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ORIGINAL PAPER

Epistemic emotions and pre-service mathematics teachers' knowledge for teaching

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Abstract

Affective and cognitive processes may be jointly researched to better understand mathematics learning, paying special interest to emotions related to knowledge acquisition. However, it remains necessary to explore these processes in studies linked to the education of pre-service mathematics teachers. This study aims to characterize epistemic emotions in different practices linked to the practice of mathematics teaching: problem-solving, anticipating what would happen with the students and reflecting on classroom implementation. It considers the theory of Mathematical Working Spaces to describe the mathematical and cognitive dimensions generated by epistemic emotions, paying special attention to the cognition-affect interaction and the workspace created. The results indicate that the epistemic emotions of the pre-service mathematics teachers associated with the distinct practices were different. Differences are observed in the interaction between emotions and cognitive epistemic actions, depending on whether the pre-service mathematics teachers analyze them within the framework of their own solving or anticipate them in their students. This reveals how personal work relates to what is considered to be suitable for students. Specifically, certain antecedents and consequences have been specified for the emotions of surprise and boredom in relation to the characteristics of the optimization problems and the cognitive activity of the subject when solving them. These results highlight the need to enhance the education of pre-service mathematics teachers through training that helps regulate their epistemic emotions and model effective strategies for regulating their own emotions and those of their students.

Keywords Epistemic emotions · Mathematical Working Spaces · Pre-service mathematics teacher · Teacher education · Surprise · Boredom

1 Introduction

The articulation between theory and practice continues to be a key issue in research, given that the knowledge of pre-service teachers and the use of this knowledge are dependent constructs. Multiple studies have suggested that the practices making up initial teacher training are an important scenario in which theoretical knowledge converges with practical knowledge (Bastian et al., 2024; Potari, 2021). Furthermore, the need to jointly consider the affect and cognitive dimensions to study the learning of PMTs has become evident (Barnes, 2021; Chamberlin & Sriman, 2019; Kuntze & Dreher, 2015; Sakiz et al., 2012).

Some authors, such as Pekrun (2014), have suggested the need for additional research on the emotions of pre-service mathematics teachers within the educational context. Emotions differ depending on the events and objects triggering them (Pekrun & Loderer, 2020). In terms of object focus, general groups of emotions and moods have been typified based on multiple inputs that may be the most important for learning: achievement emotions, general and specific moods, epistemic emotions, topic emotions, and social emotions. Few studies have examined epistemic emotions. Extensive research has been conducted on pre-service mathematics teachers' achievement emotions [e.g., Bekdemir (2010) on anxiety], social emotions [e.g., Jenßen et al. (2022) on shame] and emotions in relation to teaching mathematics

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[e.g. Marbán et al. (2021) on enjoyment]. Many of these studies have been carried out during pre-service in primary school, but very few works on epistemic emotions have been conducted on pre-service teachers in secondary school. Future research is necessary to better understand certain factors: epistemic emotions when performing mathematics, effects on pre-service teachers' knowledge of mathematics, or effects or predictions on subsequent teaching.

Recent studies on teacher assessment of students' emotions (Kanefke & Schukajlow, 2023) indicate that pre-service teachers establish differences based on the situational interest and type of mathematics tasks. They also highlight that, when faced with the same problem, the judgments made by pre-service teachers regarding students' emotions can be different from the emotions that they actually feel when solving it. These studies conclude that it is necessary to consider perspectives regarding the implications for theoretical models of pre-service teacher judgments and the consequences that they have on teacher education. Other studies (Cai & Leikin, 2020; Gómez-Chacón, 2018; Schindler & Bakker, 2020) have highlighted the need to describe the combined nature of cognition, motivation and emotion in the learning and teaching of mathematics. Therefore, it may be interesting to consider perspectives for the training of PMTs that consider the cognition-affect interaction in their professional knowledge for teaching. To recognize epistemic action when performing mathematics is to recognize cognitive epistemic actions (actions belonging to the observable mathematical knowledge of participants) and affect epistemic actions (epistemic emotions,[1] epistemic beliefs, values and epistemic goals).

Epistemic emotions are defined as emotions arising when their object is knowledge and the process of knowing (Gómez-Chacón, 2017; Muis et al., 2015; Pekrun et al., 2017). Epistemic emotions (surprise, curiosity, enjoyment, confusion, anxiety, frustration or boredom) are triggered by knowledge, the characteristics of the tasks and the cognitive activity of the subject when solving them. Distinct studies (Chevrier et. al., 2019; D'Mello & Graesser, 2012, Gómez-Chacón, 2017; Schukajlow et al., 2017) have explored how some of these epistemic emotions are beneficial or counterproductive to learning, identifying the origin that generates them. Our perspective assumes the influence of emotions in the configuration of personal mathematical work at two levels (Person and Task-Person). Emotions may differ from one person to another (trait emotions) but a specific task can also bring out state emotions, dynamics that unfold during the phases of performing a mathematical task at the Task-Person level. Theories such as control-value theory (Pekrun & Perry, 2014) suggest that the development of emotions can be explained by evaluations (appraisals) of situations. Study results (Chevrier et al., 2019; Muis et al., 2015; Pekrun et al., 2014; Schubert et al., 2023; Vogl et al., 2019) highlight control, value, novelty, complexity and achievement (or epistemic goals) as antecedents of epistemic emotions as elements for a theoretical model. They also have considered the effects on planning, motivation, cognitive and metacognitive strategies and learning outcomes, which raises the question whether, and how, they relate to each other. In the context of Mathematics Education, an attempt has been made to detail various antecedents in relation to mathematical knowledge. For example, epistemic emotions such as surprise, confusion, frustration and curiosity are considered important in students' mathematical thinking (Goldin, 2004; Gómez-Chacón, 2000; Hannula, 2012; Schukajlow et al., 2017), in the processes of demonstration (Schubert et al., 2023), in visualization processes (Gómez-Chacón, 2018) and in the use of heuristics in problem-solving (Gómez-Chacón, 2017).

Studies from the perspective of pre-service teachers are necessary to understand how emotions act on mathematical reasoning, their antecedents and consequences. Numerous studies have indicated that the epistemic emotion of surprise occurs as an initial reaction to information that is inconsistent with prior knowledge. Likewise, they highlight boredom as a crucial point at which students disengage from activities, often a consequence of the frustration emotion (D'Mello & Graesser, 2012). These results may contribute to the knowledge of the PMT for teaching. Therefore, it would be interesting to examine questions regarding the emotional experience of the PMT (self-awareness), but also with regard to the perception of the emotional experience of others in mathematics (awareness of the task and of accompanying another).

