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Limit analysis of 3D building structures

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ABSTRACT

In this work, a limit analysis of typical building frames is carried out, and the collapse load and collapse mechanism of 3D frames composed of slender steel structural elements under concentrated and uniformly distributed loads are calculated. To this end, the direct kinematic method and nonlinear optimization techniques are proposed to search for the collapse mechanism and determine the collapse load. This work has the advantage that the structure's equilibrium equations are obtained through a completely systematic formulation of the problem of interest. In many cases, the case of uniformly distributed loads is simplified by equivalent point loads. However, this work verifies that the results differ in the cases of point loads and uniformly distributed loads, both qualitatively and quantitatively.

1. Introduction

Limit analysis of building structures focuses on determining the load level at which a building structure collapses. This type of analysis uses the concept of collapse mechanism, which describes how a structure fails under certain loading conditions. Plastic hinges are used to model the zones where plastic deformations occur, allowing a more accurate representation of the structure's behavior when the load limit is reached. The collapse load factor is an essential parameter, as it indicates the maximum increase in load that can be supported before the structure collapses. This approach is essential for the evaluation of the safety and efficiency of building structures under extreme loads.

Plastic calculation is based on the theory of plasticity, which studies the behavior of materials beyond the elastic limit. In the context of bar structures, it is considered that the cross sections of the bars can develop plastic hinges, which are plastic hinges are points where plastic deformation is concentrated. Once a plastic hinge is formed, the cross section of the bar can no longer support the bending moment. The structure collapses when enough plastic hinges are formed to turn it into a mechanism. This occurs when the structure can no longer support any more load and deforms indefinitely. Steel frames show a high nonlinear behavior due to the plasticity of the material and the slenderness of the members. In general, the plastic-hinge approach is adopted to capture the inelastic of material [1].

It allows for a more efficient and economical design by taking advantage of the plastic deformation capacity of materials, optimizing the design. It provides a more realistic estimate of the ultimate load capacity of the structure, which improves structural safety.

Kazinczy (1914) was the first to investigate the reserve of plastic strength in a statically insulated beam structures, introducing the concept of plastic hinge and the collapse mechanism. The terminology plastic hinge is used to indicate a section in which all points are in plastic regime. And collapse mechanism was initially used to describe the ultimate state of a frame.

The great impulse acquired by limit analysis was possible thanks to the rigorous establishment of the basic theorems, which was carried out by Gvozdev, in 1938. In general, there are two fundamental theorems: the static and the kinematic. This gives rise to two corresponding approaches: the static approach and the kinematic approach. The latter are called direct methods and have difficulties in solving large-scale problems because direct methods are one-step methods based on the method of combining mechanisms [2–4].

Orbison (1982) presents an efficient procedure for modeling the inelastic behavior in three-dimensional finite elements of beams and columns. The formation of plastic hinges and the interaction of element forces at a hinge and the elastic unloading (yielding surface equation) of light- and medium-weight American wide-flange steel sections are taken into account [5].

In 2020, Casciaro and Garcea perform the analysis of perfect elastic–plastic structures and propose a fast incremental-iterative solution method and describe an example of its implementation for the analysis of planar structures [6].

A few years later, in 2008, the works of Hoang-Van et al. stand out, this author presents an efficient algorithm for both limit and shakedown analysis of 3-D steel frames by the kinematic method using linear programming technique. Some numerical examples are presented to

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demonstrate the robustness, efficiency of the proposed technique and computer program (CEPAO) [7,8].

The kinematic direct method has important drawbacks from the point of view of its practical application: first, it is not systematic or general; and secondly, it requires possible collapse mechanisms to be tested, which even with few plastic hinges implies many possible collapse mechanisms that will have to be tested and verified. On the other hand, the step-by-step methods based on the matrix formulation are systematic and efficient for concentrated load cases at the nodes of the structure, and they are very inefficient and imprecise for analyzing structures with uniform distributed loads [9,10].

In 2016, a new beam–column element for nonlinear analysis of planar steel frames under static loads is presented by Doan-Ngoc et al. [11] The refined plastic-hinge method is used to model the material nonlinearity to avoid the further division of the beam–columns in modeling the structure. A Matlab computer program is developed based on the combined arc-length and minimum residual displacement methods.

Shin suggests the improved model that can employ a new finding on the behavior of plastic hinges at buckled steel braces [12]. It was confirmed from the validation that the improved model with a nonlinear assumption can properly capture the changes of hinge rotations during the excursion between the maximum tensile and compressive loads and can be the best candidate for addressing the shortcoming of the existing physical theory model.

Zhou et al. propose a plastic hinge method applicable to different materials that considers strain hardening and local buckling [13]. A beam–column element was proposed, incorporating the second-order effect between axial load and bending moment. The continuum strength method was introduced into the plastic hinge model to consider the strain hardening effect of cross sections and local buckling, controlling the strength and deformability of the sections.

To address the complex construction in beam–column joints of prefabricated frame structures, Wu et at. study introduces innovative artificial plastic hinge joints (HJs), exhibited excellent seismic performance [14]. The validated finite element model reliably simulates the seismic behavior of HJs, accurately capturing their failure modes, load-bearing capacity, and energy dissipation characteristics.

Step-by-step methods, or elastic–plastic incremental methods, are based on the standard methods of elastic analysis. The loading process is divided into various steps. The step-by-step methods benefit the long experiences of the linear elastic analysis by the finite element method. For the case of arbitrary loads, the step-by-step methods are cumbersome and embed many difficulties, it is a great challenge [15].

It is also worth mentioning that current sampling-based techniques are related to machine learning-based assessment methods. Advances in artificial intelligence (AI) in the field of structural engineering offer new solutions to improve design safety, efficiency, and cost-effectiveness. Wang et al. present a novel machine learning-assisted structural reliability analysis for frame structures with functional grading under static loading [16]. Uncertain system parameters, as well as the degree of functionally graded material gradation (FGM), can be incorporated into a unified 3D structural reliability analysis framework. Gondaliya et al. present a machine learning-based approach to estimate the seismic vulnerability of reinforced concrete building frames [17].

Finally, the most important contributions of this work can be summarized in the following points:

- The calculation of the equilibrium equations has been systematized, which facilitates the application of the kinematic direct method of plastic calculation of structures, and especially in the case of space structures.
- The interaction between the stresses in the sections is taken into account by implementing the Orbison yielding function.
- The collapse load and the collapse mechanism are obtained in a single step using a non-linear optimization algorithm.
- Point or concentrated loads and uniform distributed loads are considered.
- When considering uniform distributed loads, the methodology contemplates the possibility of plastic hinges forming within the element (internal) and not only in the extreme sections of the elements.

