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Checking Orthogonal Transformations and Genetic Algorithms for Selection of Fuzzy Rules based on Interpretability-Accuracy Concepts

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***Abstract*—Fuzzy modeling is one of the most known and used techniques in different areas to emulate the behavior of systems and processes. In most cases, as in data-driven fuzzy modeling, these fuzzy models reach a high performance from the point of view of accuracy, but from other points of view, such as complexity or interpretability, the models can present a poor performance.**

Several approaches are found in the specialized literature to reduce the complexity and improve the interpretability of the fuzzy models. Here, a post-processing approach is taken into account via the definition of the rules selection criterion that aims to choose the most relevant rules according to the well-known accuracy-interpretability trade-off. This criterion is based on Orthogonal Transformations, here the QRP transformation is taking into consideration, and its parameters are tuned genetically. The main objective is to check the true significance, drawbacks and advantages the firing matrix of the rules, that is the foundation of the most usual approaches based on orthogonal transformations for the complexity reduction of the fuzzy models.

A neuro-fuzzy system, FasArt (Fuzzy Adaptive System ART based), and several case studies, data sets from the KEEL Project Repository, are used to tune and check this approach. This neuro-fuzzy system generates Mamdani fuzzy rule based systems (FRBSs), each with its own particularities and complexities from the point of view of fuzzy sets and rule generation. NSGA-II is the MOEA tool used to tune the criterion parameters based on accuracy-interpretability ideas.

***Index Terms*—Fuzzy Systems, Interpretability, Accuracy, Rule Selection, Orthogonal Transformations, Genetic Algorithm**

I. Introduction

Fuzzy modeling is one the most known approaches for a wide range of problems. Data-driven rule based fuzzy models have been used in several and very different scientific and technical areas [1], [2], [3], [4], [5].

In general, the fuzzy models taken into consideration in real world applications have been data-driven and rule based fuzzy models due to their advantages: easy use and performance. This performance has usually been evaluated on the basis of the accuracy of the models, thus minimizing the error between the real output an the estimated output generated by the fuzzy models. But other aspects have not been taken into consideration: complexity, interpretability, etc. Some of them are base principles of fuzzy logic but these fuzzy models used them as simple mathematical tools, losing their original fuzzy meaning.

Complexity is a very usual index or measure, and it is a problem in data-driven rule based fuzzy models, so the reduction of this complexity permits important aspects of the fuzzy models to be improved, gaining a better performance for these models. The question is the way in which this complexity reduction must be carried out [6], [7], [8], [9], [10], [11].

Here, the complexity reduction is carried out based on Orthogonal Transformations and accuracy-interpretability trade-off, as this approach has traditionally been focused in most works [12]. The goal of this work is a postprocessing approach to simplify a rule-based fuzzy model based on a simple $Criterion\ of\ selection$ that is tuned by a genetic approach, and, in this way, checks the possibilities and drawbacks of this type of approach.

The paper is organized as follows: first, in Section II, a brief description of alternative points of view about fuzzy modeling, interpretability and accuracy are given. Also in this section, a brief description of orthogonal transformation and, specially, QR decomposition with column pivoting - QRP, are given, and finally, several complexity and interpretability measures are proposed. In Section III, a $Criterion$ to select the best rules is described. In Section IV the methodology used in this work is described. Some experimental studies are carried out and the main results obtained are discussed in Section V. Finally, in Section VI, the most interesting conclusions obtained from this work are set out.

II. Fuzzy Modeling: Accuracy vs.Interpretability

Initially, two well known modeling approaches to generate fuzzy rules are described in the technical and scientific literature [13], [14], [15]:

1) *Precise Fuzzy Modeling*, whose main goal is to obtain as much accuracy as possible. In general, the models

generated have a good accuracy but a low level of interpretability. This modeling is popular with data-driven knowledge but expert knowledge is also considered.

2) *Linguistic Fuzzy Modeling*, these models have a good level of interpretability but poor accuracy. Here, knowledge from experts and from data guide the modeling process.

Both approaches have their own drawbacks and advantages, but there are several ways to deal with the generation of fuzzy systems whose performance includes an adequate accuracy-interpretability trade-off:

- Algorithms take into account the idea of accuracy-interpretability during the generation of the fuzzy system. i.e genetic fuzzy systems [14], [16].
- Orthogonal transformations where the interpretability is improved by complexity reduction rules [12], [17], [18], [19].

The orthogonal transformations, used traditionally, permit relevant simplifications, but the loss of accuracy is also very high. In most of the usual works about this topic and approach, the accuracy-interpretability is not truly involved, and, in addition, the experimental work is not sufficient to reach relevant conclusions.

These orthogonal transformations are used for rule selection/reduction in fuzzy modeling in two approaches [12]: the range-revealing approach (Singular Value Decomposition (SVD) or QRP) and those that evaluate the individual contributions of the rules (Orthogonal Least Squares (OLS)).

In this work, range-revealing methods are used, so the fuzzy model can be written as a linear regression problem [19]:

$$y = P * \theta + e \tag{1}$$

SVD is used to determine the effective rank of the rule firing matrix (P). This can be expressed as:

$$p_i(x) = \frac{\prod_{j=1}^{N} A_{ij}(x_j)}{\sum_{k=1}^{M} \prod_{j=1}^{N} A_{kj}(x_j)} \tag{2}$$

where $x = [x_1, .., x_N]^T$ is the input vector, $A_{i1}, ...A_{iN}$ are fuzzy sets defined in the antecedent space and M is the number of rules of the fuzzy model. The most important rules are those whose singular values are higher.