Achievement and epistemic emotions are the dominant emotions that are triggered in the solving of a specific task, dynamically influencing the regulatory phases of learning. Therefore, regarding the professional knowledge to be developed by a PMT, it is important to pay attention to the characteristics and mathematical processes of solving tasks that may be catalysts of emotions. In this study, we focus on the solving of optimization tasks. Mathematical optimization is an essential mathematical concept to address everyday problems. In the Spanish curriculum, they are studied at a high school level (16–18 years of age) (Balcaza-Bautista et al., 2017) and a PMT must be aware of this knowledge and manage it in the classroom).

The theoretical foundations of the study are presented below, describing the methodological design used as well as the results obtained in order to, finally, present conclusions derived from this research and suggestions for future studies.

[1] In this study, we focus on epistemic emotions.

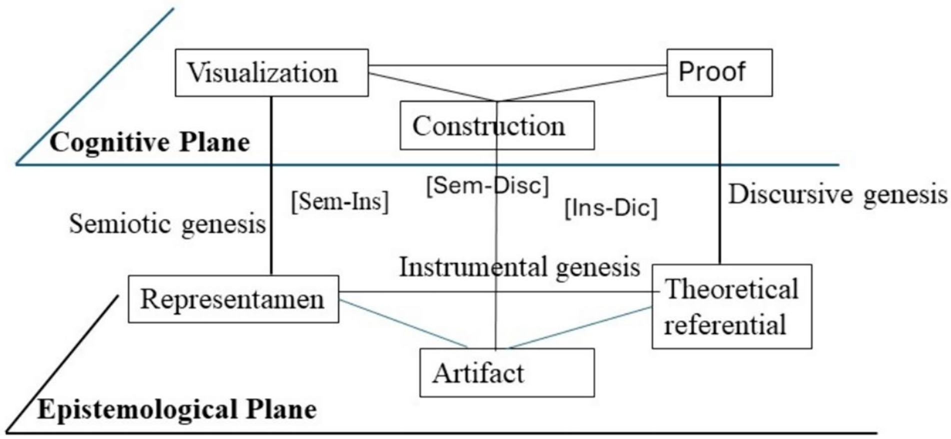


Fig. 1 Mathematical Working Spaces (MWS), geneses and vertical planes

2 Theoretical framework

Regarding the characterization of the professional development of the teacher, the theory of Mathematical Working Spaces (MWS) has been considered in distinct studies (Gómez-Chacón, 2022; Gómez-Chacón et al., 2016a, 2016b). The dynamism of the theory has been demonstrated, as well as how it can provide methodological tools to advance the description, understanding and creation of the mathematical work, establishing the connection between the practices of the teachers and the development of the students' learning in the real-world classrooms. This theory focuses on the detailed analysis of the mathematical work in which both students and teachers actively participate. It emphasizes *mathematical* content, and the term *work* involves three aspects related to its execution and development: the *goal* of the work, distinguishing between a simple activity, assigning an objective to a long-term action; the *processes* that are related to the procedures and the limitations of the execution of certain tasks and the *results* of the work that should be validated and coherent within the mathematics domain under consideration (Kuzniak et al., 2022).

Within the framework of the theory of MWS, mathematical situations/problems or tasks are considered as objects involved in mathematics practice, from whose solution the concepts and procedures emerge. In this study, the theory of MWS is used to describe the mathematical workflow in the cognitive-affect interaction. Through the lens of the MWS, the individual's development of appropriate mathematics work is a gradual and progressive process that builds bridges between the epistemological and cognitive planes. The *epistemological plane* consists of three poles (Fig. 1): *referential* (in close relation to the mathematical content, based on definitions, theorems, properties and axioms), *representamen* (semiotic signs used, which may be geometric images, algebraic symbols and graphics) and *artifacts* (material or symbolic). This plane refers to the theoretical part of the mathematical work that will support the deductive discourse of proof and justification. The *cognitive plane* refers to processes and procedures used by individuals in the task-solving activity. It is structured around three cognitive processes: visualization, construction and proving. The functioning should not be understood as the union of individual components located on the epistemological and cognitive levels, but as links activated by three geneses, semiotic, instrumental and discursive genesis, which articulate both planes. Therefore, in the configuration of objects and processes associated with a mathematical practice, different associated dimensions are established: semiotic (referring to natural language, mathematical symbols, visualization), instrumental (establishes the link between artifacts and construction processes) and discursive (mathematical reasoning in the proof, using theoretical concepts and discursive validations).

The productions of the PMTs related to the optimization problems are analyzed while considering the relationship between the research model of the Mathematical Working Spaces (MWS) (Fig. 1) and the modeling cycle (Blum & Borromeo-Ferri, 2009) (Fig. 2), which may be considered a first cycle for solving the modeling of a task. When a student begins at a given situation, it is assumed that a horizontal mathematization process starts, which is the basis for bringing the problem situation to a mathematical domain. Next, a vertical mathematization process takes place in which the MWS framework and the modeling cycle can interact with each other.

Developing modeling skills (Blum & Borromeo Ferri, 2009) mobilizes mathematical notions and objects from distinct mathematical domains, where the knowledge that can be learned by students is based on arguments belonging to different domains (analysis, algebra, geometry, etc.). This gives rise to different forms of mathematical work within an arithmetic/geometric, calculation or real analysis paradigm (see optimization task, Sect. 4.2). The modeling cycle (Fig. 2) is not taken in its entirety: the focus is on phases 3 to 5 of the cycle and the objective is to analyze the

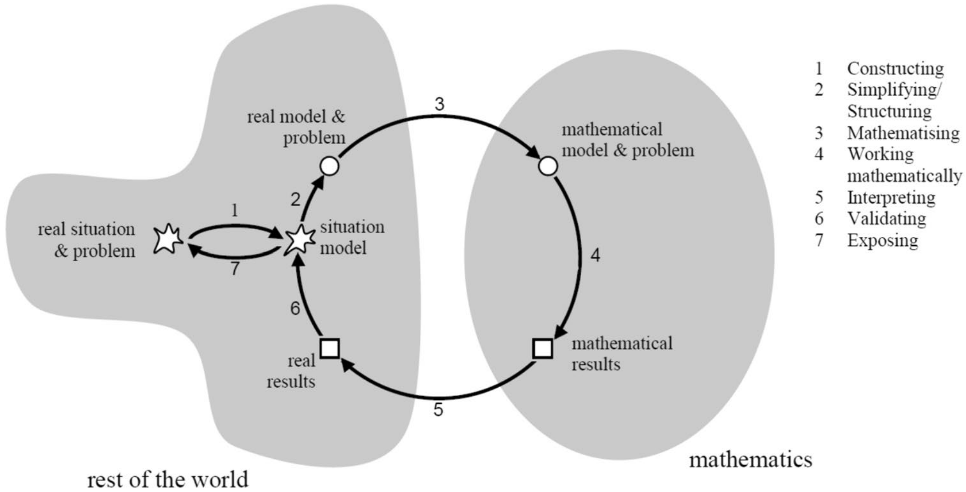


Fig. 2 The modeling cycle (Blum & Borromeo-Ferri, 2009)

