EXPANSION SPEED DETERMINATION

NOMENCLATURE

A area (m^2)
ICE Ignition Combustion Engine
m mass (kg)
p pressure (Pa)
q incoming heat per mass unit (J/kg)
$\dot{q}$ incoming heat flow per mass unite (J/kgs)
Q incoming heat to the system (J)
$\dot{Q}$ incoming heat flow to the system (J/s)
R gas ideal constant (J/kgK), radius (m)
S speed (m/s)
t time (s)
T temperature (K)
v specific volume (m^3/kg)
V volume (m^3)
x piston displacement (m)

Greek letters
γ specific heats ratio (-)

Subscripts
b burned
bb blow by
e expansion
f flame front
in incoming to the system
out outgoing from the system
p piston
t turbulent
u unburned

In Fig. 1 a scheme of the flame front position in a combustion chamber of an ICE is shown, where S_e is the expansion speed, A_{pu} is the piston area occupied by the unburned mass zone, and S_p is the piston speed.

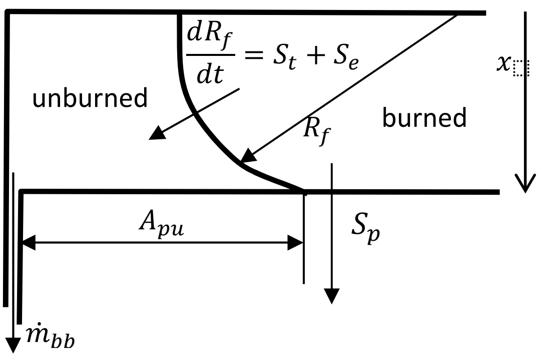

Fig. 1 Geometric approach for determining the flame front speed

The temporal variation of V_u can be expressed in two ways: first from a geometric point of view as the sum of the temporal variation of the flame front radius and the temporal variation of the volume due to the piston movement:

$$\frac{dV_u}{dt} = -\frac{dR_f}{dt}A_f + A_{pu}\frac{dx}{dt} = -\frac{dR_f}{dt}A_f + \frac{A_{pu}}{A_p}\frac{dV}{dt}$$

And the second way as bellow:

$$\frac{dV_u}{dt} = \frac{d(m_u\, v_u)}{dt} = m_u\frac{dv_u}{dt} + v_u\frac{dm_u}{dt}$$

From Eq. **¡Error! No se encuentra el origen de la referencia.**, $(dm_u)/dt =$

$-dm_b/dt - \dot{m}_{bb}$, then:

$$\frac{dR_f}{dt} A_f = v_u \frac{dm_b}{dt} + v_u \dot{m}_{bb} + \frac{A_{pu}}{A_p} \frac{dV}{dt} - m_u \frac{dv_u}{dt} \qquad (1)$$

An expression of the term dv_u/dt is in Eq. **¡Error! No se encuentra el origen de la referencia.** of the thermodynamic model. Substituting the expression in Eq. (1):

$$\frac{dR_f}{dt} A_f = v_u \frac{dm_b}{dt} + v_u \dot{m}_{bb} + \frac{A_{pu}}{A_p} \frac{dV}{dt} + \frac{m_u}{p} \left(\frac{dp}{dt} \frac{v_u}{\gamma_u} - \frac{\gamma_u - 1}{\gamma_u} \dot{q}_u \right) \qquad (2)$$

The temporal variation of the flame front depends on the combustion process (dm_b/dt), the combustion chamber volume variation (dV/dt and $\dot{m}_{bb}$), and the increment of the unburned density because of the pressure increase and heat transmission (dp/dt and $\dot{q}_u$).

On the other hand, one can say that the flame front speed is the sum of two phenomena: combustion process and the unburned products movement.

The expansion speed S_e, corresponds to the velocity in the perpendicular direction to the flame front of the unburnt products located in the position of the flame front. This expansion speed is the cause of shear forces in the unburned mixture near the walls, so that they can generate turbulence.

