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New delay-dependent finite-time stabilization of Takagi–Sugeno fuzzy approaches for delayed nonlinear systems

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Abstract

In this paper, the finite-time stability and stabilization of nonlinear systems with delays is studied, via a Takagi–Sugeno approach. By using a novel Lyapunov–Krasovskii functional and introducing some fuzzy free-weighting matrices, sufficient conditions are derived, for bounded and differentiable time-varying delays in terms of an upper bound of the delay derivatives. Then, we achieve closed-loop stabilization in finite time through an efficient parallel distributed compensation design. The sufficient conditions are formulated as linear matrix inequalities to achieve the desired performance. Finally, the proposed methodology is applied to various case studies, highlighting its significance.

KEYWORDS

finite-time stability, nonlinear systems, parallel distributed compensation, T–S fuzzy approach

1 | INTRODUCTION

The real world is nonlinear by nature, so the design and analysis of systems should take this into account. For this, the Takagi–Sugeno (T–S) fuzzy model proposed in [1] is a powerful tool to overthrow these complications. In this approach, a family of local linear models is associated with fuzzy membership functions; this is an effective method to approximate whichever smooth nonlinear system on a compact set [2–4]. Thus, this method makes the nonlinear systems similar to the linear ones in stability analysis studies and the synthesis of stabilizing controllers. Then, nonlinear systems can be successfully described by a T–S model. Many works have used this approach (see, for instance, [5–10]), leading to less conservative solutions. Then, these recent methods make it possible to reduce the conservatism inherent in this approach and thus reduce the weakness for less weak results. Therefore, an efficient method is studied here for deriving less conservative results and obtaining good performances that can ensure the asymptotic stability of the system.

This paper focuses on nonlinear systems with delays, as delays are inherent to many dynamical systems, such as chemical, metallurgical, biological, mechanical, and other areas. It is well known that the delays are a significant source to reduce the performances and even lead to instability. Thus, great efforts are being devoted to the analysis and synthesis related to time-delay systems. One of the conclusions is that linear matrix inequalities (LMIs) are a useful tool for investigating stability conditions. There are two different approaches in the literature regarding the stability analysis: (i) delay-independent stability [11–14] and (ii) delay-dependent stability [15–20]. The first does not take into account the magnitude of the delay, whereas delay-dependent criteria, as they take into account the magnitude of the delay, are less conservative than delay-independent criteria, particularly if the delay is not big. Therefore, the delay-independent techniques require more work and effort to get less conservative results and to ensure finite-time stability and stabilizability of systems when delays are considered.

Published works in stability criteria focus on Lyapunov asymptotic stability over an infinite time interval. When faced with applications in the real world, this is not enough; there are situations when the state's large values are unacceptable. So, the finite-time stability (FTS) stability concept was introduced [21], the time interval is finite and known, and, given an initial state, the system's variables must lie within preset bounds. Many results deal with FTS for delayed systems [22–25], and most of them work with LMI conditions.

Recently, in [26], the global stabilization over a finite-time interval is addressed for triangular control systems described by delayed functional differential equations and distributed delay feedback. The robust finite-time control problem for a class of uncertain switched neutral systems with unknown time-varying disturbance is investigated in [27]. In [2], the descriptor technique is applied to sensor fault estimation and finite-time state for T–S fuzzy systems. Consequently, when regarding this problem, we find that the aforementioned works still offer plenty of room for enhancement. However, the finite-time stability (FTS) stabilization has not been investigated to nonlinear time-delay systems in [28–30].

In this work, motivated by the above idea, we derive a new and improved finite-time approach for stability and stabilization of T–S fuzzy systems with time varying delay. The sufficient conditions for asymptotic stability and stabilization analysis are derived by using a new Lyapunov–Krasovskii functional method, free-weighting matrices, making use of improved technique, and using a parallel distributed compensation (PDC) controller to ensure the finite-time stability (FTS) and finite-time stabilizability of closed-loop delayed systems via T–S fuzzy models. Finally, less conservative LMI-based design conditions are suggested and calculated using the LMI Toolbox (MATLAB); the computer results present visually the bonuses and efficacy of the proposed approach.

The rest of this paper is structured as follows: Section 2 describes the theory required to understand the method. Sections 3 and 4 cover the delay-dependent FTS analysis and the delay-dependent finite-time stabilization, respectively. Section 5 deals with the numerical examples and finally, Section 6 presents the conclusions of the work.

2 | PRELIMINARIES

This section presents the basic definitions and formulations relevant to the topic of the paper and widely used by researchers.

Consider the nonlinear time-varying delay system, described by the T–S fuzzy model:

- Plant from:

R_i: IF z_1 is F_i^1 and ... and z_n is F_i^g, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-h(t)) + B_i u(t) \\ x(t) = \phi(t), \ \mathrm{t} \in [-h_m, 0] \end{cases}, \ i = 1, \ldots, r \tag{1}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the input vector; A_i, A_{di}, and B_i are appropriate dimensions matrices; F_i^1, F_i^2,..., and F_i^g are fuzzy sets; and r is the number of IF-THEN rules. The time delay $h(t)$ is a time-varying function satisfying

$$0 \le h(t) \le h_m, \ \dot{h}(\mathrm{t}) \le \rho \tag{2}$$

where h_m and ρ are constants. The initial condition $\phi(t)$ is a continuous vector-valued function of $t \in [-h_m, 0]$

Using the center average defuzzifier, the dynamic model given by (1) will be:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z)\{A_i x(t) + A_{di} x(t-h(t)) + B_i u(t)\} \tag{3}$$

where

$$\mu_i(z) = \frac{\omega_i(z)}{\sum_{i=1}^{r} \omega_i(z)},\ \omega_i(z) = \prod_{j=1}^{n} F_i^j(z_j),\ \mu_i(z) \geq 0,\ \sum_{i=1}^{r} \mu_i(z) = 1 \tag{4}$$

Let $X(t) = \sum_{i=1}^{r} \mu_i(z) X_i$ for any matrices X_i. Then, the fuzzy system (3) is reworked:

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t-h(t)) + B(t)u(t) \tag{5}$$

Definition 1 [31]**:** Given R(positive definite matrix) and c_1, c_2, T_f, positive scalars, such that $c_1 < c_2$. Then, the system given by (5) and $u(t) = 0$ is said to be finite time stable with respect to $(c_1, c_2, T_f, h_m, \mathrm{R})$ if

$$\sup_{-h_m \leq \theta \leq 0}\{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\} \leq c_1 \Rightarrow x^T(t)Rx(t) < c_2, \forall t \in [0, T_f]$$

Definition 2 [28, 30]**:** Given R (positive definite matrix) and c_1, c_2, T_f, positive scalars, such that $c_1 < c_2$. Then, the time delay fuzzy system (5) is said to be finite time stabilizable with respect to $(c_1, c_2, T_f, h_m, \mathrm{R})$, when there is a control input $u(t)$ so that:

$$\sup_{-h_m \leq \theta \leq 0}\{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\} \leq c_1 \Rightarrow x^T(t)Rx(t) < c_2,\ \forall t \in [0, T_f]$$

Lemma 1 [32]**:** Let d be the maximum number of the fuzzy rules are activated simultaneously at any instant, where $1 \leq d \leq r$. Then, the membership functions of the fuzzy rules satisfy the inequality:

$$0 \leq (d-1)\sum_{i=1}^{r} \mu_i^3(z) + (d-3)\sum_{i=1}^{r}\sum_{j \neq i}^{r} h_i^2(z)\mu_j(z) - 6\sum_{i=1}^{r}\sum_{j>i}^{r}\sum_{k>j}^{r} \mu_i(z)\mu_j(z)\mu_k(z)$$

for any z.

3 | DELAY-DEPENDENT FTS ANALYSIS

In this section, the autonomous T–S fuzzy delayed system to work with is presented:

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t-h(t)) \tag{6}$$

Let us now establish sufficient conditions to analyze the FTS of (6).

Theorem 1. *If there exist symmetric positive definite matrices $P = P^T > 0$, $Q(t) = Q^T(t) > 0$, $S_i(t) = S_i^T(t) > 0$, $Z_i(t) = Z_i^T(t) > 0$, $i = 1, 2$, positive semi-definite matrix $X(t) = X^T(t)$, appropriately sized matrices $T_i(t), i = 1, 2, 3, 4, 5, 6$, $N_j(t), j = 1, 2, 3, 4, 5$, and a scalar $\alpha \geq 0$ satisfying these conditions*:

$$\Pi(t) = \begin{bmatrix} \Pi_{11}(t) & \Pi_{12}(t) & \Pi_{13}(t) & \Pi_{14}(t) \\ * & \Pi_{22}(t) & \Pi_{23}(t) & \Pi_{24}(t) \\ * & * & \Pi_{33}(t) & \Pi_{34}(t) \\ * & * & * & \Pi_{44}(t) \end{bmatrix} + h_m X(t) < 0, \tag{7}$$

$$\Psi(t,s) = \begin{bmatrix} & -A^T(t)T_5^T(s) & -N_1(t) + N_5^T(t) - A^T(t)T_6^T(s) \\ & -A_d^T(t)T_5^T(s) & -N_2(t) - N_5^T(t) - A_d^T(t)T_6^T(s) \\ -X(t) & 0 & -N_3(t) \\ & T_5^T(s) & T_6^T(s) - N_4(t) \\ * \ * \ * \ * & (-1+\rho)\mu S_1(s) - Z_1(s) & 0 \\ * \ * \ * \ * & * & (-1+\rho)\mu S_2(s) - Z_2(s) \end{bmatrix} \leq 0, \tag{8}$$

$$\frac{e^{\alpha T_f}}{\lambda_1}\left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + h_m^2 e^{\alpha h_m}(\lambda_4 + \lambda_5) + \frac{1}{2}h_m^2(\lambda_6 + \lambda_7)\right)c_1 < c_2 \tag{9}$$