mathematization process through the MWS model when students solve a modeling task (e. g., Montoya Delgadillo et al., 2017, Nechache & Gómez-Chacón, 2022). The theoretical elements provided by the MWS model allow us to identify the mathematical domain of resolution, representations and knowledge in the work of PMTs, paying special attention to the cognition-emotion interaction.

In the theory of Mathematical Working Spaces, there are three types of mathematical work associated: *Reference MWS*, defined by theoretical criteria aimed to organize certain knowledge according to the epistemological principles of the mathematical domain studied; *personal MWS*, related to the individual's own work, without necessarily having a teaching intention, defined by the way in which the individual constructs their own mathematical work and, finally, *suitable MWS*, related to an individual (teacher or researcher) who makes sense of mathematical content designed for teaching in a given context (Kuzniak et al., 2022).

Teachers aim to configure the suitable MWS so that their students can perform mathematical work according to the reference MWS. It is at this level of the suitable MWS where mathematical learning occurs. In effect, when solving problems, both high school students and PMTs face the mathematical task with their own knowledge and cognitive and affect processes, which are shaped by their personal MWS. However, the work carried out may not conform to what is expected and defined by the educational organization. Therefore, the teacher's goal in implementing the suitable MWS is to more closely align students' personal MWS with the expectations of the teaching organization (Menares-Espinosa & Vivier, 2022).

In this context and considering the indicated concepts, issues related to the description, characterization or (trans) formation of the mathematical work into distinct practices linked to the practice of teaching mathematics are key to characterizing epistemic emotions. Analyzing the mathematical work of PMTs through the lens of the MWS allows us to reflect on the interaction between cognition and affect in a context of optimization problems. It reveals how to consider epistemic emotions in teaching practice in order to configure suitable the MWS. It also helps understand the connections between theory and practice in the affect dimension and between the personal MWS of the PMT and the configuration of the suitable MWS.

3 Research questions

This study examines how PMTs characterize the epistemic emotions associated with the different practices linked to the practice of mathematics teaching: in problem-solving (personal MWS), anticipating what was happening with the students, and reflecting on classroom implementation (configuration of the suitable MWS). Based on this objective, the following question was presented:

How do the epistemic emotions reported by the PMTs when solving optimization problems differ from those when they anticipate the emotions and strategies of high school students when solving those same problems?

This research question seeks to contribute to the understanding of PMTs regarding epistemic emotions (Kanefke & Schukajlow, 2023), as well as the knowledge that PMTs should develop regarding epistemic emotions for the practice of mathematics teaching. Therefore, it is necessary to pay attention to the cognitive and epistemic aspects of mathematics knowledge (Gómez-Chacón, 2017; Schubert et al., 2023) and to the importance of personal MWS on the establishment of suitable MWS.

4 Method

4.1 Participants and context

Given the characteristics of the research, two different participant groups were necessary. On the one hand, there was a group of 24 PMTs, all of whom were graduates in mathematics and were studying a specialized master's degree in mathematics education. And, on the other hand, there was a group of 31 high school students (16–17 years old), all of whom were part of a classroom in which one of the PMTs from the first group was teaching as part of their teaching internship.

The study was carried out in accordance with the Design Experiment principles (Cobb et al., 2003), following a clearly qualitative approach. The observation processes were associated with the university training of the PMTs and the internship teaching.

PMT training was carried out over four 90-min sessions. Three of these sessions focused on the solving of five optimization problems and the creation of a report anticipating what would happen in the classroom with a group of high school students if these same problems were applied. The fourth session was carried out after the participating PMT implemented the mentioned problems with a group of high school students. In this session, two researchers acted as non-participant observers. This fourth session was devoted to reflection on the personal MWS of the PMTs upon solving two of the optimization problems. The reflection and discussion focused on the domains and resolution paradigms used, as well as the difficulties and emotions experienced. Their solutions were compared with what was expected and with the actual teaching practice and with that reported by the two researchers during the teaching practice. This permitted reflection on the interaction between cognition and affect in a context of optimization problems and how to consider epistemic emotions in the practice of mathematics teaching in order to configure the suitable MWS. This fourth session was recorded on video, and it highlighted the connections between theory and practice in the affect dimensions and between the personal MWS of the PMT and the configuration of the suitable MWS.

4.2 Instruments

As a data collection instrument, a *Questionnaire* was developed (Fig. 3). It consisted of two parts. In the first part, the PMTs had to solve an optimization problem detailing the mathematical processes followed, the thoughts that arose, and the feelings and emotions experienced. They were explicitly asked to solve it using different methods and to describe the mathematical actions and epistemic emotions experienced.

In the second part, the PMTs were to anticipate the behavior of the high school students upon solving the problem: problem-solving strategies, difficulties and epistemic emotions. For the epistemic emotions, a Mood Map of the problems was followed (Gómez-Chacón, 2000). The Mood Map is an iconic instrument that imitates weather maps, establishing a code to express different emotional reactions experienced by the student in the course of mathematical activity. In this study, it was adapted using emoticons for the epistemic emotions. The marks registered by the student using these emoticons allow the identification of their emotions during the process of solving a mathematical problem and its origins, antecedents, and consequences.

They also had to indicate which methods they considered most appropriate in teaching and how they could help them overcome difficulties.

This problem was used in the classroom by one of the PMTs having 31 high school students (aged 16–17) in the teaching internships. This PMT proposed that this group of students solve the problem and explain the processes used in detail, as well as the emotions experienced according to the Mood Map emoticons or if they felt no emotion at all when working on the mathematical task.

