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# Selection of Rules by Orthogonal Transformations and Genetic Algorithms to Improve the Interpretability in Fuzzy Rule Based Systems

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***Abstract*—Fuzzy modeling is one of the best known techniques to model systems and processes. In most cases, as in data-driven fuzzy modeling, these fuzzy models reach a high accuracy, but show poor performance in complexity or interpretability, which are key aspects of Fuzzy Logic.**

**There are several approaches in the literature to deal with the complexity and interpretability challenges for fuzzy rule based systems (FRBSs). In this paper, a post-processing approach is proposed via a genetic rule selection based on the relevance of each rule (using Orthogonal Transformations (OTs), in this case P-QR) and the well-known accuracy-interpretability trade-off. The main objective is to check the true significance, drawbacks and advantages of the rule selection based on OTs to manage the accuracy-interpretability trade-off.**

**In order to achieve this aim, a neuro-fuzzy system (FasArt-Fuzzy Adaptive System ART based) and several case studies from the KEEL Project Repository are used to tune and check this selection of rules based on rule relevance by OTs, genetic selection and accuracy-interpretability trade-off. This neuro-fuzzy system generates Mamdani FRBSs, in an approximate way. SPEA2 is the multi-objective evolutionary algorithm (MOEA) tool used to tune the proposed rule selection, and different interpretability measures have been considered.**

***Keywords*—FRBS, Orthogonal Transformations, Interpretability, Genetic Algorithm.**

## I. Introduction

Fuzzy systems are one usual way to apply of the Fuzzy Set Theory, frequently using a model structure in the form of FRBSs. FRBSs constitute an extension to classical rule-based systems, whose antecedents and consequents are composed of fuzzy logic statements instead of classical ones. They have demonstrated their ability in very different scientific and technical areas.

In general, most of the FRBSs in real world applications have been used due to their advantages: easy use and performance. This performance has usually been evaluated on the accuracy of the models: minimizing the error between the known and the estimated output by the fuzzy models. But other aspects have not usually been taken into consideration: complexity, interpretability, etc, despite being some of the principles of fuzzy logic.

Complexity is a usual measure for FRBSs related with their interpretability. Thus, if a reduction of this complexity was reached, it could permit a better performance on interpretability. The question lies in how this complexity reduction can be carried out. Different approaches to this question can be found in [1], [2], [3], [4], and one review has been made in [5].

On the other hand, this work is devoted to complexity reduction based on the selection of rules by OTs and accuracy-interpretability trade-off using a genetic algorithm. OTs [6] have been an alternative approach for complexity reduction and interpretability improvement of fuzzy models [3], [7], [8], [9]. This approach applies OTs on the *firing strength matrix* of the fuzzy model rules as a regression problem in order to estimate the relevance of the rules, then a rule selection is carried out.

In this context, this work checks the possibilities and drawbacks of the OTs as a postprocessing approach to reduce complexity and get more interpretable scatter FRBSs. Thus, a scatter FRBS is involved with rule selection by a genetic approach subject to the rule relevance and the accuracy-interpretability trade-off, taking into account different interpretability measures.

The paper is organized as follows: first, in Section II, a brief description about interpretability and the main concepts of OTs are given. The proposal of genetic rule selection based on OTs and accuracy-interpretability trade-off is introduced in Section III. The methodology used in this work is described in Section IV, while the experimental studies are carried out and the main results obtained are discussed in Section V. Finally, in Section VI, the most interesting conclusions obtained are set out.

## II. Interpretability and Orthogonal Transformations

### A. *Interpretability: Taxonomy*

The interpretability of FRBSs is the capacity to express the behavior of the real system in an understandable way. This is a subjective property that is related to several factors, mainly the model structure, the number of input variables, the number of fuzzy rules, the number of linguistic terms, the shape of the

TABLE I. A TAXONOMY TO ANALYZE INTERPRETABILITY [5]

| | Rule Base level | Fuzzy partition level |
|---|---|---|
| Complexity-based interpretability | $Q1$<br>number of rules<br>number of conditions | $Q2$<br>number or membership functions<br>number of features |
| Semantic-based interpretability | $Q3$<br>consistency of rules<br>rules fired at the same time<br>transparency of rule structure<br>cointension | $Q4$<br>completeness or coverage<br>normalization<br>distinguishability<br>complementarity<br>relative measures |

fuzzy sets, etc. There is still no standard measure to assess how good interpretability is [2], [4], [5], [10].

In this work the taxonomy considered is [5]. This is based on a double axis: ”complexity versus semantic interpretabilyty” considering the two main kinds of measures; and ”rule base versus fuzzy partitions” considering the different components of the knowledge base (KB) to which both kinds of measures can be applied. There are four different quadrants to be analyzed:

- *Q1*: The complexity at the rule based (RB) level.
- *Q2*: The complexity at the fuzzy partition level.
- *Q3*: The semantics at the RB level.
- *Q4*: The semantics at the fuzzy partition level.

Each $Q_i$ contains several interpretability measures. Some of these measures are shown in Table I.

### B. Complexity Reduction and Orthogonal Transformations

The reduction of the complexity system can imply a better interpretability of the fuzzy system [8], [9]. OTs are used for rule selection/reduction and for reducing complexitity in FRBSs. In this context, an FRBS can be written as a linear regression problem [9] (Eq. 1)

$$y = P * \theta + e \tag{1}$$

where: $y = [y_1, y_2 ..., y_N]^T$ are the measured outputs, $\theta = [c_1, c_2, ..., c_M]^T$ are the consequents of the $M$ rules and $e = [e_1, e_2, ..., e_N]^T$ are the vectors of approximation errors. The matrix $P = [p_1, p_2, ..., p_M] \in R^{N \times M}$ contains the firing strength of all the $M$ rules for the $N$ inputs $x_k$, where $p_i = [p_{i1}, p_{i2}, ..., p_{iN}]^T$.

In this work, the P-QR Decomposition has been considered: this approach produces a rule ordering without a rank estimation. Here, P-QR is directly applied to P, obtaining a permutation matrix [6]:

The QR decomposition of P is given by $P * \Pi = Q * R$, where $\Pi \in \Re^{M*M}$ is a permutation matrix, $Q \in \Re^{N*M}$ has orthogonal columns and $R \in \Re^{M*M}$ is upper triangular (Eq. 2), such that

$$R = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{kk} \end{bmatrix} \tag{2}$$

The diagonal values of *R* are called *R-values* ($|R_{kk}|$) [3], which track the singular values $\sigma(P)$, so the most active and least redundant rules are those whose R-values are higher [9] in the original fuzzy rule space.

In order to address the rule selection in this work, concepts about interpretability, accuracy and their trade-off are used together with the orthogonal transformations.

## III. GENETIC RULE SELECTION BASED ON ORTHOGONAL TRANSFORMATIONS AND ACCURACY-INTERPRETABILITY TRADE-OFF

The main objective of this work is the selection of rules by OTs and accuracy-interpretability trade-off from a multi-objective evolutionary algorithm (MOEA). One well-known OT has been involved in this work: P-QR. The FRBSs are generated by a neuro-fuzzy system, FasArt [11], [12], which is a scatter fuzzy system. Now, in order to generate a better rule selection, this is carried out following the guidance of different points of view concerned with:

- Relevance or influence of each fuzzy rule estimated by the OTs.
- Accuracy-Interpretability trade-off in FRBSs defined by measures on both concepts.

