

Modelling of a Combinatorial Double Auction using DEVS

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Abstract Combinatorial auctions have emerged as suitable mechanisms to efficiently coordinate supply and demand in electronic markets. Most of the combinatorial auction designs found in the literature assume that all market participants are perfectly rational and thus develop centralized simulation models according to that premise. However, participants in real-world auctions tend to exhibit bounded rationality, which requires a decentralized modelling approach. An absence of specialized simulation frameworks for decentralized implementation of auctions has been noted. To cover this gap, this work presents a combinatorial iterative double auction model using the Discrete Event System Specification (DEVS) formalism that can be employed to simulate a combinatorial auction in a decentralized fashion.

Keywords: combinatorial auctions; DEVS; modelling; optimisation

1 Introduction

Auctions are well-known market mechanisms traditionally used to determine resource allocation and prices based on bids from market participants (McAfee *et al.*, 1986). The emergence of electronic platforms as a new means of conducting business transactions has fostered the proliferation of advanced auction mechanisms to

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coordinate these markets (Baranwal *et al.*, 2018). In particular, auctions allowing combinatorial bids (i.e., combinatorial auctions) are gaining prominence as market mechanisms in e-platforms (Palacios-Huerta, Parkes and Steinberg, 2022).

Existing combinatorial auction designs for e-platforms often assume the participants’ perfectly rational behaviour, which presumes that agents are perfect optimizers who always make correct bidding decisions to maximize their utilities. However, empirical studies on auctions have found that it is hardly realistic to expect all individuals to consistently behave in a perfectly rational manner (Khalid *et al.*, 2022).

Combinatorial auction proposals designed within the framework of perfect rationality are often modelled and implemented through the utilisation of mixed-integer and linear programming solvers. In these models, agent behaviour is predetermined and predictable, and the auction algorithm is implemented in an iterative and centralized manner. Consequently, the resultant simulation outcomes fail to authentically represent real-world auction environments, instead representing the resolution of an optimisation problem in an iterative way. Conversely, more realistic auction proposals with agents exhibiting bounded rationality entail a higher level of complexity due to the inherently unpredictable nature of participant behaviour. As a result, there is a rising need for more flexible modelling and simulation methodologies that can capture the nuances of bounded rationality in these decentralized systems. Within this context, the Discrete Event System Specification (DEVS) formalism (Zeigler and Muzy, 2018) can offer a useful framework for modelling and simulating the dynamics inherent to such decentralized market structures.

This work presents a formal model of an iterative combinatorial auction using the DEVS formalism. As the objective is to establish the base model for simulating such auctions, the most general scenario is addressed. Thus, the model encapsulates a combinatorial double auction framework wherein both buyers and sellers present bids for combinations of items they seek to acquire or sell. Therefore, the primary aim of this investigation is to showcase the application of the DEVS formalism in crafting a decentralized model of a combinatorial double auction.

2 Background

Combinatorial auctions have emerged as favoured market mechanisms in online markets due to their ability to efficiently coordinate supply and demand, resulting in significant reductions in transaction costs (Abedrabbah, Karaki and Al-Fagih, 2023). However, from a modelling and simulation standpoint, a fundamental challenge arises as these mechanisms are often conceived within theoretical frameworks that inadequately capture the complexities of real-world contexts (Evans and Prokopenko, 2023), often resorting to centralized models assuming rational behaviour.

At the same time, combinatorial auction mechanisms often lack dedicated simulation frameworks for decentralized implementation, particularly evident in combinatorial double auctions. While recent proposals, such as those by Jiang *et al.* (2022)

and Abedrabbah, Karaki and Al-Fagih (2023), have conducted centralized simulations, decentralized implementations are rare. Notable exceptions include the work of Umer, Nazir and Ahmad (2022), who utilized the CloudAuction extension within the CloudSim implementation tool for a decentralized auction simulation (Calheiros *et al.*, 2009). However, CloudAuction is tailored for cloud computing services and lacks formalization for a combinatorial double auction system. In this context, the DEVS formalism offers a suitable framework for developing a formal decentralized model for such auctions, facilitating extensions to analyse more realistic scenarios.

DEVS, a hierarchical and modular formalism, is tailored for modelling discrete event systems, allowing for the modular coupling of models to construct intricate systems from simpler ones (Zeigler and Muzy, 2018). It employs a rigorous methodology for representing models by decomposing complex systems into two types of models: atomic and coupled models. Within this framework, atomic models define system behaviour, while coupled models articulate the overall structure. Both atomic and coupled models possess the capability to interlink inputs and outputs, thereby facilitating the construction of complex models from simpler components.

DEVS's versatility lies in its ability to represent systems with finite states, where state transitions after events depend on the time spent in the previous state, enabling comprehensive event-driven system representation. DEVS allows building complex models by combining smaller, reusable submodels. This feature will enable us to simulate different agent behaviours by modifying only the buyer/seller submodels, while keeping the rest of the auction model unchanged.