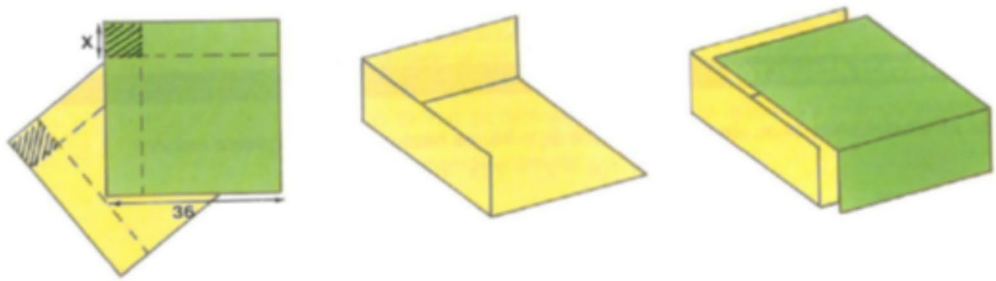
Prior analysis of the problem This optimization problem requires a geometric interpretation of what the problem asks of us. The statement does not give the direct function to optimize. The situation is realistic and easy to understand, but the translation of the geometric situation into a symbolic one requires the semiotic dimension and visual reasoning, as well as mathematical knowledge (function, formula, derivation). It may be solved using distinct domains and different strategies or methods: arithmetic/geometric, calculation techniques or real analysis techniques or using distinct artifacts such as GeoGebra to represent the optimal point of the function. A more common resolution is:

> *The volume of the box is $(36- x)\ (36-x)\ x= (36-x)^2\ x$ To actually have a box, x must be positive and less than 36. The function to be modeled is $f(x)= (36-x)^2\ x$; the critical points of the function are calculated by setting the first derivative equal to zero and using the second derivative to check if they are maximums or minimums, obtaining the y the second derivative is used to check if they are maximums or minimums, obtaining the solution $x=12$ cm.* (Analytical method resolution).

4.3 Data analysis

The data analyzed for this research are the PMT's responses to the questionnaire, the recording of the reflection session (session 4 of the training process) and the high school

The Box problem: With each of the two square pieces having 36-cm sides, the following operation is performed.



What volume should the side of the square that we cut have in order to maximize the volume of the resulting box?

Part I

1. Solve the problem in more than one way and explain the processes used, in detail.

2. Use the Mood Map with the following emotions to describe how you felt throughout the problem-solving process:

Surprise	Curiosity	Confusion	Boredom	Frustration	Enjoyment	Anxiety	Interest
1	2	3	4	5	6	7	8

3. Reflect on what you have learned from this activity.

Part II: Think about a class of 30 high school students (aged 16 to 17) and anticipate the behavior of these students when solving the problem: mathematically and emotionally. Indicate which optimization problem-solving strategy and method you would consider the most suitable for teaching.

Fig. 3 Questionnaire proposed to the PMTs

students' responses to the problem and questions indicated in the previous section.

Given that two types of data were used (the discursive responses to the questionnaire and the Mood Map responses), two analytical approaches were employed. For the discursive data, a qualitative approach was used (Strauss & Corbin, 1990) based on cross-checking of solutions by three researchers. The PMTs' responses to the questionnaire were analyzed by differentiating between two practices: resolution, referring to the process of problem-solving by the PMTs, and anticipation, referring to what the PMTs expected to find with regard to the students (Fig. 4).

Emotional episodes were classified according to the epistemic emotions expressed by the subjects, which occurred during the process of searching for the answer to the mathematical task by both the PMTs and the high school students, highlighting the alleged reasons and the mathematical work of mathematization and modeling (Figs. 4 and 5). The data

<table>
<tr><th>Extract from the response to the questionnaire by the pre-service teacher PMT24
(Resolution and anticipation practices)</th><th>Researcher-category analysis</th></tr>
<tr><td>Resolution practice by the PMT
"Trying different values of x to determine how the volume of the figure changes and since it is a not very high number it may be possible to calculate the solution, or at least have an approximate idea of what it could be:
<table>
<tr><td>x-36</td><td>35</td><td>34</td><td>33</td><td>…</td><td>29</td><td>28</td><td>27</td><td>26</td><td>25</td><td>24</td><td>23</td><td>22</td></tr>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>…</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr>
<tr><td>x·(x-36)²</td><td>1225</td><td>2312</td><td>3267</td><td>…</td><td>5887</td><td>6272</td><td>6561</td><td>6760</td><td>6875</td><td>6912</td><td>6877</td><td>6776</td></tr>
</table>
If we look at the values in the table, we see that the maximum volume is obtained when x=12 and as of that value, the volume decreases" (PMT24)</td><td>Introduction of variables (CEA 21)
Visualization of volume (CEA 1)
Representation in Table (CEA 23)</td></tr>
<tr><td>"When solving the problem, in my case, the feelings that predominated were those of curiosity and interest, upon reading the statement for the first time and considering how to solve it. Frustration, upon managing to solve it with one method but not being able to determine a different way to solve it. And, finally, enjoyment when I managed to solve it using two methods, by trial and by analysis" (PMT24).</td><td>Curiosity
Interest
Frustration
Enjoyment</td></tr>
<tr><td>Anticipation practice by the PMT
"As for the learning obtained, it is an optimization exercise which, based on the method used to solve it, may be studied at various levels. If functions and derivative are used to calculate the maximum, we will be talking about a high school level for which they will also need some basic geometry knowledge on calculating volumes and areas.</td><td>Suitable MWS</td></tr>
<tr><td>Moreover, solving it using distinct values of x is much more costly and does not serve the students' interest beyond getting an idea of what the result will be" [..].</td><td>Personal MWS-suitable MWS</td></tr>
<tr><td>"Depending on the group's academic level, they may show an interest in learning to solve a new type of problem, but at the same time, if the initial presentation is complex, it may lead to boredom and a loss of interest in following the class.
Furthermore, this specific problem is difficult since the student needs to correctly interpret the statement using the images given with regard to the construction of the box, since they cannot understand that the fragment of side x that is removed from the base would be the height". PMT24</td><td>Boredom
Interest
Determination variable X (CEA 21)
Visualization</td></tr>
</table>

Fig. 4 Analysis of the solution as reported by PMT24 (PMTs were named from PMT1 to PMT24 and the high school students from A1 to A31.) in the resolution practice and in the anticipation and categories of analysis by the research

from the mood maps of both the PMTs and the high school students were considered for the interrelation cognition and affect. The evidence from the Mood Map in the protocols (Fig. 5) reveals patterns of sequences of emotions that interact with heuristics during the resolution of mathematical problems or mathematical task processes. This permits the qualification of the interaction between cognition and affect in the characterization of the epistemic emotions.

The inductive analysis process generated the following system of categories, organized in three dimensions: epistemic emotions, paradigms and resolution methods, and epistemic cognitive actions.