where

$$\begin{aligned}
\Pi_{11}(t) = {} & -T_1(t)A(t) - A^T(t)T_1^T(t) + N_1(t) + N_1^T(t) + Q(t) \\
& + h_m S_1(t) + h_m Z_1(t) - \alpha P - N_5(t) - N_5^T(t), \\
\Pi_{12}(t) = {} & -N_1(t) + N_2^T(t) - T_1(t)A_d(t) - A^T(t)T_2^T(t) \\
& + N_5(t) + N_5^T(t), \\
\Pi_{13}(t) = {} & N_3^T(t) - A^T(t)T_3^T(t), \\
\Pi_{14}(t) = {} & N_4^T(t) + T_1(t) - A^T(t)T_4^T(t) + P, \\
\Pi_{22}(t) = {} & -T_2(t)A_d(t) - A_d^T(t)T_2^T(t) - N_2(t) - N_2^T(t) \\
& - N_5(t) - N_5^T(t), \\
\Pi_{23}(t) = {} & -N_3^T(t) - A_d^T(t)T_3^T(t), \\
\Pi_{24}(t) = {} & T_2(t) - N_4^T(t) - A_d^T(t)T_4^T(t), \\
\Pi_{33}(t) = {} & -e^{\alpha h_m}Q(t-h_m),\ \Pi_{34}(t) = T_3(t), \\
\Pi_{44}(t) = {} & T_4(t) + T_4^T(t) + h_m S_2(t) + h_m Z_2(t), \\
\lambda_1 = {} & \lambda_{\min}\left(\widetilde{P}\right),\ \lambda_2 = \lambda_{\max}\left(\widetilde{P}\right),\ \lambda_3 = \lambda_{\max}\left(\widetilde{Q}(\theta)\right), \\
\lambda_{i+3} = {} & \lambda_{\max}\left(\widetilde{S}_i(\theta)\right), \lambda_{i+5} = \lambda_{\max}\left(\widetilde{Z}_i(\theta)\right), \\
\widetilde{P} = {} & R^{-1/2}PR^{-1/2},\ \widetilde{Q}(t) = R^{-1/2}Q(t)R^{-1/2}, \\
& \widetilde{S}_i(t) = R^{-1/2}\widetilde{S}_i(t)R^{-1/2}, \widetilde{Z}_i(t) = R^{-1/2}Z_i(t)R^{-1/2},
\end{aligned}$$

$i = 1, 2$

Then, the system (6) is finite time stable with respect to (c_1, c_2, T_f, h_m, R).

Proof.

Let us consider the following Lyapunov–Krasovskii functional (LKF): $V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t))$ where

$$V_1(x(t)) = x^T(t)Px(t)$$

$$\begin{aligned}
V_2(x(t)) = {} & \int_{t-h_m}^{t} e^{\alpha(t-s)}x^T(s)Q(s)x(s)ds + \int_{t-h(t)}^{t}(s-t+h(t))e^{\alpha(t-s)}x^T(s)S_1(s)x(s)ds \\
& + \int_{t-h(t)}^{t}(s-t+h(t))e^{\alpha(t-s)}\dot{x}^T(s)S_2(s)\dot{x}(s)dsV_3(x(t)) \\
V_3(x(t)) = {} & \int_{-h_m}^{0}\int_{t+\delta}^{t} e^{\alpha(t-\theta)}x^T(\theta)Z_1(\theta)\dot{x}^T(\theta)d\theta d\delta + \int_{-h_m}^{0}\int_{t+\delta}^{t} e^{\alpha(t-\theta)}\dot{x}^T(\theta)Z_2(\theta)\dot{x}^T(\theta)d\theta d\delta
\end{aligned}$$

From this LKF, we obtain:

$$\dot{V}_1(x(t)) = 2x^T(t)P\dot{x}(t)$$

$$\begin{aligned}
\dot{V}_2(x(t)) = {} & x^T(t)Q(t)x(t) - e^{\alpha h_m}x^T(t-h_m)Q_2(t-h_m)x(t-h_m) \\
& + h(t)x^T(t)S_1(t)x(t) + \int_{t-h(t)}^{t}\left(-1+\dot{h}(t)\right)e^{\alpha(t-s)}x^T(s)S_1(s)x(s)ds \\
& + h(t)\dot{x}^T(t)S_2(t)\dot{x}(t) + \int_{t-h(t)}^{t}\left(-1+\dot{h}(t)\right)e^{\alpha(t-s)}\dot{x}^T(s)S_2(s)\dot{x}(s)ds \\
& + \alpha V_2(x(t)) \\
\leq {} & x^T(t)Q(t)x(t) - e^{\alpha h_m}x^T(t-h_m)Q_2(t-h_m)x(t-h_m) \\
& + h_m x^T(t)S_1(t)x(t) + (-1+\rho)\mu\int_{t-h(t)}^{t} x^T(s)S_1(s)x(s)ds \\
& + h_m\dot{x}^T(t)S_2(t)\dot{x}(t) + (-1+\rho)\mu\int_{t-h(t)}^{t}\dot{x}^T(s)S_2(s)\dot{x}(s)ds \\
& + \alpha V_2(x(t))
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(x(t)) = {} & h_m x^T(t)Z_1(t)x(t) - \int_{t-h_m}^{t} e^{\alpha(t-s)}x^T(s)Z_1(s)x(s)ds \\
& + h_m\dot{x}^T(t)Z_2(t)\dot{x}(t) - \int_{t-h_m}^{t} e^{\alpha(t-s)}\dot{x}^T(s)Z_2(s)\dot{x}(s)ds \\
& + \alpha V_3(x(t)) \\
\leq {} & h_m x^T(t)Z_1(t)x(t) - \int_{t-h_m}^{t} x^T(s)Z_1(s)x(s)ds \\
& + h_m\dot{x}^T(t)Z_2(t)\dot{x}(t) - \int_{t-h_m}^{t}\dot{x}^T(s)Z_2(s)\dot{x}(s)ds \\
& + \alpha V_3(x(t))
\end{aligned}$$

where $\mu = \begin{cases} 1 & \text{if } \dot{h}(t) \leq 1 \\ e^{\alpha h_m} & \text{if } \dot{h}(t) \geq 1 \end{cases}$.

Let us consider the Newton–Leibniz formula with the matrices $N_j(t), j = 1,2,3,4,5$, and $T_i(t), i = 1,2,3,4,5,6$, the following relations are true:

$$\begin{aligned}
0 = 2\Big[& x^T(t)N_1(t) + x^T(t-h(t))N_2(t) + x^T(t-h_m)N_3(t) \\
& + \dot{x}^T(t)N_4(t) + \left(\int_{t-h(t)}^{t}\dot{x}^T(s)\right)N_5(t)\Big]\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t}\dot{x}(s)ds\right]
\end{aligned} \tag{10}$$

$$\begin{aligned}
0 = 2\Big[& x^T(t)T_1(t) + x^T(t-h(t))T_2(t) + x^T(t-h_m)T_3(t) \\
& + \dot{x}^T(t)T_4(t) + \left(\int_{t-h(t)}^{t} x^T(s)T_5(s)ds\right) \\
& + \left(\int_{t-h(t)}^{t}\dot{x}^T(s)T_6(s)ds\right)\Big]\Big[\dot{x}(t) - A(t)x(t) \\
& - A_d(t)x(t-h(t))\Big]
\end{aligned} \tag{11}$$

Then:

$$
\begin{aligned}
\dot{V}(x(t)) \leq\ & x^T(t)\Big(-T_1(t)A(t) - A^T(t)T_1^T(t) + N_1(t) + Q(t) \\
& + h_m S_1(t) + h_m Z_1(t) - \alpha P\Big)x(t) \\
& + x^T(t-h(t))\Big(-T_2(t)A_d(t) - A_d^T(t)T_2^T(t) \\
& - N_2(t) - N_2^T(t)\Big)x(t-h(t)) \\
& + x^T(t-h_m)\big(-e^{\alpha h_m}Q(t-h)_m\big)x(t-h_m) \\
& + \dot{x}^T(t)\Big(T_4(t) + T_4^T(t) + h_m S_2(t) \\
& + h_m Z_2(t)\Big)\dot{x}(t) + sym\Big(x^T(t)\Big(-N_1(t) + N_2^T(t) \\
& - T_1(t)A_d(t) - A^T(t)T_2^T(t)\Big)x(t-h(t))\Big) \\
& + sym\big(x^T(t)\big(N_3^T(t) - A^T(t)T_3^T(t)\big)x(t-h)\big) \\
& + sym\big(x^T(t)\big(N_4^T(t) + T_1(t) - A^T(t)T_4^T(t) + P\big)\dot{x}(t)\big) \\
& + sym\big(x^T(t-h(t))\big(-N_3^T(t) - A_d^T(t)T_3^T(t)\big)x(t-h_m)\big) \\
& + sym\big(x^T(t-h(t))\big(T_2(t) - N_4^T(t) + A_d^T(t)T_4^T(t)\big)\dot{x}(t)\big) \\
& + sym\big(x^T(t-h_m)T_3^T(t)\dot{x}(t)\big) \\
& + sym\left(\int_{t-h(t)}^{t} x^T(t)\big(-A^T(t)T_5^T(t)\big)x(s)ds\right) \\
& + sym\left(\int_{t-h(t)}^{t} x^T(t)\big(-N_1(t) + N_5^T(t)\right. \\
& \left. - A^T(t)T_6^T(t)\big)\dot{x}(s)ds\right) \\
& + sym\left(\int_{t-h(t)}^{t} x^T(t-h(t))\big(-A_d^T(t)T_5^T(t)\big)x(s)ds\right) \\
& + sym\left(\int_{t-h(t)}^{t} x^T(t-h(t))\big(-N_2(t) - N_5^T(t) - A_d^T(t)T_6^T(t)\big)\dot{x}(s)ds\right) \\
& + sym\left(x^T(t-h_m)(-N_3(t))\int_{t-h(t)}^{t}\dot{x}(s)ds\right) \\
& + sym\left(\int_{t-h(t)}^{t}\dot{x}^T(t)\big(T_5^T(s)\big)x(s)ds\right) \\
& + sym\left(\int_{t-h(t)}^{t}\dot{x}^T(t)\big(T_6^T(s) - N_4(t)\big)\dot{x}(s)ds\right) \\
& + \left(\int_{t-h(t)}^{t}\dot{x}(s)\right)^T\big(-N_5(t) - N_5^T(t)\big)\left(\int_{t-h(t)}^{t}\dot{x}(s)\right) \\
& + \int_{t-h(t)}^{t} x^T(s)\big((-1+\rho)\mu S_1(s) - Z_1(s)\big)x(s)ds \\
& + \int_{t-h(t)}^{t}\dot{x}^T(s)\big((-1+\rho)\mu S_2(s) - Z_2(s)\big)\dot{x}(s)ds + \alpha V(x(t))
\end{aligned}
$$

It is straightforward:

$$
\begin{aligned}
&\left(\int_{t-h(t)}^{t}\dot{x}(s)\right)^T\big(-N_5(t) - N_5^T(t)\big)\left(\int_{t-h(t)}^{t}\dot{x}(s)\right) \\
&= \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} -N_5(t) - N_5^T(t) & N_5(t) + N_5^T(t) \\ * & -N_5(t) - N_5^T(t) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}
\end{aligned} \tag{12}
$$

Also, for all positive semi-definite matrices $X(t)$, this inequality holds:

$$
h_m\xi^T(t)X(t)\xi(t) - \int_{t-h(t)}^{t}\xi^T(t)X(t)\xi(t)ds \geq 0 \tag{13}
$$

where $\xi(t) = \left[x^T(t)\ x^T(t-h(t))\ x^T(t-h_m)\ \dot{x}^T(t)\right]^T$.