The QRP approach can produce a rule ordering without an rank estimation. Here, QRP is directly applied to P, obtaining a permutation matrix [20]: The QR decomposition of P is given by $P * \Pi = Q * R$, where $\Pi \in \Re^{M*M}$ is a permutation matrix, $Q \in \Re^{N*M}$ has orthogonal columns and $R \in \Re^{M*M}$ is upper triangular, such that

$$R = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{kk} \end{bmatrix} \tag{3}$$

The diagonal values of *R* are called $R-values$ ($|R(kk)|$) [17], which track the singular values $\sigma(P)$, so the most active and least redundant rules are those whose R-values are higher [19] in the original fuzzy rule space.

III. Rule Selection Based on QRP Transformations and Genetic Algorithm: a $Criterion$

The objective of this work is to try to check the orthogonal transformations, in this case QRP. To reach this goal, a simple $Criterion$ is defined in order to address the post-processing rule selection based on the relevance of each rule, then some conclusions about orthogonal transformations can be reached. In this case QRP and its R-values, in decreasing order, have been considered for this approach.

The main problem in the use of orthogonal transformations for rule ordering and selection is the selection $Criterion$. The goal of this work is to select rules automatically based on the percentage of R-values associated with the selected rules. This selection must be guided by a good trade-off between accuracy and interpretability.

A. Rule Selection: a $Criterion$

In order to address the post-processing rule selection, with the aim of reducing the complexity of the fuzzy model while preserving most of its accuracy, the next $Criterion$ has been proposed:

Given a fuzzy rule model whose complexity is to be reduced by rule selection, the rules to be taken into consideration in the new fuzzy model are those rules associated to R-values whose aggregation is greater than a threshold value, β. In this way, most of the behavior (and variability) of the rules is theoretically preserved:

Given a set of n fuzzy rules, each one associated with an R-value, and given R_{norm_i} the normalized R-value of the rule i, such that

$$R_{norm_i} = \frac{R - values_{Rule_i}}{\sum_{j=1}^{j=n} R - values_{Rule_j}}$$

then

$$Rule_1, .., Rule_k \in ReducedModel$$

if

$$\sum_{i=1}^{i=k} R_{norm_i} \geq \beta, \quad k \leq n$$

where

$$\beta = \textit{Percentage of information (rules)}$$

B. Accuracy and Interpretability Measures

The accuracy and interpretability measures considered in this work are defined in [21]. The $accuracy$ of the model is measured through its Mean Squared Error (MSE) (Eq. 4):

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (Y_i - Y_i')^2 \tag{4}$$

The $interpretability$ measure is a set of simple indexes based on similarity and complexity. In both cases, a lower value has a positive influence on reducing the complexity and improving the interpretability of the fuzzy models. These indexes are:

- *Compactness* or *Number of rules*.
- *Similarity* amongst rules.
- *Redundancy* of the fuzzy rule set.
- *Incoherency* of rules.
- *Completeness* or *No-Coverage*.

¿From these indexes, the function to measure the interpretability is (Eq. 5)

$$\begin{aligned}
&Inter_C = ArithmeticMean(\\
&\quad \lambda_{nr} * RuleNumber_{nor}, \lambda_s * Similarity_{nor}, \\
&\quad \lambda_r * Redundancy_{nor}, \lambda_i * Incoherency_{nor}, \\
&\quad \lambda_{nc} * NoCoverage_{nor}) \\
&\lambda_j \in (0, 1)
\end{aligned} \tag{5}$$

Here, λ_j = 1 because it gives equal weight to all indexes, and the normalization is (Eq. 6):

$$Index_{nor} = \frac{Index_{Current} - Index_{Original}}{Index_{Current}} \tag{6}$$

C. Genetic Algorithm and Neuro-Fuzzy Systems

A genetic algorithm and a neuro-fuzzy system are used in this work to check and analyze the proposal.

The well-known multi-objective evolutionary algorithm NSGA-II [22] is taken into account in this work. Two fitness functions from MSE and $Inter_C$ are used to reach a fuzzy model with a good accuracy-interpretability trade-off. A third fitness is used to penalize smaller R-values corresponding to less active and more redundant rules, such that,

$$Penalty_{R-values} = \sqrt[n]{\prod_{i=1}^{n} 1 - R_{norm_i}} \tag{7}$$

and a restriction is imposed to ensure an adequate percentage of information (rules).

Neuro-Fuzzy systems are very popular approaches to generate fuzzy rule based systems, taking advantage of the learning capacity of Artificial Neural Networks (ANN) and the explanatory capacity of Fuzzy Logic. In this work, the neuro-fuzzy system FasArt [23], [24] has been used, which is a neuro fuzzy system based on the Adaptive Resonance Theory (ART). FasArt introduces an equivalence between the activation function of each FasArt neuron and a membership function. In this way, FasArt is equivalent to a Mamdani fuzzy rule-based system with: Fuzzification by single point, Inference by product, and Defuzzification by average of fuzzy set centers. A full description of this model can be found in [23] and [24].

IV. TUNING THE $Criterion$: METHODOLOGY

The methodology of this work is focused on tuning the *Criterion of selection* proposed in III-A and its parameter (β). To reach this goal, the methodology used is made up of four general steps:

1) *Initial model generation.*
   For each data set to be considered, two cases of fuzzy models are generated by FasArt: compact and complex.
2) *Tuning of the $Criterion$, β parameter, by MOEA for each fuzzy model*
   This is based on the accuracy-interpretability trade-off and the R-values associated of the fuzzy rules.
   The β parameter is tuned for each fuzzy model data set, in three representative points of the Pareto Front ([6]):
   - The most interpretable model: Best $Inter_C$ (BI).
   - The most accurate model: Best Acc (BA).
   - The balanced interpretability-accuracy model: Balanced $Inter_C - Acc$ (BIA).