1. *Epistemic emotions* surprise, curiosity, confusion, boredom, frustration, enjoyment, anxiety, interest. In addition, the category none of them as considered.

2. *Paradigms and resolution methods* The methods and procedures were categorized into five sections:

- *Analytical*: Search for the "Volume of the box as a function of the side of the square to be cut" function and

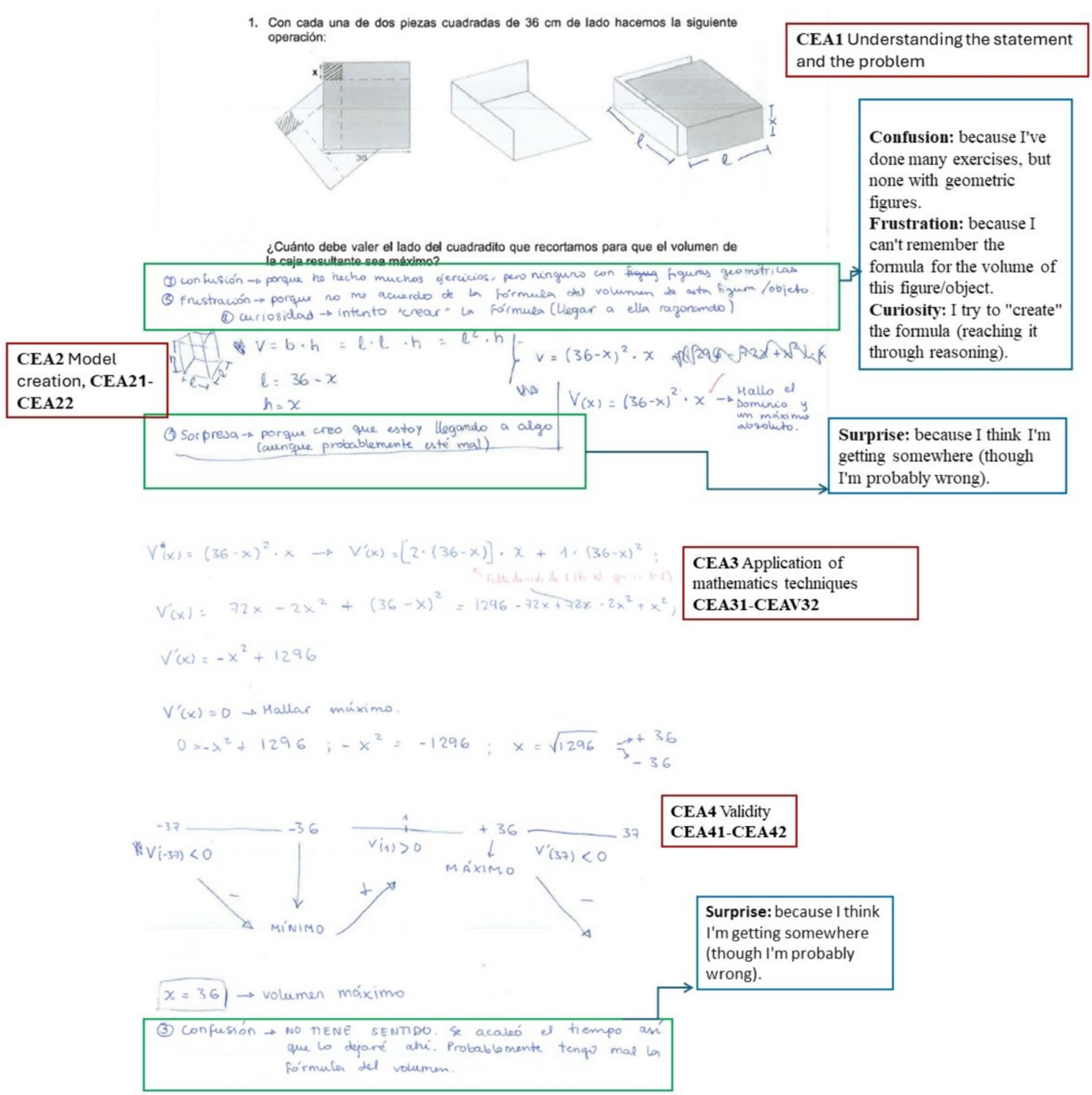


Fig. 5 Analysis of the resolution protocol of high school student A25

calculate its maximum through the derivative function (first and second derivative).

- *Algebraic*: Divide the side of the piece by three and proceed with algebraic resolution using a similar problem as a model.
- *Geometric*: Search for the "Volume of the box as a function of the side of the square to be cut" function to represent it graphically and find its maximum point, using GeoGebra in some cases.
- *Numerical-arithmetic trial*: Compare the volumes of the boxes of dimensions n·(36-n)·(36-n), with n between 0 and 18.
- *Theoretical*: This way of proceeding consists of reasoning that in order for the product to be maximum, the factors must be as large as possible.

3. Epistemic cognitive actions related to MWS and the modeling of optimization problems. Four epistemic actions are identified, some of which have specific characteristics:

Table 1 Identified paradigms and resolution methods

	Analytical	Algebraic	Geometric	Numerical-arithmetic trial	Theoretical
Percentage	100%	8.3%	29.1%	16.6%	8.3%

- *Understanding* of the statement and the problem (CEA1)
- *Representamen, semiotics and mathematical objects*: creation of a mathematical model of the situation (CEA2):
- Selection of variables and specification of what each of them represents (CEA21)
- Obtain the expression of the function to be analyzed to calculate its extreme value (CEA22)
- If the previous function is expressed with more than one variable, look for the relationship that may exist between these variables to express the function with a single variable (CEA23)
- *Mathematical content*: application of mathematical techniques to the model (CEA3):
- Calculate the derivative of the function (CEA31) and calculations to obtain the values of x that cancel it (CEA32)
- *Instrumental-discursive dimension*: translation to the real situation to analyze its validity (CEA4):
- The domain of the function (CEA41) is considered, often conditioned by the problem statement, and the values obtained in the previous step, determining the absolute extremes using the sign of the second derivative as necessary (CEA42).

5 Results

The results are organized in two sections. First, the relationship between epistemic emotions and problem-solving approaches, identifying paradigms and methods in the personal MWS of the PMTs and their relation to the suitable MWS. And, secondly, the differences between epistemic emotions in different teaching practices: resolution, anticipation and students' epistemic emotions in the implementation.