Taking into account (12) and (13), we obtain:

$$
\begin{aligned}
\dot{V}(x(t)) \leq\ & \xi^T(t)\Pi(t)\xi(t) + \int_{t-h(t)}^{t}\eta^T(s,t)\Psi(s,t)\eta(s,t)ds \\
& + \alpha V(x(t))
\end{aligned}
$$

where $\Pi(t)$ is given by (7) and $\Psi(s,t)$ by (8), and

$$
\eta(s,t) = \left[x^T(t)\ \ x^T(t-h(t))\ \ x^T(t-h_m)\ \ \dot{x}^T(t)\ \ x^T(s)\ \ \dot{x}^T(s)\right]
$$

Since the matrices (7) and (8) holds, we have $\Pi(t) < 0$ and $\Psi(s,t) \leq 0$.

Then, another straightforward conclusion:

$$
\dot{V}(x(t)) < \alpha V(x(t))
$$

Integrating both sides from 0 to $t \in [0, T_f]$:

$$
V(x(t)) < e^{\alpha t}V(x(0)) \leq e^{\alpha T_f}V(x(0))
$$

Also, we have:

$$
\begin{aligned}
V(x(0)) =\ & x^T(0)Px(0) + \int_{-h_m}^{0} e^{-\alpha s}x^T(s)Q(s)x(s)ds \\
& + \int_{-h(0)}^{0}(s+h(0))e^{-\alpha s}x^T(s)S_1(s)x(s)ds \\
& + \int_{-h(0)}^{0}(s+h(0))e^{-\alpha s}x^T(s)S_2(s)x(s)ds \\
& + \int_{-h_m}^{0}\int_{\delta}^{0} e^{-\alpha s}x^T(s)Z_1(s)x(s)dsd\delta \\
& + \int_{-h_m}^{0}\int_{\delta}^{0} e^{-\alpha s}\dot{x}^T(s)Z_2(s)\dot{x}(s)dsd\delta \\
<\ & \lambda_{\max}\left(\widetilde{P}\right)x^T(0)Rx(0) \\
& + \left(h_m e^{\alpha h_m}\lambda_{\max}\left(\widetilde{Q}(\theta)\right) + \sum_{i=1}^{2}\left(h_m^2 e^{\alpha h_m}\lambda_{\max}\left(\widetilde{S}_i(\theta)\right)\right.\right. \\
& \left.\left. + \frac{1}{2}h_m^2\lambda_{\max}\left(\widetilde{Z}_i(\theta)\right)\right)\right)\sup_{-h_m \leq \theta \leq 0}\left\{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\right\} \\
<\ & \left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + h_m^2 e^{\alpha h_m}(\lambda_4 + \lambda_5) + \frac{1}{2}h_m^2(\lambda_6 + \lambda_7)\right)c_1
\end{aligned}
$$

And $\forall t \in [0, T_f]$: $V(x(t)) \geq \lambda_{\min}\left(\widetilde{P}\right)x^T(t)Rx(t)$, which implies that

$$\Pi(t,s)=\begin{bmatrix}\Pi_{11}(t) & \Pi_{12}(t) & \Pi_{13}(t) & \Pi_{14}(t) & -h_mA^T(t)T_5^T(s) & -h_mN_1(t)+h_mN_5^T(t)-h_mA^T(t)T_6^T(s)\\ * & \Pi_{22}(t) & \Pi_{23}(t) & \Pi_{24}(t) & -h_mA_d^T(t)T_5^T(s) & -h_mN_2(t)-h_mN_5^T(t)-h_mA_d^T(t)T_6^T(s)\\ * & * & \Pi_{33}(t) & \Pi_{34}(t) & 0 & -h_mN_3(t)\\ * & * & * & \Pi_{44}(t) & h_mT_5^T(s) & h_mT_6^T(s)-h_mN_4(t)\\ * & * & * & * & (-1+\rho)\mu h_mS_1(s)-h_mZ_1(s) & 0\\ * & * & * & * & * & (-1+\rho)\mu h_mS_2(s)-h_mZ_1(s)\end{bmatrix}<0, \tag{15}$$

$$x^T(t)Rx(t)<\frac{e^{\alpha T_f}}{\lambda_1}\left(\lambda_2+h_me^{\alpha h_m}\lambda_3+h_m^2e^{\alpha h_m}(\lambda_4+\lambda_5)+\frac{1}{2}h_m^2(\lambda_6+\lambda_7)\right)c_1<c_2$$

and the proof of the theorem has been completed.

To reduce the number of LMIs, variables and remove the constraint $\Psi(t,s)\le 0$, we set:

$$X(t)=-\Theta^T(t,s)(\Omega(s))^{-1}\Theta(t,s) \tag{14}$$

where

$$\Theta^T(t,s)=\begin{bmatrix}-A^T(t)T_5^T(s) & -N_1(t)+N_5^T(t)-A^T(t)T_6^T(s)\\ -A_d^T(t)T_5^T(s) & -N_2(t)-N_5^T(t)-A_d^T(t)T_6^T(s)\\ 0 & -N_3(t)\\ T_5^T(s) & T_6^T(s)-N_4(t)\end{bmatrix},$$

$$\Omega(s)=\begin{bmatrix}(-1+\rho)\mu S_1(s)-Z_1(s) & 0\\ * & (-1+\rho)\mu S_2(s)-Z_2(s)\end{bmatrix}$$

Then, the following proposition is given to assure the finite-time stability of the system (6).

Proposition 1. If there exist some symmetric positive definite matrices $P=P^T>0$, $Q(t)=Q^T(t)>0$, $S_i(t)=S_i^T(t)>0$, and $Z_i(t)=Z_i^T(t)>0$, $i=1,2$; appropriately sized matrices $T_i(t), i=1,2,3,4,5,6$ and $N_j(t), j=1,2,3,4,5,$; and a scalar $\alpha\ge 0$ satisfying:

$$\frac{e^{\alpha T_f}}{\lambda_1}\left(\lambda_2+h_me^{\alpha h_m}\lambda_3+h_m^2e^{\alpha h_m}(\lambda_4+\lambda_5)+\frac{1}{2}h_m^2(\lambda_6+\lambda_7)\right)c_1<c_2 \tag{16}$$

then, the system (6) is finite time stable with regard to (c_1,c_2,T_f,h_m,R).

Proof.

To obtain the condition (15), we use the Schur complement and Equations (7) and (14).

From Proposition 1, we can get the following results that ensure the finite-time stability of system (6):

Theorem 2. *If there exist symmetric positive definite matrices* $\widetilde{P}=\widetilde{P}^T>0$, $Q_j=Q_j^T>0$, $S_{ij}=S_{ij}^T>0$, *and* $Z_{ij}=Z_{ij}^T>0$; *appropriately sized matrices* W, N_{ij}, $i=1,2,3,4,5, j=1,2,\ldots,r$, *and* T_{ij} $i=1,2,3,4,5,6, j=1,2,\ldots,r$; *and scalars* $\alpha\ge 0, \lambda_i>0$, $i=1,2,3,4,5,6,7$, *satisfying the following conditions*:

$$0<\lambda_1I<\widetilde{P}<\lambda_2I \tag{17}$$

$$0<\widetilde{Q}_j<\lambda_3I \tag{18}$$

$$0<\widetilde{S}_{1j}<\lambda_4I \tag{19}$$

$$0<\widetilde{S}_{2j}<\lambda_5I \tag{20}$$

$$0<\widetilde{Z}_{1j}<\lambda_6I \tag{21}$$

$$0<\widetilde{Z}_{2j}<\lambda_7I \tag{22}$$

$$\Pi_{i,i,i,k} + (d-1)W < 0 \tag{23}$$

$$\Pi_{i,i,j,k} + \frac{1}{3}(d-3)W < 0,\ i \neq j \tag{24}$$

$$\Pi_{i,j,l,k} - W < 0,\ i < j < l \tag{25}$$

$$\frac{e^{\alpha T_f}}{\lambda_1}\left(\lambda_2 + h_m e^{\alpha h_m}\lambda_3 + h_m^2 e^{\alpha h_m}(\lambda_4+\lambda_5) + \frac{1}{2}h_m^2(\lambda_6+\lambda_7)\right)c_1 < c_2 \tag{26}$$

then, the system (6) is finite time stable with respect to (c_1, c_2, T_f, h_m, R) and

$$\Pi_{i,j,l,k} = \begin{bmatrix} \Pi_{11}^{i,j,l,k} & \Pi_{12}^{i,j,l,k} & \Pi_{13}^{i,j,l,k} & \Pi_{14}^{i,j,l,k} & -h_m\Pi_{15}^{i,j,l,k} & -h_m\Pi_{16}^{i,j,l,k} \\ * & \Pi_{22}^{i,j,l,k} & \Pi_{23}^{i,j,l,k} & \Pi_{24}^{i,j,l,k} & -h_m\Pi_{25}^{i,j,l,k} & -h_m\Pi_{26}^{i,j,l,k} \\ * & * & \Pi_{33}^{i,j,l,k} & \Pi_{34}^{i,j,l,k} & h_m\Pi_{35}^{i,j,l,k} & -h_m\Pi_{36}^{i,j,l,k} \\ * & * & * & \Pi_{44}^{i,j,l,k} & h_m\Pi_{45}^{i,j,l,k} & h_m\Pi_{46}^{i,j,l,k} \\ * & * & * & * & h_m\Pi_{55}^{i,j,l,k} & h_m\Pi_{56}^{i,j,l,k} \\ * & * & * & * & * & h_m\Pi_{66}^{i,j,l,k} \end{bmatrix},$$