This paper has been organized as follows: after this brief introduction, the methodology is then applied to various types of space frames. Finally, the main conclusions and contributions of the work are summarized.

2. Methodology

In this section, the calculation hypotheses are established, the derivation of the equilibrium equations is explained in a systematic way and the resolution of the plastic calculation problem is proposed using the kinematic direct method [18,19].

2.1. Hypotheses

- Beams and columns are assumed to be free of residual stresses and/or initial deformations.
- Plastic collapse implies unlimited displacement at constant load, and the level of load that causes it is called the collapse load.
- The value of the maximum bending moment that the section can transmit is called the plastic moment (M_p) and the value of the maximum axial force is called the plastic axial force (N_p).
- When a plastic hinge is formed, the rotation of the section where it occurs can increase indefinitely.
- The plastic moment and plastic axial force depends on the material and the section.
- The formation of each plastic hinge is assumed to take place suddenly and concentrated in the section in which the Orbison yielding function is fulfilled [5].
- The hypothesis of small displacements and rotations of the sections of the structure at the moment of collapse is assumed; therefore, the accumulated rotations between beams or columns in the plastic hinges must also be small.

2.2. Equilibrium equations

The equilibrium of each beam/column of the structure is based on the stresses in extreme sections (six per node) at the local reference of each bar (see Fig. 1) and is formulated in a vectorial form [7,20].

$$f_k = \begin{pmatrix} -N_x \\ V_{yi} \\ V_{zi} \\ -M_x \\ -M_{yi} \\ -M_{zi} \\ \hline N_x \\ V_{yj} \\ V_{zj} \\ M_x \\ M_{yj} \\ M_{zj} \end{pmatrix} = \begin{pmatrix} -N_x \\ \frac{1}{L_k}\left(M_{zj} - M_{zi} + \lambda q_y \frac{L_k^2}{2}\right) \\ \frac{1}{L_k}\left(M_{yj} - M_{yi} + \lambda q_z \frac{L_k^2}{2}\right) \\ -M_x \\ M_{yi} \\ -M_{zi} \\ \hline N_x \\ -\frac{1}{L_k}\left(M_{zj} - M_{zi} - \lambda q_y \frac{L_k^2}{2}\right) \\ -\frac{1}{L_k}\left(M_{yj} - M_{yi} - \lambda q_z \frac{L_k^2}{2}\right) \\ M_x \\ -M_{yj} \\ M_{zj} \end{pmatrix} \tag{1}$$

where (N_x) is the axial force, (M_x) is the twisting moment, ($V_{yi}, V_{zi}, M_{yi}, M_{zi}$) are the shear forces and the bending moments at node i, ($V_{yj}, V_{zj}, M_{yj}, M_{zj}$) are the shear forces and the bending moments at node j, (q_y, q_z) are transversal uniform distributed loads in directions y and z, respectively; (L_k) es the length of the beam/column element k, defined from node i to node j. All magnitudes are expressed as functions of the axial force (N_x), twisting moment (M_x) and the values of the

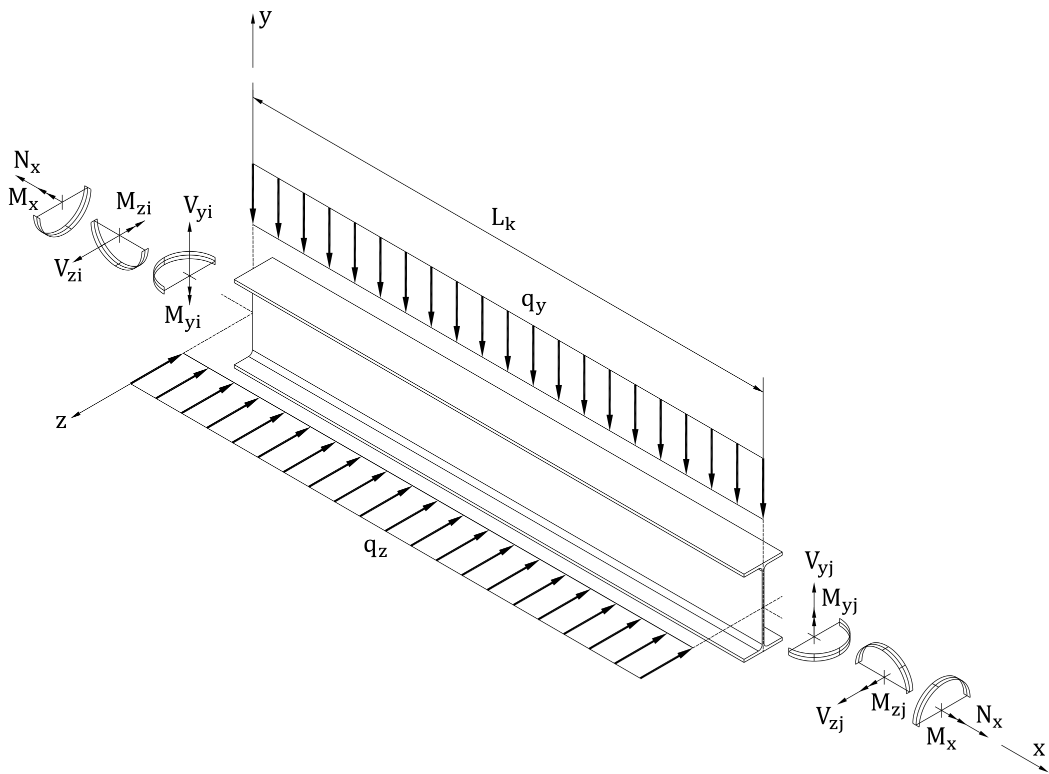


Fig. 1. Methodology. Space beam/column element.

bending moments in both end sections ($M_{yi}, M_{zi}, M_{yj}, M_{zj}$) and the applied loads (q_y, q_z) and the load factor (λ) (monotonically increasing).