   Here, the multi-objetive genetic algorithm uses the three fitness functions of Eq. 8 and the restriction of Eq. 9:

$$\begin{aligned}
&max(Accuracy) = min(MSE) \\
&max(Interpretability) = min(Inter_C) \\
&max(Penalty) = max(Penalty_{R-values})
\end{aligned} \tag{8}$$

$$\beta \in [60\% - 85\%] \tag{9}$$

   The fitness functions involve the accuracy-interpretability trade-off and penalize smaller R-values, which correspond to less active and more redundant rules. The restriction is imposed to preserve an adequate percentage of information (rules). In this work, a first run of the MOEA with the two first functions was carried out in order to estimate the restriction range of Eq. 9.
3) *Generalization of $Criterion$: Generation of the β parameter from $\beta_i's$.*
   Taking into consideration the particular tuning obtained for each fuzzy model, a general range of β is carried out for cross-validation:

for $Criterion = 1$ **do**
  for $Model = Compact{:}Complex$ **do**
    for Accur-Interpret Point=$BI{:}BA{:}BIA$ **do**
      for $N\ datasets$ **do**
        All possible combinations of 2/3 of data set, each one with its β ($\beta_i's$), are done: β's training.
        For each combination, the mean and median are done, obtaining new β's (2*num-combination).
        MSE and $Inter_C$ are calculated with these new β's, for 2/3 and the remaining 1/3: β's test.
        The mean of MSE and $Inter_C$ are done.
        The best accuracy ($min(MSE)$) and the best interpretability ($min(Inter_C)$) are selected and each one has a β associated obtaining two c.
        The final β is obtained to find the mean/median of these two β's. This midpoint gives a good accuracy-interpretability trade-off .
      end for
    end for
  end for
end for

TABLE I
DATA SETS FROM KEEL PROJECT

data sets	Variables	Records
Plastic Strength (PLA)	3	1650
Quake (QUA)	4	2178
Electrical Maintenance (ELE)	5	1056
Abalone (ABA)	9	4177
Stock prices (STP)	10	950
Weather Ankara (WAN)	10	1609
Weather Izmir (WIZ)	10	1461
Mortgage (MOR)	16	1049
Treasury (TRE)	16	1049

4) *Checking the* $Criterion$*: Accuracy and interpretability based on the* $Criterion$ *and its parameter* β.

```
for Criterion = 1 do
   for Model = Compact:Complex do
      for Accur-Interpret Point=BI:BA:BIA do
         for datasets = 1 to N do
            Calculation of MSE and Inter_C
         end for
      end for
   end for
   Analysis of results
end for
```

V. EXPERIMENTS: RESULTS AND ANALYSIS

The proposal described in previous sections is checked using nine data sets from the KEEL project [1] (Table I) [25], [26].

In accordance with the methodology described in section IV:

1) Two types of base fuzzy models are generated by FasArt for each data set: Model1-Compact and Model2-Complex.
2) MOEA has been run using several options and the best results (based on best MSE and $Inter_C$) have been obtained combining the three fitness functions formulated in Eq. 8 with the restriction of Eq. 9.
   In this work, all the $genetic\ operators$ are used in their default options:
   - Tournament without replacement for selection. Tournament size $= 2$.
   - Simulated binary for crossover. Crossover probability $= 0.9$.
   - Polynomial for mutation. Mutation probability $=$ 0.1.

   From here, the β's obtained for MOEA for each data set are shown in Table II.
   These β's of Table II are used to tune a general β by cross validation, as described in point 3 of section IV, then the best accuracy and interpretability are obtained for the β's shown in Table III.
   The final value of β, so the final tuning of the $Criterion$, is obtained by taking the mean/median of these two β's in order to obtain a compromise between the best accuracy point and the best interpretability point. The final β's are shown in Table IV.
3) MSE and $Inter_C$ are calculated for nine data sets, selecting the percentage of R-values corresponding to the final β's (Table IV).

[1] http://sci2s.ugr.es/keel/data setss.php

TABLE II
PERCENTAGE OF β OBTAINED FOR MOEA WITH THREE FITNESS FUNCTIONS AND RESTRICTION $\beta \in [60\% - 85\%]$ (%)

Model1	Best $Inter_C$	Balanced $Inter_C - Acc$	Best Acc
PLA1	63.6	68.7	70.9
QUA1	61.8	63.7	68.2
ELE1	79.1	75.3	84.0
ABA1	78.0	68.7	74.4
STP1	72.5	76.7	82.4
WIZ1	67.3	66.2	74.8
WAN1	64.3	67.8	68.8
MOR1	69.8	74.0	78.8
TRE1	68.7	72.6	78.4
Model2	$BestInter_C$	$BalancedInter_C - Acc$	$BestAcc$
PLA2	69.0	68.7	70.5
QUA2	64.5	64.1	63.1
ELE2	66.3	72.0	81.0
ABA2	63.1	63.0	66.0
STP2	74.8	72.3	72.8
WIZ2	64.6	66.7	67.8
WAN2	61.6	63.4	63.9
MOR2	74.0	74.5	76.2
TRE2	68.8	74.1	79.0

TABLE III
PERCENTAGE OF β OBTAINED FOR THE BEST MSE AND THE BEST $Inter_C$ (%)

Model1	Best $Inter_C$	Balanced $Inter_C - Acc$	Best Acc
Best MSE	72.1	73.3	78.8
Best $Inter_C$	65.7	68.0	72.6
Model2	Best $Inter_C$	Balanced $Inter_C - Acc$	Best Acc
Best MSE	69.6	72.2	74.6
Best $Inter_C$	64.6	65.4	66.9

TABLE IV
FINAL β'S (%)

	Best $Inter_C$	Balanced $Inter_C - Acc$	Best Acc
Model1	68.9	70.7	75.7
Model2	67.1	68.8	70.8

To see the results obtained for MSE and $Inter_C$ with their indexes of accuracy and interpretability, each study case is done with the new β's and the results are shown in the following subsections.

