40] considers the use of “general agreement” and “majority view” as two widespread opposite senses of the word¹. And according to Williams [103]:

Given this actual range, it is now a very difficult word to use, over a range from the positive sense of seeking general agreement, through the sense of a relatively inert or even

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¹ It is meaningful that Emerson [40] entitled his book after Tocqueville’s argument against the “tyranny of the majority” appearing in *Democracy in America*. Perhaps the current controversy on climate change can be understood as the the opposition between the majoritarian belief that global warming is significant and the opinion of those claiming that there is no a general agreement or evidence of this fact.

unconscious assent [...], to the implication of a “manipulative” kind of politics seeking to build a silent majority as the power-base from which dissenting movements or ideas can be excluded or repressed. It is remarkable that so apparently mild a word has attracted such strong feelings, but some of the processes of modern electoral and “public opinion” politics go a long way to explain this.

Emerson [39] also asserts that “the general consensus on any one issue is that which is perceived to be the agreed opinion of an overwhelming number, though there may be some who dissent”. Thus, this author points out an evident fact, i.e., that this notion is not endowed with an absolute meaning, but a relative one, and nowadays it is usual to deal with levels or measures of consensus, as several of the authors appearing next do.

The chapter, which aims to tackle different approaches to consensus, is organized as follows. In Section 2 we present some philosophical aspects of consensus essentially focused on the doctrine that men are joined together within a society by a contract with explicit or hidden agreements, as Rousseau believed. Then we outline some further developments and connections, such as the link between Rousseau and Condorcet. We also distinguish between the concept of consent and the more technical and recent idea of consensus as appearing in modern Political Science and Sociology. In Section 3 we deal with several formal approaches to consensus mainly from the Social Choice framework, and advances from distance-based, fuzzy or linguistic points of view are presented. In addition, we point out some aspects of an emergent research field focused on *judgment aggregation*. In Section 4 we include some applications as signs of the power of consensus-based methods in practice, and we refer to the way of aggregating different estimates of each candidate through a median-based voting system tested in 2007 (the *majority judgement*), to consensus as a determining condition for publishing in Wikipedia, and about how this idea naturally appears in Biology and Biomathematics. In the last section, we give our concluding remarks.

2 Philosophical Background

There is a long tradition, from Greek and Roman thinkers² to Locke, Hobbes, Paine and mainly Rousseau [93], supporting the idea that political authority relies on the consent of the governed through a *social contract*. Thus, the legitimacy of the government should be based on the *general will*³. It is interesting to point out how this concept is related to the idea of consensus as appearing in Condorcet’s [24] *jury theorem*. This fact has been explained by Grofman and Feld [54], who point out the following three elements of Rousseau’s theory then formalized by Condorcet:

² “The doctrine that society is itself a voluntary association was not unheard of in Greek and Roman times” (see Partridge [89, p. 15]).

³ See Saccamano [96] and Graham Jr. [53].

1. There is a common good.
2. Citizens are not always accurate in their judgements about what is in the common good.
3. When citizens strive to identify this common good and vote in accordance with their perceptions of it, the vote of the Assembly of the People can be taken to be the most reliable means for ascertaining the common good.

An overview of other recent authors such as Habermas, Rawls, etc. about the subject of consensus-based legitimacy can be found in Knight and Johnson [76]. And it is worth mentioning that Buchanan and Tullock consider their well-known work, *The Calculus of Consent*, as a contractual theory of the State (see Buchanan and Tullock [15, appendix 1]).

We have used the terms “consent” and “consensus” and, in fact, both notions appear jointly in Willams’ [103] *Keywords*. According to Partridge [89, p. 71] “within modern Social Theory *consent* has been mainly a term of Political Philosophy, *consensus* mainly a term in Sociology”. Indeed, according to this author, it was Comte who introduced in the XIXth century in the vocabulary of Social Sciences the notion of “consensus”, in its Latin version. It is well known that Comte founded Sociology from the basis of Biology, and in this particular case the coined term was inspired by Medicine during his age, where consensus meant solidarity among distinct parts of the human body⁴. Thus, in recent years, consensus refers to agreements which provide the conditions of political and social cohesion (on this aspect, see De Dreu and De Vries [33]).

3 Formal Approaches to Consensus

In what follows we present several aspects of consensus and how they have been developed within different formal frameworks.