The above vector is expressed in the coordinates (x, y, z) of the beam/column element, the local coordinate system of the element, but the bars can have different orientations in their arrangement in the structure, which requires considering the equilibrium of each node in a common global coordinate system (X, Y, Z), through the corresponding coordinate transformation (see Annex A):

$$\boldsymbol{F}_k = \boldsymbol{T}^T \cdot \boldsymbol{f}_k \tag{2}$$

where ($\boldsymbol{F}_k$) are the forces (and moments) at the ends of the bar k, expressed in a common system for all the members of the structure, and ($\boldsymbol{T}^T$) indicates the operation of transposing rows and columns in the matrix ($\boldsymbol{T}$) of coordinate transformation.

The internal force vector ($\boldsymbol{F}_{int}$) must be balanced the external loads ($\boldsymbol{F}_{ext}$) applied at the nodes of the structure:

$$\boldsymbol{F}_{int} = \boldsymbol{F}_{ext} \tag{3}$$

2.3. Limit analysis

This section summarizes the plastic calculation method proposed in the work. Firstly, the sections of the structure that are candidates to form a possible plastic hinge are: in the case of point loads, the nodes (connections between bars), the fixed supports, the section of application of the loads and the changes of section [21,22]; and in the case of distributed loads, additional plastic hinges can be formed in intermediate sections of the beam/column element. It requires carrying out the corresponding checks in intermediate sections of the element based on the bending moments calculated in the nodes of the structure [23,24].

The proposed methodology must take into account that if a plastic hinge occurs in an intermediate section, then its location in the element requires defining a new parameter x_k. This parameter reports the specific element where it is formed and its position relative to the first node of said beam/column element.

This work employs the kinematic direct method and the Orbison concentrated plastic hinge model that considers the interaction of the effect of axial forces and bending moments through the plastic function (see Annex B).

In Oxz bending plane:

$$\begin{aligned} M_y &= M_{yi} + \frac{M_{yj} - M_{yi} + \lambda q_z \frac{L_k^2}{2}}{L_k} x_y - \frac{\lambda q_z x_y^2}{2} \\ x_y &= \frac{M_{yj} - M_{yi} + \lambda q_z \frac{L_k^2}{2}}{\lambda q_z L_k} \end{aligned} \tag{4}$$

where (M_y) is the maximum Oxz bending moment in the beam k and (x_y) is the section position where this maximum value occurs.

For the Oxz bending plane:

$$\begin{aligned} M_z &= M_{zi} + \frac{M_{zj} - M_{zi} + \lambda q_y \frac{L_k^2}{2}}{L_k} x_z - \frac{\lambda q_y x_z^2}{2} \\ x_z &= \frac{M_{zj} - M_{zi} + \lambda q_y \frac{L_k^2}{2}}{\lambda q_y L_k} \end{aligned} \tag{5}$$

where (M_z) is the maximum Oxy bending moment in the beam k and (x_z) is the section position where this maximum value occurs.

The internal section that is a candidate for the formation of the plastic hinge is determined from the previous values relative to the Orbison yield function:

$$\begin{aligned} &\phi_{max} = \max\left(\phi_y, \phi_z\right); \ x_k = \left\{x_y \ or \ x_z\right\} \\ &\phi_y = \phi(n, m_y, m_z)|_{x_y}; \ \phi_z = \phi(n, m_y, m_z)|_{x_z} \end{aligned} \tag{6}$$

where ϕ_{max} is the maximum ϕ yield function value in the beam k and x_k is the section where the maximum value occurs.

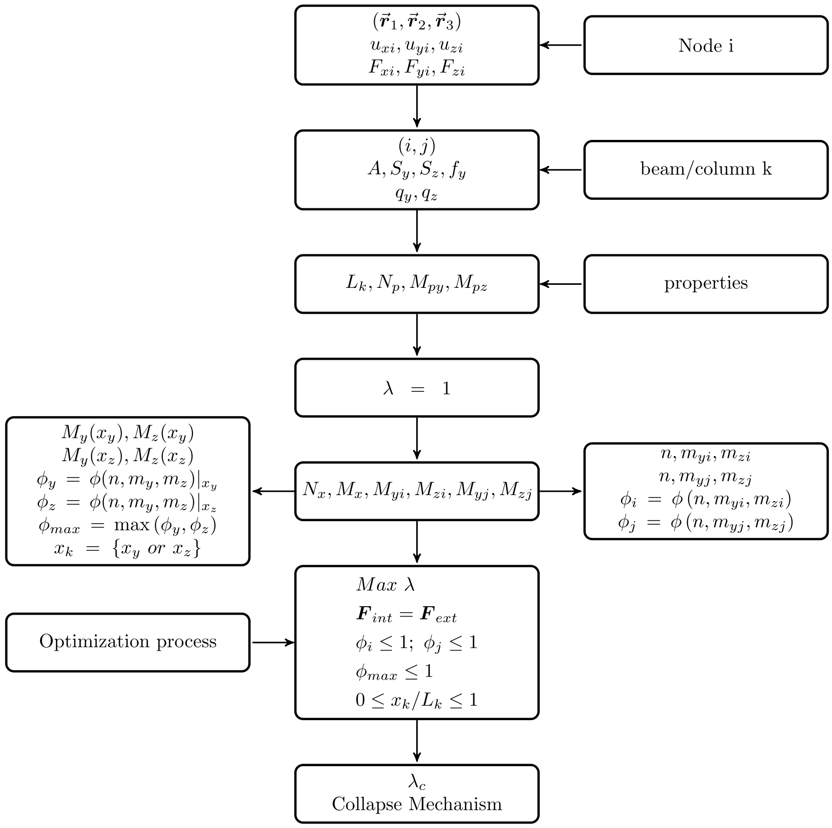


Fig. 2. Methodology flowchart.

2.4. Collapse mechanism

The search for the collapse mechanism is carried out by posing and solving an optimization problem. The objective function consists of maximizing the load factor subject to: equality constraints, the equilibrium equations of the problem; and the inequality constraints given by the yielding function in each study section.

$$\underline{\max \lambda} : \quad \begin{cases} \left.\begin{matrix} \boldsymbol{F}_{int} = \boldsymbol{F}_{ext} \\ \phi_i \leq 1; \ \phi_j \leq 1 \end{matrix}\right\} \ \forall i, \forall j \ / \ (i,j) \in k \text{ element} \\ \left.\begin{matrix} \phi_{max} \leq 1 \\ 0 \leq x_k/L_k \leq 1 \end{matrix}\right\} \ \forall k \text{ element} \end{cases} \tag{7}$$

where λ is the load factor; $\boldsymbol{F}_{int}$ are the internal forces; $\boldsymbol{F}_{ext}$ are the external loads; $\phi_i, \phi_j, \phi_{max}$ are the value of the plastic function at section i, section j and interior of beam/column k, respectively; and x_k/L_k is the relative length from the first node to the section where the internal plastic hinge of element k is formed.