3 DEVS Model for a Combinatorial Double Auction

The auction mechanism considered in this work will gather the two sides of the market: buyers requesting the acquisition of some items and sellers offering the supply of those items. Therefore, auction participants will be several distributed buyers and sellers trying to maximize their benefits.

The functioning of the auction mechanism is as follows: first, each buyer will request an item and will place a purchase offer (i.e., a bid) showing the amount of money that they are willing to pay to obtain that item; second, each seller will place a sell offer (i.e., an ask) for some combination of the buyers' items showing the amount of money that they are willing to charge for the supply of the whole bundle. After collecting bids from buyers and asks from sellers, the auctioneer will determine the temporary allocation of items to sellers, thus concluding the first round of the auction. In the subsequent round, agents (either buyers or sellers) whose bids or asks were not successful will be allowed to update their prices to increase their chances of winning in the following round. This process will be repeated in successive rounds until the stopping criterion is met, resulting in the final allocation of items to sellers. Given these features, the auction mechanism is classified as a combinatorial iterative double auction according to auction theory (Abrache, Crainic

and Gendreau, 2004). Next, we will detail the structure and components of the DEVS model for this auction.

The combinatorial iterative double auction model presented in this section is a coupled model consisting of three basic components: *Buyer* (with multiple instances), *Seller* (with multiple instances), and *Auctioneer* (one instance), as depicted in Figure 1. *Buyer* and *Seller* are analogous coupled models, each representing an individual buyer and seller in the auction, respectively. Both models can be further divided into two atomic components: *Buyer* is composed of *Filter* and *Bid*; *Seller* is composed of *Filter* and *Ask*. *Auctioneer* is an atomic model that assumes the coordinating role of the auctioneer in an auction. During the instantiation phase, the number of instances of *Buyers* and *Sellers* generated will be adjusted according to the number of buyers and sellers participating in the auction.

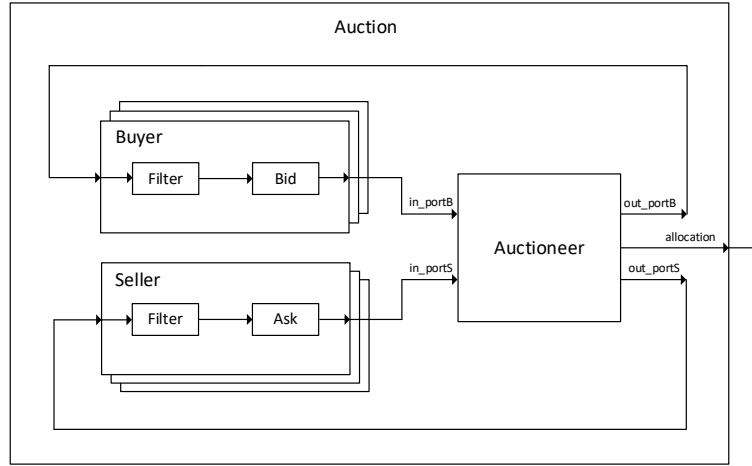


Fig. 1 DEVS model of the auction

The *Buyer* model simulates the behaviour of a buyer in an auction. Each buyer has a maximum amount that they are willing to pay (i.e., the reservation price). Initially, a buyer will bid an amount lower than their reservation price. If they do not win the current auction round, they will increase their bid. This process of updating the bid price continues until the reservation price is reached.

The aforementioned behaviour is modelled in the *Buyer* coupled model. In each round, the *Buyer* sends a message indicating the offered purchase price for the item. Following the determination of the current round's allocation, the *Buyer* receives a message indicating whether their bid was successful.

Buyer is subdivided into two atomic models: *Filter* and *Bid*. Each buyer has a unique identifier represented as a parameter of both the *Filter* and the *Bid* atomic models. Every message received by the *Buyer* includes an identifier indicating the intended recipient. *Filter* is thus used to identify among all the messages received by the model, the one/s corresponding to its associated *Buyer*. *Bid* determines the

initial purchase price for the bid and updates this price based on the feedback message received at the end of each round.

The *Seller* model is very similar to the *Buyer* model. In this case, every seller will have a minimum amount of money that they are willing to charge, known as the production cost. Initially, a seller will ask for a quantity higher than their production cost, and, if they do not win the current auction round, they will decrease their ask. This process of updating the ask price continues until the production cost is reached. To implement this behaviour, *Seller* is subdivided into *Filter* and *Ask* atomic models. The *Filter* component functions similarly to the one in the *Buyer* model. The *Ask* model is analogous to the *Bid* model but follows the seller's logic for price updates, decreasing the price instead of increasing it.