5.1 Paradigms and methods in the personal MWS of the PMTs and their relation to a suitable MWS

The actions inherent to the modeling and resolution of optimization problems can be associated with different work domains: arithmetic, geometric, calculation and real analysis. We identify five PMTs' approaches to solving modeling problems (Table 1). Table 1 shows the percentage of use of each approach of the 24 PMTs. The personal MWS differs between the PMTs, although there is a trend to use one specific method, the analytical approach which is the most institutionalized approach used in teaching. The optimization problem resolution may remain in a numerical-arithmetic trial MWS (see the solution in Fig. 4). However, the introduction of a function changes the MWS, and this involves specific analysis techniques (for example, derivation) and representations (graphs, table of values). The choice of each MWS requires the use of diverse knowledge based on the mathematical domains. It is found that various PMTs have difficulty investigating other domains of mathematical work that combine the geometric and algebraic registers. This lack of knowledge triggers more negative epistemic emotions or, as shown in the following section, the emotion of boredom.

Regarding what they consider to be suitable MWS for teaching practice, 62% of PMTs believe that the analytical approach is the best working paradigm and method since it allows for work on the official curriculum content and is considered more accurate. It refers to the application of a more *standard* technique, which simplifies the student's learning process and provides them with more *confidence*. However, almost 40% of the PMTs responded that, when solving this problem with students, other ways that encourage reasoning would also be useful. For example, PMT21 indicated: "*Different types of resolution expand our concept of the same knowledge. They help us see all its sides, relate them and become aware of the abstract structures underlying mathematics, thus providing a broader view of mathematics. As a personal opinion, I also believe that it develops the students' creativity. Therefore, it is important to consider this when teaching"* (PMT21).

In addition, the PMTs indicate that the use of a paradigm and resolution method will also depend on the students' level of knowledge. For example, two PMTs indicated the following:

"*Once the problem has been modeled, we proceed to solve it in two different ways. The first way could be done by 9th or 10th graders and is based on use of the GeoGebra mathematical tool to solve the problem. The students would do what is shown in the following image in GeoGebra [includes resolution with GeoGebra]. And a second way, which could be done by high school students, is based on the use of notions of derivatives. The high school students, upon hearing the word "maximum" will think of functions and their derivatives, in this case, the use of the derivative of* $V(x)$ *to calculate its maximum as taught in class*" (PMT23).

"Regarding the learning obtained, this is an optimization exercise which, depending on the method used to solve it, can be studied at various levels. If functions and the derivative are used to calculate the maximum, we will be talking about a high school level, since they will also need some knowledge of basic geometry of calculating volumes and areas. On the other hand, solving it by giving different values to x is much costlier and lacks interest beyond providing the students with an idea of what the result will be" (PMT24).

Table 2 Percentage of epistemic emotions indicated during resolution, anticipation and implementation

	During the PMTs resolution	PMTs anticipating the students' emotions	Students' emotions in the implementation
Surprise	12.5	37.5	42
Curiosity	37.5	37.5	22.6
Interest	45.8	41.7	9.8
Enjoyment	37.5	45.8	38.7
Confusion	25	66.7	77.41
Frustration	45.8	70.8	42
Anxiety	8.3	66.7	22.6
Boredom	20.8	45.8	6.45

5.2 Differences between resolution-anticipation-implementation

The epistemic emotions of the PMTs associated with the different practices related to the practice of mathematics teaching (resolution, anticipation and implementation) were different. According to the established categorization of cognitive actions and epistemic emotions (Sec 4.3), we proceeded to analyze the frequency of appearance and the cognitive epistemic action with which it was associated. The results are shown in Table 2 and Fig. 6, distinguishing between the practices linked to the teaching of the optimization problem. Thus, Table 2 shows the percentage of PMTs that indicate experimenting each of the emotions making up the Mood Map when solving the problem; the percentage of PMTs that anticipate those emotions in the high school students and, finally, the percentage of the high school students that indicated each of the emotions during the PMT's teaching internship. As previously mentioned, these students solved the problem and indicated the emotions experienced (see resolution protocol in Fig. 5).

Figure 6 shows the relative weight of the different emotions according to the practices related to the PMTs during the resolution as well as their anticipation of what could be expected from the students, and the high school students' epistemic emotions when solving the problem.

Figure 6 can be interpreted as a flowchart of epistemic emotions through the three analyzed practices. This suggests possible transfers from personal MWS to suitable MWS, as well as their characteristics. This transfer is exemplified, considering the specific case of the emotions of surprise and boredom, two emotions typified by different valence and activation levels. Table 3 shows the percentages of both emotions in relation to cognitive epistemic actions. In anticipation, it is the percentage of PMTs that indicate it and in implementation, it is the percentage corresponding to the high school students who indicated it when solving the problem.

Table 3 shows the differences between the emotion of *surprise* experienced by PMTs and what they anticipate that the high school students would experience. In the resolution of the PMTs, only 12.5% say that they experience this emotion (Table 2). They do, however anticipate its appearance in their students in 37.5% of the cases, and it ultimately appeared in the real implementation undertaken in the classroom in 42% of the high school students (Table 3).

Table 3 shows the differences between the emotion of *surprise* experienced by PMTs and what they anticipate that the high school students would experience. In the resolution of

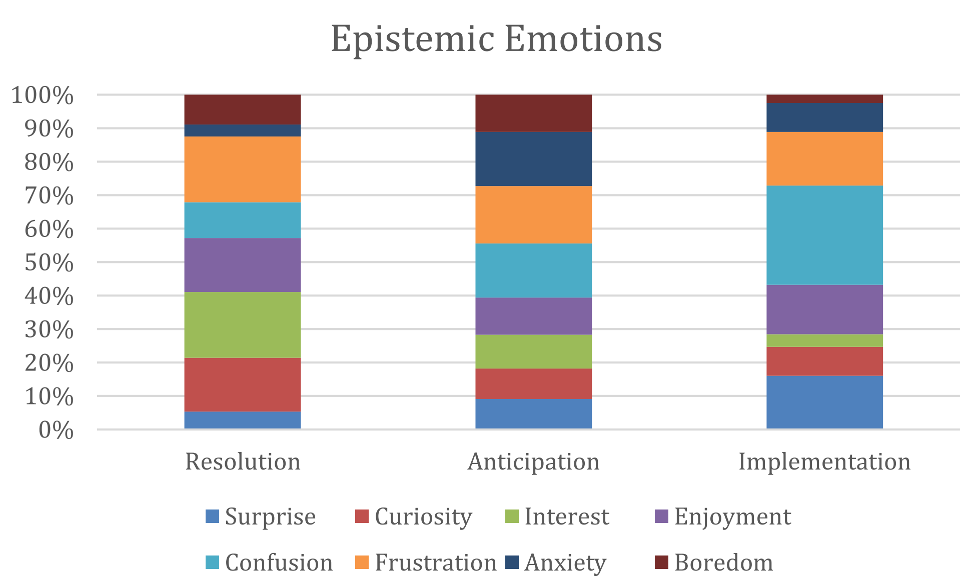


Fig. 6 Relative weights of the epistemic emotions according to the practices

Table 3 Percentage of the *surprise* and *boredom* emotions and cognitive epistemic actions in the anticipation by the PMTs and that manifested by the high school students [CEA1: understanding of the statement and the problem; CEA2: Representamen, semiotics and mathematical objects: creation of a mathematical model of the situation; CEA3: Mathematical content: application of mathematical techniques to the model, CEA32: calculations to obtain the values of x that cancel out the derivative; CEA4: Instrumental-discursive dimension: translation to the real situation to analyze its validity]