Fig. 2 includes a flowchart with the steps to follow in the methodology outlined.

The design variables of the optimization problem posed here are: N_x the axial forces and M_x the torsional moment of each bar, M_{yi}, M_{zi} the bending moments at node i and M_{yi}, M_{zi} the bending moments at node j, that is, at the extreme sections of each k bar; and the locations x_k where plastic hinges can occur at interior sections of each k bar element.

The final value taken by the objective function is the value of the load factor that causes the collapse of the structure (λ_c). In the sections where the corresponding yielding surface is reached, a plastic hinge is formed. This load value implies the formation of enough plastic hinges so that the structure cannot withstand any further load. This is the ultimate state of the structure for that load state and is called the collapse mechanism.

Although the formulation may appear to be complicated, this is not the case, since what has been called the vector method are the equilibrium equations of the matrix formulation of structures in which the degrees of freedom, and therefore, neither the corresponding stiffness matrices, are involved. This method does not require the calculation of the stiffness matrices of each bar element of the discretization. Consider that in the limiting case, the collapse mechanism is an isostatic structure that cannot withstand further loads and can therefore be solved using only the equilibrium equations. Additional compatibility equations are not necessary, unless the desired deformation of the structure at that instant is required, which is not the objective of this work. Remember that the objective here is to obtain the collapse mechanism and the associated collapse load factor for spatial structures.

2.5. Safety factor

In this section, it is interesting to calculate the safety factor (n_s), that is, to determine the quotient between the load factor associated with the plastic design (λ_c) and the load factor associated with a linear elastic design (λ_1) of the structure.

$$n_s = \frac{\lambda_c}{\lambda_1} \tag{8}$$

For nominal loads ($\lambda = 1$), a linear elastic analysis is performed, and the most stressed section is determined. Based on its internal forces, the maximum value of the yielding function (ϕ_{max}) is calculated. According to AISC standards, if loads are increased, the structure cannot withstand further loads once the first plastic hinge (λ_1) is formed:

$$\lambda_1 = \sqrt{\frac{1}{\phi_{max}}} \tag{9}$$

The safety factor provides information on the theoretical resistance reserve available to the linear elastic design compared to the plastic design of the structure.

Table 1
Resultant force in X direction.

Case	a	b	c	d
F_X (kN)	480.6	231.16	231.16	231.16
F_X/F_0	2.079	1.0	1.0	1.0

3. Numerical results and discussion

In this section, the methodology is applied to the study of three application problems: a six-story space frame, a twenty-story space frame and an industrial building [25].

For the first two examples, all the beams and columns have the same length, they are: $L_b = 7.315$ m and $L_c = 3.658$ m where L_b is the length of the beams and L_c is the length of the columns, respectively.

The following applied loads are assumed: wind $q_w = 0.96\,\mathrm{kN/m^2}$ and usage $q_u = 4.8\,\mathrm{kN/m^2}$; where q_w is the wind load and q_u is variable usage load, both are uniformly distributed loads per unit area. And four cases are studied:

- Case a: hypothesis of load, wind and variable usage load, modeled with point or concentrated loads at the nodes of the structure; wind loads $F_w = 26.7$ kN/node and usage loads $F_u = 64.211$ kN/node/bar. This case is compared with the results of other researchers [6,7].
- Case b: the wind hypothesis of case a) is unrealistic, so here the concentrated wind load is calculated as $F_w = q_w L_b L_c/4$ and is applied at each node as said value ($F_w = 6.422$ kN) multiplied by the number of surfaces ($L_b\,x\,Lc$) that meet there.
- Case c: the same load situation but modeled with uniform distributed loads per bars, in principle a more realistic hypothesis.
- Case d: in this case the wind load predominates.

3.1. Six-story space frame

In this section, the Orbison's six-story space frame is solved [6–8], the columns base are fixed, requested by the wind load q_w on wall ($X = L_b$) and by the usage load q_u on each floor (see Fig. 3). The yield strength of all members is 250 MPa and Young'modulus is 206 GPa. Two analyses are performed: case a, assumes concentrated/point loads applied at the nodes of the structure; and case b, assumes all loads are uniform distributed load type.

The methodology described in Section 2 is implemented in Matlab, which systematically solves the plastic problem in a single step using the kinematic direct method. To do this, the calculation of the equilibrium equations of the structure is first systematized and then a non-linear optimization algorithm (sequential quadratic programming, sqp) maximizes the load proportionality factor (λ) that takes the structure to its limiting load situation, it is called the collapse mechanism, which corresponds to the applied nominal loads multiplied by a factor (λ_c) called the collapse load factor.

If the resultants of the forces applied to the structures in each direction are calculated in order to compare the results (see Table 1), the following is obtained:

$$F_0 = q_w s_x L_b L_c = 231.16\,\mathrm{kN} \tag{10}$$

where s_x is the total number of rectangles ($L_b\,x\,L_c$) in the wall where the wind acts (in this case $s_x = 9$).

Fig. 4 summarizes how to apply concentrated loads at the nodes for case b. Table 2 presents the summary of results for the six-story building, and Fig. 5 shows the corresponding collapse mechanisms, where the blue circles indicate plastic hinges at the nodes and the red circles are in-element plastic hinges. The results are compared using the CEPAO [26] and SAP2000 [27] programs. The table indicates the load factor (λ_1) associated with an elastic design according to the American standard and the corresponding safety factor (n_s), which provides information on the structure's resistance reserve. The table also shows the relative error between the results obtained compared to the available value (the values being compared are indicated in blue for each row).