Based on previous concepts, a genetic approach for the rule selection is done. This provides an interesting scenario of results concerning the rule ordering and selection based on these OTs. The study of this scenario will give us a better knowledge of the scope of this selection proposal.

In the following subsections, the accuracy-interpretability measures considered are briefly described. Then, some comments and references on the genetic and neuro-fuzzy approach used in this work are introduced.

### A. Accuracy and Interpretability Measures

The *accuracy* ($Acc$) is the capability to faithfully represent the real system. Here, the accuracy of the model is measured through its Mean Squared Error ($MSE$) (Eq. 3):

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - y'_i)^2 \tag{3}$$

The *interpretability* ($Inter$) is the capacity to express the behavior of the real system in an understandable way. Here, we use some of the proposed interpretability measures in [5] (see table I). In this work, the most used measure of each quadrant have been selected:

- Q1, *Number of Rules (RN)* has been selected as the interpretability measure (Eq. 4):

$$Inter = \mathit{RN} \tag{4}$$

- Q2, *Number of Membership Functions (MFs)* (Eq. 5).

$$Inter = \mathit{Number\ of\ MF} \tag{5}$$

- Q3, *Consistency of Rules* (Eq. 6 and Eq. 7).

$$Cons(R(i), R(k)) = exp - \left( \frac{(\frac{SRP(i,k)}{SRC(i,k)} - 1.0)^2}{(\frac{1}{SRP(i,k)})^2} \right) \tag{6}$$

where SRP is the similarity of the rule premises and SRC is the similarity of the rule consequents.

$$Incons(i) = \sum_{\substack{1 \leq k \leq N \\ k \neq i}} [1.0 - Cons(R^1(i), R^1(k))] + \sum_{\substack{1 \leq l \leq L \\ i=1,2,..,N}} [1.0 - Cons(R^1(i), R^2(l))] \tag{7}$$

where $R^1$ and $R^2$ denote the rule base (RB) generated from the data and the RB extracted from prior knowledge and $N$ and $L$ are the rule numbers of $R^1$ and $R^2$. Then,

$$Inter = Incons \tag{8}$$

- Q4, *Distinguishability* has been selected as an interpretability measure. The most common measure to quantify distinguishability is similarity S [13] (Eq. 9).

$$S(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A|+|B|-|A \cap B|} \tag{9}$$

Then,

$$Inter = Similarity = S(A,B) \tag{10}$$

### B. Genetic Algorithms

The well-known MOEA SPEA2 [14] is taken into account to select a subset of cooperative rules from a set of candidate fuzzy rules, but other MOEAs can also be used [15]. Two fitness functions from $MSE$ (Eq. 3) and $Inter$ (Eqs. 4, 5, 7 and 9) are used for a better accuracy-interpretability trade-off.

A third fitness function is considered to penalize rules with lower relevance obtained by OTs. According to previous works [8], [9], these rules introduce a high level of similarity, low level of activity and high redundancy, so they must be avoided. Thus, this is implemented as follows (Eq. 11):

$$Relevance_{OT} = Relevance_{R\text{-}value} = \sqrt[n]{\prod_{i=1}^{n} (1 - R - value_{norm_i})} \tag{11}$$

where $n$ = *number of rules* and:

$$R\text{-}value_{norm_i} = \frac{R\text{-}value_{Rule_i}}{\sum_{j=1}^{n} R\text{-}value_{Rule_j}} \tag{12}$$

### C. Neuro-Fuzzy System FasArt

On the other hand, neuro-fuzzy systems are a very popular approach to generate FRBSs. In this work, the neuro-fuzzy system FasArt [11], [12], which is a neuro-fuzzy system based on the Adaptive Resonance Theory (ART) has been used. If the taxonomy for FBRSs described in [16] is taken into account, FasArt is a scatter model. Another classification can be done if [17] is considered: FasArt is a Mamdani-type FRBS for precise modeling.

TABLE II. FasArt Parameters for modeling

| | FasArt Parameters |
|---|---|
| Numbers of variables $< 9$ | $\rho_A = \rho_B = 0.7$<br>$\gamma_A = \gamma_B = 8$ |
| Numbers of variables $> 9$ | $\rho_A = \rho_B = 0.7$<br>$\gamma_A = \gamma_B = 6$ |

## IV. Experimental Methodology

In this paper, the proposed methodology checks the capabilities of the OTs for rule selection based on accuracy-interpretability trade-off and genetic tuning. This goal is carried out by a general post-processing fuzzy rule selection through a three-objective genetic approach: accuracy, interpretability and the most influential rules. In this scenario, it will be possible to check the trade-off of the FRBSs tuned by the rule selection, the rule influence level preserved in the simplified models, the level of complexity reduction achieved, the distribution of the rule influence amongst the selected rules for each FRBS, etc.

The FRBSs were generated by FasArt in five fold cross validation for each regression problem considered (see data sets in Section V). The FasArt parameters considered have been divided into two groups, depending on the numbers of antecedents and consequents of each case. Thus, the FasArt parameters considered are shown in Table II, where $\rho_A$=$\rho_B$ is the vigilance parameter used by FasArt and $\gamma_A = \gamma_B$ is the fuzzification rate in FasArt.

A general methodology description is summarized in Algorithm 1. This methodology is set out in the following sections, describing in detail the MOEA applied in the post-processing stage for this rule selection.

**Algorithm 1** Methodology for Genetic Rule Selection based on Accuracy-Interpretability Trade-Off and Orthogonal Transformations

```
for Neuro-Fuzzy Algorithm=FasArt do
  for OT=P − QR do
    for DataSet = 1 to 9 do
      for CrossValidation = 1 to 5 do
        Generation of Rule Importance Ordering by OT
        Training Neuro-Fuzzy System (see Table II)
        for Run = 1 to 6 do
          Generate Initial Population and Create the Empty External Population
          Run Genetic Algorithm SPEA2 (see Table III)
        end for
      end for
      Analysis Pareto Front (DataSet) (see Section IV-B)
    end for
  end for
end for
Non-Parametric Statistical Test
```

### A. Multi-Objective Evolutionary Algorithm for Rule Selection

The fuzzy rule selection to achieve *lower complexity and better performance on interpretability with enough accuracy* based on the influential rules is carried out by a MOEA. In order to achieve the aims commented previously, a three-objective ($Inter$, $Acc$, $Relevance_{OT}$) genetic approach is used based on the SPEA2 algorithm [14]: The SPEA2 algorithm (Strength Pareto Evolutionary Algorithm 2 for multiobjetive optimization) is one of the most used techniques for solving problems with multi-objective nature.

TABLE III. SPEA2 PARAMETERS

| Genetic operator | |
|---|---|
| Selection | Binary Tournament |
| Crossover | HUX $P_c$=0.6 |
| Mutation | Classical $P_m$=0.2 |
| **Other options** | |
| Population size | 200 |
| External Population size | 61 |
| Evaluations | 100000 |

In the next sections, the fitness functions are formulated and the genetic parameters and operators are described.