$$\begin{aligned}
\Pi_{11}^{i,j,l,k} &= -T_{1j}A_i - A_i^T T_{1j}^T + N_i + N_{1i}^T + Q_i + h_m S_{1i} + h_m Z_{1i} \\
&\quad - \alpha P - N_{5i} - N_{5i}^T, \\
\Pi_{12}^{i,j,l,k} &= -N_{1i} + N_{2i}^T - T_{1j}A_{di} - A_i^T T_{2j}^T + N_{5i} + N_{5i}^T, \\
\Pi_{13}^{i,j,l,k} &= N_{3i}^T - A_i^T T_{3j}^T, \\
\Pi_{14}^{i,j,l,k} &= N_{4i}^T + T_{1i} - A_i^T T_{4j}^T + P,\ \Pi_{15}^{i,j,l,k} = -A_i^T T_{5l}^T, \\
\Pi_{16}^{i,j,l,k} &= -N_{1i} - N_{5i}^T - A_i^T T_{6l}^T, \\
\Pi_{22}^{i,j,l,k} &= -T_{2j}A_{di} - A_{di}^T T_{2j}^T - N_{2i} - N_{2i}^T - N_{5i} - N_{5i}^T, \\
\Pi_{23}^{i,j,l,k} &= -N_{3i}^T - A_{di}^T T_{3j}^T, \\
\Pi_{24}^{i,j,l,k} &= T_{2i} - N_{4i}^T - A_{di}^T T_{4j}^T,\ \Pi_{25}^{i,j,l,k} = -A_{di}^T T_{5l}^T, \\
\Pi_{26}^{i,j,l,k} &= -N_{2i} + N_{5i}^T - A_{di}^T T_{6l}^T, \\
\Pi_{33}^{i,j,l,k} &= -e^{\alpha h_m} Q_k,\ \Pi_{34}^{i,j,l,k} = T_{3i},\ \Pi_{35}^{i,j,l,k} = 0, \\
\Pi_{36}^{i,j,l,k} &= -N_{3i}, \\
\Pi_{44}^{i,j,l,k} &= T_{4i} + T_{4i}^T + h_m S_{2i} + h_m Z_{2i},\ \Pi_{45}^{i,j,l,k} = T_{5l}^T, \\
\Pi_{46}^{i,j,l,k} &= T_{6l}^T - N_{4i}, \\
\Pi_{55}^{i,j,l,k} &= (-1+\rho)\mu h_m S_{1l} - h_m Z_{1l},\ \Pi_{56}^{i,j,l,k} = 0, \\
\Pi_{66}^{i,j,l,k} &= (-1+\rho)\mu h_m S_{2l} - h_m Z_{2l}, \\
\widetilde{P} &= R^{-1/2} P R^{-1/2},\ \widetilde{Q}_j = R^{-1/2} Q_j R^{-1/2}, \\
\widetilde{S}_{ij}(t) &= R^{-1/2} S_{ij}(t) R^{-1/2}, \\
\widetilde{Z}_{ij}(t) &= R^{-1/2} Z_{ij}(t) R^{-1/2},\ i = 1,2, j = 1, \ldots., r.
\end{aligned}$$

Proof.

Taking into account Lemma 1 and the matrix (15), we can write:

$$\begin{aligned}
\Pi(t,s) &= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r \sum_{k=1}^r \mu_i(z(t))\mu_j(z(t))\mu_l(z(t))\mu_k(z(t-h_m))\Pi_{i,j,l,k} \\
&= \sum_{k=1}^r \mu_k(z(t-h_m)) \Bigg\{ \sum_{i=1}^r \mu_i^3(z(t))\Pi_{i,i,i,k} \\
&\quad + 3\sum_{i=1}^r \sum_{j\neq i}^r \mu_i^2(z(t))\mu_j(z(t))\Pi_{i,i,j,k} \\
&\quad + 6\sum_{i=1}^r \sum_{j>i}^r \sum_{l>j}^r \mu_i(z(t))\mu_j(z(t))\mu_l(z(t))\Pi_{i,j,l,k} \Bigg\} \\
&\leq \sum_{k=1}^r \mu_k(z(t-h_m)) \Bigg\{ \sum_{i=1}^r \mu_i^3(z(t))(\Pi_{i,i,i,k} + (d-1)W) \\
&\quad + 3\sum_{i=1}^r \sum_{j\neq i}^r \mu_i^2(z(t))\mu_j(z(t))\left(\Pi_{i,i,j,k} + \frac{1}{3}(d-3)W\right) \Bigg\}
\end{aligned}$$

If the conditions (23), (24), and (25) are verified, the condition (15) is, in turn, verified. Thus, the rest of the proof can be easily deduced.

Remark 1. To the best of our method, there are no results dealing with finite-time stabilization of T–S fuzzy approach for delayed nonlinear systems. Considering what it has not been covered by literature research, our method presents a novel analysis and design technique to improve the performance over finite-time intervals with less conservative results for closed-loop T–S fuzzy delayed systems.

Remark 2. It should be noted that the number of LMIs in Theorem 2 is $r^2 + r^2(r-1) + \left(\sum_{i=3}^r \sum_{j=i}^r r(r-j+1)\right) + 1,.$ In [29, 30, 33], there are $r^4 + 2r + 2$ LMIs, which are larger than in our approach. So, this is a huge benefit of our work. Moreover, the total of LMIs number can be reduced to $r + \sum_{i=2}^r (r-i+1) + 1$ if we suppose that $Q_i = Q$, $S_{1i} = S_1, S_{2i} = S_2, Z_{1i} = Z_1, Z_{2i} = Z_2$, $i = 1,2,\ldots,r$, and use as slack variables $T_5(t), T_6(t)$ instead of $T_5(s), T_6(s)$, that is, $T_{5l} = T_{5i}$ and $T_{6l} = T_{6i}$. In this case, the fuzzy LKF $V(x(t))$ is transformed into a single LKF.

In order to show the effect of LMI reduction, a corollary is introduced.

Corollary 1. If there exist symmetric positive definite matrices $\widetilde{P}=\widetilde{P}^T>0$, $Q=Q^T>0$, $S_i=S_i^T>0$, and $Z_i=Z_i^T>0$, $i=1,2$; appropriately sized matrices W, M_{ij}, $i=1,2,3,4,5, j=1,2,\ldots,r$, T_{ij}, $i=1,2,3,4,5,6, j=1,2,\ldots,r$; and scalars $\alpha\geq 0, \lambda_i>0$, $i=1,2,3,4,5,6,7$, satisfying the following conditions:

$$0<\lambda_1 I<\widetilde{P}<\lambda_2 I \tag{27}$$

$$0<\widetilde{Q}_j<\lambda_3 I \tag{28}$$

$$0<\widetilde{S}_1<\lambda_4 I \tag{29}$$

$$0<\widetilde{S}_2<\lambda_5 I \tag{30}$$

$$0<\widetilde{Z}_1<\lambda_6 I \tag{31}$$

$$0<\widetilde{Z}_2<\lambda_7 I \tag{32}$$

$$\Pi_{i,i}+(d-1)W<0 \tag{33}$$

$$\Pi_{i,j}+\Pi_{j,i}-W<0,\ i<j \tag{34}$$

$$\frac{e^{\alpha T_f}}{\lambda_1}\left(\lambda_2+h_m e^{\alpha h_m}\lambda_3+h_m^2 e^{\alpha h_m}(\lambda_4+\lambda_5)+\frac{1}{2}h_m^2(\lambda_6+\lambda_7)\right)c_1<c_2 \tag{35}$$

then, the system (6) is finite time stable with respect to (c_1, c_2, T_f, h_m, R) where

$$\Pi_{i,j}=\begin{bmatrix} \Pi_{11}^{i,j}, & \Pi_{12}^{i,j} & \Pi_{13}^{i,j} & \Pi_{14}^{i,j} & h_m\Pi_{15}^{i,j} & h_m\Pi_{16}^{i,j} \\ * & \Pi_{22}^{i,j} & \Pi_{23}^{i,j} & \Pi_{24}^{i,j} & h_m\Pi_{25}^{i,j} & h_m\Pi_{26}^{i,j} \\ * & * & \Pi_{33}^{i,j} & \Pi_{34}^{i,j} & h_m\Pi_{35}^{i,j} & h_m\Pi_{36}^{i,j} \\ * & * & * & \Pi_{44}^{i,j} & h_m\Pi_{45}^{i,j} & h_m\Pi_{46}^{i,j} \\ * & * & * & * & h_m\Pi_{55}^{i,j} & h_m\Pi_{56}^{i,j} \\ * & * & * & * & * & h_m\Pi_{66}^{i,j} \end{bmatrix},$$