The *Auctioneer* model serves as the coordinator of the auction. At the outset, it establishes the duration of the requesting period. During this period, the *Auctioneer* receives bids from *Buyer* instances via the `in_portB` port and asks from *Seller* instances via the `in_portS` port. The *Auctioneer* extracts the bidding and asking information from these messages to determine the allocation for the current round, referred to as the temporary allocation. Subsequently, the *Auctioneer* sends individual messages through the `out_portB` and `out_portS` ports to inform each participant whether their corresponding bids or asks are winning in the current round. These ports are connected to the *Buyer* and *Seller* models, respectively. Once the messages have been dispatched, the *Auctioneer* restarts the requesting period and the next round begins. This process is repeated over multiple rounds until either all agents cease updating their prices or the maximum number of rounds specified for the auction is reached. When this occurs, the last temporary allocation is announced as the final allocation and the *Auctioneer* sends a message through the allocation port to indicate the conclusion of the auction.

4 Conclusions and further research

In this paper, a DEVS model for a combinatorial iterative double auction has been formulated. Given the scarcity of specialized simulation frameworks allowing a decentralized implementation of combinatorial and double auction mechanisms, the formal definition of the auction presented in this work aims to fill this gap. The model has been verified through its implementation in the Cadmium simulation tool (Belloli *et al.*, 2019) and the run of several test cases.

The model presented in this work provides a generic formulation that can be easily adapted to simulate the execution of combinatorial auctions in different domains. E-procurement platforms for the supply of additive manufacturing products, cloud computing resources or truckload transportation services are examples of markets employing combinatorial auctions for assignment and price determination.

Models of this nature can also be developed using Agent-Based Modelling (ABM) languages. However, in such cases, the model definition and

implementation are typically intertwined and dependent on the language, leading to a lack of separation of concerns, unlike in the DEVS formalism. Furthermore, most ABM languages follow a time-step approach, whereas the DEVS formalism is more suitable in this context since agent decisions occur within well-defined time windows rather than at predefined time steps.

Further research could develop new *Buyer* and *Seller* models with different behaviours regarding bidding and price updating decisions. Owing to the modularity property of DEVS, these new buyer and seller models can be defined without modifying the Auctioneer model, thereby allowing an incremental extension of the existing model. Additionally, some randomisation could be introduced into the decision-making process of agents. Simulating such auction scenarios can provide better insights into the expected outcomes in a real-world environment where agents may employ unpredictable strategies.

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5 References

- Abedrabbah, K., Karaki, A. and Al-Fagih, L. (2023) ‘A Combinatorial Double Auction for Community Shared Distributed Energy Resources’, *IEEE Access*, 11, pp. 28355–28369. <https://doi.org/10.1109/ACCESS.2023.3260022>.
- Abrache, J., Crainic, T.G. and Gendreau, M. (2004) ‘Design issues for combinatorial auctions’, *4or*, 2(1), pp. 1–33. <https://doi.org/10.1007/s10288-004-0033-y>.
- Baranwal, G. *et al.* (2018) *Auction based resource provisioning in cloud computing*, *Springer-Briefs in Computer Science*.
- Belloli, L. *et al.* (2019) ‘Building Devs Models with the Cadmium Tool’, *Proceedings - Winter Simulation Conference*, 2019, pp. 45–59. <https://doi.org/10.1109/WSC40007.2019.9004917>.
- Calheiros, R.N. *et al.* (2009) ‘CloudSim: A Novel Framework for Modeling and Simulation of Cloud Computing Infrastructures and Services’. <https://arxiv.org/abs/0903.2525v1>.
- Evans, B.P. and Prokopenko, M. (2023) ‘Bounded rationality for relaxing best response and mutual consistency: the quantal hierarchy model of decision making’, *Theory and Decision*, pp. 1–41. <https://doi.org/10.1007/S11238-023-09941-Z/FIGURES/15>.
- Jiang, X. *et al.* (2022) ‘Combinatorial double auction for resource allocation with differential privacy in edge computing’, *Computer Communications*, 185, pp. 13–22. <https://doi.org/10.1016/J.COMCOM.2021.11.025>.
- Khalid, A. *et al.* (2022) ‘On the Reality of Signaling in Auctions’, *Information 2022*, Vol. 13, Page 549, 13(11), p. 549. <https://doi.org/10.3390/INFO13110549>.
- McAfee, R. *et al.* (1986) ‘Auctions and Bidding’, *Journal of Economic Literature*.
- Palacios-Huerta, I., Parkes, D.C. and Steinberg, R. (2022) ‘Combinatorial Auctions in Practice’, *SSRN Electronic Journal*. <https://doi.org/10.2139/SSRN.3844338>.
- Umer, A., Nazir, B. and Ahmad, Z. (2022) ‘Adaptive market-oriented combinatorial double auction resource allocation model in cloud computing’, *Journal of Supercomputing*, 78(1), pp. 1244–1286. <https://doi.org/10.1007/S11227-021-03918-X/TABLES/8>.
- Zeigler, B. and Muzy, A. (2018) *Theory of Modeling and Simulation Discrete Event and Iterative System Computational Foundations*. Elsevier.