3.1 *Consensus and Social Choice*

In Theory of Democracy as understood by van Mill [82] there are two extremely different currents for dealing with conflicts in a legitimate way: “theories of democratic discourse” advocated mainly by Habermas [55], which try to solve them through deliberation and consensus, and “disequilibrium theories of social choice” as proposed by Black [10], Arrow [5] and Riker [92], aggregating individual preferences into a social result.

However, Social Choice is not merely a tool to decide in absence of consensus, and some authors have formalized this notion in the Social Choice context. In

⁴ It seems that this aspect was foreseen by Saint-Simon, although Comte properly developed such idea (see Campillo [22, pp. 69-71]).

this way, Samet and Schmeidler [97] have introduced and then axiomatized “consent rules, that incorporate aspects of majoritarianism and liberalism” and Ballester and García-Lapresta [9] have analyzed a recursive consensus process for selecting qualified individuals of a group⁵. On their part, Bosch [14], Alcalde-Unzu and Vorsatz [1, 2] and García-Lapresta and Pérez-Román [50] have proposed several consensus measures and they have also characterized them. On the other hand, García-Lapresta [47] and García-Lapresta and Pérez-Román [49], following Cook and Seiford [29, 30], have devised voting methods penalizing the disagreement, the key idea being the “marginal contribution to consensus” for each agent, and Mata et al. [80, 81] have analyzed strategic aspects in consensus processes. Recently, García-Lapresta et al. [48], also in a Social Choice context, have obtained the set of scoring rules that optimizes consensus among voters by maximizing the collective utility.

It is also worth noting the existence of consensus models for decision making in committees which have nothing to do with voting, for example that appearing in Eklund, Rusinowska and De Swart [38]. These authors have performed a method where a chairman suggests the decision makers adjust or modify their evaluation of some alternatives, attending several criteria in order to improve the agreement, following an idea which recurrently appears in Consensus Theory⁶. In this way, Carlsson et al. [23] have also considered “an advising monitor which tries to contract the decision makers into a mutual decision through soft enforcement”, where convergence to consensus is analyzed by means of a topological approach which can measure distances between decision makers and allow these authors to model the trade-off between “degree of consensus” and “strength of majority”.

Such metric techniques lead us to the next section, where the notion of distance becomes crucial.

3.2 *Distance-based approaches to consensus*

The idea of considering distances among preferences (already suggested in Condorcet’s writings) has revealed to be very fruitful⁷. In this way, consensus can be

⁵ The research of these authors is focused on aspects of “group identification”. An overview of selected topics on this subject in the context of Social Choice has been outlined by Dimitrov [36] in one of the chapters of this book.

⁶ For example, the possibility of changing agents’ opinions along the process, in an iterative way, is also essential in the *Delphi method* for expert decision (see Turoff and Linstone [100]). On the Delphi method as a tool for consensus-building see, for example, Hsu and Sanford [63] and the references therein.

⁷ This is, for example, the key for the Kemeny [71] rule, inspired by the treatment of statistical data. In fact, as demonstrated by Young [101], this method is just the “maximum likelihood rule” tacitly formulated by Condorcet [24]. See Klamler [74, 75] and Nurmi [87] for Kemeny-type and other distance functions on the set of choice functions, García-Lapresta and Pérez-Román [49] and the references cited there concerning other voting methods using distances for obtaining a collective outcome, and Meskanen and Nurmi [83, 84] who present a comprehensive approach to methods

somehow understood as closeness to unanimity, and several attempts has been done to minimize (from distance-based assumptions) the aggregate disagreement.

Hornik and Meyer [65] note that such “optimization approach” goes back to R gnier [90]. Nonetheless, it is fair to acknowledge the pioneer analysis of so-called “compromise solutions” (those minimizing the group regret through appropriate distance functions) by Yu [104] and Freimer and Yu [46]. On the other hand, in order to determine the closest ranking to all decision-makers, Cook and Seiford [29, 30] considered l^1 and l^2 metrics previously introduced by Kemeny and Snell [72] in an important paper and also analyzed by Bogart [11, 12].

It is worth noting that the last of the cited distances is related to the Borda rule⁸ and it was independently obtained by Kendall [73] in a statistical estimation context. On the other hand, the l^1 metric has been used by Contreras et al. [27] in order to provide a compromise method for collective decision problems especially suited for situations where the members of the group provide partial or imprecise information about their preferences.