A. Performance of the Fuzzy Models

Table V summarizes the main performance aspects of the base fuzzy models generated. The indexes shown in the tables are the squared error for training (MSE_{tra}) and the test (MSE_{tst}), the rule number (RN), the similarity (S), the redundancy (R), the incoherency (I) and the percentage of completeness (C).

The performance of these base fuzzy models are matched with Wang & Mendel Models in [6]. In general, the complex models show a higher accuracy, while the compact models have a similar performance in some cases, and in others are a little more accurate or a little worse (see Table VI). The highlighted values mean that the performance (NR/MSE) in the FasArt models is greater than in Wang & Mendel.

TABLE V
PERFORMANCE OF THE FASART FUZZY MODELS

Model	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA1	6.553	6.553	14	0.204	0	0	100
PLA2	2.498	2.498	143	0.144	0.001	0.008	100
QUA1	0.071	0.071	20	0.225	0	0.005	90.4
QUA2	0.040	0.040	310	0.255	2e-4	0.002	100
ELE1	158937	158938	25	0.212	0	0.007	100
ELE2	55102	55103	145	0.291	0.008	0.001	97.6
ABA1	8.861	8.861	36	0.342	0	0.005	100
ABA2	5.176	5.176	305	0.330	4e-4	0.001	100
STP1	3.948	3.948	15	0.266	0	0	100
STP2	0.432	0.432	165	0.187	0	3e-4	100
WIZ1	15.208	15.208	26	0.317	0	0	100
WIZ2	3.737	3.737	118	0.377	0	0	100
WAN1	19.516	19.514	32	0.285	0	0	100
WAN2	4.060	4.044	391	0.358	0	0	100
MOR1	0.146	0.146	30	0.338	0	0	100
MOR2	0.071	0.071	101	0.290	4e-4	4e-4	100
TRE1	0.234	0.234	23	0.306	0	0	100
TRE2	0.138	0.138	62	0.288	0	0	100

TABLE VI
WANG & MENDEL [6] VS NEURO-FUZZY SYSTEMS.

Compact	Wang & Mendel [27]			FasArt (Model 1)		
Models	RN	MSE_{tra}	MSE_{tst}	RN	MSE_{tra}	MSE_{tst}
PLA	14.8	6.868	7.114	14	6.553	6.553
QUA	53.6	0.0516	0.0534	20	**0.071**	**0.071**
ELE	65	115212	115868	25	**158937**	**158938**
ABA	68	16.814	16.844	36	8.861	8.861
STP	122.8	18.148	18.084	15	3.948	3.948
WIZ	104.8	13.888	14.736	26	**15.208**	**15.208**
WAN	156	32.126	32.786	32	19.516	19.514
MOR	77.6	1.97	1.946	30	0.146	0.146
TRE	75	3.272	3.262	23	0.234	0.234
Complex	**Wang & Mendel [6]**			**FasArt (Model 2)**		
Models	NR	MSE_{tra}	MSE_{tst}	NR	MSE_{tra}	MSE_{tst}
PLA	14.8	6.868	7.114	**143**	2.498	2.498
QUA	53.6	0.0516	0.0534	**310**	0.040	0.040
ELE	65	115212	115868	**145**	55102	55103
ABA	68	16.814	16.844	**305**	5.176	5.176
STP	122.8	18.148	18.084	**165**	0.432	0.432
WIZ	104.8	13.888	14.736	**118**	3.737	3.737
WAN	156	32.126	32.786	**391**	4.060	4.044
MOR	77.6	1.97	1.946	**101**	0.071	0.071
TRE	75	3.272	3.262	62	0.138	0.138

Thus, these fuzzy models can be simplified in order to reach better and less complex fuzzy models using a more adequate accuracy-interpretability trade-off based on this proposal.

B. Reducing the complexity of the Fuzzy Models: Results and Analysis

The analysis of the results is organized according to the compact base fuzzy models and the complex base fuzzy models. In both cases, the values of MSE and $Inter_C$ with their indexes are presented.

1) Compact Models: Table VII shows the averaged results obtained by the characteristic models considered in this work over 5 runs for each case study considered. Specifically, the table shows the mean squared error for the test, MSE_{tst}, and the mean of the proposed index $Inter_C$, for each one of the three characteristic models taken into account: BI, BIA and BA. The first line shows the initial/original model (I) and the second line shows the improved model performance (F).