$$\Pi_{11}^{i,j}=-T_{1j}A_i-A_i^T T_{1j}^T+N_i+N_{1i}^T+Q+h_m S_1+h_m Z_1-\alpha P$$
$$-N_{5i}-N_{5i}^T,$$
$$\Pi_{12}^{i,j}=-N_{1i}+N_{2i}^T-T_{1j}A_{di}-A_i^T T_{2j}^T+N_{5i}+N_{5i}^T,$$
$$\Pi_{13}^{i,j}=N_{3i}^T-A_i^T T_{3j}^T, \Pi_{14}^{i,j}=N_{4i}^T+T_{1i}-A_i^T T_{4j}^T+P,$$
$$\Pi_{15}^{i,j}=-A_i^T T_{5i}^T, \Pi_{16}^{i,j}=-N_{1i}-N_{5i}^T-A_i^T T_{6i}^T,$$
$$\Pi_{22}^{i,j}=-T_{2j}A_{di}-A_{di}^T T_{2j}^T-N_{2i}-N_{2i}^T-N_{5i}-N_{5i}^T,$$
$$\Pi_{23}^{i,j}=-N_{3i}^T-A_{di}^T T_{3j}^T, \Pi_{24}^{i,j}=T_{2i}-N_{4i}^T-A_{di}^T T_{4j}^T,$$
$$\Pi_{25}^{i,j}=-A_{di}^T T_{5i}^T, \Pi_{26}^{i,j}=-N_{2i}+N_{5i}^T-A_{di}^T T_{6i}^T,$$
$$\Pi_{33}^{i,j}=-e^{\alpha h_m}Q,\ \Pi_{34}^{i,j}=T_{3i},\ \Pi_{35}^{i,j}=0,\ \Pi_{36}^{i,j}=-N_{3i},$$
$$\Pi_{44}^{i,j}=T_{4i}+T_{4i}^T+h_m S_2+h_m Z_2, \Pi_{45}^{i,j}=T_{5i}^T,\ \Pi_{46}^{i,j}=T_{6i}^T-N_{4i},$$
$$\Pi_{55}^{i,j}=(-1+\rho)\mu h_m S_1-h_m Z_1, \Pi_{56}^{i,j}=0,$$
$$\Pi_{66}^{i,j}=(-1+\rho)\mu h_m S_2-h_m Z_2,$$
$$\widetilde{P}=R^{-1/2}PR^{-1/2}, \widetilde{Q}=R^{-1/2}QR^{-1/2},$$
$$\widetilde{S}_i(t)=R^{-1/2}S_i(t)R^{-1/2},$$
$$\widetilde{Z}_i(t)=R^{-1/2}Z_i(t)R^{-1/2}, i=1,2.$$

Remark 3. Compared with [30], the steps presented here provide a systematic method to ensure the finite-time stability of the system (6) and lead to more flexible matrices by introducing additional variables $T_6(t)$, $T_5(t), S_1(t), S_2(t)$, and $Z_1(t)$, resulting in less conservatism. Then, these variables are introduced so as to provide additional degrees of freedom to the resulting optimization problem, thus assuring the feasibility of the controller. In fact, the set of slack variables adopted in the problem formulation is responsible for softening and limit the existence of open-loop unstable and integrating modes.

Remark 4. As a note, reduction refers to applying simple rules to a series of matrices to change them into a simpler form. Then, any feasible LMI can be reduced by eliminating implicit equality constraints. Therefore, LMI reduction techniques aim to reduce problem complexity and calculation time while simultaneously maintaining suitable solution accuracy.

4 | DELAY-DEPENDENT FINITE-TIME STABILIZATION

This section deals with new delay dependent conditions that are developed to ensure the finite-time stabilization of the closed-loop system (5) with the memory PDC controller:

$$u(t)=\sum_{j=1}^{r}\mu_j(z(t))\,K_jx(t)=K(t)x(t) \tag{36}$$

where $K_j, j=1,\ldots,r$, are the state feedback gain matrices to be deduced. Using (36), the system (5) can be reformulated:

$$\dot{x}(t)=\overline{A}(t)x(t)+A_d(t)x(t-h(t)) \tag{37}$$

where $\overline{A}(t)=A(t)+B(t)K(t)$, $B(t)=\sum_{i=1}^{r}\mu_i(z(t))B_i$..

Theorem 3. Given the scalars $\eta_1\neq 0,\eta_2,\eta_3\neq 0,\eta_4\neq 0,\eta_5,$ and η_6, the system (37) is finite time stabilizable with respect to (c_1,c_2,T_f,h_m,R), if there exist symmetric positive definite matrices $\overline{P}=\overline{P}^T>0$, $\overline{Q}(t)=\overline{Q}^T(t)>0$, $\overline{S}_i(t)=\overline{S}_i^T(t)>0$, and $\overline{Z}_i(t)=\overline{Z}_i^T(t)>0$, $i=1,2$; appropriately sized matrices $M_i(t)$, $i=1,2,3,4,5$; and a scalar $\alpha\geq 0$ satisfying:

$$\Delta(t)=\begin{bmatrix}\Delta_{11}(t) & \Delta_{12}(t) & \Delta_{13}(t) & \Delta_{14}(t) & h_m\Delta_{15}(t) & h_m\Delta_{16}(t)\\ * & \Delta_{22}(t) & \Delta_{23}(t) & \Delta_{24}(t) & h_m\Delta_{25}(t) & h_m\Delta_{26}(t)\\ * & * & \Delta_{33}(t) & \Delta_{34}(t) & h_m\Delta_{35}(t) & h_m\Delta_{36}(t)\\ * & * & * & \Delta_{44}(t) & h_m\Delta_{45}(t) & h_m\Delta_{46}(t)\\ * & * & * & * & h_m\Delta_{55}(t) & h_m\Delta_{56}(t)\\ * & * & * & * & * & h_m\Delta_{66}(t)\end{bmatrix}<0 \tag{38}$$

$$\begin{aligned}&\left(\frac{\eta_1^{-2}}{\lambda_{\min}\left(\widehat{P}\right)}+\frac{1}{\left(\lambda_{\min}\left(\overline{P}\right)\right)^2}\left(h_me^{\alpha h_m}\lambda_{\max}\left(\widehat{Q}(\theta)\right)\right.\right.\\&+h_m^2e^{\alpha h_m}\left(\eta_1^{-2}\lambda_{\max}\left(\widehat{S}_1(\theta)\right)+\eta_4^{-2}\lambda_{\max}\left(\widehat{S}_2(\theta)\right)\right)\Big)\\&\left.+\frac{1}{2}h_m^2\left(\eta_1^{-2}\lambda_{\max}\left(\widehat{Z}_1(\theta)\right)+\eta_4^{-2}\lambda_{\max}\left(\widehat{Z}_2(\theta)\right)\right)\right)e^{\alpha T_f}c_1<c_2\frac{\eta_1^{-2}}{\lambda_{\max}\left(\widehat{P}\right)}\end{aligned} \tag{39}$$

where

$$\begin{aligned}&\widehat{P}=R^{1/2}\overline{P}R^{1/2},\ \widehat{Q}(\theta)=R^{-1/2}\overline{Q}(\theta)R^{-1/2},\\&\widehat{S}_i(\theta)=R^{-1/2}\overline{S}_i(\theta)R^{-1/2},\ \widehat{Z}_i(\theta)=R^{-1/2}\overline{Z}_i(\theta)R^{-1/2},\\&\forall i=1,2,\\&\Delta_{11}(t)=-\eta_1\overline{A}(t)\overline{P}-\eta_1\overline{PA}^T(t)+M_1(t)+M_1^T(t)+\overline{Q}(t)\\&\qquad+h_m\overline{S}_1(t)+h_m\overline{Z}_1(t)-\alpha\overline{P}-M_5(t)-M_5^T(t),\\&\Delta_{12}(t)=-\eta_2M_1(t)+\eta_2M_2^T(t)-\eta_2A_d(t)\overline{P}-\eta_1\overline{PA}^T(t)\\&\qquad+\eta_2M_5(t)+\eta_2M_5^T(t),\\&\Delta_{13}(t)=M_3^T(t)-\eta_1\eta_3\overline{PA}^T(t),\\&\Delta_{14}(t)=\eta_4M_4^T(t)+\eta_1\eta_4\overline{P}-\eta_1\overline{PA}^T(t)+\eta_4\overline{P},\\&\Delta_{15}(t)=-\eta_1\eta_5\overline{PA}^T(t),\\&\Delta_{16}(t)=-\eta_4M_1(t)+\eta_4M_5^T(t)-\eta_1\eta_6\overline{PA}^T(t),\\&\Delta_{22}(t)=-\eta_2A_d(t)\overline{P}-\eta_2\overline{P}A_d^T(t)-\eta_2^2M_2(t)-\eta_2^2M_2^T(t)\\&\qquad-\eta_2^2M_5(t)-\eta_2^2M_5^T(t),\\&\Delta_{23}(t)=-\eta_2M_3^T(t)-\eta_2\eta_3\overline{P}A_d^T(t),\\&\Delta_{24}(t)=\eta_4\overline{P}-\eta_2\eta_4M_4^T(t)-\eta_2\overline{P}A_d^T(t),\\&\Delta_{25}(t)=-\eta_2\eta_5\overline{P}A_d^T(t),\\&\Delta_{26}(t)=-\eta_2\eta_4M_2(t)-\eta_2\eta_4M_5^T(t)-\eta_2\eta_6\overline{P}A_d^T(t),\\&\Delta_{33}(t)=-e^{\alpha h_m}\overline{Q}(t-h_m),\ \Delta_{34}(t)=\eta_3\eta_4\overline{P},\ \Delta_{35}(t)=0,\\&\Delta_{36}(t)=-\eta_4M_3(t),\\&\Delta_{44}(t)=2\eta_4\overline{P}+h_m\overline{S}_2(t)+h_m\overline{Z}_2(t),\ \Delta_{45}(t)=\eta_4\eta_5\overline{P},\\&\Delta_{46}(t)=\eta_4^2\overline{P}-\eta_4^2M_4(t),\\&\Delta_{55}(t)=(-1+\rho)\mu\overline{S}_1(s)-\overline{Z}_1(s),\ \Delta_{56}(t)=0,\\&\Delta_{66}(t)=(-1+\rho)\mu\overline{S}_2(s)-\overline{Z}_2(s).\end{aligned}$$

Proof.