*1) Objectives:* The fitness functions are shown in Eq. (13). Performance desired for the FRBS is:

$$\begin{array}{l} max(Accuracy) = min(MSE_{tra}) \\ max(Interpretability) = min(Inter) \\ max(Relevance_{OT}) = min(lowerR - values) \end{array} \quad (13)$$

*2) Coding Scheme and Populations:* In order to run SPEA2, the following characterization is done:

- Individuals are coded by *binary-coding*: $C$ = $C_1$, $C_2$, .. , $C_m$, with $m$ being the number of initial rules and $C_i$ = $(c_1, .., c_m)$ | $c_i \in 0, 1$.
- The initial population is obtained so that all genes take value '1' in all their individuals to favour a progressive extraction of the worst rules.

*3) Genetic Operators: Crossover and mutation:*

- HUX [18] is used to *crossover* with probability $P_c$.
- Classical *mutation* with probability $P_m$. This operator changes a gene value at random, sets a gene to zero with probability $P_m$ and sets to one with probability $1 - P_m$ [19].

The *stopping criterion* is the number of evaluations.

Table III shows the parameters used to run SPEA2.

### B. Pareto Front Analysis

The Pareto fronts are generated for each trial and three representative points are analyzed according to the objectives:

1) According to the objetives *accuracy and information from OTs* (Plane12): the most information from OT (Best $Relevance_{OT}$), the most accurate model (Best $Acc$) and the median model (Median $Acc - Relevance_{OT}$).
2) According to the objetives *accuracy and interpretability* (Plane13) [20], [21]: the most interpretable model (Best $Inter$), the most accurate model (Best $Acc$) and the median model (Median $Acc - Inter$).
3) According to the objetives *information from OTs and interpretability* (Plane23): the most interpretable model (Best $Inter$), The most information from OT (Best $Relevance_{OT}$) and the median model (Median $Relevance_{OT} - Inter$).

TABLE IV. PERFORMANCE OF FASART MODELS (ACCORDING TO SECTION III-A)

| Model | Fasart | | | | | | |
|---|---|---|---|---|---|---|---|
| | $MSE$ | | $Inter$ | | | | $RN$ [a] |
| | $tra$ | $tst$ | $RN$ | $MF$ | $I$ | $S$ | |
| PLA | 3.792 | 3.820 | 46.8 | 86.6 | 0.006 | 0.156 | 46.8 |
| QUA | 0.049 | 0.052 | 107.4 | 315.2 | 0.005 | 0.194 | 107.4 |
| ELE | 109178 | 153749 | 81.8 | 320.2 | 0.000 | 0.178 | 81.8 |
| ABA | 8.134 | 8.585 | 45.6 | 357.8 | 0.006 | 0.315 | 45.6 |
| STP | 2.068 | 2.190 | 36.2 | 318.8 | 0.000 | 0.243 | 36.2 |
| WIZ | 7.020 | 9.970 | 83.4 | 743.6 | 0.000 | 0.360 | 83.4 |
| WAN | 8.960 | 11.647 | 93.6 | 835.4 | 0.000 | 0.323 | 93.6 |
| MOR | 0.448 | 0.503 | 22.6 | 332.0 | 0.001 | 0.230 | 22.6 |
| TRE | 0.823 | 0.858 | 25.0 | 368.0 | 0.000 | 0.236 | 25.0 |

[a] $RN$ is duplicated because $Inter$=$RN$ in Q1

## V. EXPERIMENTAL STUDY: RESULTS AND ANALYSIS

In order to check the performance of the proposal introduced in this work, nine real-world data sets from the KEEL Project [22] [1] have been used:

1) Plastic Strength (PLA): 3 variables, 1650 records.
2) Quake (QUA): 4 variables, 2178 records.
3) Electrical Maintenance (ELE): 5 variables, 1056 records.
4) Abalone (ABA): 9 variables, 4177 records.
5) Stock prices (STP): 10 variables, 950 records.
6) Weather Ankara (WAN): 10 variables, 1609 records.
7) Weather Izmir (WIZ): 10 variables, 1461 records.
8) Mortgage (MOR): 16 variables, 1049 records.
9) Treasury (TRE): 16 variables, 1049 records.

TABLE VI. WILCOXON TEST FOR FASART MODEL IN Q1-PLANE12: ORIGINAL MODEL (R+) AND IMPROVED MODEL (R-)

| Best $Rel_{OT}$ | | | | |
|---|---|---|---|---|
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 17.0 | 28.0 | Accepted | 0.515 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| **Median $Acc - Rel_{OT}$** | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 37.0 | 8.0 | Rejected | 0.086 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| **Best $Acc$** | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 42.0 | 3.0 | Rejected | 0.021 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |

First of all, the FRBSs are generated by Fasart. Next, the multi-objective rule selection is carried out, generating a Pareto Front for each dataset and trial, as shown in Algorithm 1: for each experiment, a fivefold cross validation model is adopted