So, the flame front movement is the sum of the combustion velocity and the expansion speed, Eq. (3)

$$\frac{dR_f}{dt} = S_t + S_e \qquad (3)$$

Substituting Eq. (3) in Eq. (2), the expansion speed can be obtained from the thermodynamic and geometrical variables:

$$\frac{dR_f}{dt} A_f = (S_t + S_e) A_f = S_t A_f + \frac{A_{pu}}{A_p} \frac{dV}{dt} + v_u \dot{m}_{bb} + \frac{m_u}{p} \left(\frac{dp}{dt} \frac{v_u}{p\gamma_u} - \frac{\gamma_u - 1}{\gamma_u} \dot{q}_u \right)$$

$$S_e = \frac{A_{pu}}{A_f A_p} \frac{dV}{dt} + v_u \dot{m}_{bb} + \frac{m_u}{p A_f} \left(\frac{dp}{dt} \frac{v_u}{\gamma_u} - \frac{\gamma_u - 1}{\gamma_u} \dot{q}_u \right) \qquad (4)$$

With the expression of Eq. (4), the expansion speed S_e can be calculated from the diagnostic model results. However, if a predictive model is used for calculating the pressure from the combustion rate, the expression is not explicit because dp/dt depends on the fuel burned ratio and on the piston movement.

<u>Expression of expansion speed for a predictive model</u>

For making an explicit expression of S_e, and for analyzing the causes of its evolution during the combustion process, the hypothesis of uniformity of the properties of the burned and unburned zones is considered.

The energy conservation equation for the mass in the combustion chamber is raised:

$$\frac{dU}{dt} = -p\frac{dV}{dt} + \dot{Q}$$

$$mc_v\frac{dT}{dt} = -p\frac{dV}{dt} + \dot{Q} \qquad (5)$$

On the other hand, deriving the ideal gas equation, $pV = mRT$, respect to time:

$$mR\frac{dT}{dt} = p\frac{dV}{dt} + V\frac{dp}{dt} \qquad (6)$$

Clearing dT/dt in Eq. (5) y substituting in Eq. (6):

$$\frac{mR}{mc_v}\left(-p\frac{dV}{dt} + \dot{Q}\right) = p\frac{dV}{dt} + V\frac{dp}{dt} \qquad (7)$$

Clearing dp/dt:

$$V\frac{dp}{dt} = (\gamma - 1)\dot{Q} - \gamma p\frac{dV}{dt} \qquad (8)$$

Substituting Eq. (8) in Eq. (4):

$$S_e A_f = \frac{A_{pu}}{A_p}\frac{dV}{dt} + v_u\dot{m}_{bb} + \frac{m_u}{p}\left[\left((\gamma - 1)\dot{Q} - \gamma p\frac{dV}{dt}\right)\frac{v_u}{\gamma V} - \frac{\gamma - 1}{\gamma}\dot{q}_u\right]$$

Knowing that $\dot{Q} = S_t\rho_u A_f H_p + \dot{Q}_w$

$$S_e = \frac{1}{A_f}\left[\frac{dV}{dt}\left(\frac{A_{pu}}{A_p} - \frac{V_u}{V}\right) + v_u\dot{m}_{bb}\right] + S_t\left(\frac{(\gamma - 1)m_u H_p}{\gamma pV}\right) + \frac{(\gamma - 1)}{\gamma p A_f}\left(\dot{Q}\frac{V_u}{V} - \dot{Q}_u\right) \qquad (9)$$

From Eq. (9), the expansion speed expression depends on three terms. The first is the variation of S_e due to the volume variation through the piston movement (dV/dt) and the blow by leakage ($\dot{m}_{bb}$). The second is due to the flame front advance by the combustion with a combustion speed S_t. And the third is due to the heat transfer to the walls.

Respect the first term, the expansion speed increases with the piston movement, but the variation of the combustion chamber volume is not very significant during the combustion process in an ICE.

On the other hand, the higher the combustion speed S_t the higher the expansion speed S_e, but the m_u value is important in the S_t influence over the S_e. At the beginning of the combustion, the m_u value is high and the effect of the combustion on the expansion speed is higher than at the end of combustion, when the m_u value is low.