Let $L_i(t)=T_i^{-1}(t), \forall i=1,2,\ldots,6$, the following matrices are defined:

$$\begin{aligned}&\overline{P}=L_1(t)PL_1^T(t),\ \overline{Q}(t)=L_1(t)Q(t)L_1^T(t),\\&\overline{S}_1(t)=L_1(t)S_1(t)L_1^T(t),\\&\overline{S}_2(t)=L_4(t)S_2(t)L_4^T(t),\\&\overline{Z}_1(t)=L_1(t)Z_1(t)L_1^T(t),\ \overline{Z}_2(t)=L_4(t)Z_2(t)L_4^T(t),\\&M_i(t)=L_1(t)N_i(t)L_1^T(t),\ i=1,\ldots,5\end{aligned}$$

Then, the condition (38) is obtained pre- and post-multiplying the matrix $\Pi(t,s)$ by $diag\{L_1(t),L_2(t),L_1(t-h_m),L_4(t),L_1(s)L_4(s)\}$ and its transpose, respectively, and taking the following changes of variables:

$$L_1(t)=\eta_1\overline{P},\ L_2(t)=\eta_2L_1(t),\ L_3(t)=\eta_3^{-1}L_1(t-h_m),\ L_4(t)=\eta_4L_1(t),\ L_5(t)=\eta_5^{-1}L_1(t),\ L_6(t)=\eta_6^{-1}L_1(t)$$

Using the adopted LKF, the condition (39) can be obtained where:

$$V_1(x(t)) = \eta_1^{-2} x^T(t) \overline{P}^{-1} x(t)$$

$$V_2(x(t)) = \eta_1^{-2} \int_{t-h_m}^{t} e^{\alpha(t-s)} x^T(s) \overline{P}^{-1} \overline{Q}(s) \overline{P}^{-1} x(s) ds$$
$$+ \eta_1^{-2} \int_{t-h(t)}^{t} (s - t + h(t)) e^{\alpha(t-s)} x^T(s) \overline{P}^{-1} \overline{S}_1(s) \overline{P}^{-1} x(s) ds$$
$$+ \eta_4^{-2} \int_{t-h(t)}^{t} (s - t + h(t)) e^{\alpha(t-s)} \dot{x}^T(s) \overline{P}^{-1} \overline{S}_2(s) \overline{P}^{-1} \dot{x}(s) ds$$

$$V_3(x(t)) = \eta_1^{-2} \int_{-h_m}^{0} \int_{t+\delta}^{t} e^{\alpha(t-\theta)} x^T(\theta) \overline{P}^{-1} \overline{Z}_1(\theta) \overline{P}^{-1} x^T(\theta) d\theta d\delta$$
$$+ \eta_4^{-2} \int_{-h_m}^{0} \int_{t+\delta}^{t} e^{\alpha(t-\theta)} \dot{x}^T(\theta) \overline{P}^{-1} \overline{Z}_2(\theta) \overline{P}^{-1} \dot{x}^T(\theta) d\theta d\delta$$

Also, new delay-dependent sufficient conditions for the design of a stabilizing finite-time PDC controller (38) are given by this theorem:

Theorem 4. Given scalars $\eta_1 \neq 0, \eta_2, \eta_3 \neq 0, \eta_4 \neq 0, \eta_5,$ and η_6, the system (37) is finite time stabilizable with respect to (c_1, c_2, T_f, h_m, R), if there exist symmetric positive definite matrices $\overline{P} = \overline{P}^T > 0$, $\overline{Q}_j = \overline{Q}_j^T > 0$, $\overline{S}_{ij} = \overline{S}_{ij}^T > 0$, and $\overline{Z}_{ij} = \overline{Z}_{ij}^T > 0$, $i = 1, 2$; appropriately sized matrices $M_{ij}, Y_j,$ and $\overline{W}$, $i = 1, 2, 3, 4, 5, j = 1, \ldots, r$; and a scalar $\alpha \geq 0$ fulfilling the following conditions:

$$\Delta_{i,i,i,k} + (d-1)\overline{W} < 0 \tag{40}$$

$$\Delta_{i,i,j,k} + \frac{1}{3}(d-3)\overline{W} < 0, i \neq j \tag{41}$$

$$\Delta_{i,j,l,k} - \overline{W} < 0, i < j < l \tag{42}$$

$$\Bigg(\frac{\eta_1^{-2}}{\lambda_{\min}\left(\widehat{P}\right)} + \frac{1}{\left(\lambda_{\min}\left(\overline{P}\right)\right)^2} \Bigg(h_m e^{\alpha h_m} \lambda_{\max}\left(\widehat{Q}(\theta)\right)$$
$$+ h_m^2 e^{\alpha h_m} \left(\eta_1^{-2} \lambda_{\max}\left(\widehat{S}_1(\theta)\right) + \eta_4^{-2} \lambda_{\max}\left(\widehat{S}_2(\theta)\right) \right) \Bigg)$$
$$+ \frac{1}{2} h_m^2 \left(\eta_1^{-2} \lambda_{\max}\left(\widehat{Z}_1(\theta)\right) + \eta_4^{-2} \lambda_{\max}\left(\widehat{Z}_2(\theta)\right) \right) \Bigg) \Bigg) e^{\alpha T_f} c_1 < c_2 \frac{\eta_1^{-2}}{\lambda_{\max}\left(\widehat{P}\right)} \tag{43}$$

The state-feedback gain matrices are given by $K_j = Y_j \overline{P}^{-1}, i = 1, 2, \ldots, r$. where

$$\Delta_{i,j,l,k} = \begin{bmatrix} \Delta_{11}^{i,j,l,k} & \Delta_{12}^{i,j,l,k} & \Delta_{13}^{i,j,l,k} & \Delta_{14}^{i,j,l,k} & h_m\Delta_{15}^{i,j,l,k} & h_m\Delta_{16}^{i,j,l,k} \\ * & \Delta_{22}^{i,j,l,k} & \Delta_{23}^{i,j,l,k} & \Delta_{24}^{i,j,l,k} & h_m\Delta_{25}^{i,j,l,k} & h_m\Delta_{26}^{i,j,l,k} \\ * & * & \Delta_{33}^{i,j,l,k} & \Delta_{34}^{i,j,l,k} & h_m\Delta_{35}^{i,j,l,k} & h_m\Delta_{36}^{i,j,l,k} \\ * & * & * & \Delta_{44}^{i,j,l,k} & h_m\Delta_{45}^{i,j,l,k} & h_m\Delta_{46}^{i,j,l,k} \\ * & * & * & * & h_m\Delta_{55}^{i,j,l,k} & h_m\Delta_{56}^{i,j,l,k} \\ * & * & * & * & * & h_m\Delta_{66}^{i,j,l,k} \end{bmatrix},$$

$$\widehat{P} = R^{1/2} \overline{P} R^{1/2}, \ \widehat{Q}(\theta) = R^{-1/2} \overline{Q}(\theta) R^{-1/2},$$
$$\widehat{S}_i(\theta) = R^{-1/2} \overline{S}_i(\theta) R^{-1/2}, \widehat{Z}_i(\theta) = R^{-1/2} \overline{Z}_i(\theta) R^{-1/2}, \forall \mathrm{i} = 1, 2,$$
$$\Delta_{11}^{i,j,l,k} = -\eta_1 A_i \overline{P} - \eta_1 \overline{P} A_i^T - \eta_1 B_i Y_j - \eta_1 Y_j^T B_i^T + M_{1i} + M_{1i}^T$$
$$+ \overline{Q}_i + h_m \overline{S}_{1i} + h_m \overline{Z}_{1i} - \alpha \overline{P} - M_{5i} - M_{5i}^T,$$
$$\Delta_{12}^{i,j,l,k} = -\eta_2 M_{1i} + \eta_2 M_{2i}^T - \eta_2 A_{di} \overline{P} - \eta_1 \overline{P} A_i^T - \eta_1 Y_j^T B_i^T + \eta_2 M_5 + \eta_2 M_{5i}^T,$$
$$\Delta_{13}^{i,j,l,k} = M_{3i}^T - \eta_1 \eta_3 \overline{P} A_i^T - \eta_1 \eta_3 Y_j^T B_i^T, \ \Delta_{14}^{i,j,l,k} = \eta_4 M_{4i}^T + \eta_1 \eta_4 \overline{P}$$
$$- \eta_1 \overline{P} A_i^T - \eta_1 Y_j^T B_i^T + \eta_4 \overline{P},$$
$$\Delta_{15}^{i,j,l,k} = -\eta_1 \eta_5 \overline{P} A_i^T - \eta_1 \eta_5 Y_j^T B_i^T, \ \Delta_{16}^{i,j,l,k} = -\eta_4 M_{1i} + \eta_4 M_{5i}^T - \eta_1 \eta_6 \overline{P} A_i^T$$
$$- \eta_1 \eta_6 Y_j^T B_i^T,$$
$$\Delta_{22}^{i,j,l,k} = -\eta_2 A_{di} \overline{P} - \eta_2 \overline{P} A_{di}^T - \eta_2^2 M_{2i} - \eta_2^2 M_{2i}^T - \eta_2^2 M_{5i} - \eta_2^2 M_{5i}^T,$$
$$\Delta_{23}^{i,j,l,k} = -\eta_2 M_{3i}^T - \eta_2 \eta_3 \overline{P} A_{di}^T, \ \Delta_{24}^{i,j,l,k} = \eta_4 \overline{P} - \eta_2 \eta_4 M_{4i}^T - \eta_2 \overline{P} A_{di}^T,$$
$$\Delta_{25}^{i,j,l,k} = -\eta_2 \eta_5 \overline{P} A_{di}^T, \ \Delta_{26}^{i,j,l,k} = -\eta_2 \eta_4 M_{2i} - \eta_2 \eta_4 M_{5i}^T - \eta_2 \eta_6 \overline{P} A_{di}^T,$$
$$\Delta_{33}^{i,j,l,k} = -e^{\alpha h_m} \overline{Q}_k, \ \Delta_{34} = \eta_3 \eta_4 \overline{P}, \ \Delta_{35}^{i,j,l,k} = 0, \ \Delta_{36}^{i,j,l,k} = -\eta_4 M_{3i},$$
$$\Delta_{44}^{i,j,l,k} = 2\eta_4 \overline{P} + h_m \overline{S}_{2i} + h_m \overline{Z}_{2i}, \ \Delta_{45}^{i,j,l,k} = \eta_4 \eta_5 \overline{P}, \ \Delta_{46}^{i,j,l,k} = \eta_4^2 \overline{P} - \eta_4^2 M_{4i},$$
$$\Delta_{55}^{i,j,l,k} = (-1+\rho)\mu \overline{S}_{1l} - \overline{Z}_{1l}, \ \Delta_{56}^{i,j,l,k} = 0, \ \Delta_{66}^{i,j,l,k} = (-1+\rho)\mu \overline{S}_{2l} - \overline{Z}_{2l}.$$

Proof.
We can prove Theorem 4 in the same way as Theorem 2 by replacing $\Pi_{i,j,l,k}$ by $\Delta_{i,j,l,k}$.