Tables VIII, IX and X show the mean values of some individual indexes for the final β's: the mean squared error for training (MSE_{tra}) and testing (MSE_{tst}), the mean rule number (RN), the mean similarity (S), the mean redundancy

TABLE VII
PERFORMANCE OF THE IMPROVED COMPACT FUZZY MODELS

	Best $Inter_C$		Balanced $Inter_C - Acc$		Best Acc	
Model1	MSE_{tst}	$Inter_C$	MSE_{tst}	$Inter_C$	MSE_{tst}	$Inter_C$
PLA1(I)	6.553	0.241	6.553	0.241	6.553	0.241
PLA1(F)	12.589	0.152	12.589	0.152	12.256	0.155
QUA1(I)	0.071	0.265	0.071	0.265	0.071	0.265
QUA1(F)	0.051	0.188	0.044	0.190	0.045	0.199
ELE1(I)	158938	0.244	158938	0.244	158938	0.244
ELE1(F)	992076	0.132	749610	0.137	641352	0.152
ABA1(I)	8.861	0.269	8.861	0.269	8.861	0.269
ABA1(F)	10.916	0.190	9.521	0.194	8.070	0.202
STP1(I)	3.948	0.253	3.948	0.253	3.948	0.253
STP1(F)	14.162	0.153	13.538	0.156	13.538	0.156
WIZ1(I)	15.208	0.263	15.208	0.263	15.208	0.263
WIZ1(F)	103.700	0.173	100.041	0.177	88.800	0.182
WAN1(I)	19.514	0.257	19.514	0.257	19.514	0.257
WAN1(F)	60.108	0.172	57.658	0.176	27.359	0.181
MOR1(I)	0.146	0.268	0.146	0.268	0.146	0.268
MOR1(F)	5.045	0.162	4.106	0.161	3.707	0.167
TRE1(I)	0.234	0.261	0.234	0.261	0.234	0.261
TRE1(F)	3.987	0.135	3.839	0.137	3.288	0.149

TABLE VIII
RESULTS OBTAINED WITH COMPACT FASART BEST $Inter_C$ β=68.9 %

Model1	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA1(I)	6.553	6.553	14.00	0.204	0.000	0.000	100
PLA1(F)	12.586	12.589	8.00	0.187	-	-	100
QUA1(I)	0.071	0.071	20.00	0.225	0.000	0.005	90.4
QUA1(F)	0.051	0.051	7.80	0.365	-	0.000	81.7
ELE1(I)	158937	158938	25.00	0.212	0.000	0.007	100
ELE1(F)	861883	992077	11.00	0.213	-	0.000	99.1
ABA1(I)	8.861	8.861	36.00	0.342	0.000	0.005	100
ABA1(F)	10.944	10.916	15.40	0.411	-	0.009	89.8
STP1(I)	3.948	3.948	15.00	0.266	0.000	0.000	100
STP1(F)	13.787	14.162	5.80	0.280	-	-	90.0
WIZ1(I)	15.208	15.208	26.00	0.317	0.000	0.000	100
WIZ1(F)	101.24	103.70	10.20	0.333	-	-	85.8
WAN1(I)	19.516	19.514	32.00	0.285	0.000	0.000	100
WAN1(F)	60.101	60.108	11.60	0.371	-	-	87.4
MOR1(I)	0.146	0.146	30.00	0.338	0.000	0.000	100
MOR1(F)	4.993	5.045	9.00	0.368	-	-	86.0
TRE1(I)	0.234	0.234	23.00	0.306	0.000	0.000	100
TRE1(F)	3.829	3.987	6.80	0.291	-	-	91.1

TABLE IX
RESULTS OBTAINED WITH COMPACT FASART BALANCED $Inter_C - Acc$
β=70.7 %

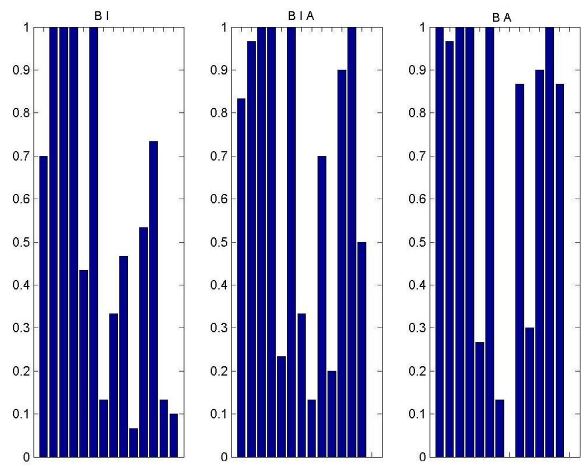
Model1	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA1(I)	6.553	6.553	14.00	0.204	0.000	0.000	100
PLA1(F)	12.586	12.589	8.00	0.187	-	-	100
QUA1(I)	0.071	0.071	20.00	0.225	0.000	0.005	90.4
QUA1(F)	0.044	0.044	8.00	0.368	-	0.000	81.7
ELE1(I)	158937	158938	25.00	0.212	0.000	0.007	100
ELE1(F)	700522	749610	11.80	0.208	-	0.000	99.5
ABA1(I)	8.861	8.861	36.00	0.342	0.000	0.005	100
ABA1(F)	9.521	9.521	16.00	0.414	-	0.008	89.8
STP1(I)	3.948	3.948	15.00	0.266	0.000	0.000	100
STP1(F)	13.359	13.538	6.00	0.285	-	-	90.4
WIZ1(I)	15.208	15.208	26.00	0.317	0.000	0.000	100
WIZ1(F)	97.331	100.041	10.60	0.334	-	-	85.8
WAN1(I)	19.516	19.514	32.00	0.285	0.000	0.000	100
WAN1(F)	57.762	57.658	12.40	0.372	-	-	87.8
MOR1(I)	0.146	0.146	30.00	0.338	0.000	0.000	100
MOR1(F)	4.090	4.106	10.00	0.369	-	-	89.5
TRE1(I)	0.234	0.234	23.00	0.306	0.000	0.000	100
TRE1(F)	3.735	3.839	7.00	0.289	-	-	91.1

(R), the mean incoherency (I) and the mean percentage of completeness ($C\%$).