A general model for distance-based consensus can be found in Cook et al. [28] (see Gonz lez-Pach n and Romero [51] for a detailed exposition and further developments). The last mentioned authors, Gonz lez-Pach n and Romero [51, 52], have considered the aforementioned problem of minimization as a Goal Programming model and have obtained consensus weights when the information taken into account by the decision maker is somehow inconsistent, i.e., does not verify some rational assumptions.

In Eckert and Klamler [37] the reader will find a survey on distance-based approaches to aggregation theory dealing with the construction of aggregation rules, the comparison of aggregation procedures and the generalizations of aggregation problems, among other topics. These authors point out that “aggregation problems arise in many other areas than social choice distance-based approaches [and] are not limited to the construction and analysis of voting rules”, as happening with the emerging literature on the subject of the next subsection.

which can be viewed as distance minimizing ones, focusing on the measurement of disagreement. Further insights of this approach can be found in Nurmi [88] and Eckert and Klamler [37] in two chapters of this book.

⁸ The Borda [13] and Condorcet [24] approaches have been called “*constructive* consensus methods” by Hornik and Meyer [65], but they assert that both rules agree in absence of cycles in the collective outcome, which is not true. There is a wide literature on Borda-Condorcet disagreement, and one of the chapters in this book deals with this topic in an experimental way (see Regenwetter and Popova [91]). At this juncture, Emerson [39, 40] advocates for the Borda rule, which promotes consensus and avoids the “tyranny of the majority”. This is the reason why he has devised a modified Borda count system of consensus voting (see Emerson [41]). It is interesting to point out that Emerson [40] also defines a “level of consensus”, but specifically intended for his system.

3.3 Judgment aggregation

According to List [77], *judgment aggregation* is concerned with how “a group of individuals aggregate the group members’ individual judgments on some interconnected propositions into corresponding collective judgments on these propositions”, and such analysis may be done from different collective decision making contexts. This research field runs parallel to Social Choice. In fact there is a *doctrinal paradox* (or *discursive dilemma*) whose homologue in Social Choice would be *Condorcet’s paradox*⁹. There is also an *Impossibility Theorem*¹⁰ and further developments to mitigate the resulting impasse, as happening with Arrow’s Theorem and its variants. A comprehensive overview on judgment aggregation can be found in List and Puppe [79] (see also List [77] and the references appearing there, all of them available online, as well as Dietrich [34], and Dietrich and List [35]).

Within this framework, Hartmann and Sprenger [63] and Hartmann et al. [56] are developing models of consensus appearance in judgment aggregation situations in order to achieve compromise-based decisions.

3.4 Fuzzy and linguistic consensus models

An updated overview on the use of fuzzy preferences and majorities in obtaining group decision making outcomes and levels of consensus has been presented by Kacprzyk et al. (see [69] and the references appearing there). As for fuzzy and linguistic approaches to consensus models for group decision making it is worth to mention Herrera-Viedma et al. [60, 61, 62], Herrera et al. [58, 59], Alonso [3], Alonso et al. [4] and Cabrerizo et al. [16, 17, 19], among others. In these papers, there usually exists a moderator in the process of obtaining a high level of consensus or agreement among the experts to decide in a group decision making problem (see also Fedrizzi et al. [44], Kacprzyk and Fedrizzi [66], Kacprzyk et al. [67, 68], and the iterative dynamical approach by Fedrizzi et al. [43]). Recently, also guided by a moderator and inspired by a data mining technique, Kacprzyk et al. [70] have provided an interesting tool for consensus reaching: action rules, which take into

⁹ As exposed by Fishburn [45] in a comprehensive paper concerning the landmarks in Social Choice, “Condorcet felt strongly that a majority candidate ought to be elected when one exists, [but] he realized that a profile need not have a majority candidate, as when m [voters]= n [candidates]=3 and the profile is $(a >_1 b >_1 c, b >_2 c >_2 a, c >_3 a >_3 b)$, in which case a has a 2-to-1 majority over b , b has a 2-to-1 majority over c , and c has a 2-to-1 majority over a . In such case the profile illustrates *Condorcet’s effect*, or the *paradox of voting*, or has *cyclical majorities*”. The implications of this kind of choice paradoxes to consensus-reaching methods have been analyzed by Nurmi [88] in the pages of this book.