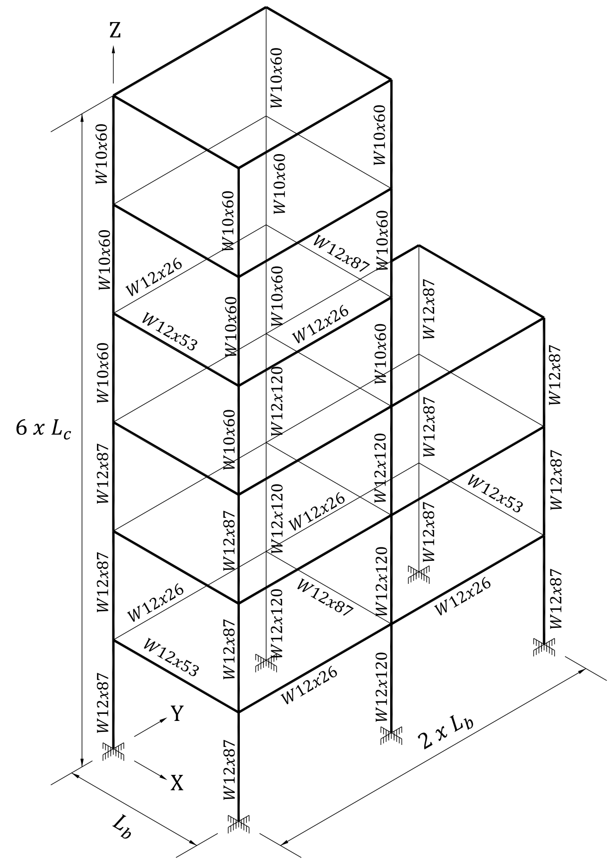


Fig. 3. Six-story space frame. 3D view.

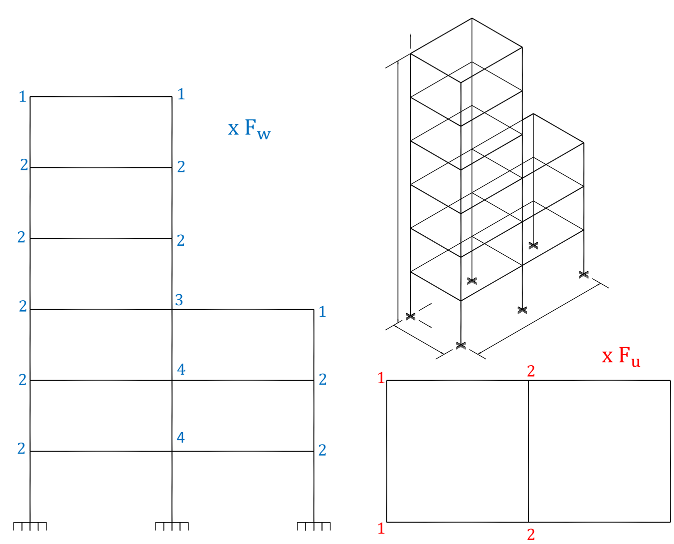
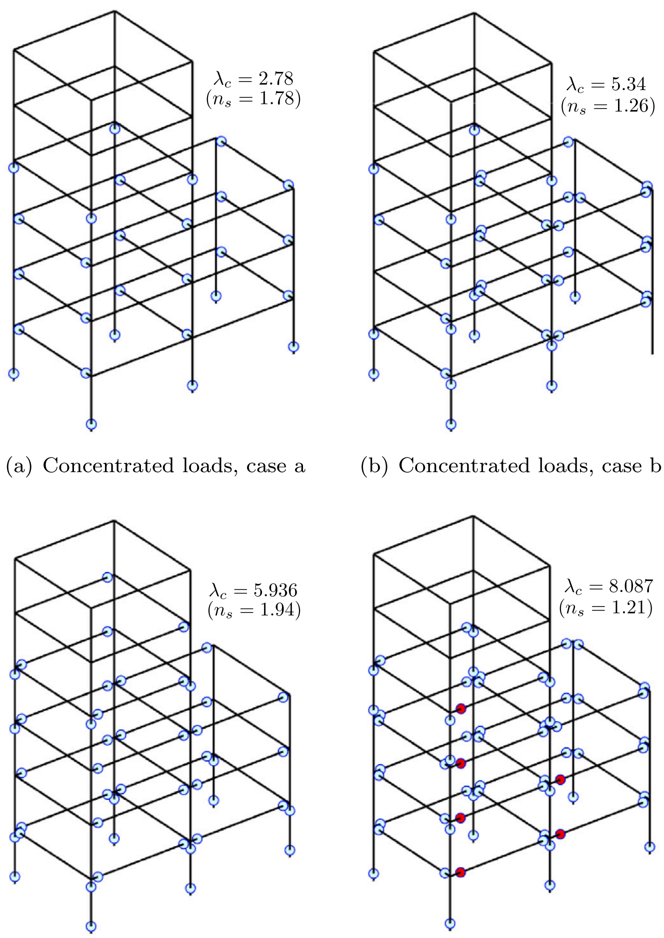


Fig. 4. Six-story space frame. Concentrated wind loads (case b).

For cases b and c, the collapse load factor is logically higher, since in Table 1 it was found that case a involves 107.9% more wind load than the problem of interest. Comparing Fig. 5.(a) with Figs. 5.(b) and (c), it can be observed that the collapse mechanism is different, since case a considers equal point wind forces at all nodes of the wall where the

Table 2
Six-story space frame. Collapse load factor.

Case	n_s	λ_1	λ_c			Difference
			This work	SAP2000 [27]	CEPAO [26]	
a	1.78	1.564	2.780	2.162	2.033	−26.87%
b	1.26	4.252	5.340	4.528	–	−15.21%
c	1.94	3.057	5.936	3.798	–	−36.02%
d	1.21	6.687	8.087	5.518	–	−31.77%



(a) Concentrated loads, case a
(b) Concentrated loads, case b
(c) Uniform distributed loads, case c
(d) Uniform distributed load, case d

Fig. 5. Six-story space frame. Collapse load factor (safety factor).

wind impacts; in case b they are different, more realistic, with higher values where there is a greater applied load; and in case c the load is applied in a uniformly distributed manner, both for the wind load and the use load.

Finally, Fig. 5.(d) shows the results when wind load predominates and the value of the usage overload is small. The red circles indicate the formation of in-element plastic hinges.

Logically, if the nominal loads to which the structure is subjected are modified, the collapse load factor is modified and the collapse mechanism could change. A new limit analysis of the structure would then have to be performed.

3.2. Twenty-story space frame

In this section, this working methodology is used to solve a large building, a twenty-story space frame (see Fig. 6). The yield strength of all members is 344.8 MPa and Young'modulus is 200 GPa. The same four cases of the previous problem are solved.

Table 3
Twenty-story space frame. Collapse load factor.

Case	n_s	λ_1	λ_c			Difference
			This work	SAP2000 [27]	CEPAO [26]	
a	2.61	0.4083	1.066	1.208	1.024	−3.94%
b	1.21	1.146	1.386	1.646	–	36.04%
c	1.36	1.088	1.485	1.454	–	−4.45%
d	1.62	0.9900	1.600	1.774	–	27.84%

Table 4
Industrial building. Collapse load factor.