These tables show that the interpretability is always improved, while the accuracy is only improved sometimes. This is because the selection of rules with orthogonal transformations is done by removing the rules whose R-values are smaller, but

TABLE X
RESULTS OBTAINED WITH COMPACT FASART BEST Acc β=75.7 %

Model1	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA1(I)	6.553	6.553	14.00	0.204	0.000	0.000	100
PLA1(F)	12.230	12.256	8.20	0.188	-	-	100
QUA1(I)	0.071	0.071	20.00	0.225	0.000	0.005	90.4
QUA1(F)	0.045	0.045	9.00	0.366	-	0.000	82.0
ELE1(I)	158937	158938	25.00	0.212	0.000	0.007	100
ELE1(F)	561249	641352	13.60	0.211	-	0.000	99.7
ABA1(I)	8.861	8.861	36.00	0.342	0.000	0.005	100
ABA1(F)	8.202	8.070	18.00	0.401	-	0.007	89.8
STP1(I)	3.948	3.948	15.00	0.266	0.000	0.000	100
STP1(F)	13.359	13.538	6.00	0.285	-	-	90.4
WIZ1(I)	15.208	15.208	26.00	0.317	0.000	0.000	100
WIZ1(F)	87.841	88.800	12.20	0.323	-	-	88.3
WAN1(I)	19.516	19.514	32.00	0.285	0.000	0.000	100
WAN1(F)	27.809	27.359	14.20	0.368	-	-	90.9
MOR1(I)	0.146	0.146	30.00	0.338	0.000	0.000	100
MOR1(F)	3.594	3.707	11.00	0.373	-	-	90.4
TRE1(I)	0.234	0.234	23.00	0.306	0.000	0.000	100
TRE1(F)	3.147	3.288	8.20	0.301	-	-	91.1



Rule ordering x-axis: [10 1 9 3 2 7 14 11 5 8 12 4 6 13]

Fig. 1. Selected Rules for MOEA in PLA1

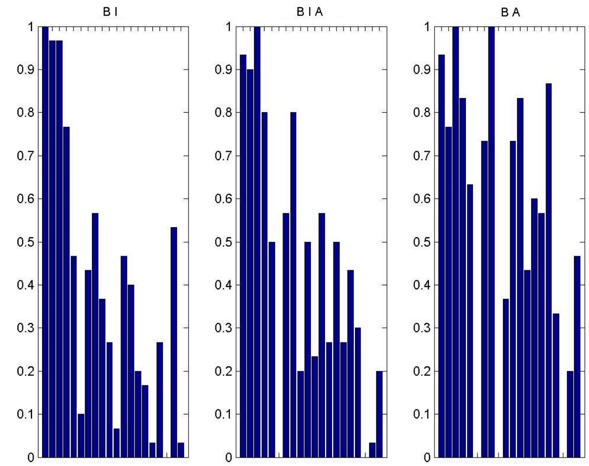
these R-values provide accuracy to the system, as can be seen in Figure 1.

This table shows the rules selected by MOEA in the case of a data set (in this example PLA1 with 14 rules) for the three points (BI, BA and BIA). In the x-axis are the rules from highest to lowest importance (from left to right), as the R-values, and the vertical axis is the percentage of times that MOEA selects each rule. Thus, 0.5 means that a rule has been selected for 50% of the time.

Figures 2, 3 and 4 show similar selections for Model1 with other data sets. In all cases, it can be seen that the rules associated to small R-values are selected, even in the best Acc point.

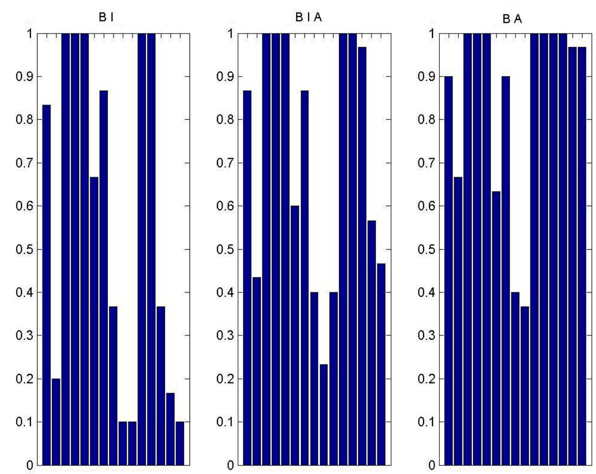
2) Complex Models: Table XI shows the averaged results obtained by the characteristic models considered in this work over 5 runs for each case study considered. As compact models, the table shows the mean squared error for the test, MSE_{tst}, and the mean of the proposed index $Inter_C$, for each one of the three points: BI, BIA and BA.