¹⁰ In fact, there are many impossibility theorems in Social Choice, but that seminally due to Arrow [5] has been considered the “Impossibility Theorem” par excellence. Again, according to Fishburn [45] “Arrow’s discovery [shows] that a few appealing criteria for social ranking methods are mutually incompatible. Its essential idea is that the problems that arise from Condorcet’s paradox of voting [...] cannot be avoided under any reasonable generalization of majority comparisons”.

account different concessions than can be offered to the individuals so that they will change their preferences in order to obtain consensus convergence; and, on their hand, Calvo and Beliakov [20] have introduced penalty functions in order to minimize deviations of individual inputs from the collective output¹¹. Advantages, drawbacks and future trends of fuzzy and linguistic approaches to consensus are analyzed in Cabrerizo et al. [18].

It is important to note here that Montero [85, 86] has analyzed the impact of fuzziness in Group Decision Making and has also proposed an escape from Arrow's impossibility result to reach a consensus-based solution respecting democratic compelling properties¹². Even more, when such impossibility result is unavoidable in some rational amalgamation operations, Cutello and Montero [31] have searched for (and then characterized) "good enough" solutions.

4 Applications Dealing with Consensus

This section displays three practical cases where the idea of consensus is taken into account for different purposes in distinct contexts.

4.1 Aggregating different assessments through majority judgement

According to Balinski and Laraki [7], Galton's only comment related to Social Choice Theory "was discarded as a small contribution by Black". These are Galton's words (cited in Black [10]), which support the idea of mitigate the influence of the extreme opinions:

Each voter [...] has equal authority with each of his colleagues. How can the right conclusion be reached, considering that there may be as many different estimates as there are members? That conclusion is clearly *not* the *average* of all the estimates, which would give a voting power to cranks in proportion to their crankiness. One absurdly large or small estimate would leave a greater impress on the result than one of reasonable amount, and the more an estimate diverges from the bulk of the rest, the more influence would it exert. I wish to point out that the estimate to which least objection can be raised is the *middlemost* estimate, the number of votes that it is too high being exactly balanced by the number of votes that it is too low. Every other estimate is condemned by a majority of voters as being either too high or too low, the middlemost alone escaping this condemnation.

¹¹ The relationship between these penalty functions and consensus is analyzed by Calvo et al. [21] in one of the chapters of this book.

¹² As pointed out in Montero [86], "the main criticism to Arrow's approach should be the underlying Boolean assumption, present in every concept in his model. This binary view is taken for granted, and therefore hidden to readers. But in Arrow's theorem every condition (modeling liberty or equality) is fuzzy in nature, as consistency is, as consensus and preferences are".

This argument, advocated by Surowiecki [99] in the first pages of his book, has also been assumed by Balinski and Laraki, the idea of considering the median (middlemost) of the voters' assessments being the key of a new voting system proposal: the *majority judgement*, which has been empirically tested at Orsay during the 2007 French presidential elections (see Balinski and Laraki [7, 8] for details both of the theoretical background and concerning the aforementioned experiment). The method has also been implemented worldwide concerning 2008 United States elections (see Balinski [6]).

4.2 Consensus for publishing in Wikipedia

It is commonly known and accepted the role of consensus in prediction markets and the development of online tools as Google or Wikipedia (see Surowiecki [99]). Here we focus on the last one, where consensus is fundamental¹³ in its editorial policy (see Figure 1, where the Wikipedia publishing process is shown through a flowchart), and might entail a rejected entry if agreement for acceptance is not present after a reasonable time period. According to Wikipedia rules “consensus need not be fully opposed; if consensus is neutral or unclear on the issue and unlikely to improve, the proposal is likewise rejected” (see [102] for more details). Thus, Wikipedia seems to respect the ancient criterion of truth¹⁴ known as *consensus gentium* (L.: Agreement of the peoples), stating that “which is universal among men carries the weight of truth” (cited in Runes [94]).

4.3 Consensus in Biology and Biomathematics

One tends to think of consensus aspects as something essentially human, but biologists have concluded that “consensus decision making is common in non-human animals, and that cooperation between group members in the decision-making process is likely to be the norm” (see Conradt and Roper [25]; a reply to this insight

¹³ This consensual nature of Wikipedia has been noted, for example, in Essential-Facts.com [42]: “Genuine consensus typically requires more focus on developing the relationships among stakeholders, so that the compromises they achieve are based on willing consent – we want to give this to you, and we want you to give that to us only because you want to. The articles of Wikipedia itself are intended to follow this kind of approach”.