Case	n_s	λ_1	λ_c		Difference
			This work	SAP2000 [27]	
a	1.65	4.278	7.029	10.30	46.54%
b	1.21	4.704	5.704	8.352	46.42%
c	1.72	3.090	5.321	4.404	−17.23%

Table 3 presents the plastic collapse load factor and the safety factor for the elastic design. The results of this work are in good agreement with those obtained using the CEPAO program (only the results for case a are available) and show discrepancies with those obtained using the commercial SAP2000 program.

It is also observed that for the results of this work, see column four of Table 3, there is a difference in considering the loads as concentrated in the nodes or uniformly distributed in the beams/columns.

Fig. 7 represents the collapse mechanism for each load case. It can be seen that if the load model of the structure changes, the final state of the structure due to plasticity also changes. In this example, the building is slender, and even changing the loads only results in the formation of plastic hinges at the nodes and not in-element hinges.

3.3. Industrial building

In this example, a gabled industrial building has been designed with the following data: $L_0 = 25$ m building span, $m_0 = 5$ m longitudinal modulus, $L_c = 7$ m height of the columns, $H = 9.5$ m height of the building (see Fig. 8). The cross-sections are W18 × 106 and W14 × 82 for the columns and beams of the intermediate frames; and W10 × 30 and W8 × 21 for the columns and beams of the two end frames. The yield strength of all members is 235 MPa and Young'modulus is 200 GPa. Regarding the applied loads, only the lateral wind action from left to right with a value of $q_w = 0.5\,\mathrm{kN/m^2}$ is considered.

For the analysis of the gabled industrial building, three calculation models are proposed:

- Case a: the structure is discretized with a bar-by-bar element and the load is modeled using equivalent point loads applied at the nodes.
- Case b: the structure is discretized with two bar-by-bar elements and the load is modeled again using equivalent point loads applied at the nodes.
- Case c: the structure is discretized as in case (a) and the load is modeled by uniform distributed loads applied to the bar elements.

Table 4 and Fig. 9 summarize the results for the cases in this example. The Table 4 presents the collapse load factor and safety factor, and Fig. 9 shows the collapse mechanism corresponding to each load application model. Fig. 9.(c) represents plastic hinges at the ends of the bars with blue circles, and internal plastic hinges within the bar element, with red circles.

After the application problems, it is found that one of the advantages of the methodology is that the uniform distributed loads case is solved using the same meshing of nodes and elements that for concentrated

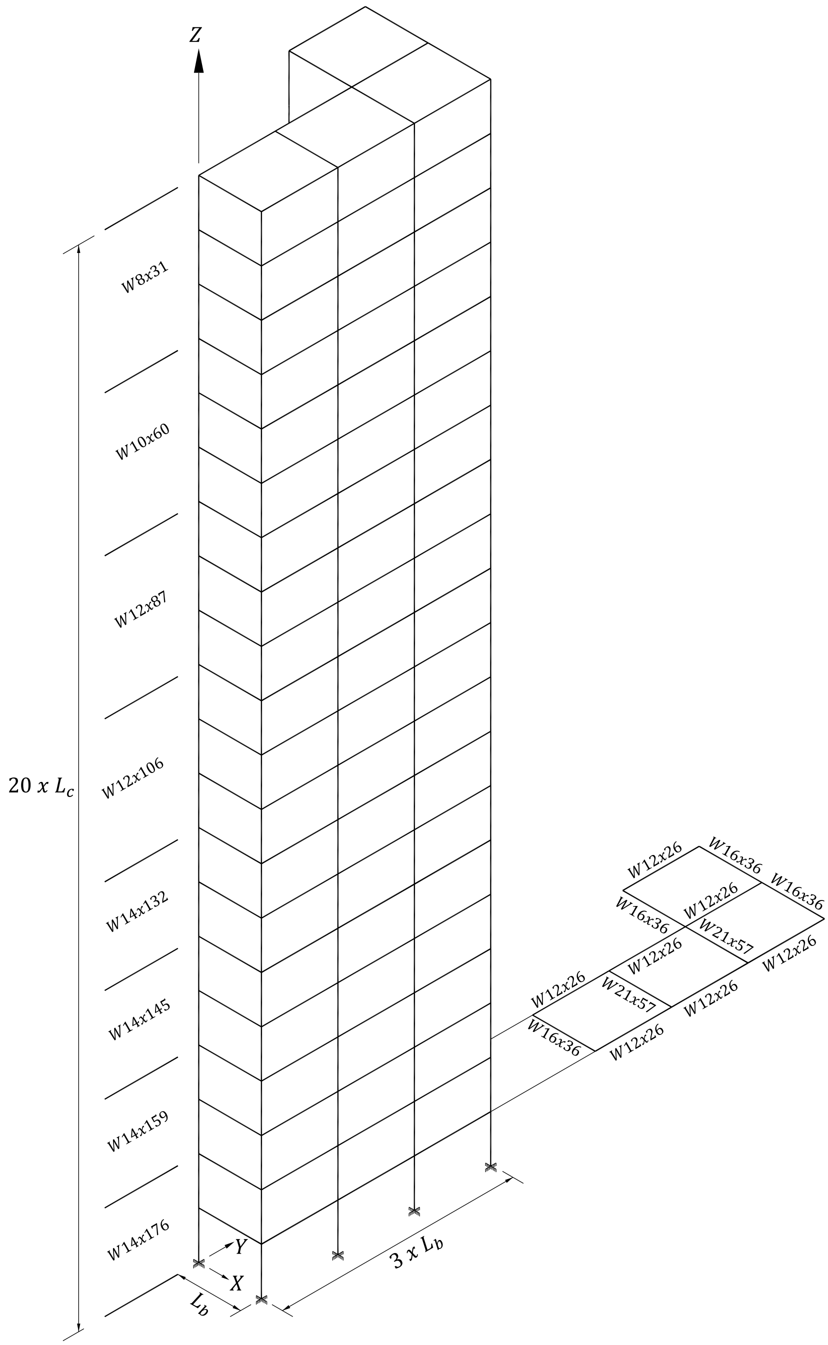


Fig. 6. Twenty-story space frame. 3D view.