[1] http://sci2s.ugr.es/keel/datasets.php

TABLE V. FASART: PERFORMANCE OF THE IMPROVED FRBSS: P-QR. $Inter$ = $RN$

| Plane12 | Best $Rel_{OT}$ | | | | | | Median $Acc$-$Rel_{OT}$ | | | | | | Best $Acc$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Plane13 | Best $Inter$ | | | | | | Median $Acc$-$Inter$ | | | | | | Best $Acc$ | | | | | |
| Plane23 | Best $Inter$ | | | | | | Median $Rel_{OT}$-$Inter$ | | | | | | Best $Rel_{OT}$ | | | | | |
| DS | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ |
| PLA ini | 46.800 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 46.800 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 46.800 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 |
| Plane12 | 9.900 | **3.603** | **3.605** | 9.9 | 0.953 | 4.581 | 15.533 | **2.507** | **2.523** | 15.5 | 0.963 | 3.633 | 2.300 | **2.300** | **2.369** | 22.3 | 0.971 | 2.918 |
| Plane13 | 9.133 | **3.450** | **3.493** | 9.1 | 0.956 | 4.380 | 14.700 | **2.510** | **2.518** | 14.7 | 0.964 | 3.556 | | | | | | |
| Plane23 | | | | | | | 9.333 | **3.602** | **3.702** | 9.3 | 0.954 | 4.514 | 9.900 | **3.603** | **3.605** | 9.9 | 0.953 | 4.581 |
| QUA ini | 107.400 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 107.400 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 107.400 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 |
| Plane12 | 46.667 | **0.037** | **0.039** | 46.7 | 0.986 | 1.369 | 57.000 | **0.035** | **0.038** | 57.0 | 0.988 | 1.234 | 67.433 | **0.034** | **0.038** | 67.4 | 0.989 | 1.091 |
| Plane13 | 44.567 | **0.037** | **0.039** | 44.6 | 0.987 | 1.324 | 55.367 | **0.035** | **0.038** | 55.4 | 0.988 | 1.196 | | | | | | |
| Plane23 | | | | | | | 45.300 | **0.037** | **0.039** | 45.3 | 0.986 | 1.351 | 46.667 | **0.037** | **0.039** | 46.7 | 0.986 | 1.369 |
| ELE ini | 81.800 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 81.800 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 81.800 | 109178 | 153749 | 81.8 | 0.988 | 1.222 |
| Plane12 | 70.867 | 122246 | 178727 | 70.9 | 0.987 | 1.302 | 75.067 | 112272 | 162213 | 75.1 | 0.987 | 1.271 | 78.800 | **108217** | 154058 | 78.8 | 0.988 | 1.234 |
| Plane13 | 69.933 | 127291 | 180517 | 69.9 | 0.987 | 1.288 | 73.933 | 111551 | 161552 | 73.9 | 0.987 | 1.263 | | | | | | |
| Plane23 | | | | | | | 70.200 | 125304 | 181415 | 70.2 | 0.987 | 1.297 | 70.867 | 122246 | 178727 | 70.9 | 0.987 | 1.302 |
| ABA ini | 45.600 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 45.600 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 45.600 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 |
| Plane12 | 24.567 | **7.235** | **7.481** | 24.6 | 0.969 | 3.088 | 29.267 | **6.267** | **6.703** | 29.3 | 0.972 | 2.785 | 34.733 | **5.994** | **6.464** | 34.7 | 0.975 | 2.424 |
| Plane13 | 22.967 | **7.244** | **7.516** | 23.0 | 0.971 | 2.911 | 28.833 | **6.142** | **6.534** | 28.8 | 0.974 | 2.621 | | | | | | |
| Plane23 | | | | | | | 23.400 | **7.496** | **7.700** | 23.4 | 0.969 | 3.025 | 24.567 | **7.235** | **7.481** | 24.6 | 0.969 | 3.088 |
| STP ini | 36.200 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 36.200 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 36.200 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 |
| Plane12 | 21.567 | 2.580 | 2.747 | 21.6 | 0.965 | 3.446 | 25.800 | 2.105 | 2.311 | 25.8 | 0.968 | 3.163 | 29.333 | **2.052** | 2.226 | 29.3 | 0.971 | 2.919 |
| Plane13 | 20.600 | 2.711 | 2.816 | 20.6 | 0.966 | 3.378 | 25.200 | 2.085 | 2.269 | 25.2 | 0.969 | 3.081 | | | | | | |
| Plane23 | | | | | | | 20.800 | 2.631 | 2.761 | 20.8 | 0.966 | 3.410 | 21.567 | 2.580 | 2.747 | 21.6 | 0.965 | 3.446 |
| WIZ ini | 83.400 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 83.400 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 83.400 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 |
| Plane12 | 50.400 | **6.335** | **9.515** | 50.4 | 0.986 | 1.400 | 53.300 | **5.779** | **8.946** | 53.3 | 0.986 | 1.357 | 55.767 | **5.600** | **8.604** | 55.8 | 0.987 | 1.311 |
| Plane13 | 49.467 | **6.480** | **9.535** | 49.5 | 0.986 | 1.375 | 52.733 | **5.701** | **8.697** | 52.7 | 0.986 | 1.342 | | | | | | |
| Plane23 | | | | | | | 49.833 | **6.579** | **9.747** | 49.8 | 0.986 | 1.389 | 50.400 | **6.335** | **9.515** | 50.4 | 0.986 | 1.400 |
| WANini | 93.600 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 93.600 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 93.600 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 |
| Plane12 | 47.300 | **7.802** | **10.735** | 47.3 | 0.986 | 1.346 | 52.467 | **6.940** | **10.484** | 52.5 | 0.988 | 1.237 | 62.367 | **6.762** | **10.273** | 62.4 | 0.989 | 1.140 |
| Plane13 | 45.667 | **7.789** | **11.217** | 45.7 | 0.987 | 1.293 | 53.967 | **6.860** | **10.404** | 54.0 | 0.988 | 1.215 | | | | | | |
| Plane23 | | | | | | | 46.167 | **7.960** | **11.022** | 46.2 | 0.987 | 1.322 | 47.300 | **7.802** | **10.735** | 47.3 | 0.986 | 1.346 |
| MORini | 22.600 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 22.600 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 22.600 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 |
| Plane12 | 13.067 | 0.608 | 0.657 | 13.1 | 0.944 | 5.536 | 15.833 | 0.461 | 0.512 | 15.8 | 0.948 | 5.136 | 18.600 | **0.377** | **0.405** | 18.6 | 0.952 | 4.714 |
| Plane13 | 12.033 | 0.806 | 0.864 | 12.0 | 0.946 | 5.297 | 15.433 | **0.436** | **0.456** | 15.4 | 0.950 | 4.950 | | | | | | |
| Plane23 | | | | | | | 12.233 | 0.779 | 0.832 | 12.2 | 0.945 | 5.415 | 13.067 | 0.608 | 0.657 | 13.1 | 0.944 | 5.536 |
| TRE ini | 25.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 25.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 25.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 |
| Plane12 | 14.167 | 1.008 | 1.082 | 14.2 | 0.948 | 5.167 | 16.600 | **0.760** | **0.825** | 16.6 | 0.953 | 4.709 | 19.200 | **0.729** | **0.777** | 19.2 | 0.957 | 4.231 |
| Plane13 | 12.833 | 1.111 | 1.186 | 12.8 | 0.950 | 5.034 | 16.100 | **0.763** | **0.829** | 16.1 | 0.953 | 4.687 | | | | | | |
| Plane23 | | | | | | | 13.333 | 1.086 | 1.172 | 13.3 | 0.949 | 5.111 | 14.167 | 1.008 | 1.082 | 14.2 | 0.948 | 5.167 |

TABLE VII. WILCOXON TEST FOR FASART MODEL IN Q1-PLANE13: ORIGINAL MODEL (R+) AND IMPROVED MODEL (R-)

| Best $Inter$ | | | | |
|---|---|---|---|---|
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 16.0 | 29.0 | Accepted | 0.441 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| Median $Acc - Inter$ | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 40.0 | 5.0 | Rejected | 0.038 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| Best $Acc$ | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 42.0 | 3.0 | Rejected | 0.021 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |

TABLE VIII. WILCOXON TEST FOR FASART MODEL IN Q1-PLANE23: ORIGINAL MODEL (R+) AND IMPROVED MODEL (R-)

| Best $Inter$ | | | | |
|---|---|---|---|---|
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 16.0 | 29.0 | Accepted | 0.441 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| Median $Rel_{OT} - Inter$ | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 16.0 | 29.0 | Accepted | 0.441 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |
| Best $Rel_{OT}$ | | | | |
| Measure | R+ | R- | Hypothesis (alpha=0.10) | p-value |
| $MSE_{tst}$ | 17.0 | 28.0 | Accepted | 0.515 |
| $NR$ | 45.0 | 0.0 | Rejected | 0.008 |
| $Inter$ | 45.0 | 0.0 | Rejected | 0.008 |

(each fold contained 20% of the records). For each of these five partitions (train/test), both stages of the algorithm were run 6 times. Therefore, we consider the average results of 30 runs on the three representative models from the Pareto front. Finally, non-parametric statistical tests are run to know the general significance of the results: non-parametric Wilcoxon's signed-rank tests [23].

### A. FasArt Fuzzy Models

The fuzzy models were generated by FasArt in fivefold cross validation for each regression problem considered. The FasArt parameters used for all the cases are shown in Table II. In Table IV, the performance of these fuzzy models is shown: it is possible to see that the accuracy of the models is high.