¹⁴ In fact, there are theories where the concept of truth crucially relies on consensus as an essential component. And nowadays consensus has become to mean “accepted subject”, specially concerning scientific matters. In this way, in the specific context of Medicine, it is common to speak about “consensus statements” on diseases like AIDS, diabetes, etc. There are strict protocols to achieve this kind of agreements. According to Wikipedia, “medical consensus is a public statement on a particular aspect of medical knowledge available at the time it was written, and that is generally agreed upon as the evidence-based, state-of-the-art (or state-of-science) knowledge by a representative group of experts in that area”.

can be found in List [78], and further advances appear in Conradt and Roper [26]). As examples, these authors cited nest choices in bees and ants, agreement about routes in navigating birds, activity timing in mammals, etc. An in-depth research of consensus decision making by fish has been experimentally carried out in Sumpter et al. [98], where the gregarious behavior of the stickleback¹⁵ has been analyzed.

In this way, according to Day and McMorris [32], who are the authors of a handbook relating Consensus Theory to Biomathematics¹⁶, the archetypal problem of aggregation concerning group choice pioneered by Arrow [5] and then refined and extended by many other authors, can be applied in areas far from their origins in Group Choice and Welfare Economics¹⁷. As pointed out by Janowitz in the foreword to the mentioned book:

Since consensus theory has its roots in the theory of elections, many contributions have been (and are being) made by political scientists, sociologists and economists. In the context of human behavior, consensus research is carried out by psychologists. Faced with conflict-

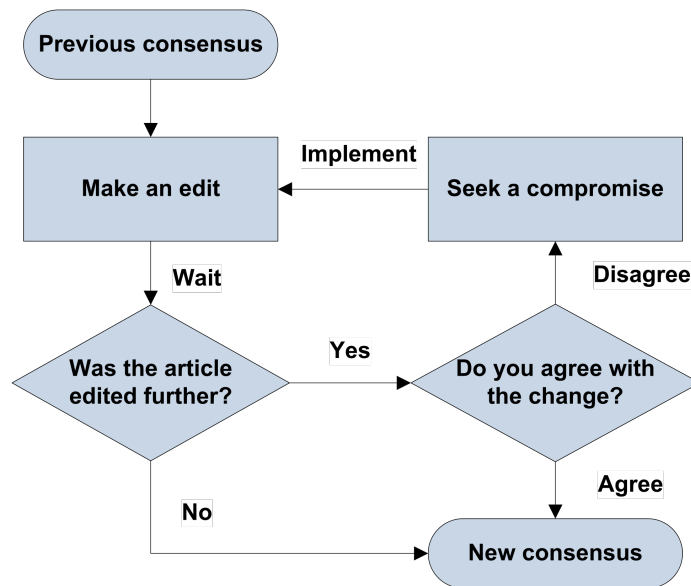


Fig. 1 Wikipedia publishing consensus process: When an edit is made, other editors have these options: accept the edit, change the edit, or revert the edit. These options may be discussed if necessary. *Source: Wikipedia.*

¹⁵ Curiously, this very fish is also subject of research by Mr. Pickwick, his paper being parodied at the beginning of Dickens’ novel as empty erudition.

¹⁶ This book has been interestingly reviewed by Saari [95] mainly attending its Social Choice background.

¹⁷ Taking into account the common structure or pattern between what they call “bioconsensus” and Social Choice, Day and McMorris [32] advocate for the existence of an Arrow’s paradigm in the Kuhn’s sense.

ing evidence on evolutionary history, systematic biologists appeal to concepts of consensus, and molecular biologists attempt to apply consensus theory in areas of DNA research. Market researchers find the discipline relevant since it can be viewed as a theory of how large organizations make decisions based on possibly conflicting lines of evidence. Contemporary applications of consensus theory involve airplane and missile navigational systems, methods to combat bioterrorism, drug development based on DNA research, marketing and manufacturing decisions of large companies, and stock market predictions.

Particularly, Biomathematics and Molecular Biology are concerned with consensus in areas where the objects of interest represent partitions of a set, hierarchical structures, phylogenetic trees and alignment of molecular sequences (for deeper explanations see Day and McMorris [32], especially chapter 1, and Janowitz et al. [64]).

5 Conclusion

In this chapter, in a non-exhaustive way, we have outlined the idea of consensus from classic authors to nowadays researchers, from theoretical philosophical points of view to practical applications. Progress has been achieved, but we can still expect new results. The list of chapters of this book well may become a guideline for further advances.

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