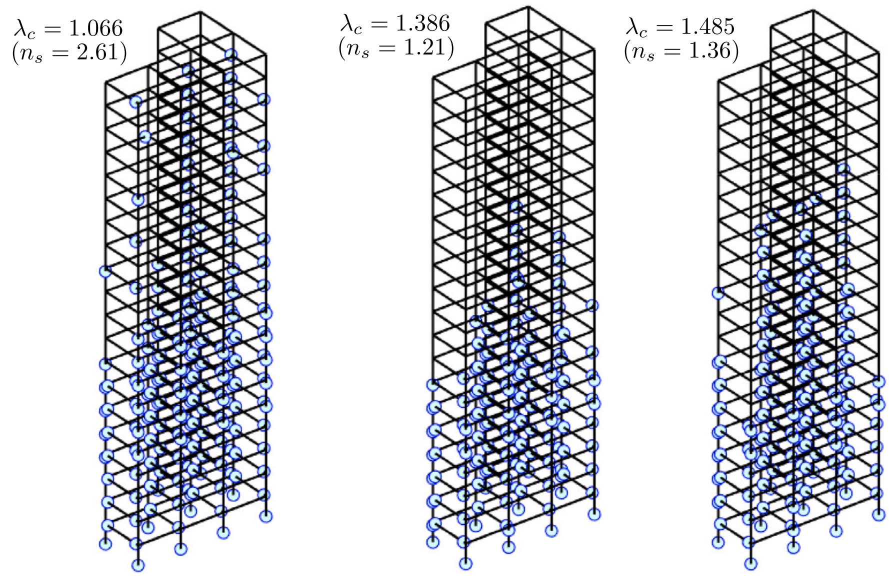
loads case. And it is possible to obtain the safety factor (n_s) of the linear-elastic design and quantify the resistance reserve available in the structure.

4. Conclusions

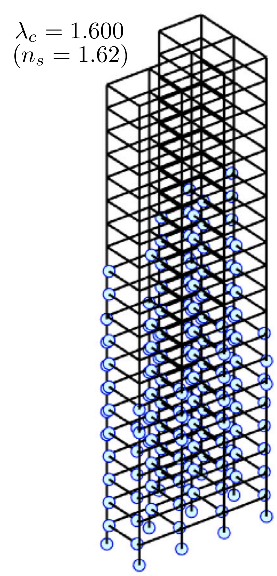
The classical formulation of plastic methods for spatial and building structures is not very systematic. The combination of mechanisms method, for example, relies on testing possible mechanisms until the final collapse mechanism is found, which may require numerous tests. It is also not very efficient in cases of distributed loads acting on beams/columns.

This paper attempts to facilitate the formulation of the equilibrium equations necessary for applying the direct kinematic method. The final state of the structure (collapse mechanism) is obtained using a nonlinear optimization method with an objective function of maximizing the load factor parameter of the structure. Equality constraints are posed, which are the equilibrium equations for the problem, and inequality equations are also posed, given by the yield function used. This methodology leads to the collapse mechanism corresponding to the structure with the given loads, geometry, and boundary conditions.

This article summarizes a spatial method for the analysis of buildings, which can be analyzed using 3D steel frames. The method allows the structure to be analyzed regardless of the load type, whether point, uniformly distributed, or both. The factor of safety for a linear-elastic



(a) Concentrated loads, case a (b) Concentrated loads, case b (c) Uniform distributed loads, case c



(d) Uniform distributed load, case d

Fig. 7. Twenty-story space frame. Collapse load factor (safety factor).

design of the structure under study can be obtained as an additional result. Another advantage is that the same dicretization of the structural members is used, regardless of the load type, whether concentrated or uniformly distributed.

The application examples show that the solution to the plastic design problem with uniformly distributed loads differs from the solution modeled with statically equivalent point loads; both the value of the collapse load factor and the resulting collapse mechanism are different.

Work is underway to include second-order analysis and stability checking as the load factor increases, which may require changing the limit analysis methodology from a direct kinematic method to a stepwise static method.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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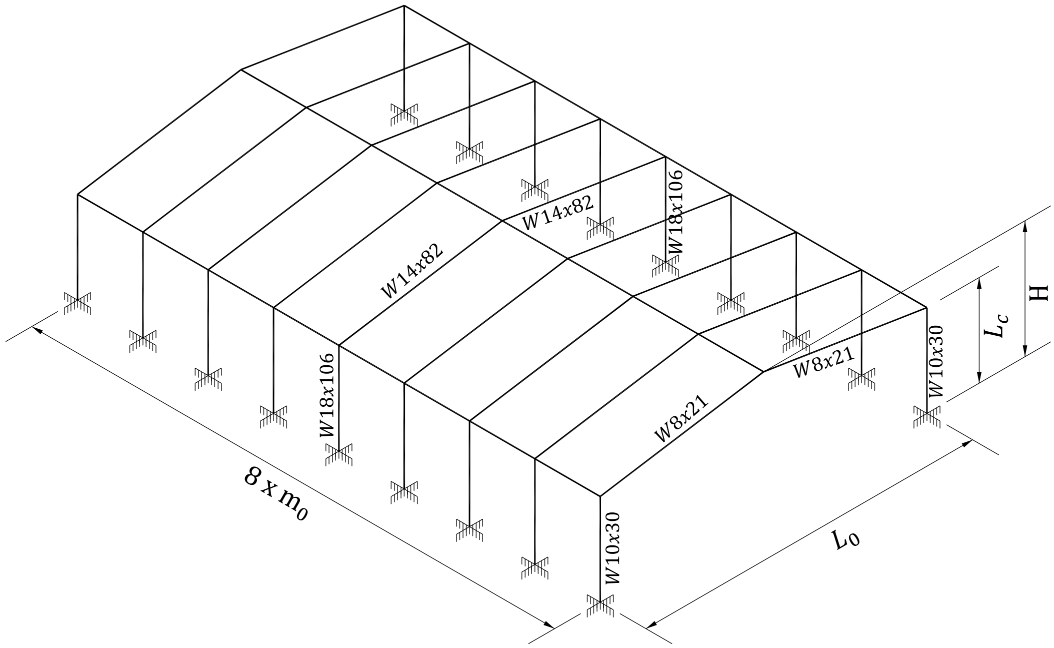
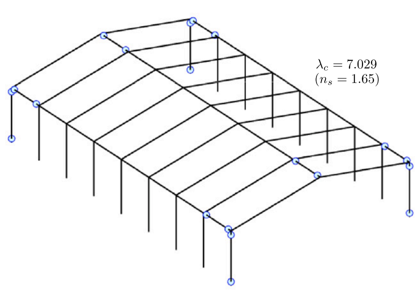
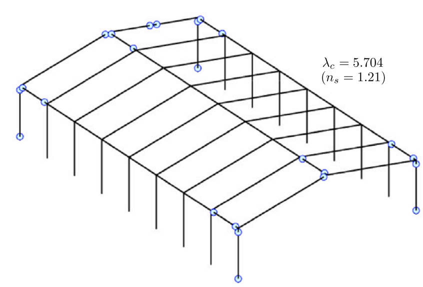