### B. Genetic Rule Selection: Results

This section shows the main results obtained by the SPEA2 genetic algorithm and the fitness-functions. Taking into account that SPEA2 has been run with four interpretability measures, one for each quadrant, the study has been made for each quadrant ($Q1$ with $Inter$=$RN$, $Q2$ with $Inter$=$MF$, $Q3$ with $Inter$=$I$ and $Q4$ with $Inter$=$S$). Moreover, taking into account that SPEA2 has been run with three objetives, in each quadrant the study has been carried out for three planes:

TABLE IX. FASART: PERFORMANCE OF THE IMPROVED FRBSS: P-QR. $Inter$ = $NumofMF$

| Plane12 | Best $Rel_{OT}$ | | | | | | Median $Acc$-$Rel_{OT}$ | | | | | | Best $Acc$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Plane13 | Best $Inter$ | | | | | | Median $Acc$-$Inter$ | | | | | | Best $Acc$ | | | | | |
| Plane23 | Best $Inter$ | | | | | | Median $Rel_{OT}$-$Inter$ | | | | | | Best $Rel_{OT}$ | | | | | |
| DS | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ |
| PLA ini | 86.600 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 86.600 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 86.600 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 |
| Plane12 | 13.667 | **3.549** | **3.556** | 10.3 | 0.955 | 4.487 | 24.467 | **2.465** | **2.510** | 15.7 | 0.964 | 3.606 | 38.067 | **2.299** | **2.349** | 22.5 | 0.971 | 2.916 |
| Plane13 | 12.667 | **3.327** | **3.393** | 9.8 | 0.957 | 4.285 | 24.133 | **2.453** | **2.509** | 15.6 | 0.965 | 3.447 | | | | | | |
| Plane23 | | | | | | | 13.000 | **3.562** | **3.588** | 10.0 | 0.955 | 4.446 | 13.667 | **3.549** | **3.556** | 10.3 | 0.955 | 4.487 |
| QUA ini | 315.200 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 315.200 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 315.200 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 |
| Plane12 | 130.100 | **0.037** | **0.039** | 45.7 | 0.986 | 1.383 | 160.700 | **0.035** | **0.038** | 55.9 | 0.987 | 1.251 | 191.800 | **0.034** | **0.038** | 66.3 | 0.989 | 1.100 |
| Plane13 | 124.800 | **0.036** | **0.039** | 43.9 | 0.987 | 1.344 | 156.400 | **0.035** | **0.038** | 54.5 | 0.988 | 1.201 | | | | | | |
| Plane23 | | | | | | | 126.200 | **0.037** | **0.039** | 44.4 | 0.986 | 1.369 | 130.100 | **0.037** | **0.039** | 45.7 | 0.986 | 1.383 |
| ELE ini | 320.200 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 320.200 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 320.200 | 109178 | 153749 | 81.8 | 0.988 | 1.222 |
| Plane12 | 277.267 | 121294 | 177692 | 71.1 | 0.987 | 1.301 | 292.467 | 112447 | 162067 | 74.9 | 0.987 | 1.272 | 309.000 | **108199** | 153760 | 79.0 | 0.988 | 1.233 |
| Plane13 | 273.000 | 122256 | 175706 | 70.0 | 0.987 | 1.292 | 290.333 | 111263 | 160243 | 74.3 | 0.987 | 1.259 | | | | | | |
| Plane23 | | | | | | | 274.200 | 121624 | 176723 | 70.3 | 0.987 | 1.298 | 277.267 | 121294 | 177692 | 71.1 | 0.987 | 1.301 |
| ABA ini | 357.800 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 357.800 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 357.800 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 |
| Plane12 | 193.267 | **7.261** | **7.552** | 25.0 | 0.969 | 3.080 | 238.067 | **6.207** | **6.657** | 30.6 | 0.972 | 2.735 | 273.000 | **5.990** | **6.487** | 35.0 | 0.976 | 2.417 |
| Plane13 | 181.000 | **7.258** | **7.581** | 23.5 | 0.970 | 2.917 | 227.933 | **6.133** | **6.532** | 29.4 | 0.974 | 2.614 | | | | | | |
| Plane23 | | | | | | | 184.733 | **7.393** | **7.732** | 24.0 | 0.969 | 3.033 | 193.267 | **7.261** | **7.552** | 25.0 | 0.969 | 3.080 |
| STP ini | 318.800 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 318.800 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 318.800 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 |
| Plane12 | 183.800 | 2.580 | 2.791 | 21.2 | 0.965 | 3.452 | 225.800 | 2.106 | 2.311 | 25.9 | 0.968 | 3.165 | 258.200 | **2.052** | 2.226 | 29.5 | 0.971 | 2.911 |
| Plane13 | 179.000 | 2.480 | 2.704 | 20.7 | 0.966 | 3.409 | 219.200 | 2.085 | 2.260 | 25.1 | 0.969 | 3.078 | | | | | | |
| Plane23 | | | | | | | 179.300 | 2.617 | 2.838 | 20.7 | 0.965 | 3.437 | 183.800 | 2.580 | 2.791 | 21.2 | 0.965 | 3.452 |
| WIZ ini | 743.600 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 743.600 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 743.600 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 |
| Plane12 | 447.500 | **6.270** | **9.478** | 50.5 | 0.986 | 1.397 | 471.200 | **5.782** | **8.913** | 53.1 | 0.986 | 1.357 | 495.800 | **5.640** | **8.691** | 55.9 | 0.987 | 1.315 |
| Plane13 | 441.800 | **6.127** | **9.292** | 49.9 | 0.986 | 1.372 | 470.600 | **5.699** | **8.677** | 53.1 | 0.986 | 1.342 | | | | | | |
| Plane23 | | | | | | | 443.300 | **6.313** | **9.569** | 50.0 | 0.986 | 1.388 | 447.500 | **6.270** | **9.478** | 50.5 | 0.986 | 1.397 |
| WANini | 835.400 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 835.400 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 835.400 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 |
| Plane12 | 434.300 | **7.764** | **10.799** | 49.0 | 0.987 | 1.329 | 483.200 | **6.905** | **10.437** | 54.5 | 0.988 | 1.216 | 562.400 | **6.772** | **10.318** | 63.3 | 0.989 | 1.133 |
| Plane13 | 421.400 | **7.525** | **11.197** | 47.6 | 0.987 | 1.289 | 493.400 | **6.854** | **10.464** | 55.6 | 0.988 | 1.201 | | | | | | |
| Plane23 | | | | | | | 424.400 | **7.765** | **10.959** | 47.9 | 0.987 | 1.307 | 434.300 | **7.764** | **10.799** | 49.0 | 0.987 | 1.329 |
| MORini | 332.000 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 332.000 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 332.000 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 |
| Plane12 | 187.000 | 0.615 | 0.659 | 12.9 | 0.944 | 5.533 | 233.000 | 0.451 | **0.484** | 16.0 | 0.948 | 5.121 | 272.000 | **0.377** | **0.405** | 18.6 | 0.952 | 4.714 |
| Plane13 | 175.000 | 0.789 | 0.812 | 12.1 | 0.945 | 5.375 | 227.000 | **0.429** | **0.449** | 15.6 | 0.950 | 4.943 | | | | | | |
| Plane23 | | | | | | | 179.500 | 0.690 | 0.732 | 12.4 | 0.944 | 5.461 | 187.000 | 0.615 | 0.659 | 12.9 | 0.944 | 5.533 |
| TRE ini | 368.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 368.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 368.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 |
| Plane12 | 203.500 | 1.017 | 1.078 | 14.0 | 0.948 | 5.170 | 244.000 | **0.752** | **0.818** | 16.7 | 0.953 | 4.671 | 281.000 | **0.729** | **0.777** | 19.2 | 0.957 | 4.231 |
| Plane13 | 184.000 | 1.129 | 1.178 | 12.7 | 0.949 | 5.039 | 235.000 | **0.761** | **0.828** | 16.1 | 0.953 | 4.685 | | | | | | |
| Plane23 | | | | | | | 192.000 | 1.109 | 1.178 | 13.3 | 0.949 | 5.121 | 203.500 | 1.017 | 1.078 | 14.0 | 0.948 | 5.170 |