Remark 5. Theorems 4 and 2 have the same number of LMIs. Sufficient to say that this enforce the benefits of our work, because compared to [29, 30], the approach of both theorems is less conservative. Then, the LMI number can be reduced to $r + \sum_{i=2}^{r} (r - i + 1) + 1$ and the fuzzy LKF $V(x(t))$ is transformed into a single LKF if we suppose that $\overline{Q}_j = \overline{Q}$, $\overline{S}_{ij} = \overline{S}_i$, and $\overline{Z}_{ij} = \overline{Z}_i$, $i = 1, 2, j = 1, 2, \ldots, r$.

To show the effect of LMI reduction, the following corollary is presented:

Corollary 2. For given scalars $\eta_1 \neq 0, \eta_2, \eta_3 \neq 0, \eta_4 \neq 0, \eta_5,$ and η_6, the system

(37) is finite time stabilizable with respect to (c_1, c_2, T_f, h_m, R), if there exist symmetric positive definite matrices $\overline{P} = \overline{P}^T > 0$, $\overline{Q} = \overline{Q}^T > 0$, $\overline{S}_i = \overline{S}_i^T > 0$, and $\overline{Z}_i = \overline{Z}_i^T > 0$, $i = 1, 2$; appropriately sized matrices $M_i, Y_j,$ and $\overline{W}$, $i = 1, 2, 3, 4, 5$, $j = 1, 2, \ldots, r$; and a scalar $\alpha \geq 0$ satisfying the following conditions:

$$\Delta_{i,i} + (d-1)\overline{W} < 0, \tag{44}$$

$$\Delta_{i,j} + \Delta_{j,i} - \overline{W} < 0, \; i < j, \tag{45}$$

$$\begin{aligned}&\left(\frac{\eta_1^{-2}}{\lambda_{\min}\left(\widehat{P}\right)} + \frac{1}{\left(\lambda_{\min}\left(\overline{P}\right)\right)^2}\left(h_m e^{\alpha h_m}\lambda_{\max}\left(\widehat{Q}(\theta)\right)\right.\right.\\ &+ h_m^2 e^{\alpha h_m}\left(\eta_1^{-2}\lambda_{\max}\left(\widehat{S}_1(\theta)\right) + \eta_4^{-2}\lambda_{\max}\left(\widehat{S}_2(\theta)\right)\right)\\ &\left.\left.+\frac{1}{2}h_m^2\left(\eta_1^{-2}\lambda_{\max}\left(\widehat{Z}_1(\theta)\right) + \eta_4^{-2}\lambda_{\max}\left(\widehat{Z}_2(\theta)\right)\right)\right)\right)e^{\alpha T_f}c_1 < c_2\frac{\eta_1^{-2}}{\lambda_{\max}\left(\widehat{P}\right)}\end{aligned} \tag{46}$$

The state-feedback gain matrices are given by $K_j = Y_j\overline{P}^{-1}, i = 1, 2, \ldots, r$. where

$$\Delta_{i,j} = \begin{bmatrix} \Delta_{11}^{i,j} & \Delta_{12}^{i,j} & \Delta_{13}^{i,j} & \Delta_{14}^{i,j} & h_m\Delta_{15}^{i,j} & h_m\Delta_{16}^{i,j} \\ * & \Delta_{22}^{i,j} & \Delta_{23}^{i,j} & \Delta_{24}^{i,j} & h_m\Delta_{25}^{i,j} & h_m\Delta_{26}^{i,j} \\ * & * & \Delta_{33}^{i,j} & \Delta_{34}^{i,j} & h_m\Delta_{35}^{i,j} & h_m\Delta_{36}^{i,j} \\ * & * & * & \Delta_{44}^{i,j} & h_m\Delta_{45}^{i,j} & h_m\Delta_{46}^{i,j} \\ * & * & * & * & h_m\Delta_{55}^{i,j} & h_m\Delta_{56}^{i,j} \\ * & * & * & * & * & h_m\Delta_{66}^{i,j} \end{bmatrix},$$

$$\begin{aligned}
\Delta_{11}^{i,j} &= -\eta_1 A_i\overline{P} - \eta_1\overline{P}A_i^T - \eta_1 B_i Y_j - \eta_1 Y_j^T B_i^T + M_{1i} + M_{1i}^T + \overline{Q}\\
&\quad + h_m\overline{S}_1 + h_m\overline{Z}_1 - \alpha\overline{P} - M_{5i} - M_{5i}^T,\\
\Delta_{12}^{i,j} &= -\eta_2 M_{1i} + \eta_2 M_{2i}^T - \eta_2 A_{di}\overline{P} - \eta_1\overline{P}A_i^T - \eta_1 Y_j^T B_i^T\\
&\quad + \eta_2 M_5 + \eta_2 M_{5i}^T,\\
\Delta_{13}^{i,j} &= M_{3i}^T - \eta_1\eta_3\overline{P}A_i^T - \eta_1\eta_3 Y_j^T B_{i\,j}^T,\\
\Delta_{14}^{i,j} &= \eta_4 M_{4i}^T + \eta_1\eta_4\overline{P} - \eta_1\overline{P}A_i^T - \eta_1 Y_j^T B_i^T + \eta_4\overline{P},\\
\Delta_{15}^{i,j} &= -\eta_1\eta_5\overline{P}A_i^T - \eta_1\eta_5 Y_j^T B_i^T,\\
\Delta_{16}^{i,j} &= -\eta_4 M_{1i} + \eta_4 M_{5i}^T - \eta_1\eta_6\overline{P}A_i^T - \eta_1\eta_6 Y_j^T B_i^T,\\
\Delta_{22}^{i,j} &= -\eta_2 A_{di}\overline{P} - \eta_2\overline{P}A_{di}^T - \eta_2^2 M_{2i} - \eta_2^2 M_{2i}^T - \eta_2^2 M_{5i} - \eta_2^2 M_{5i}^T,\\
\Delta_{23}^{i,j} &= -\eta_2 M_{3i}^T - \eta_2\eta_3\overline{P}A_{di}^T,\\
\Delta_{24}^{i,j} &= \eta_4\overline{P} - \eta_2\eta_4 M_{4i}^T - \eta_2\overline{P}A_{di}^T,\; \Delta_{25}^{i,j} = -\eta_2\eta_5\overline{P}A_{di}^T,\\
\Delta_{26}^{i,j} &= -\eta_2\eta_4 M_{2i} - \eta_2\eta_4 M_{5i}^T - \eta_2\eta_6\overline{P}A_{di}^T,\\
\Delta_{33}^{i,j} &= -e^{\alpha h_m}\overline{Q},\; \Delta_{34}^{i,j} = \eta_3\eta_4\overline{P},\; \Delta_{35}^{i,j} = 0,\; \Delta_{36}^{i,j} = -\eta_4 M_{3i},\\
\Delta_{44}^{i,j} &= 2\eta_4\overline{P} + h_m\overline{S}_2 + h_m\overline{Z}_2,\\
\Delta_{45}^{i,j} &= \eta_4\eta_5\overline{P},\; \Delta_{46}^{i,j} = \eta_4^2\overline{P} - \eta_4^2 M_{4i},\\
\Delta_{55}^{i,j} &= (-1+\rho)\mu\overline{S}_1 - \overline{Z}_1,\; \Delta_{56}^{i,j,l,k} = 0,\\
\Delta_{66}^{i,j,l,k} &= (-1+\rho)\mu\overline{S}_2 - \overline{Z}_2.
\end{aligned}$$

Remark 6. Testing the feasibility of condition (39) in numerical examples is not possible, because there is a nonlinear term. For this reason, in order to simplify or eliminate this nonlinearity entirely, we can use these conditions:

$$0 < \lambda_1 I < \overline{P} < \lambda_2 I \tag{47}$$

$$0 < \overline{Q}_j < \lambda_3 I \tag{48}$$

$$0 < \overline{S}_{1j} < \lambda_4 I \tag{49}$$

$$0 < \overline{S}_{2j} < \lambda_5 I \tag{50}$$

$$0 < \overline{Z}_{1j} < \lambda_6 I \tag{51}$$

$$0 < \overline{Z}_{2j} < \lambda_7 I \tag{52}$$

$$\begin{bmatrix} \eta_1^{-2}\lambda_1 + h_m e^{\alpha h_m}\lambda_3 + h_m^2 e^{\alpha h_m}\left(\eta_1^{-2}\lambda_4 + \eta_4^{-2}\lambda_5\right) + \frac{1}{2}h_m^2\left(\eta_1^{-2}\lambda_6 + \eta_4^{-2}\lambda_7\right) & \lambda_1 \\ * & \left(e^{\alpha T_f}\eta_1^{-2}\frac{c_2}{c_1}\right)^{-1}\lambda_2 \end{bmatrix} < 0 \tag{53}$$

5 | NUMERICAL EXAMPLES

Example 1:

This example will show that our method is less conservative and more efficient than those in the literature. We consider the T–S fuzzy system (1) where $u(t) = 0$ and

$$A_1 = \begin{bmatrix} 0.0 & 0.6 \\ 0.0 & 1.0 \end{bmatrix}, A_2 = \begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}, A_3 = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 1.5 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0.0 & 2.0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.9 & 0.0 \\ 1.0 & 1.6 \end{bmatrix}, A_{d3} = \begin{bmatrix} 0.2 & 1.0 \\ 0.5 & 1.0 \end{bmatrix}.$$

Then, we choose the following parameters:

$$c_1 = 1, \ c_2 = 1.5, \ R = I, \ T_f = 10, \ \alpha = 0.01$$

Applying Theorem 2, the maximum upper bound h_m of time delay is presented in Table 1 for $\rho = 1.2$..

As shown in Table 1, the obtained value of h_m is larger than those obtained in [30, 33], and then, the results are significantly improved. Thus, it is clear that the FTS criterion proposed in this paper is less conservative than those given in the literature. On the other hand, the evolution of the state variable trajectories is shown in Figure 1 where the initial values are [1 1]. Then, it can be seen that these trajectories converge quickly to zero showing that very good transient responses are obtained. These results demonstrate the accuracy and the efficiency of the proposed approach for which the closed-loop system is asymptotically stable.