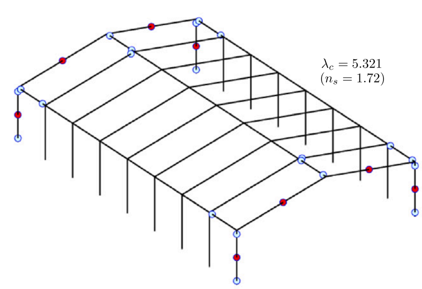
Fig. 8. Industrial building.



(a) Concentrated loads, case a



(b) Concentrated loads, case b



(c) Uniform distributed loads, case c

Fig. 9. Industrial building. Collapse load factor (safety factor).

Annex A. Coordinate transformation matrix

This section summarizes the transformation of coordinates between Cartesian systems, specifically from the global coordinate system (of the structure) to the local coordinate system (of each bar): $(X, Y, Z) \rightarrow (x, y, z)$ (see Fig. 10). For this purpose, it is necessary to define each bar element by three nodes (1,2,3) and calculate the following unit vectors:

$$\begin{aligned} \vec{\boldsymbol{u}}_x &= \frac{(\vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1)}{|\vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1|} \\ \vec{\boldsymbol{u}}_y &= \frac{(\vec{\boldsymbol{r}}_3 - \vec{\boldsymbol{r}}_1)}{|\vec{\boldsymbol{r}}_3 - \vec{\boldsymbol{r}}_1|} \\ \vec{\boldsymbol{u}}_z &= \vec{\boldsymbol{u}}_x \times \vec{\boldsymbol{u}}_y \end{aligned} \tag{11}$$

It allows to define the change of basis matrix $\boldsymbol{R}$ $[3 \times 3]$:

$$\boldsymbol{R} = \left[\vec{\boldsymbol{u}}_x; \, \vec{\boldsymbol{u}}_y; \, \vec{\boldsymbol{u}}_z\right]^T \tag{12}$$

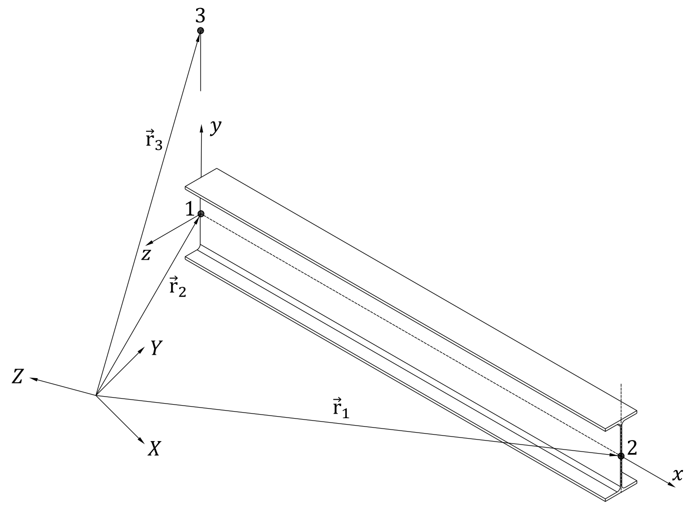


Fig. 10. Cartesian coordinate transformation.

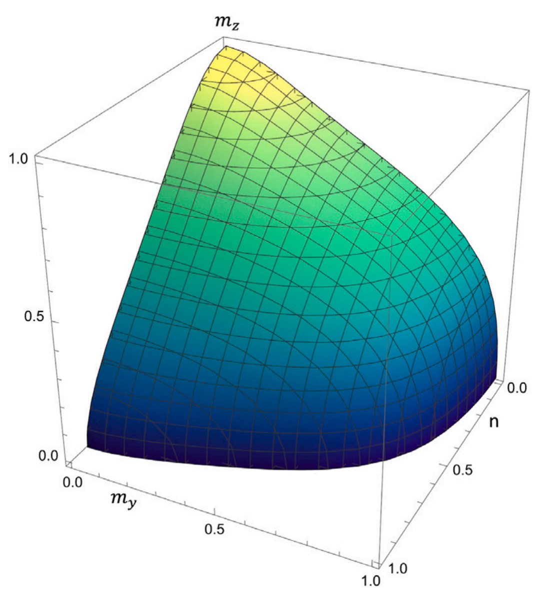


Fig. 11. Yield surface ($\phi = 1$).

And the coordinate transformation matrix $\boldsymbol{T}$ [12 × 12] is based on the matrix $\boldsymbol{R}$ [3 × 3] organized by blocks on the diagonal in the following way:

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{R} & & & \\ & \boldsymbol{R} & & \\ & & \boldsymbol{R} & \\ & & & \boldsymbol{R} \end{pmatrix} \tag{13}$$

Annex B. Yielding function

In this work, the Orbison plastic hinge model is used, an efficient procedure for modeling inelastic behavior in three-dimensional beam/column elements (see Fig. 11). Plastic hinge formation and the interaction of element forces at a hinge are taken into account. A single-equation, stress-resultant yield surface has been developed [5,10].

$$\phi(n, m_y, m_z) = 1.15n^2 + m_z^2 + m_y^4 + 3.67n^2m_z^2 + 3n^6m_y^2 + 4.65m_z^4m_y^2 \tag{14}$$

where $n = N_x/N_p$ is the ratio of the applied axial force to the plastic axial force ($N_p = Af_y$), $m_y = M_y/M_{py}$ is the ratio of the weak axis bending moment to the corresponding plastic moment ($M_{py} = S_y f_y$) and $m_z = M_z/M_{pz}$ is the ratio of the strong axis bending moment to the corresponding plastic moment ($M_{pz} = S_z f_y$), with A the cross-sectional area, S_y and S_z the corresponding plastic modules of the section and f_y the elastic limit of the material.

Data availability

No data was used for the research described in the article.

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