1) Plane 12: $MSE$ and $Relevance_{OT}$
2) Plane 13: $MSE$ and $Inter$
3) Plane 23: $Relevance_{OT}$ and $Inter$

The best popular plane is *Plane 13* because this plane takes into account accuracy and interpretability whose trade-off is wanted. The results of these three planes for each quadrant are described in the next subsections.

*1) Q1: $Inter$=$RN$:* Table V shows the averaged results obtained from the Pareto Frontwork over 30 runs for each case study: the MSE for training ($MSE_{tra}$) and testing ($MSE_{tst}$), the interpretability ($Inter$), the rule relevance obtained by OTs ($Rel_{OT}$=$Relevance_{OT}$) and the amount of averages information (relevance) for rule in % ($Rel_{Rule}$=$Relevance_{OT}$/Num.of Rules). Values in bold indicate a better performance.

In order to check the scope of this work, the Wilcoxon test is run on error and interpretability/complexity indices for the three characteristic models from the Pareto front.

The Wilcoxon test for the three planes (Tables VI, VII and VIII) accepts that: In general, results on three Pareto Front points analyzed for each plane show that the interpretability have been improved, reducing the complexity and the number of rules of the FRBSs and the accuracy of the models has been preserved or, in some cases, the accuracy has also been improved. On the other hand, $Rel_{OT}$ has decreased, which means that the genetic algorithm has selected the most relevant rules and this also can be seen because the information for rule ($Rel_{Rule}$) has increased.

*2) Q2: $Inter$=Number of MF:* Table IX shows the averaged results obtained from the Pareto Front work over 30 runs for each case study.

The Wilcoxon test has been run for the three planes and the results obtained are very similar to Q1.

*3) Q3: $Inter$=$Inconsistency$:* Table X shows the averaged results obtained from the Pareto Front work over 30 runs for each case study.

The Wilcoxon test has been run for the three planes and the results obtained are very similar to Q1. Here, the interpretabiliy is preserved in some cases because initial Inconsistency = 0 for several data sets.

*4) Q4: $Inter$=$Similarity$:* Table XI shows the averaged results obtained from the Pareto Front work over 30 runs for each case study.

The Wilcoxon test has been run for the three planes and the results obtained are very similar to Q1.

## VI. CONCLUSIONS

This work is focused on the checking of the capacities and drawbacks of OTs for complexity reduction and interpretability of FRBSs. This aim is carried out by rule selection using a genetic algorithm subject to accuracy-interpretability trade-off, and the rule influence provided by OTs. P-QR orthogonal

TABLE X. FASART: PERFORMANCE OF THE IMPROVED FRBSS: P-QR. $Inter$ = $Inconsistency$