Respect the third term of the S_e expression of Eq. (9) depending on heat flux. The total wall heat flux $\dot{Q}$, is the sum of the wall heat flux of the unburned zone $\dot{Q}_u$, plus the one of the burned zone $\dot{Q}_b$:

$$\dot{Q} = \dot{Q}_u + \dot{Q}_b$$

The wall heat fluxes of burned and unburned mass zones are calculated as the total heat flux proportional part depending on the temperature and the volume fraction of the two zones:

$$\dot{Q}_u = \dot{Q}\frac{T_u \frac{V_u}{V}}{T_u \frac{V_u}{V} + T_b \frac{V_b}{V}}$$

$$\dot{Q}_b = \dot{Q}\frac{T_b \frac{V_b}{V}}{T_u \frac{V_u}{V} + T_b \frac{V_b}{V}}$$

Then, the third term of Eq. (9) can be written as:

$$\left(\dot{Q}\frac{V_u}{V} - \dot{Q}_u\right) = \left((\dot{Q}_u + \dot{Q}_b)\frac{v_u}{V} - \dot{Q}_u\right) = \left(\dot{Q}_u\,\frac{V_u - 1}{V} + \dot{Q}_b\frac{V_u}{V}\right) = \left(\dot{Q}_u\,\frac{V_b}{V} + \dot{Q}_b\frac{V_u}{V}\right) \qquad (10)$$

The heat flux of the burned mass is higher than the one of the unburned mass because of the higher temperature in burned mass, but the value of the two terms of Eq. (10) also depends on the volume fractions. The wall heat fluxes are negative during combustion, so the higher the wall heat fluxes the smaller the expansion speed, but it also depends on the volume occupied by the unburned and burned mass. For example, at the beginning of the combustion, $\dot{Q}_u$ is high but V_b is low, and $\dot{Q}_b$ is low but V_u is high. At the end of the combustion, $\dot{Q}_u$ is low but V_b is high, and $\dot{Q}_b$ is high but V_u is low.

In Fig. 2, the flame front speed dR_f/dt, the turbulent combustion speed S_t, and the expansion speed S_e versus dimensionless flame front radius R_f/R_p, for a mixture of 50% Hydrogen and 50% natural gas, at 1750 rpm engine speed, are shown. The expansion speed is higher at small radii, when the V_u is high, and is lower at the end of combustion when the V_u is small and has low capacity to vary its volume so the expansion speed greatly decreases.

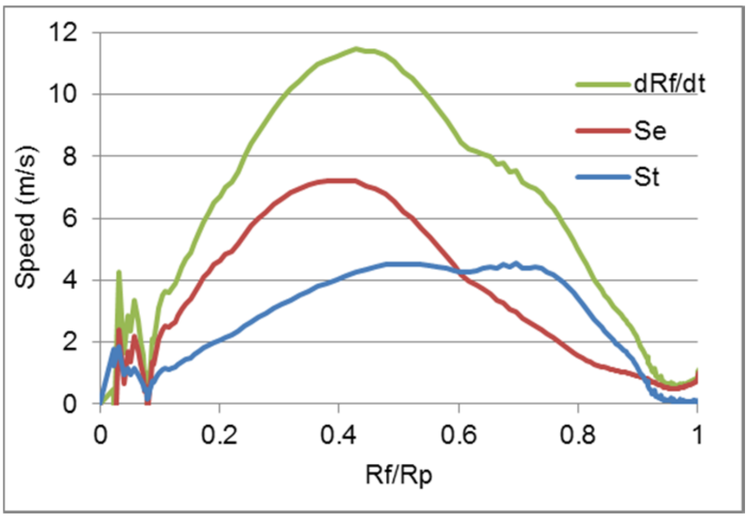

Fig. 2. Flame front speed dR_f/dt, turbulent combustion speed S_t, and expansion speed S_a versus dimensionless flame front radius R_f/R_p. 50%H2, 1750 rpm.