	Anticipation emotions	Anticipation Cognitive Epistemic Action	Implementation. High school students' Emotions	Implementation. High school students' Cognitive Epistemic Action
Surprise	37.5%	CEA1 (8.3%), CEA3 (12.5%), CEA 4(20.8%)	42%	CEA1 (25.8) CEA2 (6.5%) CEA32 (3.2%) Not specified (6.5%)
Boredom	45.8%	CEA1 (8.3%), CEA2 (4.2%), CEA3 (8.3%), CEA4 (12.5%), Other reasons (12.5%)	6.45%	CEA32 (3.2%), CEA4 (3.2%)

the PMTs, only 12.5% say that they experience this emotion (Table 2); however, they anticipate its appearance in their students in 37.5% of the cases, and it ultimately appeared in the real implementation undertaken in the classroom in 42% of the high school students (Table 3).

Although the epistemic emotion of *surprise* is considered to be an initial reaction to information that is incongruent with prior knowledge (CEA1), in the anticipation made by PMTs they also relate it more broadly to cognitive processes. They also refer to it in the application of techniques to the model (CEA3 and CEA32)—ignorance of the concept of derivative and calculation techniques—and in the analysis of the solution's validity (CEA4). Furthermore, of the indicated epistemic actions that generate surprise, one PMT points out the discrepancy that the high school student may experience with their beliefs with regard to this type of problem.

In the implementation, surprise is indicated with a greater frequency by high school students in the creation of the model to be optimized (CEA2) and in the execution of calculation techniques (CEA3.2).

If we are to further examine what was expressed by the PMTs during the problem resolution, the emotion of surprise is linked to the onset and to the understanding of the problem (semiotic and visualization dimension) and also to the determination of model variables and the validation and confirmation of the technique. For example, according to PMT23: "I was surprised *to discover that there were equivalent resolution methods. I saw that I could make this problem equivalent to one that I could solve numerically and by arithmetic trial. Its statement would be: "Decompose number 36 into two positive addends to maximize the product of the first and the square of the second", this would be easier for the students to solve. This has been interesting for my future work with the students since at first glance, they appear to be totally different problems and I think this is what the students would feel ".* In this case (PMT23), it is interesting to observe where the surprising or significant event takes place, the confirmation of an intuition and the establishment of a technique. In the anticipation reported by PMT23, this will be significant in terms of performing epistemic actions with the high school students. She seeks to find and re-experience surprise as a reaction to something that has become routine and obvious, and to use it to decide what to emphasize, which can make a big difference in the development of a topic.

In the anticipation, some of the PMTs describe surprise as an emotion that can be generated in validation and in the link to self-confidence: "*The most important thing at the beginning is to have a spatial view of how the box is assembled. Students who get overwhelmed and do not know how to plan the exercise well will feel anxiety and frustration over not knowing how to advance. Either nothing occurs to them or the approach that they have taken is incorrect, and therefore, they will get confused and even bored since they don't know what to do or how to move beyond this mental block. On the other hand, those who were able to solve everything more or less the first time will feel enjoyment and satisfaction, and even surprise if they did not have much faith in themselves."* (PMT4).

Regarding the epistemic emotion of *boredom*, considered a crucial point where students disengage from activities and often a consequence of the emotion of frustration, we find that, in the implementation, only 6.45% of the high school students noted this emotion. Some of them associated it with performing the operations to find the maximum of the objective function, while the others mentioned it when concluding the problem and validating the results. The latter, even though they knew that their solution was incorrect, spent time looking for the error in their resolution process. However, in the anticipation made by the PMTs, they consider that with this problem, there will be an increased appearance of the epistemic emotion of boredom, which is linked to the following epistemic actions: the understanding of the statement and the problem (CEA1), the execution of a sequence of steps in the calculation technique (CEA3, CEA32) -ignorance of these-, the execution of calculations, the recognition that it is similar to other optimization problems (validation phase (CEA4)). In addition, three PMTs mention other reasons that may cause boredom, such as the belief regarding its usefulness (believing that these problems are useless) and the situating of the problem to the student's level.

In the PMT's resolution practice, this emotion was linked to the execution of calculations, to being unable to solve it using various methods and to the acceptance that repetition can be of interest to strengthen the acquisition of mathematics procedures and techniques. In their own words: "*When I solved it I felt interested, then a little bored and frustrated with the calculations. Also, I'm not able to find a second method of solving it that isn't conceptually equivalent, looking for function endpoints or something like that. This makes me anxious and frustrated.*" (PMT5). The PMTs mention how for them, this repetition leads to boredom: "*I don't see the point in having to do it again, this problem is formulated geometrically, but it is the same as another one that we performed in an arithmetic formulation*" (PMT14). However, it can be seen that they did not go into detail regarding the understanding of the mathematical work paradigms and some of these techniques and their didactic qualification for teaching practice.

6 Discussion

This study explores how PMTs characterize epistemic emotions in different practices linked to the practice of mathematics teaching: in the resolution of optimization problems and in the anticipation of the epistemic emotions that may arise in high school students when solving these same problems. In addition, we compare the PMTs' expectations with the epistemic emotions indicated by the high school students upon solving the same problem.

The epistemic emotions of the PMTs that were associated with the three practices were different. The study reveals some significant differences in the interaction between emotions and epistemic actions according to practices of solving and anticipating and provides evidence of the influence of personal MWS on a suitable MWS.