The first line shows the initial/original model (I) and the second line shows the improved model performance (F) for



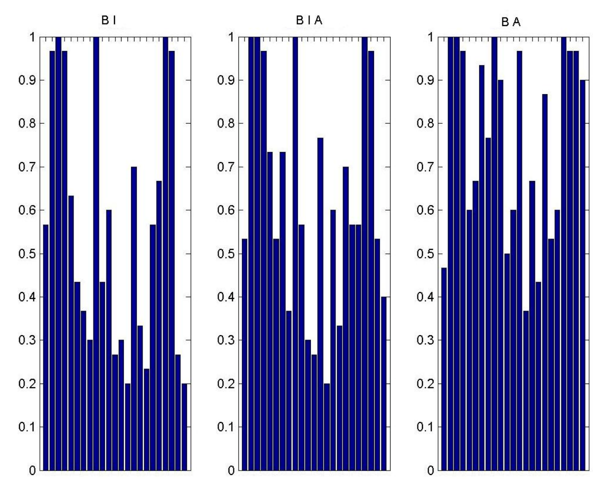
Rule ordering x-axis: [6 9 4 1 3 20 5 7 19 16 11 12 10 8 14 15 2 18 17 13]

Fig. 2. Selected Rules for MOEA in QUA1



Rule ordering x-axis: [4 9 7 11 5 3 2 1 15 14 6 8 10 12 13]

Fig. 3. Selected Rules for MOEA in STP1



Rule ordering x-axis: [17 19 10 2 13 11 22 23 8 18 20 14 15 6 9 5 12 3 4 7 1 21 16]

Fig. 4. Selected Rules for MOEA in TRE1

TABLE XI
PERFORMANCE OF THE IMPROVED COMPLEX FUZZY MODELS

	Best $Inter_C$		Balanced $Inter_C - Acc$		Best Acc	
Model2	MSE_{tst}	$Inter_C$	MSE_{tst}	$Inter_C$	MSE_{tst}	$Inter_C$
PLA2(I)	2.498	0.231	2.498	0.231	2.498	0.231
PLA2(F)	8.471	0.081	8.355	0.083	8.391	0.088
QUA2(I)	0.040	0.251	0.040	0.251	0.040	0.251
QUA2(F)	0.051	0.115	0.045	0.118	0.039	0.121
ELE2(I)	55103	0.265	55103	0.265	55103	0.265
ELE2(F)	1079317	0.100	1063736	0.103	1035999	0.123
ABA2(I)	5.176	0.266	5.176	0.266	5.176	0.266
ABA2(F)	6.669	0.157	6.686	0.161	6.715	0.166
STP2(I)	0.432	0.238	0.432	0.238	0.432	0.238
STP2(F)	12.957	0.131	12.529	0.132	11.793	0.135
WIZ2(I)	3.737	0.275	3.737	0.275	3.737	0.275
WIZ2(F)	18.279	0.175	17.644	0.177	17.326	0.180
WAN2(I)	4.044	0.272	4.044	0.272	4.044	0.272
WAN2(F)	9.992	0.147	9.866	0.146	9.298	0.149
MOR2(I)	0.071	0.258	0.071	0.258	0.071	0.258
MOR2(F)	3.622	0.146	3.458	0.149	3.311	0.153
TRE2(I)	0.138	0.258	0.138	0.258	0.138	0.258
TRE2(F)	3.916	0.158	3.713	0.158	3.138	0.161

TABLE XII
RESULTS OBTAINED WITH COMPLEX FASART BEST $Inter_C$ β=67.1 %

Model2	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA2(I)	2.498	2.498	143.00	0.144	0.001	0.008	100
PLA2(F)	8.538	8.471	38.20	0.137	0.001	0.000	100
QUA2(I)	0.040	0.040	310.00	0.255	0.000	0.002	100
QUA2(F)	0.050	0.051	89.40	0.260	0.000	0.002	97.5
ELE2(I)	55102	55103	145.00	0.291	0.008	0.001	97.6
ELE2(F)	910201	1079317	39.20	0.188	0.011	0.001	97.2
ABA2(I)	5.176	5.176	305.00	0.330	0.000	0.001	100
ABA2(F)	6.570	6.669	120.40	0.339	0.001	0.001	95.0
STP2(I)	0.432	0.432	165.00	0.187	0.000	0.000	100
STP2(F)	12.654	12.957	55.20	0.254	-	0.002	93.6
WIZ2(I)	3.737	3.737	118.00	0.377	0.000	0.000	100
WIZ2(F)	18.037	18.279	36.60	0.424	-	-	86.0
WAN2(I)	4.060	4.044	391.00	0.358	0.000	0.000	100
WAN2(F)	8.575	9.992	134.00	0.357	-	-	96.4
MOR2(I)	0.071	0.071	101.00	0.290	0.000	0.000	100
MOR2(F)	3.090	3.622	37.40	0.320	0.001	0.000	96.2
TRE2(I)	0.138	0.138	62.00	0.288	0.000	0.000	100
TRE2(F)	4.054	3.916	22.80	0.321	-	-	90.0

TABLE XIII
RESULTS OBTAINED WITH COMPLEX FASART BALANCED $Inter_C - Acc$ β=68.8 %

Model2	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA2(I)	2.498	2.498	143.00	0.144	0.001	0.008	100
PLA2(F)	8.418	8.355	40.00	0.136	0.001	0.000	100
QUA2(I)	0.040	0.040	310.00	0.255	0.000	0.002	100
QUA2(F)	0.049	0.045	93.80	0.259	0.000	0.001	97.5
ELE2(I)	551024	55103	145.00	0.291	0.008	0.001	97.6
ELE2(F)	859980	10637365	41.20	0.192	0.012	0.001	97.2
ABA2(I)	5.176	5.176	305.00	0.330	0.000	0.001	100
ABA2(F)	6.569	6.686	125.80	0.339	0.001	0.001	95.0
STP2(I)	0.432	0.432	165.00	0.187	0.000	0.000	100
STP2(F)	11.898	12.529	57.20	0.252	-	0.002	93.9
WIZ2(I)	3.737	3.737	118.00	0.377	0.000	0.000	100
WIZ2(F)	17.303	17.644	38.40	0.425	-	-	86.4
WAN2(I)	4.060	4.044	391.00	0.358	0.000	0.000	100
WAN2(F)	7.894	9.866	139.60	0.352	-	-	97.9
MOR2(I)	0.071	0.071	101.00	0.290	0.000	0.000	100
MOR2(F)	2.994	3.458	39.00	0.322	0.001	0.000	96.3
TRE2(I)	0.138	0.138	62.00	0.288	0.000	0.000	100
TRE2(F)	3.780	3.713	23.80	0.317	-	-	91.0