Consider now the two-rule fuzzy system that has been studied in [34–38] where $A_3 = 0$, $A_{d3} = 0$, and $B_1 = B_2 = [1 \ 1]^T$. For different methods, the delay bounds h (it is given as τ in these papers) are shown in Table 2 taking into account the same parameters.

Applying our method, the maximum upper bound of time delay is $h_m = 1.7563$ for $\rho = 1.2$. Then, the results are significantly improved taking into account that the time delay $h(t)$ in this paper is a time-varying function. Therefore, it is clear that these results are less conservative than those obtained in the literature, and so, LMI reduction techniques provide additional degrees of freedom to the resulting optimization problem, thus assuring the feasibility of the controller. These techniques reduce problem complexity and calculation time while simultaneously maintaining suitable solution accuracy. Finally, the proposed approach reduces the conservatism as much as possible and it is more effective.

TABLE 1 Comparisons results for $\rho = 1.2$.

Methods	Maximum allowed h_m
Theorem 2 [33]	0.4125
Theorem 3.2 [30]	0.4416
Theorem 2 (this paper)	0.4837

TABLE 2 Comparison results.

Methods	Maximum allowed h
[36]	1.6499
[37]	1.6499
[35]	1.6421
[38]	1.4257
[34]	1.4214

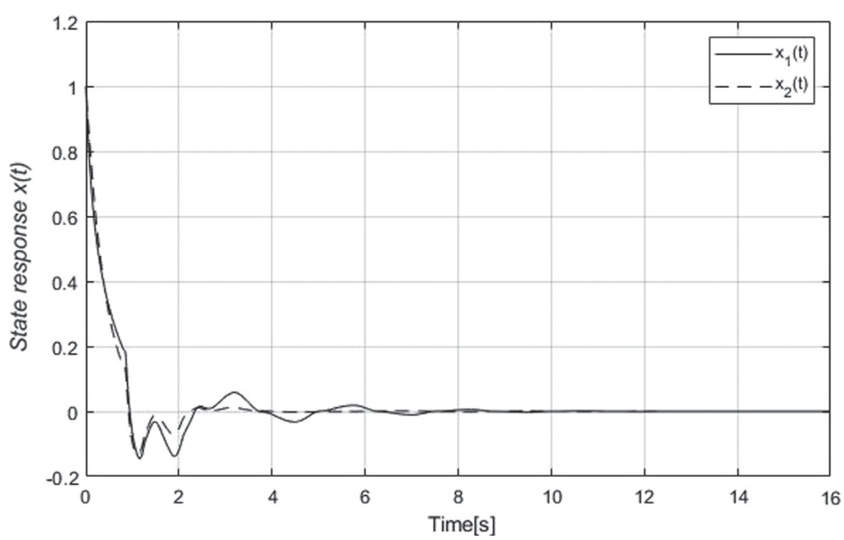

FIGURE 1 Closed-loop fuzzy system's state response.

Example 2:

As an example of nonlinear system, let us consider a continuous stirred tank reactor. As shown in [30, 39, 40], these equations define the system:

$$\dot{x}_1(t) = -\frac{1}{\lambda}x_1(t) + D_a(1-x_1(t))\exp\left(\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right) + \left(\frac{1}{\lambda}-1\right)x_1(t-h(t))$$

$$\dot{x}_2(t) = -\left(\frac{1}{\lambda}+\beta\right)x_2(t) + HD_a(1-x_1(t))\exp\left(\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right) + \left(\frac{1}{\lambda}-1\right)x_2(t-h(t)) + \beta u(t)$$

$$\dot{x}_i(t) = \varphi_i(t),\ for\ t \in [-h_m 0],\ \text{i} = 1,2$$

where $0 \le x_1(t) \le 1$ is the reactor's conversion rate and $x_2(t)$ is the dimensionless temperature $D_a = 0.072$, $\lambda = 0.8, \beta = 0.3, H = 8, \gamma_0 = 20$.

The IF-THEN rules are:

R_1: If x_2 is about 0.8862
Then $\dot{x}(t) = A_1x(t) + A_{d1}x(t-h(t)) + B_1u(t)$
R_2: If x_2 is about 2.7520
Then $\dot{x}(t) = A_2x(t) + A_{d2}x(t-h(t)) + B_2u(t)$
R_3: If x_2 is about 4.7052
Then $\dot{x}(t) = A_3x(t) + A_{d3}x(t-h(t)) + B_3u(t)$

where $x(t) = [x_1(t)\ \ x_2(t)]^T$ and

$$A_1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix},\ A_2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6268 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & -0.9387 \end{bmatrix},$$
$$A_{d1} = A_{d2} = A_{d3} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}, B_1 = B_2 = B_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}.$$

The membership functions are:

$$h_1(x_2) = \begin{cases} 1, & x_2(t) \le 0.8862 \\ 1 - \dfrac{x_2 - 0.8862}{2.7520 - 0.8862}, & 0.8862 < x_2(t) < 2.7520; \\ 0, & x_2(t) \ge 2.7520 \end{cases}$$

$$h_3(x_2) = \begin{cases} 1, & x_2(t) \le 2.7520 \\ 1 - \dfrac{x_2 - 2.7520}{4.7052 - 2.7520}, & 2.7520 < x_2(t) < 4.7052 \\ 0, & x_2(t) \ge 4.7052 \end{cases}$$

$$h_2(x_2) = \begin{cases} 1 - h_1(x_2), & x_2(t) < 2.7520 \\ 1 - h_3(x_2), & x_2(t) \ge 2.7520 \end{cases}$$

Thus, the parameters for the simulation are: $c_1 = 1.5, c_2 = 2, R = I, T_f, h_m = 1, \rho = 1.01,\quad \eta_1 = -2, \eta_2 = 1, \eta_3 = -1, \eta_4 = -2, \eta_5 = -0.8, \eta_6 = -0.8$. Then, using Theorem 4 and LMI Toolbox (MATLAB), the controller gains are:

$$K_1 = [6.8467\ \ -2.4137],\ K_2 = [16.3138\ \ -7.3844],$$
$$K_3 = [84.0808\ \ -7.6761]$$

or the initial values $[0.5\ \ -1]$; the evolution of the state variables trajectories is shown in Figure 2. Also, the evolution of a norm square of the state vector is given in Figure 3.

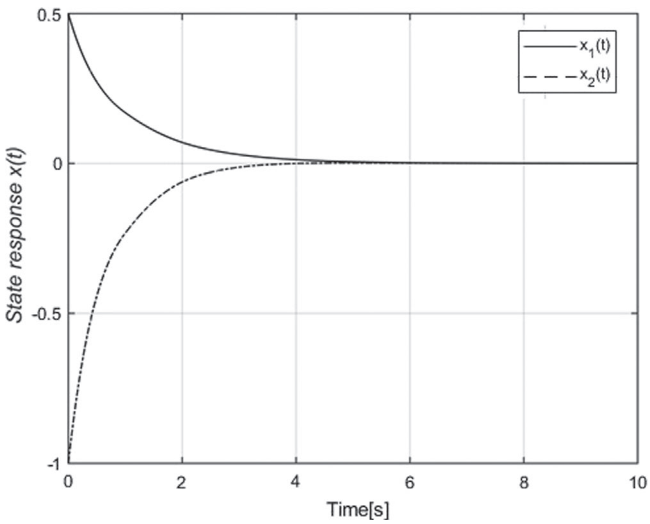

FIGURE 2 Closed-loop fuzzy system's state response.

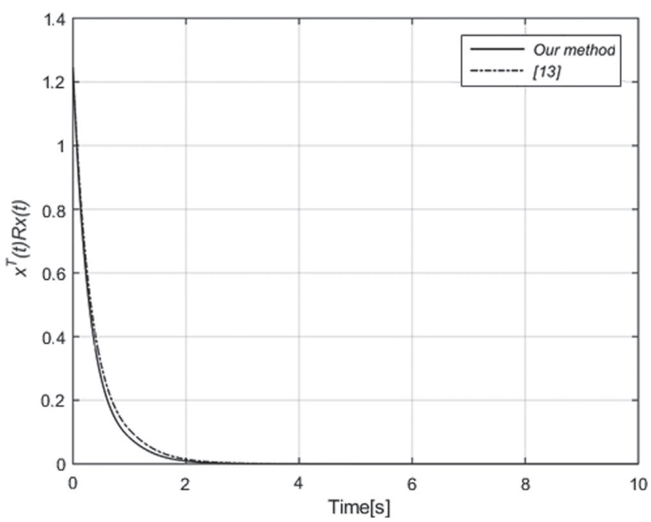

FIGURE 3 Time history of $x^T(t)Rx(t)$.

Figure 2 shows how the system's state responses converge to the equilibrium point reaching the desired tracking performance. Observing Figure 3, we can conclude the system is finite time stabilizable with respect (c_1, c_2, T_f, R, h_m). It is important to mention that using our approach, the convergence is faster than in [30]. Summing up, these results highlight the efficacy and accuracy of the methodology proposed in this paper for which the closed-loop system is stable.

Before concluding, it is better to highlight the importance of our method for conservatism reduction, which has been demonstrated by the above examples.

6 | CONCLUSIONS

The research presented here proposes a new strategy to ensure FTS. The results of this work are obtained from delay-dependent LMI conditions where a new synthesis of the PDC controller is also proposed using some free-weighting matrices to offer more flexibility and achieve good performance. Using T–S fuzzy models and a new LKF, the method is used for systems with delay where the value of our LMI-based algorithm is shown. The main conclusion is that the proposed design methodology is very useful and less conservative compared to those previously published.

The investigation is not closed; there are other topics and subjects that can benefit from this approach such as data samples and discrete time systems among others.

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AUTHOR CONTRIBUTIONS

Nabil El Fezazi: Conceptualization; formal analysis; investigation; writing—original draft. **Mohamed Fahim:** Investigation; software. **Rashid Farkous:** Formal analysis; investigation; writing—original draft. **Said Idrissi:** Methodology. **Teresa Alvarez:** Funding acquisition; validation; writing—original draft. **El Houssaine Tissir:** Conceptualization; investigation; supervision.

CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

DATA AVAILABILITY STATEMENT

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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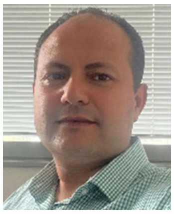

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