6.1 Personal MWS of the PMTs and suitable MWS

The results regarding the epistemic emotions experienced during the resolution and the anticipation of how they would be generated in high school students when solving the same problem provide us with information on the personal MWS of the PMTs and their relations to a suitable MWS. The PMTs refer to the analytical method as the one that would offer them greater confidence since it has more institutional recognition and is considered to be an application of techniques, making it the prescribed procedure for use with students. With this method, the introduction of a function as a mathematical modeling object marks the MWS with the derivation techniques and the graph and table of values. The personal MWS involved also differs from one PMT to another. Several of the PMTs have difficulty investigating other domains of mathematical work that combine geometric and algebraic registers, opting to use the analytical method, which is considered to generate the most confidence. The lack of knowledge to solve the problem using other methods triggers negative emotions in the PMTs, and when they anticipate it in the students, they expect that it will generate similar emotions. However, they suggest a positive value for learning when solving the problem through distinct methods. This leads us to question whether the modeling process for some PMTs is a routine that has been *normalized* by the institution, especially for certain types of problems and if it is associated with a positive epistemic emotion.

6.2 Epistemic emotions and epistemic cognitive actions

The results obtained are in line with those from previous studies regarding the distance in the emotions that PMTs analyze in themselves and those that they expect from their students (Kanefke & Schukajlow, 2023). The PMTs valued enjoyment, interest, boredom with the situation and considered it important to avoid frustration in the development of high school students. The study has provided empirical evidence on the emotions of surprise and boredom as applied to themselves or in the anticipation made regarding what the high school students will possibly experience. The analysis from the theory of MWS allows us to consider the mathematical concepts and objects from mathematization identified in the modeling process in detail. This offers additional knowledge and specificity to the results of other studies on epistemic emotions (D'Mello et al., 2012; Munzar et al., 2021) regarding the task characteristics and the subject's cognitive activity when solving them. For example, when personal knowledge conflicts with external knowledge, that is, when a cognitive incongruence arises, emotions may be triggered, not only due to the nature of the knowledge but also due to gaps in the same. In the study, they are associated with the determination of the model, the concept of derivative, the execution of a sequence of steps in the calculation technique and with validation actions.

As for the epistemic emotions and cognitive interactions in the personal MWS, the results show how certain influences of the reference MWS become evident in the personal MWS of the PMTs, who, in turn, project this onto the suitable MWS for teaching. Regarding the cognitive epistemic actions, interactions with emotions were identified in the creation of a model of the situation, the application of techniques and the translation to the real situation to analyze its validity.

However, the possibility that different MWS are generated in optimization problems from the mathematization process makes it an attractive but complex task to control for the PMT, especially for a second modeling cycle. In fact, this study focuses on a first cycle in which PMTs are asked to solve a modeling task. This reveals the difficulty for a practical teaching internship to make the combined nature of cognition, motivation and emotion on mathematical work explicit, although it is a challenge that has been considered in distinct studies (Cai & Leikin, 2020; Gómez-Chacón, 2018; Schindler & Bakker, 2020).

Finally, the results of this study may help PMTs connect specific knowledge on epistemic emotions with their professional teaching knowledge. The results highlight three dimensions: knowledge and awareness of oneself, awareness of the mathematical task and knowledge and awareness of how to accompany others. The study highlights the contrast between what they experience as PMTs and what they anticipate from their students. Two implications for PMTs are identified from the proposed PMT training: an increased mathematical knowledge (awareness of the discipline) and the reorganization of knowledge in the modeling cycle to consider the cognition-affect interaction for suitable MWS.

7 Conclusions

The research described in this article has addressed the characterization of epistemic emotions by PMTs in the teaching internship. We have identified epistemic emotions in optimization problems, analyzing them with regard to the associated personal and suitable Mathematical Working Spaces (MWS). Thus, the identification of epistemic emotions has been established in distinct practices linked to mathematics teaching: in solving problems and anticipating what would happen with the students and reflecting on classroom implementation.

The results indicate that PMTs display a variety of paradigms and methods for solving the optimization problem, although the analytical method predominates. Furthermore, the influence of the institutionalized reference MWS on the personal MWS of the PMTs has been confirmed. They, in turn, project it on the suitable MWS anticipated for the students, considered the best for fostering the confidence emotion. The lack of knowledge of several PMTs when faced with solving the problem using a variety of methods triggers more negative epistemic emotions.

PMTs rely on distinct perspectives when analyzing emotions in themselves and when anticipating possible emotions in high school students. They positively value enjoyment and interest in high school students and consider it important to avoid frustration and boredom during classes. The study also shows that the perception and interpretation of emotions such as surprise and boredom vary in how PMTs personally experience them and how they expect for them to arise in students. These findings expand upon past works (D'Mello et al., 2012; Munzar et al., 2021) on the cognition-affect interaction. In the specific case of the epistemic emotion of surprise, in addition to being an initial reaction to information that is incongruent with prior knowledge, as typified in past studies, it is also identified with respect to cognitive processes and linked to gaps in knowledge regarding the application of techniques to the model and the analysis of the validity of the solution.

Both conclusions suggest the need to reinforce teacher training with educational processes that will help them face their own epistemic emotions and model appropriate strategies to regulate their own emotions and those of their students.

This study has certain limitations that should be considered. First, the specific typology of problems, in our case optimization problems, so further studies were required to address a wide variety of tasks and to contrast how the suitable MWS is important for the understanding of dominant epistemic emotions by PMTs.

In relation to the extension of the work at the methodological level, it can be implemented with several groups of high school students. The implementation has been carried out with a High School group by only one of the PMTs, so as to better understand the process of anticipating the possible emotions of the students. At a methodological level, this would permit a multiple analysis of cases so as to obtain information on the demands associated with classroom teaching. Also, another limitation to note is in relation to the PMT self-reported classroom observed emotion data. The external observation made of their performance could be analyzed in more depth and contrasted with their statements to identify biases related to selective or attribution memory, among others. Finally, a limitation exists with regard to the data coding process. This coding was carried out without using triangulation processes supported by intercoder reliability indicators, based on consensual decisions made by the participating researchers. The study may be considered exploratory and is an initial phase of a sequential mixed strategy. The next stage would be quantitative in nature, in order to generalize or expand upon the findings of the qualitative phase.

Specifically, if confirmed, these findings may have certain implications on educational interventions at the PMT university training level. They also may indicate relevant future research lines. Given the relationship between emotions and mathematical knowledge, these results may be of interest to generalize aspects related to the role of epistemic emotions and their impact on the generation of knowledge for teaching. Furthermore, when reflecting on a suitable MWS, further study may be needed on the origin of the strong link made by teachers in initial training programs between certain epistemic emotions such as boredom and confusion as limitations of mathematical learning. We believe that both issues may help to design practical education interventions that can develop awareness on accompanying others.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest.

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