| | Plane12 | Best $Rel_{OT}$ | | | | | | Median $Acc$-$Rel_{OT}$ | | | | | | Best $Acc$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Plane13 | Best $Inter$ | | | | | | Median $Acc$-$Inter$ | | | | | | Best $Acc$ | | | | | |
| | Plane23 | Best $Inter$ | | | | | | Median $Rel_{OT}$-$Inter$ | | | | | | Best $Rel_{OT}$ | | | | | |
| DS | | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ |
| PLA | ini | 0.006 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 0.006 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 0.006 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 |
| | Plane12 | 0.000 | **3.054** | **3.039** | 14.0 | 0.959 | 4.049 | 0.001 | **2.442** | **2.487** | 17.9 | 0.965 | 3.487 | 0.003 | **2.305** | **2.367** | 22.9 | 0.971 | 2.928 |
| | Plane13 | 0.000 | **2.330** | **2.420** | 22.3 | 0.971 | 2.912 | 0.003 | **2.315** | **2.408** | 24.5 | 0.971 | 2.848 | | | | | | |
| | Plane23 | | | | | | | 0.000 | **3.054** | **3.039** | 14.0 | 0.959 | 4.049 | 0.000 | **3.054** | **3.039** | 14.0 | 0.959 | 4.049 |
| QUA | ini | 0.005 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 0.005 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 0.005 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 |
| | Plane12 | 0.000 | **0.038** | **0.039** | 58.7 | 0.987 | 1.301 | 0.002 | **0.035** | **0.038** | 65.3 | 0.988 | 1.209 | 0.003 | **0.035** | **0.038** | 74.9 | 0.989 | 1.073 |
| | Plane13 | 0.000 | **0.036** | **0.038** | 65.3 | 0.988 | 1.181 | 0.001 | **0.035** | **0.038** | 70.6 | 0.989 | 1.134 | | | | | | |
| | Plane23 | | | | | | | 0.000 | **0.038** | **0.039** | 59.7 | 0.987 | 1.290 | 0.000 | **0.038** | **0.039** | 58.7 | 0.987 | 1.301 |
| ELE | ini | 0.000 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 0.000 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 0.000 | 109178 | 153749 | 81.8 | 0.988 | 1.222 |
| | Plane12 | 0.000 | 120552 | 175328 | 72.0 | 0.987 | 1.297 | 0.000 | 111692 | 160710 | 75.5 | 0.987 | 1.269 | 0.000 | **108244** | **152794** | 79.8 | 0.988 | 1.230 |
| | Plane13 | 0.000 | 111567 | 159047 | 78.3 | 0.987 | 1.242 | 0.000 | 109495 | 155028 | 79.1 | 0.988 | 1.238 | | | | | | |
| | Plane23 | | | | | | | 0.000 | 120552 | 175328 | 72.0 | 0.987 | 1.297 | 0.000 | 120552 | 175328 | 72.0 | 0.987 | 1.297 |
| ABA | ini | 0.006 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 0.006 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 0.006 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 |
| | Plane12 | 0.004 | **7.865** | **7.887** | 26.6 | 0.970 | 2.974 | 0.006 | **6.676** | **6.898** | 29.6 | 0.972 | 2.766 | 0.006 | **6.308** | **6.642** | 35.7 | 0.976 | 2.404 |
| | Plane13 | 0.002 | **6.781** | **7.013** | 31.7 | 0.975 | 2.513 | 0.004 | **6.499** | **6.776** | 35.2 | 0.976 | 2.390 | | | | | | |
| | Plane23 | | | | | | | 0.002 | **7.776** | **7.827** | 26.8 | 0.970 | 2.957 | 0.004 | **7.865** | **7.887** | 26.6 | 0.970 | 2.974 |
| STP | ini | 0.000 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 0.000 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 0.000 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 |
| | Plane12 | 0.000 | 2.403 | 2.576 | 23.4 | 0.966 | 3.364 | 0.000 | 2.092 | 2.287 | 26.9 | 0.969 | 3.107 | 0.000 | **2.052** | 2.226 | 29.3 | 0.971 | 2.923 |
| | Plane13 | 0.000 | **2.052** | 2.226 | 29.6 | 0.971 | 2.909 | 0.000 | **2.052** | 2.226 | 29.7 | 0.971 | 2.907 | | | | | | |
| | Plane23 | | | | | | | 0.000 | 2.403 | 2.576 | 23.4 | 0.966 | 3.364 | 0.000 | 2.403 | 2.576 | 23.4 | 0.966 | 3.364 |
| WIZ | ini | 0.000 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 0.000 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 0.000 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 |
| | Plane12 | 0.000 | **6.093** | **9.294** | 52.3 | 0.986 | 1.387 | 0.000 | **5.735** | **8.836** | 54.2 | 0.986 | 1.349 | 0.000 | **5.639** | **8.686** | 56.9 | 0.987 | 1.308 |
| | Plane13 | 0.000 | **5.639** | **8.686** | 57.0 | 0.987 | 1.307 | 0.000 | **5.639** | **8.686** | 57.0 | 0.987 | 1.307 | | | | | | |
| | Plane23 | | | | | | | 0.000 | **6.093** | **9.294** | 52.3 | 0.986 | 1.387 | 0.000 | **6.093** | **9.294** | 52.3 | 0.986 | 1.387 |
| WAN | ini | 0.000 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 0.000 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 0.000 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 |
| | Plane12 | 0.000 | **7.575** | **10.673** | 53.4 | 0.987 | 1.290 | 0.000 | **6.856** | **10.366** | 56.3 | 0.988 | 1.205 | 0.000 | **6.769** | **10.311** | 64.1 | 0.989 | 1.127 |
| | Plane13 | 0.000 | **6.769** | **10.311** | 64.4 | 0.989 | 1.125 | 0.000 | **6.769** | **10.311** | 64.4 | 0.989 | 1.125 | | | | | | |
| | Plane23 | | | | | | | 0.000 | **7.575** | **10.673** | 53.4 | 0.987 | 1.290 | 0.000 | **7.575** | **10.673** | 53.4 | 0.987 | 1.290 |
| MOR | ini | 0.001 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 0.001 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 0.001 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 |
| | Plane12 | 0.000 | 0.608 | 0.655 | 13.8 | 0.944 | 5.510 | 0.001 | **0.438** | **0.472** | 16.5 | 0.949 | 5.066 | 0.001 | **0.377** | **0.405** | 18.6 | 0.952 | 4.713 |
| | Plane13 | 0.000 | **0.380** | **0.410** | 18.6 | 0.952 | 4.725 | 0.001 | **0.379** | **0.406** | 18.8 | 0.952 | 4.713 | | | | | | |
| | Plane23 | | | | | | | 0.000 | 0.608 | 0.655 | 13.8 | 0.944 | 5.510 | 0.000 | 0.608 | 0.655 | 13.8 | 0.944 | 5.510 |
| TRE | ini | 0.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 0.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 0.000 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 |
| | Plane12 | 0.000 | 0.862 | 0.922 | 15.1 | 0.949 | 5.045 | 0.000 | **0.745** | **0.814** | 17.3 | 0.954 | 4.597 | 0.000 | **0.729** | **0.779** | 19.2 | 0.957 | 4.230 |
| | Plane13 | 0.000 | **0.729** | **0.779** | 19.3 | 0.957 | 4.218 | 0.000 | **0.729** | **0.779** | 19.2 | 0.957 | 4.230 | | | | | | |
| | Plane23 | | | | | | | 0.000 | 0.862 | 0.922 | 15.1 | 0.949 | 5.045 | 0.000 | 0.862 | 0.922 | 15.1 | 0.949 | 5.045 |

transformation has been considered with different interpretability measures: RN (Q1), Number of MFs (Q2), Inconsistency (Q3) and Similarity (Q4).

In order to check this, nine regression problems have been involved in the experimental work. The results achieved by the genetic selection on complexity, interpretability and accuracy show, in general, that the number of rules and the interpretability have always been improved. The accuracy of the models has been preserved or, in many cases,improved.

In general, there are no significant differences through the interpretability measures considered in this work:

Plane $MSE$-$Inter$: the interpretability has been improved, reducing the complexity and the number of rules of the FRBSs on three Pareto Front points analyzed, and the accuracy of the models has been preserved on the point of Best $Inter$ and improved in the rest of the points.

Plane $MSE$-$Rel_{OT}$: the accuracy is better when some not so relevant rules (smaller R-values) are selected.

Plane $Rel_{OT}$-$Inter$: the interpretality is better when relevant rules (larger R-values) are selected.

The experiments have shown that the rules associated with lower influence values by OTs, not relevant rules, have relevance from the accuracy point of view.

Another point to be analyzed is the level of influence preserved for rule ($Rel_{Rule}$): the obtained level is always better than the initial model because the genetic algorithm selects the most influential rules from the OTs. Moreover, the most interpretable model achieved a higher level of relevancy: the interpretabiliy is better when larger R-values are selected by OT's, and larger values preserve more information and permit a reduction in the number of rules, resulting in better interpretability. The most accurate model shows a lower level of relevancy: the accuracy is better when smaller R-values are selected by OT's, and these smaller values preserve less information (although higher than in the initial model).

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TABLE XI. FASART: PERFORMANCE OF THE IMPROVED FRBSS MODELS: P-QR. $Inter$ = $Simitarity$