TABLE XIV
RESULTS OBTAINED WITH COMPLEX FASART BEST Acc β=70.8 %

Model2	MSE_{tra}	MSE_{tst}	RN	S	R	I	$C(\%)$
PLA2(I)	2.498	2.498	143.00	0.144	0.001	0.008	100
PLA2(F)	8.362	8.391	42.20	0.134	0.001	0.000	100
QUA2(I)	0.040	0.040	310.00	0.255	0.000	0.002	100
QUA2(F)	0.040	0.039	99.40	0.258	0.000	0.001	97.5
ELE2(I)	55102	55103	145.00	0.291	0.008	0.001	97.6
ELE2(F)	790187	1035999	43.20	0.189	0.011	0.002	97.2
ABA2(I)	5.176	5.176	305.00	0.330	0.000	0.001	100
ABA2(F)	6.576	6.715	132.80	0.336	0.001	0.001	95.4
STP2(I)	0.432	0.432	165.00	0.187	0.000	0.000	100
STP2(F)	11.003	11.793	59.80	0.254	-	0.002	94.2
WIZ2(I)	3.737	3.737	118.00	0.377	0.000	0.000	100
WIZ2(F)	16.882	17.326	40.40	0.421	-	-	86.5
WAN2(I)	4.060	4.044	391.00	0.358	0.000	0.000	100
WAN2(F)	7.379	9.298	146.40	0.349	-	-	98.1
MOR2(I)	0.071	0.071	101.00	0.290	0.000	0.000	100
MOR2(F)	2.808	3.311	41.00	0.319	0.001	0.000	96.3
TRE2(I)	0.138	0.138	62.00	0.288	0.000	0.000	100
TRE2(F)	3.194	3.138	24.80	0.316	-	-	91.2

these three characteristic models.

Tables XII, XIII, XIV show the mean and individual values for some indexes considered for the final β's: the mean squared error for training (MSE_{tra}) and testing (MSE_{tst}), the mean rule number (RN), the mean similarity (S), the mean redundancy (R), the mean incoherency (I) and the mean percentage of completeness ($C\%$).

These tables show that the interpretability is always improved, while the accuracy is only improved for one data set and for the best Acc point. This is normal because complex models have an original high precision.

However, the accuracy is also reduced by the rule selection, and this is connected with orthogonal transformations: the firing matrix used to obtain the R-values only takes into account the antecedents of the rules. Thus, there are important rules for the accuracy of the system which are removed. These rules can correspond to small R-values and Figures 1, 2, 3 and 4 show these drawbacks: rules with small R-values are selected by the MOEA to provide accuracy and to obtain a good interpretability-accuracy trade-off.

3) Global Analysis: In general, the results show the advantages and some of the drawbacks obtained when orthogonal transformation are applied to the firing matrix of the fuzzy rule based model: a high reduction of the complexity, so the levels of interpretability under fuzzy criteria are increased (see Tables VII and XI).

However, on the other hand, the model accuracy is decreased in most of the cases. This reduction of the accuracy can be highly relevant due to the hard simplification achieved by orthogonal transformations. This happens, even taking into consideration the most relevant rules based on their $R{-}values$. The cause for this is: some of the rules removed correspond to the smaller R-values, or those near to zero, and these R-values correspond to non active and redundant rules. Redundant rules, however, can significantly contribute to the output system.

This can be shown by looking at the selection of rules that MOEA carries out according to the restrictions commented in point 2 of section IV: although smaller R-values are penalized, the algorithm continues to select some of the small R-values, to provide the system with accuracy (see Figures 1 to 4).

VI. CONCLUSIONS

This work deals with the complexity reduction of fuzzy models via Orthogonal Transformations, QRP: The goal is to check the limitations of the traditional application of this type of transformation to the firing matrix of a fuzzy model. In order to check this, a $Criterion$ for fuzzy rule selection has been defined based on R-values, and its parameter has been tuned by a MOEA. This tuning has been carried out taking into consideration the accuracy-interpretability trade-off.

In all cases, interpretability has been improved considerably. The number of rules is always reduced, to 70% in some cases, and the rest of the indexes defined to measure the interpretability are also reduced in most cases.

However, in most cases, there is a loss of accuracy. This is due to the fact that orthogonal transformations for rule reduction are usually based on a firing matrix (P) involving the rule antecedent [19], [12]. The rules associated with the smaller R-values are removed because they are considered not relevant, but these rules usually provide a better accuracy.

In this work, the R-values for rule selection are obtained from the firing matrix (P). The results show that this matrix is not sufficient for this selection of rules because the rules associated with the small R-values are removed, and some of these rules can be important to obtain a good accuracy.

Future work is focused on a more realistic *"firing matrix"*, or index-values for rule selection, considering other elements of the fuzzy models such as the inference mechanism or consequents.

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