| Plane12 | Best $Rel_{OT}$ | | | | | | Median $Acc$-$Rel_{OT}$ | | | | | | Best $Acc$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Plane13 | Best $Inter$ | | | | | | Median $Acc$-$Inter$ | | | | | | Best $Acc$ | | | | | |
| Plane23 | Best $Inter$ | | | | | | Median $Rel_{OT}$-$Inter$ | | | | | | Best $Rel_{OT}$ | | | | | |
| DS | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ | $Inter$ | $MSE_{tra}$ | $MSE_{tst}$ | $RN$ | $Rel_{OT}$ | $Rel_{Rule}$ |
| PLA ini | 0.191 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 0.191 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 | 0.191 | 3.792 | 3.820 | 46.8 | 0.978 | 2.137 |
| Plane12 | 0.156 | 4.087 | 4.157 | 9.9 | 0.954 | 4.550 | 0.172 | **2.551** | **2.620** | 15.3 | 0.962 | 3.718 | 0.175 | **2.299** | **2.332** | 23.2 | 0.971 | 2.830 |
| Plane13 | 0.128 | 4.585 | 4.586 | 10.7 | 0.963 | 3.654 | 0.147 | **2.793** | **2.943** | 14.9 | 0.966 | 3.370 | | | | | | |
| Plane23 | | | | | | | 0.138 | 4.553 | 4.665 | 10.4 | 0.958 | 4.120 | 0.156 | 4.087 | 4.157 | 9.9 | 0.954 | 4.550 |
| QUA ini | 0.194 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 0.194 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 | 0.194 | 0.049 | 0.052 | 107.4 | 0.991 | 0.931 |
| Plane12 | 0.170 | **0.039** | **0.042** | 57.3 | 0.987 | 1.274 | 0.177 | **0.036** | **0.038** | 62.8 | 0.988 | 1.215 | 0.175 | **0.035** | **0.037** | 74.1 | 0.989 | 1.072 |
| Plane13 | 0.152 | **0.041** | **0.044** | 61.5 | 0.989 | 1.075 | 0.158 | **0.036** | **0.039** | 66.8 | 0.989 | 1.061 | | | | | | |
| Plane23 | | | | | | | 0.159 | **0.041** | **0.043** | 58.1 | 0.988 | 1.194 | 0.170 | **0.039** | **0.042** | 57.3 | 0.987 | 1.274 |
| ELE ini | 0.178 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 0.178 | 109178 | 153749 | 81.8 | 0.988 | 1.222 | 0.178 | 109178 | 153749 | 81.8 | 0.988 | 1.222 |
| Plane12 | 0.174 | 126752 | 184389 | 69.8 | 0.987 | 1.310 | 0.176 | 114532 | 166142 | 73.9 | 0.987 | 1.282 | 0.178 | **108315** | **152850** | 79.8 | 0.988 | 1.230 |
| Plane13 | 0.171 | 125201 | 176799 | 72.4 | 0.987 | 1.286 | 0.174 | 114193 | 164344 | 75.7 | 0.987 | 1.265 | | | | | | |
| Plane23 | | | | | | | 0.172 | 125466 | 180020 | 71.4 | 0.987 | 1.298 | 0.174 | 126752 | 184389 | 69.8 | 0.987 | 1.310 |
| ABA ini | 0.315 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 0.315 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 | 0.315 | 8.134 | 8.585 | 45.6 | 0.978 | 2.193 |
| Plane12 | 0.294 | 8.140 | **8.218** | 22.7 | 0.969 | 3.257 | 0.313 | **6.760** | **6.960** | 27.3 | 0.971 | 2.933 | 0.310 | **6.294** | **6.627** | 32.0 | 0.975 | 2.576 |
| Plane13 | 0.265 | 8.937 | 9.037 | 22.7 | 0.974 | 2.804 | 0.283 | **6.974** | **7.278** | 27.4 | 0.975 | 2.684 | | | | | | |
| Plane23 | | | | | | | 0.275 | 8.633 | 8.716 | 22.4 | 0.971 | 3.063 | 0.294 | 8.140 | **8.218** | 22.7 | 0.969 | 3.257 |
| STP ini | 0.243 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 0.243 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 | 0.243 | 2.068 | 2.190 | 36.2 | 0.972 | 2.762 |
| Plane12 | 0.258 | 4.104 | 4.807 | 19.3 | 0.964 | 3.569 | 0.259 | 2.551 | 3.068 | 22.6 | 0.966 | 3.371 | 0.255 | 2.095 | 2.461 | 28.9 | 0.970 | 2.968 |
| Plane13 | 0.231 | 4.026 | 4.449 | 26.4 | 0.971 | 2.900 | 0.238 | 2.611 | 3.028 | 28.3 | 0.971 | 2.928 | | | | | | |
| Plane23 | | | | | | | 0.239 | 4.225 | 4.916 | 22.5 | 0.968 | 3.230 | 0.258 | 4.104 | 4.807 | 19.3 | 0.964 | 3.569 |
| WIZ ini | 0.360 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 0.360 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 | 0.360 | 7.020 | 9.970 | 83.4 | 0.988 | 1.199 |
| Plane12 | 0.354 | **6.606** | **9.581** | 51.8 | 0.986 | 1.383 | 0.355 | **5.971** | **9.012** | 54.4 | 0.986 | 1.358 | 0.354 | **5.634** | **8.653** | 57.9 | 0.987 | 1.293 |
| Plane13 | 0.336 | 8.421 | 11.427 | 53.7 | 0.987 | 1.271 | 0.342 | **6.132** | **9.124** | 56.8 | 0.987 | 1.286 | | | | | | |
| Plane23 | | | | | | | 0.342 | 7.298 | 10.403 | 52.5 | 0.986 | 1.335 | 0.354 | **6.606** | **9.581** | 51.8 | 0.986 | 1.383 |
| WAN ini | 0.323 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 0.323 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 | 0.323 | 8.960 | 11.647 | 93.6 | 0.989 | 1.068 |
| Plane12 | 0.316 | **8.763** | 12.031 | 59.7 | 0.988 | 1.220 | 0.319 | **7.415** | **10.715** | 63.8 | 0.988 | 1.180 | 0.315 | **6.915** | **10.647** | 67.8 | 0.989 | 1.110 |
| Plane13 | 0.290 | 13.631 | 16.998 | 59.1 | 0.989 | 1.101 | 0.298 | **7.893** | **11.625** | 63.6 | 0.989 | 1.106 | | | | | | |
| Plane23 | | | | | | | 0.298 | 10.266 | 13.519 | 59.4 | 0.988 | 1.170 | 0.316 | **8.763** | 12.031 | 59.7 | 0.988 | 1.220 |
| MOR ini | 0.230 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 0.230 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 | 0.230 | 0.448 | 0.503 | 22.6 | 0.955 | 4.425 |
| Plane12 | 0.218 | 0.731 | 0.788 | 12.5 | 0.942 | 5.704 | 0.215 | 0.471 | 0.523 | 15.6 | 0.947 | 5.224 | 0.221 | **0.376** | **0.402** | 18.6 | 0.952 | 4.708 |
| Plane13 | 0.199 | 1.128 | 1.153 | 13.2 | 0.949 | 5.059 | 0.209 | 0.479 | 0.506 | 15.8 | 0.949 | 5.051 | | | | | | |
| Plane23 | | | | | | | 0.204 | 0.919 | 0.945 | 13.1 | 0.945 | 5.406 | 0.218 | 0.731 | 0.788 | 12.5 | 0.942 | 5.704 |
| TRE ini | 0.236 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 0.236 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 | 0.236 | 0.823 | 0.858 | 25.0 | 0.960 | 4.000 |
| Plane12 | 0.214 | 1.102 | 1.094 | 13.2 | 0.947 | 5.535 | 0.215 | **0.744** | **0.776** | 15.3 | 0.952 | 5.077 | 0.223 | **0.712** | **0.758** | 17.4 | 0.957 | 4.539 |
| Plane13 | 0.200 | 1.224 | 1.344 | 13.3 | 0.953 | 5.092 | 0.211 | **0.786** | **0.814** | 15.8 | 0.955 | 4.834 | | | | | | |
| Plane23 | | | | | | | 0.205 | 1.157 | 1.186 | 13.5 | 0.950 | 5.293 | 0.214 | 1.102 | 1.094 | 13.2 | 0.947 | 5.535 |


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